

Article

Model-Based Analysis of the Accuracy of Tracking Control and Energy Efficiency of a Six-Wheeled Skid-Steered Robot

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Abstract: This article concerns the modeling and motion control of a mobile robot with six independently driven and non-steered wheels. The main research issue is analyzing the influence of the structure of the control system and wheel track on the control accuracy and energy efficiency during robot motion on horizontal paved ground. For this purpose, the kinematic relationships for the robot are discussed and a simplified dynamics model for control applications is developed. The robot's dynamics model takes into account the most important phenomena of the wheel interaction with the paved ground, including slip. In addition, it is supplemented with a model of the robot's drive units. Two versions of the control system were adopted for analysis, i.e., with the wheels' controller only and additionally equipped with a pose controller. Simulation studies were carried out for the developed robot dynamics model and the analyzed versions of the control system in order to investigate the influence of the track width of the wheels and the structure of the control system on motion accuracy and energy efficiency. In order to quantitatively compare the results for the analyzed solutions, quality indices were introduced. The results of the simulation research indicate the influence of the track width of the wheels on the accuracy of motion when using the wheels' controller, as well as its impact on energy efficiency. Moreover, they show that it is possible to significantly improve the accuracy of motion by using an additional pose controller, which allows limiting the impact of the non-optimal geometric parameters of the robot and the slip of the wheels on trajectory tracking errors. However, the addition of the pose controller does not significantly affect the energy efficiency during the robot's motion, which may be even worse in this case.



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Keywords: wheeled mobile robot; wheel slip; dynamics model; drive unit model; trajectory tracking control; wheels' controller; pose controller; motion accuracy; energy efficiency; simulation studies

1. Introduction

Model-based simulation studies can be the basis for selecting the optimal parameters of a mobile robot, so they are an important tool in the process of designing new solutions. The robot model can be used both at the design stage and in the case of existing robots when designing and optimizing their motion control systems, as well as for energy-efficient path planning.

Knowledge of the real parameters of the robot may allow for increasing the accuracy of simulation research. These parameters can be obtained in the identification process.

The key geometric parameters determining the driving properties of a wheeled vehicle are those related to the number of wheels and the wheel arrangement, as well as their diameter and width. In turn, an optimally selected control system can allow for the accurate implementation of a desired vehicle movement.

One of the key questions that arise at the design stage of wheeled mobile robots is what kinematic structure of the robot should be adopted. This has a decisive impact on the driving properties of the robot and is closely related to the area of its application.

As indicated in Trojnacki and Dąbek, 2019 [1], modern wheeled mobile robots belong to one of the following groups: differentially driven, skid-steered, car-like or Ackermann steering, omnidirectional, and rover-type.

Skid-steered mobile robots deserve special attention due to their universal application, as they can be used both indoors and outdoors. For this reason, they are most often used in robots for special applications, e.g., for neutralizing improvised explosive devices. However, they have a fundamental drawback—wheel slip is their inherent feature when turning or rotating in place, which adversely affects the accuracy of the movement and energy efficiency. The question arises as to how these negative aspects can be reduced by appropriately selecting the wheel arrangement.

Therefore, the aim of this study is to investigate the influence of the wheel arrangement and control system structure of a six-wheeled robot with non-steered wheels on the tracking control accuracy and energy efficiency.

To investigate the influence of the wheel arrangement, it would be necessary to analyze a different ratio of the wheel track to the wheel axle distance. For this purpose, a constant value of the wheel axle distance is assumed and various values in terms of the wheel track are analyzed. In addition, the same wheel axle distance is adopted for all robot wheels. The different ratio of the wheel track to the wheel axle distance could also be studied analogously, assuming a constant wheel track and changing the wheel axle distance.

As far as the structure of the control system is concerned, one can primarily take into account the control of only the robot's wheels, based on the measurement of the angular velocities of wheel spins, which may allow for achieving a high accuracy of movement in the case of negligible slip of the wheels. Such a control system can also be supplemented with an additional pose controller, i.e., used to control the position and course of the robot based on their measurement or estimation. This may allow for minimizing tracking errors in the case of wheel slip and, consequently, the deviation of the actual robot trajectory from the desired trajectory.

In order to achieve the aim of the article, the following key research questions must be answered:

1. How does the wheel arrangement affect the accuracy of the robot's movement?
2. How does the wheel arrangement affect the energy efficiency?
3. What impact does the choice of control system structure have on the above issues, taking into account the wheels' controller and, additionally, the pose controller?

To answer the above research questions using a model-based approach, it is necessary to analyze the robot's kinematics and develop its dynamics model. It is also necessary to define the control system for both the analyzed structures and adopt a model of the robot's drive units, which may allow for the analysis of the energy efficiency of individual solutions.

Therefore, the research methodology adopted in this work assumes the following:

- Adopting the nominal parameters of the robot and the desired motion trajectory for simulation studies, as well as selecting the parameters of the control system;
- Introducing the quality indices for a quantitative assessment of the research results;
- Conducting simulation studies for the assumed range of the ratio of the wheel track to the wheel axle distance;
- Discussing the obtained results, taking into account the obtained motion path, and time histories of the selected physical quantities and quality indices;
- Formulating conclusions and answers to research questions;
- Indicating directions for further scientific work based on the obtained research results.

The paper concerns the optimization of the design of a hypothetical robot, so its subject is in the area of applied sciences.

The main contributions of the paper include the following:

- The development of a robot dynamics model that can be used to optimize the robot design and test various solutions of control systems;

- The application of a modified version of the Kiencke tire model, which is less common than other tire models, such as Pacejka [2];
- The development of a new variant of the pose controller;
- Simulation studies based on the developed robot model and the adopted control system structures, allowing us to answer the research questions regarding the influence of the robot's wheel track on the accuracy of movement and energy efficiency.

The content of the further sections of the paper is as follows:

- Section 2 includes an introduction to the state of the art in the field of the modeling of wheeled mobile robots, in particular with non-steered wheels and taking into account the slip of the wheels. This section also covers a brief introduction to the state of the art in the area of tracking control of wheeled mobile robots.
- Section 3 concerns modeling issues and a discussion of the kinematics of the analyzed six-wheeled skid-steered mobile robot, and the derivation of the robot dynamics model based on the Newton–Euler formalism, as well as supplementing the robot model with a model of drive units.
- Section 4 concerns the problem of the motion tracking control of the analyzed mobile robot and covers a general description of the desired robot motion trajectory, as well as defining the structures of the robot control system, including the wheels' controller and an additional pose controller.
- Section 5 focuses on simulation studies and includes the assumption of the nominal parameters of the robot, the type of desired motion trajectory, the selection of the settings of the wheels' and pose controllers, the adoption of quality indices, the presentation of the simulation study results, including the motion path of the selected robot point, and time histories of the selected physical quantities and quality indices, as well as a discussion of the obtained results.
- Section 6 is a summary of the work and includes answers to key research questions, as well as an indication of directions for potential further scientific work.

2. State of the Art Review

2.1. Modeling of Wheeled Mobile Robots

The modeling of wheeled mobile robots covers kinematics and dynamics issues. From the point of view of robot control, it is important to solve the inverse kinematics problem, which allows us to determine what the wheel motion parameters should be for the desired robot motion parameters. This problem can be unambiguously solved assuming no wheel slip during robot movement. In the case of skid-steered robots that typically move in wheel-slipping conditions, this solution is subject to error. The real movement parameters can then be estimated based on the robot dynamics model, which takes into account the interaction of the wheels with the ground and the phenomenon of wheel slip.

The basics of the modeling and testing of automotive vehicles, taking into account the wheel slip phenomenon, are presented in the book by Pacejka, 2005 [2]. In turn, the basics of soil mechanics and the modeling of off-road vehicles on various grounds, i.e., terramechanics, are formulated in the book by Bekker, 1960 [3]. Therefore, depending on whether the wheeled vehicle is traveling on paved or unpaved ground, a partially different approach should be used.

Various formalisms can be used to determine the dynamic equations of motion. The most frequently used are the Newton–Euler formalism and formalisms based on the Lagrange equations, an example of which is the work of Kozłowski and Pazderski, 2004 [4].

The article by Alexa et al., 2023 [5] is devoted to a review of mathematical models of wheeled and tracked vehicles, which discusses selected issues of kinematics and dynamics. In the field of wheeled vehicles, the car-like kinematic structure and a variant of this structure with independently steered four wheels, as well as the skid-steered structure are considered.

In turn, the work by Moshayedi et al., 2022 [6] contains a review of mathematical models of the service robots. The kinematics and dynamics of these robots are taken

into account, including the phenomenon of wheel slip. The review covers differentially driven, skid-steered, and car-like kinematic structures with various numbers of wheels. Additionally, models of tracked robots as well as legged and snake robots are discussed.

The article by Zhang et al., 2021 [7] concerns the issues of the modeling and control of a six-wheeled skid-steered robot. This work focuses on the issues of the driving force distribution and control strategy of such a robot, which aim to improve the vehicle maneuverability and stability. It also contains the results of experimental studies of the robot using the analyzed solutions of the control system.

When modeling the movement of wheeled mobile robots, it is usually assumed that they move on horizontal ground. However, one can also find fewer works that take into account the case of sloped terrain, an example of which is the article by Ordóñez et al., 2012 [8]. In such a case, the robot dynamics model is more complex, but more general and allows for taking into account additional phenomena resulting from the slope of the terrain.

In order to obtain the actual parameters of the dynamic system, an identification process is carried out based on the measurement of the excitations and responses of the object. This process can take place both offline, as in the paper by Lichota, 2023 [9], and online, as in the work by Hendzel and Trojnecki, 2014 [10], where the parameters of the object are determined in real time. The first approach allows for a more comprehensive analysis, while the second has the advantage that, in the event of a change in the parameters of the robot's model, e.g., related to a change in the type of ground or the transported mass, information about the current parameters can be used by the control system. The robot's parameters can also be estimated by the control system if an adaptive control strategy based on the known structure of the object model is applied.

In order to estimate the wheel slip, various measurement techniques are applied during research of wheeled mobile robots. In particular, the IMU-based technique is used, an example of which is the article by Yi et al., 2007 [11]. An alternative approach may include, for example, a laser scanner, which is reflected in the work by Wang et al., 2015 [12].

The dynamics model of a wheeled mobile robot can be applied at the path-planning stage, e.g., to optimize it in terms of energy efficiency, an example of which is the paper by Jaroszek and Trojnecki, 2014 [13]. The dynamics model of a robot is also used when designing motion control systems and this model can even be a part of the structure of the control system, which allows for achieving a higher accuracy of the tracking control.

2.2. Motion Control of Wheeled Mobile Robots

The motion control system of wheeled mobile robots often has a hierarchical structure, which may include a wheels' controller and a mobile platform controller. The mobile platform controller may be based on the measurement of linear and angular velocities or on the measurement of position and course, i.e., pose. The pose controller is also called the kinematic controller, e.g., in Hassani and Rekik, 2023 [14], because it uses only the kinematic parameters of the robot's movement.

It is more advantageous to use the robot's pose measurement and rely on the pose controller, because, in the case of a velocity controller, errors related to the estimation of the robot's pose based on the velocity measurement accumulate over time. Therefore, the robot may deviate from the desired path, which may not be noticed by the velocity controller of the mobile platform.

The wheels' controller may be based on PID control or have a more complex structure, for instance taking into account the dynamics model of the robot. In such a case, this type of controller can be called a dynamic controller—see Hassani and Rekik, 2023 [14]. Using the dynamics model and methods such as robust or adaptive control, one can achieve a high accuracy of the tracking control in the absence of wheel slip. For example, in the paper by Viadero-Monasterio et al., 2023 [15], robust control was proposed for the path tracking control of autonomous vehicles.

The movement of mobile robots is often related to the occurrence of variable working conditions, which may include variable ground properties, changes in the mass and position

of the mass center of the robots due to the transport of goods, etc. For example, in the article by Trojnecki, 2013 [16], the influence of the change of the type of ground on the robot's movement is analyzed. From the point of view of motion control, such changes can be interpreted as parametric disturbances. This is another argument for using control systems based on the robot dynamics model, such as adaptive control. This method enables the estimation of the current parameters of the robot model, reducing the model uncertainty.

Artificial intelligence techniques can also be applied to approximate the nonlinear dependencies occurring in the dynamics model, including those based on fuzzy logic systems, artificial neural networks, and genetic algorithms, an example of which is the paper by Hassani and Rekik, 2023 [14]. In the case of movement in slipping conditions, relying only on the wheels' control is insufficient, as shown in the article by Trojnecki, 2013 [16].

In addition, control systems using artificial intelligence techniques have an advantage over those using an analytical approach in that they can additionally provide resistance to non-parametric disturbances resulting from the inaccurate knowledge of the structure of the robot model. For example, in the case of artificial neural networks, a robot dynamics model can be obtained as a result of the learning process.

In the paper by Hendzel and Trojnecki, 2023 [17], in addition to the kinematic pose controller, the adaptive fuzzy control method is applied to control the movement of a four-wheeled skid-steered robot. This approach allowed for achieving a high accuracy of tracking control despite the occurring slip of wheels. In this case, the control system allows for the compensation of nonlinearities resulting from the dynamics of the robot.

In the paper by Gao et al., 2021 [18], a kinematic robot pose controller, a controller based on robot dynamics, and a nonlinear disturbance observer are used. The proposed approach, as a result of the inclusion of the wheels' skidding and slipping in the control system, allowed us to achieve a high accuracy of the robot's movement, which was confirmed at the experimental research stage.

In the work by Liang et al., 2023 [19], a model-based tracking control method for skid-steered mobile robot is proposed. In this case, a hierarchical structure of the control system is applied, which includes the adaptive robust control of the upper-level robot chassis, the middle-level control allocation approach, and the adaptive robust control of the bottom-level timing-belt servo system.

The article by Tu et al., 2009 [20] is devoted to the issues of motion control and the stabilization of a four-wheeled skid-steered mobile robot, taking into account various types of ground. The control system uses a high-order differential feedback controller (HODFC), which co-operates with the ratio controller for motion control. Moreover, vehicle states are estimated by a high-order differential (HOD) observer. The developed control system also uses the conversion of position control to velocity control.

In turn, in the article by Lin et al., 2007 [21], the adaptive critic anti-slip control method is used to control a wheeled robot taking into account the slip of wheels. The proposed control system includes an action network, critic network, and verification network, which are based on artificial neural networks.

Another example is the paper by Tilahun et al., 2023 [22], in which adaptive fuzzy control with a neuro-fuzzy system is used to control the movement of a three-wheeled differentially driven robot. In this case, no lateral slip motion and a pure rolling constraint are assumed, which is reasonable due to the kinematic structure of the robot and works well in the case of low velocities of the robot's movement.

Another important issue related to the control of wheeled robots is the issue of operation in the event of a failure of actuators or sensors. For example, in the work by Alshorman et al., 2020 [23], a fuzzy-based fault-tolerant control system for an omnidirectional mobile robot is proposed, which ensures the correct control of the robot's movement towards the target in the presence of obstacles and in a situation when the drive of one of the wheels fails. In turn, the article by Bigaj et al., 2014 [24] presents the results of the research on

the influence of the failure of environmental sensors on the autonomous movement of the robot.

3. Model of the Robot

3.1. Kinematics of the Robot

The kinematic structure of the robot and the distribution of the velocity vectors are shown in Figure 1. It is possible to distinguish the following main components of the robot: 0—mobile platform; and 1–6—wheels.

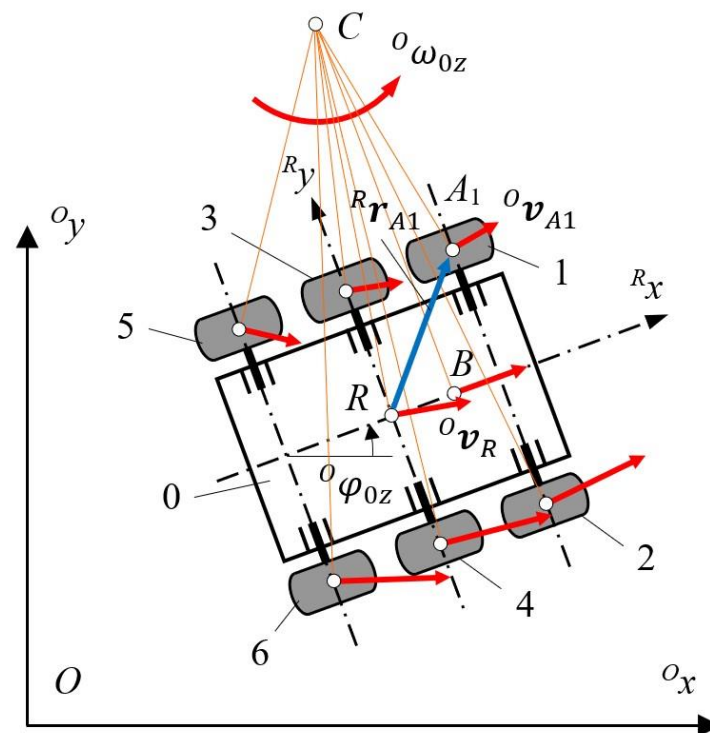


Figure 1. Kinematic structure of the robot and distribution of the velocity vectors during turning.

The following designations for the i th wheel have been introduced in the robot model: A_i —geometric center; r_i (m)—geometric (unloaded) radius; L_f (m), L_b (m)—distances of the front and back axles from the middle axle, respectively; W (m)—track width; and θ_i (rad)—spin angle.

It is assumed that the robot motion is performed in the Oxy plane of the fixed co-ordinate system $\{O\}$. In turn, a moving co-ordinate system $\{R\}$ is associated with the robot and its origin lies halfway between the centers of wheels 3 and 4.

The robot's actual pose (or posture) in the $\{O\}$ co-ordinate system, including its position and course, can be presented in the form of the vector of generalized co-ordinates:

$${}^O \mathbf{q} = [{}^O x_R, {}^O y_R, {}^O \varphi_{0z}]^T, \quad (1)$$

where ${}^O x_R$ (m), ${}^O y_R$ (m) are co-ordinates defining the position of point R of the robot, and ${}^O \varphi_{0z}$ (rad) is its course angle, that is, the rotation angle of the mobile platform about the ${}^O z$ axis.

In turn, the vectors of generalized velocities of the robot, respectively, in the $\{O\}$ and $\{R\}$ co-ordinate systems can be written as:

$${}^O \dot{\mathbf{q}} = [{}^O v_{Rx}, {}^O v_{Ry}, {}^O \omega_{0z}]^T, \quad {}^R \dot{\mathbf{q}} = [{}^R v_{Rx}, {}^R v_{Ry}, {}^R \omega_{0z}]^T, \quad (2)$$

where ${}^O v_{Rx} = {}^O \dot{x}_R$ (m/s), ${}^O v_{Ry} = {}^O \dot{y}_R$ (m/s) are co-ordinates of the linear velocity vector of point R of the robot and ${}^O \omega_{0z} = {}^O \dot{\varphi}_{0z}$ (rad/s) is the co-ordinate of the angular velocity vector of the mobile platform, both in the fixed co-ordinate system $\{O\}$, and ${}^R v_{Rx}$ (m/s), ${}^R v_{Ry}$ (m/s) and ${}^R \omega_{0z}$ (rad/s) are similar co-ordinates of velocity vectors, but described in the robot co-ordinate system $\{R\}$.

Those two vectors of generalized velocities satisfy the following relationship:

$${}^O \dot{\mathbf{q}} = {}^O_R \mathbf{J} {}^R \dot{\mathbf{q}}, \quad (3)$$

where matrix ${}^O_R \mathbf{J}$ has the following form:

$${}^O_R \mathbf{J} = \begin{bmatrix} \cos({}^O \varphi_{0z}) & -\sin({}^O \varphi_{0z}) & 0 \\ \sin({}^O \varphi_{0z}) & \cos({}^O \varphi_{0z}) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (4)$$

The subject of the research is a skid-steered robot in which the slip of the wheels is an inherent feature of its motion when turning and rotating in place. This means that the location of the instantaneous center of rotation of the robot's mobile platform results from dynamics, i.e., from the interaction of the wheels with the ground.

An example location of the instantaneous center of rotation of the robot's mobile platform, i.e., point C , is illustrated in Figure 1. In turn, the projection of this point on the Rx axis is indicated with point B .

The velocity vectors of characteristic points A_i of the geometric centers of the robot's wheels can be determined from the distribution of velocity vectors, an example of which is shown in Figure 1. They can be calculated based on the knowledge of the linear velocity vector ${}^R \mathbf{v}_R$, and the angular velocity vector ${}^R \boldsymbol{\omega}_0$ of the mobile platform from the following equation:

$${}^R \mathbf{v}_{Ai} = {}^R \mathbf{v}_R + {}^R \boldsymbol{\omega}_0 \times {}^R \mathbf{r}_{Ai}, \quad (5)$$

where ${}^R \mathbf{r}_{Ai} = [{}^R x_{Ai}, {}^R y_{Ai}, {}^R z_{Ai}]^T$ is the position vector of point A_i in the co-ordinate system associated with the robot $\{R\}$, $i = 1, \dots, n$, and $n = 6$ is the number of wheels.

In Figure 1, in order to keep the drawing clear, only one position vector ${}^R \mathbf{r}_{A1}$, i.e., for wheel 1, is illustrated.

Moreover, projections of the velocity vector of point A_i , which belongs to the i th wheel geometric center, on the ${}^R x$ and ${}^R y$ axes of the robot co-ordinate system $\{R\}$ depend on the angular velocity of wheel spin ω_i and the velocity of the slip.

Thus, the following relationship is satisfied:

$${}^R \mathbf{v}_{Ai} = [\omega_i r_i + {}^R v_{S_{ix}}, {}^R v_{S_{iy}}, 0]^T, \quad (6)$$

where ${}^R v_{S_{ix}}$ (m/s) and ${}^R v_{S_{iy}}$ (m/s) are, respectively, the longitudinal and lateral co-ordinates of the slip velocity vector in the $\{R\}$ co-ordinate system, that is, the velocity vector of motion of the point of the wheel which is in contact with the ground with respect to the ground.

3.2. Dynamics of the Robot

The robot is under the action of the following external forces and moments of force:

- Ground reaction forces ${}^R \mathbf{F}_{Ai} = [{}^R F_{A_{ix}}, {}^R F_{A_{iy}}, {}^R F_{A_{iz}}]^T$ acting on each wheel after their reduction to the geometric center of the wheel;
- Moments of force ${}^R \mathbf{T}_{Ai} = [{}^R T_{A_{ix}}, {}^R T_{A_{iy}}, {}^R T_{A_{iz}}]^T$ resulting from the forces and moments of force acting at the points of contact of wheels with the ground, also after their reduction to the geometric center of the wheel;
- Gravity force ${}^R \mathbf{G} = m_R {}^R \mathbf{g}$, where m_R (kg) denotes the total mass of the robot.

It is assumed that the robot moves on horizontal non-deformable ground, which is associated with a fixed co-ordinate system $\{O\}$.

Reaction forces ${}^R F_{Aix}$ and ${}^R F_{Aiy}$, $i = 1, \dots, n$, acting on the robot are shown in Figure 2.

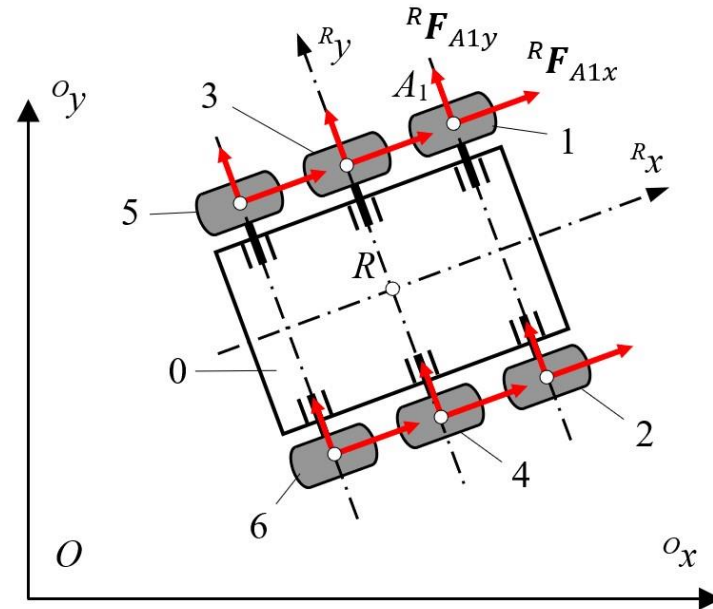


Figure 2. Reaction forces acting on the robot in the plane of wheel–ground contact.

Force of gravity vector ${}^R G$ is a function of the gravitational acceleration vector, that is, ${}^R g = [{}^R g_x, {}^R g_y, {}^R g_z]^T$, and it is applied at the robot mass center, whose position is described by the vector ${}^R r_{CM} = [{}^R x_{CM}, {}^R y_{CM}, {}^R z_{CM}]^T$.

The Newton–Euler formalism is used to describe the dynamics of the robot. Based on this formalism, the following dynamic equations of motion can be written for the robot in the co-ordinate system associated with the robot $\{R\}$:

$$m_R {}^R a_{CM} = \sum_{i=1}^n {}^R F_{Ai} + m_R {}^R g, \quad (7)$$

$$I_R {}^R \varepsilon_0 + {}^R \omega_0 \times I_R {}^R \omega_0 = \sum_{i=1}^n \left({}^R T_{Ai} + \left({}^R r_{Ai} - {}^R r_{CM} \right) \times {}^R F_{Ai} \right). \quad (8)$$

where I_R —the inertia tensor of the robot, ${}^R a_{CM}$ —the acceleration vector of the robot's center of mass, and ${}^R \varepsilon_0$ —the angular acceleration vector of the robot's mobile platform.

In order to determine the robot dynamics model in the simplest possible form, convenient for the synthesis of control systems, the following simplifying assumptions are made:

- The robot moves on horizontal ground and the tilt of the robot during movement is negligible; hence, ${}^R g = [0, 0, -g]^T$, where g (m/s²) is the value of gravitational acceleration;
- There is no robot's center of mass movement in the vertical direction, i.e., ${}^R a_{CMz} = 0$;
- Points of wheel–ground contact lie in one plane;
- Wheel tires deflect directly proportional to reaction forces acting along a direction normal to the ground and inversely proportional to their stiffness k (N/m), which is assumed to be constant and the same for all wheels.

In the case of the analyzed robot, there are a large number of unknown reaction forces related to the contact of the wheels with the ground. However, adopting the simplifying assumptions, as in the article by Trojnecki, 2015 [25], it is possible to determine the normal components of the ground reaction forces ${}^R F_{Aiz}$ (N) for all wheels.

Assuming that $L_f = L_b = L$ (m), the solution is obtained in the following form:

$${}^R F_{Aiz} = m_R \left(g^R y_{CM} - {}^R a_{CM y} (r + {}^R z_{CM}) \right) \text{sgn}({}^R y_{Ai}) / (3W) + m_R \left(g^R x_{CM} - {}^R a_{CM x} (r + {}^R z_{CM}) \right) \text{sgn}({}^R x_{Ai}) / (4L) + m_R g / 6. \quad (9)$$

The general solution, that is, for $L_f \neq L_b$, is much more complex.

In practical applications, one can find examples of robots with various L_f and L_b configurations, as well as center of mass positions. In particular, the position of the center of mass may change if the robot is equipped with a manipulator that modifies its configuration. However, to simplify the interpretation of the results, the analysis of the influence of the robot's parameters on the tracking control accuracy and energy efficiency in this paper is limited only to the case of $L_f = L_b$ and one selected position of the robot's center of mass.

Knowing the normal co-ordinates of the ground reaction forces ${}^R F_{Aiz}$ (N) and the wheel slip velocities, one can determine the tangential co-ordinates ${}^R F_{Aix}$ (N) and ${}^R F_{Aiy}$ (N).

In the general case, the slip velocity vector for the i th wheel is equal to the following:

$${}^R v_{Si} = \left[{}^R v_{Aix} - v_{oi}, {}^R v_{Aiy}, 0 \right]^T, \quad (10)$$

where $v_{oi} = \omega_i r_i$ (m/s) is the value of the velocity at the wheel circumference, and ${}^R v_{Aix}$ (m/s) and ${}^R v_{Aiy}$ (m/s) are, respectively, the longitudinal and lateral co-ordinates of the velocity vector of the wheel geometric center.

The actual value of longitudinal slip ratio λ_i (%) for the i th wheel is then determined based on the following relationship:

$$\lambda_i = \begin{cases} 0 & \text{if } \max(|{}^R v_{Aix}|, |v_{oi}|) = 0, \\ -({}^R v_{Aix} - v_{oi}) / \max(|{}^R v_{Aix}|, |v_{oi}|) & \text{otherwise.} \end{cases} \quad (11)$$

The actual value of the lateral slip angle α_i (rad) for the i th wheel, that is, the angle between the velocity vector ${}^R v_{Ai}$ and the wheel plane, is calculated from the following formula:

$$\alpha_i = \begin{cases} 0 & \text{if } |{}^R v_{Aiy}| = 0, \\ \arctan2({}^R v_{Aiy}, |{}^R v_{Aix}|) & \text{otherwise.} \end{cases} \quad (12)$$

Based on the known longitudinal slip ratio, the actual value of the tire adhesion coefficient on longitudinal direction for this wheel can be determined. For this purpose, one can use the Kiencke tire model described in the book by Kiencke and Nielsen, 2005 [26] and applied in the automotive industry. However, this model is also sometimes used for modeling of wheeled mobile robots, an example of which can be found in the doctoral thesis by Ping, 2009 [27]. As rightly noted in the paper by Majdoub et al., 2010 [28], the Kiencke tire model provides the best accuracy/simplicity compromise. Due to its effectiveness, its use in the design of control systems for wheeled vehicles is justified.

For this reason, a modified version of the Kiencke tire model is used in this work, that is, in the following form:

$$\mu_{ix} = \begin{cases} (2\mu_p \lambda_p \lambda_i) / (\lambda_p^2 + \lambda_i^2) & \text{if } |\lambda_i| \leq \lambda_p, \\ a_{\lambda x} \lambda_i + b_{\lambda x} \text{sgn}(\lambda_i) & \text{otherwise,} \end{cases} \quad (13)$$

where λ_p (%) denotes the value of the longitudinal slip ratio corresponding to the value of the maximum tire adhesion coefficient μ_p (-).

Coefficients $a_{\lambda x}$ (–) and $b_{\lambda x}$ (–) in the relationship (13) are described by the following formulae:

$$a_{\lambda x} = (\mu_p - \mu_k) / (\lambda_p - \lambda_{max}), \quad b_{\lambda x} = \mu_p - a_{\lambda x} \lambda_p, \quad (14)$$

where μ_k (–) is a coefficient of the kinetic friction between the tire and ground, and $\lambda_{max} = 100\%$.

The modified version of the Kiencke tire model allows for a more precise description of the characteristics of $\mu_{ix}(\lambda_i)$ for $|\lambda_i| > \lambda_p$, including obtaining $\mu_{ix} = \mu_k$ for $|\lambda_i| = \lambda_{max}$.

Knowing the lateral slip angle, it is possible to calculate the actual value of the adhesion coefficient in the lateral direction for the i th wheel. For this purpose, the following simplified relationship can be used:

$$\mu_{iy} = -\mu_{ymax} \sin(\alpha_i), \quad (15)$$

where μ_{ymax} (–) denotes the maximum value of adhesion coefficient in the lateral direction, which is assumed to be equal to μ_k (–).

It is assumed that the tangential co-ordinates of the ground reaction forces, that is, ${}^R F_{Aix}$ (N) and ${}^R F_{Aiy}$ (N), are proportional to the normal co-ordinates of the ground reaction forces ${}^R F_{Aiz}$, and the actual values of the tire adhesion coefficient in the longitudinal and lateral directions, respectively. Thus, the following dependencies are satisfied:

$${}^R F_{Aix} = \mu_{ix} {}^R F_{Aiz}, \quad {}^R F_{Aiy} = \mu_{iy} {}^R F_{Aiz}. \quad (16)$$

Strictly speaking, in the case of combined slip conditions, including both longitudinal and lateral slip, the value of the resultant tangential component of the ground reaction force should lie inside the friction ellipse. This would require a reduction in the value of lateral force ${}^R F_{Aiy}$. However, in the developed robot model, for simplicity, this aspect is omitted because it does not significantly affect the robot's movement.

Figure 3 shows the ${}^R F_{Aix}(\lambda_i)$ characteristic resulting from the adopted modified version of the Kiencke tire model.

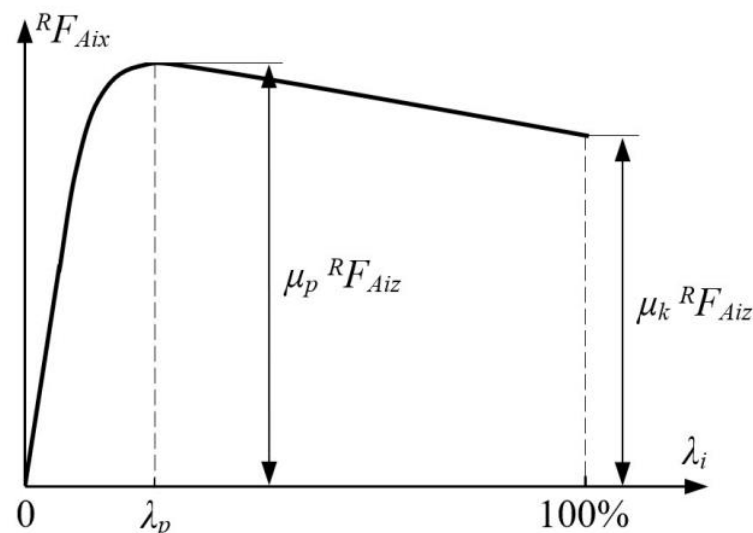


Figure 3. Illustration of longitudinal force vs. longitudinal slip ratio dependency.

As a result of the forces acting on the robot, it moves with linear acceleration ${}^R a_{CM}$, whose co-ordinates result from the following formulae:

$${}^R a_{CMx} = \sum_{i=1}^n {}^R F_{Aix} / m_R, \quad {}^R a_{CMy} = \sum_{i=1}^n {}^R F_{Aiy} / m_R, \quad {}^R a_{CMz} = 0. \quad (17)$$

If the robot turns or rotates in place and all its wheels are driven, then the longitudinal forces ${}^R F_{Aix}$ (N) cause movement, while the lateral forces ${}^R F_{Aiy}$ (N) counteract it. It is,

therefore, intuitive that, in such a case, the wider the wheel track is in relation to the axle distance, the more advantageous it will be.

It can also be assumed that the values of the so-called self-aligning torques ${}^R T_{Aiz}$ (Nm) are negligibly small in comparison to the values of the remaining moments of force acting about the ${}^R z$ axis; therefore, they can be omitted in the robot's dynamics model.

Therefore, the ${}^R \varepsilon_{0z}$ (rad/s²) co-ordinate of the robot's angular acceleration vector results from the equation:

$${}^R \varepsilon_{0z} = (\sum_{r \in \mathbb{R}} {}^R F_{Arx} w_r - \sum_{l \in \mathbb{L}} {}^R F_{Alx} w_l + \sum_{f \in \mathbb{F}} {}^R F_{Afy} l_f - \sum_{b \in \mathbb{B}} {}^R F_{Aby} l_b) / I_{Rz}, \quad (18)$$

where the sets $\mathbb{L}_w = \{1, 3, 5\}$, $\mathbb{R}_w = \{2, 4, 6\}$, $\mathbb{F}_w = \{1, 2\}$, and $\mathbb{B}_w = \{5, 6\}$ define the indices for the left, right, front, and back wheels, respectively.

The individual distances between the centers of these wheels and the center of mass of the robot, measured along the ${}^R x$ and ${}^R y$ axes, are, respectively, equal to $l_f = (L_f - {}^R x_{CM})$, $l_b = (L_b + {}^R x_{CM})$, $w_r = (W/2 + {}^R y_{CM})$, and $w_l = (W/2 - {}^R y_{CM})$.

Moreover, I_{Rz} (kg m²) is the mass moment of inertia of the robot about an axis parallel to ${}^R z$ and passing through its center of mass.

Knowing the forces acting on the robot's wheels, one can determine the angular acceleration of their spin based on the following equation:

$$\varepsilon_i = (\tau_i - {}^R F_{Aix} r_i - {}^R F_{Aiz} r_i f_r \operatorname{sgn}(\omega_i)) / I_{Wy}, \quad (19)$$

where τ_i (Nm) is the value of the driving torque, the expression $-{}^R F_{Aiz} r_i f_r \operatorname{sgn}(\omega_i)$ reflects the value of the rolling resistance torque acting on the i th wheel, and $f_r (-)$ is the rolling resistance coefficient.

Equation (19) omits the moments of motion resistance occurring in kinematic pairs, in particular, those related to the phenomenon of friction in the bearings. This is due to the fact that they are negligible in relation to the remaining moments of force acting on wheels. They can only have a noticeable effect at a very low velocity of robot movement, when they can be comparable to driving torques.

In order to take into account the gradual change of the rolling resistance moment, instead of the function $\operatorname{sgn}(\omega_i)$, the function $\tan(\beta \omega_i)$ can be introduced, where the greater the value of the coefficient $\beta > 0 (-)$ is, the closer this function will be to $\operatorname{sgn}(\omega_i)$.

3.3. Model of the Robot's Drive Units

The driving torques acting on the robot's wheels can be determined based on the model of the drive units.

In the case of the analyzed robot, the following is assumed:

- All robot wheels are independently driven and each drive unit contains an identical DC motor, encoder, and transmission system;
- The drive units are not self-locking; that is, they can freely turn under the influence of external moments of force;
- The mass moments of inertia of the rotating elements of the servomechanisms are small in comparison to the mass moments of inertia of the driven wheels, which is why they are neglected;
- The maximum motor voltage input is U_{max} (V), which corresponds to the maximum motor rotational speed n_{max} (rpm) and the resulting maximum angular velocity of wheel spin ω_{max} (rad/s).

Taking the above assumptions into account, the following dependencies can be written for the i th power unit:

$$dI_i/dt = (U_i - k_e n_d \omega_i - R_d I_i) / L_d, \quad \tau_i = \eta_d n_d k_m I_i, \quad (20)$$

where U_i (V)—the motor voltage input; I_i (A)—the rotor current; L_d (mH), R_d (Ω)—the inductance and resistance of the rotor, respectively; k_e (Vs/rad)—the electromotive force constant; k_m (Nm/A)—the motor torque coefficient; and n_d (–), η_d (–)—the gear ratio and efficiency factor of the transmission system, respectively.

Therefore, in order to determine the driving torque τ_i (Nm) for the i th drive unit, it is necessary to know the motor voltage input U_i (V) coming from the wheels' controller.

In turn, the relationship between the motor rotational speed n_i (rpm) and the angular velocity of wheel spin ω_i (rad/s) for the i th driven wheel is as follows:

$$\omega_i = \pi n_i / (30 n_d). \quad (21)$$

Knowing the motor voltage input U_i (V) and the rotor current I_i (A), it is possible to determine the electric power p_i (W) necessary to rotate the i th wheel with angular velocity ω_i (rad/s) and to generate driving torque τ_i (Nm) for this wheel by the drive unit.

For this purpose, one can use the following formula:

$$p_i = |U_i I_i|. \quad (22)$$

In the relationship (22), there is a rotor current I_i (A), which takes into account the efficiency factor of the transmission system η_d (–), so the mechanical power resulting from driving the drive unit is correspondingly lower and is equal to $\tau_i \omega_i$ (W).

Assuming that no electric energy recuperation is applied when braking the robot, the electric energy E_D (J) for all drive units can be calculated for period T from the following relationship:

$$E_D = \sum_{i=1}^6 \int_0^T p_i dt. \quad (23)$$

4. Motion Control System

4.1. Desired Motion of the Robot

The desired motion of the robot can be described by the vector of the robot's desired pose, which has the form of the vector of generalized co-ordinates:

$${}^O q_d = [{}^O x_{Rd}, {}^O y_{Rd}, {}^O \varphi_{0zd}]^T, \quad (24)$$

where ${}^O x_{Rd}$ (m), ${}^O y_{Rd}$ (m) are the desired co-ordinates of the position of point R of the robot, and ${}^O \varphi_{0zd}$ (rad) is a desired course of the robot's mobile platform with respect to the ${}^O z$ axis.

It should be noted that the analyzed robot is a non-holonomic system and is characterized by the fact that it cannot move in the lateral direction, except in the case of movement under the condition of the slip of the wheels. Therefore, it is assumed that ${}^R v_{Ryd} = 0$.

Thus, the desired motion of the trajectory of the robot is represented in the form of the vector of desired generalized velocities:

$$v_d = [{}^R v_{Rd}, {}^R \omega_{0zd}]^T = [v_{Rd}, \omega_{0zd}]^T, \quad (25)$$

where ${}^R v_{Rxd} = v_{Rd}$ (m/s) is a value of the desired linear velocity of the characteristic point R of the robot and ${}^R \omega_{0zd}$ (rad/s) is a value of the desired angular velocity of its mobile platform, both in the robot co-ordinate system $\{R\}$.

The relationship between the generalized desired velocities of the robot in the fixed co-ordinate system $\{O\}$ and the one associated with the robot $\{R\}$ can be presented in the following form:

$${}^O\dot{\mathbf{q}}_d = \begin{bmatrix} {}^O v_{Rxd} \\ {}^O v_{Ryd} \\ {}^O \omega_{0zd} \end{bmatrix} = \begin{bmatrix} {}^O \dot{x}_{Rd} \\ {}^O \dot{y}_{Rd} \\ {}^O \dot{\varphi}_{0zd} \end{bmatrix} = \begin{bmatrix} \cos({}^O \varphi_{0zd}) & 0 \\ \sin({}^O \varphi_{0zd}) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^R v_{Rd} \\ {}^R \omega_{0zd} \end{bmatrix}. \quad (26)$$

It can be noticed that, if the robot moves on horizontal ground, then, even though the co-ordinates of the velocity vectors ${}^R \mathbf{v}_{Rd}$ and ${}^O \mathbf{v}_{Rd}$ have different values, their modules are equal, that is, ${}^R v_{Rd} = {}^O v_{Rd} = v_{Rd}$ (m/s). From the fact that the axes of both reference systems are parallel to each other, it also follows that ${}^R \omega_{0zd} = {}^O \omega_{0zd} = \omega_{0zd}$ (rad/s). The same applies to accelerations, i.e., ${}^R a_{Rd} = {}^O a_{Rd} = a_{Rd}$ (m/s²) and ${}^R \varepsilon_{0zd} = {}^O \varepsilon_{0zd} = \varepsilon_{0zd}$ (rad/s²). Therefore, in the analyzed case, the desired values of the robot's generalized velocities and accelerations do not depend on whether they are expressed in the fixed co-ordinate system $\{O\}$ or in the moving co-ordinate system associated with the robot $\{R\}$.

In the case of the plane motion of the robot's mobile platform, the desired velocity of the point R , that is, ${}^R v_{Rxd} = v_{Rd}$ (m/s), depends on the desired angular velocity ${}^R \omega_{0zd} = \omega_{0zd}$ (rad/s) and the radius of curvature of the path R_z (m) according to the following formula:

$$v_{Rd} = \omega_{0zd} R_z. \quad (27)$$

4.2. Wheels' Controller

As mentioned earlier, various control system structures and techniques can be used for the trajectory tracking control of wheeled robots. However, this problem ultimately involves controlling the movement of the robot's wheels based on the desired motion parameters, which, in particular, may come from a higher layer of the control system.

Let us assume that the wheels' controller is the only layer of the control system, and the desired robot movement is based on the robot's generalized velocity vector $\mathbf{u}_s$. Then this vector is equal to the vector of desired generalized velocities $\mathbf{v}_d$, that is:

$$\mathbf{u}_s = \mathbf{v}_d = [v_{Rd}, \omega_{0zd}]^T. \quad (28)$$

In order to control the motion of the wheels, the vector $\mathbf{u}_s$ must be transformed into a vector of the desired angular velocities of the spin of wheels $\boldsymbol{\omega}_d$.

Therefore, in this case, the trajectory tracking control problem is to determine the control vector $\mathbf{u}(t)$ such that $\boldsymbol{\omega}(t) \rightarrow \boldsymbol{\omega}_d(t)$ for $t \rightarrow \infty$.

Assuming that the same desired angular velocity of spin ω_{ld} is used for all left wheels of the robot, and the same ω_{rd} for all right wheels, the following relationship can be used to determine these velocities:

$$\begin{bmatrix} \omega_{ld} \\ \omega_{rd} \end{bmatrix} = \frac{1}{r} \begin{bmatrix} 1 & -W/2 \\ 1 & W/2 \end{bmatrix} \mathbf{u}_s. \quad (29)$$

The relationship (29) results from the kinematics of the robot for the case of motion without slip of the wheels and the use of differential steering, and can be applied to wheeled robots with various kinematic structures. In the case of the analyzed six-wheeled robot with non-steered wheels, the desired angular velocities ω_{id} (rad/s) for individual wheels are equal to the following:

$$\omega_{id} = \begin{cases} \omega_{ld} & \text{for } i \in \mathbb{L}_w, \\ \omega_{rd} & \text{for } i \in \mathbb{R}_w, \end{cases} \quad (30)$$

where $\mathbb{L}_w = \{1, 3, 5\}$, $\mathbb{R}_w = \{2, 4, 6\}$ are the indices for the left and right wheels, respectively.

In order to execute the trajectory tracking control of the robot, let us introduce the following error vector of the angular velocities of the spin of the wheels $\mathbf{e}_\omega = [e_{\omega 1}, \dots, e_{\omega n}]^T$ and the error vector of the rotation angles of these wheels $\mathbf{e}_\theta = [e_{\theta 1}, \dots, e_{\theta n}]^T$:

$$\mathbf{e}_\omega = \boldsymbol{\omega}_d - \boldsymbol{\omega} = \dot{\boldsymbol{\theta}}_d - \dot{\boldsymbol{\theta}}, \quad \mathbf{e}_\theta = \boldsymbol{\theta}_d - \boldsymbol{\theta}. \quad (31)$$

Let us also assume that a linear controller is used for the motion control of the robot's wheels. Then, the control vector $\mathbf{u} = [U_1, \dots, U_n]^T$ can be determined based on the following relationship:

$$\mathbf{u} = (k_\omega \mathbf{e}_\omega + k_\theta \mathbf{e}_\theta) |\operatorname{sgn}(\boldsymbol{\omega}_d)|, \quad (32)$$

where k_ω (Vs/rad) and k_θ (V/rad) are the controller gains, and U_i (V) is the i th motor voltage input, $i = 1, \dots, n$.

The control vector $\mathbf{u}$ is further limited using the saturation function so that individual control signals do not exceed the maximum motor voltage input U_{max} (V).

The expression $|\operatorname{sgn}(\boldsymbol{\omega}_d)|$ was introduced to force the robot to stop when the desired angular velocities $\boldsymbol{\omega}_d$ are equal to zero and the trajectory tracking errors $\mathbf{e}_\omega$ or $\mathbf{e}_\theta$ are non-zero.

The schematic diagram of the analyzed control system is illustrated in Figure 4.

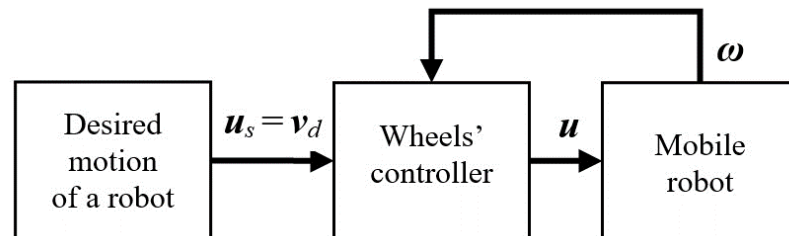


Figure 4. Schematic diagram of the control system with wheels' controller only.

It should be noted that such a control system does not take into account the actual pose of the robot or its real velocities. Therefore, it is not able to compensate for trajectory tracking errors resulting from the slip of the wheels, which may cause such errors to accumulate over time. Moreover, the proposed controller does not take into account the robot's dynamics model; thus, it does not allow for the compensation of its nonlinearities. Therefore, it does not allow for ensuring a high accuracy of the robot's movement and the robustness of the control system to changing robot operating conditions.

4.3. Pose Controller

In order to take into account the actual pose of the robot in the control system, let us introduce the pose error vector ${}^O\mathbf{q}_e$ in the fixed co-ordinate system $\{O\}$ in the following form:

$${}^O\mathbf{q}_e = \begin{bmatrix} {}^Oe_x \\ {}^Oe_y \\ {}^Oe_\varphi \end{bmatrix} = {}^O\mathbf{q}_d - {}^O\mathbf{q} = \begin{bmatrix} {}^Ox_{Rd} - {}^Ox_R \\ {}^Oy_{Rd} - {}^Oy_R \\ {}^O\varphi_{0zd} - {}^O\varphi_{0z} \end{bmatrix}. \quad (33)$$

Since it is convenient to control the robot motion in a moving co-ordinate system $\{R\}$, let us transform the pose error vector ${}^O\mathbf{q}_e$ to such a system using the following relationship:

$${}^R\mathbf{q}_e = \begin{bmatrix} {}^Re_x \\ {}^Re_y \\ {}^Re_\varphi \end{bmatrix} = \begin{bmatrix} \cos({}^O\varphi_{0z}) & \sin({}^O\varphi_{0z}) & 0 \\ -\sin({}^O\varphi_{0z}) & \cos({}^O\varphi_{0z}) & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^O\mathbf{q}_e. \quad (34)$$

Figure 5 illustrates the following quantities for the robot: the desired path resulting from the desired motion trajectory ${}^O\mathbf{q}_d(t)$, the desired pose vector ${}^O\mathbf{q}_d = [{}^Ox_{Rd}, {}^Oy_{Rd}, {}^O\varphi_{0zd}]^T$ and the desired velocity vector ${}^O\mathbf{v}_{Rd}$ at the analyzed moment of time, and the actual pose vector

${}^O\mathbf{q} = [{}^Ox_R, {}^Oy_R, {}^O\varphi_{0z}]^T$ and the actual velocity vector ${}^O\mathbf{v}_R$, as well as the vectors of pose errors ${}^O\mathbf{q}_e = [{}^Oe_x, {}^Oe_y, {}^Oe_\varphi]^T$ and ${}^R\mathbf{q}_e = [{}^Re_x, {}^Re_y, {}^Re_\varphi]^T$, expressed in the fixed co-ordinate system $\{O\}$ and in the robot co-ordinate system $\{R\}$, respectively.

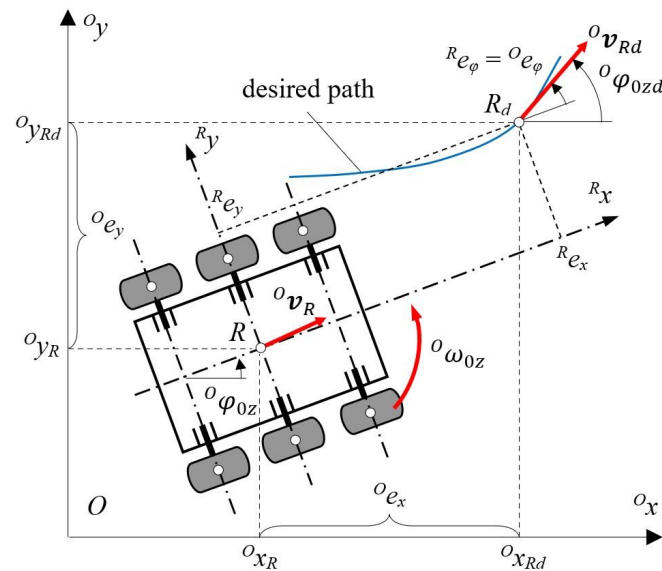


Figure 5. Illustration of pose errors of the robot.

Additionally, one can calculate the distance of the characteristic point R of the robot from its actual to the desired position:

$${}^Oe_d = {}^Re_d = \sqrt{({}^Oe_x)^2 + ({}^Oe_y)^2} = \sqrt{({}^Re_x)^2 + ({}^Re_y)^2}. \quad (35)$$

In this case, the trajectory tracking control problem is to determine the control vector $\mathbf{u}_s$, such that ${}^O\mathbf{q}(t) \rightarrow {}^O\mathbf{q}_d(t)$ for $t \rightarrow \infty$.

Figure 6 shows a schematic diagram of the proposed control system with a pose controller, which also includes the wheel's controller. In the analyzed case, the pose controller generates a control vector $\mathbf{u}_s$ for the lower layer of the control system, i.e., for the wheels' controller, for which this vector is an input vector, and, so, equivalent to the vector of the desired generalized velocities $\mathbf{v}_d$, and its output vector is the control vector $\mathbf{u}$ for the robot's drive units.

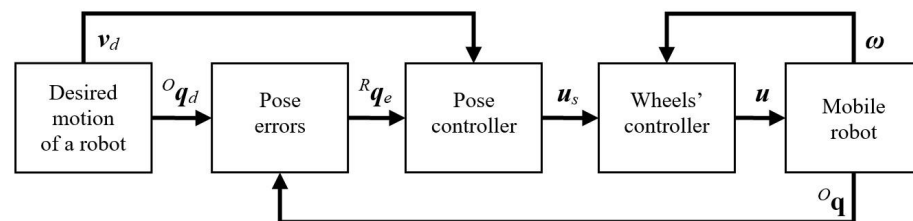


Figure 6. Schematic diagram of control system with wheels' and pose controllers.

In this case, the desired wheel spin angular velocities ω_d are calculated by the wheels' controller based on the control vector $\mathbf{u}_s$. Various solutions are used as a pose controller. A comparative analysis of selected solutions of this type was the subject of the article by Trojnecki et al., 2016 [29]. Newer solutions are proposed, among others, in the works of Axenie and Saveriano, 2023 [30], Hassani and Rekik, 2023 [14], Wu et al., 2023, [31] and Hassan et al., 2024, [32].

In this paper, the pose controller is used in the following form:

$$\mathbf{u}_s = \begin{bmatrix} v_s \\ \omega_s \end{bmatrix} = \begin{bmatrix} v_{Rd} \cos(R e_\varphi) + k_x \tanh(k_v R e_x) |\operatorname{sgn}(v_{Rd})| \\ \omega_{0zd} + k_y v_{Rd} R e_y + k_\varphi \tanh(k_\omega R e_\varphi) |\operatorname{sgn}(v_{Rd})| \end{bmatrix}, \quad (36)$$

where k_x (m/s), k_y (rad/m²), and k_φ (rad/s) are the chosen positive parameters responsible for eliminating pose errors $R e_x$, $R e_y$, and $R e_\varphi$, respectively.

The adopted controller is similar to the solution presented in Jiang et al., 2001 [33]. The following improvements were made compared to the original version:

- The expression $|\operatorname{sgn}(v_{Rd})|$ was added to enforce robot stopping when the desired velocity v_{Rd} (m/s) is equal to zero despite the $R e_x$ (m) and $R e_\varphi$ (rad) errors being different than zero;
- The parameters k_v (rad/m) and k_ω (–) were added for the hyperbolic tangent function, which allows the operation of these parts of the controller to be adjusted to the assumed maximum values for the errors $R e_x$ (m) and $R e_\varphi$ (rad), respectively,
- The part of the controller responsible for eliminating the $R e_y$ (m) error was significantly simplified, and, at the same time, a gain k_y (rad/m²) was added for it, following, in this respect, the controller solution discussed in the article by Kanayama et al., 1990 [34].

Moreover, the method described below for selecting the controller parameters was proposed.

Suppose that the absolute values of the robot's maximum generalized velocities v_{Rmax} (m/s) and ω_{0zmax} (rad/s) are known. Then, it can be assumed that the following dependencies are met in relation to the control vector of the pose controller $\mathbf{u}_s$:

$$\max(|v_{Rd}|) \leq v_{smax} \leq v_{Rmax} \text{ and } \max(|\omega_{0zd}|) \leq \omega_{smax} \leq \omega_{0zmax}. \quad (37)$$

During the choice of the parameters of the pose controller, it is assumed that the controller should generate maximum values of velocities, that is, v_{smax} (m/s) or ω_{smax} (rad/s), for minimum or critical pose errors. Moreover, it is assumed that, in case of their surpassing, the controller should still generate maximum velocities v_{smax} (m/s) or ω_{smax} (rad/s) as a result of the use of the saturation function.

Taking into account the form of the adopted pose controller, described by Formula (36), the following inequalities for linear and angular control are obtained:

$$|v_{Rdmax}| \cos(R e_{\varphi min}) + k_x \tanh(k_v R e_{xmax}) |\operatorname{sgn}(v_{Rdmax})| \leq v_{smax}, \quad (38)$$

$$|\omega_{0zdmax}| + k_y |v_{Rdmax}| R e_{ymax} + k_\varphi \tanh(k_\omega R e_{\varphi max}) |\operatorname{sgn}(v_{Rdmax})| \leq \omega_{smax}, \quad (39)$$

where $|\operatorname{sgn}(v_{Rdmax})| = 1$ and $R e_{\varphi min} = 0$; hence, $\cos(R e_{\varphi min}) = 1$.

From the inequality for the linear velocity control (38), the constraint for the k_x (m/s) parameter is obtained in the following form:

$$k_x \leq (v_{smax} - |v_{Rdmax}|) / \tanh(k_v R e_{xmax}), \quad (40)$$

In turn, from the inequality for the angular velocity control (39), it is possible to determine two parameters k_y (rad/m²) and k_φ (rad/s) by introducing an additional assumption.

Since the robot's angular velocity control law includes parts responsible for minimizing the lateral position error $R e_y$ (m) and the course error $R e_\varphi$ (rad), the resulting control ω_s (rad/s) must be a compromise between minimizing both of these errors.

To decide how much the pose controller should minimize the lateral position error $R e_y$ (m), let us introduce the parameter η (–). Then, based on the inequality for the

angular velocity control (39), the parameter k_y (rad/m²) can be determined from the following relationship:

$$k_y \leq \eta (\omega_{smax} - |\omega_{0zmax}|) / (|v_{Rdmax}|^R e_{ymax}), \quad (41)$$

where $0 < \eta < 1$.

Finally, for the part of the pose controller responsible for minimizing the course error ${}^R e_\varphi$ (rad), the following constraint for the k_φ (rad/s) parameter can be introduced:

$$k_\varphi \leq (\omega_{smax} - |\omega_{0zmax}| - k_y |v_{Rdmax}|^R e_{ymax}) / \tanh(k_\omega {}^R e_{\varphi max}), \quad (42)$$

5. Simulation Studies

5.1. Robot and Environment Parameters

The previously described robot model and control system were implemented in the MATLAB/Simulink package in order to carry out simulation studies. The ode4 fixed-step solver was used in the simulations, which implements the classic Runge–Kutta method. A step time of $dt = 0.01$ ms was assumed, which is a compromise between the simulation accuracy and the simulation time.

For the simulation studies, the following basic parameters of the robot are assumed:

- Dimensions: $L_f = L_b = L = 0.250$ m, $W = 0.520$ m, and $r_i = r = 0.0965$ m;
- Masses: $m_R = 45.4$ kg, $m_i = 1$ kg;
- Mass center co-ordinates: ${}^R x_{CM} = -0.02$ m, ${}^R y_{CM} = 0$ m, and ${}^R z_{CM} = 0.04$ m;
- Mass moments of inertia: $I_{Wy} = 0.01$ kg·m², and $I_{Rz} = 3.1$ kg·m²;
- Drive units: $L_d = 0.0823$ mH, $R_d = 0.317$ Ω , $k_e = 0.0301$ Vs/rad, $n_d = 53$, $\eta_d = 0.8$, $k_m = 0.0302$ Nm/A, $U_{max} = 32$ V, and $\omega_{max} = 15.8$ rad/s.

As for the track width W (m), in accordance with the topic of the article, it is also the subject of further analyses in which the impact of its different values on the accuracy of tracking control and energy efficiency is analyzed.

The robot's motion on concrete ground is analyzed and the following tire–ground contact parameters are adopted: $\mu_p = 0.85$, $\mu_k = 0.75$, $f_r = 0.03$, and $\lambda_p = 16.5\%$. Moreover, the value of gravitational acceleration $g = 9.81$ m/s² and the coefficient $\beta = 2$ s^{−1} for the function $\tan(\beta\omega_i)$ approximating the function $\text{sgn}(\omega_i)$ are assumed.

5.2. Desired Motion and Initial Conditions

It is assumed that the robot's desired motion consists of three phases:

- Accelerating with maximum acceleration a_{Rmax} (m/s²) on the distance of l_r (m);
- Steady motion with constant velocity v_{Ru} (m/s);
- Braking with maximum deceleration a_{Rmax} (m/s²) on the distance of l_h (m).

In turn, the desired motion path contains the following:

- A straight line segment of length L_r (m);
- A circular arc of radius R_z (m);
- The second straight line segment of length L_h (m).

As a result of turning with maximum angular velocity $\omega_{0zu} = v_{Ru} / R_z$ (rad/s) and maximum angular acceleration ε_{0zmax} (rad/s²), the robot should turn by the angle φ_{0zmax} (rad).

The convention is that a positive value of the turning radius R_z (m) means turning to the left (rotation of the robot in accordance with the direction of the ${}^R z$ axis), and a negative value to the right. The desired time histories of the robot's motion parameters are generated in the form of polynomials; i.e., for accelerations, a second-degree polynomial is assumed with respect to the time t (s), and, for velocities, a third-degree polynomial.

Assuming lengths of acceleration and braking distances $l_r = l_h = l$ (m), the necessary maximum acceleration value a_{Rmax} (m/s²) can be determined for the given velocity profile. However, it cannot exceed the maximum value resulting from the dynamics of the drive

units. In turn, the maximum value of the angular acceleration ε_{0zmax} (rad/s²) can be selected in such a way that, in the case of rotation in place, it corresponds to analogous values of accelerations of the geometric centers of the middle wheels a_{Am} (m/s²).

Therefore, for the adopted method of generating the desired robot motion parameters, one can use the following dependencies:

$$a_{Rmax} = 3(v_{Ru})^2 / (4l), \quad \varepsilon_{0zmax} = 2a_{Rmax} / W. \quad (43)$$

It can be noticed that a large value of W (m) is disadvantageous due to ε_{0zmax} (rad/s²) and, consequently, ω_{0zmax} (rad/s).

Next, based on the knowledge of velocities v_{Ru} (m/s) and ω_{0zu} (rad/s), accelerations a_{Rmax} (m/s²) and ε_{0zmax} (rad/s²), and lengths L_r (m) and L_h (m), as well as angle φ_{0zmax} (rad), the characteristic time instants are determined.

5.3. Controller Settings

The wheels' controller settings were selected iteratively in such a way so as to obtain small tracking errors and maintain the stability of the control system. As a result, the gains $k_\omega = 10$ Vs/rad and $k_\theta = 30$ V/rad were selected.

For the pose controller in the form (36), the previously derived Formulae (40)–(42) defining the limit values of the settings were applied, the result of which is described below.

Let us assume that the maximum absolute values of pose errors are ${}^R e_{xmax} = 0.5$ m, ${}^R e_{ymax} = 0.5$ m, and ${}^R e_{\varphi max} = \pi/4$ rad. Thus, the gains k_v (rad/m) and k_ω (–) for the analyzed pose controller are, respectively, equal to $k_v = 2\pi$ rad/m and $k_\omega = 4$. In such a case, $\tanh(\pi) \approx 1$.

Then, based on the dependencies (40)–(42) on the limit values of the controller settings, the following is obtained:

$$k_x = v_{smax} - |v_{Rdmax}|, \quad (44)$$

$$k_y = \eta (\omega_{smax} - |\omega_{0zdmax}|) / (|v_{Rdmax}| {}^R e_{ymax}), \quad (45)$$

$$k_\varphi = \omega_{smax} - |\omega_{0zdmax}| - k_y |v_{Rdmax}| {}^R e_{ymax}. \quad (46)$$

Assuming that $v_{smax} = v_{Rmax} = \omega_{max}r$ (m/s) and $\omega_{smax} = 2v_{smax}/W$ (rad/s) and parameter $\eta = 0.2$, the final settings of the pose controller are selected depending on the assumed maximum desired motion parameters. Taking into account the constant settings $|v_{Rdmax}| = 1$ m/s and $|\omega_{0zdmax}| = 2|v_{Rdmax}|/W$ (rad/s) for all analyzed desired motion trajectories and the nominal value of the wheel track W (m), the following final settings are obtained: $k_x = 0.53$ m/s, $k_y = 0.81$ rad/m², and $k_\varphi = 1.62$ rad/s.

5.4. Quality Indices

In order to evaluate the analyzed geometric configurations of the robot and control system solutions from the point of view of the accuracy of tracking control and energy efficiency, quality indices are adopted.

The accuracy of the tracking control by the wheels' controller is assessed based on the maximum angular velocities' errors of individual wheels and their average for all n wheels using the following formulae:

$$e_{\omega imax} = \max_{t \in (0, T)} (|e_{\omega i}|), \quad e_{\omega max} = \sum_{i=1}^n e_{\omega imax} / n. \quad (47)$$

The integrals for the errors (47) are also calculated in relation to the simulation period T (s) for individual wheels and their average for all n wheels from the following relationship:

$$E_{\omega i} = \sqrt{\frac{1}{T} \int_0^T (e_{\omega i})^2 dt}, \quad E_{\omega} = \sum_{i=1}^n E_{\omega i} / n. \quad (48)$$

In terms of assessing the accuracy of movement using the pose controller, the following quality indices are applied, analogous as before, separately with respect to the distance error ${}^R e_d$ and course ${}^R e_\varphi$:

$${}^R e_{kmax} = \max_{t \in \langle 0, T \rangle} (|{}^R e_k|), \quad E_k = \sqrt{\frac{1}{T} \int_0^T ({}^R e_k)^2 dt}, \quad (49)$$

where $k \in \{d, \varphi\}$.

To assess the energy efficiency, the following quality indices relating to electrical power and electrical energy consumed by the robot drives are introduced:

$$p_{imax} = \max_{t \in \langle 0, T \rangle} (\text{pos}(p_i)), \quad p_{max} = \sum_{i=1}^n p_{imax} / n, \quad (50)$$

$$E_{Di} = \int_0^T \text{pos}(p_i) dt, \quad E_D = \sum_{i=1}^n E_{Di} / n, \quad (51)$$

where function

$$\text{pos}(p_i) = \begin{cases} 0 & \text{if } p_i \leq 0, \\ p_i & \text{otherwise} \end{cases} \quad (52)$$

allows one to take into account the lack of electric energy recuperation.

As before, for the quality indices p_{imax} (W) and E_{Di} (J), their averages are calculated for all n wheels.

5.5. Results of Simulation Studies

As part of the simulation studies, various sets of desired motion parameters were analyzed, taking into account motion with different maximum values of velocities and accelerations, as well as turning with different radii.

In the first stage, simulation studies were carried out for the nominal geometric parameters of the robot, including the track width W (m).

The characteristic motion parameters of the robot for the selected cases presented in this article are summarized in Table 1. In turn, Figure 7 shows the exemplary time histories of the desired motion parameters corresponding to extreme cases, i.e., 1 and 3. They are desired linear velocity and acceleration of the robot's point R (Figures 7a and 7b, respectively) as well as desired angular velocity and acceleration of the robot's mobile platform (Figures 7c and 7d, respectively). In the simulation studies, zero initial conditions were assumed in terms of the robot's pose and velocities.

Table 1. Characteristic motion parameters of the robot for the selected cases.

Case No.	v_{Ru} (m/s)	a_{Rmax} (m/s ²)	L_r (m)	L_h (m)	l (m)	R_z (m)	ω_{0zu} (rad/s)	ε_{0zmax} (rad/s ²)	φ_{0zmax} (rad)
1	0.3	0.675	0.5	0.75	0.1	−0.6	−0.5	2.60	$2/3 \pi$
2	0.6	2.700	0.5	0.75	0.1	−0.6	−1.0	10.38	$2/3 \pi$
3	0.9	6.075	0.5	0.75	0.1	−0.6	−1.5	23.37	$2/3 \pi$

Moreover, in addition to the cases of desired motion, the research considered two cases regarding the structure of the control system, including the following:

- The control system with the wheels' controller only;
- The control system with the wheels' controller and pose controller.

Figure 8 shows the results obtained from the simulation of the robot's motion using the wheels' controller for the desired motion cases 1 and 3.

Based on the analysis of the motion paths of the robot's R point (Figure 8a), it can be seen that, after making a turn, the deviation of the actual motion path from the desired one increases, which is related to the occurrence of wheel slip during turning. This effect is

more visible when the robot moves at a higher velocity, but it is expected for each case that this error increases as the robot moves.

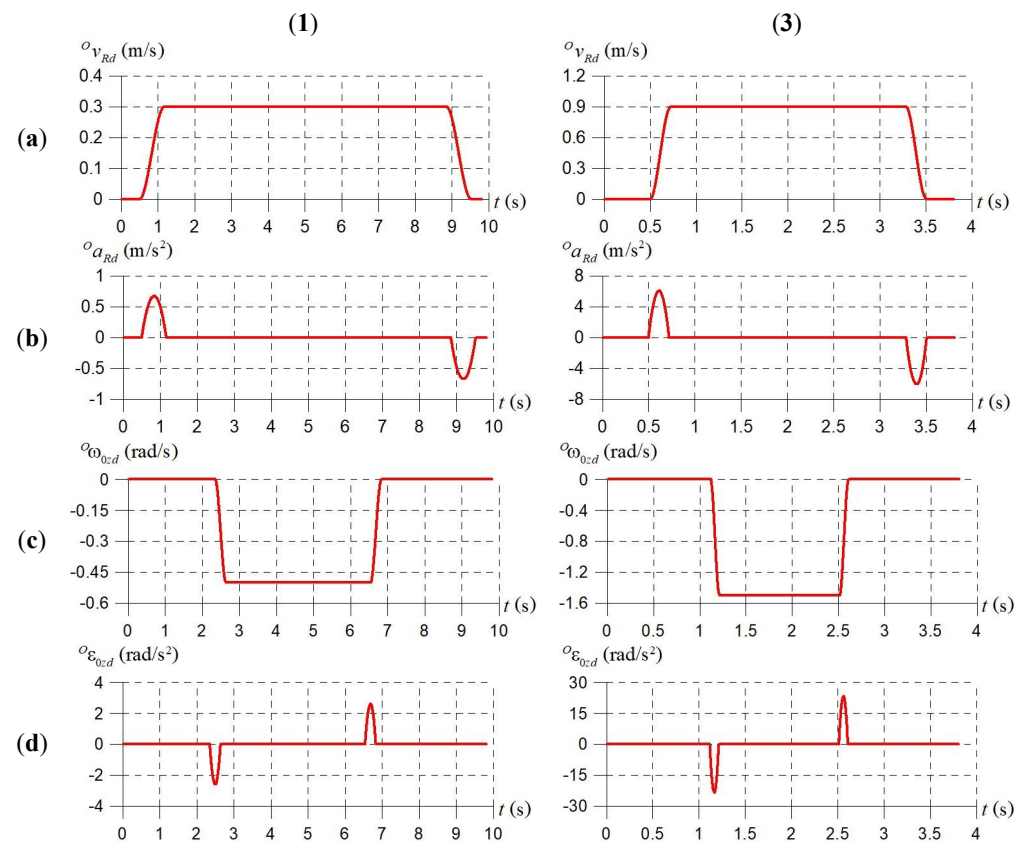


Figure 7. Time histories of the desired motion parameters for cases 1 and 3.

In addition to wheel slip, other factors influencing the tracking errors are the wheels' controller itself and the dynamic properties of the drive units, which are reflected in the time histories of the wheel spin angular velocities (Figure 8b). The largest errors in the angular velocities of wheel spin (Figure 8c) occur at the moments of the highest linear and angular accelerations, i.e., during acceleration and braking. Additionally, in the case of higher velocities, the process of reducing tracking errors takes much longer.

It should be noted that a simple linear controller is used as a wheels' controller, which is not able to compensate for the nonlinearities occurring in the robot dynamics model. This functionality can be achieved by using a controller based on the robot dynamics model, e.g., an adaptive or robust controller.

Moreover, due to the fact that the actual robot pose is not transmitted to the wheels' controller, it is unable to minimize the pose error, which is confirmed by the time histories of the pose errors (Figure 8d).

It can be seen that the highest values of driving torques occur when accelerating the robot and while turning (Figure 8e). In particular, the driving torques obtain high values all the time during turning, which is related to the lateral forces acting on the wheels for this type of kinematic structure of the robot. Therefore, the driving torques during turning have the greatest impact on energy consumption, as can be seen from the time histories of the electrical power for the robot's drive units (Figure 8f).

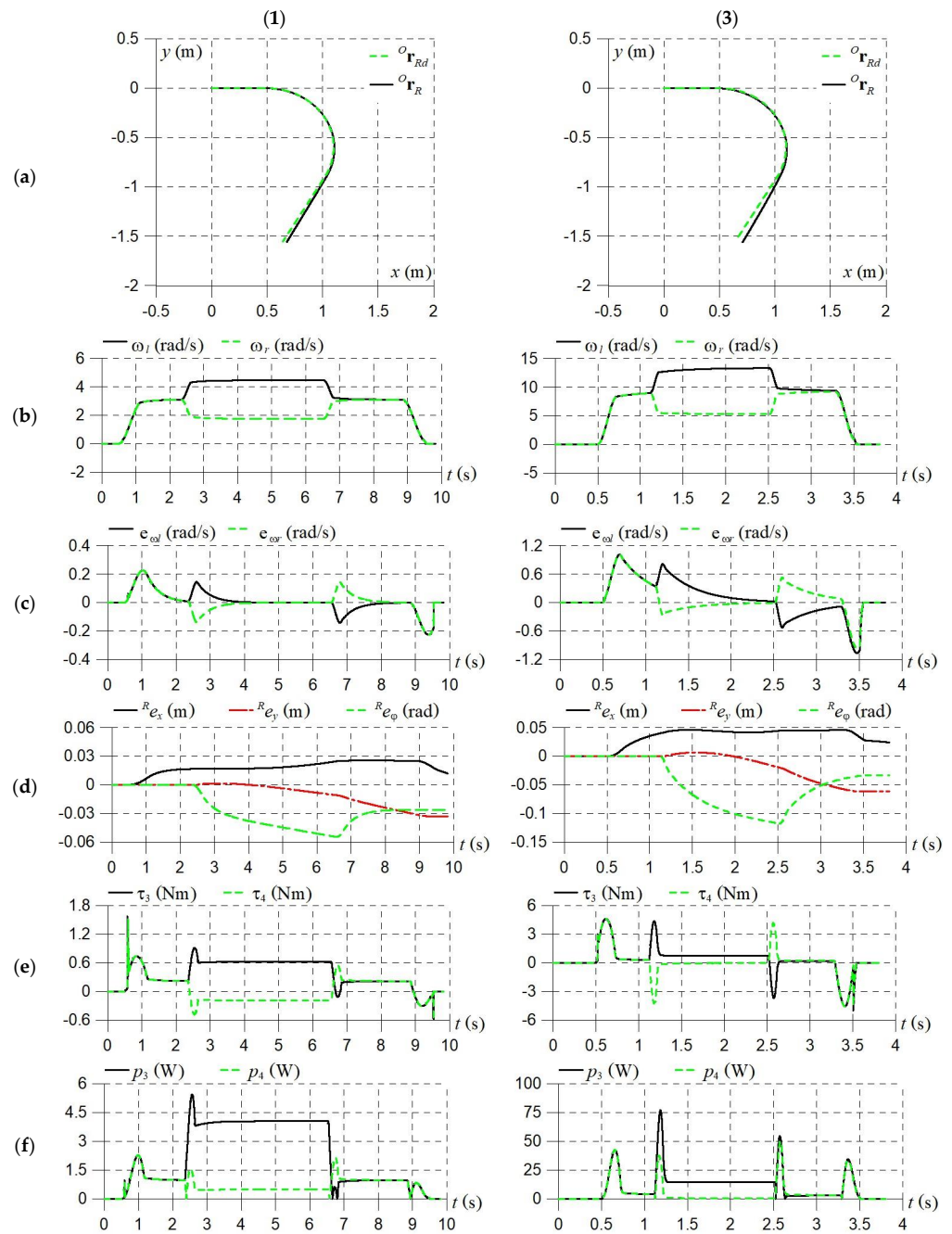


Figure 8. Robot motion simulation results with wheels' controller for cases 1 and 3.

Figure 9 shows the results of the simulation of the robot's movement using a wheels' controller and, additionally, a pose controller, for cases 1 and 3.

It can be noticed that, this time, the use of an additional pose controller allows for a significant increase in the accuracy of tracking the motion path, even for a higher velocity (Figure 9a). This is due to the relatively quick minimization of pose errors during the robot's movement (Figure 9d).

It is worth emphasizing that the generalized velocities of the robot included in the control vector for the pose controller, i.e., $u_s = [v_s, \omega_s]^T$, are also the desired velocities from the point of view of the wheels' controller (see Figure 6), which, based on relationships (29) and (30), converts them into the desired wheel spin angular velocities ω_d . Moreover, these generalized velocities differ slightly from the desired velocities resulting from the desired motion trajectories (compare Figure 7a,c with Figure 9b), which results from the fact that

the pose controller, while minimizing pose errors, must modify the original values of the robot's desired generalized velocities, i.e., ${}^O v_{Rd}$ (m/s) and ${}^O \omega_{0zd}$ (rad/s).

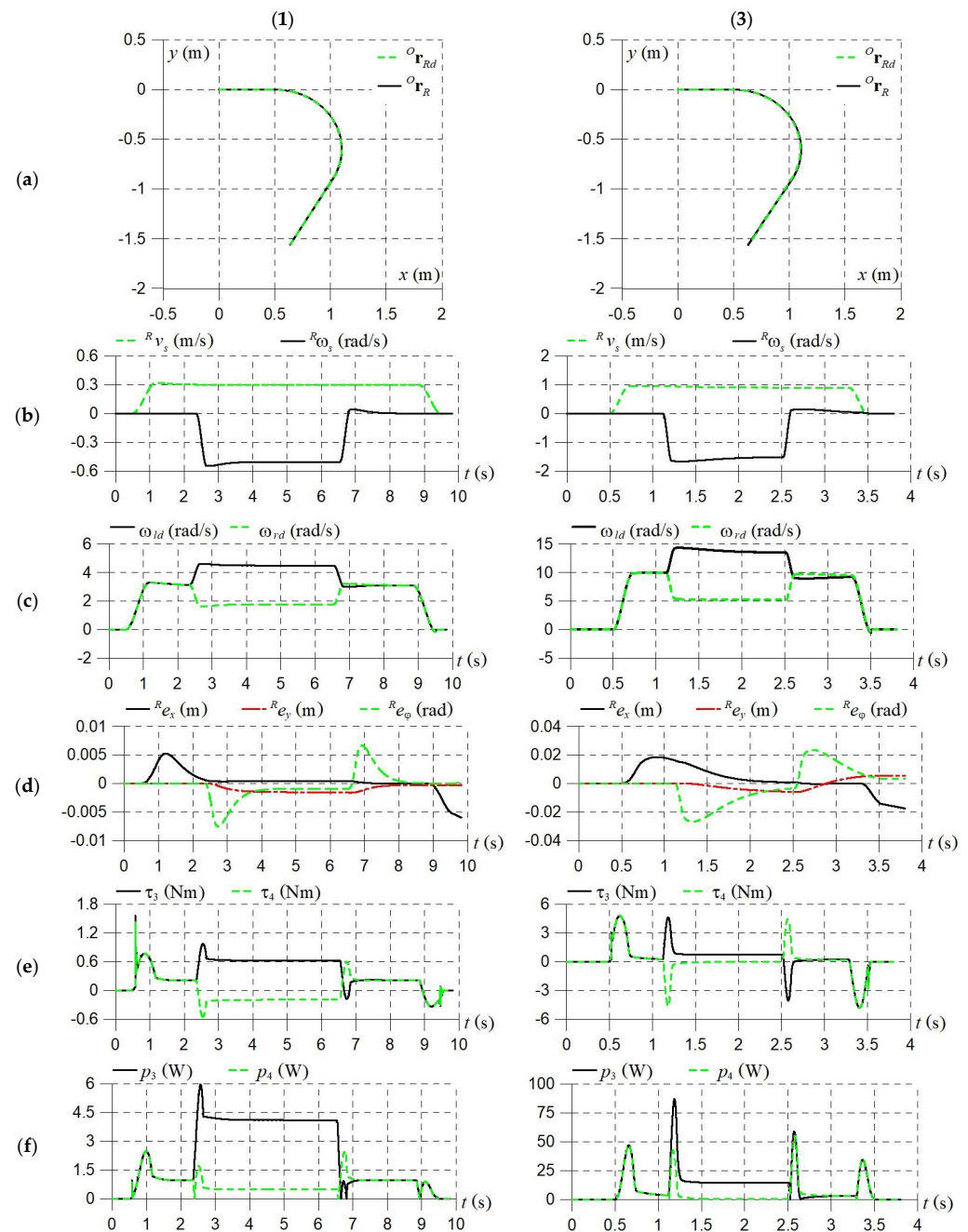


Figure 9. Robot motion simulation results with wheels' controller and pose controller for cases 1 and 3.

The obtained time histories of wheel spin angular velocities (Figure 9c) are the result of many factors, including the operation of the wheels' controller and the pose controller, the properties of the drive units, and the occurring wheel slips.

As for the issue of driving torques and electrical power for the robot's drive units, their time histories are similar to those obtained when only the wheels' controller is used (compare Figure 8e,f with Figure 9e,f). The key conclusion is, therefore, that adding a pose controller does not significantly affect the robot's energy efficiency.

In the second stage of simulation studies, the influence of the wheel track W (m) on the accuracy of tracking control and energy efficiency was analyzed. As a consequence,

the ratio of the wheel track to the distance of the front wheel axle from the rear wheel axle $W/(2L)$ (–) is also analyzed. The research results at this stage are presented for the control system with the wheels' controller only in Table 2, and for the control system with an added pose controller in Table 3.

Table 2. Quality indices obtained for the wheels' controller and various values of the wheel track W .

Case No.	W (m)	$W/(2L)$ (–)	$e_{\omega max}$ (rad/s)	E_{ω} (rad/s)	Re_{dmax} (m)	E_d (m)	$Re_{\phi max}$ (rad)	E_{ϕ} (rad)	p_{max} (W)	E_D (J)
1	0.364	0.728	0.2274	0.0705	0.0685	0.0373	4.540	2.695	5.77	16.11
	0.442	0.884	0.2274	0.0722	0.0506	0.0290	3.655	2.146	4.51	14.84
	0.520	1.040	0.2274	0.0742	0.0405	0.0244	3.139	1.832	3.85	14.08
	0.598	1.196	0.2274	0.0765	0.0343	0.0218	2.817	1.640	3.59	13.68
	0.676	1.352	0.2274	0.0788	0.0303	0.0201	2.606	1.517	3.44	13.54
2	0.364	0.728	0.6049	0.1964	0.0870	0.0509	6.498	3.648	35.47	22.40
	0.442	0.884	0.6058	0.2014	0.0671	0.0411	5.465	3.052	26.79	20.69
	0.520	1.040	0.6066	0.2071	0.0556	0.0358	4.865	2.712	20.90	19.61
	0.598	1.196	0.6074	0.2136	0.0486	0.0327	4.490	2.502	17.90	18.92
	0.676	1.352	0.6082	0.2207	0.0440	0.0308	4.243	2.366	16.43	18.53
3	0.364	0.728	1.0343	0.3468	0.1155	0.0686	8.909	4.788	87.86	33.35
	0.442	0.884	1.0382	0.3554	0.0897	0.0551	7.544	4.038	77.24	31.03
	0.520	1.040	1.0420	0.3655	0.0748	0.0477	6.753	3.608	64.26	29.50
	0.598	1.196	1.0459	0.3770	0.0656	0.0434	6.266	3.345	56.02	28.50
	0.676	1.352	1.0498	0.3896	0.0595	0.0408	5.945	3.173	51.50	27.87

Table 3. Quality indices obtained for the wheels' controller and pose controller for various values of the wheel track W .

Case No.	W (m)	$W/(2L)$ (–)	$e_{\omega max}$ (rad/s)	E_{ω} (rad/s)	Re_{dmax} (m)	E_d (m)	$Re_{\phi max}$ (deg)	E_{ϕ} (deg)	p_{max} (W)	E_D (J)
1	0.364	0.728	0.2443	0.0744	0.0083	0.0022	0.422	0.119	6.68	16.75
	0.442	0.884	0.2433	0.0761	0.0083	0.0022	0.421	0.124	5.06	15.32
	0.520	1.040	0.2434	0.0782	0.0083	0.0022	0.432	0.131	4.23	14.49
	0.598	1.196	0.2431	0.0805	0.0083	0.0022	0.448	0.140	3.92	14.05
	0.676	1.352	0.2439	0.0829	0.0083	0.0023	0.465	0.150	3.75	13.88
2	0.364	0.728	0.6355	0.2074	0.0127	0.0053	1.079	0.344	42.20	23.83
	0.442	0.884	0.6356	0.2123	0.0127	0.0053	0.938	0.349	30.48	21.77
	0.520	1.040	0.6358	0.2183	0.0128	0.0054	0.927	0.368	23.11	20.52
	0.598	1.196	0.6361	0.2251	0.0129	0.0055	0.948	0.392	19.58	19.75
	0.676	1.352	0.6363	0.2325	0.0129	0.0056	0.979	0.418	17.79	19.31
3	0.364	0.728	1.0791	0.3682	0.0185	0.0094	2.327	0.732	100.58	36.53
	0.442	0.884	1.0825	0.3765	0.0185	0.0094	1.739	0.686	88.09	33.36
	0.520	1.040	1.0862	0.3869	0.0185	0.0095	1.537	0.693	72.03	31.41
	0.598	1.196	1.0902	0.3989	0.0185	0.0096	1.516	0.723	61.77	30.20
	0.676	1.352	1.0944	0.4122	0.0185	0.0097	1.541	0.763	56.39	29.46

In both cases, the results are presented in the form of quality indices obtained for various wheel track values and for the analyzed cases of the desired motion trajectories.

In the case of the wheels' controller, changing the wheel track does not significantly affect the quality indices related to the wheel angular velocity errors, which reach slightly higher values for a larger wheel track (see Table 2). Increasing the wheel track, however, has a positive effect on the robot's pose errors, as can be seen from the quality indices associated with them—some of which decrease even twice. Increasing the wheel track also significantly reduces the maximum demand for electrical power of the drives, but the savings on the overall demand for electrical energy are not as large. This is due to the fact that the greatest demand for electrical power is when the robot is turning, when the greatest driving torques occur. Since turning is only one phase of the robot's movement, in general, it does not significantly affect the demand for electrical energy in the considered motion scenario.

Partially similar conclusions as in the case of the research for the wheels' controller can be reached for the control system with an additional pose controller (see Table 3). In particular, it can be seen that the quality indices related to the errors of the angular velocities of the wheels, as well as the electrical power and the electrical energy of the drives, reach

similar values (compare Tables 2 and 3). In turn, the quality indices related to pose errors change only slightly with the change in wheel track. However, when comparing the quality indices related to pose errors for the control system with the wheels' controller and with the additional pose controller, it can be noticed that, this time, they reach values lower by an order of magnitude. Therefore, the introduction of a pose controller significantly improves the accuracy of tracking control. However, it does not significantly affect the demand for electrical power and electrical energy of the drives.

6. Conclusions and Directions for Further Works

As part of this work, a six-wheeled skid-steered robot dynamics model was developed and two versions of the control system were formulated for it, i.e., one based on wheel control only and one with an additional pose controller. Based on the dynamics model and both versions of the control system, simulation studies were executed in the MATLAB/Simulink package.

The performed simulation studies allowed us to answer all the defined research questions. In particular, the following was found:

1. Increasing the wheel track significantly increases the accuracy of the robot's movement when using the wheels' controller only, which is reflected in the quality indices related to robot pose errors. However, changing the wheel track does not significantly affect the accuracy of movement if an additional pose controller is used. In both cases, this change does not have a significant impact on wheel velocity errors.
2. Increasing the wheel track has a positive impact on energy efficiency. The maximum values of electrical power necessary for the desired movement are then significantly reduced. The savings on the overall demand for electrical energy during the entire analyzed robot's motion are smaller. This is due to the fact that the maximum values of electrical power occur when the robot is turning, which is only one of the phases of the robot's movement.
3. The choice of control system structure has a significant impact on the accuracy of robot pose tracking. In particular, the use of a pose controller in addition to the wheels' controller allows the maximum pose errors to be reduced by an order of magnitude. Comparing the results for both analyzed control system structures, it can be seen that they do not have a significant impact on the wheel spin velocities' errors and energy efficiency. Therefore, adding a pose controller does not significantly affect the robot's energy efficiency. It can also be noted that errors related to the angular velocities of wheel spin are not significant from the point of view of the accuracy of tracking control when an additional pose controller is used in the control system.

It should be noted that the main limitation of the developed robot's dynamics model is that it only takes into account movement on horizontal non-deformable ground. In addition, it neglects the compliance of the tires and the resulting tilting of the robot due to the accelerations.

In turn, the wheels' controller is a linear controller, so it does not allow for the compensation of nonlinearities resulting from the robot's dynamics model, ensuring resistance to changing robot operating conditions (e.g., changes in the properties of the ground or the mass of the transported load).

Directions for further research work may cover the following:

- The development of a dynamics model of the analyzed robot for the case of movement on unpaved ground, such as sand, gravel, etc., in which the model of the wheel interaction with the soil should be additionally taken into account;
- An analysis of the influence of the robot's geometric parameters on the movement accuracy and energy efficiency in the case of robot movement on unpaved ground;
- The development of more advanced robot motion control systems based on a dynamics model, for instance, using robust or adaptive control, and a comparison of the effectiveness of various control strategies;

- The development of a motion control system for the analyzed robot, minimizing energy consumption during robot movement through the optimal control of individual wheels, taking into consideration various types of the ground.

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Nomenclature

$\{O\}$	fixed (stationary) co-ordinate system
$\{R\}$	moving co-ordinate system associated with the robot
L_f, L_b	distances of the front and back axles of the robot from the middle axle, respectively
W	track width
r_i	geometric (unloaded) radius of the wheel
θ_i, ω_i	spin angle and angular velocity of the i th wheel, respectively
${}^O q, {}^O q_d$	robot's actual and desired pose vectors in $\{O\}$ co-ordinate system, respectively
${}^O \dot{q}, {}^R \dot{q}$	robot's generalized velocity vectors in $\{O\}$ and $\{R\}$ co-ordinate systems, respectively
${}^O \dot{q}_d$	robot's desired generalized velocity vector in $\{O\}$ co-ordinate system
${}^R r_{Ai}$	position vector of point A_i of the i th wheel in $\{R\}$ co-ordinate system
${}^R r_{CM}$	position vector of the robot's mass center in $\{R\}$ co-ordinate system
${}^R v_R$	linear velocity vector of point R of the robot in $\{R\}$ co-ordinate system
${}^R \omega_0$	angular velocity vector of the robot's mobile platform in $\{R\}$ co-ordinate system
${}^R v_{Ai}$	velocity vector of point A_i of the i th wheel in $\{R\}$ co-ordinate system
${}^R v_{Si}$	slip velocity vector for the i th wheel in $\{R\}$ co-ordinate system
v_d	vector of desired generalized velocities of the robot, including desired linear velocity v_{Rd} and desired angular velocity ${}^R \omega_{0zd}$
ω_d	vector of desired angular velocities of spin of wheels
λ_i, α_i	longitudinal slip ratio and lateral slip angle for the i th wheel, respectively
μ_{ix}, μ_{iy}	tire adhesion coefficients in longitudinal and lateral direction, respectively
${}^R g$	gravitational acceleration vector in $\{R\}$ co-ordinate system
${}^R F_{Ai}$	ground reaction force vector acting on the i th wheel in $\{R\}$ co-ordinate system
${}^R T_{Ai}$	moment of force vector resulting from the interaction of the i th wheel with the ground in $\{R\}$ co-ordinate system
${}^R G$	gravity force vector of the robot in $\{R\}$ co-ordinate system
I_R	inertia tensor of the robot
${}^R a_{CM}$	acceleration vector of the robot's center of mass in $\{R\}$ co-ordinate system
${}^R \varepsilon_0$	angular acceleration vector of the robot's mobile platform in $\{R\}$ co-ordinate system
U_i, I_i	motor voltage input and rotor current, respectively
L_d, R_d	inductance and resistance of the rotor, respectively
k_e, k_m	electromotive force constant and motor torque coefficient, respectively
n_d, η_d	gear ratio and efficiency factor of the transmission system, respectively
n_i	motor rotational speed for the i th wheel
τ_i	driving torque for the i th wheel
p_i, E_{Di}	electric power and electric energy for the i th wheel drive unit
e_θ	error vector of the rotation angles of wheels
e_ω	error vector of the angular velocities of spin of wheels
${}^O q_e, {}^R q_e$	robot's pose error vectors in $\{O\}$ and $\{R\}$ co-ordinate systems, respectively
${}^O e_d = {}^R e_d$	distance of the point R of the robot from its actual to the desired position
u_s	generalized velocity vector for wheels' controller
u	control vector for robot's drives
k_ω, k_θ	wheels' controller gains
k_x, k_y, k_φ	gains of the pose controller, responsible for eliminating pose errors

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