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# Research on Intraparticle to Interparticle Entanglement Swapping Protocols

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**Abstract:** Entanglement is one of the most striking features of quantum systems, whereby its non-classical correlation is an essential resource in numerous quantum protocols. Entanglement can be divided into two categories: interparticle and intraparticle entanglement. There are both distinctions and similarities between these two kinds of entangled states. This work delves into these distinctions and similarities from the following aspects: correlation and non-locality, robustness, the mechanisms of generation and separation, and practical applications. Entanglement swapping is a technique based on quantum entanglement. As entanglement has different categories, entanglement swapping also has various types, including interparticle to interparticle and intraparticle to interparticle. Swapping protocols from intraparticle entanglement to interparticle entanglement can be applied to super quantum dense encoding, quantum information transmission, quantum teleportation, etc. Thus, this work proposes three swapping protocols, from spin–orbit intraparticle entanglement to spin–spin interparticle entanglement, based on Bell state joint measurement, the cross-Kerr medium, and linear optical elements. This work can help us better understand entanglement by analyzing the differences and similarities between the two types of entangled states. It can also enhance entanglement swapping protocols, from spin–orbit intraparticle to spin–spin interparticle entanglement, for use in quantum information transfer.

**Keywords:** intraparticle entangled states; distinctions and similarities; entanglement swapping

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## 1. Introduction

Quantum technology is currently receiving substantial worldwide attention and accounts for a major national strategy. Quantum technology is a new interdisciplinary field combining quantum physics and information technology. Entanglement is one of the most striking features of quantum systems, whereby its non-classical correlation is an essential resource in quantum technology. Many quantum protocols rely on non-classical correlations, such as quantum teleportation (QT) [1], quantum cryptography [2,3], and quantum random number generators [4], etc.

In addition to entwining at one degree of freedom, two systems can also entangle at various degrees of freedom (hybrid entanglement) [5–7]. Hybrid entanglement opens the door to intraparticle entanglement. Intraparticle entanglement was first proposed by Knight in 1998 [8]. The study highlighted entanglement involving at least two commutation observables, which can also belong to a particle or multi-particle system. The commutation observables of intraparticle entanglement belong to a particle. Various degrees of freedom in a single system are commutation observables, which can form intraparticle entangled states, such as the polarization–momentum entanglement of a photon [9,10], the spin–path

entanglement of a neutron [11–13], and the spin–orbit entanglement of a photon [14–16], etc. Intraparticle entanglement has a stronger robustness property under decoherence and dephasing and consumes fewer resources than interparticle entanglement. It can also be applied to numerous endeavors, such as superdense coding [15], QT [17,18], information transfer [19], etc.

Interparticle and intraparticle entanglements are two types of entanglement. While interparticle entanglement has received the most attention in current research, intraparticle entanglement has also received considerable attention. The two forms of entanglement have differences and similarities. Analyzing and contrasting these similarities and differences can help us understand entanglement.

Entanglement swapping is a technique based on quantum entanglement. It can be applied in many quantum protocols. It especially plays a vital role in the quantum repeater [20], as it can distribute the entanglement over a greater distance. Entanglement consists of two types: intraparticle and interparticle entanglement. Entanglement swapping also consists of various kinds, including interparticle to interparticle and intraparticle to interparticle. Interparticle entanglement swapping has been extensively studied, such as in [21–24]. The following studies have been conducted to swap entanglement from intraparticle to interparticle. A swapping protocol was established from path–spin intraparticle to spin–spin interparticle entanglement by using the Mach–Zehnder interferometer, the Stern–Gerlach measuring device, and the corresponding unitary operations [25]. Another swapping protocol was accomplished from the intraparticle to interparticle hybrid entanglement by using two indistinguishable particles and suitable unitary operations [26]. Another swapping protocol was achieved from intraphoton path–polarization (spin) to interphoton entanglement by using linear optical devices [27]. The authors of [28] achieved an entanglement swapping protocol from path–spin intraparticle to spin–spin interparticle entanglement by using simple optical devices.

The spin–orbit entanglement of photons has significant academic and practical value and has acquired extensive attention. It is frequently used in quantum protocols based on the experimental progression of manipulating photon spin–orbit degrees of freedom [29,30]. Thus, this study focuses on the entanglement swapping from orbit–spin intraparticle to orbit–orbit interparticle entanglement. This study parallels previous studies about intraparticle to interparticle entanglement swapping and does not contrast with prior studies. There are currently no specific studies on this matter, so this work preventatively focuses on entanglement swapping on intraparticle degrees of freedom of spin and orbit.

To sum up, this work designed three entanglement swapping protocols from spin–orbit intraparticle to spin–spin interparticle entanglement based on six years of studying intraparticle and interparticle entanglement and familiarizing ourselves with the characteristics of optical devices, such as the cross-Kerr medium [29], beam splitters (BSs), polarization beam splitters (PBSs), and half-wave plates (HWPs), etc. This work has apparent originality.

This work is organized as follows. Firstly, the distinctions and similarities between the two kinds of entangled states are analyzed. Then, three entanglement swapping methods are designed from spin–orbit intraparticle to spin–spin interparticle entanglement based on Bell state joint measurement, the cross-Kerr medium, and linear optical elements. Finally, the results are discussed, and conclusions are drawn.

## 2. The Distinctions and Similarities between the Two Kinds of Entangled States

Interparticle and intraparticle entanglement have both distinctions and similarities. Comparing these distinctions and similarities can help us better understand entanglement. This section delves into these distinctions and similarities from the following aspects: correlation, non-locality, robustness, the methods of generation and separation, and the advantages and drawbacks in practical applications.

### 2.1. Correlation and Non-Locality

As shown in Figure 1, interparticle and intraparticle entanglement both have a correlation. When two parts are entangled, one's state changes, regardless of the distance separating the two, and the other's state correspondingly alters. Interparticle entanglement possesses a non-local correlation, which has been proven in extensive experiments [31–33]. Non-locality is meaningless for intraparticle entangled states, given that their kind of entanglement is established on various degrees of freedom within a single particle. However, a correlation has been verified in experiments [9,14] for intraparticle entangled states.

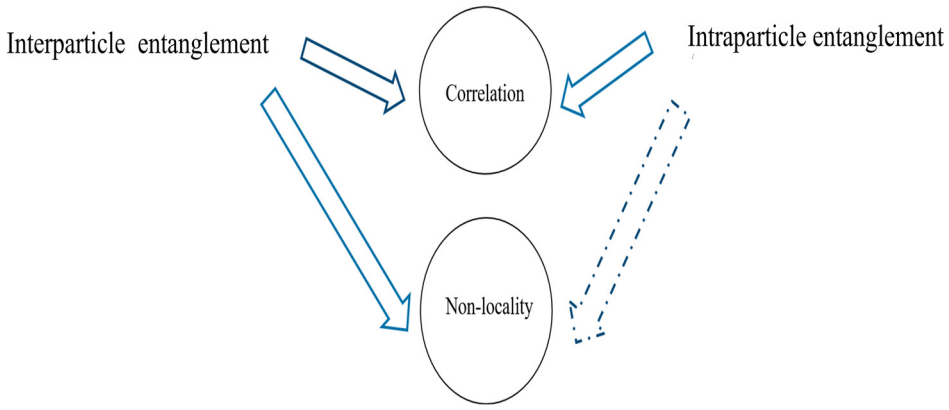



**Figure 1.** Analysis of the correlation and non-locality of the two types of entangled states.

### 2.2. Robustness

Intraparticle entangled states possess a stronger robustness than interparticle entangled states. Their quantum system has a smaller interface, with the external environment based on a single particle, and therefore experiences less noise interference from the external environment during the communication process. Quantum systems have a more extensive interface, with the external environment based on multiple particles for interparticle entangled states. Therefore, the system suffers more noise interference, which leads to weak robustness [34,35].

### 2.3. Generation and Separation Methods

This section compares the generation and separation methods of two spin–orbit entangled states. The experimental setups to generate and separate spin–orbit intraparticle entangled states are displayed in Figure 2a [16]. The experimental setups to generate and separate interparticle spin–orbit entangled states are exhibited in Figure 2b [36] and Figure 2c [37], respectively. According to Figure 2b and the left side of Figure 2a, there are just two steps required to generate intraparticle spin–orbit entangled states, which makes them easier to generate and more convenient than interparticle spin–orbit entangled states.

The right side of Figure 2a,c illustrate how challenging it is to detect and separate intraparticle spin–orbit entangled states as opposed to interparticle spin–orbit entangled states. The former requires more instruments, such as Michelson interferometers [36], quarter-wave plates (QWPs), HWPs, and a circularly polarized beam splitter (CBS). The latter uses relatively simple instruments, such as PBSs, BSs, and photodetectors [37].

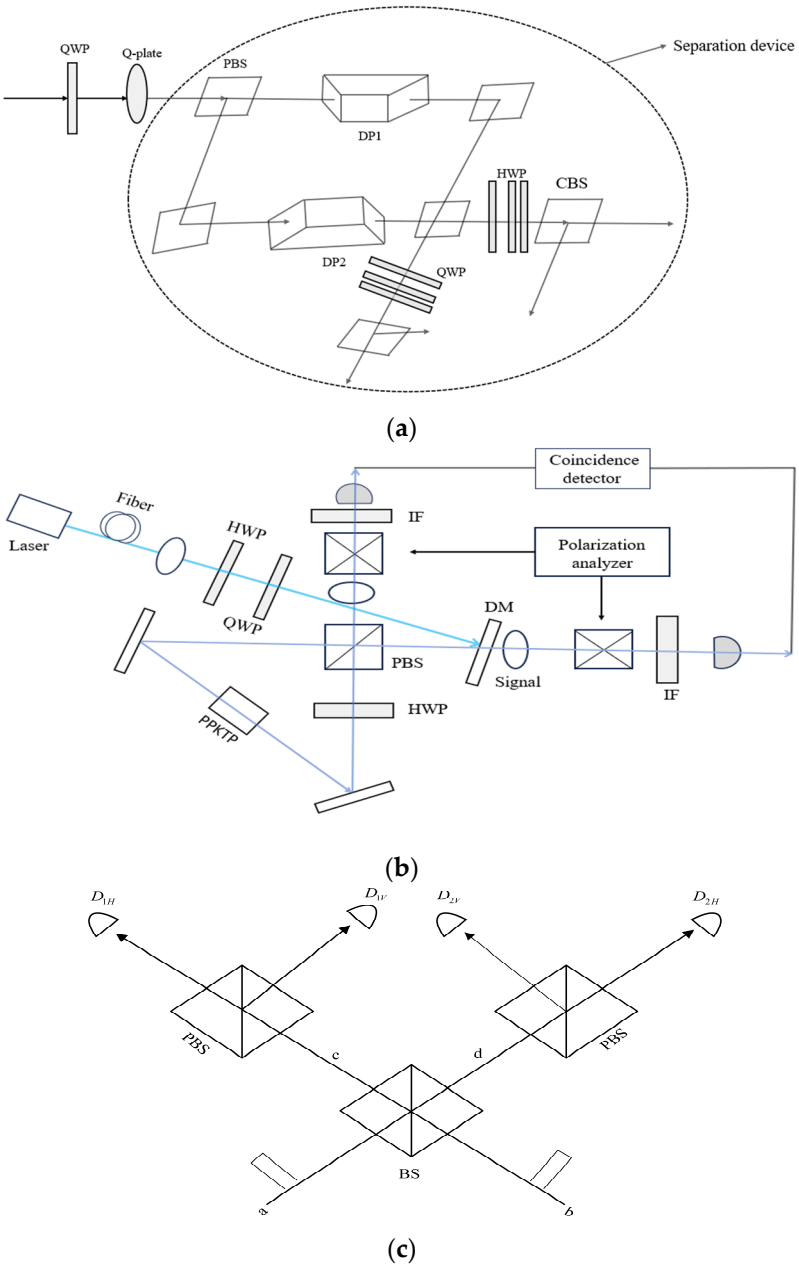



**Figure 2.** Experimental setups for generating and separating intraparticle and interparticle entangled states. In (**a**), the experimental setup for generating and separating intraparticle entangled states is shown [16]. QWP represents the quarter wave plate, and the Q-plate is an optical instrument (modulating the polarization state of light). PBS is the polarization beam splitter, DP is the Michelson interferometer, and HWP represents the half-wave plate. CBS represents the circularly polarized beam splitter. In (**b**), the experimental setup for generating interparticle entangled states is shown. PPKTP is an efficient non-linear artificial crystal (generating entangled photon sources), DM is the reflector mirror, IF is 1-nm interference filter [36]. In (**c**), the experimental setup for separating interparticle entangled states is shown. $D_{1H}$, $D_{2H}$, $D_{1V}$, and $D_{2V}$ are the detectors, and BS represents the beam splitter.

### *2.4. Practical Application*

This section compares the practical application of the two types of entangled states. Interparticle entangled states can be used in many state-of-the-art technologies, including quantum key distribution (QKD), quantum dense coding, quantum secure direct communication (QSDC), and QT. Intraparticle entangled states can be applied to quantum-dense

encoding, entanglement switching, QT, and QKD. Intraparticle entanglement states exhibit stronger robustness and a greater capacity for information carrying than interparticle entangled states [15] in communication processes. However, separating and detecting intraparticle entangled states is a relatively challenging task. High levels of security are demonstrated for the two types of entanglement states [15,17,38].

All in all, this section compared and analyzed the distinctions and similarities between the two kinds of entangled states. These findings show that interparticle and intraparticle entanglement both have a correlation and that interparticle entanglement is non-local; however, non-locality has no meaning for intraparticle entanglement. Intraparticle entanglement has a stronger robustness and higher capacity to carry information, and it is easier to generate and more convenient to detect and separate than interparticle entanglement in real-world applications.

## 3. Methods

### *3.1. Entanglement Swapping Protocols*

After comparing the distinctions and similarities between two types of entangled states, and this section will present three entanglement swapping methods based on Bell state joint measurement, the cross-Kerr medium, and linear optical elements. The structure of this section is as follows. Firstly, the principle of entanglement swapping is illustrated in Section 3.1.1. Then, three entanglement swapping protocols are displayed in Sections 3.1.2–3.1.4.

#### 3.1.1. Entanglement Swapping Principle

The principle of entanglement swapping is illustrated in Figure 3. Particles 1 and 2 are entangled in their respective spin–orbit angular momentum degrees of freedom. The initially unrelated two degrees of freedom of orbital angular momentum can be entangled with one another between two particles by performing certain related unitary operations.

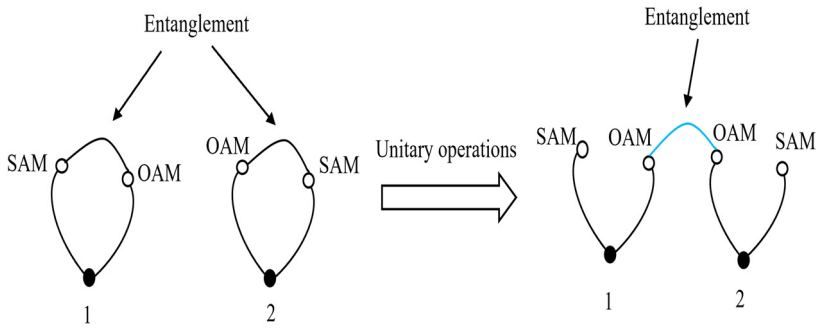



**Figure 3.** Entanglement swapping principle. SAM represents the spin angular momentum, and OAM represents the orbital angular momentum. One circle represents one degree of freedom, and a black dot represents a particle.

#### 3.1.2. Entanglement Swapping Protocols Based on Bell State Joint Measurement

This section presents an entanglement swapping protocol from spin–orbit intraparticle to spin–spin interparticle entanglement based on Bell state joint measurement. We suppose that the spin–orbit intraparticle entangled states are

$$|\varphi\rangle_1 = \frac{1}{\sqrt{2}}(|0\rangle_{1s}|0\rangle_{1o} + |1\rangle_{1s}|1\rangle_{1o}), \tag{1}$$

and

$$|\varphi\rangle_2 = \frac{1}{\sqrt{2}}(|0\rangle_{2s}|0\rangle_{2o} + |1\rangle_{2s}|1\rangle_{2o}), \tag{2}$$

where $1s, 1o$ and $2s, 2o$ represent the spin and orbital angular momentum degrees of freedom of photons 1 and 2, respectively. The target entangled state between two photons is as follows:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{1o}|0\rangle_{2o} + |1\rangle_{1o}|1\rangle_{2o}), \tag{3}$$

and the joint state of Equations (1) and (2) can be written as shown here:

$$|\varphi\rangle_3 = \frac{1}{2}\begin{pmatrix} |0\rangle_{1s}|0\rangle_{1o}|0\rangle_{2s}|0\rangle_{2o} + |0\rangle_{1s}|0\rangle_{1o}|1\rangle_{2s}|1\rangle_{2o} \\ +|1\rangle_{1s}|1\rangle_{1o}|0\rangle_{2s}|0\rangle_{2o} + |1\rangle_{1s}|1\rangle_{1o}|1\rangle_{2s}|1\rangle_{2o} \end{pmatrix}. \tag{4}$$

United measurement is executed on the spin degree of freedom of two photons by using the Bell basis. The Bell basis is shown in the following equations:

$$\begin{array}{l} |\mathrm{g}^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{1s}|0\rangle_{2s} \pm |1\rangle_{1s}|1\rangle_{2s}) \\ |k^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{1s}|1\rangle_{2s} \pm |1\rangle_{1s}|0\rangle_{2s}) \end{array}. \tag{5}$$

Equation (4) can be rewritten as (6) using the Bell basis, as follows:

$$|\varphi\rangle_4 = \frac{1}{2\sqrt{2}}\begin{bmatrix} |\mathrm{g}^{+}\rangle(|1\rangle_{1o}|1\rangle_{2o} + |0\rangle_{1o}|0\rangle_{2o}) + |\mathrm{g}^{-}\rangle(|1\rangle_{1o}|1\rangle_{2o} - |0\rangle_{1o}|0\rangle_{2o}) \\ +|k^{+}\rangle(|0\rangle_{1o}|1\rangle_{2o} + |1\rangle_{1o}|0\rangle_{2o}) + |k^{-}\rangle(|0\rangle_{1o}|1\rangle_{2o} - |1\rangle_{1o}|0\rangle_{2o}) \end{bmatrix}. \tag{6}$$

According to (6), the desired entanglement swapping state $|\psi\rangle$ can be directly obtained when the joint measurement result is $|g^{+}\rangle$. If the joint measurement result is otherwise, a corresponding unitary operation is carried out on the orbital angular momentum degree of freedom of photon 2 to obtain the necessary entangled state. The measurement results and corresponding unitary operation operators are shown in Table 1.

**Table 1.** Measurement results and corresponding unitary operation operators.

| Measurement Results | Corresponding Unitary Operators |
|---|---|
| $\left\|g^{+}\right\rangle$ | $I$ |
| $\left\|g^{-}\right\rangle$ | $\sigma_z$ |
| $\left\|k^{+}\right\rangle$ | $\sigma_x$ |
| $\left\|k^{-}\right\rangle$ | $i\sigma_y$ |

The expression of these unitary operators is as follows:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, i\sigma_y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{7}$$

3.1.3. Entanglement Swapping Protocols Based on the Cross-Kerr Medium

The entanglement swapping protocol from spin–orbit intraparticle to spin–spin interparticle entanglement can be achieved using the cross-Kerr medium. We suppose that the intraparticle entangled states of photons 1 and 2 are as follows:

$$|\varphi\rangle_1' = \alpha|0\rangle_{1s}|0\rangle_{1o} + \beta|1\rangle_{1s}|1\rangle_{1o}, \tag{8}$$

and

$$|\varphi\rangle_2' = \alpha|0\rangle_{2s}|0\rangle_{2o} + \beta|1\rangle_{2s}|1\rangle_{2o}, \tag{9}$$

where $|\alpha|^2 + |\beta|^2 = 1$, $|\alpha|^2$ and $|\beta|^2$ represent the probability of obtaining each part, respectively. To obtain the target entangled state (10), the protocol, as shown in Figure 4, is used to implement the entanglement swapping as is shown here:

$$|\psi\rangle' = \frac{1}{c}(|0\rangle_{1o}|0\rangle_{2o} + |1\rangle_{1o}|1\rangle_{2o}), \tag{10}$$

where $\frac{1}{|c|^2}$ represents the probability of obtaining each part, $\frac{2}{|c|^2} = 1$.

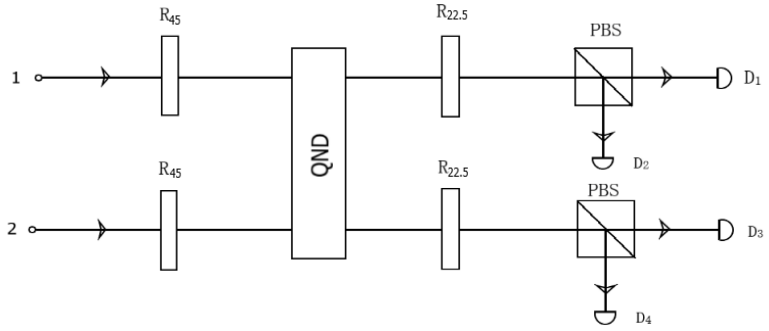



**Figure 4.** The experimental scheme of entanglement swapping based on the cross-Kerr medium. $R_{45}$ represents a half-wave plate placed at 45°, $R_{22.5}$ represents a half-wave plate placed at 22.5°, QND represents quantum nondemolition measurement, PBS represents polarization beam splitter, and $D_{i(i=1,2,3,4)}$ represents detectors.

The joint state $|\varphi\rangle_3'$ composed of $|\varphi\rangle_1'$ and $|\varphi\rangle_2'$ is as shown here:

$$|\varphi\rangle_3' = \alpha^2|00\rangle_{1so}|00\rangle_{2so} + \alpha\beta(|00\rangle_{1so}|11\rangle_{2so} + |11\rangle_{1so}|00\rangle_{2so}) + \beta^2|11\rangle_{1so}|11\rangle_{2so}, \quad (11)$$

where $|\alpha|^4 + |\beta|^4 + 2|\alpha|^2|\beta|^2 = 1$. To achieve entanglement swapping, a half-wave plate placed at 45° is used to perform a spin–flip operation on the spin degrees of freedom of photons 1 and 2, and $|\varphi\rangle_3'$ becomes $|\varphi\rangle_4'$, which is represented by the following equation:

$$|\varphi\rangle_4' = \alpha^2|10\rangle_{1so}|10\rangle_{2so} + \alpha\beta(|10\rangle_{1so}|01\rangle_{2so} + |01\rangle_{1so}|10\rangle_{2so}) + \beta^2|01\rangle_{1so}|01\rangle_{2so}. \quad (12)$$

Next, photons 1 and 2 pass through the QND [39], and $|\varphi\rangle_4'$ becomes $|\varphi\rangle_5'$, as is represented by the following equation:

$$|\varphi\rangle_5' = \left(\alpha^2|10\rangle_{1so}|10\rangle_{2so} + \beta^2|01\rangle_{1so}|01\rangle_{2so}\right)\left|he^{i\theta}\right\rangle + \alpha\beta|h\rangle\left(|10\rangle_{1so}|01\rangle_{2so} + |01\rangle_{1so}|10\rangle_{2so}\left|e^{2i\theta}\right\rangle\right), \quad (13)$$

where $|h\rangle$ represents incident light in QND, and $\theta$ represents the phase shift.

The corresponding unitary operations are performed on the QND to make $\theta = \pi$. Photons pass through the QND. If the measurement result of phase shift is 0 or $2\pi$, which indicates a difference in polarization states between two photons [37], $|\varphi\rangle_5'$ collapses to $|\varphi\rangle_6'$, as shown in the following equation:

$$|\varphi\rangle_6' = \sqrt{(1-\alpha^4-\beta^4)}(|10\rangle_{1so}|01\rangle_{2so} + |01\rangle_{1so}|10\rangle_{2so}). \quad (14)$$

Next, photons 1 and 2 pass through a half-wave plate (placed at 22.5°) that can achieve a Hadamard operation on the spin degree of freedom of the photons. The Hadamard's effect is as follows:

$$H|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \quad (15)$$

and

$$H|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle). \quad (16)$$

After photons pass through the half-wave plate placed at 22.5°, $|\varphi\rangle_6'$ will become $|\varphi\rangle_7'$, and the equation is as follows:

$$|\varphi\rangle_7' = \frac{\sqrt{(1-\alpha^4-\beta^4)}}{2}[(|01\rangle_{1o2o} + |10\rangle_{1o2o})(|00\rangle_{1s2s} - |11\rangle_{1s2s}) + (|01\rangle_{1o2o} - |10\rangle_{1o2o})(|01\rangle_{1s2s} - |10\rangle_{1s2s})], \quad (17)$$

where $1o2o$ and $1s2s$ represent the orbital and spin angular momentum degrees of freedom of photons 1 and 2, respectively. Next, photons 1 and 2 pass through a PBS, and then four photon detectors $D_1, D_2, D_3, D_4$ are used to detect photons. Based on detected results, the entangled state of the two photons is determined.

If $D_1, D_3$ or $D_2, D_4$ detectors respond at the same time, the entangled state $|\varphi\rangle_7'$ collapses as $|\varphi\rangle_8'$, which is shown by the following equation:

$$|\varphi\rangle_8' = \frac{\sqrt{(1-\alpha^4-\beta^4)}}{2}(|01\rangle_{1o2o} + |10\rangle_{1o2o}). \tag{18}$$

Performing the unitary operation $\sigma_x$ on the orbital angular momentum degree of freedom of photon 2 will provide the desired entangled state. The equation is as follows:

$$|\varphi\rangle_9' = \frac{\sqrt{(1-\alpha^4-\beta^4)}}{2}(|00\rangle_{1o2o} + |11\rangle_{1o2o}). \tag{19}$$

If $D_1, D_4$ or $D_2, D_3$ detectors respond at the same time, the entangled state $|\varphi\rangle_7'$ collapses as $|\varphi\rangle_{10}'$, which is represented by the following equation:

$$|\varphi\rangle_{10}' = \frac{\sqrt{(1-\alpha^4-\beta^4)}}{2}(|01\rangle_{1o2o} - |10\rangle_{1o2o}). \tag{20}$$

For the same reason, the entangled state desired will be obtained by performing the unitary operation $i\sigma_y$ on the orbital angular momentum degree of freedom of photon 2.

3.1.4. Entanglement Swapping Protocols Based on Linear Optical Elements

We assume that the intraparticle entangled states of photons 1 and 2 are (8) and (9). To obtain the desired entangled state (10), the protocol, as shown in Figure 5, is used to implement entanglement swapping. The joint state $|\varphi\rangle_3''$ composed of (8) and (9) is as follows:

$$|\varphi\rangle_3'' = \alpha^2|00\rangle_{1so}|00\rangle_{2so} + \alpha\beta(|00\rangle_{1so}|11\rangle_{2so} + |11\rangle_{1so}|00\rangle_{2so}) + \beta^2|11\rangle_{1so}|11\rangle_{2so}. \tag{21}$$

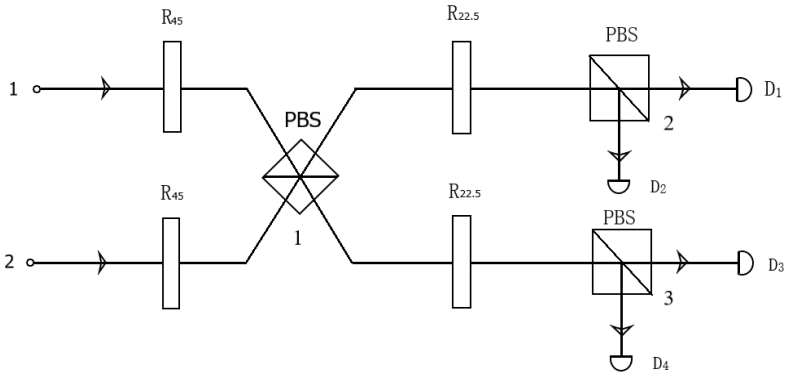



**Figure 5.** The experimental schematic of entanglement swapping based on linear optical elements. $R_{45}$ represents a half-wave plate placed at 45°, $R_{22.5}$ represents a half-wave plate placed at 22.5°, PBS represents the polarization beam splitter, and $D_{i(i=1,2,3,4)}$ represents detectors.

The spin–flip operation is performed on the spin degrees of freedom of photons 1 and 2 by the half-wave plates placed at 45°, and therefore (21) becomes (22), as is shown by the following equation:

$$\begin{aligned}|\varphi\rangle_4'' = {} & \alpha\beta(|1\rangle_{1s}|0\rangle_{2s}|0\rangle_{1o}|1\rangle_{2o} + |0\rangle_{1s}|1\rangle_{2s}|1\rangle_{1o}|0\rangle_{2o}) \\ & \alpha^2|1\rangle_{1s}|1\rangle_{2s}|0\rangle_{1o}|0\rangle_{2o} + \beta^2|0\rangle_{1s}|0\rangle_{2s}|1\rangle_{1o}|1\rangle_{2o}\end{aligned}. \tag{22}$$

In the above equation, the first and second terms will cause the transmission and reflection spatial modes of PBS1 to output photons. The third and fourth terms can only cause one spatial mode (transmission or reflection) of PBS1 to output photons. Thus, if photons output from both the transmission and reflection spatial modes of PBS1, $|\varphi\rangle_4''$ becomes $|\varphi\rangle_5''$.

$$|\varphi\rangle_5'' = \sqrt{(1-\alpha^4-\beta^4)}(|1\rangle_{1s}|0\rangle_{2s}|0\rangle_{1o}|1\rangle_{2o} + |0\rangle_{1s}|1\rangle_{2s}|1\rangle_{1o}|0\rangle_{2o}). \tag{23}$$

Next, photons pass through a half-wave plate placed at 22.5°, which can achieve a Hadamard operation for the spin degree of freedom of photons, and $|\varphi\rangle_5''$ becomes $|\varphi\rangle_6''$, which is represented by the following equation:

$$|\varphi\rangle_6'' = \frac{\sqrt{(1-\alpha^4-\beta^4)}}{2}[(|01\rangle_{1o2o} + |10\rangle_{1o2o})(|00\rangle_{1s2s} - |11\rangle_{1s2s}) + (|01\rangle_{1o2o} - |10\rangle_{1o2o})(|01\rangle_{1s2s} - |10\rangle_{1s2s})]. \tag{24}$$

In Equations (23) and (24), the subscripts $1s2s$ and $1o2o$ represent the spin and orbital angular momentum degrees of freedom of photons 1 and 2, respectively. Next, photons pass through a PBS, and four photon detectors are used to determine the results. If one of the four detectors rings twice, the entangled state $|\varphi\rangle_6''$ will collapse, as is represented by the following equation:

$$|\varphi\rangle_7'' = \frac{\sqrt{(1-\alpha^4-\beta^4)}}{2}(|01\rangle_{1o2o} + |10\rangle_{1o2o}). \tag{25}$$

The unitary operator $\sigma_x$ is performed on the orbital angular momentum degree of freedom of photon 2; the desired entangled state is obtained as the following:

$$|\varphi\rangle_8'' = \frac{\sqrt{(1-\alpha^4-\beta^4)}}{2}(|00\rangle_{1o2o} + |11\rangle_{1o2o}). \tag{26}$$

If detectors $D_1, D_2$ or $D_3, D_4$ ring, the entangled state $|\varphi\rangle_6''$ will collapse, as is represented by the following equation:

$$|\varphi\rangle_9'' = \frac{\sqrt{(1-\alpha^4-\beta^4)}}{2}(|01\rangle_{1o2o} - |10\rangle_{1o2o}). \tag{27}$$

The desired entangled state will be obtained by performing the unitary operation $i\sigma_y$ on the orbital angular momentum degree of freedom of photon 2.

All in all, this section explained the principle of entanglement swapping, and designed three entanglement swapping protocols, from spin–orbit intraparticle to spin–spin interparticle entanglement, based on Bell state joint measurement, the cross-Kerr medium, and linear optical elements.

## 4. Conclusions

This work analyzed the distinctions and similarities between the two kinds of entangled states. The findings demonstrate that intraparticle and interparticle entanglement both have a correlation, that interparticle entanglement is non-local but non-locality has no meaning for intraparticle entanglement, that intraparticle entanglement has a greater capacity for information carrying than interparticle entangled states in practical applications, and that, while the creation of intraparticle entangled states is easier, the detection and separation of intraparticle entangled states is more complex. Furthermore, three swapping protocols were devised, from intraparticle spin–orbit to spin–spin interparticle entanglement, in accordance with the entanglement swapping principle and based on Bell state joint measurement, the cross-Kerr medium, and linear optical elements. The three entanglement swapping protocols correspond to the level of implementation complexity. Laboratory equipment might be considered when selecting these protocols in particular circumstances.

This work contributes to the further understanding of entanglement by examining the distinctions and similarities between the two types of entangled states. Based on these

entanglement swapping protocols, this work may help enhance intraparticle entangled states for use in quantum information transfer. In the future, this can be further explored by the application of these entanglement swapping protocols to the quantum repeater.

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