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Mathematical Modeling and Experimental Tests of Working Parameters of Piston-Radial Pumps in Hydraulic Systems of Mobile Machines

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Abstract: Modern methods of designing and constructing hydraulic pumps can no longer be imagined without the use of appropriate mathematical models, phenomena and processes that take place in concrete constructions. A mathematical model of a process represents an analytical interpretation with certain assumptions. Obtaining a mathematical model requires detailed theoretical research, based on knowledge of the laws of fundamental sciences and scientific disciplines, in order to fully understand and interpret the process, upon which assumptions are adopted and model equations are defined. As mobile machines continue to evolve, so do the demands on their hydraulic systems. Efficiency, automation and other major industry trends present ongoing development opportunities for the hydraulic systems used in mobile applications. Hydraulic systems remain an important component in a range of mobile applications, from construction and agricultural equipment to heavy trucks, because of the power density they can provide for a range of work functions. The power density of hydraulics remains unmatched in many applications. The pump, as an essential hydraulic component, converts mechanical energy into hydraulic energy with a relatively small amount and speed of fluid.

Keywords: mobile equipment; piston-radial pump; mathematical modeling; hydrodynamic processes; pressure; power; energy recovery; sensors; efficiency

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1. Introduction

When a hydraulic pump is running, it performs two functions in the hydraulic system. First, its mechanical action creates a vacuum at the pump inlet and allows atmospheric pressure to force fluid from the reservoir into the pump inlet line [1–4]. Second, its hydraulic action delivers fluid to the pump outlet and pushes it into the hydraulic system.

Several factors influence the design of today's hydraulic components and systems:

- Digitization, which includes the addition of sensors to products and efficiency, the trends that most influence the design of hydraulic components.
- Efficiency, particularly significant due to customers' emphasis on reducing energy consumption leading to cost and emissions savings. Efficiency a key driver as there is a greater focus on the environmental impact of machines and their components shaping the way hydraulics are designed

These and many other factors affecting hydraulic system design are linked to major trends affecting numerous industries, including fluid power and mobile equipment [5,6].

In this study, the identification and optimization of the working and structural parameters of the radial piston pump in the hydraulic circuit for energy recovery in mobile machines (Figure 1) was carried out [7–10].

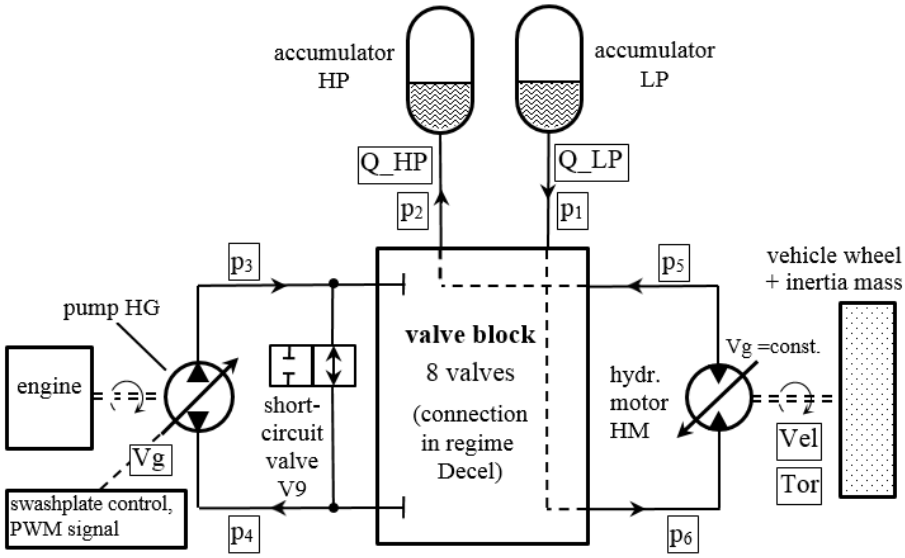


Figure 1. Hydraulic circuit for energy recovery in mobile machines.

After formulating the mathematical model, we performed a mathematical interpretation of the phenomena that occur in the entire physical model, i.e., a simulation of the physical system was performed. When analyzing a concrete physical system, the boundaries of the system, the subsystems that make it up and their mutual connections, as well as the processes that take place in it, are defined. Then assumptions and mathematical models of all previously defined processes are determined. The resulting systems of equations and their interconnections represent a simulation of the physical model. Finally, experimental verification of the obtained results are performed and their accuracy analyzed [11].

The hydrodynamics of radial-piston pumps includes current and hydrodynamic processes in the pump cylinder, suction and discharge piping, suction and discharge chamber and pump distribution organs. Current processes in the pump cylinder during the high-pressure part of the cycle were analyzed due to the occurrence of pressure pulsation [12]. Current processes in the suction and discharge pipeline, suction and discharge chamber and distribution organs (valves) were analyzed in order to determine the influence of the structure of the suction and discharge pipeline and the law of opening distribution organs (valves) for filling and emptying the pump cylinder [13,14].

2. Mathematical Modeling

Modeling the main parts of a hydraulic radial piston pump involves understanding its key components and their interactions.

By systematically modeling each component and their interactions, a comprehensive simulation of the performance of a hydraulic radial piston pump can be created. This approach enables a better understanding and optimization of hydraulic systems in practical applications [6].

The basic approach to cylinder-piston modeling involves the use of a set of differential equations, which represent the motion of the pistons based on fluid dynamics and pressure differences. Modeling the swashplate that drives the reciprocating motion of the pistons as the rotor rotates represents the swashplate angle and its effect on piston stroke length. The modeling of the rotor driven by the engine that creates the centrifugal force and moves the pistons is related to the speed of rotation and its relation to flow and pressure [15,16]. The hydraulic fluid inlet and outlet are modeled by implementing a fluid flow equation (like Bernoulli's equation) to describe the flow dynamics through these connections. The pump housing, which contains the components and directs the flow of hydraulic fluid, is modeled with a fixed volume in the fluid pressure and flow equations.

To model the pump function, it is necessary to define parameters such as fluid viscosity, density, piston diameter, swashplate angle, etc. To model the dynamics of the fluid, the continuity equation and the Bernoulli equation are used. It is necessary to establish kinematic equations that regulate the movement of the pistons based on the rotation of the rotor and the angle of the tilting plate [17]. To calculate the forces acting on the pistons and the torque on the rotor, pressure differences and mechanical losses are taken into account [18].

2.1. Mathematical Modeling of Valves

The role and location of the suction and discharge valves can be seen in Figure 2, where the pump is shown in isometry and in section. The first phase of the mathematical modeling of the valve refers to the determination of the sizes of the flow sections of the valve. For the valve separation of fluid in radial-piston pumps, the size of the geometric flow section depends on the stroke of the valve parameters and then the differences are highlighted in terms of their functions.

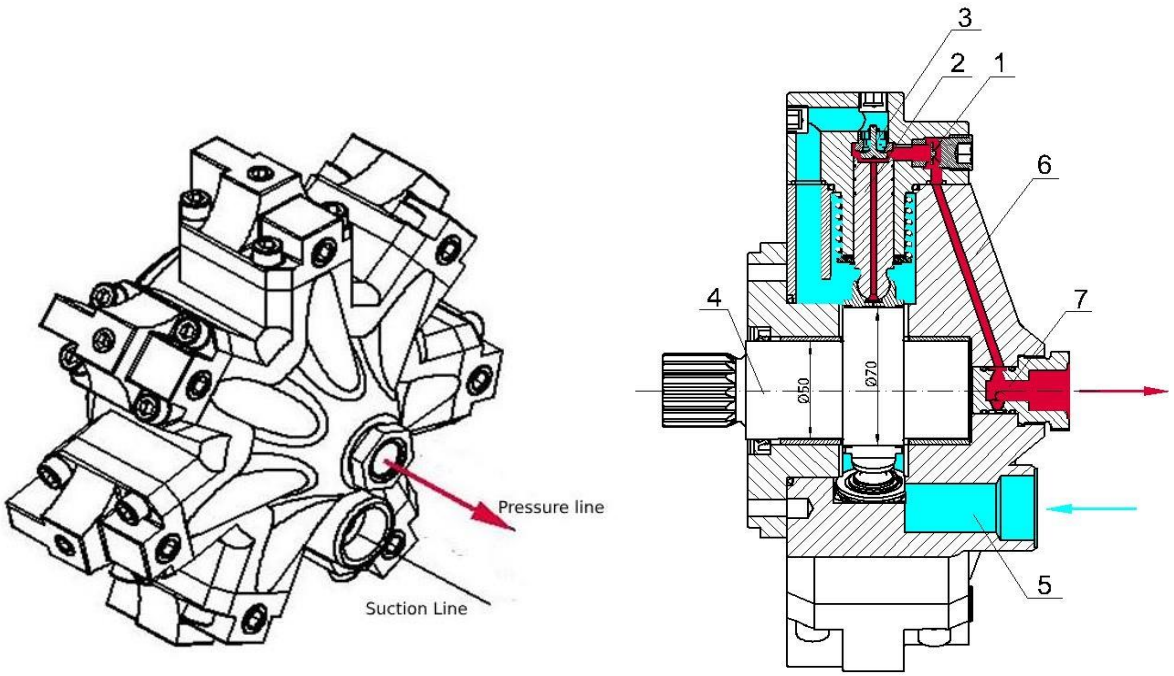


Figure 2. Radial piston pump. 1. pressure valve, 2. piston, 3. suction valve, 4. the pump shaft, 5. intake space of the pump, 6. cylinder block, 7. discharge space of the pump.

The size of the geometrical flow pressure of the valve is determined by the expression:

$$A_v = \pi d_v x_v \tag{1}$$

With the connection $d\varphi = \omega dt$, the differential equation of movement of the moving mass of the valve is arrived at, to which 1/3 of the mass of the valve spring is added, in the form:

$$\frac{dv_v}{d\varphi} = \frac{F_v}{\omega m_v} \tag{2}$$

where F_v is the resultant of all forces acting on the moving mass of the valve m_v, which depend on the position of the moving mass. All elements are shown in Figure 3.

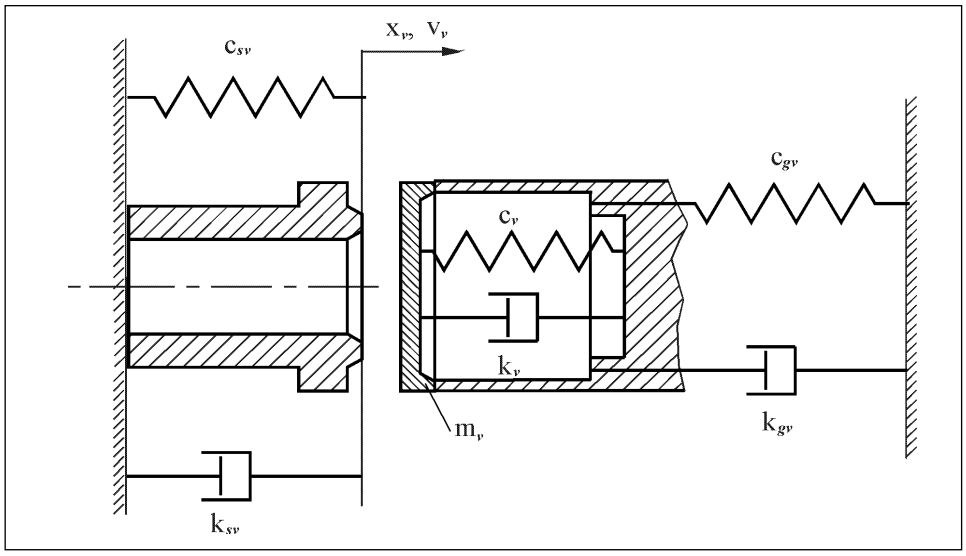


Figure 3. Dynamic model of the valve of the radial-piston pump.

During valve stroke $0 < x_v < h_v$, the resultant of all forces acting on the valve consists of fluid pressure force, spring force, friction force proportional to the first degree of speed and constant friction force:

$$F_v = A_v(p_c - p_v) - c_v(x_v + x_0) - k_v v_v - F_{tr}\sigma \tag{3}$$

where:

$\sigma = 1$ for $v_v > 0$
$\sigma = -1$ for $v_v < 0$
h_v—maximum valve stroke.

When the stroke of the valve is $x_v > h_v$, the valve is additionally acted upon by a force originating from the deflection of the valve stop and the damping force that additionally occurs during the deformation of the stop (it is assumed that this force is proportional to the first degree of the deformation rate):

$$F_v = A_v(p_c - p_v) - c_v(x_v + x_0) - k_v v_v - F_{tr}\sigma - \sigma_{gv}\left[c_{gv}(x_v - h_v) + k_{gv}v_v + F_{trgv}\sigma\right] \tag{4}$$

where:

$\sigma_{gv} = 0$ for $x_v \leq h_v$
$\sigma_{gv} = 1$ for $x_v > h_v$.

In the case when $x_v < 0$, the valve is additionally acted upon by a force originating from the deformation of the valve seat and the damping force that additionally occurs during the deformation of the valve seat (proportional to the first degree of deformation rate):

$$F_v = A_v(p_c - p_v) - c_v(x_v + x_0) - k_v v_v - F_{tr}\sigma - \sigma_{sv}[c_{sv}x_v + k_{sv}v_v + F_{trsv}\sigma] \tag{5}$$

where:

$\sigma_{gv} = 0$ for $x_v \leq 0$
$\sigma_{gv} = 1$ for $x_v > 0$.

The differential equation of the valve path is:

$$\frac{dx_v}{d\varphi} = \frac{v_v}{\omega} \tag{6}$$

The effective flow section of the valve through which the fluid flows in or out is obtained as a product of the geometric flow section and the corresponding flow coefficient, i.e.,

$$A_{\mu v} = A_v \mu_{u(i)} \tag{7}$$

When it is a suction or discharge valve, all the expressions (1) to (7) given above receive the corresponding indices (u—when it is a suction, that is, when it is a discharge valve).

This is the first phase of modeling and represents the basis for all other phases, both modeling and design of the valves of the suction and discharge lines of the radial-piston pump.

2.2. Mathematical Model of a Pump Process

2.2.1. Processes in the Suction Chamber of the Pump

The mass flow m through opening, at the point of entry of the suction pipe into the suction chamber of the pump, during quasi-stationary discharge is calculated from the known relation for stationary discharge of an incompressible fluid:

$$\frac{dm_1}{dt} = \sigma_1 \mu_1 A_1 \sqrt{2\rho_s |p_u - p_s|} \tag{8}$$

where:

p—pressure,
ρ—fluid density,
A—geometric flow section,
μ—flow coefficient constant.

The index "1" refers to the opening, the index "u" to the suction, the index "s" to the suction chamber.

The magnitudes of the constant σ_1 are:

$\sigma_1 = 1$ for $p_u \geq p_s$
$\sigma_1 = -1$ for $p_u < p_s$

In a similar way, the mass flow of the fluid m through the distribution organ of the pump (distribution plate or valves) when filling one cylinder of the pump is calculated:

$$\frac{dm_u}{dt} = \sigma_u \mu_u A_u \sqrt{2\rho_s |p_s - p_c|} \tag{9}$$

where are:

$\sigma_u = 1$ for $p_s \geq p_c$
$\sigma_u = -1$ for $p_s < p_c$.
A_u—geometrical flow section, intake opening of the distribution board or intake valve opening.
The index "c" refers to the cylinder.

The mass balance of the suction chamber reads:

$$\frac{dm_s}{dt} = \frac{dm_1}{dt} - \sum_{j=1}^{z_c} \frac{dm_{u,j}}{dt} \tag{10}$$

where:

$j = 1, 2, ..., z_c$—serial number of the cylinder,
z_c—total number of cylinders.

The density of the fluid in the suction chamber of the pump is:

$$\rho_s = \frac{m_S}{V_S} \tag{11}$$

V_S is the volume of the suction chamber.

Logarithmic differentiation of this equation by time yields:

$$\frac{1}{\rho_S}\frac{d\rho_S}{dt} = \frac{1}{m_s}\frac{dm_s}{dt} - \frac{1}{V_S}\frac{dV_S}{dt} \tag{12}$$

The equation of state of the fluid for pressure p and temperature T can be written in the form:

$$p = p(\rho, T) \tag{13}$$

The total derivative of pressure with time during an isothermal change of state ($T = const$) can be written in the form:

$$\frac{dp}{dt} = \left(\frac{\partial p}{\partial \rho}\right)_T \frac{d\rho}{dt} \tag{14}$$

For $m = \rho V$ the following differential can be written:

$$d(\rho V) = \rho dV + V d\rho \tag{15}$$

The isothermal coefficient of compressibility of the fluid ε_r is defined by the equation:

$$\varepsilon_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \tag{16}$$

Based on Equations (14)–(16), the following relationship between pressure, density and isothermal compressibility coefficient is obtained:

$$\left(\frac{dp}{dt}\right)_T = \left(\frac{\partial p}{\partial \rho}\right)_T = \frac{1}{\varepsilon_T \rho} \tag{17}$$

In the case of isentropic change (s = const) the equation of state can be written in the form:

$$p = p(\rho, s) \tag{18}$$

The total pressure time derivative in this case is:

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial \rho}\right)_S \frac{d\rho}{dt} \tag{19}$$

The isentropic coefficient of fluid compressibility ε_s is:

$$\varepsilon_s = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_s \tag{20}$$

Based on Equations (15), (19) and (20), the relationship between pressure, density and isentropic compressibility coefficient is obtained:

$$\left(\frac{dp}{d\rho}\right)_s = \left(\frac{\partial p}{\partial \rho}\right)_s = \frac{1}{\varepsilon_s \rho} \tag{21}$$

Comparing Equations (17) and (21) shows a complete similarity, so a general equation for the relationship between pressure, density and compressibility coefficient can be written:

$$\frac{dp}{d\rho} = \frac{\partial p}{\partial \rho} = \frac{1}{\varepsilon \rho} \tag{22}$$

i.e., in the form:

$$\frac{dp}{dt} = \frac{1}{\varepsilon \rho}\frac{dp}{dt} \tag{23}$$

wherein:

$\varepsilon = \varepsilon_T$—for isothermal change of state
$\varepsilon = \varepsilon_S$—for an isentropic change of state.

Using Equations (10), (12) and (23), the differential pressure equation in the suction chamber of the pump is obtained in the form:

$$\frac{dp_s}{dt} = \left[\frac{1}{\rho_s}\left(\frac{dm_1}{dt} - \sum_{j=1}^{z_c}\frac{dm_{u,j}}{dt}\right) - \frac{dV_s}{dt}\right] \tag{24}$$

where:

$j = 1, 2, ..., z_c$ serial number of the cylinder,
z_c—total number of cylinders.

Since the volume of the suction chamber is constant, and the elementary rotation angle φ of the drive shaft can be expressed through the angular velocity ω ($d\varphi = \omega dt$), and that the relation between the compressibility coefficient and the modulus of elasticity is $\varepsilon = 1/E$, the differential equation of the pressure in the suction chamber of the pump, expressed through the rotation angle of the drive shaft and the mass flow is:

$$\frac{dp_s}{d\varphi} = \frac{E}{V_s\rho_s}\left(\frac{dm_1}{d\varphi} - \sum_{j=1}^{z_c}\frac{dm_{u,j}}{d\varphi}\right) \tag{25}$$

2.2.2. Processes in the Pump Cylinder

The mass flow of the fluid flowing into the cylinder through the hole in the piston is defined by Equation (9), and the mass flow of the fluid flowing out of the cylinder is:

$$\frac{dm_i}{dt} = \sigma_i\mu_i A_i\sqrt{2\rho_c|p_c - p_v|} \tag{26}$$

where:

$\sigma_1 = 1$ for $p_c \geq p_v$
$\sigma_1 = -1$ for $p_c < p_v$
A_i—geometric flow section of the pressure opening of the distribution board or pressure valve opening. The index "i" refers to outflow.

Applying the law of conservation of mass to the pump cylinder, it follows that the change in mass in the cylinder caused by the flow of fluid into the cylinder and the flow out of the cylinder will be:

$$\frac{dm_c}{dt} = \frac{dm_u}{dt} - \frac{dm_i}{dt} \tag{27}$$

The density of the fluid in the cylinder is

$$\rho_c = \frac{m_c}{V_c} \tag{28}$$

Logarithmic differentiation of this equation by time yields:

$$\frac{1}{\rho_c}\frac{d\rho_c}{dt} = \frac{1}{m_c}\frac{dm_c}{dt} - \frac{1}{V_c}\frac{dV_c}{dt} \tag{29}$$

Using relation (23) and Equations (27) and (29), the differential pressure equation in the pump cylinder is obtained in the form:

$$\frac{dp_c}{dt} = \frac{1}{\varepsilon V_c}\left[\frac{1}{\rho_c}\left(\frac{dm_u}{dt} - \frac{dm_i}{dt}\right) - \frac{dV_c}{dt}\right] \tag{30}$$

The current volume of the cylinder is:

$$V_c = V_{cmin} + V_{cx} \quad V_{cx} = A_c x_k \tag{31}$$

The change in the volume of the pump cylinder caused by the movement of the piston is:

$$\frac{dV_c}{dt} = -A_c v_k \tag{32}$$

where:

x_k—current stroke of the piston calculated from the outer dead center.

By replacing the relation (32), introducing the connection $d\varphi = \omega dt$ and the modulus of elasticity E, the differential Equation (30) expressed through the mass flow and the rotation angle of the drive shaft reads:

$$\frac{dp_c}{d\varphi} = \frac{E}{V_c}\left[\frac{A_c v_k}{\omega} + \frac{1}{\rho_c}\left(\frac{dm_u}{d\varphi} - \frac{dm_i}{d\varphi}\right)\right] \tag{33}$$

2.2.3. Processes in the Pressure Chamber of the Pump

Applying the law of mass maintenance to the pressure chamber of the pump, it follows that the change in mass is caused by the flow of fluid from cylinders through the distribution bodies (distribution plate or pressure valves) and flow through the pressure pipeline.

The mass balance of the pressure chamber reads:

$$\frac{dm_v}{dt} = \sum_{j=1}^{z_c} \frac{dm_{i,j}}{dt} - \frac{dm_2}{dt} \tag{34}$$

where:

$j = 1, 2, ..., z_c$ serial number of the cylinder,
z_c—total number of cylinders.

The mass flow flowing out of the thrust chamber into the thrust pipeline is

$$\frac{dm_2}{dt} = \sigma_2 \mu_2 A_2 \sqrt{2\rho_t |p_v - p_n|} \tag{35}$$

where:

$\sigma_2 = 1$ for $p_v \geq p_{vn}$
$\sigma_2 = -1$ for $p_v < p_n$.

A_2-geometric flow section of the pressure pipeline

Analogous derivation of the differential pressure equation in the suction chamber, using Equations (34), (35) and Equation (23) with appropriate index notation, leads to the differential pressure equation in the pump discharge chamber:

$$\frac{dp_v}{d\varphi} = \frac{E}{V_v \rho_v}\left(\sum_{j=1}^{z_c} \frac{dm_{i,j}}{d\varphi} - \frac{dm_2}{d\varphi}\right) \tag{36}$$

The constant size of the pressure chamber V_v is defined by the construction of the pump.

2.2.4. Structure and Organization of a Computer Program

For the simulation of the radial piston pump, the original computer program AKSIP was developed, which is specific due to a number of details, ease of use and integration with other programs. It can create dynamic simulations and modeling of fluid systems. It can create custom models and use toolboxes for hydraulic systems. It is excellent for fluid dynamics simulations, allowing detailed analysis of flow characteristics inside a pump. It offers multiphysics simulations, combining fluid dynamics with structural mechanics,

which can be helpful for analyzing interactions inside a pump. It is useful for 3D modeling and fluid flow analysis, especially if SolidWorks is already being used for design. It is specially designed for pump simulations, focusing on performance and efficiency. It focuses on hydraulic systems and can be a good option for modeling different types of pumps, including radial piston pumps. It is open source, which can be adapted to specific simulation needs, although it may require more setup and expertise. It includes menus, toolbars, and various panels for setting up simulations, defining geometry, and visualizing results. Users can create geometries within AKSIP or import them from CAD software.

The AKSIP program is modularly designed and consists of the main AKSIP program and its modules.

The AKSIP program module performs loading and printing of input data and determination of constants. For further processing calls the ECITER module.

In case of deterministic model selection, the desired model is specified when creating the input data file AKSIP.DAT. Stochastic model selection is of particular interest in the statistical analysis of the reliability of modeling results.

AKSIP allows users to create structured or unstructured meshes for the simulation domain. The mesh must be carefully defined to accurately capture the flow characteristics. Users define simulation physics, such as fluid properties, turbulence models. Defining how the fluid interacts with the domain boundaries and controlling the solution, including time-stepping methods and convergence criteria, are essential. AXIP can use multiple processors to speed up simulations, making it suitable for large and complex models. Users can visualize flow patterns and other results via contour plots, vector plots, and streamlines.

In the AKSIP program, for now, the following possibilities and corresponding identifiers are foreseen when choosing a mathematical model:

(a) Pressure variability in the suction chamber IS=?

S = 0—the pressure in the suction chamber is constant or IS=1—the pressure in the suction chamber is variable.

(b) Pressure variability in the pressure chamber IT=?

IT = 0—the pressure in the pressure chamber is constant or IT=1—the pressure in the pressure chamber is variable.

(c) Selection of suction distribution organ IU=?

IU = 1—the suction distribution board is activated or IU=11—the suction valve is activated.

(d) Selection of pressure distribution body II=?

II = 1—the pressure distribution board is activated or II=11—the pressure valve is activated.

(e) Selection of the method of determining the flow coefficient IMDMI=?

IMDMI = 0—flow coefficients are given tabularly as constants,
IMDMI = 1—the flow coefficient is specified as a function of pressure,
IMDMI = 2—the flow coefficient is specified as a function of the number of revolutions.
IMDMI = 3—the flow coefficient is specified as a function of pressure and speed.

(f) Selection of a model that takes into account the mass inertia of the working fluid IMIN=?

IMIN = 0—the inertia of the mass of the working fluid is not taken into account, IMIN = 1—the inertia of the mass of the working fluid is taken into account.

The structure and organization of AKSIP enable users to efficiently run complex simulations. Understanding the function and workflows helps to streamline the workflow and improve the accuracy of the simulations. AKSIP provides tools for analyzing simulation data, such as statistics and custom reports. AKSIP provides extensive documentation and instructions to help users implement algorithms for solving the governing equations of

fluid dynamics (Navier–Stokes equations) using methods such as finite volume, finite element, or spectral methods. The graphical user interface is user friendly to easily set up simulations, view results and manage projects. Efficient storage and retrieval of grid files, solutions and results is enabled. Validation and verification are possible by comparing simulation results with analytical solutions or experimental data to confirm the accuracy of the model.

3. Experimental Testing

Model of the pump was prepared in such a way as to enable the connection of the necessary measuring transducers necessary for the measurement of characteristic parameters. A schematic view of the test installation and data acquisition system is shown in Figure 4.

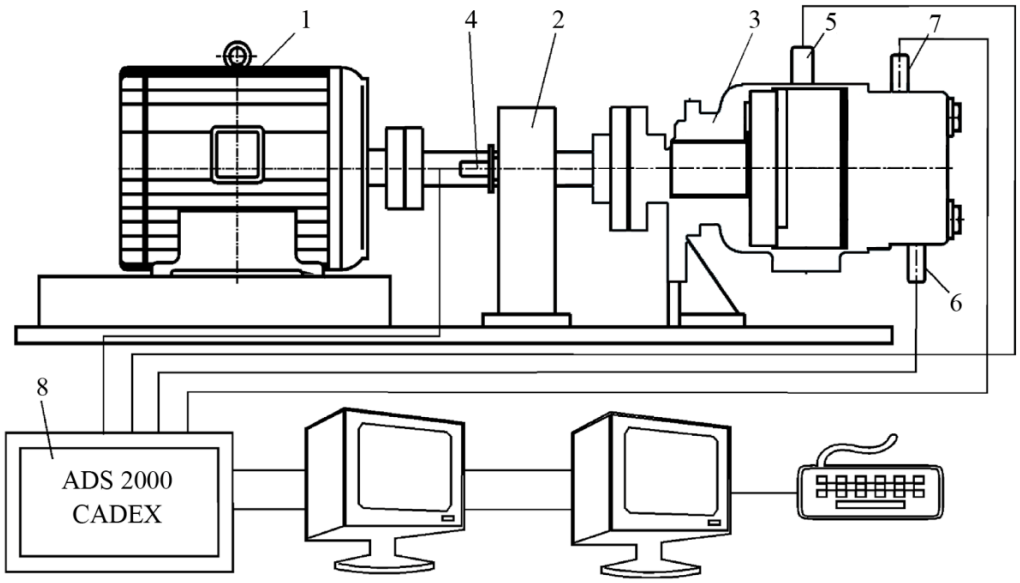


Figure 4. Testing installation and data acquisition system. 1. Electric motor power 137 kW, 1450 min^{-1} with speed and torque regulation, 2. Reducer, 3. Piston-radial pump, 4. Angular marker, 5. Vibration measurement transducer, 6. Pressure measurement transducer in the pressure chamber, 7. Pressure measurement transducer in the cylinder, 8. ADS 2000-CADEX measurement and acquisition system.

The following sizes were measured:

- Pressure p_c in the cylinder as a function of the pump drive shaft angle
- Pressure p_v in the valve chamber as a function of the pump drive shaft angle
- Pressure p_n in the pressure pipeline as a function of the angle of the pump drive shaft
- Amplitude of vibration of the pump casing as a function of the angle of the pump drive shaft.
- Angle φ of the pump drive shaft
- Pump flow Q_p
- Temperature of the working fluid T
- Number of revolutions n of the drive shaft of the pump.

Structure of the ADS 2000-CADEX measurement and acquisition system. It is given in Figure 5. VME-bus ADC contains two A/D converters with simultaneous operation of the speed twice 350 kHz at 12 bit, one timer, the start of each conversion is performed via the pulse of the angle encoder or the timer, with hardware registration of the reference brand of the incremental angle encoder in order to 100% control of the correctness of the operation of the angle encoder in real time. Up to 6 A/D cards = 12 A/D converters can be integrated into the system. Two VME-bus ADC modules are installed in the implemented system.

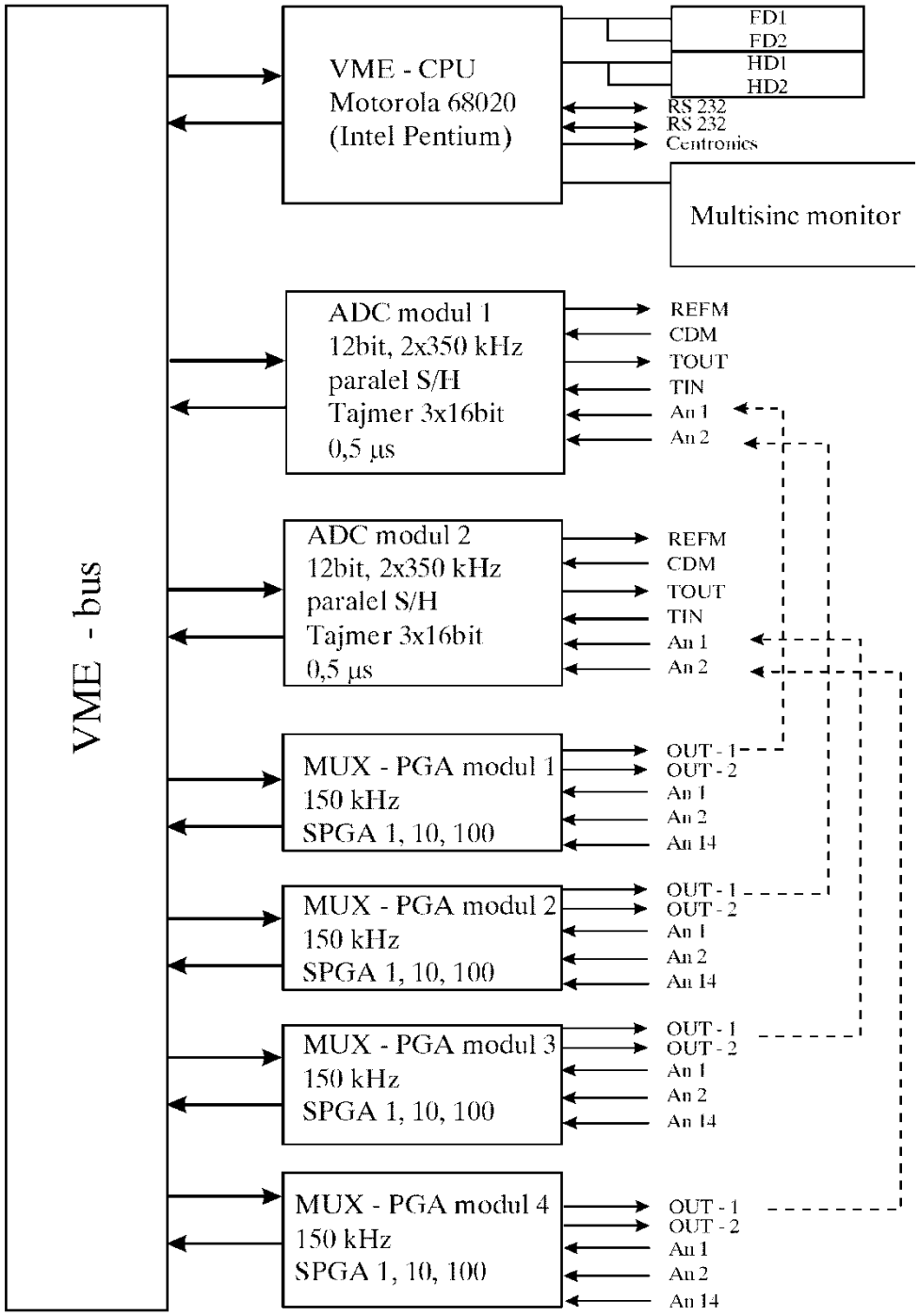


Figure 5. Ultra-fast measuring system ADS 2000.

Implementation of VME-bus PGA multiplexer for experiments with hydraulic radial piston pumps is very useful for monitoring and controlling various parameters. Used to handle multiple input signals from sensors that monitor parameters such as pressure, flow rate, and temperature in a hydraulic system. For pressure sensors, pressure transducers are installed to measure the pressure inside the hydraulic system. Flow sensors use flow meters to monitor the flow rate of hydraulic fluid. Temperature sensors include temperature probes to ensure the fluid is operating within safe limits. The PGA will amplify the signals from the sensors, especially if they are emitting low voltage signals, ensuring the data is suitable for processing. An A/D converter connected to the VME is used to acquire the

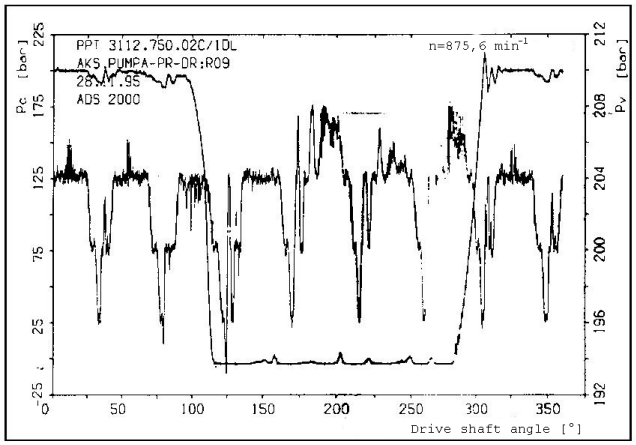
data. bus to digitize amplified signals. A PGA multiplexer can switch between different sensors based on control signals, allowing efficient data collection from multiple sources.

The applied measurement system enables simultaneous measurement on four fast analog channels each, with parallel measurement of time intervals from the angle mark to the mark of the incremental angle encoder.

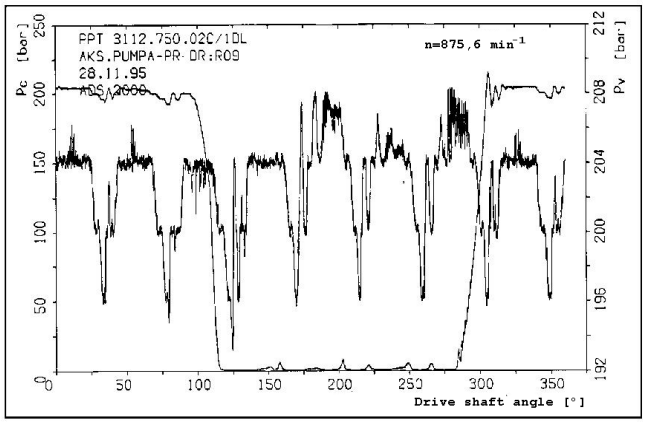
4. Results of Mathematical Modeling and Experimental Testing

Comparison of the results of mathematical modeling and experimental testing of radial piston pumps includes several key aspects. The predicted efficiency from the model was compared with experimental data. The accuracy of the predicted output pressures and flow rate was analyzed. It was established how the variations of these parameters affect the predictions of the model in relation to the experimental results. Performance under different load scenarios and their impact on modeling and experimental results were evaluated. The geometrical parameters of the piston, clearances and tolerances, changes in the design as well as their influence on the performance of the radial piston pump were analyzed. An evaluation of the dynamic behavior was performed to see how well the models predicted the dynamic behavior with the experimental behavior. Vibrations and noise were analyzed and compared with those observed in the experiments. Using the experimental results, the assumptions and equations used in the mathematical models were confirmed. The efficiency of hydraulic systems in mobile machines can be significantly affected by the operating parameters of radial piston pumps. Higher pressure can lead to increased efficiency, but also increases the risk of cavitation and pump wear. Balancing the pressure setting is critical for optimal performance. Minimized pressure losses in hoses and fittings can improve overall system efficiency. Keeping the pump flow within its design specifications ensures efficient operation. Flow rates that are too low or too high can reduce efficiency. Using variable displacement pumps enables better adaptation to different load conditions, improving efficiency.

At the pump operating mode $n = 875.6$ min^{-1} and $p_n = 200$ bar, the results of the measured pressures and the pressures obtained by mathematical modeling as a function of the rotation angle of the pump drive shaft are given in comparison Figure 6. Figure 6 shows the flow of pressure: a) in a cylinder (pc) and a pressure chamber (pv) for one cycle of the experiment; (b) in a pressure chamber (pv), to simulate one cycle; (c) pressure in the cylinder (pc) in the interval 120–270, for one cycle of the experiment; (d) in the cylinder (pc) in the interval 120–270, for one simulation cycle; (e) in the cylinder (pc) and the valve chamber (pv) in the interval 278–307, for one cycle of the experiment; (f) in cylinder (pc) and valve chamber (pv) in the interval 278–307, for one simulation cycle.



(**a**)



(**b**)

Figure 6. *Cont.*

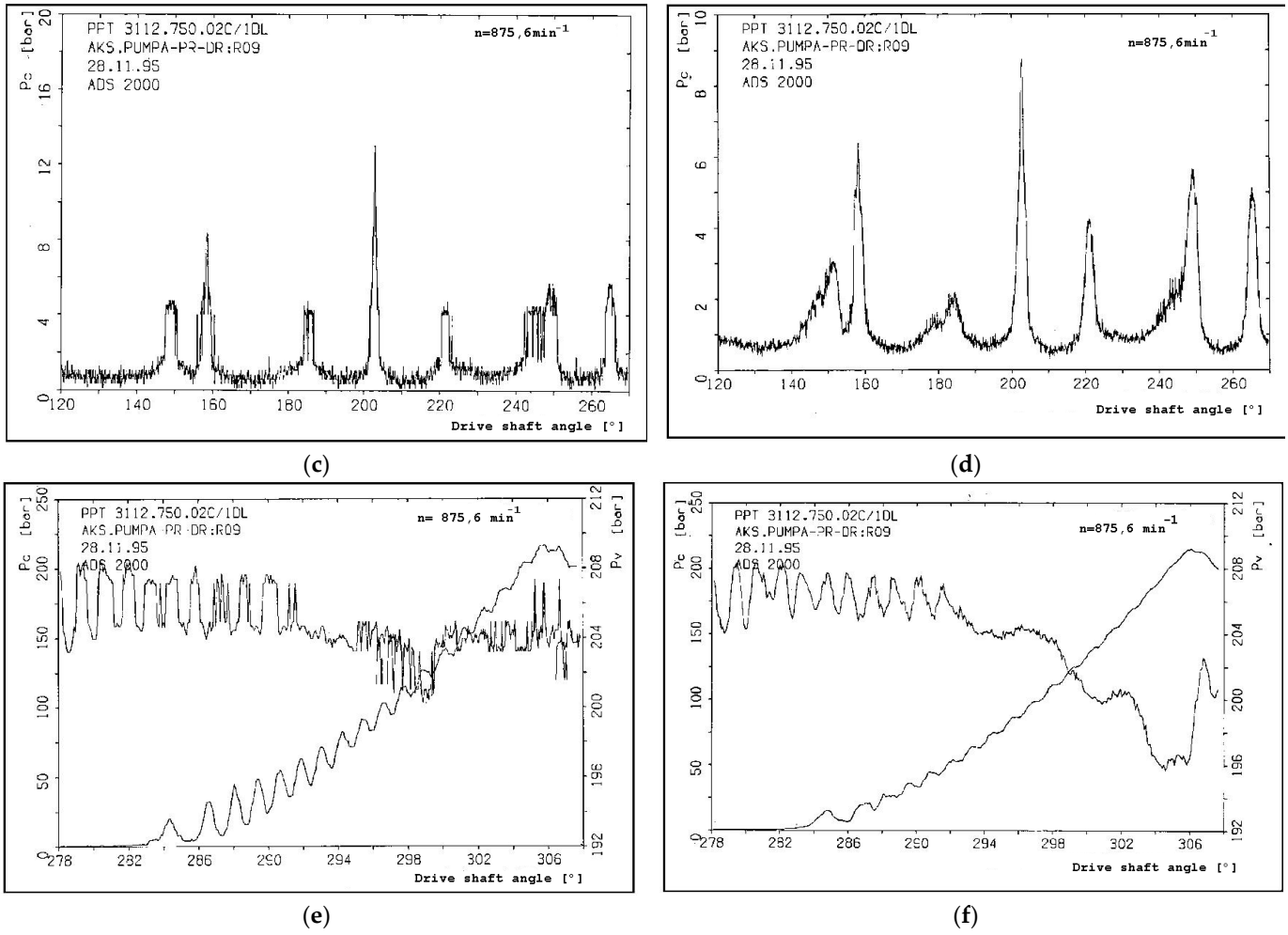


Figure 6. Diagrams of measured and modeled pressures in the operating mode $n = 875.6\ \text{min}^{-1}$ and $p_n = 200$ bar.

The number of pistons can significantly affect the performance of radial piston pumps, including pressure variations. Increasing the number of pistons generally results in higher compression capability. More pistons can distribute the load more evenly, allowing more pressure to be generated without excessive wear. With more pistons, pressure pulses can become smoother, reducing the fluctuation between pressure peaks and valleys. This can lead to more stable operation at high pressures. More pistons means more theoretical flow at a given speed, since each piston contributes to the total displacement. With fewer pistons, the pump can be more efficient at low pressures, because there is less internal friction and fewer moving parts. More pistons can increase complexity, size, and weight. Design considerations must balance these factors with performance requirements. Radial piston pumps can experience vibrations that are affected by several factors, including the number of pistons. More pistons generally result in better balance and symmetry in pump operation. This can reduce pulsations and vibrations as the forces are more evenly distributed. More pistons can lead to smoother operation by reducing the amplitude of pressure fluctuations, as more pistons can create a more continuous flow.

The vibration level is defined by the amplitude ratio as follows

$$N[dB] = 20\log_{10}\frac{A}{A_{ref}} \tag{37}$$

where are:

N—number of decibels
A—measured level
$A_{ref} = 10^{Kref}$—reference level.

The reference level is marked with the $Kref$ exponent on the corresponding diagrams.

From the attached diagrams of the harmonic analysis of the pressures in the pressure chamber, a dominant row for the maximum pressure amplitudes can be seen, which contains a module equal to the number of cylinders $\sigma = z{\cdot}n$, where z is the number of cylinders and $n = 1, 2, 3\ldots$ number of cycles.).

The magnitudes of the measured vibrations of the pump casing for the middle of ten consecutive cycles shown in Figure 7 also indicate the occurrence of amplitude pulsations at harmonics $\sigma = z{\cdot}n$.

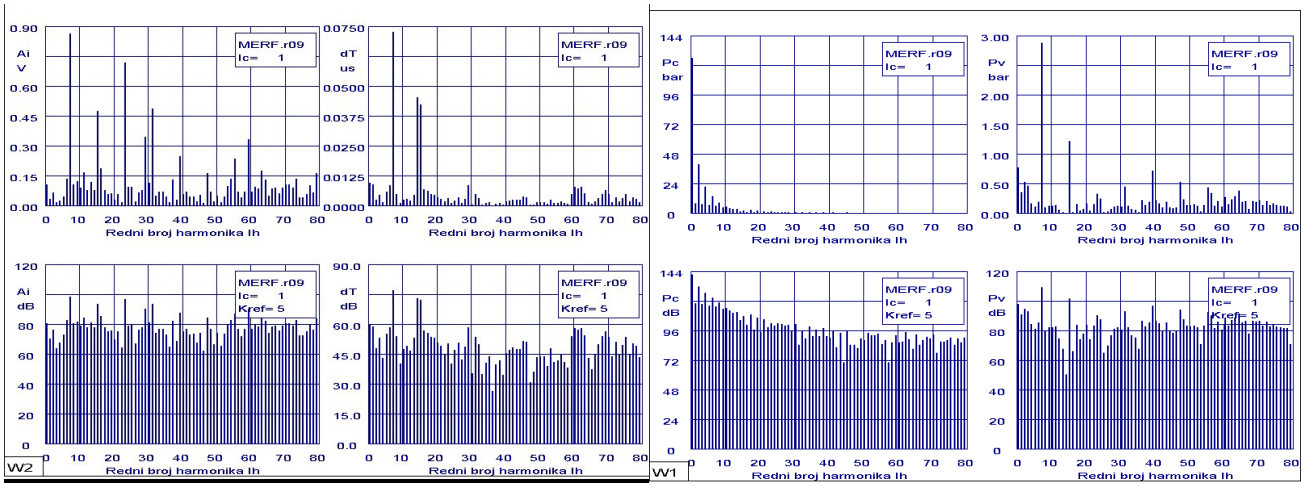


Figure 7. Results of harmonic analysis of measured pressures, vibrations and time intervals for one cycle at $n = 875.6$ min^{-1} and $p = 200$ bar with a graphic display of 80 harmonics.

These statements are in principle valid for all tested modes of operation of the piston-radial pump.

5. Conclusions

By measuring the performance of piston-radial pumps, it was concluded that their efficiency can generally be evaluated according to their maximum working pressure capacity and flow rate at a certain drive speed.

The parameters of the hydrodynamic process of the piston-radial pump (pressure flow, suction flow and compression flow) cannot be determined accurately enough purely experimentally, nor purely mathematically. Sufficiently accurate parameters can be obtained by the combined application of pressure flow measurement in the cylinder, mathematical modeling of the actual hydrodynamic process and nonlinear optimization methods, whereby systematic measurement errors and unknown parameters can be simultaneously determined.

Radial piston pumps are commonly used in mobile machinery due to their efficiency and compact design. Understanding their mechanical, volumetric, and overall efficiency is essential to optimizing performance. Mechanical efficiency measures how effectively a pump converts input power into hydraulic power, taking into account losses due to friction, wear, and mechanical components. The mechanical efficiency of radial piston pumps typically ranges from 85% to 95%. Volumetric efficiency measures the pump's efficiency in moving liquid compared to its theoretical displacement. It calculates losses due to leakage and compressibility of liquids. The volumetric efficiency of radial piston pumps can vary widely, generally falling between 90% and 98%. Overall efficiency combines both mechanical and volumetric efficiency, providing a comprehensive measure of pump performance. Overall efficiency can be quite high for radial piston pumps, often exceeding 80% when both mechanical and volumetric efficiency are optimized. This efficiency can

be affected by design factors, operating conditions and the quality of materials used in construction. Regular maintenance and proper operation are also critical to maintaining these levels of efficiency in mobile machines.

The piston-radial pump is an important component in hydraulic systems in mobile applications, from construction and agricultural equipment to heavy trucks, because of its sufficient power density for a range of work functions. Optimizing the operating and structural parameters of piston-radial pumps can directly influence the efficiency of mobile machines.

Further directions of research are possible in the construction of piston pumps in terms of: separation of the working fluid, dynamics of the cylinder block and hydrodynamic processes in the gaps between the piston and the cylinder in the cylinder block.

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References

1. Vacca, A.; Franzoni, G. *Hydraulic Fluid Power: Fundamentals, Applications, and Circuit Design*; Wiley: Hoboken, NJ, USA, 2021.
2. Akers, A.; Gassman, M.; Smith, R. *Hydraulic Power System Analysis*, 1st ed.; CRC Press: Boca Raton, FL, USA, 2006. [CrossRef]
3. Manrig, N. *Fluid Power Pumps and Motors: Analysis, Design and Control*, 1st ed.; McGraw Hill: New York, NY, USA, 2013.
4. Matthies, H.J.; Renius, K.T. *Einführung in die Ölhydraulik*; GWV Fachverlage GmbH: Wiesbaden, Germany, 2008.
5. Yang, H.; Pan, M. Engineering research in fluid power: A review. *J. Zhejiang Univ. Sci.* **2015**, *16*, 427–442. [CrossRef]
6. Petrović, R. Mathematical Modeling and Experimental Research of Characteristic Parameters Hydrodynamic Processes of a Piston Axial Pump. *Stroj. Vestn. —J. Mech. Eng.* **2009**, *55*, 222–229.
7. Bech, T.; Olsen, S.; Klit, P. Design of Pumpsfor Water Hydraulic Systems. In Proceedings of the 6th Scandinavian International Conference on Fluid Power, SICFP'99, Tampere, Finland, 26–28 May 1999.
8. Jourani, A.; Hagege, B.; Bouvier, S.; Bigerelle, M.; Zahouani, H. Influence of abrasive grain geometry on friction coefficient and wear rate in belt finishing. *Tribol. Int.* **2013**, *59*, 30–37. [CrossRef]
9. Yang, H.; Yang, J.; Zhou, H. Research on materials of piston and cylinder of waterhydraulic pump. *Ind. Lubr. Tribol.* **2003**, *55*, 38–43.
10. Songlin, N.; Ming, G.; Fanglong, Y.; Hui, J.; Zhonghai, M.; Zhen, H.; Xin, Z. Research on fluid-structure interaction for piston/cylinder tribopair of seawater hydraulic axial piston pump in deep-sea environment. *Ocean Eng.* **2019**, *219*, 108222. [CrossRef]
11. Qun, C.; Junhui, Z.; Bing, X.; Qiannan, W.; Hsinpu, H. Test rigs and experimental studies of the slipper bearing in axial piston pumps. *Measurement* **2019**, *132*, 135–149. [CrossRef]
12. Vacca, A.; Klop, R.; Ivantysynova, M. A numerical approach for the evaluation of the effects of air release and vapour cavitation on effective flow rate of axial piston machines. *Int. J. Fluid Power* **2010**, *11*, 33–45. [CrossRef]
13. Zhou, J.; Zhou, J.; Jing, C. Experimental research on the dynamic lubricating performance of slipper/swash plate interface in axial piston pumps. *Chin. J. Mech. Eng.* **2020**, *33*, 25. [CrossRef]
14. Spencer, N.A. Design and Development of a Novel Test Method to Measure the Slipper/Swashplate Interface Fluid Film in a Positive Displacement Machine. Master's Thesis, Purdue University, West Lafayette, IN, USA, 2014.
15. Ham, Y.B.; Lee, Y.B.; Park, K.M.; Choi, B.O. A study on the application of birfield joint to a water hydraulic piston pump for low leakage and low friction pumping. In Proceedings of the 6th JFPS International Symposium on Fluid Power, Tsukuba, Japan, 7–10 November 2005.
16. Canbulut, F.; Sinanoğlu, C.; Koc, E. Experimental analysis of frictional power loss of hydrostatic slipper bearings. *Ind. Lubr. Tribol.* **2009**, *61*, 123–131. [CrossRef]

17. Załuski, P. Experimental Research of an Axial Piston Pump with Displaced Swash Plate Axis of Rotation. In *Advances in Hydraulic and Pneumatic Drives and Control NSHP 2020*; Springer: Berlin/Heidelberg, Germany, 2020; pp. 135–145. [CrossRef]
18. Manring, N.D.; Wray, C.L.; Dong, Z.L. Experimental studies on the performance of slipper bearings within axial-piston pumps. *J. Tribol.* **2004**, *126*, 511–518. [CrossRef]

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