

Article

The Dynamic Soil–Foundation–Structure Interaction Problem in the Time Domain Using a Discrete Element Model

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Abstract: This study investigates the influence of the soil–structure interaction (SSI) on the seismic performance of structures, focusing on the effects of foundation size, soil type, and superstructure height. While the importance of SSI is well recognized, its impact on structural behavior under seismic loads remains uncertain, particularly in terms of whether it reduces or amplifies structural demands. A simplified dynamic model, incorporating both the mechanical behavior of the soil and structural responses, is developed and validated to analyze these effects. Using a discrete element approach and the 1940 El Centro earthquake for validation, the study quantitatively compares the response of soil-interacting structures to those with fixed bases. The numerical results show that larger foundation blocks (20 m × 20 m and 30 m × 30 m) increase the seismic response values across all soil types, causing the structure to behave more like a fixed-base system. In contrast, reducing the foundation size to 10 m × 10 m increases the flexibility of structures, particularly buildings built on soft soils, which affects the displacement and acceleration response spectra. Softer soils also increase natural vibration periods and extend the plateau region in regard to spectral acceleration. This study further finds that foundation thickness has a minimal impact on spectral displacement, but structures on soft soils show more than a 15% reduction in spectral displacement (SD) compared to those on hard soils, indicating a dampening effect. Additionally, increasing the building height from 7 to 21 m results in a more than 20% decrease in SD for superstructures with natural vibration periods exceeding 2.4 s, while taller buildings with longer natural vibration periods exhibit opposite trends. Structures built on soft soils experience larger foundation-level displacements, absorbing more seismic energy and reducing earthquake accelerations, which mitigates structural damage. These results highlight the importance of considering SSI effects in seismic design scenarios to achieve more accurate performance predictions.

Keywords: dynamic soil–foundation–structure interaction; discrete element model; central finite difference; response spectra



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1. Introduction

The dynamic soil–foundation–structure interaction (SFSI) is a complex, multidisciplinary field that encompasses civil, structural, and earthquake engineering. Due to its interdisciplinary nature, particular attention must be given to SFSI problems. The interaction between the soil and the structure influences the overall dynamic response, and neglecting this interaction can lead to inaccurate predictions on structural behavior, especially in terms of stress and strain levels. Additionally, during an earthquake, energy is dissipated through both the soil and the structure as the ground shakes. This dissipation reduces the forces transmitted to the structure, particularly in soft soils that deform and absorb more energy. Dynamic SFSI analysis is essential for developing designs that are both safe and cost effective. Recent studies on the SFSI have advanced readers' understanding of how soil dynamics influence structural responses during earthquakes [1–5]. Various physical and numerical models have been used to address these issues. The key

challenge in dynamic SFSI analysis is the representation of unbounded soil behavior at infinity. Earlier research developed simple physical models to represent unbounded soil [6]. Physical models have been categorized into two types: cone models and lumped-parameter models. These models are particularly useful because they allow for pure time-domain computation, avoiding the abstraction of frequency-domain transformation and enabling the consideration of nonlinear structural behavior. Time-domain analysis also facilitates the assessment of how various key parameters influence the soil–structure interaction (SSI) response.

To conduct SSI analysis, numerical methods that integrate boundary element (BE), finite element (FE), and scaled boundary finite element approaches are employed. A coupled FE–BE method was proposed by [7] for studying SSI issues in the time domain using real earthquake records. In this approach, a concrete dam–foundation system was divided into a near field and a far field, where the FE approach was used to model the near field and the BE approach was used for the far field. Equilibrium and compatibility conditions at the soil–structure interface linked these two methods. A hybrid approach, combining BE and FE, was used to analyze the dynamic interaction between soil and a cooling tower [8] with the numerical results compared to findings from full-scale tests. A dynamic SFSI study was presented by [9], where the soil was modeled with BE, and the structure and foundation were modeled with FE. The coupling of these elements at the soil–structure interface was achieved through equilibrium and compatibility conditions. The combined finite difference/finite element (FD/FE) method developed by [10] employed a FD in the time domain to compute the soil stresses and velocities for each cycle, and the displacement of soil grid points along the soil–structure boundary. These displacements were then used by the FE algorithm to calculate soil–structure nodal forces based on the stress state of the deformed structures, which were subsequently fed back into the FD code for use in the next time step. For frequency-domain solutions to dynamic SSI problems, ref. [11] investigated a coupled FE and fast BE approach. Numerical examples were used to assess the performance of direct, iterative, and monolithic coupling procedures. A coupled model combining FEM, BEM, and scaled boundary FEM was developed to assess the dynamic response of two-dimensional structures on layered soil media [12]. This model used a substructure method to link these approaches, and a computer program was created to perform the harmonic analysis of SSI systems. Using FEM and Minimax Probability Machine Regression (MPMR), ref. [13] studied the effects of the dynamic SSI on multi-story buildings with varying height-to-width ratios, modeling different soil types and conducting time history analysis for the El Centro and Bhuj earthquakes. The MPMR technique was found to be fast, while FEM analysis required considerable computational time. By considering the dynamic stiffness of soil, ref. [14] examined the SSI effects on structural dynamic responses using a substructure approach. The unbounded soil region was analyzed in the time domain using a coupled model, based on FE and scaled boundary FE methods, and the results from two-dimensional frameworks were compared with those from the cone model.

The cone model offers practical and efficient approximation for SSI analysis. In this model, the soil underlying the structure is idealized as a truncated, semi-infinite elastic cone, which is represented by frequency-dependent dashpots and springs for each degree of freedom. Both translational and rotational cone models are available. The cone is treated as a rod (bar), with its cross-sectional displacement pattern determined by the corresponding value on the axis of the cone, where plane cross-sections remain planar. The soil half-space outside the cone is disregarded.

The cone model performs well in low-to-intermediate frequency ranges, such as those associated with earthquakes and machine vibrations [15]. According to [16], a semi-infinite truncated cone can approximately represent the soil in SSI analysis. The cone model has been used to study torsional rotation, rocking, and vertical and horizontal translations. Using the approximate cone model method, ref. [17] investigated the dynamic analysis of a foundation resting on saturated porous media under torsional and horizontal vibra-

tions. This study analyzed the response of foundations to torsional and shear vibrations, considering parameters such as density, porosity, shear modulus, and Poisson's ratio. The study by [18] modeled foundations on deformable soil with a simple spring–dashpot–mass system using frequency-independent coefficients. For both homogeneous half-space sites and homogeneous layers fixed at their base, the provided tables list coefficients for various parameters, including dimensional ratios and Poisson's ratio. The study considered both embedded and surface rigid foundations for translational and rotational motions. Using a cone model in the time domain, ref. [19] investigated SSI effects on rigid blocks and single-degree-of-freedom superstructures, comparing results from time and frequency domain analyses. The study also examined the impact of the reflection coefficient under two different soil layering configurations. To estimate the dynamic behavior of simple structures resting on an elastic half-space, ref. [20] developed and applied discrete models. The study derived precise analytical expressions for the fundamental natural period and corresponding damping coefficients as functions of geometry and frequency-dependent foundation impedances. It also introduced the concept of statically and geometrically equivalent SSI systems and compared the radiation damping produced by a single pile and a footing under harmonic loading. An overview of the available modeling techniques for addressing SSI issues and the impact of dynamic SSI on building structures is provided in [21].

While there has been extensive research on soil–structure interaction (SSI), several key gaps remain. (i) Many studies acknowledge that SSI effects are significant in soft soils, yet there is no clear consensus on how these effects influence seismic performance across various types of structures. (ii) Some studies suggest that SSI can reduce structural demand, while others indicate it may amplify forces, particularly in high-rise or flexible structures. (iii) Furthermore, accurately modeling the interaction between soil and structure remains challenging. (iv) Although most studies rely on specific soil models, soil behavior is inherently complex and variable under dynamic conditions.

These gaps limit researchers' ability to verify the accuracy of numerical solutions and the underlying assumptions in SSI analyses. This study aims to fill these gaps by investigating how soft soil interactions influence seismic performance using a simplified dynamic model. Developing and validating simple mechanical models that accurately capture SSI behavior under seismic conditions represents a key advancement in SSI research. Additionally, this study seeks to clarify how various factors—such as foundation block dimensions, variations in superstructure height, and different soil types—affect structural behavior under seismic loads, providing insights to improve reliability and uniformity in engineering applications. This work focuses on quantitatively investigating how the response of soil-interacting structures differs from that of fixed-base (non-interacting) structures. Another unique aspect of this research is its ability to systematically examine the time-domain response of the dynamic SFSI system. The increased flexibility and natural vibration period of the dynamic system in the presence of soil are presented in detail through examples. Dynamic SFSI analysis is performed by modeling the behavior of unbounded soil extending to infinity using a discrete-element approach. For a rigid basemat, the degrees of freedom at the structure–soil interface are considered as those of a rigid body. Specifically, for this configuration, physical approximations, such as truncated, semi-infinite one-dimensional cones, are used to represent unbounded soil. The dynamic system is analyzed by considering the coupling between rocking and horizontal motions. In the context of SSI and dynamic analysis, coupled rocking and horizontal motions provide a more realistic representation, as soil behavior typically involves both vertical and horizontal movements simultaneously. This approach more accurately reflects the physical interactions that occur during seismic or dynamic loading events, where rocking and horizontal motions occur simultaneously due to the complex nature of the soil response.

This analysis is conducted using the 1940 El Centro earthquake ($M_w = 6.9$), focusing solely on the inertial interaction while excluding the effects of seismic input modification

on structural response. Dynamic SFSI analysis is performed using the central Finite Difference (FD) method in the time domain. This method is advantageous for its simplicity and efficiency, as it bypasses the need for computational complexities, saving both time and effort. Unlike traditional frequency-domain studies, which require computationally intensive Fourier transformations, the proposed time-domain approach is faster. The proposed time-domain computational technique is both precise and well suited for SSI calculations. Many SSI problems involve dynamic, time-dependent behavior, especially under seismic loading or other transient events. The FD method is well suited for explicit time-stepping schemes [22]. When implemented with small time steps, the FD method can accurately capture high-frequency wave components, which are often critical in SSI problems, particularly during seismic events. This enables the precise resolution of small, rapid changes in stress or strain, providing accurate predictions of wave propagation and soil–structure interactions [23].

2. Dynamic Model of SFSI for Coupled Horizontal and Rocking Motions

Consider a superstructure modeled as a damped, one-degree-of-freedom soil-interaction system. This system consists of a mass m , a dashpot with damping coefficient c , and a static spring with stiffness k . The mass–spring–dashpot system is connected to a rigid bar of height H , as shown in Figure 1.

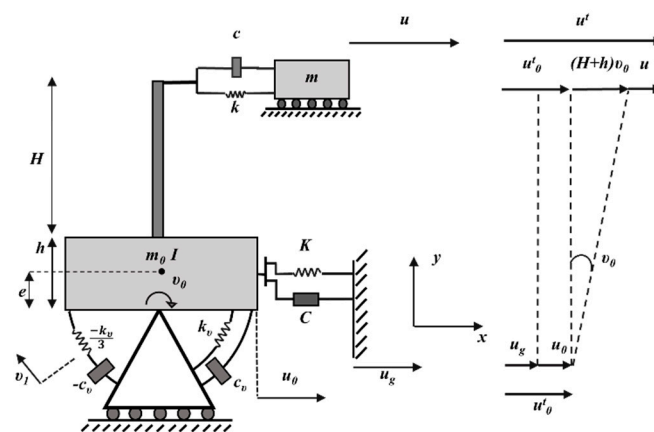


Figure 1. Dynamic model of the SFSI system (including rocking and horizontal motions).

The connection between the soil and the superstructure is provided by a foundation block. This rigid foundation block has dimensions of length ($2a$), width ($2b$), and thickness (h), forming a prismatic shape, as shown in Figure 2. The mass of the foundation block is m_0 , and its mass moment of inertia relative to the rotation axis passing through the center of gravity is I (as illustrated in Figure 1). The corresponding mass moment of inertia is

$$I = \frac{1}{12} m_0 \left[(2a)^2 + h^2 \right] \quad (1)$$

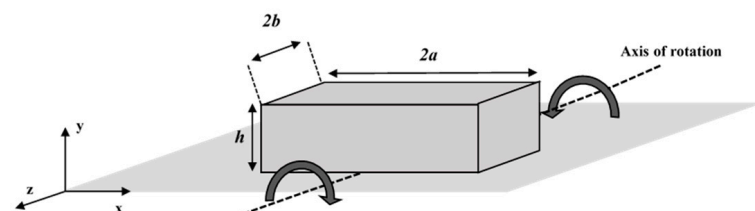


Figure 2. Geometry of the foundation block.

The explicit computation of wave displacements propagating through unbounded soil is generally unnecessary for practical applications. Instead, the focus is placed on the

interaction force–displacement relationship at the foundation block. In the translational cone model, a dashpot with coefficient C and a spring with a static stiffness coefficient K are arranged in parallel. In the rotational cone model, the high-frequency limit of radiation damping c_v of the rotational dashpot and the rotational spring with static stiffness coefficient k_v are also connected in parallel.

In this model, the foundation node, with a rocking degree of freedom v_0 , is connected to a roller support. A rotational dashpot with coefficient $-c_v$ connects the roller support to an additional internal rotational degree of freedom v_1 , which is linked to the foundation by a rotational spring with coefficient $-k_v/3$.

The dynamic SFSI system depicted in Figure 1 accounts for the coupling between rocking and horizontal motions. Since the center of the foundation block–soil interface is located at a vertical distance $e = 0.5 h$ from the foundation block’s center of gravity, the rocking and horizontal motions are coupled [24].

The amplitudes of the relative displacement between the base and the mass, referred to as the free-field motion (u_g), and those in a moving frame of reference attached to the rigid base are denoted as u_0 and u , respectively. In Figure 1, the total lateral displacements of the mass and the foundation are denoted as u^t and u_0^t , respectively. The unlayered soil is idealized as a homogeneous, linearly elastic, semi-infinite medium with mass density ρ , and material damping is not considered for the soil. The cones addressed are analyzed under the a priori assumption that the soil is a perfectly elastic medium, which dissipates energy only through the radiation of waves toward infinity. The second important source of energy dissipation—material damping—is also generally present. Material damping affects the spring and damping coefficients of the dynamic stiffness. It causes a pseudo-hyperbolic increase in the low-frequency damping coefficient and a pseudo-linear decrease in the high-frequency spring coefficient. In engineering practice, material damping is typically negligible and does not need to be considered for vibration problems that usually induce rotation angles several orders of magnitude smaller [24]. In contrast, large-amplitude rocking occurs during strong-motion earthquakes. For such cases, it is appropriate to include material damping in the analysis and update its value at each time step. In this study, it is assumed that the spring coefficient is unaffected by material damping and that material damping does not modify radiation damping in the damping coefficient. This form of the dynamic-stiffness coefficient will be used to derive simple expressions for the properties of an equivalent one-degree-of-freedom system that models the dynamic interaction between the structure and the soil. The fundamental soil properties in cone models include the propagation velocities of shear waves (c_s) and dilatational waves (c_p). The following relationships can be used to calculate these fundamental properties:

$$c_s = \sqrt{\frac{G}{\rho}} \quad (2)$$

$$c_p = \sqrt{\frac{E_c}{\rho}} \quad (3)$$

Here, E_c and G represent the elastic constrained modulus and the shear modulus, respectively. The relationship between E_c and G is expressed as follows:

$$E_c = 2G \left(\frac{1-\nu}{1-2\nu} \right) \quad (4)$$

Here, ν represents Poisson’s ratio. The static stiffness coefficient K for the translational cone in horizontal motion can be determined using the following expression [25]:

$$K = \frac{Gb}{2-\nu} \left[6.8 \left(\frac{a}{b} \right)^{0.65} + 2.4 \right] \quad (5)$$

Here, a represents half the length of the rigid foundation block, and b represents half its width. Additionally, the static stiffness coefficient k_v for the rotational cone's rocking motion is expressed as follows [25]:

$$k_v = \frac{Gb^3}{1-\nu} \left[3.73 \left(\frac{a}{b} \right)^{2.4} + 0.27 \right] \quad (6)$$

The constant dashpot coefficient C for translational motions can be expressed as the product of the density ρ , the shear-wave propagation velocity c_s , and the area of the foundation block A_0 , as follows:

$$C = \rho c_s A_0 \quad (7)$$

The constant dashpot coefficient for rocking motions can be expressed as follows:

$$c_v = \rho c I_0 \quad (8)$$

Although c (corresponding to the wave propagation velocity) must be restricted to a value of $2c_s$ for nearly incompressible soil ($1/3 < \nu \leq 1/2$), it is equal to c_p in the compressible range (when $\nu \leq 1/3$) for rocking motion. Here, I_0 is the moment of inertia of the foundation (basemat) about the axis of rotation, and it is given by

$$I_0 = \frac{1}{12} (2a)^3 2b \quad (9)$$

3. Methodology

Dynamic System Coupling Rocking and Horizontal Motions: For the dynamic system shown in Figure 1, the equations of motion include two translational and two rotational components:

$$m[\ddot{u}_g + \ddot{u}_0 + \ddot{u} + (H+h)\ddot{v}_0] + m_0(\ddot{u}_g + \ddot{u}_0 + e\ddot{v}_0) + Ku_0 + C\dot{u}_0 = 0 \quad (10)$$

$$m[\ddot{u}_g + \ddot{u}_0 + \ddot{u} + (H+h)\ddot{v}_0] + ku + c\dot{u} = 0 \quad (11)$$

$$m[\ddot{u}_g + \ddot{u}_0 + \ddot{u} + (H+h)\ddot{v}_0](H+h) + I_e\ddot{v}_0 + c_v\dot{v}_0 + k_v v_0 - \frac{k_v}{3}(v_0 - v_1) - Ce\dot{u}_0 - Keu_0 = 0 \quad (12)$$

$$-c_v\dot{v}_1 - \frac{k_v}{3}(v_1 - v_0) = 0 \quad (13)$$

where I_e is the mass moment of inertia of the system, which comprises the structure and foundation block. It is calculated relative to the rotation axis and considers the mass of the superstructure (m) and the eccentricity (e). The corresponding relation is

$$I_e = I + m_0 e^2 + m(h+H)^2 \quad (14)$$

Linear SSI analyses offer significant advantages in many engineering contexts, though they have limitations. They are valuable because they provide an efficient means of assessing structural responses, particularly under lower-level seismic events where nonlinear effects may not be prominent. Linear analysis can serve as a preliminary tool, allowing for simpler, approximate solutions that facilitate early-stage design. It also serves as a supporting framework for interpreting complex nonlinear results and can yield insights into systems with strong nonlinearities, as the results of nonlinear analyses sometimes mirror those of a linear model. However, linear analysis alone cannot fully capture the behavior of systems under high-intensity seismic events or other extreme loads, where soil and structural nonlinearities play a crucial role. Nonlinear effects are essential for accurately representing complex interactions, such as sliding, soil–structure separation, and base isolation behavior during severe earthquakes. For maximum credible and other high-seismic events, nonlinear modeling becomes necessary to accurately reflect the behav-

ior of the soil and structure. Future SSI analyses may incorporate nonlinearity by using advanced soil models, integrating material and geometric nonlinearities in the structure, and employing coupled nonlinear numerical techniques such as finite element or finite difference methods.

Although the developed dynamic model is approximate, it provides sufficient accuracy for practical purposes. Given the inherent uncertainties discussed, a deviation of $\pm 20\%$ from the rigorous solution for a single set of parameters is typically acceptable, as supported by the dynamic stiffness coefficients presented in [24]. Furthermore, for transient loads such as earthquakes, these deviations are distributed over the frequency range, reducing the impact of the maximum error at any single frequency. While simplified physical models may lose some precision compared to boundary element or finite element methods, they offer substantial practical advantages, as highlighted in this work.

To replace each derivative in the differential equations given in Equations (10)–(13) with an appropriate difference quotient approximation, the following central-difference formulas are used:

$$\dot{U}(x) = \left(\frac{dU}{dx} \right) = \frac{1}{2\Delta x} \{U(x + \Delta x) - U(x - \Delta x)\} \quad (15)$$

$$\ddot{U}(x) = \left(\frac{d^2U}{dx^2} \right) = \frac{1}{(\Delta x)^2} \{U(x + \Delta x) - 2U(x) + U(x - \Delta x)\} \quad (16)$$

Here, Δx represents a constant step size, where $x_j - x_{j-1} = \Delta x$, and the derivatives of the $U(x)$ function are single-valued, finite, and continuous. The finite difference (FD) method uses discrete approximations for the derivatives in the differential equation with these approximations derived from Taylor series expansions. There are three types of approximations: central, forward, and backward. The central difference formula, which yields better results, is commonly used. In contrast to the forward and backward schemes, where the error decreases linearly, the error in the central difference method decreases quadratically as the step size decreases [26,27].

In the FD approximation, the values of the dependent variable at a point in the solution region are related to the values at neighboring points. The solution region is divided into n subintervals, each of length Δx . For an accurate approximation, the step size should be sufficiently small. In practice, large step sizes can accelerate simulations, but they may also lead to instability. To determine the appropriate step size for the central FD method and demonstrate its applicability, the forced vibration of a damped one-degree-of-freedom system is analyzed using the piecewise linear interpolation method. A semi-sinusoidal load, which is a piecewise function and shown in Figure 3, is applied to the system as an external load.

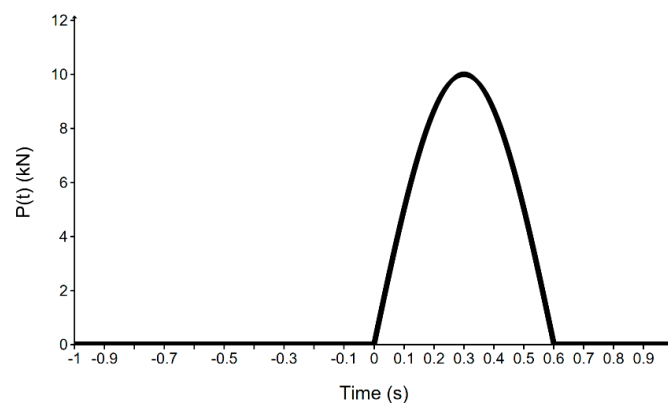


Figure 3. A semi-sinusoidal, piecewise loading function.

The load applied to the system is given by the following form:

$$P(t) = \begin{cases} 0, & t < 0 \\ 10\sin\left(\frac{\pi t}{0.6}\right), & 0 \leq t \leq 0.6 \\ 0, & t > 0.6 \end{cases} \quad (17)$$

The equation of motion for the forced vibration of the damped, one-degree-of-freedom system is as follows:

$$m + c\dot{u} + ku = P(t) \quad (18)$$

The recurrence formulas for evaluating the dynamic state (u_{n+1} , $\dot{u}_{n+1}$) at time t_{n+1} , given the state (u_n , $\dot{u}_n$) at time t_n , are expressed as follows:

$$u_{n+1} = Au_n + B\dot{u}_n + C(P_{n-1}\Delta t) + D(P_n\Delta t) \quad (19)$$

$$\dot{u}_{n+1} = A'\dot{u}_n + B'\dot{u}_n + C'(P_{n-1}\Delta t) + D'(P_n\Delta t) \quad (20)$$

The coefficients A through D' for the underdamped case ($\zeta = 0.05$) are as follows:

$$A = e^{-\zeta\omega_n\Delta t} \left(\frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_D\Delta t + \cos\omega_D\Delta t \right) \quad (21)$$

$$B = e^{-\zeta\omega_n\Delta t} \left(\frac{1}{\omega_D} \sin\omega_D\Delta t \right) \quad (22)$$

$$C = \frac{1}{k} \left\{ \frac{2\zeta}{\omega_n\Delta t} + e^{-\zeta\omega_n\Delta t} \left[\left(\frac{1-2\zeta^2}{\omega_D\Delta t} - \frac{\zeta}{\sqrt{1-\zeta^2}} \right) \sin\omega_D\Delta t - \left(1 + \frac{2\zeta}{\omega_n\Delta t} \right) \cos\omega_D\Delta t \right] \right\} \quad (23)$$

$$D = \frac{1}{k} \left[1 - \frac{2\zeta}{\omega_n\Delta t} + e^{-\zeta\omega_n\Delta t} \left(\frac{2\zeta^2-1}{\omega_D\Delta t} \sin\omega_D\Delta t + \frac{2\zeta}{\omega_n\Delta t} \cos\omega_D\Delta t \right) \right] \quad (24)$$

$$A' = -e^{-\zeta\omega_n\Delta t} \left(\frac{\omega_n}{\sqrt{1-\zeta^2}} \sin\omega_D\Delta t \right) \quad (25)$$

$$B' = e^{-\zeta\omega_n\Delta t} \left(\cos\omega_D\Delta t - \frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_D\Delta t \right) \quad (26)$$

$$C' = \frac{1}{k} \left\{ -\frac{1}{\Delta t} + e^{-\zeta\omega_n\Delta t} \left[\left(\frac{\omega_n}{\sqrt{1-\zeta^2}} + \frac{\zeta}{\Delta t\sqrt{1-\zeta^2}} \right) \sin\omega_D\Delta t + \frac{1}{\Delta t} \cos\omega_D\Delta t \right] \right\} \quad (27)$$

$$D' = \frac{1}{k\Delta t} \left[1 - e^{-\zeta\omega_n\Delta t} \left(\frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_D\Delta t + \cos\omega_D\Delta t \right) \right] \quad (28)$$

The values of u and $\dot{u}$ in Equations (19) and (20) represent the displacement and velocity of the system at any given time, respectively. The system is solved separately for two time intervals (step sizes): $\Delta t = 0.1$ s and 0.01 s. The values used in the analysis are provided in Table 1 and were calculated using the following relations:

$$\omega_n = \sqrt{\frac{k}{m}} \quad (29)$$

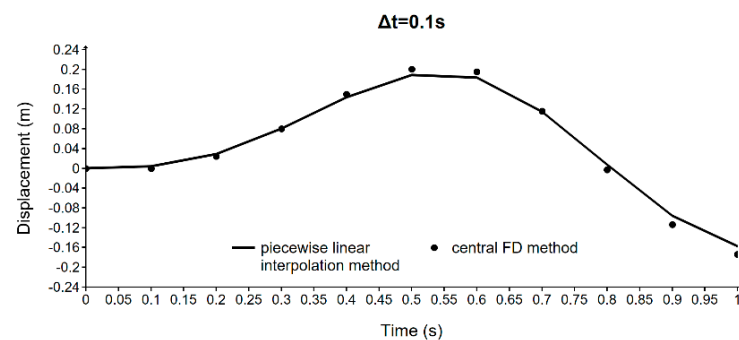
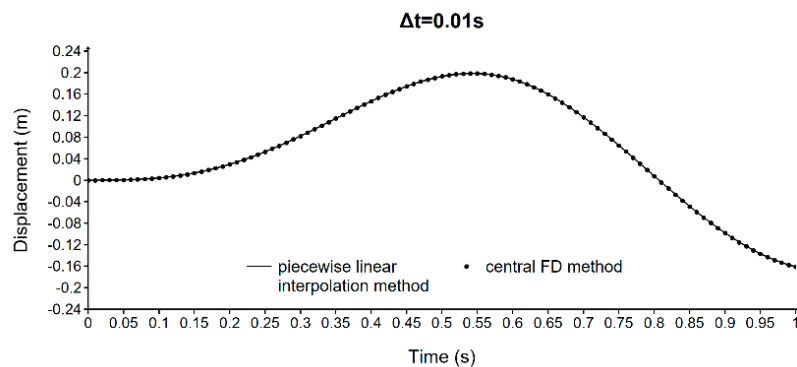
$$c = 2\zeta m\omega_n \quad (30)$$

$$\omega_D = \omega_n \sqrt{1-\zeta^2} \quad (31)$$

Table 1. Properties of the damped one-degree-of-freedom system.

Mass (m) (kg)	Natural Period (T) (s)	Damping Ratio (ζ)	Angular Frequency (ω_n) ($\frac{\text{rad}}{\text{s}}$)	Damped Angular Frequency (ω_D) ($\frac{\text{rad}}{\text{s}}$)	Stiffness Coefficient (k) ($\frac{\text{kg}}{\text{s}^2}$)	Damping Coefficient (c) ($\frac{\text{kg}}{\text{s}}$)
2	1	0.05	6.283	6.275	78.957	1.257

The displacement vs. time graphs, obtained using the piecewise linear interpolation and central FD methods for time steps $\Delta t = 0.1$ and 0.01 s, are compared in Figures 4 and 5, respectively.

**Figure 4.** Comparison of results obtained using piecewise linear interpolation and the central FD method for $\Delta t = 0.1$ s.**Figure 5.** Comparison of results obtained using piecewise linear interpolation and the central FD methods for $\Delta t = 0.01$ s.

As clearly shown in Figures 4 and 5, as the time step decreases, the results obtained from both methods converge. For $\Delta t = 0.01$ s, the convergence of the results is satisfactory for the central FD method. The subsequent analysis is performed with $\Delta t = 0.002$ s, starting with vanishing initial conditions, to obtain an accurate solution. By reducing Δt from 0.01 to 0.002 s, the number of time steps increases by a factor of 5, which also raises the computational time required for each iteration. However, earthquake data are provided at a finer time resolution ($\Delta t = 0.002$ s). Therefore, using a coarser time step ($\Delta t = 0.01$ s) could lead to aliasing or inaccuracies in how the earthquake data are applied. Using $\Delta t = 0.002$ s ensures that the model accurately captures the variations in the earthquake input.

4. Examples of Seismic Analysis of the SFSI System

In this study, the dynamic SFSI system is analyzed under seismic excitation. In the analysis, the north–south acceleration component of the 1940 El Centro earthquake, with a magnitude of $M_w = 6.9$, is used (see Figure 6).

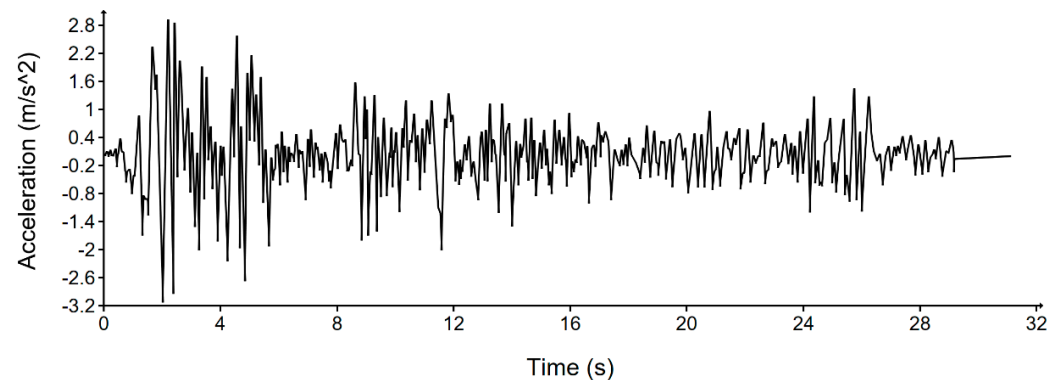


Figure 6. The north–south acceleration component of the 1940 El Centro earthquake.

The Turkish Building Seismic Code (TBSC 2018) classifies soil into six groups based on geological subsurface conditions [28]: ZA, ZB, ZC, ZD, ZE, and ZF. These categories are defined as follows: ZA (solid hard rocks), ZB (strong rocks), ZC (weak rocks), ZD (solid clay), ZE (soft clay), and ZF (highly sensitive clays). This study performs a dynamic analysis based on four different soil conditions, classified as ZB, ZC, ZD, and ZE, each with different mass densities ρ . Based on the approximate values of Poisson’s ratios for various soils, a Poisson’s ratio of $\nu = 1/3$ is assumed for all soil types.

The soil properties, including the unit mass density and the propagation velocities of shear and dilatational waves for the four different soil types, are provided in Table 2. The constrained elastic modulus (E_c) and shear modulus (G) values are calculated using the propagation velocities of dilatational waves (c_p) and shear waves (c_s) for each local soil class.

Table 2. Soil characteristics.

Soil Type	Mass Density ρ (kg/m ³)	Horizontal Translation:	Rocking:	Constrained Elastic Modulus (E_c) (N/m ²)	Shear Modulus (G) (N/m ²)
		Shear Waves Velocity (c_s) (m/s)	Dilatational Waves Velocity (c_p) (m/s)		
ZB	2200	1130	2260	1.124×10^{10}	2.809×10^9
ZC	2000	560	1120	2.509×10^9	6.272×10^8
ZD	1900	270	540	5.54×10^8	1.385×10^8
ZE	1800	90	180	5.832×10^7	1.458×10^7

5. Results and Discussion

SFSI analyses are performed for different foundation block sizes ($2a \times 2b$) with dimensions of (10 m $\times$ 10 m), (20 m $\times$ 20 m), and (30 m $\times$ 30 m) to examine the effect of foundation block size on the dynamic behavior of the superstructures. The mass density of the foundation block, with a constant height of $h = 1$ m, is $\rho_{block} = 2.5$ t/m³. In the analyses, the height of the superstructures is kept constant at $H = 14$ m.

In the interaction between the soil and the superstructure, reducing the foundation block size causes significant changes in SD values (Figure 7). When the analyses are conducted for a foundation block of (10 m $\times$ 10 m), SD values vary according to soil type. As shown in Figure 7, the SD values for superstructures interacting with hard soils (such as ZB and ZC) are higher compared to those on soft soils (such as ZD and ZE). This difference arises because the equivalent earthquake loads are reduced by the increased flexibility and damping in soil-interacting structures on relatively soft soils. This result is consistent with findings from published studies [28–31]. It is observed that this flexibility is more pronounced in buildings with smaller foundations situated on soft soils. As the foundation block dimensions increase from (10 m $\times$ 10 m) to (20 m $\times$ 20 m) and (30 m $\times$ 30 m), the spectral displacement (SD) values for each soil type converge for all superstructure systems

(Figures 8 and 9). For superstructures with larger foundation blocks, the response values increase regardless of soil type. The primary reason for this increase is the higher mass of the foundation block, which results in larger earthquake forces. This increase suppresses the damping effect of the soil and causes the structure to behave like a fixed-base structure.

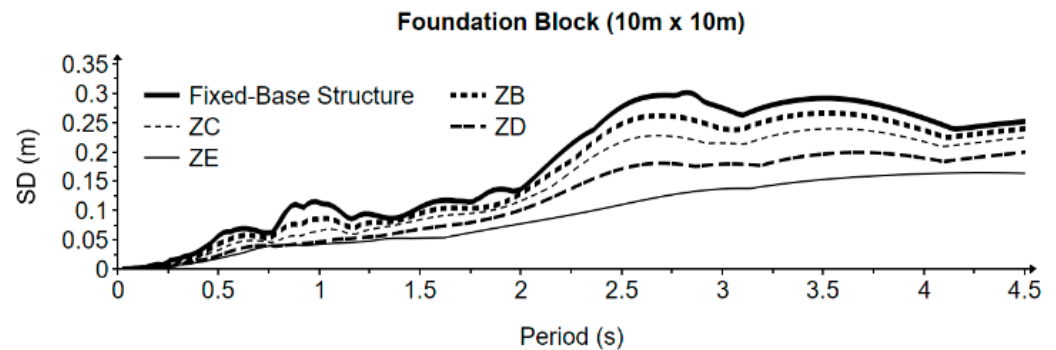


Figure 7. Effect of soil type on the SD response of structures with (10 m × 10 m) foundation blocks.

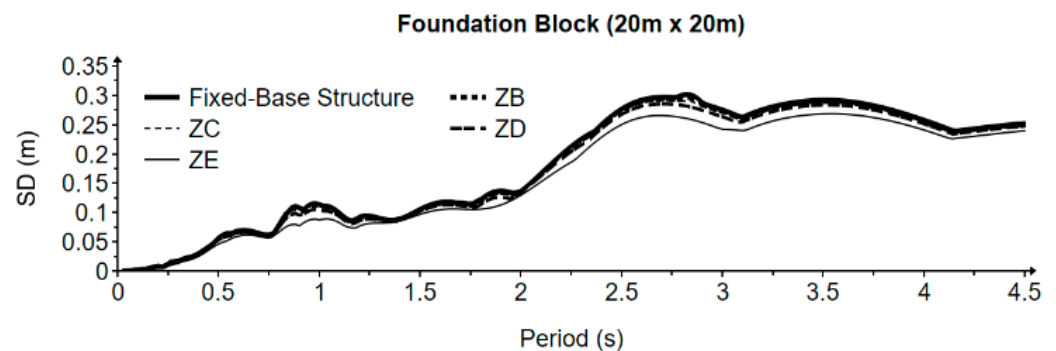


Figure 8. Effect of soil type on the SD response of structures with (20 m × 20 m) foundation blocks.

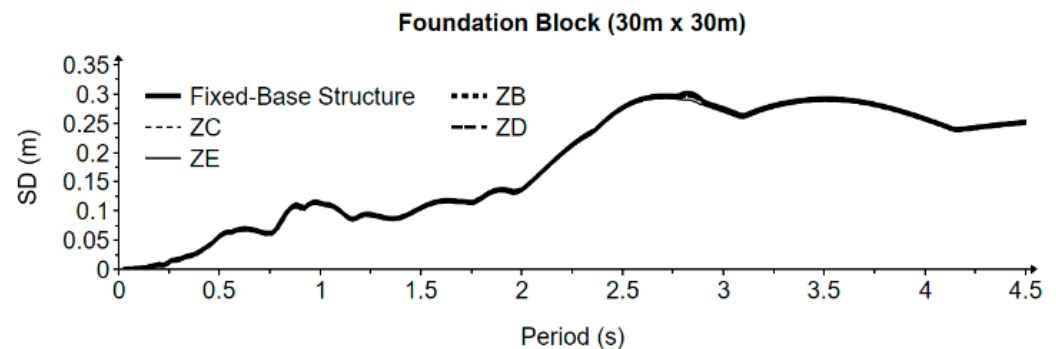


Figure 9. Effect of soil type on the SD response of structures with (30 m × 30 m) foundation blocks.

In soil-interacting structures, the system's flexibility increases compared to a fixed-base structure, altering the natural vibration periods of the structures. Analyzing the spectral acceleration (SA) values for a structure with a smaller foundation size (10 m × 10 m) shows that as the soil becomes softer, the spectral characteristic periods (or corner periods, T_A and T_B), which define the boundary values of the plateau region in the acceleration response spectra, increase (Figure 10). Moreover, for systems with small foundations on soft soils, the plateau region of the spectrum is extended compared to that of the non-interacting system (represented by the thick continuous line in Figure 10). This extension implies that the period value used in the design of the non-interacting system would fall within the plateau region of the soft-soil interacting system.

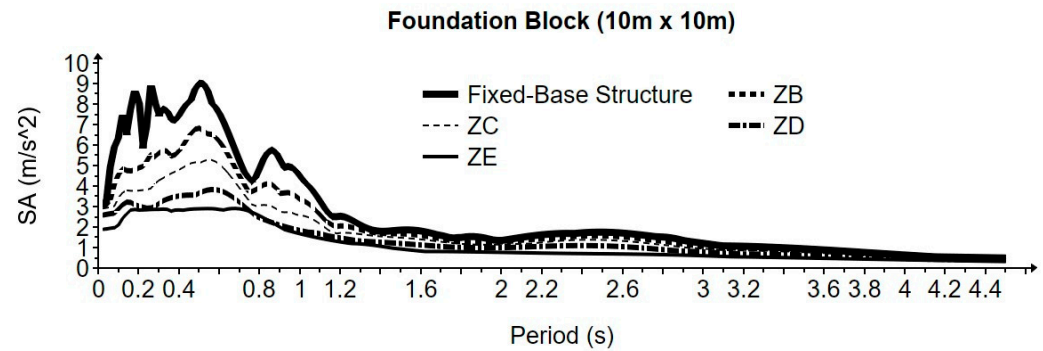


Figure 10. Effect of soil type on the SA response of structures with (10 m × 10 m) foundation blocks.

To examine the impact of foundation block thickness on the SD values, superstructures with a height of $H = 21\text{m}$ and small foundations (10 m × 10 m) placed on both hard (ZB and ZC) and soft (ZD and ZE) soils are analyzed. The foundation block thickness values considered are $h = 0.5\text{ m}$, 1.5 m , and 2.5 m . This analysis investigates how changes in foundation block thickness affect the dynamic SFSI behavior across different soil types. The SD values of the structures do not vary with changes in foundation thickness ($h = 0.5\text{ m}$, 1.5 m , and 2.5 m) for any soil type (ZB, ZC, ZD, and ZE), as shown in Figures 11–14. However, for soil types ZD and ZE, the SD values of superstructures decrease by more than 15% compared to those on ZB and ZC soils. As expected, dynamic SFSI leads to significant changes in the behavior of superstructures on soft soils. When considering SFSI effects (compared to fixed-base structures), the increased flexibility results in a reduction in the seismic loads experienced by the superstructure. Structures on soft soils experience greater vertical settlements but suffer less structural damage, while higher relative displacement values are observed for structures on hard soils compared to those on soft soil.

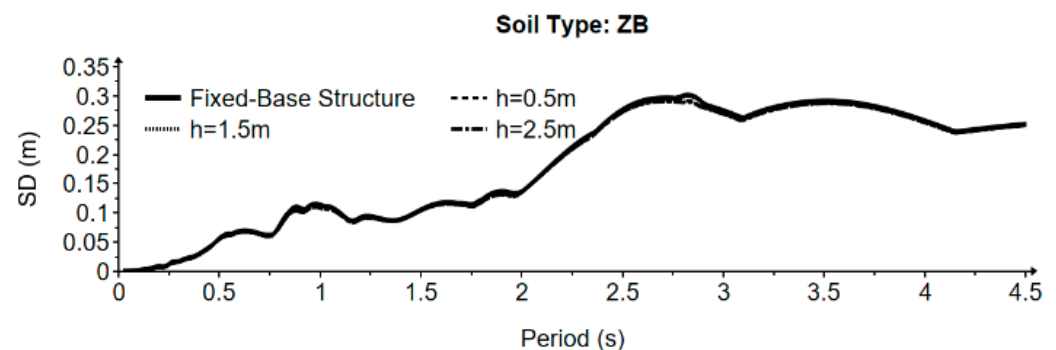


Figure 11. Effect of varying foundation block thicknesses on the SD response of structures on ZB soil.

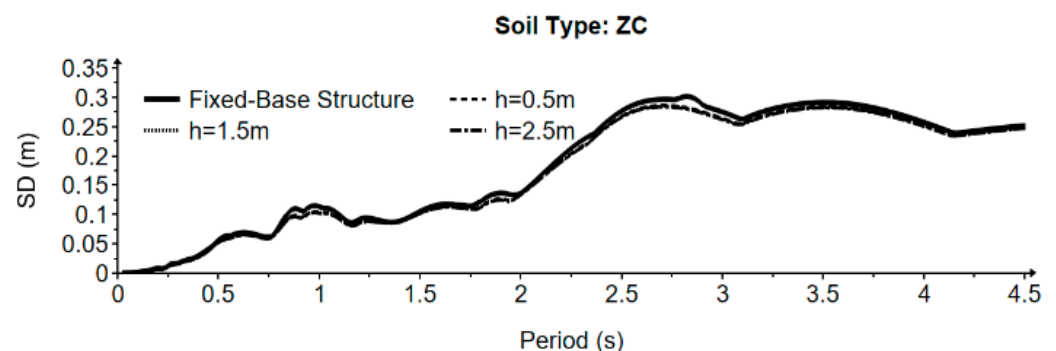


Figure 12. Effect of varying foundation block thicknesses on SD response of structures on ZC soil.

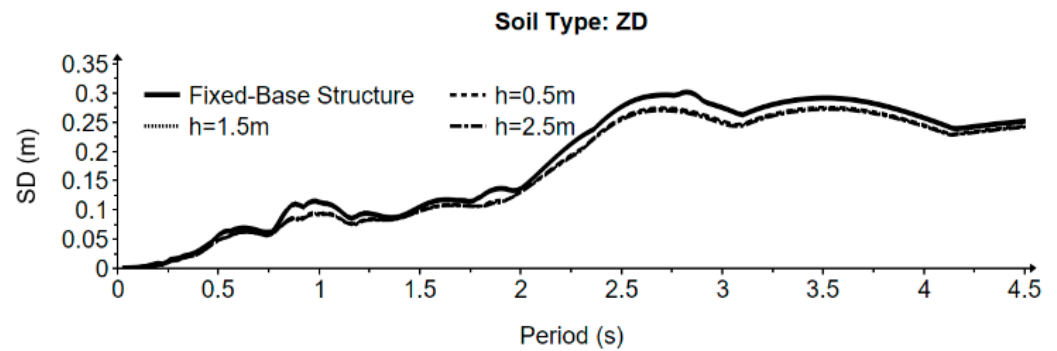


Figure 13. Effect of varying foundation block thicknesses on SD response of structures on ZD soil.

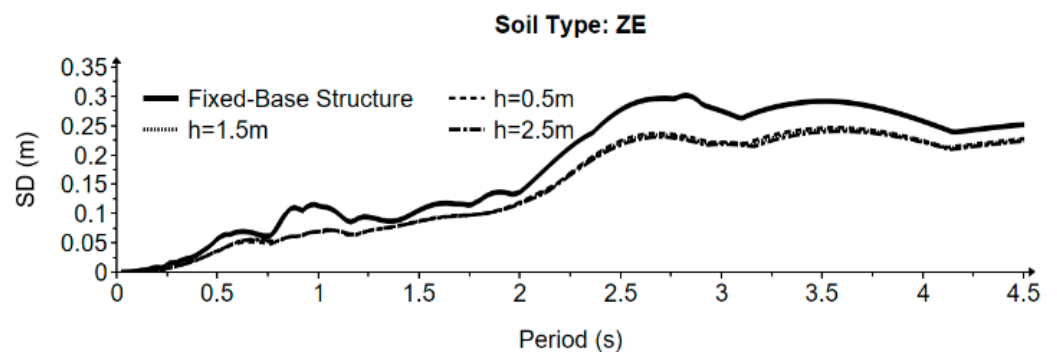


Figure 14. Effect of varying foundation block thicknesses on SD response of structures on ZE soil.

To evaluate the effects of different building heights ($H = 7, 14$, and 21 m) on acceleration and displacement response spectra, analyses are conducted on soil-interacting structures with foundations ($10 \text{ m} \times 10 \text{ m} \times 1.5 \text{ m}$) situated on different soil types. As shown in Figures 15–17, SD values significantly decrease as the building height increases for ZB, ZC, and ZD soil classes. When the height of the superstructure increases from 7 to 21 m, the SD values for superstructures with a natural vibration period exceeding 2.4 s decrease by more than 20% . Although the percentage decrease varies for the specified soil types, no consistent pattern is observed for the ZE soil type, particularly in cases where the building heights, with periods longer than 3 s, increase from 14 to 21 m (Figure 18).

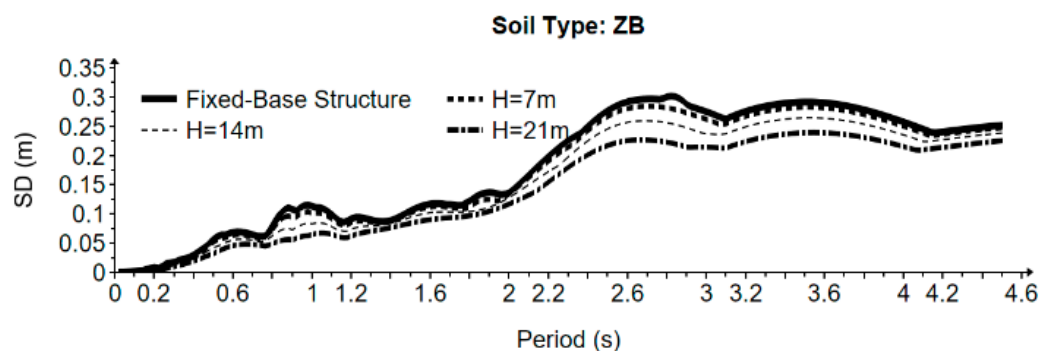


Figure 15. Effect of changes in building height on the SD response of structures on ZB soil.

The decreasing trend in SA values continues as building height increases. Additionally, as the building height on soft soil (ZE) increases from 14 to 21 m, the plateau region expands compared to the response of buildings on hard soil (ZB), as shown in Figure 19. This result indicates that the period value used for design in the hard soil system may fall within the risky plateau region for the soft soil system. Therefore, it is crucial for structural design to account for soil type to ensure safety and optimal performance.

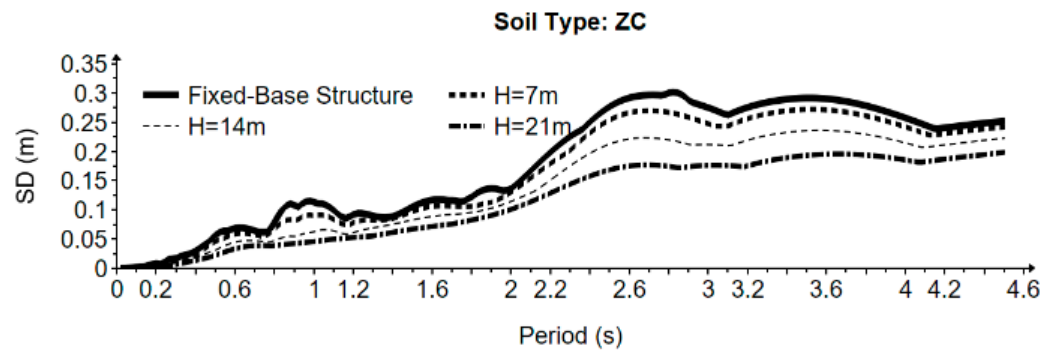


Figure 16. Effect of changes in building height on the SD response of structures on ZC soil.

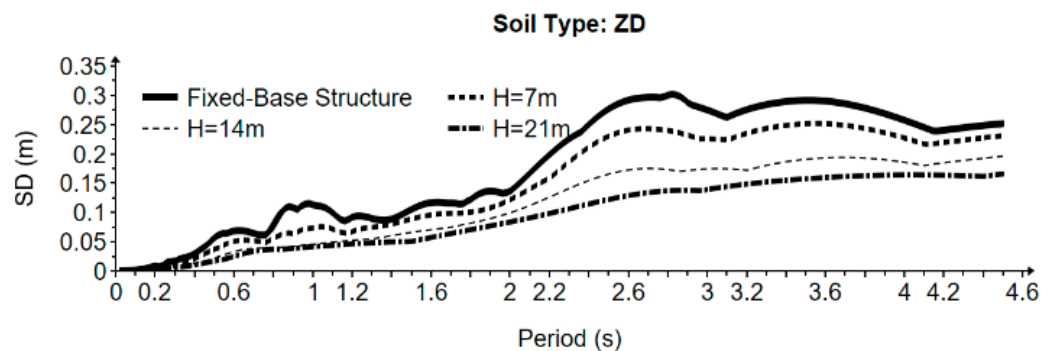


Figure 17. Effect of changes in building height on the SD response of structures on ZD soil.

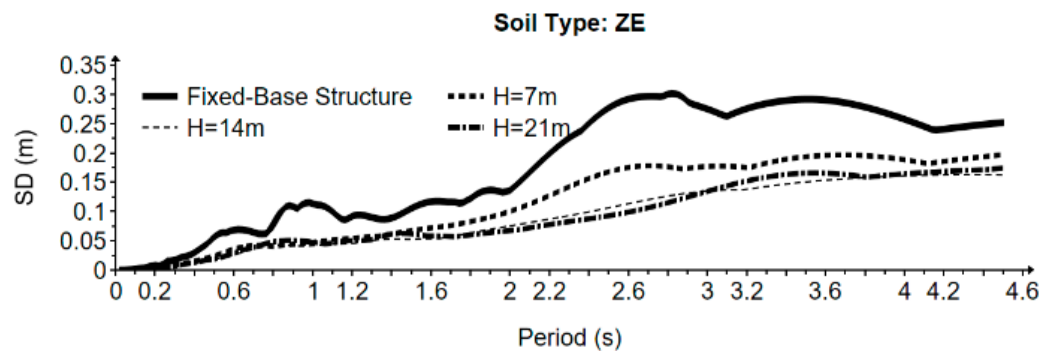


Figure 18. Effect of changes in building height on the SD response of structures on ZE soil.

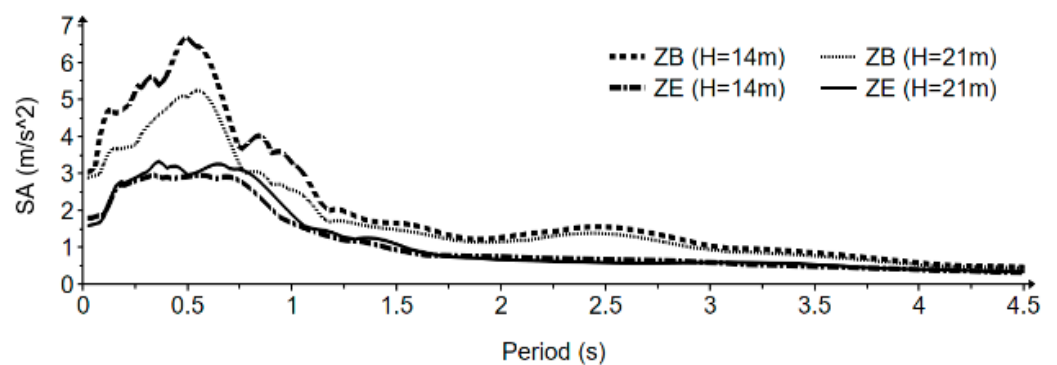


Figure 19. Effect of changes in building height on the SA response of structures on hard (ZB) and soft (ZE) soils.

When a structure is built on soft soil, the displacements at the foundation level are significantly larger compared to those on hard soil. For example, when examining the displacement values at $t = 2$ s, as shown in Figure 20, the displacement of the foundation block

on ZD soil is approximately four times greater than that on ZC soil, while the displacement on ZC soil is about three times greater than that on ZB soil. This behavior is consistently observed at other time instances as well. In structures on soft soil, a large portion of the earthquake's energy is absorbed at the foundation level, resulting in lower earthquake accelerations being transmitted to the superstructure. This implies that structural damage is generally greater in systems built on hard soils. Soils with lower bearing capacity, often referred to as soft soils, can act as natural dampers during an earthquake, absorbing more energy. As a result, the earthquake accelerations reaching the superstructure are reduced, leading to reduced structural damage. However, the term “structural damage” here refers specifically to damage that results in the loss of life, not to the idea that soft soils are inherently better for construction. Structures on soft soils can experience issues such as sinking or tilting due to high displacements, which could potentially render them dangerous or unusable after an earthquake.

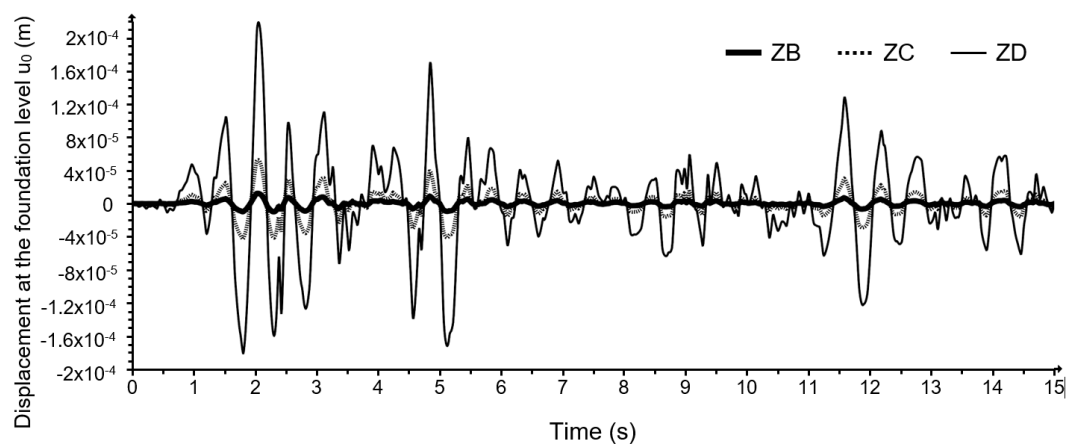


Figure 20. Displacement variation at the foundation level for a superstructure ($H = 7$ m, $T = 0.5$ s) with a $(10 \text{ m} \times 10 \text{ m} \times 0.5 \text{ m})$ foundation.

6. Conclusions

In structural engineering, accurately calculating the seismic loads on a structure is essential to ensuring its safety under dynamic loads. The most critical parameter for determining the seismic load imposed on a structure is the acceleration of earthquake ground motion. Since these accelerations cannot be predicted in advance, various approaches are adopted in structural design. One of the most important approaches is the creation of displacement and acceleration design spectra based on previously recorded earthquakes. Design spectra under different soil conditions are provided in building codes. Numerous studies have examined the dynamic interaction between structure, soil, and foundation. When a structure is placed on soft soil, displacements at the foundation level caused by inertial forces (resulting from rotation and translation) dampen seismic energy. Increased damping and decreased natural vibration frequencies significantly alter the structure's dynamic behavior. As damping and the period of the structure change due to the structure–soil interaction, the design force required for analysis becomes greater than that for fixed-base structures. This study investigates the dynamic interaction between the soil, superstructure, and rigid foundation block under seismic loading. Four types of soil are considered: strong rocks (ZB), weak rocks (ZC), solid clay (ZD), and soft clay (ZE). The goal of this study is to clarify the significance of SSI effects and to examine how key parameters, such as the dimensions of the foundation block (including its base area and thickness) and variations in the height of the superstructure, affect the displacement and acceleration response spectra of structures. The main findings of the study are as follows:

- For superstructures with large foundation blocks, such as $(20 \text{ m} \times 20 \text{ m})$ and $(30 \text{ m} \times 30 \text{ m})$, response values increase regardless of soil type. This increase is primarily due to the larger foundation mass, which amplifies earthquake forces. Consequently,

the effect of the soil is dampened, and the structure behaves more like a fixed-base structure (as shown in Figures 8 and 9).

- When the foundation block size is reduced to (10 m × 10 m), significant changes in spectral displacement (SD) and spectral acceleration (SA) values occur. The system's flexibility increases compared to a fixed-base structure, altering its natural vibration periods. This flexibility is more pronounced in buildings with smaller foundations situated on soft soils (as shown in Figure 7).
- The flexibility of soil-interacting structures alters their natural vibration periods with softer soils increasing the spectrum characteristic periods and extending the plateau region in spectral acceleration (as shown in Figure 10). This shift highlights the importance of accounting for soil–structure interaction effects in design, as assuming a non-interacting system may underestimate the dynamic response.
- Changes in foundation thickness ($h = 0.5$ m, 1.5 m, and 2.5 m) do not significantly affect the spectral displacement (SD) values for any soil type (as shown in Figures 11–14). However, structures on soft soils (ZD and ZE) exhibit a decrease in more than 15% in SD values compared to those on hard soils (ZB and ZC). This is due to the increased flexibility of soft soils, which reduces seismic loads on the structure. Superstructures on soft soils experience greater vertical settlements but less structural damage, while those on hard soils show higher relative displacement values.
- The analysis of the effects of different building heights ($H = 7$, 14, and 21 m) on acceleration and displacement response spectra reveals significant findings. For soil-interacting structures with foundations (10 m × 10 m × 1.5 m) situated on various soil types, the SD values notably decrease as the building height increases for the ZB, ZC, and ZD soil classes (as shown in Figures 15–17).
- Specifically, when the building height increases from 7 to 21 m, SD values for superstructures with a natural vibration period exceeding 2.4 s decrease by more than 20%. However, for superstructures with periods longer than 3 s, this trend does not hold when the building height increases from 14 to 21 m (as shown in Figure 18).
- The decreasing trend in SA values continues as building height increases. Notably, as the building height on soft soil (ZE) rises from 14 to 21 m, the plateau region expands, contrasting with the response of buildings on hard soil (ZB), as shown in Figure 19.
- Structures built on soft soils experience larger displacements at the foundation level compared to those on hard soils. For example, when examining the displacement values at $t = 2$ s, as shown in Figure 20, the displacement of the foundation block on ZD soil is approximately four times greater than that on ZC soil, while the displacement on ZC soil is about three times greater than that on ZB soil. This behavior is consistently observed at other time instances. The energy absorbed by the soft soil reduces the earthquake accelerations transmitted to the superstructure, leading to less structural damage, especially in terms of life-threatening consequences. While soft soils act as natural dampers by absorbing more energy, they can cause problems such as sinking or tilting, potentially making the structure unsafe after an earthquake.

The existing literature shows an increasing reliance on sophisticated models to solve SSI problems with various coupled approaches used to model the unbounded soil medium and the structure. These methods require careful attention to equilibrium and compatibility conditions at the interface between the coupled systems, often leading to a significant increase in analysis time. Additionally, the boundary conditions applied to each subdomain can affect the efficiency of coupling algorithms. In contrast, the proposed dynamic SSI system offers a more efficient approach by eliminating the need for compatibility and equilibrium linking conditions at the soil–structure interface, unlike the combined finite element (FE), boundary element (BE), and scaled boundary FE methods often used in similar studies. This study addresses SSI by assuming linear behavior for both the soil and the structure. Linear SSI analyses offer significant advantages in many engineering contexts, where nonlinear effects may not be prominent. Linear analysis serves as a valuable initial tool in design processes, offering simplified, approximate solutions that are useful in the

early stages. It also helps interpret complex nonlinear behavior, as the results of nonlinear systems may sometimes align with those predicted by linear models. The procedure outlined in this study can serve as a foundation for future research, which will help address its current limitations.

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