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Three-Dimensional Broad Learning Gravity Data Inversion Using Single-Anomaly Training Samples

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Abstract: Gravity data inversion is of critical importance in geophysics, encompassing a range of applications, such as the exploration of geological resources, the identification of geological structures, and the detection of groundwater resources. This study proposes a three-dimensional (3D) machine learning approach to enhance the efficiency of the aforementioned exploration tasks by leveraging gravity data. The mapping relationship between gravity data and subsurface density structures is modeled by the broad learning network, distinguished by its high training efficiency and robust modeling capability. Notably, the proposed inversion method obviates the constraints on the number of anomalies prior to the inversion process. This is achieved by setting one anomaly with varied locations for different training samples. Numerical and field data applications demonstrate the efficiency of the proposed 3D machine learning gravity data inversion method, especially in automatically determining the number of anomalies. In particular, the proposed method produced accurate density inversion results in the field application, aiding in the identification of potential oil and gas reservoirs in the target region and offering the potential for broader application in other resource exploration. The proposed inversion method can promote the construction of density structures of subsurfaces based on gravity data.

Keywords: gravity data inversion; machine learning; broad learning network; single-anomaly training samples; oil and gas resources



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1. Introduction

Gravity inversion plays a crucial role in geophysics, enabling the determination of subsurface density structures for various practical applications, including the exploration of geological resources [1–4], the identification of geological structures [5,6], the investigation of groundwater resources [7,8], the delineation of oil and gas [9], the revelation of hazardous phenomena [10], etc. Due to the complexity and heterogeneity of subsurface materials, a two-dimensional perspective often results in simplified and distorted representations of density structures, potentially overlooking important structural features [11,12]. Therefore, three-dimensional (3D) inversion has become essential and increasingly popular in gravity data inversion for retrieving subsurface density structures with a high spatial resolution and precision [13,14]. However, the increase in complexity due to higher-dimensional

scales imposes greater demands on the computational efficiency and global optimization capabilities of the algorithms applied in the inversion process. Advancing inversion algorithms to handle such complexity is crucial for efficiently achieving inversion results that accurately reflect reality [15–18].

The development and introduction of global inversion algorithms have consistently been key research priorities in 3D gravity data inversion, overall, encompassing the traditional global optimization algorithms (e.g., the genetic algorithm, simulated annealing algorithm, particle swarm optimization algorithm, etc.) and emerging machine learning algorithms. During the past more than 20 years, traditional inversion algorithms have been widely employed in conducting 3D gravity data inversion. Ref. [19] employed the simulated annealing algorithm for 3D gravity data inversion to investigate the density anomalies of underground materials in the Gulf of Mexico. Ref. [20] used a 3D perspective to identify the crustal structures of Fuerteventura Island, where the genetic algorithm undertook the optimization tasks. Ref. [21] developed a novel gravity inversion method by integrating the simulated annealing algorithm with the genetic algorithm to enhance the retrieval of 3D density anomalies. Ref. [22] performed 3D gravity inversion and uncertainty analysis using the particle swarm optimization algorithm, producing an estimated gravity profile that effectively reflects the subsurface conditions of the Argeles-Gazost basin in France. Even now, traditional inversion algorithms continue to be utilized for identifying optimal solutions in 3D gravity data inversion due to their global optimization capabilities [23–25]. In contrast, machine learning algorithms have emerged and rapidly developed in geophysical inversion in recent years, gaining popularity due to their real-time processing, strong global optimization, and generalization capabilities and, consequently, their potential to improve the efficiency and accuracy of the 3D gravity inversion process. As a representative of machine learning, a deep learning network has been introduced to address the optimization challenges of 3D gravity data inversion. For example, refs. [26–29] utilized deep learning networks for 3D gravity data inversion, achieving satisfactory results in modeling subsurface density structures. Currently, the application of machine learning in gravity inversion is continually evolving to address the growing demands for enhanced inversion performance.

Deep learning has recently garnered more attention from researchers for various modeling problems compared with traditional global optimization algorithms due to its real-time property and optimization abilities [30–32]. Once network training is complete, the inversion results can be determined immediately. Additionally, deep learning is widely regarded as having superior functional modeling capabilities compared with traditional algorithms. However, there are two main challenges that need to be addressed in most 3D deep learning gravity inversion studies [26,33], i.e., the time-consuming problem of network training and the issue of anomaly number representation in training samples [34–36]. The first challenge limits the inversion efficiency, particularly when network retraining could not be avoided in certain situations (e.g., cross-zone gravity exploration). Regarding the second challenge, most studies implicitly assume that the maximum number of anomalies in the training samples is greater than the actual number. Undoubtedly, these two drawbacks hinder the broader application of machine learning in 3D gravity data inversion. In this study, these two obstacles are addressed by drawing on approaches from other geophysical problems.

To address the time-consuming disadvantage of a deep learning network, in ref. [37], the authors proposed a broad learning (BL) network, which utilizes a single-layered neural architecture. A BL network increases complexity in width dimension rather than depth, providing a significant advantage in training speed while maintaining functional mapping capabilities comparable to a deep learning network. BL networks have been applied in some modeling problems and achieved satisfactory prediction results [38–40]. Regarding its application in geophysics, the advantages of a BL network in terms of both time efficiency and optimization performance have been demonstrated in surface wave inversion [41–44] and resistivity inversion [45]. As for the anomaly number problem, 3D resistivity inver-

sion similarly faces constraints regarding prior information on the number of anomalies. Ref. [45] proposed a machine learning inversion method that incorporates a single anomaly with varying locations across different training samples. The inversion results demonstrate the effectiveness of using single-anomaly training samples for network training. It is evident that the application of a BL network and single-anomaly training samples can be extended to 3D gravity inversion.

To tackle the challenges of time consumption and the representation of anomaly numbers in training samples, this study proposes a novel 3D gravity data inversion method. First, we use a single anomaly to generate training samples based on the linear superposition of potential fields [46]. Second, we introduce a BL network to map the relationship between gravity anomaly data and density models, thereby reducing the time required for the training process. The feasibility of the proposed method is then validated through three synthetic models. Additionally, the method demonstrates improved detection performance for the Vinton salt dome, which is crucial for locating oil and gas resources, as the salt dome plays a significant role in the formation of structural traps.

2. Three-Dimensional Broad Learning Gravity Data Inversion Method

2.1. Inversion Model

The functional relationship between the measured gravity data and the subsurface density model can be expressed as follows:

$$\mathbf{d} = F(\mathbf{m}), \quad (1)$$

where $\mathbf{d} = [d_1, d_2, \dots, d_{N_d}]^T \in \mathbb{R}^{N_d}$ is the measured gravity data vector, $\mathbf{m} = [m_1, m_2, \dots, m_{N_m}]^T \in \mathbb{R}^{N_m}$ is the density model vector, N_d is the number of data points of the measured gravity data, N_m is the number of grid points of the density model, and $F(\cdot)$ is the functional relationship to be modeled. The objective of gravity inversion is to estimate the optimal density model $\hat{\mathbf{m}}$, which serves as an approximation of the actual density model $\mathbf{m}$. To quantitatively evaluate the performance of the inversion, the following objective function is utilized during the inversion process:

$$F_{Obj} = \frac{1}{N_m} \|\mathbf{m} - \hat{\mathbf{m}}\|_2^2 \quad (2)$$

2.2. Broad Learning Network for Inversion

This study utilizes a BL network [37] to model the functional mapping $F(\cdot)$ in Equation (1). Figure 1 shows the configuration of the BL network. As a supervised learning model, the BL network explores the functional mapping between the input and output datasets to approximate the actual underlying relationship. Given the number of training samples is N , it uses $\mathbf{D}_t = [d_{t1}, d_{t2}, \dots, d_{tN}]^T \in \mathbb{R}^{N \times N_d}$ ($d_{ti} \in \mathbb{R}^{N_d}$ is an input training sample) and $\mathbf{M}_t = [m_{t1}, m_{t2}, \dots, m_{tN}]^T \in \mathbb{R}^{N \times N_m}$ ($m_{ti} \in \mathbb{R}^{N_m}$ is an output training sample) to represent the input gravity training dataset and output density model training dataset, respectively. It is noted that the forward modeling of gravity data is applied for the generation of training samples:

$$d_{ti} = T(m_{ti}), \quad i = 1, 2, \dots, N, \quad (3)$$

where $T(\cdot)$ indicates the forward modeling function for the gravity data using an arbitrary density model. The functional mapping between $\mathbf{D}_t$ and $\mathbf{M}_t$, constructed by the BL network to approximate the function $F(\cdot)$, can be illustrated as:

$$\mathbf{M}_t = [\mathbf{F}^{N_F}, \mathbf{E}^{N_E}] \mathbf{W}^B, \quad (4)$$

where $\mathbf{F}^{N_F} = [F_1, F_2, \dots, F_{N_F}]$ denotes the mapped features, $\mathbf{E}^{N_E}$ reflects the enhancement nodes, and $\mathbf{W}^B$ indicates the network weighting matrix, which is the only unknown

parameter to be determined for the network training of the broad learning network. Each group of mapped features of the BL network can be shown as:

$$F_i = \phi(D_i W_{F_i} + \beta_{F_i}), i = 1, 2, \dots, N_F, \quad (5)$$

where ϕ is the feature mapping function performed by a sparse autoencoder [32] for data dimensionality reduction, W_{F_i} is the i th weight matrix with random values, β_{F_i} is the i th bias with random values, and N_F is the number of groups of mapped features. Based on the mapped features F^{N_F} , each group of enhancement nodes can be calculated by:

$$E_j = \xi(F^{N_F} W_{E_j} + \beta_{E_j}), j = 1, 2, \dots, N_E, \quad (6)$$

where ξ is a nonlinear activation function (the hyperbolic tangent applied in this study), W_{E_j} is the j th random weight matrix similar to W_{F_i} , β_{E_j} is the j th random bias similar to β_{F_i} , and N_E is the number of groups of enhancement nodes. The following equation can be applied to calculate the only unknown parameter W^B for the BL network:

$$W^B = A^+ M_t, \quad (7)$$

where $A = [F^{N_F}, E^{N_E}]$. The calculation of pseudoinverse A^+ is accomplished by the Ridge regression [37].

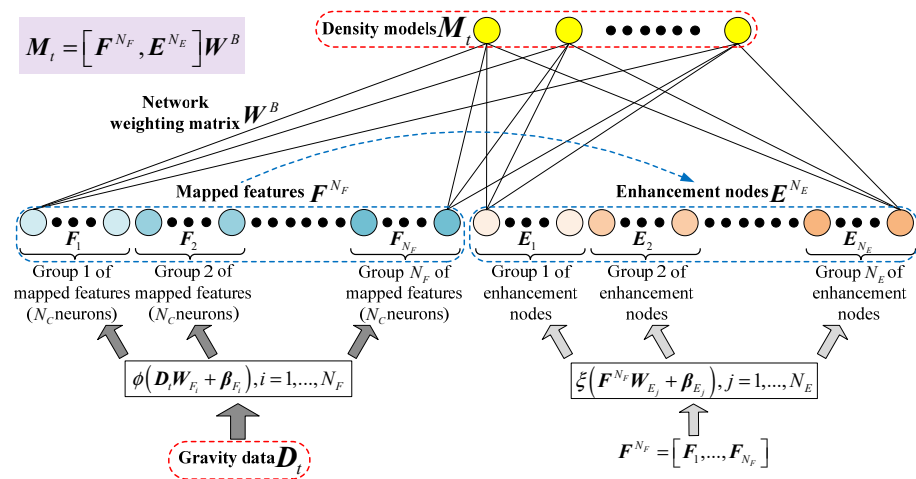


Figure 1. Configuration of the broad learning network [37,41,42].

The training process is complete once the weighting matrix W^B is determined. Using the trained network, the optimal density model $\hat{m}$ can be obtained by inputting the measured gravity data vector $\hat{d}$ into the network as follows:

$$\hat{m} = [F'^{N_F}, E'^{N_E}] W^B, \quad (8)$$

where $F'^{N_F} = [\phi(dW_{F_1} + \beta_{F_1}), \phi(dW_{F_2} + \beta_{F_2}), \dots, \phi(dW_{F_{N_F}} + \beta_{F_{N_F}})]$ and $E'^{N_E} = [\xi(F'^{N_F} W_{E_1} + \beta_{E_1}), \xi(F'^{N_F} W_{E_2} + \beta_{E_2}), \dots, \xi(F'^{N_F} W_{E_{N_E}} + \beta_{E_{N_E}})]$.

2.3. Generation of Single-Anomaly Training Samples

The BL network relies on input–output training sample pairs for functional modeling. This study employs the sample generation method from resistivity inversion proposed by ref. [45], which involves using single anomalies with varying locations across different training samples for network training. As a result, this approach imposes no restrictions on the number of anomalies prior to inversion. Figure 2 illustrates the generation of an input–

output training sample pair through the forward modeling of the gravity data. The density model is constructed with a single, colored, cube-shaped anomalous block embedded in a homogeneous background, as shown in Figure 2a. The density model is discretized into 50, 50, and 12 grid cells in the X-, Y-, and Z-directions, respectively. The size and spatial position of the anomalous block are randomly varied in each model, with the density value of the anomaly randomly selected within a range of -2 to 2 g/cm^3 . In particular, the coordinated rotations around the density model's center point are also performed to generate some single samples oriented in different directions. The rotation method is as follows:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & -\sin \gamma & \cos \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad (9)$$

where $[x, y, z]^T$ represents the coordinates of the density model, $[x', y', z']^T$ is the corresponding coordinates after rotation, and the angles α , β , and γ result from rotations along the Z-, Y-, and X-axes. Utilizing the single-anomaly density model (Figure 2a) and the gravimeter deployments (Figure 2b), the surface gravity data (Figure 2c) are simulated through forward gravity modeling, resulting in the generation of an input training sample (Figure 2d).

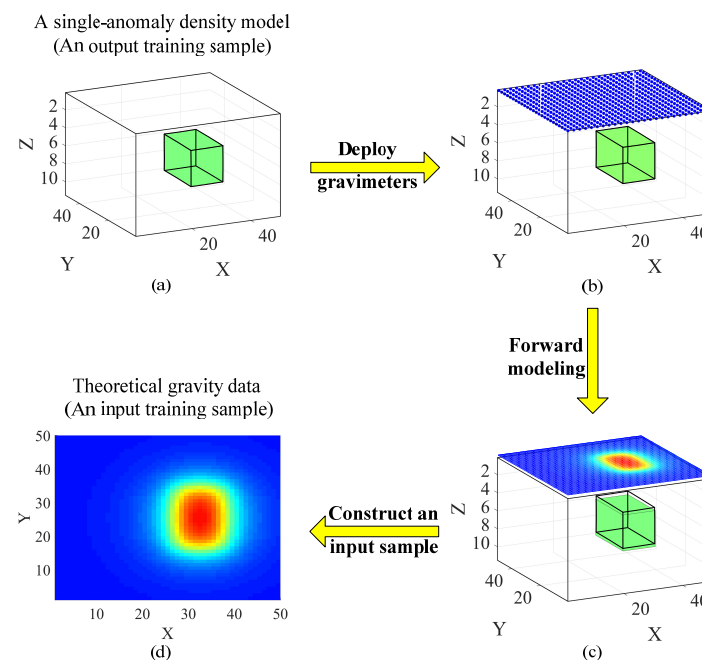


Figure 2. Generation of an input–output training sample pair based on forward modeling of gravity data. (a) An output training sample with a single anomaly. (b) Gravimeter deployment points. (c) Forward modeling of gravity data. (d) An input training sample.

2.4. Network Complexity Selection

There are three hyperparameters for the BL network, namely, N_C (the number of neurons in each group of mapped features), N_F (the number of groups of mapped features), and N_E (the number of groups of enhancement nodes). Different combinations of these three parameters result in varying network complexities, each reflecting distinct functional mapping capabilities. Therefore, selecting an appropriate mapping complexity (i.e., a suitable combination of hyperparameters) is essential for the BL gravity data inversion.

A grid search is employed to select the most suitable hyperparameter values for N_C , N_F , and N_E within the ranges $[20:20:200] \times [20:20:200] \times [50:50:500]$ ($[s:e]$ represents a range starting from s , ending at e , with an interval of i). In this study, 30,000 samples from the training dataset are used to train the BL networks, while 3000 samples from the valida-

tion dataset are employed to optimize the network complexity for 3D gravity inversion. Specifically, the objective function defined in Equation (2) is used to quantify the inversion performance for each sample in the validation dataset. The optimal network complexity is identified as the one that yields the minimum mean objective function value across all samples in the validation dataset.

2.5. Implementation Procedure of Broad Learning Gravity Data Inversion

The implementation steps of BL gravity data inversion can be summarized as:

1. Generate 33,000 random single-anomaly training sample pairs, as described in Section 2.3;
2. Select 30,000 samples to form the training dataset and 3000 samples for the validation dataset;
3. Use the training dataset to train a series of BL networks with varying network complexities;
4. Apply the validation dataset to identify the BL network with the most appropriate complexity;
5. Input the measured gravity data into the selected trained network to predict the optimal density model.

In particular, the computer for inversion tasks in this study is equipped with an Intel 11th Gen Core i7-11700KF @ 3.60 GHz and 64.00 GB (3200 MHz) RAM. Once the network training is complete, the gravity anomaly data inversion is completed within 0.2 s.

3. Results and Evaluation of Performance

Candidate BL networks with varying complexities were trained for network complexity selection by adhering to implementation steps 1 to 4. Figure 3 illustrates the objective function values of these BL networks on the validation dataset. This process resulted in the identification of the optimal hyperparameters: $N_C = 160$, $N_F = 100$, and $N_E = 100$. Using these hyperparameters, three numerical examples and one field data application were conducted to assess the effectiveness of the proposed inversion method.

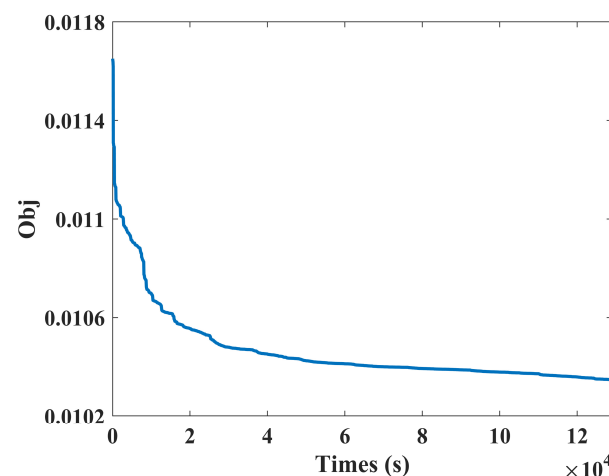


Figure 3. Objective function curves under varying network complexities.

3.1. Numerical Simulations

This section introduces three numerical density models to verify the proposed inversion method. The first model consists of multiple anomalies with rectangular shapes, designed to assess the proposed method's ability to identify the number of anomalies accurately. The second one emphasizes evaluating the capability to recover the irregular shapes of the anomalies. The third model comprises anomalies with different shapes, densities, and positions, aimed at evaluating the robustness of our approach.

As shown in Figure 4, the simulated gravity data (Figure 4b) for a density model with multiple regularly shaped anomalies (Figure 4a) are first applied to examine the proposed inversion method. The length, width, and height of each anomaly are 1000 m,

1000 m, and 600 m, respectively, with a density of 1.0 g/cm^3 . These surface gravity data, generated through the forward modeling of the numerical model, were used as the input to the trained network constructed using single-anomaly training samples. The network subsequently produced the corresponding inverted density model. Figure 5a provides 3D views of the inverted density model. The results clearly demonstrate that the trained BL network successfully identified four density anomalies without any prior assumption regarding the number of anomalies. It also shows a good recovery of the shape of the identified anomalies in the inverted results compared with the actual model. Additionally, the objective function value between the actual and inverted models is 0.0249, showcasing satisfactory performance in terms of inverting gravity anomaly values. To provide a more comprehensive evaluation of the inversion results, we calculated the coefficient of determination (R^2) between the observed gravity anomaly (Figure 4b) and the gravity anomalies derived from the inversion results (Figure 5b), yielding a value of 0.9999. The residuals between the inverted and observed gravity anomaly data were also computed, as shown in Figure 5c. The inversion residuals are predominantly concentrated within the range of -1.0 to 1.0 mGal , which are relatively small compared with the gravity anomalies in Figure 4b. Undoubtedly, the functional mapping modeled by the trained BL network using single-anomaly training samples effectively captures the relationship between a multiple-anomaly density model and its measured gravity data. It is reasonable to analyze that the success of this approach lies in the variation in anomaly size and location across the training samples and, more importantly, in the BL network's ability to generalize to different numbers of anomalies. This numerical example demonstrates the effectiveness of the proposed inversion method when applied to a subsurface density model with multiple regularly shaped anomalies.

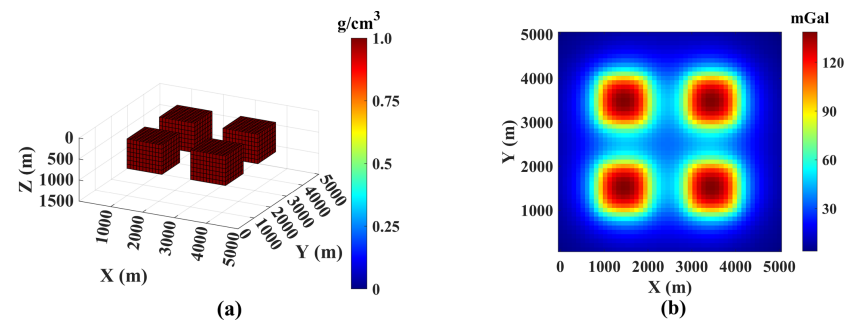


Figure 4. Numerical density model with four rectangular-shaped anomalies and corresponding surface gravity anomaly observations. (a) Three-dimensional representation of the actual density model. (b) Surface gravity anomaly observations.

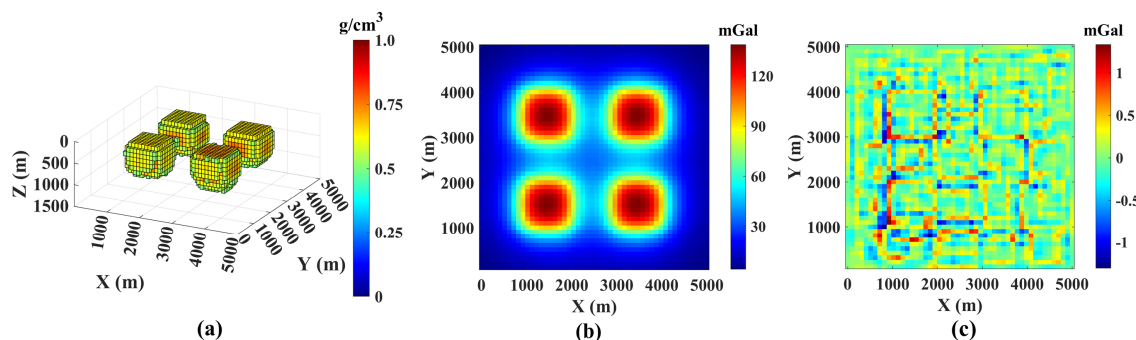


Figure 5. Inversion results of the numerical density model with four rectangular-shaped anomalies. (a) Three-dimensional representation of the inverted density model. (b) Gravity anomaly data forwarded from the inversion result. (c) Residuals between inverted and observed surface gravity anomaly data.

The effectiveness of the proposed inversion method is further assessed using a numerical earth model with irregularly shaped anomalies. Figure 6 shows the actual density model

and corresponding theoretical surface gravity anomaly data. Unlike the first numerical model, this model contains two irregularly shaped anomalies with a density of 1.0 g/cm^3 . The dimensions of the first anomaly are 2000 m in length, 500 m in width, and 600 m in height, with a center depth of 500 m. The dimensions of the second irregular anomaly are 2400 m in length, 600 m in width, and 300 m in height, with a center depth of 600 m. This numerical example aims to explore the inversion performance to predict the irregular shapes of anomalies. With gravity data as input to the trained network, the resulting inverted density model was promptly obtained, as shown in Figure 7a. Compared with the actual density anomalies, the shapes of the anomalies were well predicted, provided that the number of anomalies was accurately estimated. Regarding the inverted density values, the objective function value between the inverted and actual density models is 0.0198, indicating a strong recovery of density values within the anomalies. In particular, each training sample used for the network training contained only a single anomaly. It is obvious that the generalization capability of the proposed gravity inversion method is not limited to identifying the number of anomalies but also extends to predicting the shapes of irregularly shaped anomalies. To provide a more detailed assessment of the inversion accuracy, the differences between the inverted and observed gravity anomaly data were quantified, as illustrated in Figure 7c. The inversion residuals are largely confined to the range of -1.0 to 1.5 mGal . The R^2 -value between the observed gravity anomaly and the gravity anomalies (Figure 7b) forwarded from the inversion results is 0.9999, indicating minimal discrepancies compared with the gravity anomalies shown in Figure 6b. The proposed inversion method, trained on single-anomaly samples, produces accurate results even when applied to irregularly shaped anomalies. This numerical example further confirms the effectiveness of the proposed method.

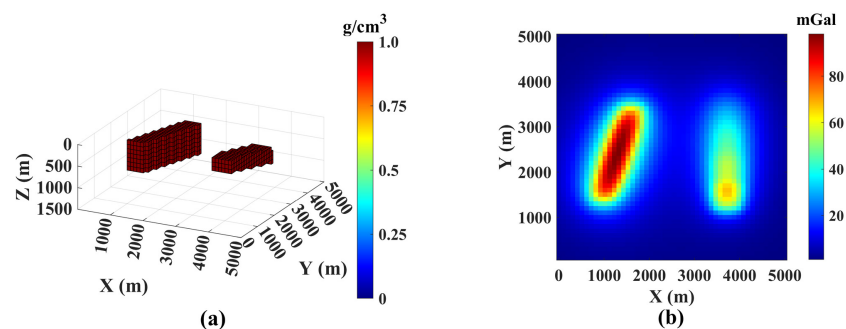


Figure 6. Numerical density model with irregular-shaped anomalies and corresponding surface gravity anomaly observations. (a) Three-dimensional representation of the actual density model. (b) Surface gravity anomaly observations.

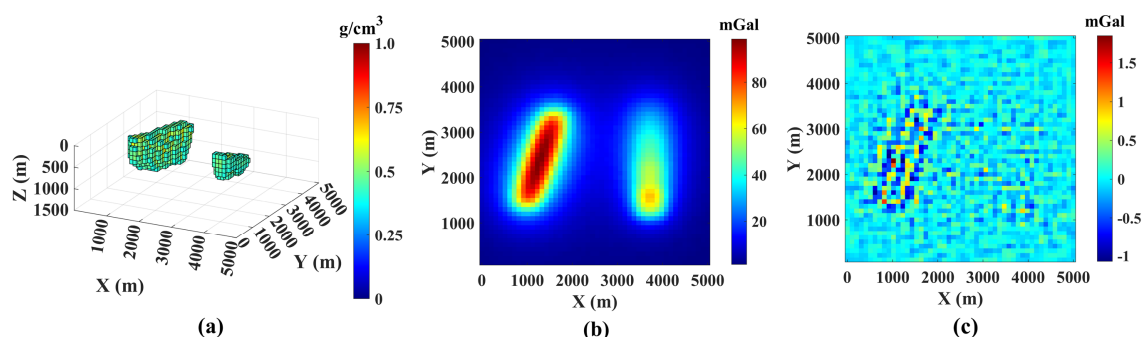


Figure 7. Inversion results of the numerical density model with irregular-shaped anomalies. (a) Three-dimensional representation of the inverted density model. (b) Gravity anomaly data forwarded from the inversion result. (c) Residuals between inverted and observed surface gravity anomaly data.

The robustness of the BL network is further evaluated using a model composed of anomalies with varying shapes, densities, and positions. As shown in Figure 8a, the test

model includes a T-shaped anomaly with a density of 0.5 g/cm^3 , a rectangular anomaly with a density of 0.75 g/cm^3 , and a cubic anomaly with a density of 1.0 g/cm^3 . The inversion results, presented in Figure 9a, demonstrate that the density structures with different shapes are well-recovered. The objective function value between the inverted and actual density models is 0.0323. The R^2 between the observed gravity anomaly (Figure 8b) and the gravity anomalies (Figure 9b) forwarded from the inversion results is 0.9989. The differences between the inverted and observed gravity anomaly data, shown in Figure 9c, fall within the range of -2.0 to 1.5 mGal . All quantitative metrics validate the accuracy and stability of the proposed method.

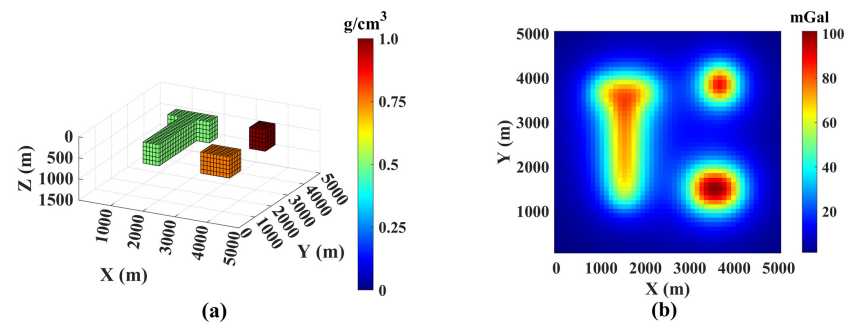


Figure 8. Numerical density model with varied shapes, densities, and positions and corresponding surface gravity anomaly observations. (a) Three-dimensional representation of the actual density model. (b) Surface gravity anomaly observations.

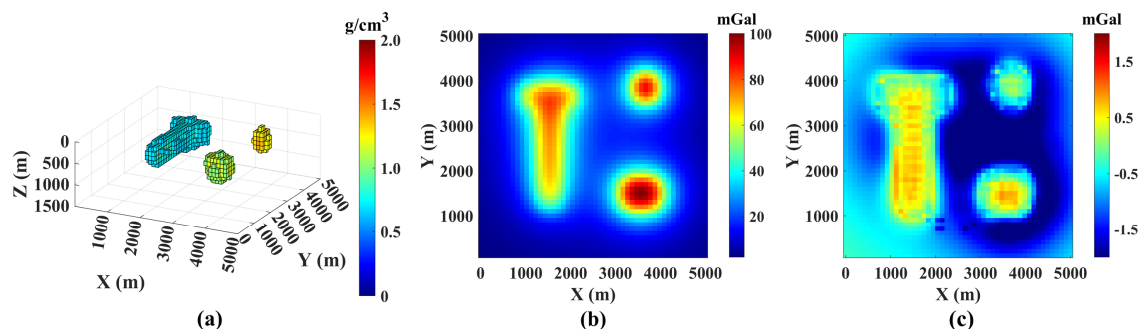


Figure 9. Inversion results of the numerical density model with varied shapes, densities, and positions. (a) Three-dimensional representation of the inverted density model. (b) Gravity anomaly data forwarded from the inversion result. (c) Residuals between inverted and observed surface gravity anomaly data.

3.2. Field Data Application

This section further examines the inversion capability of the proposed 3D gravity data inversion method through a real-world application. The field data were obtained from a gravity survey conducted by Bell Geospace over the Vinton salt dome, located in southwest Louisiana, USA. The salt dome lies within the onshore Gulf of Mexico region, which has served as a major oil and gas production area for over a century [47,48]. Most reservoirs in this region are predominantly found along the flanks of salt domes, as structural traps are formed by the upward movement of the salt. The ability to accurately detect the size and shape of the salt dome significantly affects the analysis of these structural traps, which is critical for identifying and locating oil and gas reservoirs. In particular, the proposed inversion method was applied for the detection of the size and shape of the salt dome. Figure 10 presents the surface gravity anomaly map of the Vinton salt dome, where the observation area is limited to $4 \text{ km} \times 4 \text{ km}$. As shown in the figure, a density anomaly with a radius of approximately 1 km is clearly visible.

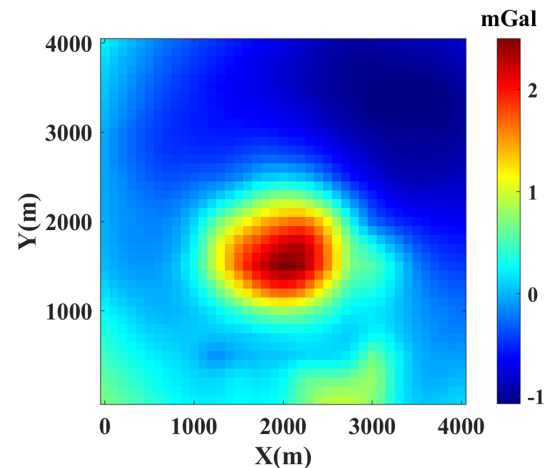


Figure 10. Surface gravity anomaly observations over the Vinton salt dome.

Like the numerical simulations, the inverted density model of the subsurface was instantly obtained by inputting the surface gravity anomaly data into the trained network. Figure 11 presents the inversion results of the field data over the Vinton salt dome. As illustrated in Figure 11a, a density anomaly ranging from 0. to 0.45 g/cm^3 was identified. Furthermore, the inverted model was visualized in 2D through both a horizontal slice (Figure 11b) and two vertical slices (Figure 11c), offering further insights into the model. It is evident that the depths to the top and bottom of the density anomaly are 150 m and 400 m, respectively. To further evaluate the inverted model, theoretical surface gravity data (Figure 11d) were generated by inputting the inverted density model into the forward modeling function. This allowed for the calculation of the residual map (Figure 11e) and the coefficient of determination (R^2) between the inverted and observed surface gravity anomaly data. The inversion residuals primarily range from -0.1 to 0.1 mGal , and the R^2 value is 0.9931. The inverted gravity anomaly ranges from -1.0 to 2.5 mGal , indicating strong consistency with the gravity anomalies shown in Figure 10. To validate the accuracy of the inverted results in this study, a comparison was made between the inversion results for the field data over the Vinton salt dome and those from previous works, as shown in Table 1. It can be observed that the top depths of the salt dome obtained by different methods are generally consistent, while there are significant differences in the bottom depths. This discrepancy occurs because the gravity field decreases quadratically with increasing source depth, which leads to reduced depth resolution in gravity inversion [46]. In particular, the bottom boundary obtained from our results is closer to the geological data [49]. This comparison confirms the reasonableness of the inverted results. The application of field data reinforces the effectiveness of the proposed 3D gravity inversion method.

Table 1. Comparison of inversion results for field data over the Vinton salt dome between this study and previous studies [49].

Work	Depth to Top of Density Anomaly	Depth to Bottom of Density Anomaly
Oliveira et al. [48] by inversion	160 m	460 m
Gao and Huang [49] by inversion	200 m	450 m
Ennen [50] by geological data	160 m	360 m
This study by inversion	150 m	400 m

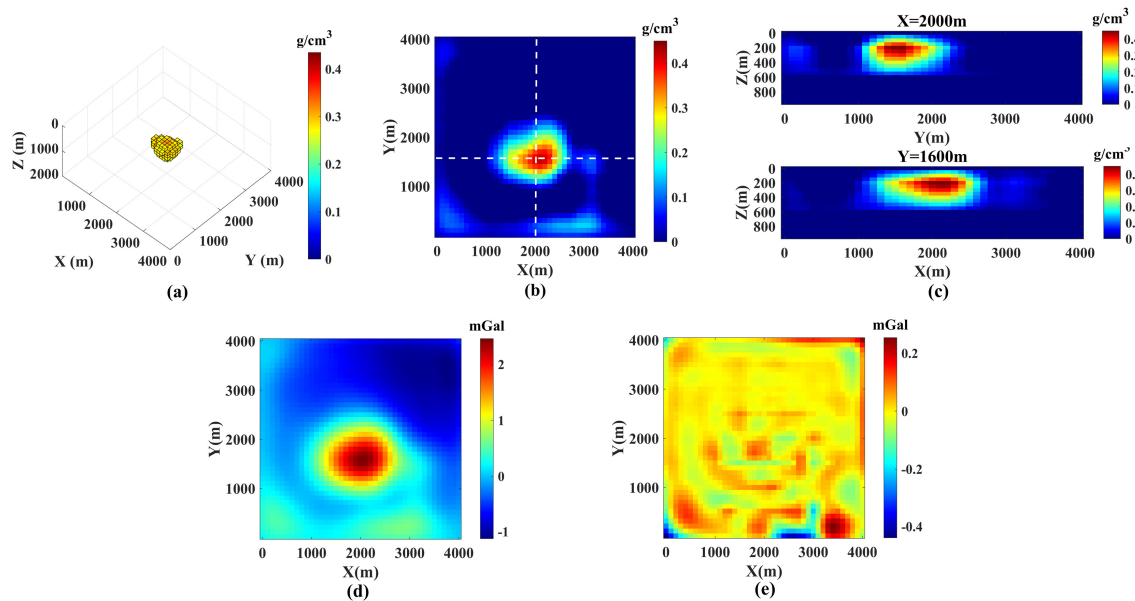


Figure 11. Inversion results of the field data over the Vinton salt dome. (a) Three-dimensional representation of the inverted density model. (b) Horizontal slice at a depth of $Z = 300$ m. (c) Vertical slices at $X = 2000$ m and $Y = 1600$ m. (d) Gravity anomaly data forwarded from the inversion result. (e) Residuals between inverted and observed surface gravity anomaly data.

4. Discussion

This study proposes a novel method for 3D gravity data inversion. The single-layered architecture of the BL network facilitates a time-efficient training process. The introduction of the BL network enables the proposed method to perform real-time inversion, providing superior time efficiency compared with traditional inversion algorithms. Additionally, by using single-anomaly training samples, the proposed method eliminates the need to consider the number of anomalies during the training process. This section explores these three aspects in detail.

The only unknown in the BL network is the network weighting matrix, which involves far fewer unknown parameters compared with a deep learning network. As a result, the training process of the BL network-based inversion is more efficient than that of the deep learning network-based inversion. In fact, this has been validated in previous surface wave inversion studies. For example, ref. [42] provides a comparison between the BL network and the Levenberg–Marquardt algorithm-based deep learning network for inverting subsurface shear wave velocities, showing that the BL network-based inversion requires much less training time for modeling. In real-world applications, cross-zone gravity exploration is often unavoidable, necessitating network retraining due to changes in the functional mapping relationship. A gravity data inversion method with a time-efficient training process can significantly improve the efficiency of field exploration, and the method proposed in this study is precisely such an approach.

The functional mapping learned from the training samples can approximate the relationship between the measured surface gravity data and the density model. As a result, the machine learning-based inversion method can achieve real-time inversion capabilities. To better evaluate the performance of the proposed method, the inversion of the first numerical density model in Section 3.1 was also conducted using the Gauss–Newton algorithm for comparison purposes. After an inversion time of 4709 s, the resulting inverted density model was determined, as shown in Figure 12a. The objective function value between the actual and inverted models is 0.0316, which is close to the value obtained by the proposed method (0.0249). The residuals between the inverted and observed surface gravity anomaly data are distributed within the range of -0.1 to 0.1 mGal (Figure 12c), which is slightly smaller than those produced by the proposed method. The inversion results

shown in Figure 12 suggest that the Gauss–Newton algorithm demonstrates a comparable ability to search for the optimal value relative to the proposed method. Additionally, the boundary detection capabilities for the anomalies were compared between the traditional and proposed methods for further evaluation. Figure 13 presents the vertical slices at $Y = 1500$ m (Figure 13a for the traditional method and Figure 13b for the proposed method) and the density value curves (Figure 13c) along a straight line passing through the centers of two density anomalies on the vertical slices. The density value curves from the actual model, the traditional method, and the proposed method in Figure 13c clearly show that the curve from the proposed method more closely matches the actual values than that of the traditional method. The results demonstrate that the proposed method outperforms the traditional method in terms of boundary detection capabilities. Particularly, the proposed method can predict the subsurface density model based on the measured gravity data in real time, without requiring more than an hour to compute the inversion results. Undoubtedly, the proposed inversion method is more suitable for gravity inversion than the traditional method.

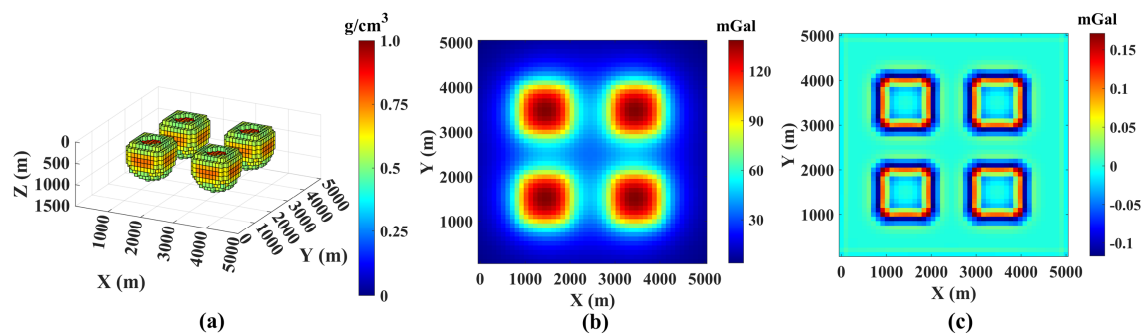


Figure 12. Inversion results of the numerical density model with four rectangular-shaped anomalies by the traditional inversion method. (a) Three-dimensional representation of the inverted density model. (b) Gravity anomaly data forwarded from the inversion result. (c) Residuals between inverted and observed surface gravity anomaly data.

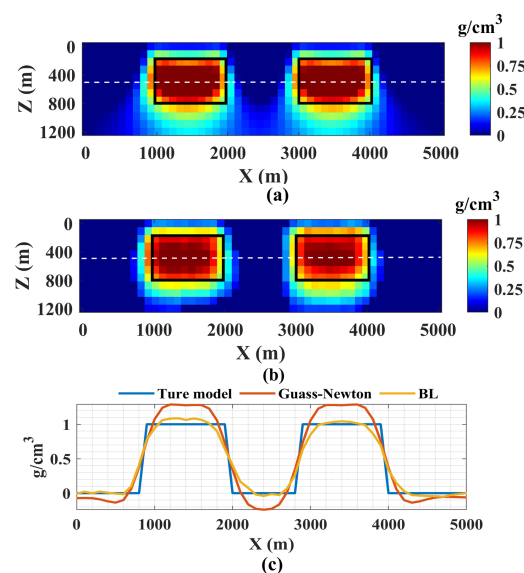


Figure 13. Comparison of boundary detection capability for anomalies between the traditional and proposed methods. (a) Vertical slice at $Y = 1500$ m by the traditional method. (b) Vertical slice at $Y = 1500$ m by the proposed method. (c) Comparison of boundary detection capability for anomalies. Note: Each black rectangular box in (a,b) represents the actual boundary of an anomaly, while the density value curve in (c) depicts the density values along the white dashed line in (a,b).

It is known that machine learning methods rely on input–output training samples to learn the functional relationship needed for various modeling tasks, including gravity inversion, which is also a modeling problem. The number of density anomalies is an inevitable consideration during the construction of training samples. This study proposes constructing single-anomaly training samples to eliminate the need for such consideration, removing the dependency on prior information about the number of anomalies. The numerical and field data examples showcased the effectiveness of using single-anomaly training samples for gravity inversion. The success of this sample construction approach depends on the generalization ability of machine learning, which is precisely what modeling problems expect from machine learning. The linear superposition property of gravity anomalies is specifically considered to investigate the underlying rationale behind the expected generalization ability of single-anomaly training samples. Figure 14 visualizes this property, illustrating that the surface gravity anomaly produced by two anomalous bodies is equivalent to the sum of the gravity anomalies generated by each individual body. Evidently, this linear superposition relationship can be leveraged by machine learning. Consequently, the proposed approach for generating training samples enables the automatic determination of the number of anomalies.

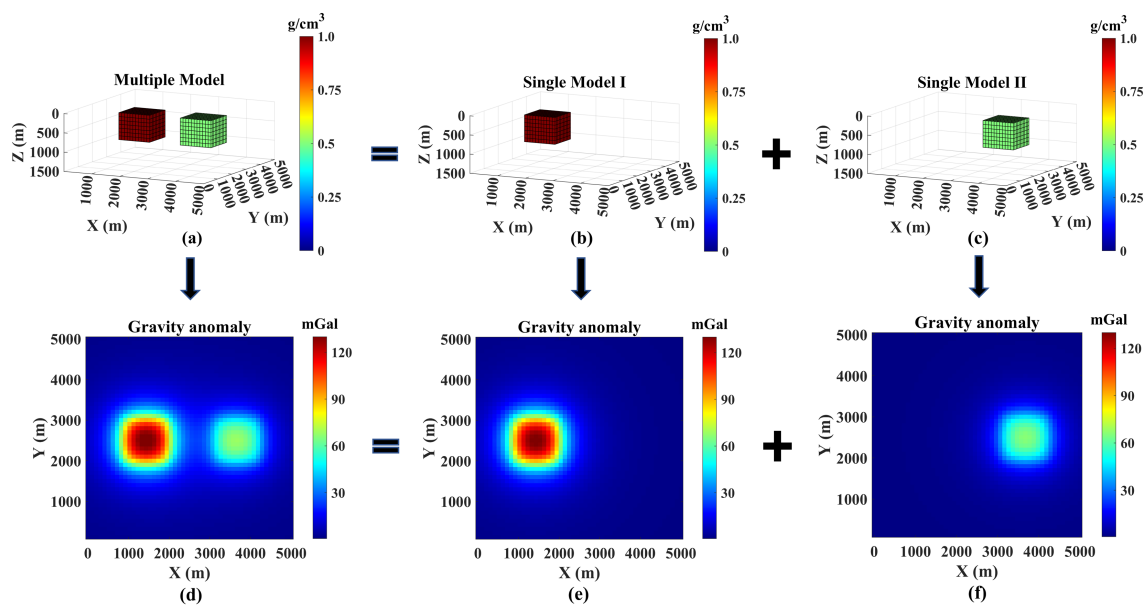


Figure 14. Visualization of the linear superposition property of gravity anomalies. (a,d) Density model with two anomalies and the corresponding theoretical surface gravity anomaly observations. (b,e) Density model with the first of the two anomalies and its theoretical surface gravity anomaly observations. (c,f) Density model with the second of the two anomalies and its theoretical surface gravity anomaly observations. The theoretical surface gravity anomaly observations of each model were calculated by the forward modeling function for gravity data.

Meanwhile, the BL method still faces limitations in terms of depth resolution due to inherent non-uniqueness [11,46,48,49]. The inversion results for the Vinton salt dome indicate that, although there is an improvement in depth resolution compared with traditional inversion methods, discrepancies remain when compared with the geological data, particularly at the bottom boundary. Reference [43] proposed a sample selection method to enhance the mapping capabilities of a BL network. This suggests that the performance of our method could be further improved by incorporating this strategy.

5. Conclusions

This study developed a 3D BL approach for gravity data inversion to enhance gravity exploration capabilities. The proposed approach was validated using both synthetic and

field gravity anomaly data. The use of single-anomaly training samples allows for the automatic determination of the number of anomalies in machine learning-based inversion. The high efficiency of this method enables flexible network creation, thereby reducing the computation time. Compared with traditional inversion methods, the results obtained from the BL network offer higher depth resolution and greater consistency with geological data. The inversion method proposed in this study improves subsurface density imaging, providing geologists and geophysicists with enhanced tools for locating geological resources.

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