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Computational Fluid Dynamics Analysis of Slip Flow and Heat Transfer at the Entrance Region of a Circular Pipe

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Abstract: In the era of sustainable development goals (SDGs), energy efficient heat transfer systems are a must. Convective heat transfer within circular pipes is an important field of research on a rarely addressed limitation of fluid flows. Vacuum solar tubes is one of many applications that could benefit from the existence of nanoparticles, Al_2O_3 , for example, to enhance the heating of air or water steam. The current research investigates the impacts of the Reynolds number (Re), Prandtl number (Pr), Knudsen number (Kn), aspect ratio (x/D_h), and volume fraction of Al_2O_3 nanoparticles (ϕ) on the Nusselt number (Nu) under constant wall heat flux conditions. An axisymmetric computational fluid dynamics (CFD) analysis of the nanofluid flowing at the entrance region of a circular pipe was conducted under a slip flow at steady-state developing laminar conditions using the Ansys-Fluent 2018 software package. A mesh sensitivity analysis was conducted, and a proper number of mesh elements was selected. The results showed that an increasing Re and/or ϕ would result in an increasing Nu. The dependence of Nu on Kn was strong due to the high slip values and temperature jump. An increasing x/D_h ratio resulted in reduced Nu values. The major impact was due to Kn, which caused a reduction of up to 40% in the Nu value due to slip conditions. However, there was an enhancement of 2.5% in the heat transfer due to the addition of nanoparticles, which was found at Re = 250, Kn = 0.1, and $\phi = 0.1$ (Pr = 0.729). Finally, Nu_{avg} , Nu_x , U/U_m , and ReC_f were correlated with Kn, Pr, Re, and x/D_h with proper coefficient of determination (R^2) values.

Keywords: slip flow; CFD; nano particles; Nusselt number; entrance region; energy efficiency



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1. Introduction

Convective heat transfer is one of the main mechanisms in heat transfer for flow in pipes, such as heat exchangers. It could be improved by changing thermophysical properties of working fluids and/or geometrical change in flow or boundary conditions. One of the methods to accomplish this is adding nanoparticles to the base fluid [1]. This innovative idea was put forth by Maxwell in 1873 [2], which has generated a great interest for researchers to develop high-performance heat transfer fluids [3]. The effects of nanoparticles on heat transfer enhancement, however, have produced challenges, regardless of their positive effects [4]. These challenges could result from parameters that affect heat transfer enhancement, such as volume fractions, aspect ratio, base fluids, and entrance effects, among others, which are still unclear and require more research.

Advanced technology applications that have adopted nanofluids have attracted a wide spectrum of research. Nanofluids could become very significant when acting as

smart fluids and applied to biomedical, heating, and automotive, among other, applications [5]. The development of artificial organs has highlighted the importance of biofluid mechanics, which is a promising field of research for nanofluids, for example, in the blood vessels/respiratory tract/lymphatic and gastrointestinal systems/urinary tract zones [6,7]. The design of electronic thermal management systems critically depends on the nanofluid type used [8] and is considered very important to the electronic cooling industry [9]. Also, nanofluids were applied to solar thermal collectors with attention to environmental concerns and production costs [10,11] and applied to solar energy desalination to overcome the relatively lower efficiency and lower yield of solar stills [12]. Nanofluids in automobiles have significant opportunities in areas such as radiator coolants, lubricants, brake fluid, and shock absorbers [13]. The list of applications is long and forces the current work to focus on nanofluid applications to gaseous laminar flow through pipes.

Vishwanadula and Nsofor [14] reported, experimentally, that the use of nanoparticles caused an increase in the heat transfer performance in pipe flows, and they developed a correlation in terms of the Reynolds number (Re), Prandtl number (Pr), and Nusselt number (Nu). Likewise, Kaur and Gangacharyulu [15] demonstrated that including a small-volume fraction of aluminum oxide (Al_2O_3) with fluids has enhanced the particle-fluid mixture's thermal conductivity when compared with those of base fluids like air or water. Furthermore, a study on the convective heat transfer of $\gamma\text{-Al}_2\text{O}_3$ nanoparticles and deionized water flowing through a tube in the laminar flow regime was published by Wen and Ding [16]. Their results presented that the use of $\gamma\text{-Al}_2\text{O}_3$ nanoparticles significantly enhanced the entrance region convective heat transfer. The reported enhancement was a function of nanoparticle volume fraction and axial distances. These results were confirmed by the study by Vafaei and Wen [17], which was supported by a detailed literature review. Trivedi and Johansen [18] worked on forced convective heat transfer in Al_2O_3 –air nano-aerosol. They reported that a small particle mass loading is required to increase the value of Nu. Moghadassi et al. [19] investigated how a horizontal circular tube's laminar forced convective heat transfer would be affected by nanoparticles by utilizing the computational fluid dynamics (CFD) approach. They showed that the addition of nanoparticles caused an increase in the pressure drop, the friction factor coefficient, and the coefficient of convective heat transfer. Moreover, Kima et al. [20] studied, using an experimentally validated CFD model, the convective heat transfer and flow characteristics of nanofluids and indicated that the heat transfer coefficient and Nu increased with an increase in the Re value.

The no-slip wall condition is applicable to Newtonian fluids when the Knudsen number (Kn) is smaller than 0.001. According to Maxwell's kinetic theory [2], slip flow is observed when the characteristic flow length scale is within the order of the mean free path of the gas molecules. When Kn is less than 0.1 and larger than 0.01, the temperature jump and velocity slip appear, and continuum models remain valid. Therefore, Navier–Stokes equations could still be used to model the laminar flow inside circular pipes [21]. Based on this approach, Vocale and Morini [22] utilized the CFD tool to investigate a rarefied, laminar, fully developed gaseous flow within a rhombic microchannel, taking into account temperature jump conditions. They confirmed the dependence of Nu on the microchannel geometry and Kn. Spiga and Vocale [23] investigated numerically rarefied fluid flowing in a micro-duct with an elliptical cross-section under slip flow. The steady-state, laminar, and fully developed flow under an axial uniform linear heat flux boundary condition was examined. They validated their model against published results from the literature. Their developed numerical code was reported to be capable of obtaining velocities and temperatures of the flowing fluid inside the elliptical cross-sectional pipes. Akbarinia et al. [24] conducted a numerical simulation to evaluate volume fractions of Al_2O_3 nanoparticles and inlet velocity effects on enhancing heat transfer in slip and no-slip nanofluid flow regimes. Their findings demonstrated that the major improvement in Nu was caused by an increase in the inlet velocity rather than a rise in the volume fraction of nanoparticles. Recently, Siham et al. [25] utilized Python software to study the rarefied flow with Cu– H_2O nanofluid particles through a rectangular microchannel. They confirmed the

significant effect of Re , Kn , and the nanoparticle volume fraction (ϕ) on the heat transfer within the system.

Rovenskaya and Groce [26] investigated heat transfer in rough wall surface microchannels with a slip flow regime. They considered incompressible gas flow for a range of Kn [0.01–0.1]. They reported that Nu is inversely proportional to Kn for both smooth and rough channels. This was confirmed by Ameer et al.'s [27] analytical solution of a fully developed laminar flow of gas in microtubes under constant wall heat flux conditions, in which they reported that Nu was inversely dependent on Kn and this dependence increased as a function of temperature jump. Furthermore, Larrode et al. [28] stressed that neglecting the temperature jump could lead to a misleading conclusion of the increase in Nu as a function of Kn .

Researchers such as Tunc and Bayazitoglu [29,30] analytically investigated the heat transfer within microtubes under laminar and fully developed gaseous flows under uniform heat flux boundary conditions. The effects of axial conduction and viscous dissipation within the fluid on the same problem were solved by Jeong and Jeong [31]. It was reported that axial conduction within the fluid was constant under constant wall heat flux boundary conditions. Consequently, Nu was not affected, and it became independent of the Péclet number ($Pe = Re Pr$) [29]. Ramadan [32] highlighted the significance of shear work in the convective heat transfer of gas flowing in a microtube. He stated that shear work could be as high as the heat transfer via conduction at the upper limits of the slip flow regime. He added that the effect of shear work consists of viscous dissipation and pressure work combined effects. Knupp et al. [33] investigated the problem of the conjugate heat transfer of gaseous flow inside a microtube under slip flow regime. They stressed the importance of including the effects of axial diffusion and wall conjugation on the value of Nu at a low Péclet number ($Pe < 20$).

In an earlier work, Alkhouz et al. [34] conducted a numerical study of Al_2O_3 –air nanofluid laminar flow in pipes, taking into consideration the velocity slip and temperature jump. A correlation for the average Nu was developed under a constant surface temperature. The study showed that an increase in temperature jump and slip velocity led to a reduction in average Nu . Sun et al. [35] conducted a numerical study of a laminar gaseous flow through a pipe under constant wall heat flux and slip flow conditions. They linked the resulting Nusselt number with Knudsen and Brinkman numbers at the developing and fully developed regions. In their comprehensive study, the effects of the addition of nanoparticles to the base fluid was not investigated. Moreover, Su et al. [36] conducted an analytical study on a fully developed laminar slip flow inside an elliptical microchannel with constant wall heat flux conditions. Their fully developed Nusselt number values were correlated to an air-specific heat ratio, as well as Knudsen and Prandtl numbers. Their work failed to report effects of nanofluids at the entrance region of a circular pipe.

Although there are many studies that have dealt with heat transfer enhancement through nanofluids in pipes, the use of slip flow in engineering applications is not widely accepted by the academic community. This is due to the small length scale of the slip flow regime compared with the length scale of the flow/system. Therefore, the rarefied flow of gaseous nanofluids under slip boundary conditions at the entrance region is a fundamental study on a rarely addressed limitation of the fluid flow and requires further attention.

Recently, sustainable development goals (SDGs) have dictated that energy efficiency must be increased from its current level to almost double. Enhancing heat transfer within engineering systems directly supports energy efficiency goals by reducing energy consumption and fostering technological innovation across industrial sectors. In this research, a mixture of air and Al_2O_3 flow at the circular pipe entrance region under slip boundary conditions was investigated. Developing laminar flows with different values of constant wall heat flux were simulated using the CFD approach based on the Ansys-Fluent 2018 software package. The CFD model investigated the effects of Pr , Re , wall heat flux (q''), Kn , aspect ratio (x/D_h —the distance traveled by the nanofluid along the pipe axis to the hydraulic diameter), and ϕ of Al_2O_3 on the Nusselt number (Nu), velocity ratio (U/U_m),

and skin friction coefficient presented by (ReC_f). Based on the comprehensive CFD simulations conducted, the correlations of Nu , U/U_m , and ReC_f as functions of other parameters were presented.

2. Modeling and Computational Details

The exact Boltzmann equation requires great computational effort that is not justified in practical and engineering calculations. The CFD approach was utilized to model a steady-state, laminar, and forced convection flow inside a circular pipe under a slip flow regime and constant wall heat flux conditions, as shown by Figure 1. The adopted CFD approach was based on the usage of the Ansys Fluent 2018 software package. The semi-implicit technique for pressure-linked equations (SIMPLE) algorithm was used to link the continuity equation with the pressure field in the CFD domain. Moreover, the PRESTO algorithm was used to calculate the pressure field. A hybrid second-order accuracy scheme of central and upwind difference was used to differentiate the convective terms. The maximum of the normalized absolute residual across all nodes was taken to be less than 10^{-6} and was considered as a condition for convergence criteria.

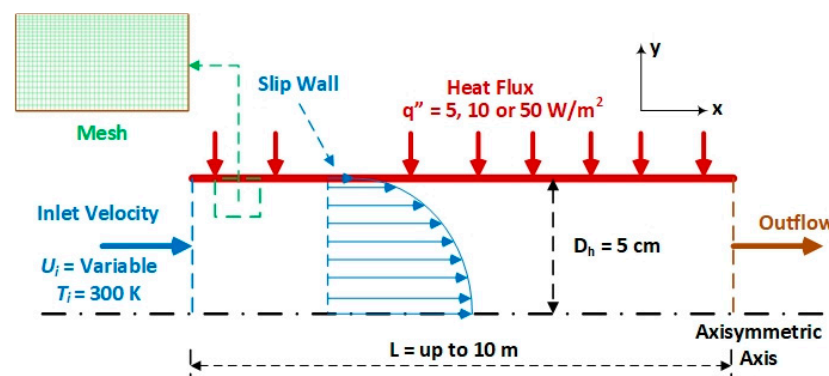


Figure 1. Schematic of the simulated pipe with boundary and axisymmetric details.

2.1. Nanoparticle Flow Modeling

It is widely accepted to ignore the axial conduction for fluid flows in a channel when the Pe value is greater than 100 [37]. Furthermore, pressure work and viscous dissipation were neglected in the current study due to the low resulted value of the Brinkman number of the investigated cases ($Br = \mu u_m^2 / q_w R < 0.01$). The following single-phase, axisymmetric governing equations of continuity (Equation (1)), momentum (Equations (2) and (3)), and energy (Equation (4)) were adopted in the current study.

$$\frac{1}{r} \frac{\partial(vr)}{\partial r} + \frac{\partial u}{\partial z} = 0 \quad (1)$$

$$\rho_{nf} \left(u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} \right) = -\frac{\partial p}{\partial z} + \mu_{nf} \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (2)$$

$$\rho_{nf} \left(u \frac{\partial v}{\partial z} + v \frac{\partial v}{\partial r} \right) = -\frac{\partial p}{\partial r} + \mu_{nf} \left(\frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} \right) - \frac{v}{r^2} \quad (3)$$

$$\rho_{nf} C_{p,nf} \left(u \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial r} \right) = k_{nf} \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \quad (4)$$

The assumption of single-phase simulation was adopted because air velocity was more than one million times the Al_2O_3 nanoparticles' relative velocities. Moreover, the resulted Stokes number (St) was much less than one, which confirmed that the particle will follow the air flow very closely [38]. Therefore, it was valid to assume that Al_2O_3 nanoparticles and air were in homogenous and stable states and could be treated as a new fluid with new thermophysical properties like those listed in Table 1. The properties of the resulting

air–Al₂O₃ nanofluid were calculated based on Equations (5)–(9) [4,34]. These include the density (ρ_{nf}), specific heat (C_{Pnf}), thermal conductivity (k_{nf}), viscosity (μ_{nf}), and thermal expansion coefficient (β_{nf}) as a function of volume fraction (ϕ) of the Al₂O₃ nanoparticles to the air. These values were considered constant during the CFD simulation, apart from ρ_{nf} values, which were estimated based on the Boussinesq approximation.

$$\rho_{nf} = (1 - \phi)\rho_f + \phi \rho_s \quad (5)$$

$$C_{Pnf} = (1 - \phi)C_{Pf} + \phi C_{Ps} \quad (6)$$

$$k_{nf} = k_f \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \quad (7)$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (8)$$

$$\beta_{nf} = \frac{\phi \rho_s \beta_s}{\phi \rho_s + (1 - \phi)\rho_f} + \frac{(1 - \phi)\rho_f \beta_f}{(1 - \phi)\rho_f + \phi \rho_s} \quad (9)$$

Table 1. Properties of the utilized base fluid and nanoparticles.

Property	Air	Al ₂ O ₃
Specific Heat: C_p (J/kg·K)	1006	765
Density: ρ (kg/m ³)	1	3970
Thermal Conductivity: k (W/m ² ·K)	0.025	40
Thermal Diffusivity: α (m ² /s)	19.00×10^{-6}	13.17×10^{-6}
Thermal Expansion Coefficient: β (1/K)	3.33×10^{-3}	8.50×10^{-6}

2.2. Slip Flow Treatment

The traditional Navier–Stokes equations have been used comfortably to describe macro-scale flows. However, they lack accuracy at local rarefied gas flow areas. To bridge this gap, a slip boundary condition at the walls has been developed and imposed. At low Kn values ($Kn < 0.1$), Loussif and Orfi [39] worked on the developing the regional heat transfer of a circular duct under temperature jump and slip velocity conditions with constant wall temperature. They reported that the second-order slip flow model was not significant. Therefore, the first-order slip condition proved to be valid [26]. The rarefaction of gas flow was measured using the Kn, in accordance with Equation (10), as follows:

$$Kn = \frac{\lambda}{D_h} \quad (10)$$

where D_h is the pipe diameter with a constant value of 0.05 m. Moreover, λ is the local mean free path for a solid spherical molecule. It is defined as the average distance a molecule moves before it collides with another molecule, as defined in Equation (11). The value of the Lennard–Jones characteristic length of air (σ) was manipulated based on the operated pressure and temperature of Ansys-Fluent to force the value of Kn to be (0.01, 0.03, 0.05, 0.07, or 0.1) at a constant Boltzmann constant (k_B) value of 1.38066×10^{-23} J/K. In this way, the effects of the absolute pressure value on the σ value became insignificant, and the derivation of pressure with respect to the traveled distance remains important to study. This approach is called the low-pressure slip flow method for laminar flow in Ansys-Fluent, where σ is a constant value input into Equation (11), and the temperature and pressure are extracted from the CFD simulation in order to estimate λ .

$$\lambda = \frac{16\mu}{5\rho} \sqrt{\frac{m}{2\pi kT}} = \frac{Tk_B}{\sqrt{2}\pi\sigma^2 P} \quad (11)$$

In a pure convection problem, heat transfer through the wall of a tube is characterized by a suitable thermal boundary condition, which is directly or indirectly specified at the interface of the wall–fluid. The set of conditions known as thermal boundary conditions describe the temperature and/or heat flow at the tube’s inner wall. Since the velocity and temperature are only determined in the fluid region, it is not necessary, in such problems, to provide a solution to the temperature problem for the solid wall. Consequently, the adopted slip and temperature jump boundary conditions followed the first-order Maxwell’s model [2] with wall perpendicular velocity ($u_g = 0$), as follows:

$$u_w - u_g = \left(\frac{2 - \sigma_v}{\sigma_v} \right) Kn D_h \frac{\partial u}{\partial n} \approx \left(\frac{2 - \sigma_v}{\sigma_v} \right) \frac{\lambda}{\delta} (u_g - u_c) \quad (12)$$

$$T_w - T_g = 2 \left(\frac{2 - \sigma_T}{\sigma_T} \right) \frac{2\gamma}{\gamma + 1} \frac{k}{\mu C_v} Kn D_h \frac{\partial T}{\partial n} \approx 2 \left(\frac{2 - \sigma_T}{\sigma_T} \right) \frac{2\gamma}{\gamma + 1} \frac{k}{\mu C_v} \frac{\lambda}{\delta} (T_g - T_c) \quad (13)$$

where δ is the distance from cell center to the wall, σ_v and σ_T represent the momentum and thermal accommodation coefficients. The value of σ_v and σ_T for many engineering applications of gas–surface material is between 0.8 and 1.0. For air, it is often assumed that $\sigma_v = \sigma_T \approx 1$ [28]. For all currently simulated Re values, the resulting error in estimating the value of Nu was less than 7% at $\sigma_v = \sigma_T$ values between 0.9 and 1.0. Consequently, the value of $\sigma_v = \sigma_T = 1.0$ was adopted throughout the current study.

Throughout the current study, the average Nu was calculated based on Equation (14) by integrating the local Nu in the flow direction. The average Nu depends on many parameters, as follows: the applied heat flux on the pipe walls ($q'' = 5 \text{ W}$), $D_h = 0.05 \text{ m}$, k_{nf} , the fluid temperature at the wall ($T_w(x)$), and the fluid mean temperature at a given location in the flow direction ($T_m(x)$). Under the constant properties assumption, ρ , C_p , and the horizontal axial velocity (U) were taken at the inlet of the pipe with the constant pipe radius (R). $T_w(x)$ values were extracted from the CFD model for each different simulation, while $T_m(x)$ values were calculated based on Equation (15).

$$Nu = \int_0^L Nu(x) dx = \int_0^L \frac{h(x) D_h}{k_{nf}} dx = \int_0^L \frac{q'' D_h}{k_{nf} (T_w(x) - T_m(x))} dx \quad (14)$$

$$T_m(x) = T_{m, inlet} + \frac{2q''}{R U \rho C_p} x \quad (15)$$

2.3. Validation

In order to design a nanofluid energy efficient heat sink system represented by the Nu value, key performance parameters must be studied and understood. These parameters could be the required pressure for pumping the nanofluid and represented by (ReC_f); the mass flow rate of the nanofluid and represented by (Re); and the velocities and temperatures at the wall or within the nanofluid and could be represented by (U/U_m) and (Kn) for the slip flow, among other parameters. In accordance with this, the current CFD model was validated against the published works of other researchers. These works were basically analytical results of fully developed slip flow due to the absence of results for the entrance region of slip flow within circular pipes. The following different parameters were benchmarked in the current study:

- Velocity ratio (U/U_m) based on the work of Sun et al. [35] and defined in Equation (16). The fully developed velocity profile of steady and fully developed laminar flow of a viscous fluid was derived by solving the equation of motion with the use of the slip velocity and the symmetry at the centerline of the pipe;
- The skin friction coefficient (C_f) based on the work of Su et al. [36] and defined in Equation (17) and the Nusselt number (Nu) based on the work of Su et al. [36] and defined in Equation (18). The derived C_f and Nu of the incompressible, laminar, fully developed with constant thermophysical properties gas flow was based on ignoring

the effects of viscous dissipation, natural convection, radiation heat transfer, the mass-weighted average bulk fluid temperature, and the first-order temperature jump boundary condition.

The value of R was constant at 2.5 cm, and the values of Kn , the specific heat ratio (γ), and Pr were variables based on the specific simulated case and nanoparticle volume fraction, as listed in Table 1. Furthermore, the value of Kn_T was connected with the value of Kn based on the assumption of $\sigma_v = \sigma_T = 1.0$.

$$\frac{U}{U_m} = \frac{2[1 - (r/R)^2] + Kn}{1 + 8Kn} \quad (16)$$

$$ReC_f = \frac{16}{8Kn + 1} \quad (17)$$

$$Nu = \frac{1}{\frac{1}{48(8Kn+1)^2} + \frac{1}{12(8Kn+1)} + \frac{1}{8} + \frac{2\gamma Kn_T}{(1+\gamma)Pr}} \quad (18)$$

Structured mesh elements were dependent on the density of elements in the perpendicular direction, with respect to the pipe wall. Five different mesh sizes, as listed in Table 2, were utilized for the mesh sensitivity analysis at six values of Kn , at $Re = 1750$, which lies within the laminar flow region inside the pipe, and at $Pr = 0.765$. The number of mesh elements varied between 3000 (AS0) and 450,000 (AS4). They corresponded to perpendicular distances of 1.042 mm (AS0) and 83.3 μm (AS4) between the first cell center in touch with the pipe wall and the cell face center of the wall. These distances are important in solving gas slip velocities and temperature jump values at cells in touch with constant heat flux pipe walls. It is worth mentioning that Nu is independent of the value of wall heat flux when a no-slip wall boundary condition is utilized.

Table 2. Mesh details utilized in the current simulation.

	AS0	AS1	AS2	AS3	AS4
No. in X	250	500	1000	2000	3000
No. in Y	12	25	50	100	150
No of Elements	3000	12,500	50,000	200,000	450,000
Cell to wall distance (m)	1.042×10^{-3}	5.000×10^{-4}	2.500×10^{-4}	1.250×10^{-4}	8.333×10^{-5}

All simulated mesh sizes showed similar behavior of decreasing values with the increase in Kn values, as shown in Figure 2. For (U/U_m) and (ReC_f) simulations, as shown in Figure 2a,b, the maximum error in (U/U_m) corresponded to AS0 mesh, with values between 0.6 and 0.7%, while the minimum error in (U/U_m) corresponded to AS3 and AS4 meshes, with values between 0 and 0.4% for Kn values between 0.0 and 0.1, respectively. The difference in the reported errors between AS0 and AS3 was 0.6% at $Kn = 0.0$ and reduced to 0.3% at $Kn = 0.1$ in favor of mesh AS3 and AS4. Similar results were reported of the error values for (ReC_f) simulations, with values between 1 and 1.5% at AS0 mesh and 0.0 and 1.2% at AS3 and AS4 meshes. The difference in reported errors between AS0 and AS3 was 1.0% at $Kn = 0.0$ and reduced to 0.3% at $Kn = 0.1$ in favor of mesh AS3 and AS4. Therefore, it was clear that relatively fine meshes AS3 and AS4 provided more accurate results in comparison with the coarse mesh AS0.

The slight differences between different meshes showed insensitivity toward mesh element sizes. The dependence of U/U_m and ReC_f , mainly, on Kn has enhanced the tendency toward mesh insensitivity. Furthermore, the assumption of constant properties has reduced the dependency on mesh element sizes. The mesh insensitivity was confirmed by the case of Nu , when all mesh sizes produced same results at all Kn values for $Re = 1750$, as shown in Figure 2c. This meant that the current mesh element sizes were relatively fine enough for the current investigation.

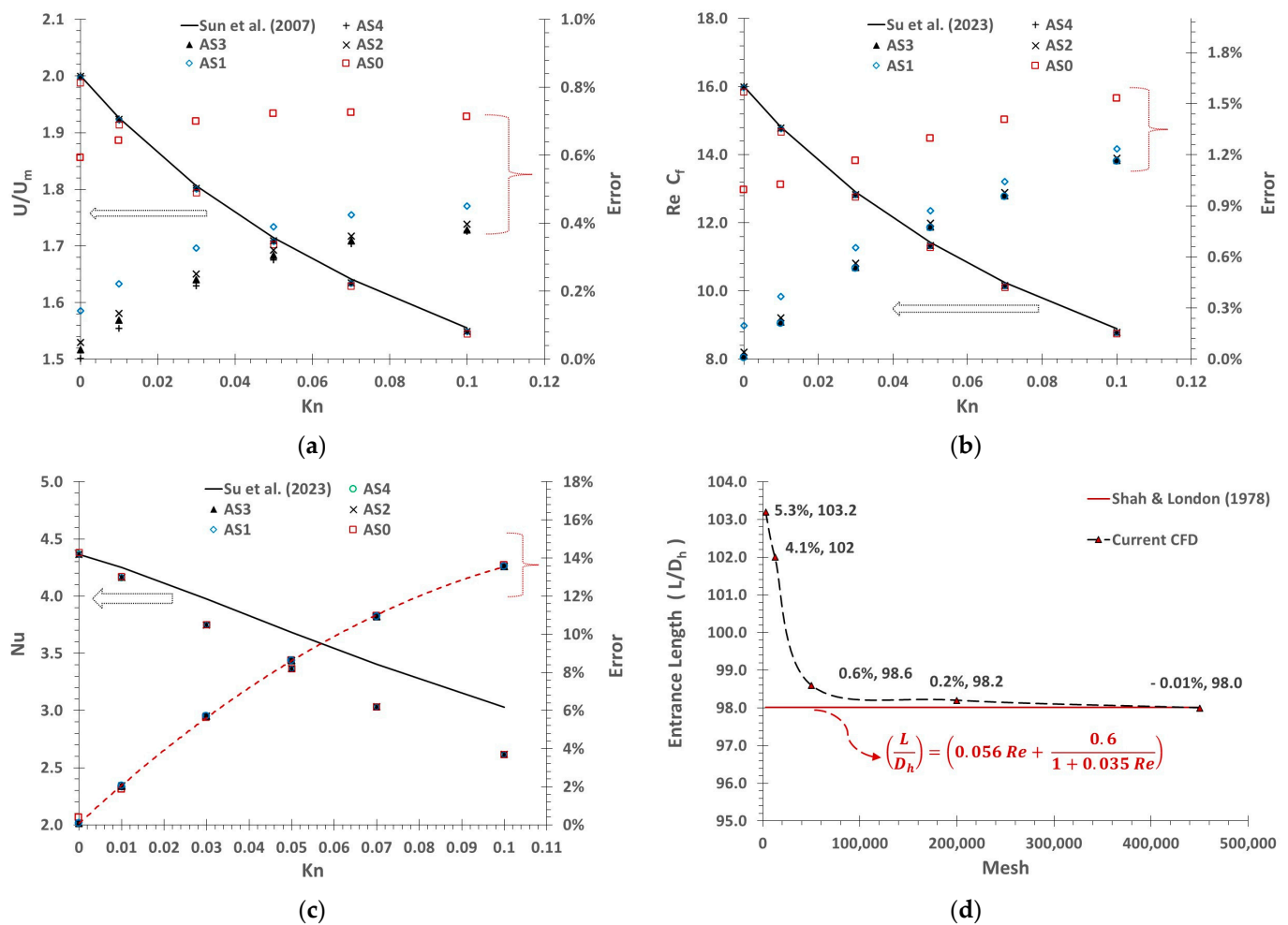


Figure 2. Mesh sensitivity and validation analysis for the main benchmarked parameters, as follows: (a) Velocity ratio (U/U_m), compared with Sun et al. [35]; (b) skin friction coefficient ($Re C_f$), compared with Su et al. [36]; (c) Nusselt number (Nu), compared with Su et al. [36]; (d) entrance length (L/D_h), compared with Shah and London [40].

Comparing the entrance length (L/D_h) values to those of the no-slip conditions of Shah and London [40] produced different value functions of the utilized mesh element sizes. Mesh AS0 resulted in an error of 5.3%, while mesh AS4 produced an error value equal to 0.01% and mesh AS3 resulted in an error of 0.2%. The main advantage of AS3 and AS4 meshes was the short distance between the wall and the adjacent cell center. Therefore, mesh AS3 was adopted to conduct the rest of simulations of the current study due to the relatively shorter simulation time and the acceptable accuracy.

The validation of the current study requires that mesh AS3 would result in an acceptable value of error when compared with reliable published results. Figure 2 showed how the error value increased with the increase in Kn value until it reached the maximum value at $Kn = 0.1$. When compared to the published analytical results of Sun et al. [35], for values of U/U_m , AS3 produced a very close results, with a maximum error of 0.4% at $Kn = 0.1$. Similarly, when compared to the published analytical results of Su et al. [36], for values of $Re C_f$, AS3 produced close results, with a maximum error of 1.2% at $Kn = 0.1$. Furthermore, when compared to the published analytical results of Su et al. [36], for values of Nu , AS3 produced results, with a maximum error of 13.6% at $Kn = 0.1$, which is considered a relatively high error.

The relatively high error value could be due to the dependence of Nu on Pr , γ , and Kn , as defined in Equation (18). This dependence is defined by the last term at the denominator and increases as the value of Kn increases. For constant properties, the slight difference

between the constant value of Kn and estimated value by the CFD simulation defined in Equations (10) and (11) resulted in a high error value. It is well-known that as the velocity slip increases, due to the increase in Re and/or Kn , the convective heat transfer increases, too. The values of λ and, consequently, Kn depend on the nanofluid bulk temperature (T_m) value, proportionally. The current study was based on a constant T_m value for all cases, which contributed to this high value. However, the current simulation results could be considered acceptable and valid based on the total validation study.

3. Results and Discussion

The effects of the Reynolds number ($Re = 250, 500, 1000, 1500$, or 1750), Al_2O_3 nanoparticle volume fraction ($\phi = 0, 0.01, 0.03, 0.05$, or 0.1) or Prandtl number ($Pr = 0.765, 0.759, 0.750, 0.742$, or 0.729), Knudsen number ($Kn = 0, 0.01, 0.03, 0.05, 0.07$, or 0.1), and aspect ratio ($x/D_h > 200$) on the entrance length (L/D_h), velocity ratio (U/U_m), skin friction coefficient (C_f), and Nusselt number (Nu) are presented in this section at the entrance region. The entrance region was decided based on the value of (x/D_h) that corresponded to the fully developed value (U/U_m), as defined in Equation (16). Results from within the entrance region were considered in the current study only.

The slip flow was modeled based on Equations (12) and (13), and it was dependent on the value of the mean free path value (λ) that is defined in Equation (11). The temperature value in Equation (11) was dependent on the constant value of wall heat flux (q'') that was provided at the wall boundary condition. Therefore, three different values of $q'' = 5, 10$, and 50 W/m^2 were studied, and the results are shown in Figures 3–7. The value of U/U_m as a function of the inverse of the Graetz number ($Gz^{-1} = x/(D_h Re Pr)$) increased with an increase in the axial distance from a value of 1.0 at $Gz^{-1} = 0.0$ to a value of 1.53 at $Gz^{-1} = 0.07$ for $q'' = 5 \text{ W/m}^2$, as shown in Figure 3a. At $q'' = 5$ or 10 W/m^2 , the value of U/U_m was similar, with minimal differences. This meant that the increase in the fluid temperature in Equation (11) was small enough not to create a major effect on the resulting value of λ , which was reflected in the value of slip velocity defined in Equation (12). However, this was not the case for $q'' = 50 \text{ W/m}^2$, where the increase in the nanofluid temperature resulted in a noticeable difference in U/U_m in comparison with the other two values. The highest difference occurred at $Kn = 0.1$ and was less than 1% when compared with the results at $q'' = 5 \text{ W/m}^2$.

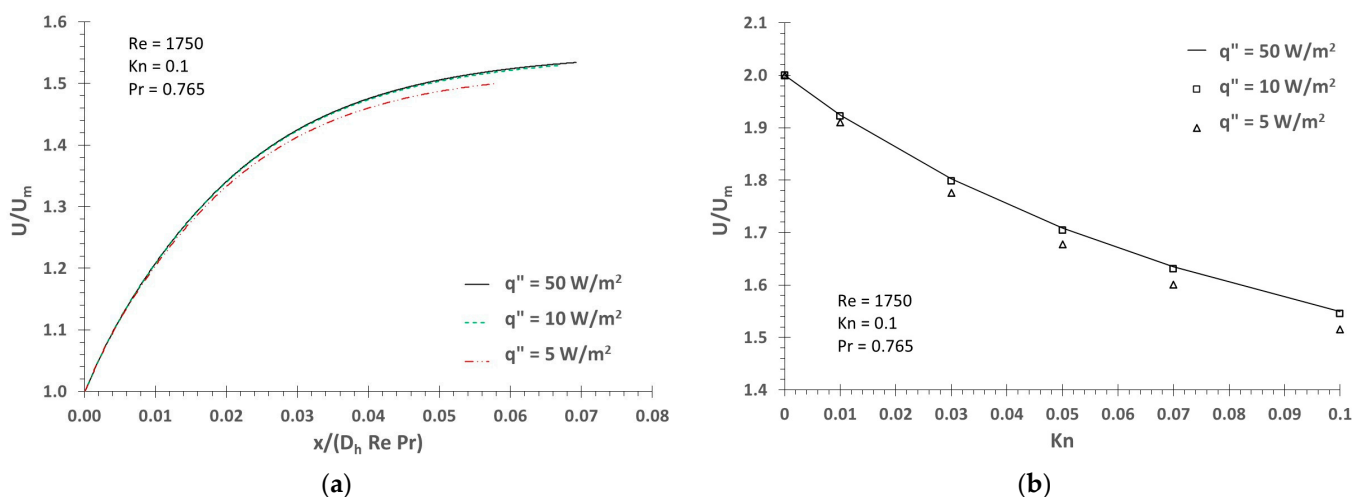


Figure 3. Velocity ratio (U/U_m) at different wall flux (q'') value functions of (a) $x/(D_h Re Pr)$; (b) Kn .

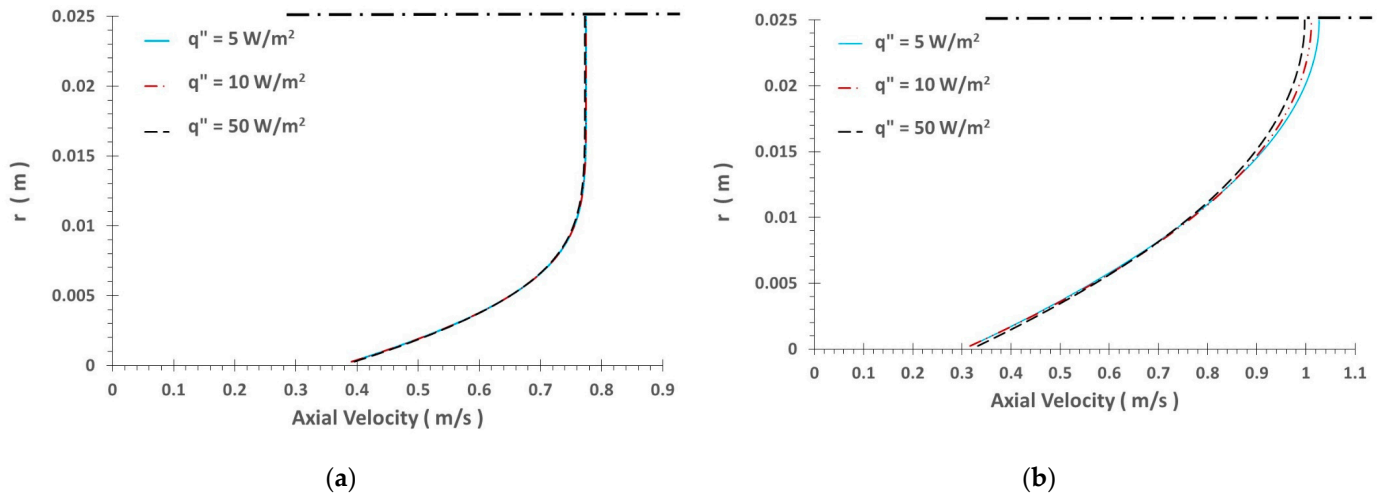


Figure 4. Velocity profile at two distances from inlet: (a) 10D; (b) fully developed.

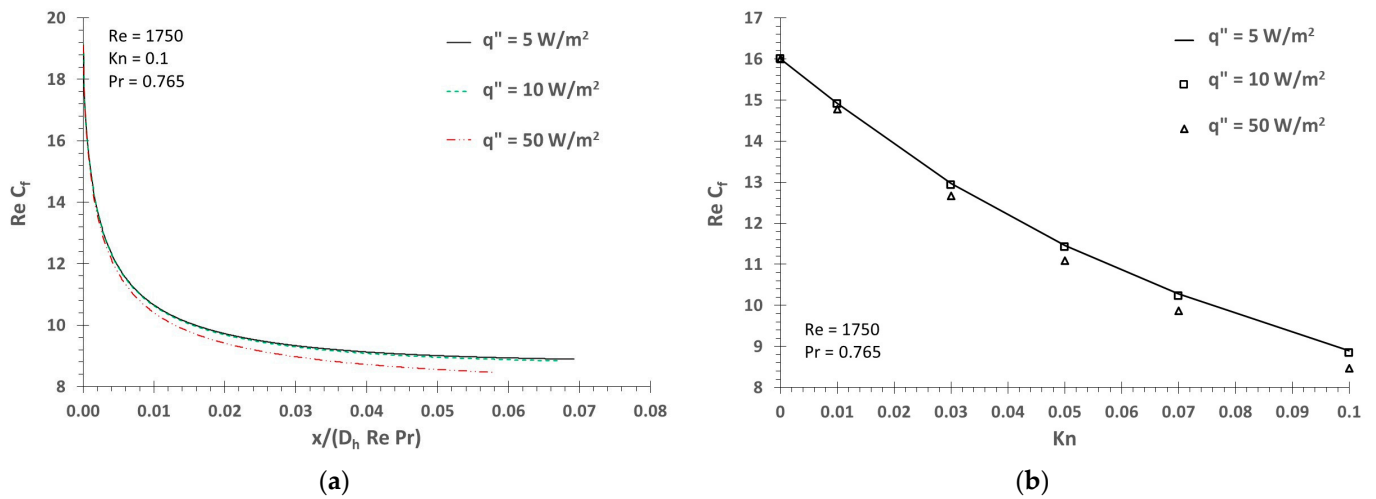


Figure 5. Values of $(Re C_f)$ at different wall flux (q'') value functions of (a) $x/(D_h Re Pr)$; (b) Kn .

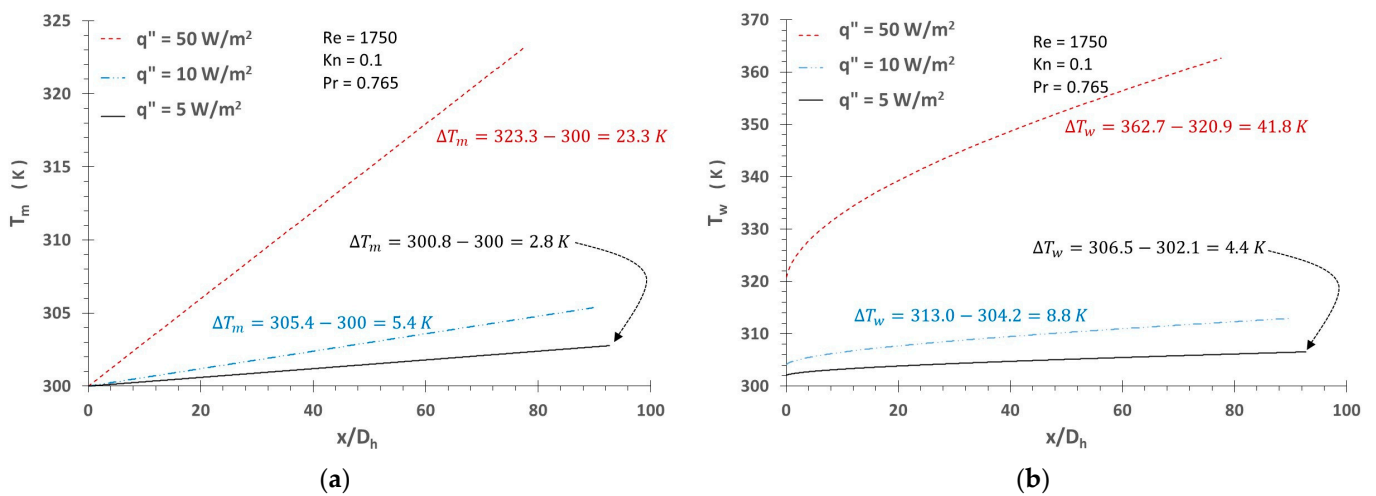


Figure 6. Nanofluid temperatures at different wall flux (q'') value functions of aspect ratio (x/D_h): (a) nanofluid bulk temperature (T_m); (b) pipe wall temperature (T_w).

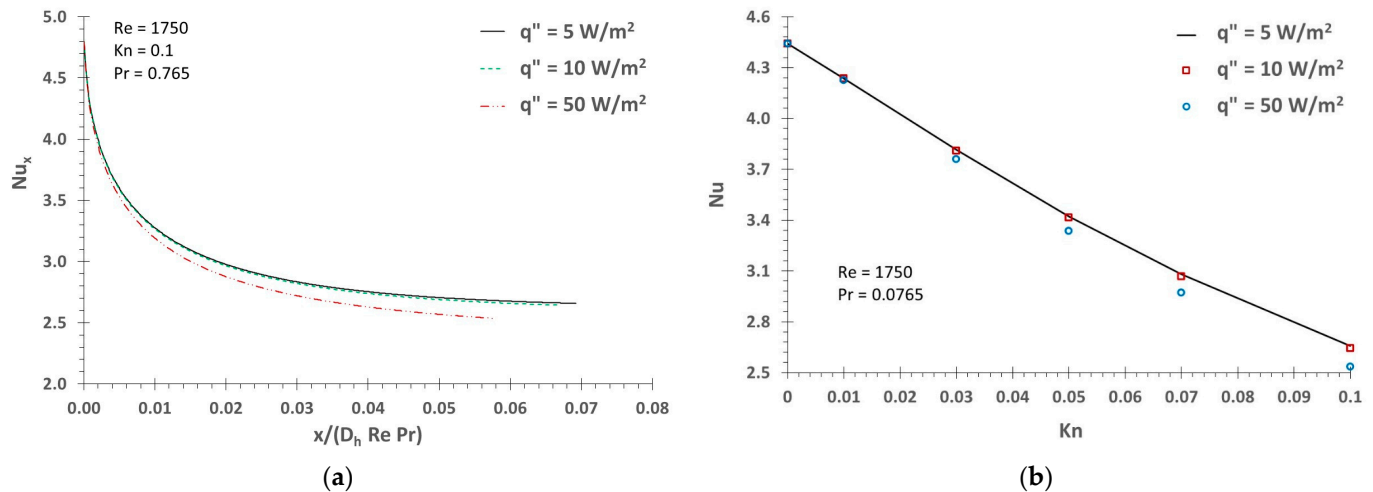


Figure 7. Values of (Nu) at different wall flux (q'') value functions of (a) $x/(D_h \text{ Re Pr})$; (b) Kn.

Figure 3b shows that the slip wall boundary represented by Kn affected the fully developed value of U/U_m by lowering its value from 2.0 at $\text{Kn} = 0.0$ (no-slip wall) to the value of 1.55 at $\text{Kn} = 0.1$ (slip wall). The other values were found to be located in between these values in a nonlinear pattern due to the conservation of mass. Consequently, a constant average velocity was maintained because of the constant thermophysical property assumption. The combination of the continuity and the momentum equations caused the non-linearity in the U/U_m vs. Kn curve. As the slip velocity value increased due to the increase in the Kn value, the fluid velocity value at the wall increased and became higher than zero. Therefore, the maximum velocity value at the pipe centerline would decrease, resulting in a lowered velocity ratio value, as shown in Figure 4.

At an entrance length of 10 pipe diameters (10D), the differences in the velocity profile were minimal for all investigated wall heat fluxes due to the relatively low increase in the nanofluid temperature, as shown in Figure 4a. However, the nanofluid temperature began to increase as the fluid traveled along the pipe axis toward the fully developed point due to the heat transferred from the pipe wall. The increase in the temperature is proportional to the value of the wall heat flux. As the q'' value increased, the λ value increased and resulted in a higher slip velocity. Consequently, the nanofluid axial velocity would decrease to fulfill the constant mass flow rate restrictions in accordance with the conservation of mass law, as shown in Figure 4b.

At constant properties, the skin friction coefficient (C_f) is defined in Equation (19) and is dependent on the variation in wall shear stress, which is correlated to the viscosity, velocity gradient at the wall, and Kn of the slip wall. As shown in Figure 5, the highest value of ($\text{Re}C_f$) corresponded to the flow with no-slip conditions ($\text{Kn} = 0.0$), which resulted in the highest velocity gradient at the wall. As the value of Kn increased, the slip velocity value increased, resulting in an increased wall velocity and a reduced velocity gradient at the wall. Consequently, this lowered the shear stress value and the skin friction coefficient (C_f) as shown in Figure 5a. The highest value was $C_f = 16$ at $\text{Kn} = 0.0$, which was decreased to $C_f = 8.9$ at $\text{Kn} = 0.1$, as shown in Figure 5b. However, the effect of increasing the value of q'' from 5 W/m^2 to 50 W/m^2 was significant and resulted in reducing the value of C_f by 4%. This is due to the increase in the λ value, which resulted in an increased slip effect at the wall and a reduced value of shear stress, which was more than expected. The 4% reduction at $\text{Kn} = 0.1$ remains an acceptable variation, and the results from the current simulations remain valid at the entrance region of the circular pipe.

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} \quad (19)$$

The estimation of Nu based on the current CFD simulation was performed by collecting the (T_w) value from the CFD simulation and calculated nanofluid bulk temperature (T_m) values based on Equation (15). The value of T_m depends on the wall heat flux and nanofluid properties that could be represented by Re and Pr , at which it changed linearly with the axial distance (x/D_h) traveled along the pipe by the nanofluid, as shown in Figure 6a. The maximum increases in T_m at the simulated entrance region, in comparison with the inlet temperature, were 2.8, 5.4, and 23.3 K for $q'' = 5, 10$, and 50 W/m^2 and occurred at $x/D_h = 92.6, 89.8$, and 77.6 , respectively. These increased T_m values resulted in an increased λ value, according to Equation (11), and caused an increase in Kn values based on Equation (10). Therefore, the increase in λ or Kn affected the estimation, by CFD, of the required values of T_w needed to maintain a constant value of $q'' = 5, 10$, or 50 W/m^2 . The maximum nonlinear increases in T_w values, in comparison with the inlet temperature, were 4.4, 8.8, and 41.8 K for $q'' = 5, 10$, and 50 W/m^2 and occurred at $x/D_h = 92.6, 89.8$, and 77.6 , respectively, as shown by Figure 6b. Consequently, the value of Nu defined in Equation (14) became dependable on the wall heat flux value.

The maximum value of Nu_x at the entrance of the pipe was 4.8 for all simulated q'' values, as shown in Figure 7a. The value decreased at different rates based on the applied q'' until it reached the fully developed values of 2.64, 2.64, and 2.53 for $q'' = 5, 10$, and 50 W/m^2 , respectively, at $Kn = 0.1$. This variation in Nu_x values was due to the variation in temperature jump values at the wall, as defined in Equation (13), as a result of the variation in the λ , Kn , and T_w values. However, the variation in the Nu_x of Nu was less than 3%, and it was considered valid to adopt the current approach of defining Kn to simulate the slip flow or temperature jump at the wall boundary condition.

The slip flow at the wall was dominated by Kn , which changed the value of slip velocity, as defined in Equation (12) and shown in Figure 8. At the no-slip condition ($Kn = 0$), the U/U_m value reached the well-known maximum value of two for a laminar flow inside a circular pipe at the fully developed flow point. Similar trends were observed for the slip flow at which the U/U_m ratio increased from a value of one to a maximum value at the fully developed flow point. As the Kn value increased, the maximum value of U/U_m was found to decrease, as shown in Figure 8a. The drop in the U/U_m ratio reached a minimum value of 1.55 at $Kn = 0.1$ for all values of Re and Pr . Therefore, the effects of Re or Pr on the U/U_m ratio at a constant Kn value were negligible, as shown in Figure 8b,c.

Figure 9 shows that the slip velocity value increased due to the increase in the Kn value, which caused the nanofluid velocity value at the wall to increase and become higher than 0 m/s. Therefore, the maximum velocity value at the pipe centerline of two would decrease to maintain constant mass flow rates in accordance with the conservation of mass law which resulted in lowering the U/U_m ratio. At $Kn = 0.01$ and 10 diameters from the inlet, for instance, the value of the velocity at the wall was equal to 0.01 m/s and increased to be equal to 0.39 m/s at $Kn = 0.1$, as shown in Figure 9a. The opposite behavior was observed at the centerline velocity, at which the velocity reduced from 0.92 m/s at $Kn = 0.01$ to 0.77 m/s at $Kn = 0.1$. The general form of the velocity curves was not parabolic and confirmed that the fully developed flow point is way beyond 10 diameters from the pipe inlet. Figure 9b shows parabolic nanofluid velocity curves resulting from the state of fully developed flow. When compared with the 10 diameter results, the values of the velocity at the wall were reduced to 0.075 m/s and 0.315 m/s at $Kn = 0.01$ and $Kn = 0.1$, respectively. However, the centerline velocity values were increased to 1.263 m/s and 1.019 m/s at $Kn = 0.01$ and $Kn = 0.1$, respectively. As a result, the U/U_m ratio increased as the nanofluid traveled downstream from the inlet, and the maximum value that could be reached decreased with the increase in the Kn value.

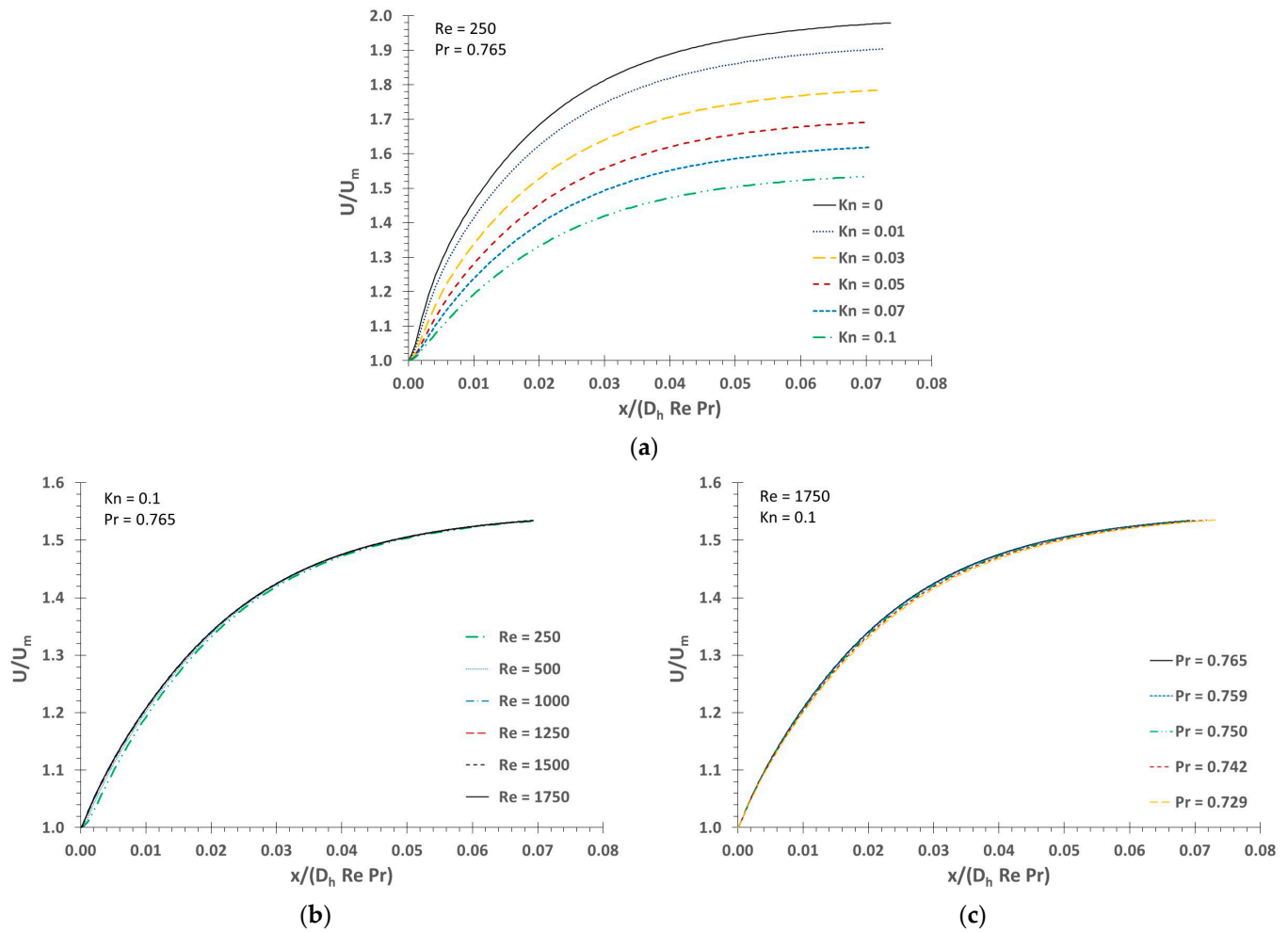


Figure 8. Values of (U/U_m) at different values of (a) Kn ; (b) Re ; and (c) Pr .

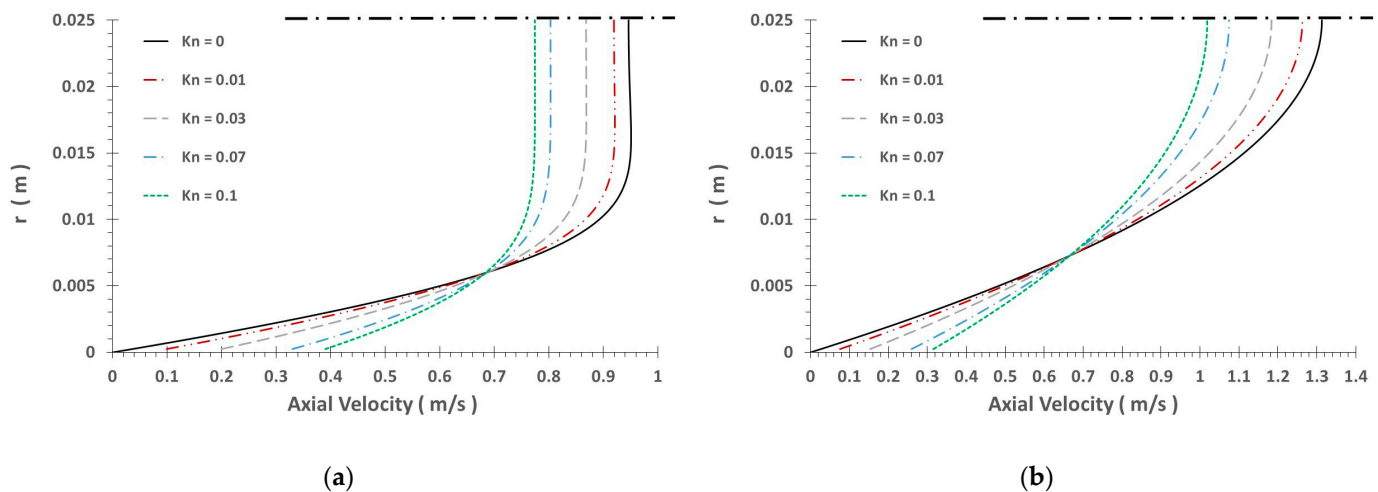


Figure 9. Axial velocity profile distribution function of pipe radial distance (r) at two distances from inlet: (a) $10D$; (b) fully developed.

It is well-known that the hydraulic entrance length (L/D_h) depends on Re proportional manners for pipe flow at no-slip conditions ($Kn = 0$). The value of (L/D_h) , based on current CFD results, was compared with the well-known equation of Shah and London [40] and shown in Figure 10a. The results showed the compliance of CFD results with the equation with a high degree of accuracy for all investigated volume fractions of Al_2O_3 nanoparticles.

With the already validated CFD results under the slip flow, as shown in Figure 2a, the effects of the slip flow on the value of (L/D_h) were examined and compared with the no-slip condition results for the same Re and Pr values ($\Delta(L/D_h) = (L/D_h)_{no\ slip} - (L/D_h)_{slip}$), as shown in Figure 10b. It can be seen that as the Kn value increased, the reduction in the (L/D_h) value increased due to the decreased value of U/U_m for all investigated Re values. Therefore, the distance required for the nanofluid to travel toward the fully developed value of U/U_m was decreased. Consequently, the hydrodynamic entrance length became shorter with the increase in Kn values when compared with the no-slip conditions, starting from a reduction of about 0.8 diameters at $Kn = 0.1$ for $Re = 250$ to a maximum value of around five diameters at $Kn = 0.1$ for $Re = 1750$. Therefore, the maximum reduction due to the increase in the Kn value was 5 diameters at $Re = 1750$ and due to the increase in Re value was $5 - 0.8 = 4.2$ diameters at $Kn = 0.1$, as shown in Figure 10b. Therefore, the presence of a slip wall reduced the hydraulic entrance length with the increase in the Kn and Re values.

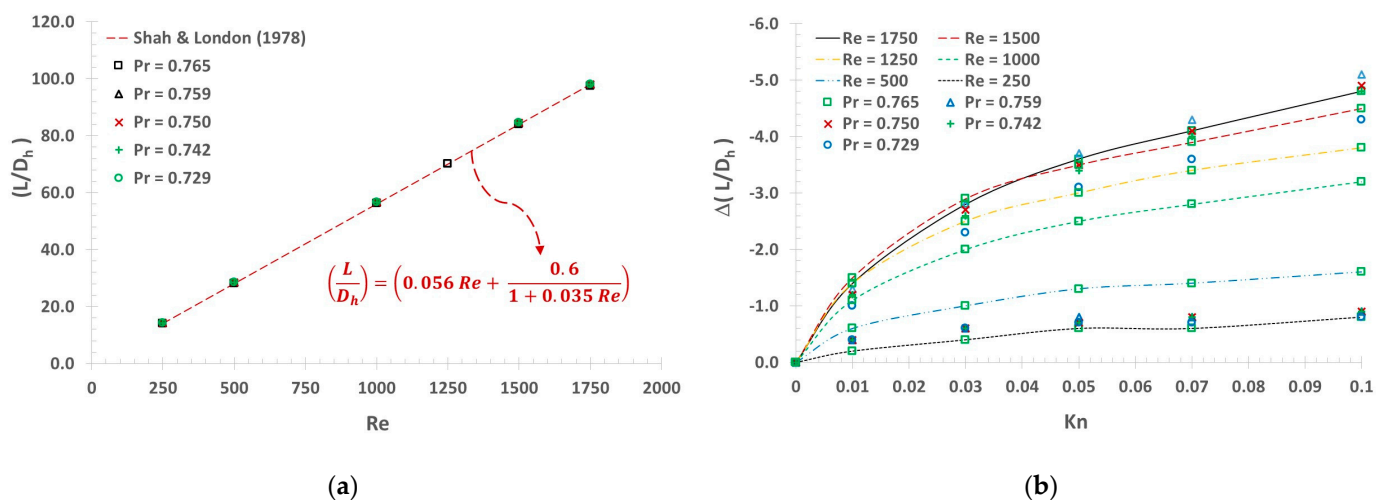


Figure 10. Effects of nanoparticles volume fraction represented by (Pr) on (a) hydrodynamic entrance length (L/D_h) at $Kn = 0$, compared with Shah and London [40]; and (b) variations in hydrodynamic entrance length ($\Delta(L/D_h) = (L/D_h)_{no\ slip} - (L/D_h)_{slip}$) at $0.01 \leq Kn \leq 0.1$.

The values of C_f obtained from CFD simulations were estimated based on Equation (19) and shown in Figure 11. The highest C_f value was at the inlet of the pipe due to the highest velocity gradient at the wall, which resulted in the highest wall shear stress. These values decreased as the nanofluid traveled downstream the pipe, causing the velocity gradient to decrease until it reached the fully developed value of $16/Re$. As the slip flow became active, with Kn values greater than 0.01 and less than 0.1, the C_f value started to drop below the no-slip condition. The lowest ReC_f value obtained at $Kn = 0.1$ was around 8.79, and the highest value under slip flow conditions was equal to 14.78 at $Kn = 0.01$, as shown in Figure 11a. Similar to the U/U_m results, it has been found that ReC_f was insensitive to the value of $x/(D_h Re Pr)$, resulting from the addition of nanoparticles to the base fluid with different percentages, as shown in Figure 11b,c. As explained earlier, the change in λ value has to be significant to cause a significant change in Kn that would increase the slip velocity value. The increase in slip velocity would decrease the wall shear stress, resulting in a decrease in the value of C_f .

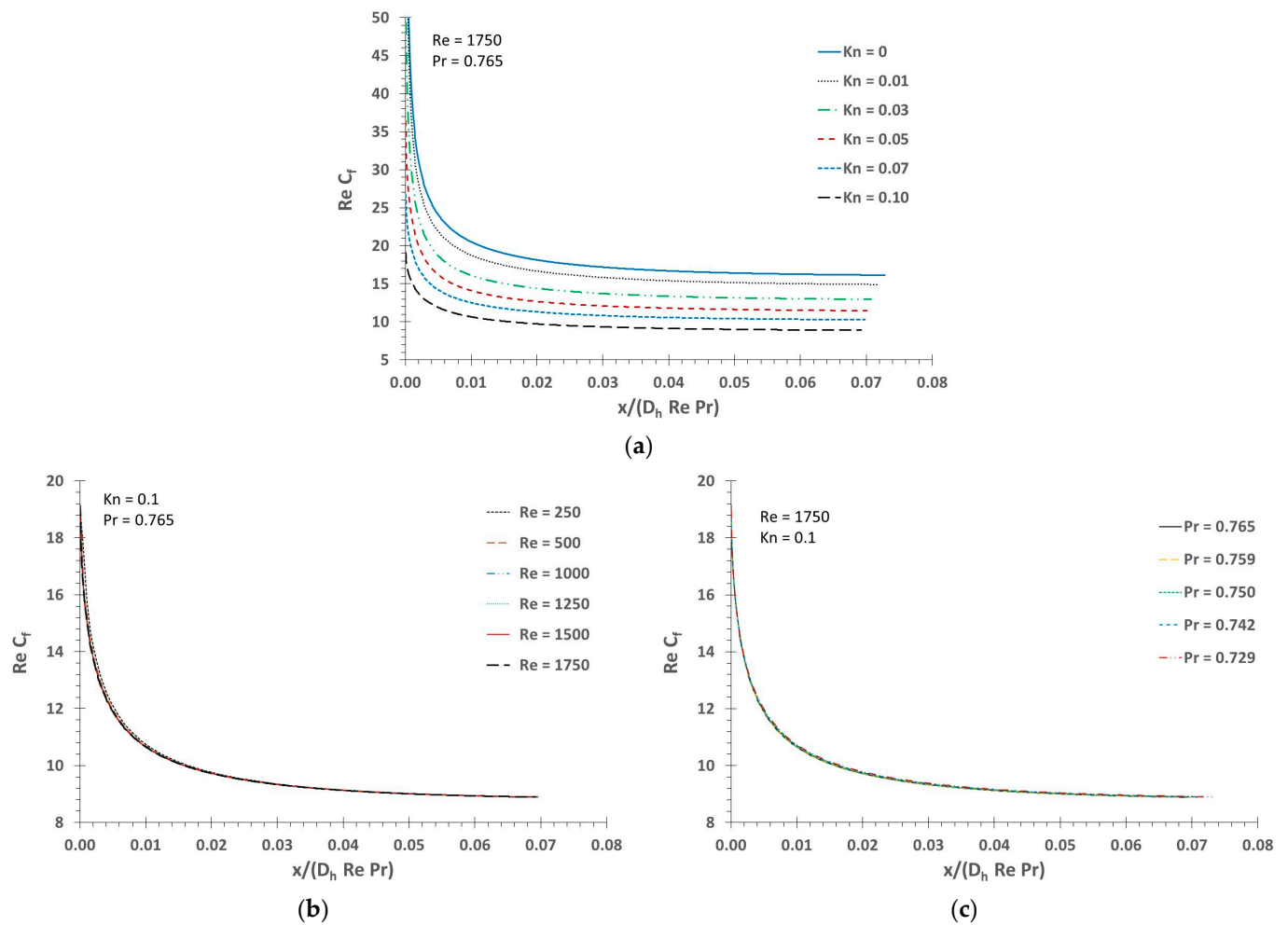


Figure 11. Values of $(Re C_f)$ at different values of (a) Kn ; (b) Re ; and (c) Pr .

The addition of nanoparticles to air would change the thermophysical properties of the resulting nanofluid, the value of Pr , and the flow inlet velocity. These changes would result in changing the magnitude of nanofluid bulk temperature (T_m) in accordance with Equation (15) and shown in Figure 12a. Increasing the nanoparticle volume fraction would result in a decrease in Pr and an increase in the specific heat of the nanofluid (C_p), which increases the ability of the nanofluid to absorb more heat. Therefore, T_m decreased with the decrease in Pr and increased in the flow direction due to the increase in the traveled axial distance. Consequently, the wall temperature (T_w) would decrease to maintain the enforced constant q'' value at the wall boundary condition, as shown in Figure 12b. The nanofluid temperature at the centerline of the pipe (T_{min}) was extracted from the CFD simulations directly without any further calculations and followed the same trend of T_m and T_w , as shown in Figure 12c.

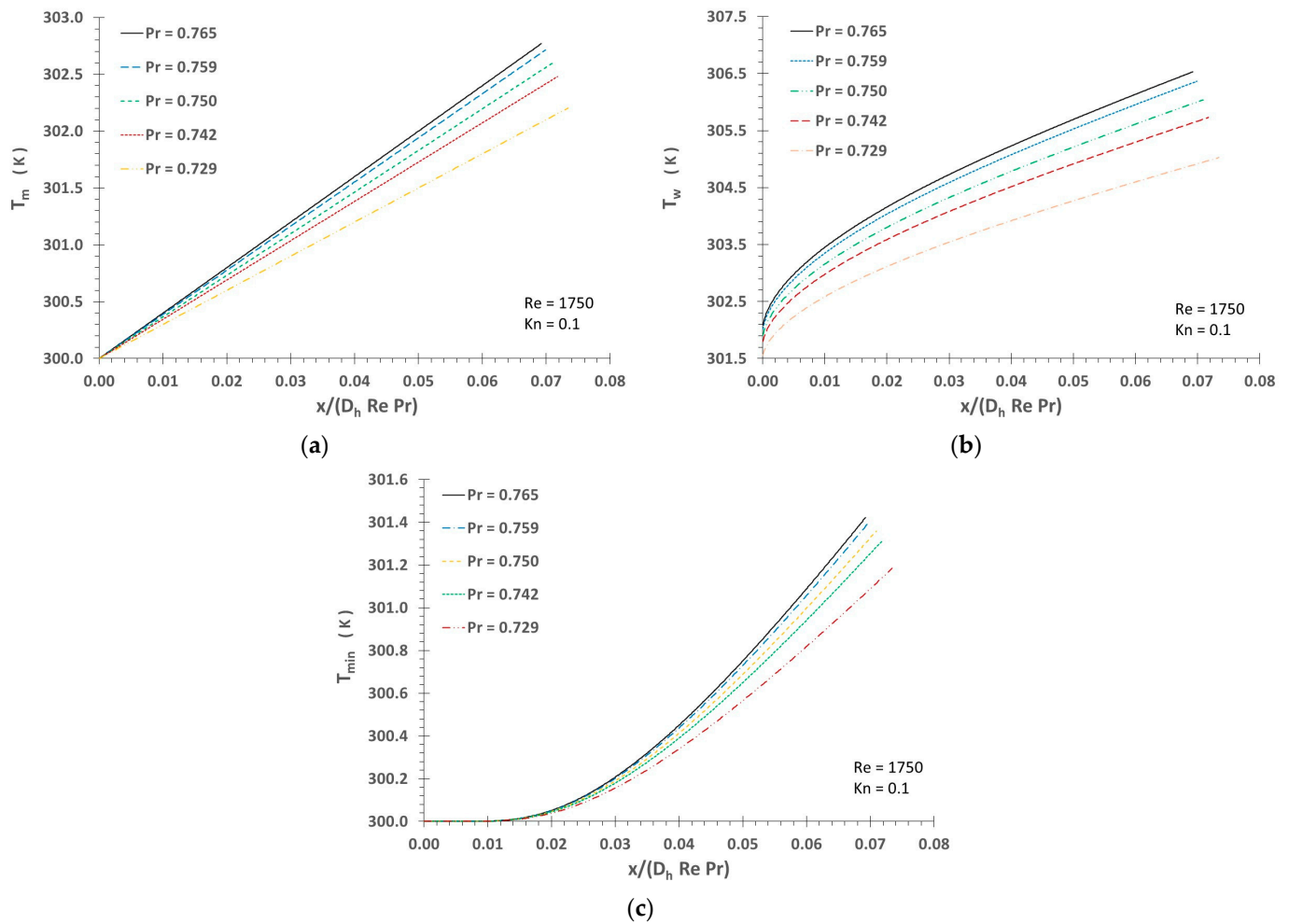


Figure 12. Temperature profiles due to different nanoparticle volume fractions: (a) nanofluid bulk temperature (T_m); (b) wall temperature (T_w); (c) pipe centerline temperature (T_{min}).

The dual variation in T_m and T_w as a function of Pr resulted in making the dependence of Nu_x on Pr at a constant Kn negligible, as shown in Figure 13a. A similar trend was found for different Re values, as shown in Figure 13b. At low Re values, the nanofluid velocity was low, which resulted in a higher T_m slope, based on Equation (15), at a relatively short entrance distance. As the Re value increased, the nanofluid velocity increased and resulted in a low T_m slope at a relatively long entrance distance, as shown in Figure 13c. The fully developed value of Nu_x was reached at different entrance distances with similar trends. This explains why all simulated Re values followed the same curve of Nu_x vs. ($x/(D_h Re Pr)$) at a constant Kn , as shown in Figure 13b. Moreover, the dependence of Nu_x on Kn was more significant than the dependence on Pr or Re , as shown in Figure 13d. It was confirmed that as Kn increased, the value of Nu_x decreased along the axial distance traveled by the nanofluid. The slip flow and temperature jump effects at the wall were caused by different values of Kn . As a result, they changed both the shear stress values and the T_w values, which resulted in a decrease in the value of Nu_x . At $Kn = 0$, the fully developed value of Nu_x was equal to 4.36, and the minimum Nu_x value obtained was equal to 2.5 at $Kn = 0.1$ for all investigated Pr and Re values. Therefore, Kn has negatively affected both the value of Nu_x and the value of the entrance length (L/D_h).

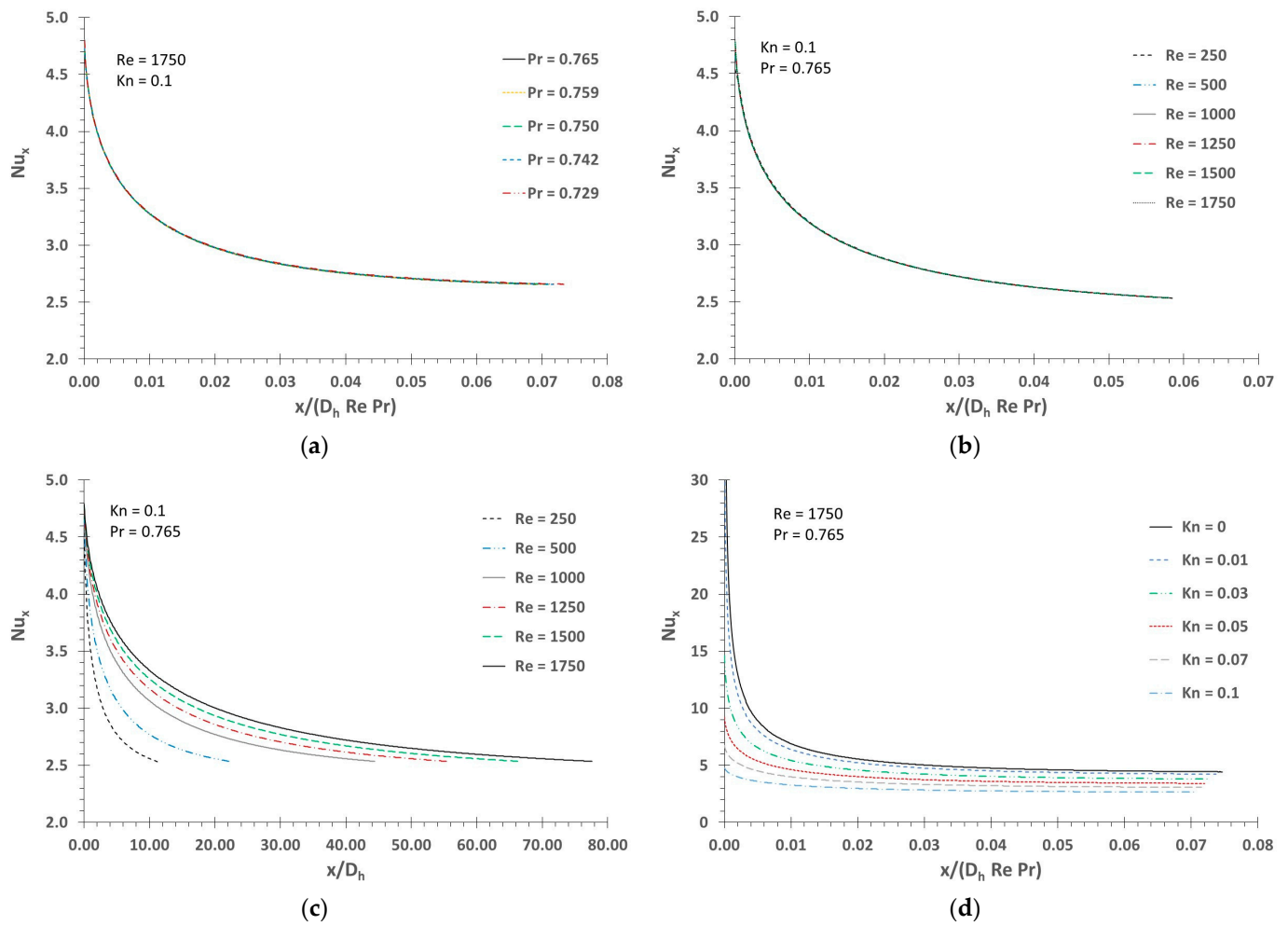


Figure 13. Local Nusselt number (Nu_x) at different values of (a) Pr ; (b) Re ; (c) Re function of x/D_h ; and (d) Kn .

The thermal entrance length $(L/D_h)_{th}$ was defined as the hydrodynamic entrance length multiplied by Pr , and it was related to the development of the thermal boundary layer inside the pipe. For the current study, the flow was considered thermally fully developed when the temperature ratio (θ), defined in Equation (20), was equal to 0.8. The inlet temperature (T_{in}) was constant at 300 K, while T_w and T_{min} were extracted from the CFD simulations. The current approach was validated against the published work of Shah and London [40] for the no-slip flow cases and shown in Figure 14a. As can be seen, the CFD results fitted well with the published correlation for all the investigated nanofluid volume fractions under the no-slip condition ($Kn = 0$). Building upon this conclusion, the variation in $(L/D_h)_{th}$ was examined and presented in Figure 14b as $\Delta(L/D_h)_{th} = (L/D_h)_{th-slip} - (L/D_h)_{th-no\ slip}$. The increase in $\Delta(L/D_h)_{th}$ with Re , when compared with same conditions at no-slip conditions, was clear, with a maximum value of 14.3 diameters. This is translated to an increase in the thermal entrance length by 14.3 diameters for the slip flow, with $Kn = 0.1$ at $Re = 1750$. At $Re = 250$ and $Kn = 0.1$, however, the maximum increase was around 2.0 diameters. For the remaining Kn values, the maximum increase in the thermal entrance length was between 2 diameters and 14.3 diameters. Hence, the presence of a slip wall resulted in a proportional increase in the thermal entrance length as both the Kn and Re values increased.

$$\theta = \frac{T_w - T_{min}}{T_w - T_{in}} \quad \{Fully\ developed\ condition: \theta = 0.8\} \quad (20)$$

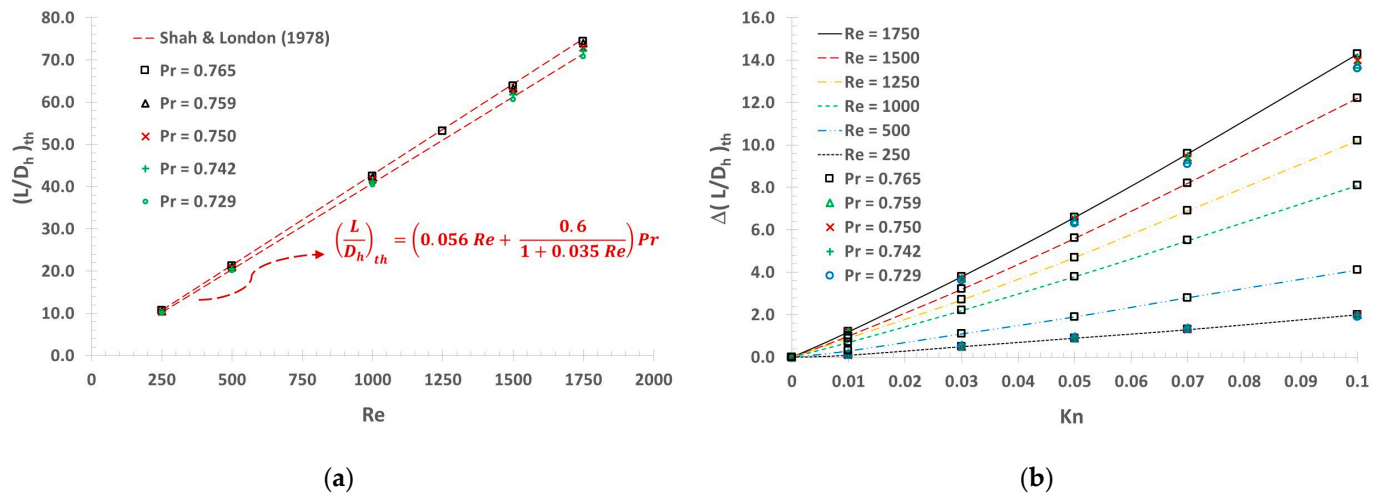


Figure 14. Thermal entrance length $(L/D_h)_{th}$ due to different nanoparticle volume fractions as functions of (a) Re under no-slip conditions at $Kn = 0.0$, compared with Shah and London [40]; (b) slip condition values $\Delta(L/D_h)_{th} = (L/D_h)_{th-slip} - (L/D_h)_{th-no\ slip}$ at $0.01 \leq Kn \leq 0.1$.

The increase in Kn caused the wall-adjacent cell temperature to decrease due to the temperature jump effect, as defined in Equation (13) and shown in Figure 15. At the inlet of the pipe, θ was equal to one and decreased as the nanofluid traveled downstream along the axial direction. Both effects required the nanofluid to travel longer downstream distances to achieve the T_{min} value that satisfied the condition of $(\theta = 0.8)$ for a thermally fully developed flow. With the increase in Re , the amount of nanofluid flowing inside the pipe increased, which caused a decrease in T_{min} and resulted in an increase in the value of $(L/D_h)_{th}$.

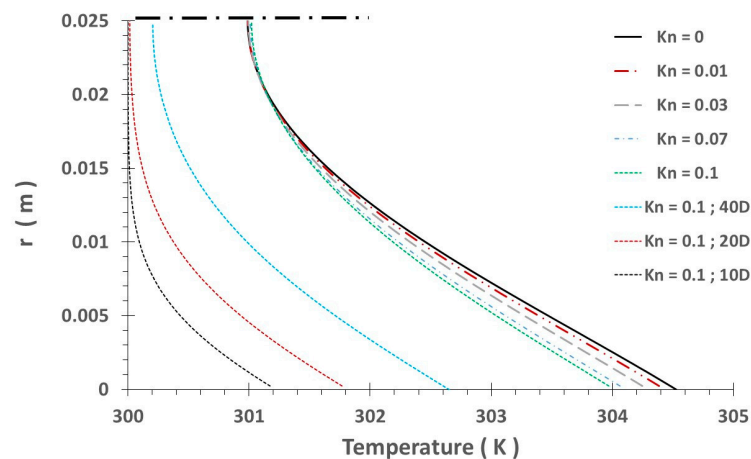


Figure 15. Temperature profiles at different sections, ranging from 10D to the fully developed flow at $Re = 1750$.

The enhancement in heat transfer due to the inclusion of nanoparticles (Al_2O_3) into air was represented by values of the local Nusselt number, as defined in Equation (21). The equation presents how much more heat was transferred from the pipe walls to the nanofluid due to the addition of nanoparticles under slip flow conditions when compared with no-slip conditions. Based on the current CFD simulations, the maximum enhancement due to the addition of nanoparticles was found to be 2.5% at $Kn = 0.1$, $Re = 250$, and $\phi = 10\%$ or $Pr = 0.729$, as shown in Figure 16. The variation in the simulated Pr was around -4.7% , between the maximum volume fraction of nanoparticles within the nanofluid and air. Also, increases in the nanofluid conductivity and viscosity of around 33% and 30%, respectively, were reported. Therefore, the increase in thermal conductivity would enhance

the heat transfer rate, while the increase in viscosity would reduce the rate of heat transfer. Furthermore, with regard to the reduction in nanofluid-specific heat due to the addition of nanoparticles, the convective heat transfer tends to increase and cause an overall heat transfer enhancement. Therefore, there was a compromise in the resulted heat transfer enhancement for the simulated values in favor of a slight enhancement.

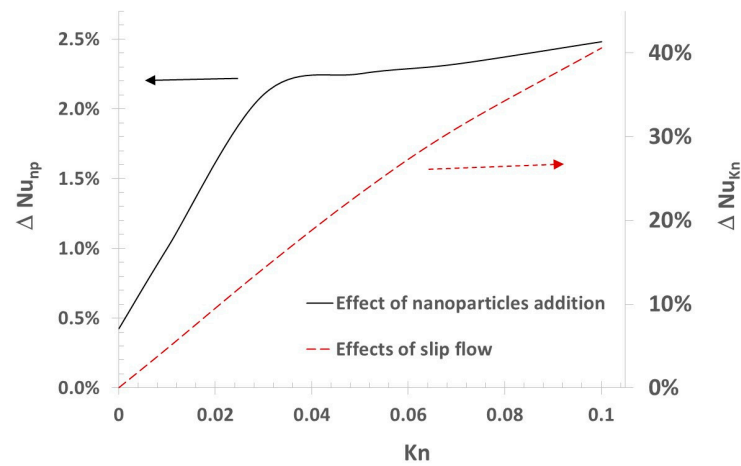


Figure 16. Maximum enhancement due to the addition of nanoparticles (ΔNu_{np}) and maximum reduction due to slip flow (ΔNu_{kn}).

The effects of slip flow conditions on the local Nusselt number were defined in Equation (22). The equation presents how much less heat was transferred from the pipe walls to the nanofluid due to the increase in the slip wall conditions (increase in Kn value) when compared with no-slip conditions. As the Kn value increased, the resulted slip velocity increased, which increased the convective effect of heat transfer. However, higher Kn values caused higher temperature jump values, which resulted in a significant reduction in heat transfer from the pipe walls to the flowing nanofluid. The results showed a relatively linear degradation in the amount of heat transfer as a function of the Kn value in a proportional manner. As a result, the highest reduction due to slip flow conditions was found to be 40% at $Kn = 0.1$. This conclusion highlights the importance of Kn over the volume fraction of nanoparticles for slip flow conditions inside a pipe for air and Al_2O_3 nanoparticles.

$$\Delta Nu_{np} = \frac{Nu_{nanofluid} - Nu_{air}}{Nu_{air}} \quad (21)$$

$$\Delta Nu_{Kn} = \frac{Nu_{no\ slip} - Nu_{slip}}{Nu_{no\ slip}} \quad (22)$$

The comprehensive CFD study conducted has resulted in many data that are suitable for developing a correlation between U/U_m , ReC_f , and Nu_x and the key investigated parameters (x/D_h , Re , Pr , Kn). CurveExpert Professional version 2.7.2. is a reliable software that was used to develop the required nonlinear correlations. The general form of the correlation is defined in Equation (23), and the related constants are listed in Table 3. These correlations were based on the following ranges of the investigated parameters: $0 \leq Kn \leq 0.1$, $250 \leq Re \leq 1750$, $0.729 \leq Pr \leq 0.765$, $0 \leq \phi \leq 0.1$, and $0 \leq x/(D_h Re Pr) \leq 0.0785$. The resulting correlations were limited to the adopted ranges of the steady-state laminar flow inside a circular pipe under constant wall heat flux. Moreover, the developed correlations

were generated to fill the gap in this area and produce a reliable correlation to be used in the field of the slip flow of nanofluid at the entrance region of a circular pipe.

$$y = \frac{a + bKn + c \left[\frac{x}{D_h Re Pr} \right]}{1 + dKn + e \left[\frac{x}{D_h Re Pr} \right]} \quad (23)$$

Table 3. Constants utilized in Equation (23).

y	Nu _{avg}	Nu _x	U/U _m	Re C _f
a	51.852499	69.008891	0.982444	34.438304
b	−231.989750	−333.523624	57.947560	−182.496147
c	1485.741079	9873.998042	212.973373	990.447757
d	89.668426	106.415922	53.569393	7.232419
e	445.780079	2729.351897	101.564399	96.175369
Coefficient of Determination (R ²)	0.93	0.92	0.99	0.87

4. Conclusions

The application of nanofluid is directed toward enhancing heat transfer within engineering systems in an effort to save energy. Improving heat transfer efficiency can make energy more affordable and accessible, particularly in developing regions where energy costs are high relative to income. These efforts could contribute to enhancing the quality of life and compliance with sustainable development goals (SDGs). With this motivation in mind, the effects of Al₂O₃ nanoparticles and rarefied flow at the entrance of a circular pipe on heat transfer under constant wall flux conditions were estimated using the CFD approach. The steady-state, two-dimensional, and single-phase simulations were validated against published research, and the results were satisfying. Based on the comprehensive study, the following conclusions were made:

- The adopted approach of the low-pressure slip flow method for the laminar flow of Ansys-Fluent was proven to be sensitive to the value of q'' at the wall that causes a change in the wall temperature value. The change in λ resulting from the change in the wall temperature changes the value of Kn along the line of the nanofluid flow and becomes variable rather than constant. Therefore, changes in the resulting fluid bulk temperature have to be insignificant for the approach to be valid, and extra care should be taken when adopting this approach in order to avoid producing inaccurate results on the basis of constant Kn values. Consequently, the highest error obtained from current simulations was 4% when the wall heat flux increased from 5 W/m² to 50 W/m²;
- The effects of different Pr values on U/U_m and ReC_f values were negligible. However, the effects of Kn values on the aforementioned parameters were significant;
- The hydrodynamic entrance length (L/D_h) became shorter with the increase in Kn and Re when compared to the no-slip flow case. However, the thermal entrance length (L/D_h)_{th} became longer with the increase in Kn and Re when compared to the no-slip flow case;
- Increasing Re and/or nanoparticle volume fraction values resulted in increased Nu_x values due to rises in nanofluid thermal conductivity values. However, increasing x/D_h values caused the Nu_x values to decrease due to the dissipation effects at the entrance region. Consequently, the effects of slip flow on heat transfer, represented by Nu, in a pipe was adverse and reduced heat transfer from the pipe wall to moving nanofluids by 40% at Kn = 0.1. The maximum heat transfer enhancement due to the addition of nanoparticles was found to be minimal, with a 2.5% increase at $\phi = 0.05$, Kn = 0.1, and Re = 250;
- Correlations between U/U_m, ReC_f, and Nu_x and the key investigated parameters (x/D_h , Re, Pr, Kn) with a high coefficient of determination (R²) were developed to

produce a reliable correlation and to be used in the field of the laminar slip flow of nanofluid inside the entrance region of a circular pipe.

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