

Article

Revealing Coupled Periodicities in Sunspot Time Series Using Bispectrum—An Inverse Problem

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Abstract: Sunspot daily time series have been available for almost two centuries providing vast and complicated information about the behavior of our star and especially the interaction of the motion of the planets and other possible interstellar phenomena and their effects on the surface of the Sun. The main result obtained from the sunspot time series analysis is the imprint of various periodicities, such as the planets' orbital periods and the planetary synodic periods on the sunspots signature. A detailed spectrum representation is achieved by means of a periodogram and a virtual extension of the time length segments with zeroed samples for longer representations. Furthermore, the dependence or coupling of these periodicities is explored by means of a bispectrum. We establish the exact interdependencies of the periodic phenomena on the sunspot time series. Specific couplings are explored that are proved to be the key issues for the coupled periodicities on the sunspot time series. In this work, contrary to what has been presented in the literature, all periodic phenomena are limited within the time period of an 11-year cycle as well as the periodicities of the orbits of the planets. The main findings are the observed strong coupling of the Mercury, Venus, and Mars periodicities, as well as synodic periodicities with all other periodicities that appear on the sunspot series. Simultaneously, the rotation of the Sun around itself (25.6 to 33.5 days) provides an extensive coupling of all recorded periodicities. Finally, there is strong evidence of the existence of a quadratic mechanism, which couples all the recorded periodicities, but in such a way that only frequency pairs that sum up to specific periods are coupled. The justification for this kind of coupling is left open to the scientific community.



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Keywords: sunspot series; planetary orbital periods; synodic periods; coupled periodicities; bispectral content; 11-year cycle; nonlinearities

1. Introduction

The Sun undergoes cyclical changes, and one well-known cycle is the 11-year cycle, as documented by Gnevyshev [1]. This cycle is closely linked to the frequency of sunspots that manifest on the Sun's surface. During the solar maximum, which marks the peak of each cycle, sunspots become more prevalent, while during the solar minimum, at the cycle's end (and the beginning of a new one), sunspots are scarce and can even vanish entirely. The recognition of this cycle dates back to the late 18th century when the Danish astronomer Christian Horrebrow observed a recurring pattern of sunspot numbers fluctuating over several years. Since then, the Sun has completed 24 full cycles, and the 25th cycle, our current one, commenced in December 2019, with the predicted maximum expected in July 2025. The 26th cycle is projected to start in 2030. Although the concept of the solar cycle is straightforward, the underlying mechanisms that propel it are intricate and not fully comprehended. However, at its core lies the Sun's magnetic field.

The Sun rotates around its axis, but owing to its lack of a solid surface (as well the convective zone up to the tachocline), distinct regions rotate at varying speeds. Specifically, the equator experiences a faster rotation (25.6 days) compared to the poles (33.5 days). This

rotational difference results in the Sun's magnetic field becoming twisted, causing it to penetrate the solar surface and give rise to sunspots. The Sun's magnetic field activity correlates with the appearance of sunspots—the more pronounced the magnetic field, the more sunspots manifest. Therefore, the solar cycle essentially coincides with the cycle of the Sun's magnetic field. Approximately every 11 years, the magnetic field experiences a quiet phase, known as the solar minimum. In the middle of this period, the solar maximum occurs [1], which corresponds to the most turbulent phase of the magnetic field. The cycle concludes, initiating a new one with another solar minimum. The generation of the magnetic field itself is attributed to the movement of charged particles (plasma) at the convective part of the Sun. As these particles interact, they create electrical currents, generating magnetic fields. However, the precise causes of these fluctuations in the magnetic field remain somewhat enigmatic. Despite the Sun being 93 million miles (150 million kilometers) away, it exerts a noticeable influence on Earth, and alterations in solar activity can impact our planet.

Recent endeavors to model solar total and spectral irradiance values across time spans ranging from days to centuries and beyond are detailed in the work by Krivova et al. [2]. Extending these models further into the distant past is crucial for providing inputs to climate simulations. Initial strides have also been made in reconstructing solar total and spectral irradiance values over millennia. Valentina Zharkova et al. [3] conducted a comprehensive exploration of the periodicities of solar activity and radiation, reaching back centuries and millennia. This research incorporates the data from various sources, including the Holocene data of solar irradiance derived from the abundance of the ^{14}C isotope.

Ilya Usoskin [4] provides a comprehensive review of the current understanding of solar activity's long-term behavior on a multi-millennial scale, reconstructed using the indirect proxy method. Jacob Oloketuyi et al. [5] conducted an analysis of daily data from January 1995 to December 2018, exploring the periodic behavior and the relationship between sunspot numbers, cosmic ray intensity, and solar wind speed. Weizheng Qu et al. [6] presented arithmetic expressions for the quasi-11-year cycle, 110-year century cycle of relative sunspot numbers, and quasi-22-year cycle of solar magnetic field polarity through a spectrum analysis. Furthermore, M. G. Ogurtsov et al. [7] confirmed the distinguishability of two long-term variations in solar activity, namely the Gleissberg and Suess cycles, at least over the last millennium. The results indicate that the century-type cycle of Gleissberg exhibits a double structure with periodicities of 50–80 and 90–140 years, while the Suess cycle has a less complex structure with a variation period of 170–260 years.

Abreu et al. [8] reported correlations between direct solar activity indices and planetary configurations. While no successful physical mechanism was proposed to explain these correlations, the potential link between planetary motion and solar activity has been largely overlooked. Despite energy considerations indicating that planets cannot directly cause solar activity, the question of whether planets can perturb the solar dynamo operation remains open. The authors used a 9400-year solar activity reconstruction from cosmogenic radionuclides to explore this hypothesis. They found a remarkable agreement between long-term cycles in solar activity proxies and periodicities in planetary torque. Some periodicities remained phase-locked over the 9400-year period. This led to the hypothesis that long-term solar magnetic activity is modulated by planetary effects, with potential implications for solar physics and the solar–terrestrial connection. Galactic cosmic rays produce cosmogenic radionuclides, such as ^{10}Be and ^{14}C , when entering Earth's atmosphere. The solar magnetic field in the heliosphere modulates cosmic rays, influencing the production rate of these radionuclides. Various periodicities, ranging up to 2200 years, have been reported based on radionuclides [8]. The idea that planetary motions can influence solar activity has a long history, dating back to the late 1850s with Rudolf Wolf. While energy considerations rule out planets as the direct cause of solar activity, the question of whether planets can affect the solar dynamo operation remains an open area of investigation [8].

However, in [9], Cauquoin et al. found no support for the hypothesis of a planetary influence on solar activity and raised the question of whether the centennial periodicities of

solar activity observed during the Holocene were representative of solar activity variability in general. Their results do suggest, however, that it is important to test other proxy records of solar variability for periods other than the Holocene. If the observed periodicities in variability are not constant and stationary over time, this rules out any regular forcing factor, such as planetary positions, having a significant influence on solar activity. In [10], the authors used a more conservative non-parametric random-phase method and found that the long-period coherence between planetary torque and heliospheric modulation potential became insignificant. Thus, they concluded that the considered hypothesis of planetary tidal influence on solar activity was not based on solid ground. Javaraiah [11] suggested that there could be an influence of some specific configurations of the giant planets in the origin of the 12-year and 50-year periodicities of the north–south asymmetry on solar activity. Cameron and Schussler [12] showed that the statistical test presented in [8] to demonstrate a causal link between the planetary orbits and the level of solar activity was conceptionally flawed and biased. Furthermore, their execution of the test contained severe technical errors. A corrected test revealed that the period coincidences reported by [8] were statistically insignificant.

Hathaway, in [13], reviewing the 11-year cycle of solar activity, commented that it was characterized by the rise and fall in the numbers and surface area of sunspots. Furthermore, a number of other solar activity indicators also vary in association with the sunspots, including the 10.7 cm radio flux, the total solar irradiance, the magnetic field, flares and coronal mass ejections, geomagnetic activity, galactic cosmic ray fluxes, and radioisotopes in tree rings and ice cores. The study by Okhlopkov [14] used the evolution of an index that characterized the relative position of the planets Venus, Earth, and Jupiter. Their linear configurations had an 11-year cycle. The index was compared with solar activity, and it was shown that the average 11-year periodicity in the index and in solar activity over a 1000-year time interval coincided to two decimal places. Finally, in [15], the authors applied the DCCA correlogram to study time-series memories (on time scale). The authors performed an analysis of the sunspot, and they identified the solar cycle as $n = 132$ months (11 years). However, this value disappears for large time scales, because of the solar phenomenon.

The studies by Vinay Kumar et al. [16] as well as by Maxim Ogurtsov et al. [17] and Potrzeba-Macrina and Zurbenko [18] highlight the future challenges and research directions, emphasizing how regional-scale atmospheric circulation can aid in understanding solar-induced climate variability. Furthermore, Wei Lu et al. [19] conducted a study to explore the potential connection between the summer temperature distribution over Eurasian land and solar activity, while Thomas and Abraham [20] focused on investigating the periodicities of rainfall over Kerala, India, and sunspot numbers, and explored the possible associations between these factors.

In the present work, we explicitly demonstrate that, using the power spectrum, the planetary periodicities as well as the synodic periodicities are mirrored by the sunspot activities on the Sun's surface. In a fine and detailed spectrum calculation procedure, it is possible to have all planetary periodic phenomena present on the surface of the Sun. The contribution of the present work is to demonstrate that simple periodicities obtained from planets' orbits are clearly present on the sunspots time series, probably manifesting the tidal effect (even too weak) of the planets on the Sun's plasma. Furthermore, coupled periodicities are also detected on the Sun's surface. The bispectrum [21,22] is the best tool for this purpose. Periodicities (e.g., $(f_1 + f_2)$) that are created based on the coupling of two distinct frequencies, f_1 and f_2 , do not appear in the bispectrum. We have only a peak at the position (f_1, f_2) without any indication at the summation of the frequencies. The amazing finding of our bispectral results is that a quadratic coupling mechanism bonds all the periodicities, which add up to specific summations. All the coupled periodicities on the sunspot time series are extensively discussed.

This work is organized as follows. In Section 2, the employed data are analyzed and, simultaneously, the planetary periods and the synodic periods are presented according to

the well-known astronomical data. The basic concepts of the bispectrum are presented in Section 3. The periodicities as extracted from the sunspot data are revealed in Section 4 using the power spectrum. The coupled periodicities are demonstrated using the bispectral representation in Section 5. Finally, the conclusions are drawn in Section 6.

2. Sunspot SILSO Data and Planetary Orbits

Sunspot SILSO data (described in detail in Section 2.1) and planetary orbit data (described in detail in Section 2.2) are analyzed to present the existence of the signatures of the periods of the planets on the Sun's surface. Then the bispectrum is applied (details in Section 3) to the derived periodicities in order to prove that the coupling of these periodicities is a reality.

2.1. Daily Total Sunspot Data—SILSO

The daily total sunspot data were obtained for more than two centuries, thus providing an important tool to work on our star and planetary system [23]. The date from which the sunspot data were strictly recorded on a daily basis was the 1 January 1818. The daily total sunspot number derived was obtained by the formula $R = N_s + 10 \times N_g$, with N_s representing the number of spots and N_g the number of groups counted over the entire solar disk. Before 1818, the data acquisition was not systematic and the records were sparse and not taken into consideration. The Solar Influences Data Analysis Center is a World Data Center for the production, preservation, and dissemination of the international sunspot number (<https://www.sidc.be/SILSO/home> (accessed on 1 September 2023)), SILSO, Sunspot Index and Long-Term Solar Observations. SILSO data are under the CC BY-NC4.0 license, which means that you can:

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In this study, the sunspot data series provided publicly, along with the Higher-Order Spectral Analysis Toolbox [24], were employed. These data are known to have an approximate 11-year cycle. This is demonstrated in Figure 1, where the 11-year cycle is prominent and, simultaneously, a square pulse of a 4000 sample semi-period is shown to fit this 11-year period.

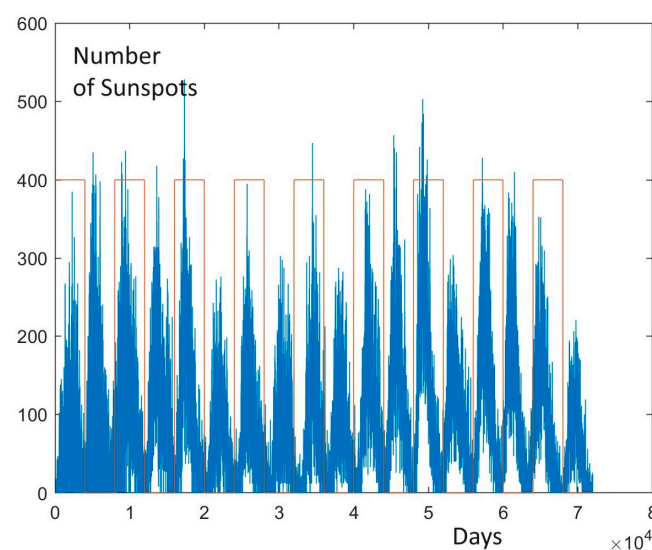


Figure 1. The 11-year cycle of sunspot data starting from 1818. A pulse with a 4000 sample semi-period (10.95 years) seems to fit the data perfectly. The horizontal axis represents days.

Before any kind of processing, the data were filtered with an 11-point median filter. This was necessary in order to eliminate unwanted outliers and obtain, later on, a fine spectral analysis of the sunspot series. The result is demonstrated in Figure 2.

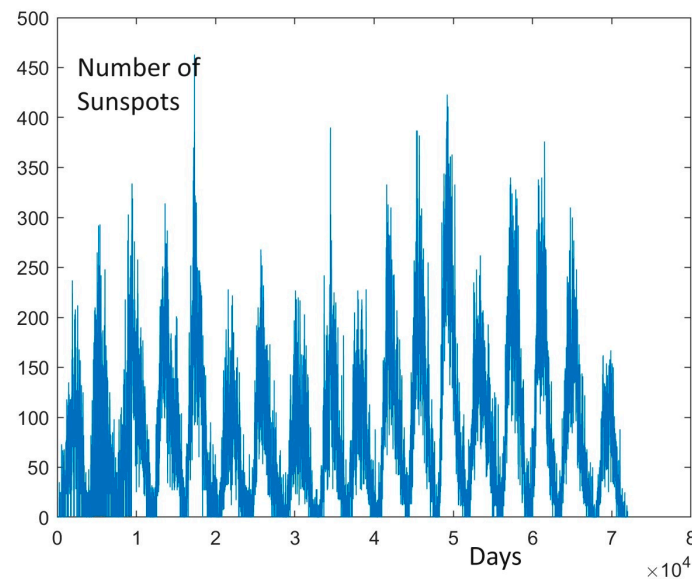


Figure 2. The 11-years cycle of sunspot data having been processed with a median filter length of 11 days. The horizontal axis represents days.

2.2. Planetary Orbital and Synodic Periods

In order to explain the periodicities that appeared in the spectrum of the sunspot time series, it was necessary to know exactly the orbital periods of the planets as well as the synodic periods of most pairs of planets, especially those that were close to the Sun and provided harmonics longer (smaller periods) than the eleven-year circle (~4000 days). The orbital periods of the planets are described by Kepler's laws and can be found in any astronomical handbook. Additionally, the synodic periods can also be found in astronomical handbooks and can be evaluated by the following relationship:

$$S/P_1 - S/P_2 = 1 \quad (1)$$

where P_1 is the period of one planet, P_2 is the (longer) period of the second planet moving around the Sun, and S is the synodic period of the two planets. The synodic period of two planets is considered the time required for the centers of the two planets to be in alignment with the center of the Sun for two consecutive times. Table 1 presents the orbital and synodic periods, the values of which are at most of the order of the eleven-year circle. In the third column in Table 1 is the position of the corresponding frequency (the inverse of the planet or synodic periods) along the power spectrum as it is depicted in the Figures in Section 4.

Table 1. Orbital and synodic periods of the planets in days. As synodic period of two planets is considered the time required for the centers of the two planets to be in alignment with the center of the Sun for two consecutive times.

#	Planets' Periods Planets' Synodic Periods	Orbital or Synodic Periods: P	Position in the 32,000 Harmonics Spectrum, Including Position 1 the DC (Position 32,000/P)
1	Uranus	30,589	2.04
2	Saturn	10,747	3.96
3	Jupiter-Saturn	7254	5.41

Table 1. Cont.

#	Planets' Periods Planets' Synodic Periods	Orbital or Synodic Periods: P	Position in the 32,000 Harmonics Spectrum, Including Position 1 the DC (Position 32,000/P)
4	Jupiter–Uranus	5045	7.34
5	Jupiter–Neptune	4669	7.85
6	Jupiter	4331	8.4
7	Jupiter–Mars	816.52	41.2
8	Mars–Earth	780	42.0
9	Mars–Saturn	733.92	44.6
10	Mars–Uranus	702.78	46.5
11	Mars–Neptune	694.98	47.0
12	Mars	687	47.6
13	Earth–Venus	584	55.8
14	Earth–Jupiter	399	81.2
15	Earth–Saturn	378	85.6
16	Earth–Uranus	370	87.4
17	Earth–Neptune	367	88.2
18	Earth	365.25	88.6
19	Venus–Mars	333.91	96.8
20	Venus–Jupiter	237	136
21	Venus–Saturn	229.5	140.4
22	Venus–Uranus	226.36	142.4
23	Venus–Neptune	225.55	142.8
24	Venus	224.7	143.4
25	Mercury–Venus	141.46	227.2
26	Mercury–Earth	116	276.9
27	Mercury–Mars	100.89	318.2
28	Mercury–Jupiter	88.95	360.7
29	Mercury	87.9691	364.8
30	Sun rotation period	25.6–33.5	956.2–1251

3. Basics of the Bispectrum

The bispectrum, obtained by applying a two-dimensional Fourier Transform to the third-order autocorrelation function, $R_{xxx}(\tau_1, \tau_2)$, proved to be effective in identifying and measuring the Quadratic Phase Coupling (QPC) [21,25,26]. Concurrently, the bispectrum serves to elucidate the statistical associations among various frequency components within a signal. Its formal definition is as follows:

$$B(f_1, f_2) = E\{X(f_1)X(f_2)X^*(f_1 + f_2)\} \quad (2)$$

For the bispectrum to manifest non-zero values at frequencies (f_1, f_2) , it necessitated that the Fourier Transforms at the individual frequency components of f_1 , f_2 , and $f_1 + f_2$ were non-zero. Additionally, these three spectral components must exhibit correlations. It is essential to highlight that, following the expectation process, the bispectrum becomes zero due to a phase randomization. However, when the phases are coupled, this does not hold true. In contrast to the power spectrum, the bispectrum is a complex component, even when the signal is real-valued.

The symmetric regions of the bispectrum are illustrated in Figure 3 [21,25,26]. Consequently, the analysis typically focused on a single, non-redundant region. In this paper, the notation

$B(f_1, f_2)$ was employed to represent the bispectrum in the non-redundant triangular region shown in blue in Figure 3. It is defined in $\{(f_1, f_2) : 0 \leq f_2 \leq f_1 \leq f_e/2; f_1 + f_2 \leq f_e/2\}$, where f_e is the sampling frequency. Further details about the computational regions can be found in [21,25].

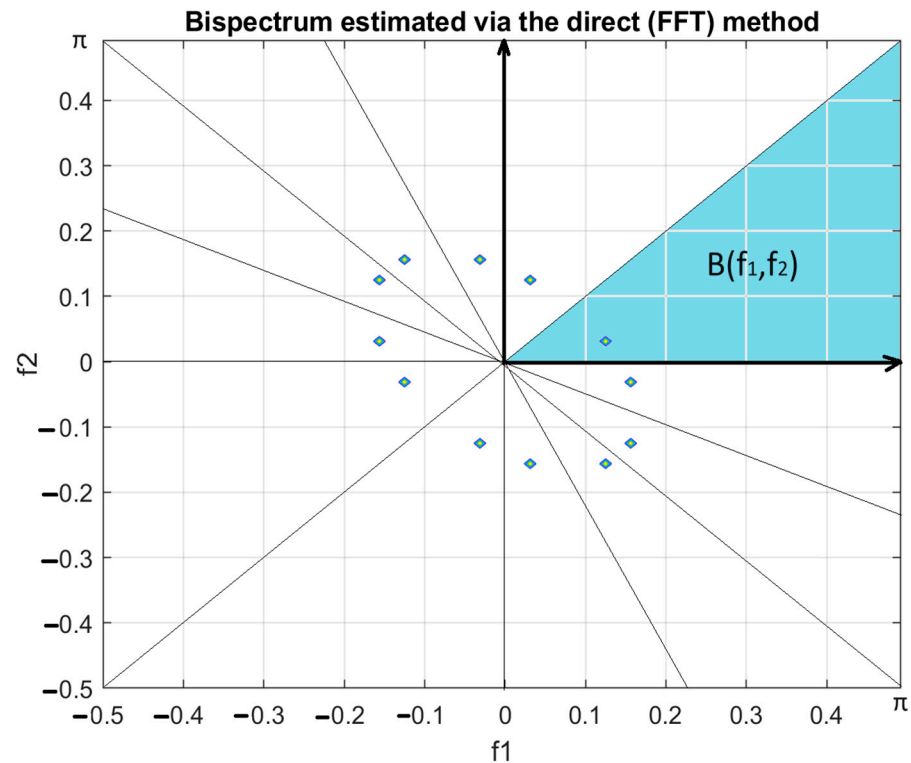


Figure 3. The symmetrical structure of the bispectrum. The blue region is the space that we have to investigate in order to recognize the frequency coordinates that contribute to the formation of coupled components. Frequencies in both axes are normalized in the sampling frequency F_s or 2π .

The bispectrum proves to be a valuable tool for effectively addressing practical problems, as exemplified by [21,27–29]:

- In the case of a stationary zero-mean Gaussian process, $x(n)$ its bispectrum consistently remains zero.
- Unlike the power spectrum, which discards phase information, the bispectrum retains it.

The bispectrum, serving as a quantified expression of Higher-Order Statistics (HOSs), represents the Fourier Transform of the third-order cumulant or moment. Nonlinearity influences these cumulants, and the bispectrum captures such effects. The estimated bispectrum $\hat{B}(f_1, f_2)$ is defined as:

$$\hat{B}(f_1, f_2) = \frac{1}{M} \sum_{k=1}^M X_k(f_1) X_k(f_2) X_k^*(f_1 + f_2) \approx E \left\{ X(f_1) X(f_2) X^*(f_1 + f_2) \right\} \quad (3)$$

The expectation operation holds significant importance in this context, particularly for the QPC detection. It implies “ensemble averaging” for an estimate: when phases are random, the bispectrum tends towards zero, whereas if phases are coupled, it does not.

Case of frequency and phase summation:

Consider three signals, $x_1 = \cos(\lambda_1 x + \varphi_1)$, $x_2 = \cos(\lambda_2 x + \varphi_2)$, and $x_3 = \cos((\lambda_1 + \lambda_2)x + (\varphi_1 + \varphi_2))$, where $\lambda_1 < \lambda_2$. The phase of signal x_3 arises from the sum of the phases of the signals x_1 and x_2 , indicating an expected coupling edge at the coordinates (λ_1, λ_2) of the bispectrum. For instance, if $\lambda_1 = 4$ and $\lambda_2 = 16$, coupled components can

appear at $\lambda_3 = 20$ (see Figure 4a). The coupling edge is anticipated at coordinates (4, 16) in the bispectrum, as is demonstrated in Figure 4b in the red region.

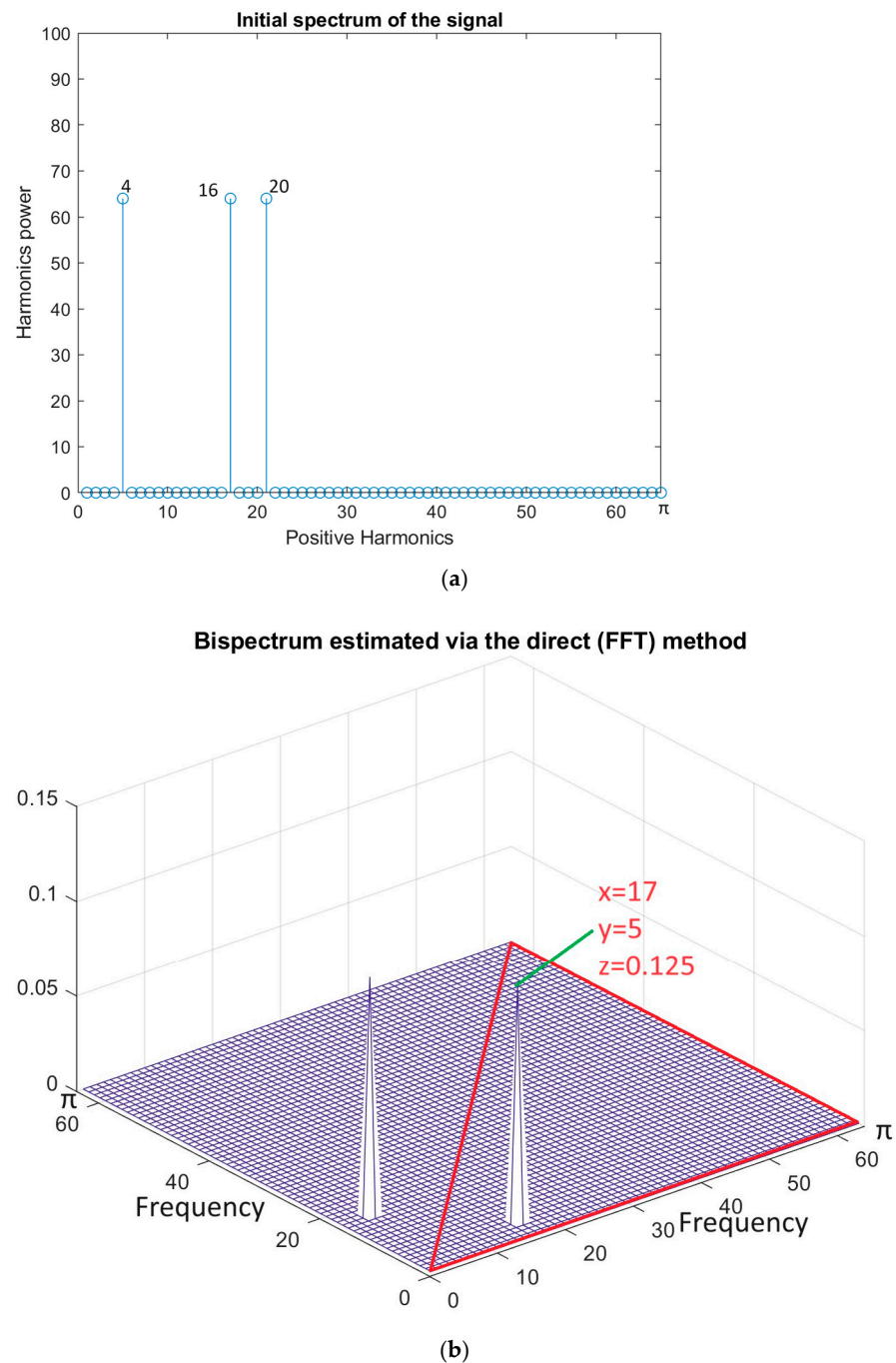


Figure 4. (a). The three harmonics that appear at positions 4, 16, and 20 (dc is at position 1). From the power spectrum, it is not possible to distinguish that the 20th harmonic is the result of coupling the other two. The 64th harmonic is at position 65, which coincides with half of the sampling frequency, $F_s/2$ or π . (b). Since (1,1) is the position of dc, the coupling edge is present at coordinates (5, 17), i.e., for coupled frequencies 4 and 16. The created 20th harmonic does not present any footprint in the region of the bispectrum, which is enclosed in the red line. On both axes, the 64th harmonic is at position 65, which coincides with half of the sampling frequency, $F_s/2$ or π .

Case of frequency and phase subtraction:

Consider three signals, $x_1 = \cos(\lambda_1 x + \varphi_1)$, $x_2 = \cos(\lambda_2 x + \varphi_2)$, and $x_3 = \cos((\lambda_2 - \lambda_1)x + (\varphi_2 - \varphi_1))$, where $\lambda_1 < \lambda_2$. The phase of the signal x_3 results from

the subtraction of the phases of the signals x_1 and x_2 , suggesting that a coupling edge is anticipated at the coordinates $(\lambda_1, \lambda_2 - \lambda_1)$. For example, if we have $\lambda_1 = 4$ and $\lambda_2 = 16$, a coupling edge is expected at coordinates $(4, 12)$. See Figure 5.

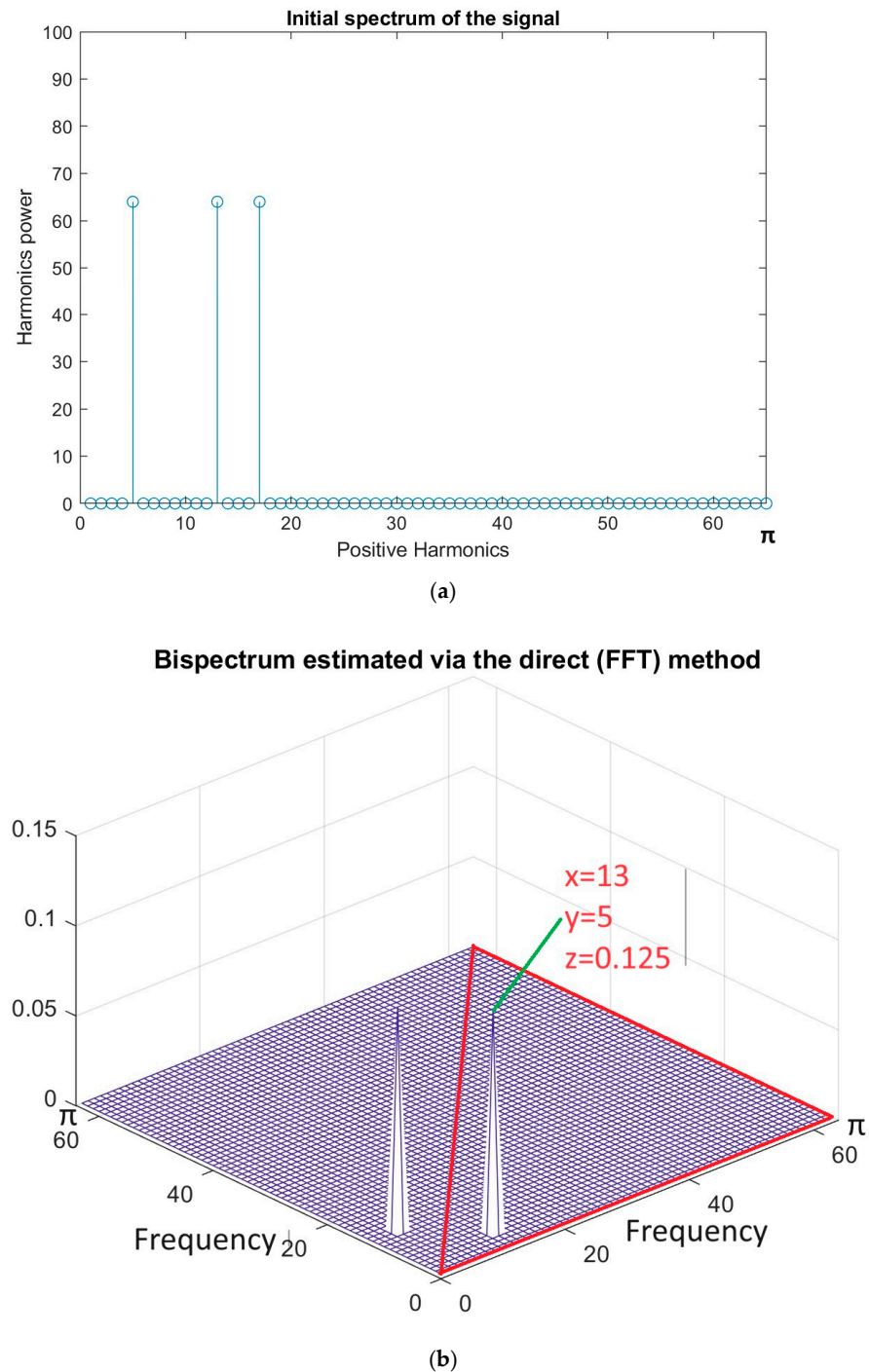


Figure 5. (a). The three harmonics that appear at positions 4, 12, and 16 (dc is at position 1). From the power spectrum, it is not possible to distinguish the coupled harmonic. The 64th harmonic is at position 65, which coincides with half of the sampling frequency, $F_s/2$ or π . (b). Since (1,1) is the position of dc, the coupling edge is present at coordinates $(5, 13)$, i.e., for coupled frequencies 4 and 12. The original 16th harmonic does not present any footprint in the region of the bispectrum, which is enclosed in the red line. On both axes, the 64th harmonic is at position 65, which coincides with half of the sampling frequency, $F_s/2$ or π .

It is worth noting that, in the case of a subtraction, the result aligns with the case of adding signals x_1 and x_3 . In other words, if initially we had the signals $x_1 = \cos(\lambda_1 x + \varphi_1)$, and $x_3 = \cos((\lambda_1 + \lambda_2)x + (\varphi_1 + \varphi_2))$, where $\lambda_1 < \lambda_2$, the phase of the signal $x_2 = \cos(\lambda_2 x + \varphi_2)$ can be derived by adding the phases of the signals x_1 and x_3 ; then, a coupling edge is expected at the coordinates $(\lambda_1, \lambda_2 - \lambda_1)$.

Quadratic Phase Coupling

Quadratic phase coupling is the most well-understood [21,25], non-linear procedure that creates a periodicity as the summation of two other harmonics. Although all three harmonic components appear in the power spectrum, in the bispectrum, there is only a peak at the coordinates determined by the two initial harmonics. This is explained in the subsequent sections in the paper:

There must exist a significant quadratic coupling nonlinearity, which applies to almost all periodicities that are found in the sunspot time series in such a way that intensive coupled harmonics appear only for specific summations, namely $f_1 + f_2 = f_{140}$, $f_1 + f_2 = f_{340}$ and $f_1 + f_2 = f_{1200}$.

These couplings become obvious from the intense spikes that appear in the diagonal ellipses in the Figures in the Section 5.

In summary, for three signals, where one has a phase resulting from the addition or subtraction of the phases of the other two, coupling edges appear at the coordinates corresponding to the two lowest frequencies of these signals.

In the field of signal processing, the phenomenon of coupling edges, as described in the context of signal addition and subtraction, highlights the complex relationships between signal phases. When the signals x_1 and x_2 are combined to yield x_3 , the coupling edge appears at specific coordinates, revealing a harmonic interaction between the frequencies λ_1 and λ_2 . This coupling effect, visualized in Figure 4b, provides information on the synchronization of the signal phases.

Conversely, in the signal subtraction scenario, where its phase, φ_3 , results from the differentiated interaction between φ_1 and φ_2 , the resulting coupling edge is realized in coordinates determined by $(\lambda_1, \lambda_2 - \lambda_1)$. This complex dance of frequencies, exemplified in Figure 5, shows the delicate balance required to de-phase the signal to yield a coherent result. Moreover, the interesting surprise is that, in the case of signal subtractions, the results mirror those of signal additions, which highlights the symmetric nature of these signal functions. This symmetric behavior, represented by coupling spikes at fixed coordinates, speaks to the underlying mathematical elegance that governs signal interactions. It is important to note that these observations extend to three-signal scenarios, where the phase relationships are more complex. In cases where the phase of one signal is obtained by adding or subtracting the phases of two other signals, the coupling spikes appear fixed at coordinates corresponding to the two smaller phases. This generalization adds a level of universality to the coupling phenomenon, demonstrating its applicability to various signal processing scenarios.

In conclusion, exploring the coupling edges in signal processing reveals a rich fabric of relationships where the addition or subtraction of signal phases orchestrates complex patterns at specific frequency coordinates. These insights not only deepen our understanding of signal behavior, but also open avenues for further explorations of the colorful field of signal processing.

It is worth mentioning that the strength of the coupling is expressed by the function of bicoherence [5,21], which is actually the bispectrum normalized by the amplitude, i.e., the strength of the spectral lines that contribute to the value of the bispectrum at the specific position. Most of the time, the value of bicoherence is large, which means that the coupling is strong, regardless of the strength of the bispectrum.

4. Spectral Analysis of the Sunspot Series

The spectral analysis of the sunspot time series incorporated five different preprocessing steps in order to achieve an expressive spectral representation. Accordingly:

1. The whole time series was filtered with an 11-point median filter so that outliers were rejected and weak harmonics were easily revealed (Figure 2).
2. Taking into consideration that the period of the 11-year circle is approximately 4000 days, we partitioned the time series into sections of 8000 days so that each section contained two 11-year circles.
3. Using an overlapping of 4000 samples, we obtained a total of 17 sections of 8000-day samples.
4. Each segment of 8000 samples was extended to 32,000 samples by adding 12,000 zero samples to each edge. This made the final spectrum finer with a distance between harmonics of $-1/32,000$ causing the first harmonic after the dc to represent a period of 32,000 days and, accordingly, the 11-year circle to be represented by the 9th harmonic. At the 16,001 position of the spectrum, we obtained the highest frequency, which represented a 2-day period.
5. The periodogram of the abovementioned 17 time-series pieces of 32,000-length samples was calculated in order to obtain the final spectrum.

Accordingly, the total spectrum was calculated as an average of 17 spectra. Any frequency that would have come from an artifact would have vanished by the averaging process.

In Figures 6–8, three parts of the evaluated spectrum are provided. In Figure 6, the low-frequency part of the sunspot series spectrum, which involves frequencies from dc to 100, which corresponds to a 320-day period, is presented. As depicted in Figure 6, a strong peak is present at spectral position 8 with almost equivalent spectral energy values at position 7 and 9, which correspond to the orbital period of Jupiter and its synodic periods with Saturn and Uranus. The 11-year cycle is also represented by the 8th spectral component, which lies at position 9.

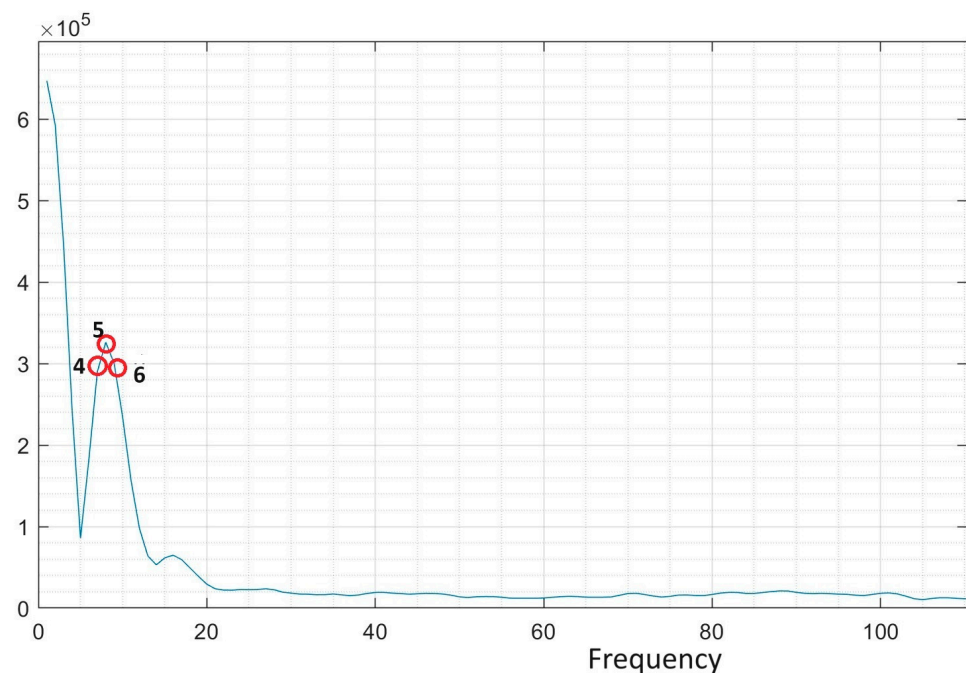


Figure 6. The first 100 harmonics of the sunspot time series from a total of 16,001. Since at position 16,001 we have the harmonic that corresponds to 2 days, the harmonic at position 101 corresponds to $32,000/100 = 320$ days. Six is the 6th periodic planetary movement presented in Table 1, which corresponds to Jupiter's rotation. It lies at the 8th harmonic position, i.e., it corresponds to a 4000-day period (11-year cycle). Positions 4 and 5 are the synodic periods of Saturn and Uranus with Jupiter, respectively. Their strength is greater than the period of Jupiter.

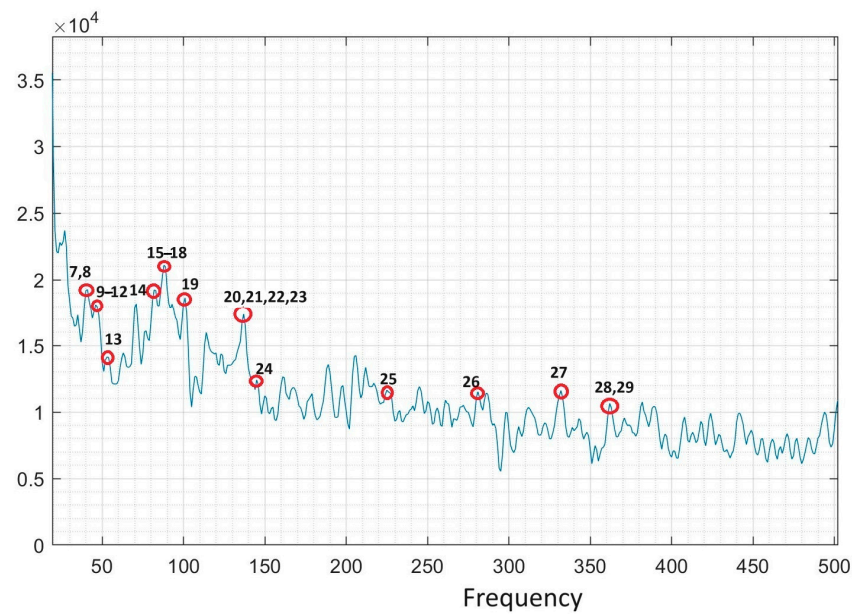


Figure 7. Spectrum of the sunspot time series with harmonics from 20 to 500 from a total of 16,001. Since at position 16,001 we have the harmonic that corresponds to 2 days, the harmonic at position 501 corresponds to $32,000/500 = 64$ days. The numbers above the red circles correspond to the serial numbers in Table 1. Indicatively, number 12 is the orbital period of Mars and corresponds to the 48th harmonic. Eighteen corresponds to Earth's orbital period, which is the 89th harmonic, while numbers 28 and 29 represent the positions of the 361 and 365 harmonics, which represent the synodic periods of Mercury and Jupiter, and the orbital period of Mercury correspondingly. All the relative information is presented in Table 1.

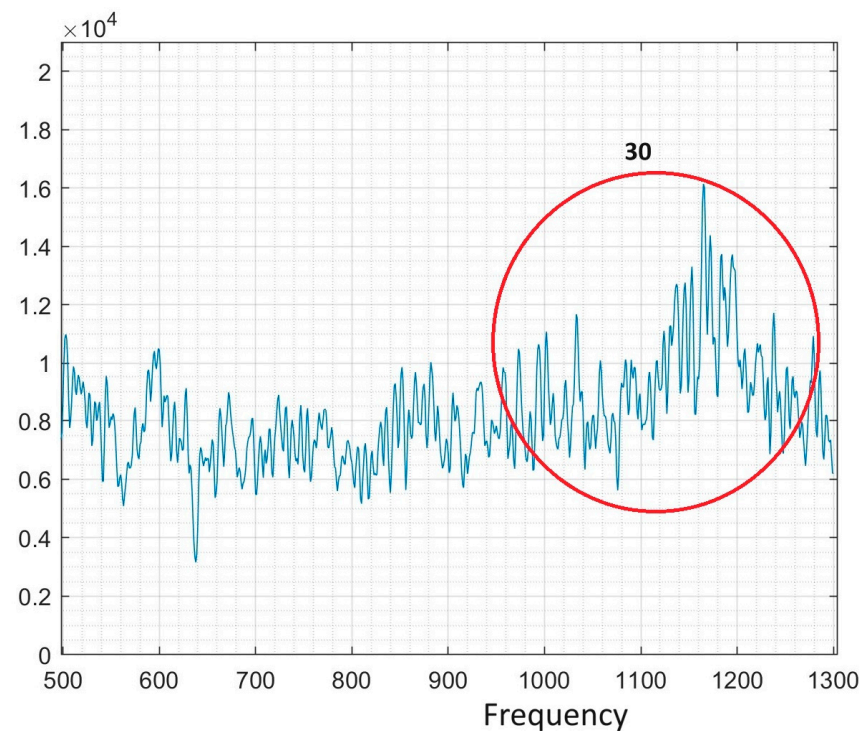


Figure 8. Spectrum of the sunspot time series with harmonics from 500 to 1300 from a total of 16,001. Since at position 16,001 we have the harmonic that corresponds to 2 days, the harmonic at position 1301 corresponds to $32,000/1300 = 24.6$ days. Harmonics 950 to 1250, in the circle indicated as 30 from the corresponding information in Table 1, represent the rotational behavior of the body of the Sun, as it rotates at different periods in its equator (25.6 days) from its mass near the poles (33.5 days).

Figure 7 presents the harmonics up to a value of 400. The reader can find the peaks corresponding to various orbital planetary and synodic periods. The exact correspondence of these peaks with specific orbits can be obtained if the number above each peak corresponds to the serial number of the orbits given in Table 1. The orbits of Mercury, Venus, Earth, and Mars are easily detected with the serial numbers 12, 18, 24, and 29, respectively.

Finally, in Figure 8, the harmonics from 950 to 1250 correspond to the differential rotation values of Sun, which has a 25.6-day rotation period at its equator and 33.5 days near its poles. We have to clarify that, in Figure 8, the strong frequencies are those at the upper end of the spectrum, ~ 1200 , which corresponds to the equatorial rotation (~ 25.6 days). Rotation at the poles is represented by frequencies at position ~ 950 , which are obviously much weaker in amplitude. Since it is well known that sunspots are not located near the poles, the differential rotation of the surface of the Sun has a strong effect on the sunspots that are far away from the equator, decelerating their rotation.

5. Coupled Periodicities Using a Bispectrum

Analyzing the bispectral contents of the sunspot series described in Section 2, we created three bispectral representations, which are demonstrated in Figures 9–11, and are actually one subset of the next representation. This was necessary in order to be able to follow the bispectral content as we zoomed out of larger areas of frequencies. This bispectral representation was preprocessed with the point operation $\log_e(1 + x)$ in order to suppress very large components and simultaneously enhance weak peaks.

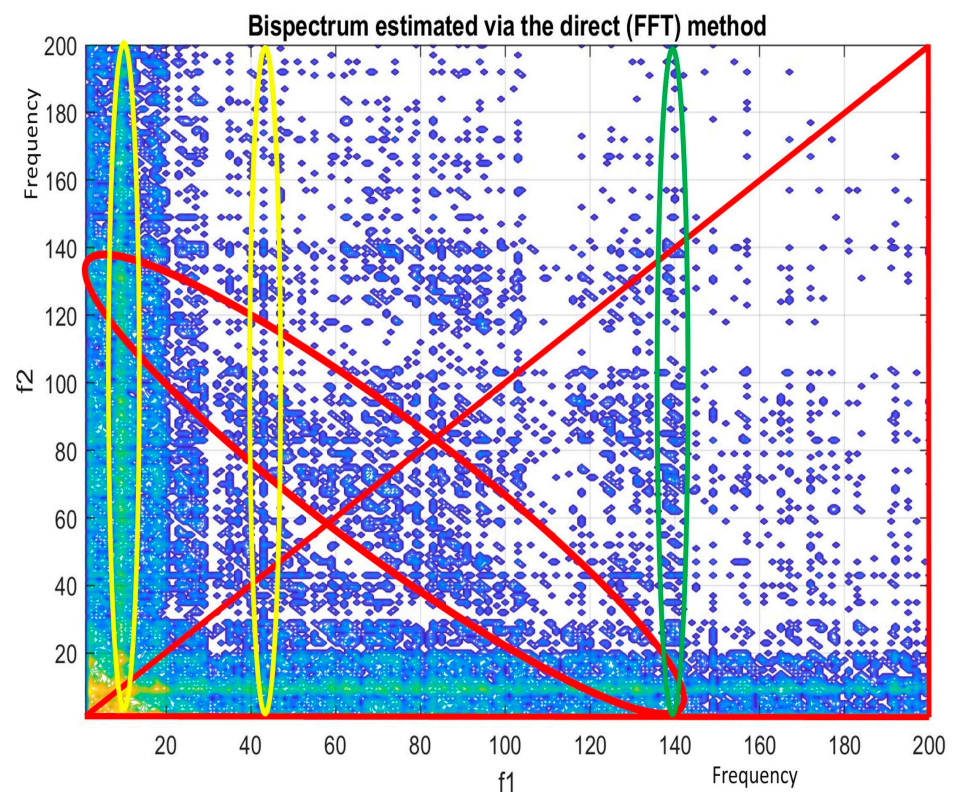


Figure 9. Bispectrum of the sunspot series containing the first 200 harmonics. On both axes, the 200th harmonic corresponds to $32,000/200 = 160$ days. We obtained symmetry with respect to the diagonal line with an origin at (1,1). All frequency coupling edges that we are interested in lie in the orthogonal red triangle. The four elongated ellipses contain specially coupled groups of periodicities, as explained in the text.

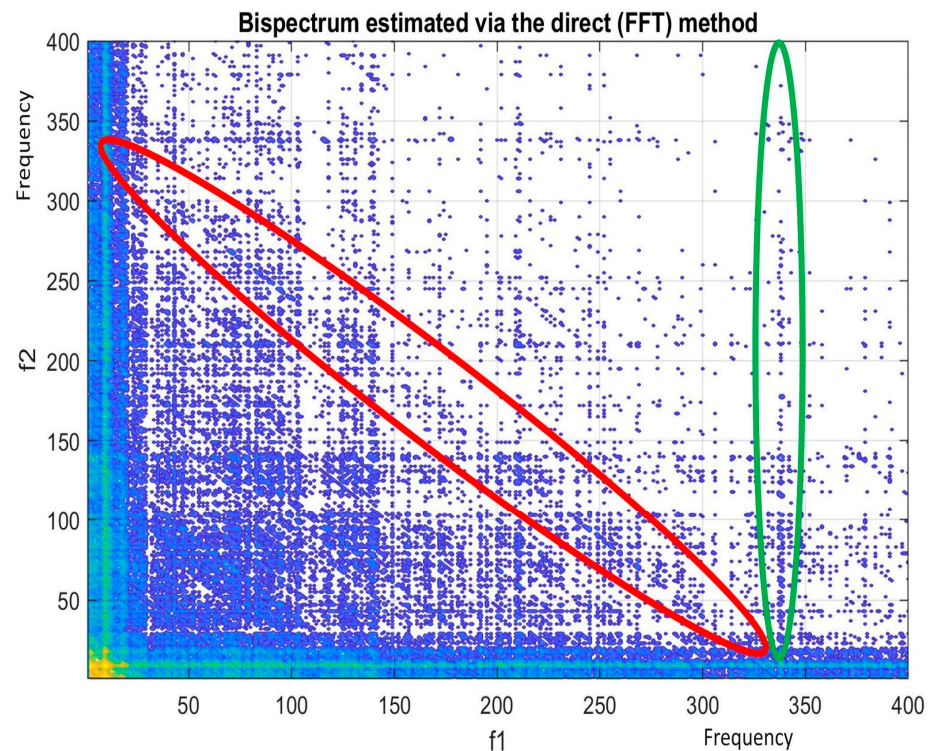


Figure 10. Bispectrum of the sunspot series containing the first 400 harmonics. On both axes, the 400th harmonic corresponds to $32,000/400 = 80$ days. We achieve symmetry with respect to the diagonal line with its origin at (1,1).

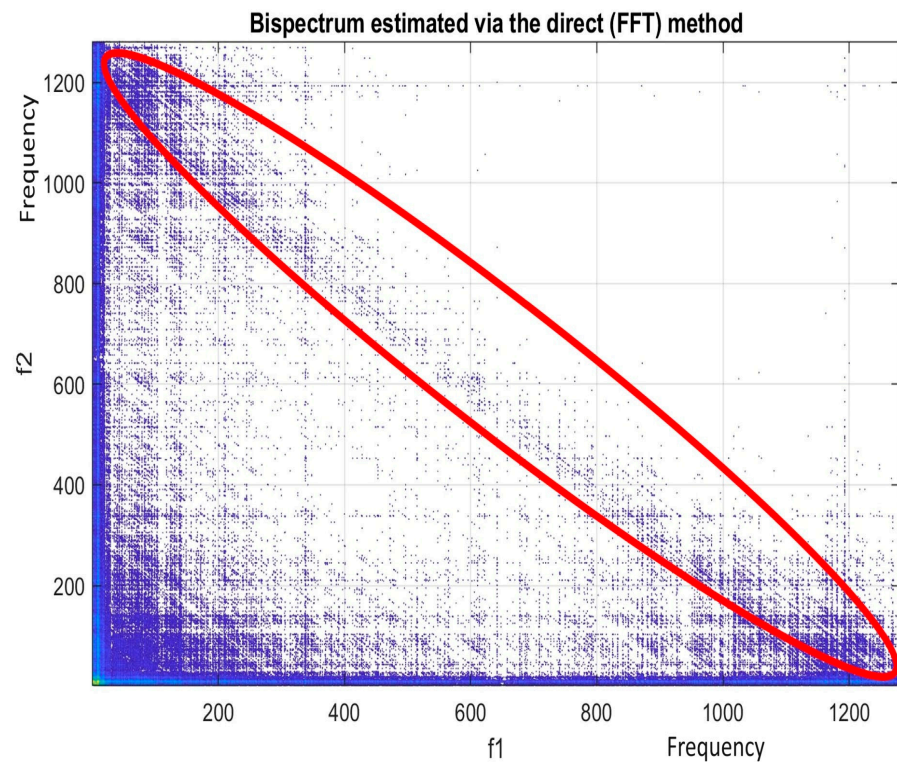


Figure 11. Bispectrum of the sunspot series containing the first 1300 harmonics. On both axes, the 1200th harmonic corresponds to $32,000/1200 = 26.7$ days (rotation of the Sun at its equator). We achieve symmetry with respect to the diagonal line with its origin at (1,1).

Accordingly, the bispectral representation in Figure 9 provides frequencies that correspond to periodicities up to those of Earth and Venus (frequency lines 88 and 143, respectively). The peaks in this figure correspond to a large variety of frequency couplings. We concentrated our attention on four different sets of them, which presented particular groupings.

Findings:

- a. Peaks that appear in the vertical, green ellipse in Figure 9. These peaks correspond to couplings of the frequency component 140 with most of the lower frequencies and some of the higher ones. Frequency component 140 corresponds to Venus' period and simultaneously to the synodic periods of Venus with large planets (Jupiter, Saturn, Uranus, and Neptune). This coupling is very strong and probably imposed by the Sun's interior plasma circulation. Obviously, the strong signature of the periodic movements of Venus and the corresponding couplings on the bispectral representation are strictly related to the strong signature of the same periods on the spectrum of the sunspot data as they are depicted in Figure 7 (peaks with numbers in the range of 20–24).
- b. Peaks that appear in the vertical yellow ellipse in Figure 9 at position 42. These peaks correspond to couplings of the frequency component 42 with most of the frequencies present in the spectrum. Frequency component 42 corresponds to Mars' synodic periods with Earth and Jupiter. This coupling is very strong and probably imposed by the Sun's interior plasma circulation. Obviously, the strong signature of the synodic movements of Mars and the corresponding couplings on the bispectral representation are strictly related to the strong signature of the same periods on the spectra of the sunspot data as they are depicted in Figure 7 (peaks with numbers 7 and 8).
- c. Peaks that appear in the vertical yellow ellipse in Figure 9 at position 8. These peaks mainly correspond to the coupling of the eleven-year cycle with all frequencies present in the sunspot time-series spectrum. Simultaneously, these couplings contribute the periodic movement of Jupiter as well as the synodic periods of Jupiter with Neptune and Uranus.
- d. It is worth mentioning that the bispectrum from 1 (dc) to the 7th frequency component is very rich since it represents the couplings of all periodic movements of the spectra with the periods and synodic periods of the large and distant planets (Jupiter, Saturn, Uranus, and Neptune).
- e. Peaks that appear in the diagonal red ellipse in Figure 9. These bispectral peaks can be described by the equation $f_1 + f_2 = f_{140}$. According to the material presented in Section 4, the "Case of frequency and phase summation" of all frequencies, f_1 and f_2 , which provide couplings with their summations at frequency component f_{140} also present bispectral peaks at the positions with coordinates f_1 and f_2 .

The last case presents strong evidence that the role of Venus in forming coupled periodicities on the sunspots' behavior is very definitive among the influences of the remaining, small interplanets. A specific strong quadratic coupling behavior must lie behind this phenomenon.

The bispectral representation in Figure 10 contains frequencies that correspond to faster rotations (smaller periods), i.e., the rotation of Mercury at line 384.

Findings:

- a. Peaks that appear in the vertical green ellipse in Figure 10 at position 340. These peaks correspond to couplings of the frequency component 340 with most of the frequencies present in the spectrum. Frequency component 340 corresponds to Mercury periods with Jupiter. This coupling is very strong and probably imposed by the Sun's interior plasma circulation.
- b. Peaks that appear in the diagonal red ellipse in Figure 10. These bispectral peaks can be described by the equation $f_1 + f_2 = f_{340}$. According to the material presented in Section 4, the "Case of frequency and phase summation" of all frequencies, f_1 and f_2 ,

which present couplings with their summations at frequency component f_{340} also present bispectral peaks at positions with coordinates (f_1, f_2) .

The last case presents strong evidence that the role of Mercury in forming coupled periodicities regarding the sunspots' behavior is very definitive among the influences of the rest of the small interplanets. A specific strong quadratic coupling behavior must lie behind this phenomenon.

Finally, in Figure 11, the rich bispectral content from 950 to 1250 spectral lines corresponds to the rotation of the Sun around its axis, and it is quite broad since the Sun rotates at 25.6-day period at its equator and 33.5 days near its poles.

Findings:

Peaks that appear in the diagonal red ellipse in Figure 11. These bispectral peaks can be described by the equation $f_1 + f_2 = f_{1200}$. According to the material presented in Section 4, the "Case of frequency and phase summation" of all frequencies, f_1 and f_2 , which present couplings with their summations at frequency component f_{1200} also present bispectral peaks at the position with coordinates (f_1, f_1) . A specific strong quadratic coupling behavior must lie behind this phenomenon, which only permits coupling that sums up to f_{1200} .

6. Conclusions

A detailed investigation of the SILSO sunspot time series was carried out based on two well-established frequency analysis tools, namely the power spectrum and the bispectrum. The material in each section was different; there were different types of processing steps and we reached conclusions that were not strictly related. The first one has to do with the spectral content of the SILSO data, while the second has to do with the bispectral content of the SILSO data and the revealed coupled periodicities.

The power spectrum was employed in its periodogram version; simultaneously, padding zeros at both sides of the data segments were used so that a high-resolution spectral analysis was performed. The results obtained from the spectral density demonstrate that the planetary periodicities as well as the synodic periodicities are mirrored by the sunspot activities on the Sun's surface. In this fine and detailed spectrum calculation, we obtained all the planetary periodic phenomena present on the surface of the Sun. The main contribution of this work is that the periodic phenomena revealed by the sunspot data are limited within the time period of the 11-year cycle as well as the periodicities of the orbits of the planets, showing the possible effect of the orbits of the planets on the Sun's plasma. This contribution fills a gap that exists in the research in relation to the appearance of planetary periodicities in sunspot time series, since, to date, dominant studies have dealt with much longer periodicities (e.g., millennia or centuries) and were based on the way the solar magnetic field in the heliosphere modulates cosmic rays, influencing the production rate of radionuclides on Earth [4–12].

As a general inference, even though there is approximately the same number of previous references ([8–15]) "for" and "against" our main claim, we observed the connection between the sunspot periods and characteristic periods of planets. However, additional examinations and analyses are needed to confirm that certain periodicities in the sunspot data are really the consequence of the tidal effects of the planets on solar plasma.

Coupled periodicities were detected on the sunspot time series by means of a bispectrum. The bispectrum revealed the coupling of two distinct frequencies, f_1 and f_2 , which contributes to a new frequency of the spectrum at position $(f_1 + f_2)$, by presenting a peak at coordinates (f_1, f_2) of the bispectrum. Accordingly, Mercury, Venus, and Mars periodicities present couplings with the vast majority of the strong line frequencies of the spectrum (groups in vertical ellipses in Figures 9 and 10). Furthermore, the groups of bispectral spikes lying in the diagonal ellipses provide us with the excellent information that all these spikes come from the spectral equation $f_1 + f_2 = f_{coupled}$. This $f_{coupled}$ corresponds to frequencies of 140, 340, and 1200. According to these observations:

There must exist a significant quadratic coupling nonlinearity, which applies to almost all periodicities that are found in the sunspot time series in such a way that intensive coupled harmonics appear only for specific summations, namely $f_1 + f_2 = f_{140}$, $f_1 + f_2 = f_{340}$ and $f_1 + f_2 = f_{1200}$.

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