

Article

High-Resolution Identification of Sound Sources Based on Sparse Bayesian Learning with Grid Adaptive Split Refinement

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Abstract: Sound source identification technology based on a microphone array has many application scenarios. The compressive beamforming method has attracted much attention due to its high accuracy and high-resolution performance. However, for the far-field measurement problem of large microphone arrays, existing methods based on fixed grids have the defect of basis mismatch. Due to the large number of grid points representing potential sound source locations, the identification accuracy of traditional grid adjustment methods also needs to be improved. To solve this problem, this paper proposes a sound source identification method based on adaptive grid splitting and refinement. First, the initial source locations are obtained through a sparse Bayesian learning framework. Then, higher-weight candidate grids are retained, and local regions near them are split and updated. During the iteration process, Green's function and the source strength obtained in the previous iteration are multiplied to get the sound pressure matrix. The robust principal component analysis model of the Gaussian mixture separates and replaces the sound pressure matrix with a low-rank matrix. The actual sound source locations are gradually approximated through the dynamically adjusted sound pressure low-rank matrix and optimized grid transfer matrix. The performance of the method is verified through numerical simulations. In addition, experiments on a standard aircraft model are conducted in a wind tunnel and speakers are installed on the model, proving that the proposed method can achieve fast, high-precision imaging of low-frequency sound sources in an extensive dynamic range at long distances.

Keywords: sound source identification; sparse Bayesian learning; grid adaptive split refinement; far-field measurement



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1. Introduction

Sound source identification technology extracts the location and strength information of the sound source from the acoustic signal measured by the microphone array. Accurate identification of sound sources is crucial in sound field characterization and noise control in aerospace [1–4], power transformers [5,6], high-speed trains [7,8], and other scenarios [9–11]. In these application scenarios, the noise distribution area is wide, and the sound field propagation characteristics are complex. In addition, for safety operational reasons, microphone arrays are often placed in the far field for measurements [12]. The above characteristics bring the following difficulties to the sound source identification process. First, due to the extensive focus area being discrete, it is difficult to ensure that the discrete grid points exactly cover the actual sound source locations, which may cause basis mismatch errors [13]. More importantly, due to the limited array aperture size and number of microphones, it will be much more difficult to obtain high-resolution sound source identification results during far-field measurements when multiple low-frequency

sound sources are distributed nearby in the space [14]. In addition, path effects during far-field propagation can amplify the impact of measurement noise [15].

In recent years, compressive beamforming has been widely used to improve sound source identification performance. The method based on the compressed sensing theory first assumes different types of equivalent sources. Then, it applies sparsely constrained prior information [16] to the source distributions in various domains, such as the frequency domain [17] and the spatial-temporal domain [18]. Finally, the optimal sound source distribution from the measured signal is sought. Compared with the traditional beamforming method, the compressive beamforming method can achieve higher resolution based on fewer measurement samples. At the same time, such methods possess a higher dynamic range and allow for dealing with correlated/uncorrelated and sparse/non-sparse distributed sources, as well as obtaining quantitative results on the strength of the sources. However, this is an ill-posed problem in the Hadamard sense. The linear operator matrix in the inverse problem is usually irreversible and ill-conditioned. It has a large number of conditions, which makes the signal reconstruction extremely sensitive to small changes in the initial conditions [19], leading to an unstable solution under small perturbations in the measurement noise.

Many methods have been developed to ensure stable solutions for ill-conditioned inverses. Orthogonal matching pursuit (OMP) [20] can effectively approximate L_0 norm minimization, but it does not perform well under conditions where the constraint boundaries cannot be guaranteed. Therefore, the original non-convex L_0 norm minimization problem is usually reduced to an L_1 norm relaxation optimization problem. Methods such as the minimum-angle regression lasso [21] algorithm and the generalized inverse beamforming (GIBF) proposed by Suzuki [22] can both achieve L_1 norm minimization. Presezniak [23] proposed a weighted matrix method to increase the dynamic range of GIBF, thereby improving the sound source identification resolution. In addition, Ping [24] proposed an iteratively reweighted L_1 norm minimization compressive beamforming (IR-CSB) method to replace the relaxed L_1 norm penalty, effectively improving the performance of source strength estimation. Yu [25] established an acoustic inverse problem framework based on the alternating direction method of multipliers, using the L_2 norm penalty based on the minimum energy assumption and the L_p norm penalty based on the sparsity assumption to solve, respectively, which can be applied to different types of sound sources. However, a common problem with the above methods is difficulty in selecting an appropriate regularization strategy. No regularization parameter selection method is currently suitable for all problems [26], which significantly impacts severely ill-conditioned acoustic inverse problems.

To improve regularization methods, Bayesian inference becomes a powerful tool for enhancing sparsity and adaptive parameter estimation. Under the Bayesian framework, an iterative algorithm is derived based on Bayesian optimization criteria such as maximum posterior probability and minimum KL divergence. This can adaptively estimate regularization parameters and significantly improve calculation accuracy while ensuring improved computational efficiency. Wipf [27] conducted an in-depth theoretical study on the sparse Bayesian learning (SBL) algorithm and found that it has significant advantages. The SBL has demonstrated its ability to obtain genuinely sparse solutions, showing insensitivity to high correlations between columns in the sensing matrix. Moreover, the SBL exhibits flexibility in utilizing the structural information of the solution, displaying inherent parameter adaptability. Based on the advantages of the SBL, some scholars have redefined the inverse problem of sound source identification as a Bayesian inference problem. Bai [28] proposed a high-resolution DOA estimation algorithm based on spatially alternating a multi-snapshot SBL. Gerstoft [29] also proposed an SBL algorithm for multi-snapshot high-resolution DOA estimation. The difference is that this algorithm uses derivative-based evidence maximization to estimate source power and noise variance. Hu [30] proposed a fast sparse reconstruction method based on Bayesian compressed sensing (BCS), which is applied to the sound field reconstruction after sparsely decomposing the sound field using

singular value decomposition. This method applies to spatially sparsely distributed and spatially extended sources. Considering that in the process of solving the inverse problem, the mismatch between the acoustic inversion model and the actual physical model will cause uncertainty in the sound source identification problem, Gilquin [31] compared traditional beamforming and Bayesian focusing through sensitivity analysis. The results showed that Bayesian focusing is overall more resilient to sources of uncertainty than conventional beamforming.

The above sound source identification methods are all implemented by dividing the search area into discrete grids. The crucial assumption is that potential sound sources are located at the search points. However, when the sources span a wide range, it is difficult to ensure that all the sources fall precisely on the grid positions, and then the received data cannot consist of a base of candidate discrete points. The difference between the actual sound source location and the nearest grid point will produce a basis mismatch, thereby reducing the accuracy of the sound source identification estimation. A direct way to alleviate basis mismatch is to refine the mesh, but at the cost of a sharp increase in computational effort and amplification of reconstruction errors due to a high correlation between adjacent basis points. Therefore, some scholars consider optimizing the mathematical model. According to different mathematical models, compressive beamforming can be divided into fixed on-grid [32], off-grid [13], and grid-free [33] methods. The off-grid and grid-free methods are introduced below.

Instead of discretizing the focal region, the grid-free method establishes source sparsity metrics (such as atomic norm [34–37] and total variational norm [38]) and directly processes the focus region as a continuum, thus theoretically avoiding the basis mismatch problem. However, grid-free methods require a priori signal-to-noise ratio (SNR) information. They are more suitable for linear arrays [34] or uniform rectangular [33] arrays, requiring extensive computation to extend to arbitrary arrays [39,40]. Off-grid methods mainly include two categories [41]: off-grid compressive beamforming that uses Taylor expansion to compensate for grid deviation and off-grid compressive beamforming that uses Newtonized orthogonal matching pursuit to search for the actual sound source location. The first type of method was proposed by Yang [42], which uses grid-based DOA estimation and two-dimensional off-grid DOA compensation as blocks to establish a block-sparse off-grid compressive beamforming model and solve it to obtain source coordinate estimates. Park [13], Sun [43], Wang [44], and others used the CVX toolbox, group minimax concave penalty function, and weighted block L_1 norm regularization to solve this problem, respectively. These methods effectively compensate for off-grid bias. However, the complexity of the model will be increased due to the introduction of additional grid variables. This method is not fully applicable, especially when multiple sound sources exhibit large spatial dispersion, resulting in many discrete points on the source surface. The second type of method [45] takes source coordinates and source strength as parameters to establish a maximum-likelihood estimate. It uses the initial estimate and subsequent Newton optimization to obtain accurate source coordinates and source strength estimates. This type of method is robust and computationally efficient, but its resolution performance is limited. If there are low-frequency sound sources that are closely spaced, the solution of the technique will seriously deviate from the actual sound source location.

In summary, among the existing compressive beamforming methods, the fixed on-grid methods suffer from the defect of base mismatch. Grid-free methods require prior information and stringent requirements for array microphone placement. Off-grid methods introduce additional grid position offsets during the solution process, increasing the solution's variable dimensions and model complexity. For source identification problems with significant source surface spans and many potential discrete points, large approximation errors can result. Therefore, a new strategy needs to be proposed.

Zan [46] proposed a variable-scale grid sound source localization method. This method first uses a relatively sparse calculation grid to generate a beam map quickly, retains the calculation grids with higher weights, and then uses the deconvolution method to locate

the sound source, effectively improving efficiency. Inspired by this, this paper proposes an off-grid compressive beamforming method based on sparse Bayesian learning and grid adaptive split refinement, which can be applied to a large focal region, effectively overcomes the basis mismatch problem, and can be applied to any microphone array. The difference is that Zan mainly uses the reduction in the number of grid points to improve the computational efficiency of the deconvolution method, while this paper mainly uses the reduction in the number of grid points to reduce the column coherence of the sensing matrix, thereby weakening the ill-posed inverse problem solving and improving the robustness. Further, the base mismatch problem is solved by continuously adjusting the positions of the grids, which improves the accuracy.

This paper proposes a sound source identification method based on sparse Bayesian learning of grid splitting refinement. The initial source locations are obtained based on a sparse Bayesian learning framework. Then, candidate grids with higher weights are retained, local areas near them are refined, and Green's functions and source strength matrices are re-established. The robust principal component analysis model replaces the sound pressure matrix obtained in the previous iteration with a low-rank matrix. The actual source location and source strength estimates are gradually approximated through the dynamically adjusted sound pressure low-order matrix and grid iteration matrix. Unlike the current conventional off-grid or grid-free methods, the method proposed in this paper is a new strategy used to solve the problem of algorithm performance degradation when the number of grid points is significant.

The main contributions of this paper are as follows.

(1) Initially, relatively sparse computational grids are established, and a statistical model describing the radiated sound field is utilized. Prior information and posterior probabilities are integrated to provide the initial values of source strength obtained from the sparse Bayesian learning. Subsequently, the previously established computational grid model is iteratively adjusted, retaining computation grids with higher weights. Fission updates are performed in their vicinity, gradually shrinking the discrete region, increasing grid density, and updating source strength values until the grid is fine enough. During the iteration process, low-rank sparse decomposition is used simultaneously to eliminate the influence of noise. This method is called VG-SBL.

(2) Simulation and wind tunnel experiments verify the superiority of the proposed method in low-frequency and high-resolution sound source identification. In particular, the proposed method can generate the most accurate and clear imaging maps compared with the traditional methods in the case of far measurement distances, low source frequencies, close source spacing, and high dynamic ranges, thus showing the robustness of the method and its potential for application in wind tunnel research.

The rest of the paper is organized as follows. In Section 2, the sound forward propagation model is presented. In Section 3, the principle of the VG-SBL method is presented. Simulations are performed in Section 4. Section 5 comprises the analysis of measured wind tunnel data. Finally, Section 6 concludes this paper.

2. Problem Statement

Assuming that multiple sound sources are radiating to the surroundings in space, the sound signals are acquired synchronously using a microphone array consisting of M channels, $r_s, s = 1, 2, \dots, N$ denoting the discrete grid point position, that is, the potential location of the sound sources, $r_i, i = 1, 2, \dots, M$ denoting the location of the microphones and Γ denoting the spatial domain consisting of discrete sound sources. Considering the relationship between the source distribution $q(r_s)$ and the radiated sound pressure as an integral Fredholm form, the sound pressure $p(r_i)$ measured by the microphone array can be expressed as:

$$p(r_i) = \int_{\Gamma} G(r_i r_s) q(r_s) d\Gamma(r_s) + e_i, i = 1, 2, \dots, M, s = 1, 2, \dots, N \quad (1)$$

where $G(\mathbf{r}_i|\mathbf{r}_s)$ denotes Green's function between the source location $\mathbf{r}_s$ and the measurement point $\mathbf{r}_i$ in the free field, and e_i denotes the measurement noise received by each microphone.

The free-field Green function is calculated as follows:

$$G(\mathbf{r}_i|\mathbf{r}_s) = \exp(ik|\mathbf{r}_i - \mathbf{r}_s|) / (4\pi|\mathbf{r}_i - \mathbf{r}_s|) \quad (2)$$

where k is the wave number. The measurement signals of M microphones are stacked and converted into column vectors, and the system of equations is expressed in a simple form:

$$\mathbf{p} = \int_{\Gamma} G(\mathbf{r}_s)q(\mathbf{r}_s)d\Gamma(\mathbf{r}_s) + \mathbf{e} \quad (3)$$

The i -th element of the column vector $G(\mathbf{r}_s)$ is $G(\mathbf{r}_i|\mathbf{r}_s)$, and $\mathbf{e}$ denotes the column vector of the noise signal.

Since measuring the acoustic field over the entire surface surrounding the radiator is impossible, this is an ill-posed problem in the Hadamard sense. To solve this problem, an estimate of the source strength is returned as a linear combination of the measurements [47].

Consider L_{snap} snapshots and rewrite the Equation (3) in matrix form:

$$\mathbf{P} = \mathbf{G}\mathbf{Q} + \mathbf{E} \quad (4)$$

$\mathbf{P} = [\mathbf{p}_1, \dots, \mathbf{p}_l, \dots, \mathbf{p}_{L_{snap}}] \in \mathbb{C}^{M \times L_{snap}}$, $\mathbf{p}_l = [p_1, \dots, p_i, \dots, p_M] \in \mathbb{C}^{M \times 1}$, and $\mathbf{P}$ represent M microphones' complex sound pressure matrix under all L_{snap} snapshots.

$\mathbf{Q} = [\mathbf{q}_1, \dots, \mathbf{q}_l, \dots, \mathbf{q}_{L_{snap}}] \in \mathbb{C}^{N \times L_{snap}}$, $\mathbf{q}_l = [q_1, \dots, q_s, \dots, q_N] \in \mathbb{C}^{N \times 1}$, and $\mathbf{Q}$ represent N focused grid points' unknown source strength distribution matrix under all L_{snap} snapshots.

$\mathbf{E} = [\mathbf{e}_1, \dots, \mathbf{e}_l, \dots, \mathbf{e}_{L_{snap}}] \in \mathbb{C}^{M \times L_{snap}}$, $\mathbf{e}_l = [e_1, \dots, e_i, \dots, e_M] \in \mathbb{C}^{M \times 1}$, and $\mathbf{E}$ represent the noise matrix of M microphones under all L_{snap} snapshots. The essence of the source identification problem is to recover the source signal $\mathbf{Q}$ from the measured value $\mathbf{P}$, which is an ill-posed problem in the Hadamard sense.

However, it is important to note that when the real sound sources are located at $\mathbf{r}_{source}$ and not on the discrete grid points $\mathbf{r}_s$, it will lead to a mismatch between the distance $|\mathbf{r}_i - \mathbf{r}_{source}|$ from the microphones to the sources and the distance $|\mathbf{r}_i - \mathbf{r}_s|$ from the microphones to the discrete grid points, as shown in Figure 1, which will result in the base mismatch error. Therefore, how to improve the accuracy of sound source identification under model base mismatch needs to be investigated.

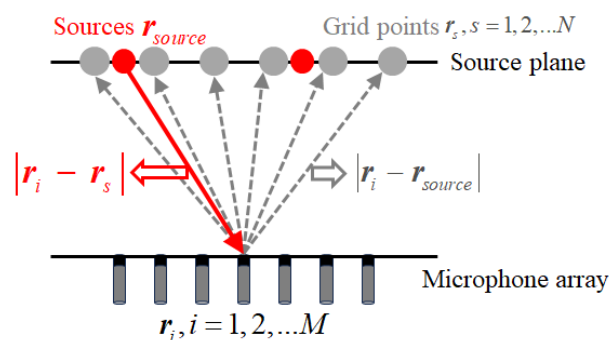


Figure 1. Microphone array measurement model. The red circles represent the actual positions of sound sources, the grey circles indicate the positions of discrete grids, the red arrows illustrate the radiation process from the sound source to the microphone array, and the grey arrows depict the sound source identification process.

3. Inverse Solving Process

3.1. Sparse Bayesian Learning Solver

A statistical model is used to describe the radiated sound field, and a Bayesian framework is used to express the inverse problem of sound source identification. The SBL method can be divided into two steps [48]. Firstly, the posterior distributions of the model parameters are deduced. Then, the maximum-likelihood method is used to learn the hyperparameters, which adaptively adjust the prior model to the sparse model. The two-level hierarchical Bayesian inference provides a good prospect for studying the sparse recovery problem.

Assuming that the different row vectors of $\mathbf{Q}$ have the same temporal correlation model and that the vectors of different rows are not correlated with one another, each snapshot datum (column vector) is sparse, and the prior model can be expressed by:

$$\mathcal{P}(\mathbf{Q}) = \prod_{l=1}^{L_{snap}} \mathcal{N}(\mathbf{q}_l; 0, \mathbf{\Gamma}) \quad (5)$$

where $\mathbf{\Gamma} = \text{diag}^{-1}(\gamma) = \text{diag}^{-1}\{\gamma_1, \dots, \gamma_s, \dots, \gamma_N\}$, and γ represents the correlation between the source strength and the observed signal.

It is assumed that the measured sound pressure $\mathbf{P}$ follows the probability density distribution:

$$\mathcal{P}(\mathbf{P}|\beta) = \prod_{l=1}^{L_{snap}} \mathcal{N}(\mathbf{p}_l; \mathbf{G}\mathbf{q}_l, \beta^{-1}\mathbf{I}) \quad (6)$$

The observation noise $\mathbf{E}$ is assumed to be independently and identically distributed, satisfying $\mathcal{P}(\mathbf{E}) = \mathcal{N}(0, \beta^{-1}\mathbf{I})$, where β^{-1} is the noise variance and $\mathbf{I}$ is the unit matrix.

Based on the signal and observation models, the posterior probability density, which corresponds to the solution of the inverse problem in the Bayesian setup, can be obtained using Bayes's rule and the Gaussian constant equation.

$$\mathcal{P}(\mathbf{Q}|\mathbf{P}; \mathbf{\Gamma}, \beta) = \frac{\mathcal{P}(\mathbf{P}|\mathbf{Q}; \beta)\mathcal{P}(\mathbf{Q}; \mathbf{\Gamma})}{\mathcal{P}(\mathbf{P}; \mathbf{\Gamma}, \beta)} \propto \mathcal{P}(\mathbf{P}|\mathbf{Q}; \beta)\mathcal{P}(\mathbf{Q}; \mathbf{\Gamma}) = \mathcal{N}(\mathbf{Q}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (7)$$

The likelihood function is:

$$\mathcal{P}(\mathbf{P}|\mathbf{\Gamma}, \beta) = \mathcal{N}(\mathbf{P}|0, \mathbf{C}) \quad (8)$$

Of these:

$$\boldsymbol{\mu} = \boldsymbol{\Sigma}\mathbf{G}^T\beta\mathbf{P} \quad (9)$$

$$\boldsymbol{\Sigma}^{-1} = \mathbf{\Gamma}^{-1} + \mathbf{G}^T\beta\mathbf{G} \quad (10)$$

$$\mathbf{C} = \beta^{-1}\mathbf{I} + \mathbf{G}\mathbf{\Gamma}\mathbf{G}^T \quad (11)$$

To estimate the covariates $\mathbf{\Gamma}$ and β , the cost function is obtained using a maximum-likelihood approach:

$$\mathcal{L}(\mathbf{\Gamma}, \beta) \triangleq -2\log \mathcal{P}(\mathbf{P}|\mathbf{\Gamma}, \beta) = L_{snap} \log|\mathbf{C}| + \text{Tr}[\mathbf{P}^T\mathbf{C}^{-1}\mathbf{P}] \quad (12)$$

Define:

$$\mathbf{C}_{-s} \triangleq \beta^{-1}\mathbf{I} + \sum_{j \neq s} \mathbf{G}_j\gamma_j\mathbf{G}_j^T \quad (13)$$

By using Woodbury's equation, it can be obtained that:

$$|\mathbf{C}| = |\gamma_s||\mathbf{C}_{-s}| \left| \gamma_s^{-1} + \mathbf{s}_s \right| \quad (14)$$

where

$$\mathbf{C}^{-1} = \mathbf{C}_{-s}^{-1} - \mathbf{C}_{-s}^{-1} \mathbf{G}_s (\gamma_s^{-1} + \mathbf{s}_s)^{-1} \mathbf{G}_s^T \mathbf{C}_{-s}^{-1} \quad (15)$$

By using the fast edge likelihood maximization method to perform Woodbury decomposition on the cost function, the formula for updating parameters can be obtained as follows:

$$\mathbf{s}_s \triangleq \mathbf{G}_s^T \mathbf{C}_{-s}^{-1} \mathbf{G}_s \quad (16)$$

$$\mathbf{t}_s \triangleq \mathbf{G}_s^T \mathbf{C}_{-s}^{-1} \mathbf{P} \quad (17)$$

The iterative updating of the parameters begins below. Making $\frac{\partial L}{\partial \gamma_s} = 0$, one obtains:

$$\gamma_s = \frac{\mathbf{t}_s^T \mathbf{t}_s / L_{snap} - \mathbf{s}_s}{\mathbf{s}_s^2} \quad (18)$$

In each iteration, after selecting the signal with the most significant reduction in the cost function, the corresponding parameter needs to be recalculated, and the parameter update consists of three types: signal addition, signal revaluation, and signal deletion [49].

The convergence condition of this algorithm is that the normalized rate of change of the correlation coefficient γ_s in two adjacent iterations is less than the threshold η , and the updated value after iteration is used as the source strength Q .

3.2. Signal Separation Based on Robust Principal Component Analysis

The robust principal component analysis (RPCA) [50] method can decompose the sound pressure matrix $\mathbf{P}$ into a low-rank matrix and a sparse matrix. Combined with Equation (4), it can be expressed as:

$$\mathbf{P} = \mathbf{G}\mathbf{Q} + \mathbf{E} = \mathbf{R} + \mathbf{E} \quad (19)$$

$\mathbf{R}$ is a low-rank matrix representing the signal component and $\mathbf{E}$ is a sparse matrix representing the noise component. Considering the complex composition of real noise components, Zhao [51] modeled noise as a mixture of Gaussians (MoG), which can approximate any continuous distribution, and developed a generative RPCA model under a Bayesian framework.

Assume that each component e_i in $\mathbf{E}$ follows the MoG distribution, that is:

$$e_i \sim \sum_{k=1}^K \pi_k N(e_i | \mu_k, \tau_k^{-1}) \quad (20)$$

K is the number of Gaussian distributions, $\pi_k \geq 0$ is the weight, and $\sum_{k=1}^K \pi_k = 1$, μ_k and τ_k^{-1} represent the mean and standard deviation of the Gaussian distribution, respectively. The conjugate prior parameters μ_k, τ_k and weights $\boldsymbol{\pi} = (\pi_1, \dots, \pi_K)$ can be expressed as:

$$\mu_k, \tau_k \sim \mathcal{N}(\mu_k | \mu_0, (\beta_0 \tau_k)^{-1}) \text{Gam}(\tau_k | c_0, d_0) \quad (21)$$

$$\boldsymbol{\pi} \sim \text{Dir}(\boldsymbol{\pi} | \alpha_0) \quad (22)$$

$\text{Gam}(\tau | c_0, d_0)$ is the gamma distribution of parameters c_0, d_0 , and $\text{Dir}(\boldsymbol{\pi} | \alpha_0)$ represents the Dirichlet distribution of $\alpha_0 = (\alpha_{01}, \dots, \alpha_{0K})$ parameterization.

A low-rank component model is established based on the automatic correlation determination method, and the matrix $\mathbf{R}_{M \times N}$ with rank $r \leq \min(M, N)$ is expressed as the product of $\mathbf{U}_{M \times H}$ and $\mathbf{V}_{N \times H}$, that is:

$$\mathbf{R} = \mathbf{U}\mathbf{V}^T = \sum_{h=1}^H \mathbf{u}_{.h} \mathbf{v}_{.h}^T \quad (23)$$

where $H > r$, $\mathbf{u}_{.h}$ and $\mathbf{v}_{.h}$ are the h -th columns of $\mathbf{U}$ and $\mathbf{V}$, respectively. Column sparsity is applied to $\mathbf{U}$ and $\mathbf{V}$ to ensure the low rank of $\mathbf{R}$:

$$\mathbf{u}_{.h} \sim \mathcal{N}(\mathbf{u}_{.h}|0, \zeta_h^{-1}\mathbf{I}_M), \mathbf{v}_{.h} \sim \mathcal{N}(\mathbf{v}_{.h}|0, \zeta_h^{-1}\mathbf{I}_N) \quad (24)$$

$\mathbf{I}_M$ and $\mathbf{I}_N$ are the identity matrices. The conjugate prior for each accuracy variable ζ_h is:

$$\zeta_h \sim \text{Gam}(\zeta_h|a_0, b_0) \quad (25)$$

Combining Equations (19), (20) and (25), a complete Bayesian robust principal component analysis model based on MoG is obtained, and the goal is to infer the posterior of all relevant variables.

3.3. Sound Source Identification Method Based on Grid Splitting and Refinement

A sound source identification method based on grid split refinement is proposed in this paper, and the process is shown as follows.

The superscript brackets indicate the number of iterations, and the initial value of the iteration number “*iter*” is set to 0.

Step 1: The decay factor δ , and the iteration stopping threshold σ are set, and the dynamic range is set to D_{range} . These parameter settings can be determined according to the situation. In this paper, σ is set to 0.001 and D_{range} is set to 30 dB. The selection of the decay factor δ will be introduced later.

Step 2: Collect the snapshot sound signal $\mathbf{P}^{(iter)}$, discretize the sound source plane into a grid surface with $A_s^{(iter)}$ as the aperture and $d_s^{(iter)}$ as the spacing between adjacent points, and the grid points $[\mathbf{X}^{(iter)}, \mathbf{Y}^{(iter)}]$ are used as the scanning point grid. Construct Green’s function $\mathbf{G}^{(iter)}$ as shown in Equation (2).

Step 3: According to Equations (12)–(18), the initial source strength $\mathbf{Q}^{(iter)}$ is obtained by sparse Bayesian learning.

Step 4: The grids are gradually refined and updated near the local maximum of the source strength. For the source strength $\mathbf{Q}^{(iter)}$, The maximum value of the average of its columns and corresponding coordinate location $q_{\max}^{(iter)}(x_{\max}^{(iter)}, y_{\max}^{(iter)})$ is searched. Following this, the set $\phi_{\max}^{range(iter)} = \{q_{\max}^{(iter)}, \dots, q_{\max-D_{range}}^{(iter)}\}$ is determined, consisting of grid positions corresponding to all local maxima within the descending dynamic range D_{range} of the maximum value $q_{\max}^{(iter)}$. These positions are selected as the centers of potential grid splits in the next step.

Step 5: Each element in the set $\phi_{\max}^{range(iter)}$ is used as the geometric center of grid fission, and the new scan points $[\mathbf{X}^{(iter+1)}, \mathbf{Y}^{(iter+1)}]$ are obtained by fission with $A_s^{(iter+1)} = A_s^{(iter)} \times \delta^{iter}$ as the aperture of the scanning surface and $d_s^{(iter+1)} = d_s^{(iter)} \times \delta^{iter}$ as the interval between adjacent points, where δ^{iter} represents δ raised to the “*iter*” power. Remove the duplicate grid points obtained after fission and construct the transfer model $\mathbf{G}^{(iter+1)}$ between the new grid points and the measurement surface.

Step 6: Determine whether $d_s^{(iter+1)}$ obtained in Step 5 reaches the threshold σ . If it has reached the threshold, output the source strength $\mathbf{Q}^{(iter)}$ calculated in Step 3 and the algorithm ends. If the threshold is not reached, combine the $\mathbf{G}^{(iter)}$ and $\mathbf{Q}^{(iter)}$ obtained in Step 2 and Step 3, respectively, to form a new sound pressure $\mathbf{P}^{(iter+1)} = \mathbf{G}^{(iter)} \mathbf{Q}^{(iter)}$, and enter it into the MoG-RPCA model. Replace the new sound pressure with the separated low-rank matrix, that is, $\mathbf{P}^{(iter+1)} = \mathbf{U}^{(iter+1)} \mathbf{V}^{(iter+1)}$. Then, add one to the number of iterations.

Step 7: The new transfer model $\mathbf{G}^{(iter+1)}$ calculated through Step 5 and the new sound pressure $\mathbf{P}^{(iter+1)}$ calculated through Step 6 are entered into Step 3 to obtain the updated source strength $\mathbf{Q}^{(iter+1)}$.

Step 8: Repeat the above Steps (3)–(7) until the iteration is jumped out in the judgment of Step 6 and the final source strength is output.

The principle of this method is illustrated in Figure 2.

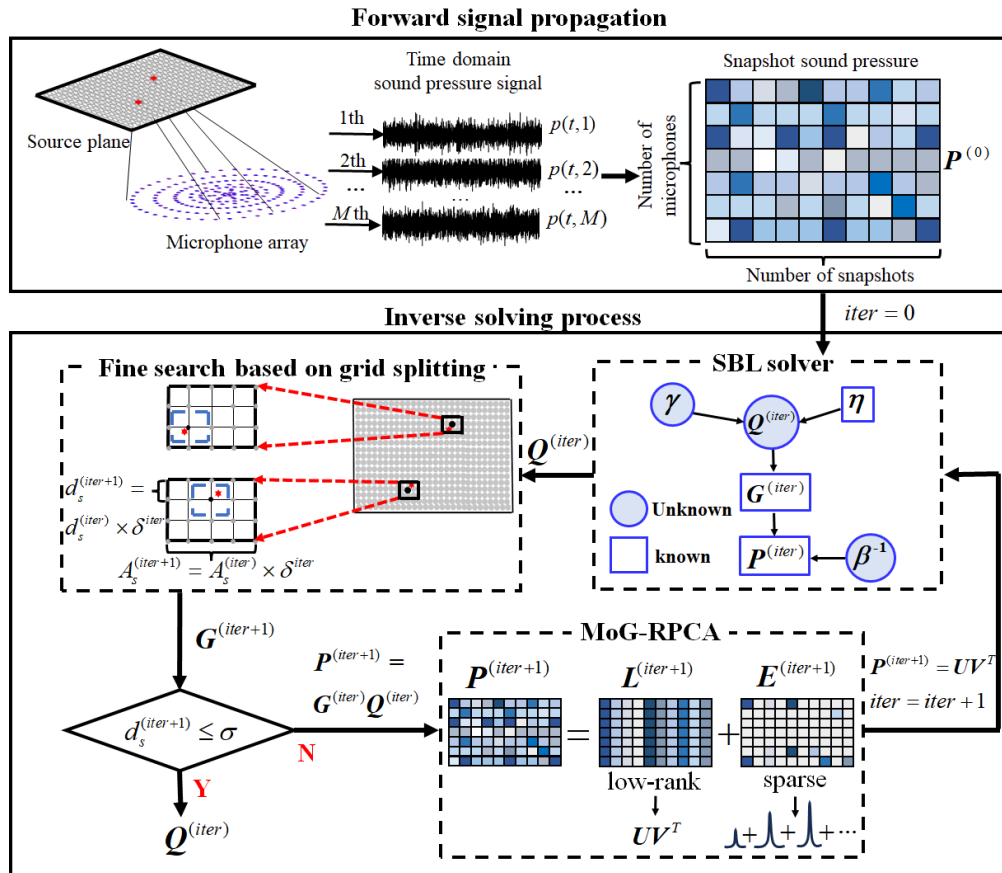


Figure 2. Schematic diagram of the method. In the figure, the red stars represent the locations of the real sound sources, and the arrow represents the process guidance of the method.

The pseudo-code for VG-SBL is given as follows (Algorithm 1).

Algorithm 1 VG-SBL Algorithm.

Input: The snapshot sound pressure: P .

- 1: Discretize the source plane into a grid surface with A_s as the aperture and d_s as the spacing.
- 2: Establish the transfer matrix G between the sound pressure and the source surface.
- 3: The initial value of the iteration number “ $iter$ ” is set to 0. The decay factor δ and the iteration stopping threshold σ are set.
- 4: while $d_s^{(iter+1)} > \sigma$.
- 5: The initial source strength $Q^{(iter)}$ is obtained by sparse Bayesian learning.
- 6: Split the meshes to update near the local maximum of $Q^{(iter)}$, with $A_s^{(iter+1)} = A_s^{(iter)} \times \delta^{iter}$ and $d_s^{(iter+1)} = d_s^{(iter)} \times \delta^{iter}$.
- 7: Remove duplicate grid points and construct a new transfer model $G^{(iter+1)}$.
- 8: Form a new sound pressure $P^{(iter+1)} = G^{(iter)} Q^{(iter)}$, and enter it into the MoG-RPCA model.
- 9: Replace the new sound pressure with the separated low-rank matrix: $P^{(iter+1)} = U^{(iter+1)} V^{(iter+1)}$.
- 10: $iter = iter + 1$.
- 11: end while

Output: The estimation of source strength Q .

In general, this paper proposes a new strategy for the problem of high-resolution sound source identification with a wide range of source distribution under far-field measurement, which is different from the traditional fixed on-grid, off-grid, or grid-free methods. By

adaptively optimizing and adjusting the grid scale and center location while dynamically removing the influence of noise, this provides an effective way to solve the problem of sound source identification under far-field large-aperture array measurement.

4. Simulation

4.1. Simulation Experiment Setup

There are two main difficulties for the acoustic imaging problem in far-field measurements due to the limitations of microphone array aperture and microphone spacing [52]. On the one hand, the spatial resolution of the low-frequency sources is poor, and it is challenging to realize the correct identification and separation of multiple sources with close location distribution. On the other hand, due to long-distance propagation, the SNR is reduced, and side lobes or false sources are prone to exist in the cloud image, reducing the dynamic range. The above problems will seriously affect the accurate identification of real sound sources. In this section, the issue of sound source identification under complex working conditions at long measurement distances, low source frequencies, and close source spacing is discussed to compare the performance differences of various methods.

In this section, the source localization process is simulated through a series of simulation experiments. To achieve high-precision identification of far-field low-frequency sound sources, in terms of the layout of the microphone array, an array evaluation function is constructed with spatial resolution and dynamic range as the goals, and an adaptive optimization design method for microphone arrays is designed. This method derives the response analytical expression and resolution constraint function of the annular microphone array, constructs the performance evaluation function of the microphone array composed of multi-circle annular arrays, and establishes an array optimization algorithm based on the annular array, which can achieve high-resolution and high-dynamic-range measurement of wideband noise sources. The microphone array used in the simulation is optimized in this way. It is a circular array consisting of 193 array elements [53], and the distribution of array element positions is shown in Figure 3a. The array aperture is 8 m, and the distance from the array to the sound source plane is 7 m. The sound source plane is set to be a rectangular plane of 8×6 m, uniformly discretized into 81×61 grid points (the spacing of adjacent discretized points in both directions is 0.1 m). The schematic diagram of the microphone array and sound source arrangement for the simulation experiment is shown in Figure 3b. Three monopole sources are set up, as shown in Figure 3c, with Source A, Source B, and Source C located at (0.36, −1.24) m, (0.54, −1.48) m, and (0.86, 1.6) m, respectively, and the amplitudes and phases of the sources are randomly assigned to ensure the generality of the methods. Complex Gaussian noise with a certain SNR is added linearly to the target signal to simulate the actual test environment.

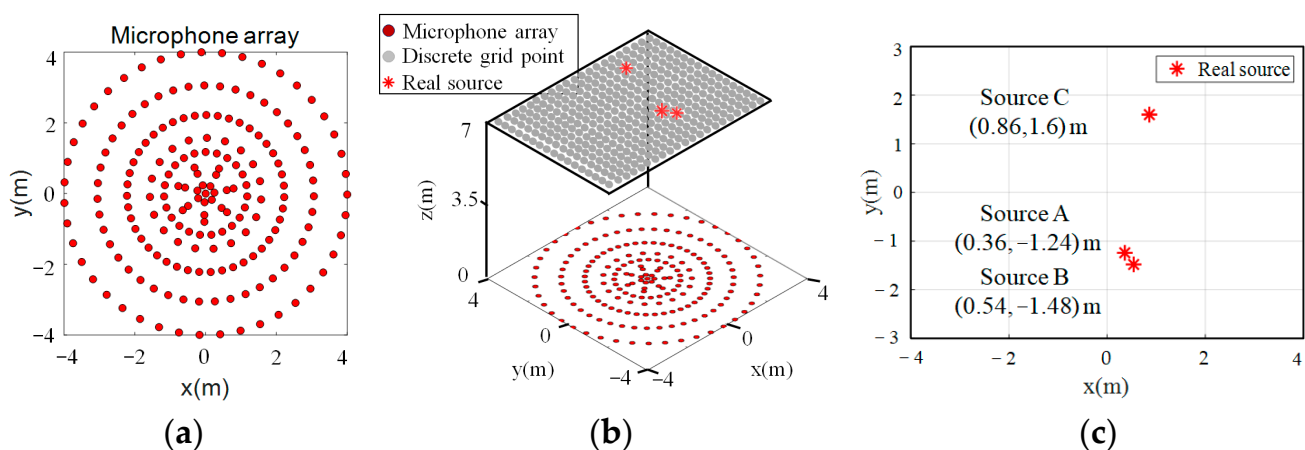


Figure 3. Simulation experiment setup. (a) Distribution of microphones; (b) Measurement schematic. (c) The locations of the three sound sources.

The signal generation method is shown in Equation (4). First, generate the complex source strength snapshot Q that obeys Gaussian distribution, and then multiply it by Green's function G to obtain the snapshot matrix under noise-free conditions. On this basis, add complex interference noise E . It is known that the signal-to-noise ratio (SNR) is defined as:

$$\text{SNR} = 10 \log_{10} \frac{P_s}{P_e} \quad (26)$$

where P_s is the power of the sound source signal and P_e is the power of the noise signal. Therefore, according to the definition of SNR, the noise component E under a given SNR can be obtained as follows [54]:

$$E = \frac{P_s^{(1/2)}}{10^{(\text{SNR}/10)}} e^{i\theta} \quad (27)$$

where $\frac{P_s^{(1/2)}}{10^{(\text{SNR}/10)}}$ denotes the amplitude of the E and θ is the phase of E .

4.2. Discussion of Results

In this section, the effect of a critical parameter in the proposed method on the identification results is analyzed through numerical simulations, and the appropriate parameter selection scheme is established. Further, the effects of source frequency and SNR on the source identification results are investigated, and the VG-SBL method proposed in this paper is compared with the OMP [55], FISTA [56], SBL [49], RPCA-SBL, and off-grid-GL₁ [13] methods. RPCA-SBL is a method that applies RPCA [50] to perform noise reduction preprocessing on SBL.

Three monopole sources are set up with source locations at (0.36, −1.24) m, (0.54, −1.48) m, and (0.86, 1.6) m. The sound is generated as independent and identically distributed complex Gaussian, while 20 dB Gaussian white noise is added to the simulated signal. To quantitatively compare different algorithms, the root mean square error for location (RMSEL) and the root mean square error for magnitude (RMSEM) are used as metrics for evaluating the performance of source identification [43]. These two metrics are defined as:

$$\text{RMSEL} = \sqrt{\frac{1}{S} \sum_{s=1}^S \left\| \mathbf{r}_s^{\text{identified}} - \mathbf{r}_s^{\text{actual}} \right\|_2^2} \quad (28)$$

$$\text{RMSEM} = \sqrt{\frac{1}{S} \sum_{s=1}^S (SPL_s^{\text{identified}} - SPL_s^{\text{actual}})^2} \quad (29)$$

where $\mathbf{r}_s^{\text{identified}} = [x_s^{\text{identified}}, y_s^{\text{identified}}, z_s^{\text{identified}}]^T$ and $\mathbf{r}_s^{\text{actual}} = [x_s^{\text{actual}}, y_s^{\text{actual}}, z_s^{\text{actual}}]^T$ are the identified location and the real location of the s -th sound source, respectively, $SPL_s^{\text{identified}}$ and SPL_s^{actual} are the identified sound pressure level and the real sound pressure level of the s -th sound source, respectively, and S is the number of sources.

The average results of multiple calculations are given to avoid random errors. If pseudo-sources occur, the first S sources with the highest amplitude are calculated.

4.2.1. Parameter Discussion

In the VG-SBL algorithm, there is an uncertain parameter, the decay factor δ , which significantly impacts the sound source identification results. Different choices of this parameter directly determine the number of times the algorithm jumps out of the iteration loop, which in turn affects the identification accuracy of the final results. Simulation comparison experiments are carried out to analyze the influence of the decay factor δ on the sound source identification results.

In the simulation, the frequency of 500 Hz is taken as an example, and the SNR is taken as 10 to 40 dB, with an interval of 10 dB. The decay factor δ is adjusted from 0.1 to 0.9, with an interval of 0.1, and the higher the value, the slower the decay speed is. The results

of RMSEL and RMSEM are calculated for different values of δ , as shown in Figure 4a, representing the average of all SNRs. Figure 4b shows the corresponding calculation time and total number of iterations under different values of δ .

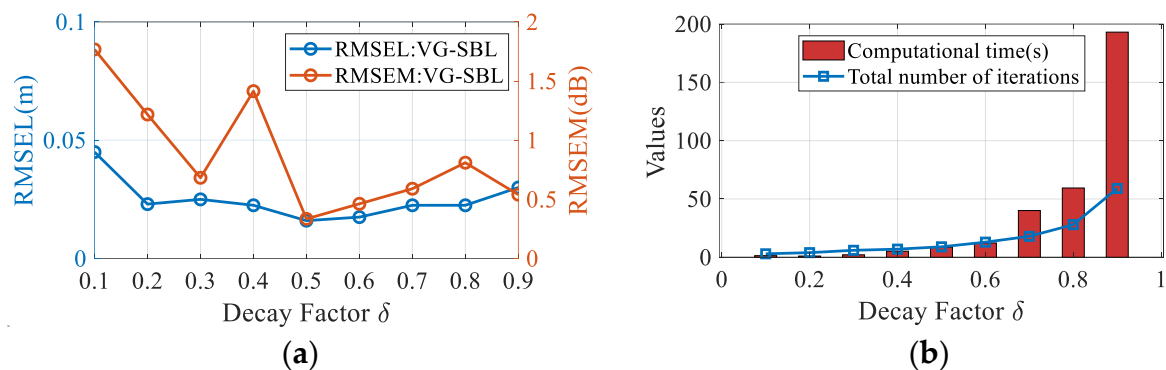


Figure 4. Analysis of the impact of decay factor δ on sound source identification results. (a) Impact on RMSEL and RMSEM. (b) Impact on the number of iterations and calculation efficiency.

As shown in Figure 4, when δ is gradually increased from 0.1, the source identification error of VG-SBL goes through a process of decreasing and then increasing. This is because VG-SBL updates the grid with denser spacing and a smaller aperture centered on the sound source location in the previous iteration result, moving towards the minimum value of the objective function. If the mesh decay rate is too slow or fast, it is easy to fail to converge to the actual sound source location, leading to significant source identification errors. At the same time, too large a δ also leads to slower iteration speed and longer computation time. Therefore, the above results show that a reasonable choice of δ can improve the accuracy of localization and strength estimation at the same time. When δ is set at 0.5, accurate identification results with high probability can be obtained, so δ is selected as 0.5 in this paper.

When the adjacent grid spacing d reaches σ , the stopping condition of jumping out of the iteration is reached and the loop is ended. For the actual test situation, when the spacing between adjacent grid points reaches about 0.001 m, the grid division is already detailed enough, and the accuracy of source identification is already high enough, so there is no need to continue the subdivision. σ is set to 0.001 m in this paper.

4.2.2. Method Performance Discussion

In this section, the effects of source frequency, SNR, and microphone array on the performance of the sound source identification results are analyzed using the same simulation conditions as in Section 4.2.1 and compared with the conventional methods SBL, OMP, FISTA, off-grid-GL₁, and RPCA-SBL. The source frequency ranges from 100 to 1000 Hz with an interval of 100 Hz, and the SNR ranges from 10 to 40 dB with an interval of 5 dB. For the OMP method, the sparsity is assumed to be five here.

(1) Influence of source frequency

The RMSEL and RMSEM of the sound source identification results for different methods at multiple frequencies are shown in Figure 5, and the average values are shown in Table 1. It should be noted that the error comparison figures below are shown using the log vertical axis to demonstrate the differences.

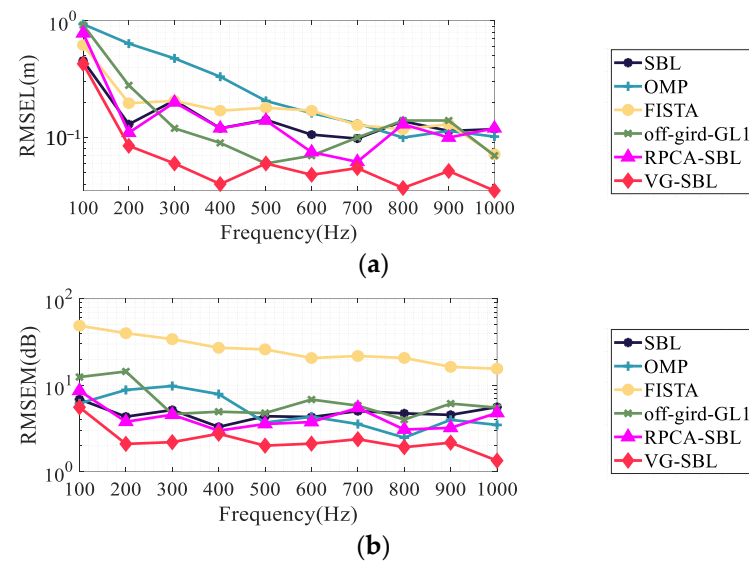


Figure 5. Identification error statistics of the methods at different frequencies for the SNR of 20 dB. (a) RMSEL. (b) RMSEM.

Table 1. Average RMSEL (m) and RMSEM (dB) between 100 and 1000 Hz, SNR = 20 dB.

Method	Average RMSEL (m)	Average RMSEM (dB)
SBL	0.16	4.82
OMP	0.32	5.42
FISTA	0.20	27.11
Off-grid-GL ₁	0.20	6.94
RPCA-SBL	0.18	4.38
VG-SBL	0.09	2.45

As shown in Figure 5a, among all methods, OMP shows the most significant localization error, followed by FISTA. Off-grid-GL₁ performs poorly for low-frequency sources, SBL and RPCA-SBL perform similarly, while VG-SBL shows the smallest error. For the minimum frequency of 100 Hz, the errors of all methods increase significantly, while for source frequencies of 200 Hz and above, the performance of all methods improves. The reason for this phenomenon can be explained from the following two angles. On the one hand, the low-frequency sound source signal has a longer wavelength, the effective sampling volume is small, and the truncation effect inherent in digital signal processing is serious. On the other hand, the condition number of the transfer matrix increases significantly as the frequency decreases, making low-frequency sound source identification quite challenging at such a long measurement distance. To mitigate these errors, a possible approach could be to optimize the array configuration or improve the signal processing algorithms, for example, using a microphone array with a larger aperture [57,58] or further proposing a sound source identification algorithm with higher-resolution performance [59–61]. In addition, the method can also be used to extend the lower limit of the working frequency by utilizing asynchronous measurement techniques [62–64]. However, implementing these improvements might present certain challenges, such as practical limitations in the experimental setup or increased complexity in data processing. Therefore, how to further improve the recognition resolution of low-frequency sound sources is the next key research direction.

The resolution performance of FISTA and OMP methods for low-frequency sound sources is insufficient. Thus, there is a significant discrepancy between the identified sound sources and their actual locations. For the off-grid-GL₁ method, since the L_1 norm is prone to additional bias and improper group sparsity penalty, multiple interference sources exist near the real sound source, and source positioning is inaccurate. Owing to the iterative estimation of the likelihood and a priori hyperparameters, SBL and RPCA-SBL provide

the most robust and accurate results among the compared methods. Since RPCA mainly removes ghost noise in cloud images, it has little effect on improving the accuracy of location recognition. However, it can be seen that the RMSEL of VG-SBL is consistently maintained within 0.1 m for sources at frequencies of 200 Hz and above. This shows that VG-SBL effectively avoids basis mismatch errors while weakening the influence of noise and improving the accuracy of sound source location identification.

As shown in Figure 5b, the amplitude error of FISTA is the largest, followed by OMP and off-grid-GL₁, then SBL, and again RPCA-SBL, while VG-SBL has the slightest error. The root causes of the above phenomena are analyzed as follows.

The main flaps obtained by the FISTA method are wide, with many false ghost points and very serious energy dissipation, which results in low amplitude energy at the source point. In addition, FISTA also incorrectly estimates two low-frequency sources that are closely spaced from each other, and it cannot successfully separate the two sources, leading to a large deviation between the estimated amplitude and the actual amplitude. The first step of OMP can be understood as a conventional beamforming process [59]. The amplitude error is large for low-frequency sound sources that are closely spaced because the main lobes are fused together. In the solution vector of off-grid-GL₁, there is also the problem of underestimated source strength due to multiple interference sources. For the SBL method, the error mainly comes from the basis mismatch problem. The sound source energy is dispersed to several adjacent grid points, which leads to a certain deviation in the source strength quantification. RPCA-SBL can effectively remove part of the noise and make the energy more concentrated, but it still cannot avoid the problem of basis mismatch. VG-SBL effectively avoids the above issues, and the RMSEL of the method is consistently maintained within 5 dB for sources at 200 Hz and above. In summary, the VG-SBL method obtains the most accurate source identification results in terms of localization and strength estimation in the entire range of frequency bands analyzed.

(2) Influence of SNR

The performance of different methods under noise interference was further studied to explore whether the methods have practical application value. The RMSEL and RMSEM of the sound source identification results of the methods under different SNRs are shown in Figure 6, and the average errors are shown in Table 2.

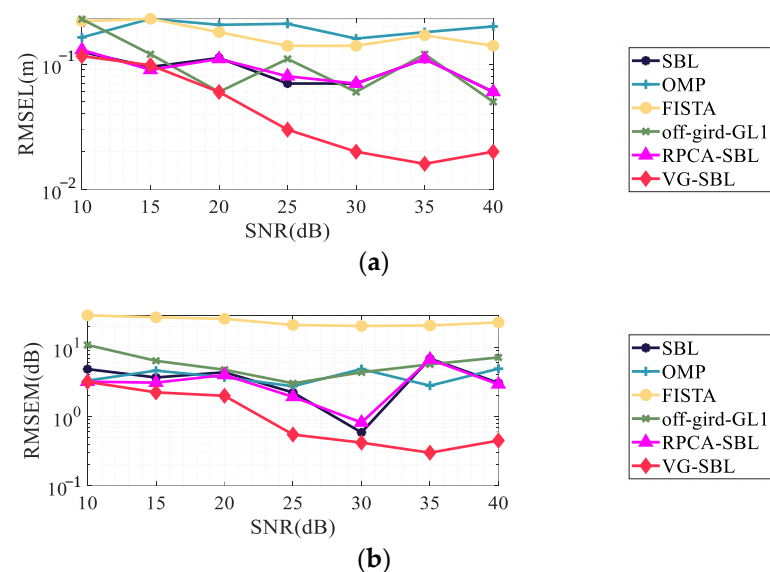


Figure 6. Identification error statistics of the methods at different SNRs for the frequency of 500 Hz. (a) RMSEL. (b) RMSEM.

Table 2. Average RMSEL (m) and RMSEM (dB) between 10–40 dB, $f = 500$ Hz.

Method	Average RMSEL (m)	Average RMSEM (dB)
SBL	0.09	3.67
OMP	0.19	3.86
FISTA	0.17	23.99
Off-grid- GL_1	0.11	6.04
RPCA-SBL	0.09	3.24
VG-SBL	0.05	1.31

Under the influence of noise, the identification errors of the FISTA and off-grid- GL_1 methods increase significantly due to the difficulty in reasonably selecting regularization parameters. OMP selects the basis with the highest correlation with the residuals in each iteration and updates it into the support matrix. If two sources are closely correlated, OMP will select one source and overestimate its strength. This also causes OMP to easily select the wrong basis vector under intense noise, leading to higher source identification errors. SBL and RPCA-SBL perform the best in comparison, and VG-SBL performs better than OMP. VG-SBL can obtain the most accurate sound source localization and strength estimation results in the whole SNR range, and its sensitivity to noise is significantly lower than traditional methods. When the SNR is 10 dB and above, RMSEL and RMSEM can be maintained within 0.1 m and 3.5 dB, respectively. On the one hand, during the iterative process, the initial grid points dynamically approximate the real sound source location under the constraints of the sparse Bayesian learning framework. On the other hand, the low-rank Gaussian mixture model effectively removes the influence of noise.

(3) The Impact of Microphone Arrays

In practical applications, it is sometimes difficult to arrange a large microphone array for measurement. Therefore, the impact of the number of microphones and array aperture on the identification performance of the method is further studied to explore whether the method has practical application value.

On the basis of the original array, the outermost rings are removed in turn to reduce the number of microphones and reduce the array aperture. The compared array is shown in Figure 7, which contains five arrays. The number of microphones is reduced from 193 to 45, and the aperture is reduced from 4 m to 1.2 m. The RMSEL and RMSEM of the sound source identification results of each method under different microphone arrays are shown in Figure 8.

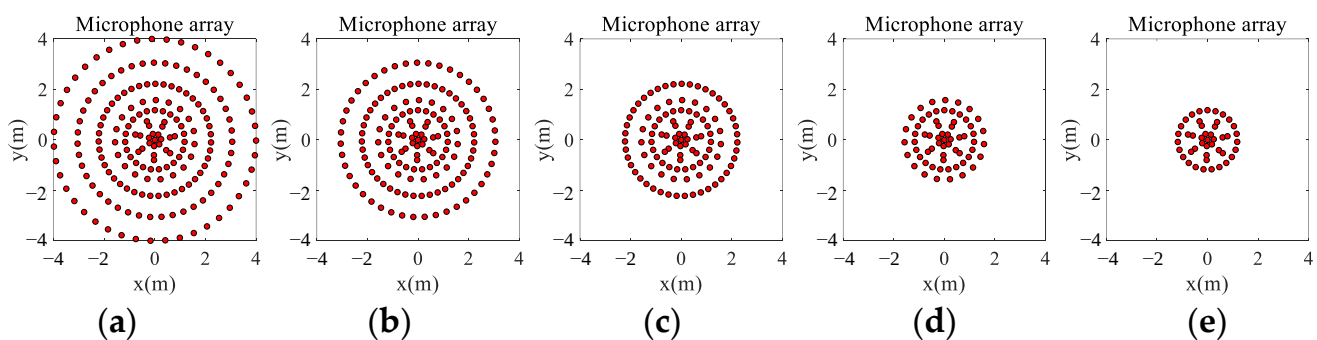


Figure 7. Comparison of arrays: (a) 4 m, 193 microphones; (b) 3 m, 150 microphones; (c) 2.2 m, 107 microphones; (d) 1.6 m, 64 microphones; (e) 1.2 m, 45 microphones. The red dots represent the locations of the microphones.

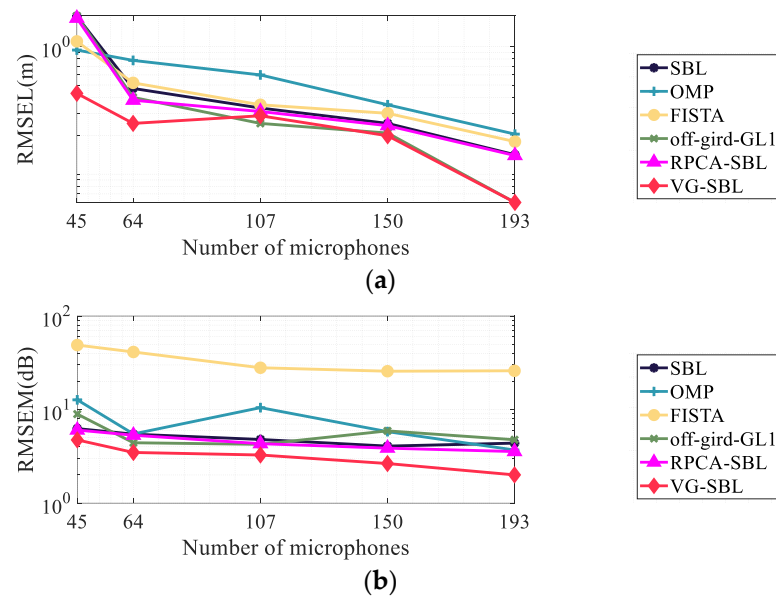


Figure 8. Error statistics of the method identification for $f = 500$ Hz, $SNR = 20$ dB. (a) RMSEL. (b) RMSEM.

As shown in Figure 8, when the number of microphones decreases and the aperture of the microphone array decreases, the sound source identification errors of different methods all have an obvious upward trend. The reason for this phenomenon is that when the array aperture decreases, the resolution performance will be significantly reduced. Resolution refers to the ability of the algorithm to distinguish the minimum distance between sound sources that are close to each other. According to the Rayleigh criterion, the minimum resolvable source distance can be given by the following formula [60]:

$$Resolution = \alpha \frac{z\lambda}{D} \quad (30)$$

where α is a constant, which is 1.44 for the circular array, z is the measurement distance, λ is the wavelength of the sound signal, and D is the aperture of the microphone array. It can be seen that for the problem of far-field, low-frequency sound source identification, due to the large values of z and λ , *Resolution* increases, while increasing the array aperture helps enhance the resolution performance of low-frequency sound sources. In addition, the spatial aliasing effect must also be considered. Spatial aliasing occurs when the signal is not fully sampled in space. One way to reduce spatial aliasing is to perform spatial sampling at intervals not exceeding half a wavelength. Therefore, the number of microphones should not be too small. The average error is shown in Table 3.

Table 3. Average RMSEL (m) and RMSEM (dB) for different numbers of microphone arrays.

Method	Average RMSEL (m)	Average RMSEM (dB)
SBL	0.54	4.99
OMP	0.58	7.64
FISTA	0.49	33.99
Off-grid-GL ₁	0.53	5.66
RPCA-SBL	0.55	4.62
VG-SBL	0.25	3.23

For the above reasons, when the number of microphones decreases and the aperture becomes smaller, the overall performance of the algorithm decreases. However, it can be seen from Table 3 that when the number of microphones is reduced, the proposed method still has obvious advantages. In the five different array cases, the average position and

amplitude errors of the algorithm are 0.25 m and 3.23 dB, respectively, which shows the robustness and applicability of the algorithm.

4.2.3. Image Results of Different Methods

To more clearly compare the differences in sound source imaging visualization between different methods, a working condition with sound source frequency of 500 Hz and SNR of 20 dB is used as an example to show the comparison results. The actual values of randomly generated sound source amplitudes are listed in Table 4.

Table 4. The actual sound pressure levels of the three sound sources.

Source	Sound Pressure Level (dB)
Source A	88.8
Source B	92.3
Source C	90.5

The source identification cloud maps of the different methods are shown in Figure 9, with RMSEL and RMSEM marked at the top of the figures. The actual source locations are marked with red boxes.

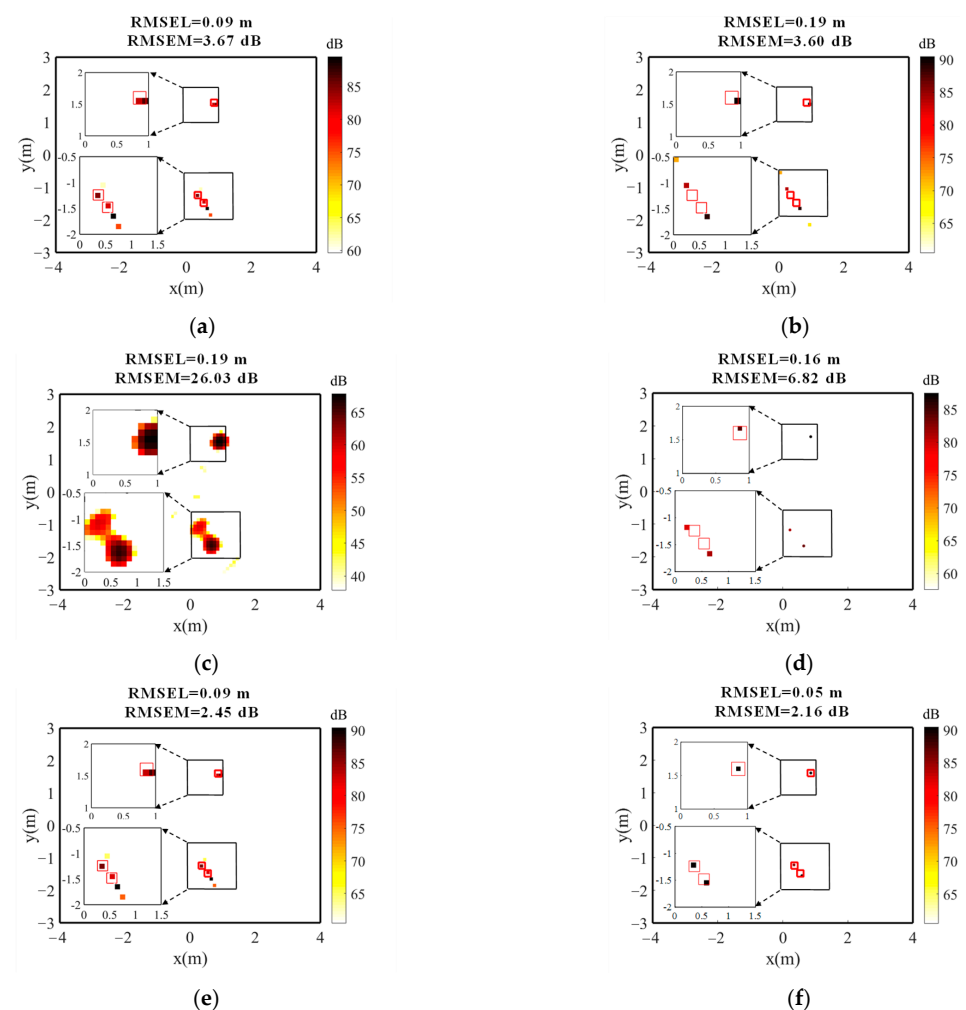


Figure 9. Sound source identification results of $f = 500$ Hz, SNR = 20 dB. (a) SBL; (b) OMP; (c) FISTA; (d) off-grid-GL₁; (e) RPCA-SBL; (f) VG-SBL. The different colors in the figure correspond to the colorbar on the right side of the figure, representing different amplitudes.

As seen in Figure 9, SBL can relatively accurately identify the locations of the three sound sources. However, due to the basis mismatch error, there is a certain deviation between the identified and actual source locations, and pseudo-sources appear near the sources. Due to the limitation of the resolution, OMP has a large identification deviation for Source A and Source B, which are closely distributed, and there are several fake sources. The main lobe of FISTA is wide and the resolution is low, so obtaining accurate source locations is challenging. Off-grid-GL₁ cannot accurately identify Source A or Source B either, and the identified source locations deviate severely from the actual locations. RPCA-SBL improves the amplitude accuracy based on SBL, but is ineffective in improving location accuracy. The VG-SBL method can identify the locations of the three sources relatively accurately, jumping out of the location constraints of the initial meshes without sidelobe contamination and with relatively minimal location and amplitude errors.

4.2.4. Calculation Time Comparison

In addition to sound source identification error, computational efficiency is an essential indicator for evaluating algorithm performance. Taking the comparison conditions in the previous section as an example, Table 5 compares the average calculation times of different methods. It should be noted that factors such as the location, frequency, and amplitude of the sound source will not affect the calculation time. Although the SNR will make a difference in the calculation time, it will not affect the efficiency comparison conclusion between different methods.

Table 5. Average computation time for different methods at $f = 500$ Hz, SNR = 20 dB.

Method	Time (s)
SBL	1.51
OMP	0.04
FISTA	16.84
Off-grid-GL ₁	28.88
RPCA-SBL	1.71
VG-SBL	7.80

As can be seen from Table 5, the calculation time of off-grid-GL₁ and FISTA is relatively long, while other methods are within 10 s. Although VG-SBL requires multiple iterative calculations and takes longer than the original SBL, its calculation time is still acceptable based on significantly improving calculation accuracy.

5. Experimental Validation

5.1. Experimental Design

To verify the effectiveness of the proposed method, test experiments were carried out in an FL-13 acoustic wind tunnel at the China Aerodynamic Research and Development Center (CARD C), and the actual test diagrams are shown in Figure 10. The FL-13 is a large-scale, low-speed wind tunnel with direct current and closed series dual test sections. The test section size is 15 m (length) $\times$ 8 m (width) $\times$ 6 m (height).

A civil aviation standard high-lift configuration CAE-AVM-HL aircraft standard model with a scale of 1:5.6 is placed in the open test section of the airflow for testing, with the aircraft model facing the incoming airflow. Three Bluetooth sound sources with the same frequency response are installed at the bottom of the leading edge of the wing. The two equal-strength speakers on the left are separated by 0.3 m to verify the resolution performance of the sound source imaging. At the same time, the one on the right is the speaker whose sound pressure level is 20 dB lower than that on the left side, which is used to verify the dynamic range performance of sound source imaging. The locations of the three sound sources remain consistent with Section 4 above. At the same time, a large microphone array is arranged 7 m directly below the aircraft standard model. The microphone array is a large circular array with a diameter of 8 m, containing nine rings

and a total of 193 array elements, and the microphone distribution is also consistent with that in Section 4 above. The microphones used are GRAS 40 PH type 1/4 inch free-field microphones, which can measure a frequency range from 10 Hz to 20 kHz, dynamic response range from 33 dB (A) to 135 dB, and sensitivity of 50 mV/Pa, which can meet the measurement requirements. A 256-channel dynamic data acquisition system based on NI.PXIe-4499 is used for multi-channel data synchronous acquisition.

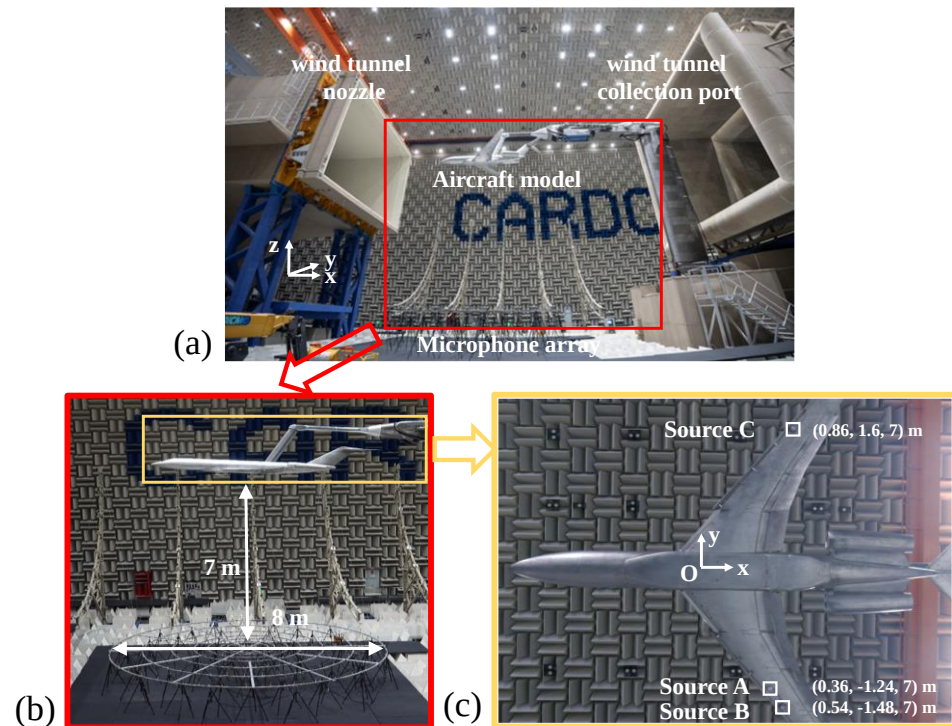


Figure 10. The experimental setup. (a) Relative locations between the wind tunnel, the aircraft standard model, and the microphone array. (b) Localized view of aircraft standard model and the microphone array. (c) Arrangement of the three Bluetooth speakers on the aircraft standard model.

Two working conditions are set up for the test, the stationary no-flow condition and the 40 m/s flow condition, and three sound sources simultaneously generate 300 Hz and 500 Hz frequency signals under the two test conditions. The actual sound pressure levels of the loudspeakers at different frequencies are collected separately based on separate microphones, as shown in Table 6. The aerodynamic noise generated by the aircraft model in the presence of flow velocity can be regarded as the background noise in the loudspeaker identification process.

Table 6. Actual sound pressure levels of loudspeakers at different frequencies.

Frequency (Hz)	300	500
Sound pressure level of Source A (dB)	106.7	107.0
Sound pressure level of Source B (dB)	106.7	107.0
Sound pressure level of Source C (dB)	86.7	87.0

5.2. Experimental Results

The sampling frequency is 51,200 Hz and the acquisition time is 15 s. Intercepting 2 s of data in the acquisition duration for calculation, each snapshot has a length of 0.1 s, has a Hanning window weighting, and corresponds to a frequency resolution of 10 Hz. Setting the overlap of two consecutive snapshots to 50% results in 39 snapshots.

When sound passes through the shear layer of the flow field, the sound propagation route will be deflected due to the refraction effect of sound. The sound propagates from

the sound-generating point in the jet to the receiving point outside the jet, and a certain modified equivalent uniform flow field can be used instead of the non-uniform flow field [65]. Assuming that the speed of sound remains constant in the uniform flow field and no-flow field, the new flow velocity in the uniform flow field is replaced by an equivalent flow velocity that makes the sound source propagate directly along a straight line to the receiving point without sound deflection. In the preprocessing stage, the equivalent flow velocity method corrects the acoustic refraction phenomenon caused by flow velocity.

5.2.1. Signal Time–Frequency Analysis

Firstly, the time–frequency waveform of the signal is plotted with the 40 m/s@300 Hz test condition as an example. The microphone signal at the center of the microphone array is selected to plot the time–frequency spectrum as shown in Figure 11. It shows that in the case of a 40 m/s flow rate, the aircraft standard model produces a wide frequency distribution of aerodynamic noise dominated by low- and medium-frequency energy, and the energy shows a gradual attenuation trend as the frequency rises, while the 300 Hz single-frequency noise produced by the loudspeaker can be highlighted, and the SNR of the signal satisfies the analysis requirements.

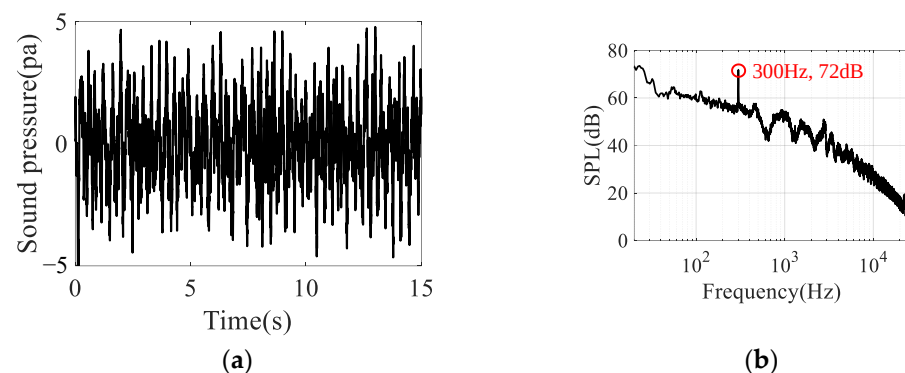


Figure 11. Time–frequency spectrum of a single channel signal under the working condition of 40 m/s@300 Hz. (a) Time domain spectrum. (b) Power spectrum. 300 Hz is marked with a red circle.

5.2.2. Sound Source Imaging Cloud Maps

In this section, the identification results of three sound sources at frequencies of 300 Hz and 500 Hz using different methods are presented under the conditions of no flow velocity and 40 m/s flow velocity, respectively. The working condition, 40 m/s@300 Hz, is used as an example to give the sound source imaging cloud maps of different methods, shown in Figure 12.

As shown in Figure 12, the sound source location and amplitude identification results of the different methods are significantly different.

SBL successfully identifies the locations of the sound sources near the three real sources. However, due to the base mismatch error, the sound sources are located at grid points near the real sound sources, resulting in certain location and amplitude errors. In addition, there are some pseudo-source interferences.

Constrained by its limited resolution performance, OMP shows serious location deviations for Source A and Source B, which are closely distributed, and the total amplitude error is also relatively large.

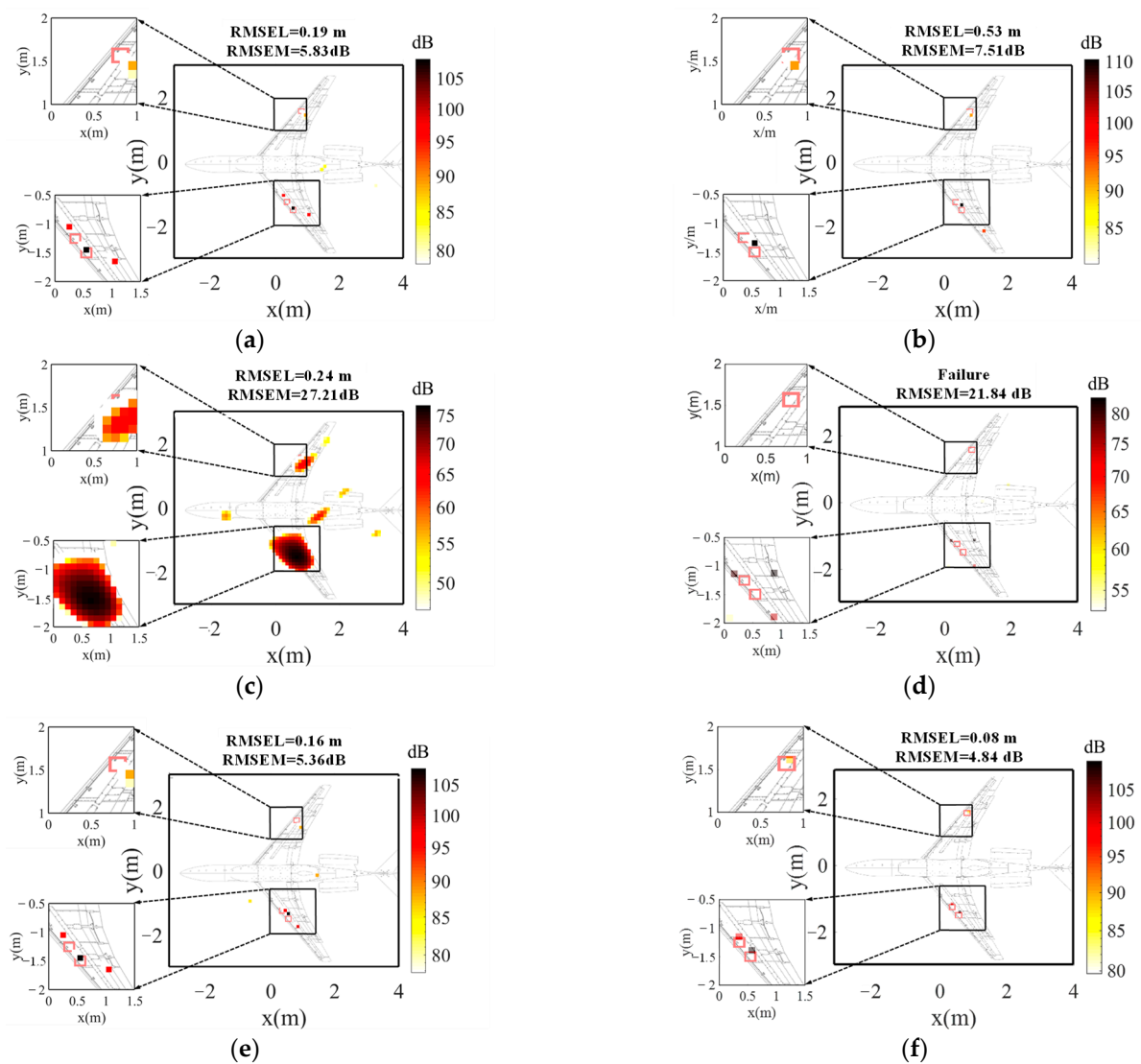


Figure 12. Sound source identification results of 40 m/s@300 Hz: (a) SBL; (b) OMP; (c) FISTA; (d) off-grid- GL_1 ; (e) RPCA-SBL; (f) VG-SBL. The different colors in the figure correspond to the colorbar on the right side of the figure, representing different amplitudes.

FISTA can find the general locations of the source distributions, but the hotspot areas are too large and the source locations are ambiguous. The width of the main flap is far beyond the actual area of the source distributions and there are many pseudo-sources, making it difficult to recognize the specific locations of the real sources accurately. The amplitude reconstruction error is significant due to severe energy dissipation.

For the off-grid- GL_1 method, there is a significant difference between the identified and actual source locations and a loss of Source C. In addition, the strength of sources is much lower than the actual strength. This is due to the uneven penalty of the L_1 norm regularization. Moreover, due to the increase in the dimension of the variables to be solved, the off-grid method is more sensitive to actual noise interference.

RPCA-SBL performs best among the comparison methods. Due to the removal of random noise interference, its sound source identification position and amplitude errors are lower than SBL. However, its search range is still limited to the grid position.

VG-SBL breaks through the limitations of the fixed grid model on sound source locations and removes pseudo-source interference. In the iterative process, the distribution of real sound sources is gradually approximated, thus achieving the lowest source identification error and better visualization.

5.2.3. Results

In this section, the statistical results of all methods under four working conditions are given, and their performance differences are comprehensively analyzed.

(1) Dynamic range

As shown in Table 5, the amplitude of Source A is the same as that of Source B, while the amplitude of Source C is relatively low, by 20 dB. The amplitude differences of the identified sources under the four conditions are plotted to analyze the dynamic range performance. According to Equation (31), D_{range} is calculated for different methods, and the closer to 20 dB, the better the dynamic range performance. If the source location in the vicinity of any of the three real sources cannot be successfully identified, it is labeled “Failure.”

$$D_{range} = \left| \frac{SPL_{SourceA}^{identified} + SPL_{SourceB}^{identified}}{2} - SPL_{SourceC}^{identified} \right| \quad (31)$$

It should be noted that the dynamic range here refers to the difference in amplitude between the mean values of sources A and B, and the amplitude of source C, as identified by the method under the test condition, rather than the ability of the algorithm to identify the difference in amplitude between the largest and smallest sources. Since the true value of this value is 20 dB, the value of “ D_{range} ” in Figure 13 is calculated to be as close to 20 dB as possible, rather than as large as possible. As shown in Figure 13, the dynamic range of source identification of the VG-SBL method is overall closest to 20 dB, which demonstrates the ability of the method to accurately reconstruct the amplitude difference between the sources. This is of great significance for analyzing the source contribution amount, as well as for practical engineering guidance, such as in structural noise reduction design. For the method labeled “Failure,” it means that the method does not have the ability to identify noise sources with 20 dB amplitude difference at the same time.

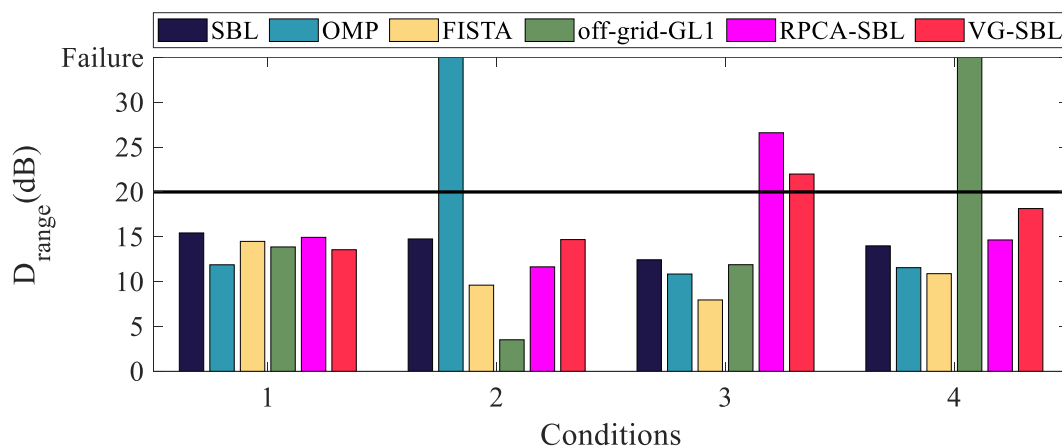


Figure 13. Dynamic range of sound source identification.

(2) Location distribution of sound sources

The distribution of the three sources obtained by all methods under four working conditions is given. To more intuitively display the difference in location distribution, the real locations of Source A, Source B, and Source C and the source locations identified by different methods (represented as scatter points) are plotted in one figure. At the same time, take the real locations of Source A, Source B, and Source C as the center of the circle, and use dotted lines to draw multiple concentric circles with a radius of 0.2–1 m and a radius of 0.2 m apart, so that readers can more easily observe the distances between the identified source locations and the real source locations. When the distribution of scatter points is more concentrated near the center of the circle, the source location is identified

more accurately. When the method is not successful in identifying a source, the location of the scatter point is still plotted to standardize the number of points. Still, the scatter point is just randomly plotted outside of the circle with a radius of 1 m, which means failure of the method. The results are shown in Figure 14.

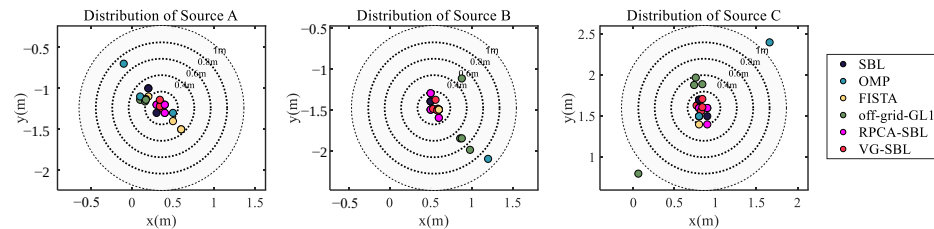


Figure 14. The schematic distribution of the identified locations of the three sound sources for the different methods.

As shown in Figure 14, among all methods, the sound source distributions of OMP and off-grid-GL₁ clearly show apparent discrete character, while the RPCA-SBL and the VG-SBL method are the most consistent and concentrated with the actual position distribution.

Among the three sound sources, there is no significant difference in the location identification errors between Source A and Source B. The OMP and off-grid-GL₁ methods have more significant identification errors for these two sources than other methods. This is because they do not have enough resolution to accurately identify the two sources, resulting in the identified locations being significantly shifted to the outside. In addition, because the amplitude of Source C is 20 dB lower than the other two sources, the OMP and off-grid-GL₁ methods fail to successfully identify Source C under certain working conditions, which shows that the methods have failed.

(3) Amplitude error in identification of sound sources

Next, the amplitude identification of the three sources is further analyzed. The four test conditions of 0 m/s@500 Hz, 0 m/s@300 Hz, 40 m/s@500 Hz, and 40 m/s@300 Hz are named Conditions 1, 2, 3, and 4, respectively. The amplitude errors of the three sources by different methods are depicted in histograms for comparison, as shown in Figure 15. When a source is not successfully identified, “Failure” is displayed.

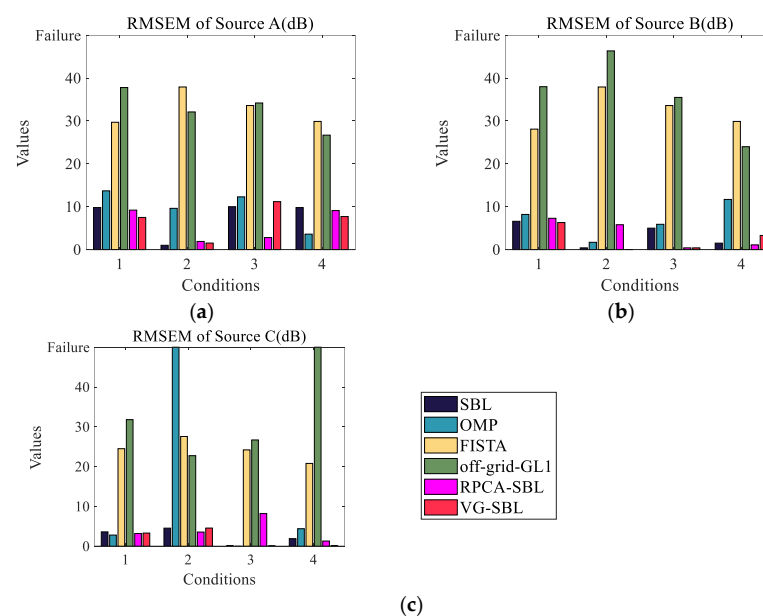


Figure 15. The amplitude identification errors of three sound sources by different methods: (a) Source A; (b) Source B; (c) Source C.

As seen from Figure 15, due to severe energy dissipation, the amplitude errors of FISTA and off-grid-GL₁ are significantly higher than other methods, while the amplitude error of VG-SBL is the smallest.

Overall, the amplitude error of Source C is smaller than the other two sources. This is due to the close spatial proximity of Source A and Source B, which can lead to difficulties in accurately assigning the amplitudes of Source A and Source B when the energy is concentrated and gathered near that region.

(4) RMSEL and RMSEM

Finally, the overall errors of different methods are given. The RMSEL and RMSEM values under the four working conditions are plotted in Figure 16 as box plots, and the mean values are marked above the boxes. The results are calculated based on Equations (28) and (29) and are the combined error of considering the three sound sources simultaneously. It should be noted that RMSEL and RMSEM cannot be calculated for cases where the source is not successfully identified, but for ease of comparison, RMSEL and RMSEM are set to 1 m and 50 dB, respectively.

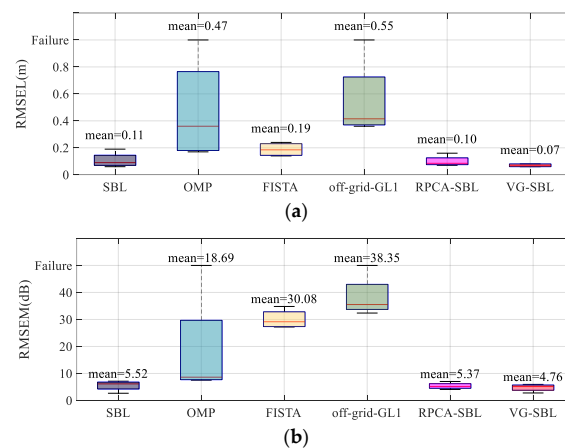


Figure 16. The error box plots. (a) RMSEL. (b) RMSEM. To facilitate distinction, different colors are used to represent different methods.

It can be seen that the average RMSEL of the traditional methods is above 0.1 m and the RMSEM is above 5 dB. The average RMSEL and RMSEM of the best-performing RPCA-SBL are 0.1 m and 5.37 dB, respectively.

Compared with the conventional method, VG-SBL can effectively reduce the source identification error, with the mean values of RMSEL and RMSEM reduced to 0.07 m and 4.76 dB, respectively. This can be attributed to the result of learned inference based on probabilistic statistical rules, which allows the mesh to be dynamically approximated to the actual sound source and the robust robustness enhancement due to matrix factorization.

6. Conclusions

A sparse Bayesian learning method, VG-SBL, based on grid adaptive split refinement for high-resolution identification of sound sources is proposed. This method adopts a significantly advantageous sparse Bayesian learning framework, which uses discrete grid iterative adjustment to retain the computing grids with high weights, performs fission updates near them, and removes the effects of noise during the iteration process by utilizing a low-rank sparse decomposition. It continuously approaches the actual location of the sound source at any location dynamically until the grid parameter variation is less than the threshold, and finally obtains the estimation of the sound source location and strength.

The proposed method is compared with five conventional methods: the on-grid compressive beamforming methods SBL, OMP, FISTA, and RPCA-SBL, and the block-sparse off-grid compressive beamforming method off-grid-GL₁. The simulation results

show that compared with the conventional fixed-grid and off-grid methods, the source identification accuracy of the VG-SBL is significantly improved over a wide range of SNRs and source frequencies.

Tests were carried out in a wind tunnel using speakers on a standard aircraft model as the sound source. The results showed that the traditional methods had different degrees of failure within a flow velocity of 40 m/s, wherein the mean values of RMSEL and RMSEM of the best-performing RPCA-SBL were 0.1 m and 5.37 dB, respectively. The mean values of RMSEL and RMSEM of VG-SBL were able to be reduced to 0.07 m and 4.76 dB, proving that VG-SBL can effectively solve the base mismatch error and improve the sound source identification accuracy. In addition, the amplitude differences between the reconstructed sources of the VG-SBL method are closest to the true values, indicating that it has the ability to rank the acoustic energy contributions.

The proposed VG-SBL method can be extended to more complex actual device sound source recognition scenarios in future work.

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