

Article

Three-Dimensional Guidance Laws for Spacecraft Propelled by a SWIFT Propulsion System

Alessandro A. Quarta 

Department of Civil and Industrial Engineering, University of Pisa, I-56122 Pisa, Italy;
alessandro.antonio.quarta@unipi.it

Abstract: This paper discusses the optimal control law, in a three-dimensional (3D) heliocentric orbit transfer, of a spacecraft whose primary propulsion system is a Solar Wind Ion Focusing Thruster (SWIFT). A SWIFT is an interesting concept of a propellantless thruster, proposed ten years ago by Gemmer and Mazzoleni, which deflects, collects, and accelerates the charged particles of solar wind to generate thrust in the interplanetary space. To this end, the SWIFT uses a large conical structure made of thin metallic wires, which is positively charged with the aid of an electron gun. In this sense, a SWIFT can be considered as a sort of evolution of the Janhunen's E-Sail, which also uses a (nominally flat) mesh of electrically charged tethers to deflect the solar wind stream. In the recent literature, the optimal performance of a SWIFT-based vehicle has been studied by assuming a coplanar orbit transfer and a two-dimensional scenario. The mathematical model proposed in this paper extends that result by discussing the optimal guidance laws in the general context of a 3D heliocentric transfer. In this regard, a number of different forms of the spacecraft state vectors are considered. The validity of the obtained optimal control law is tested in a simplified Earth–Venus and Earth–Mars transfer by comparing the simulation results with the literature data in terms of minimum flight time.

Keywords: Solar Wind Ion Focusing Thruster; three-dimensional heliocentric transfer; optimal guidance; interplanetary trajectory optimization



Citation: Quarta, A.A. Three-Dimensional Guidance Laws for Spacecraft Propelled by a SWIFT Propulsion System. *Appl. Sci.* **2024**, *14*, 5944. <https://doi.org/10.3390/app14135944>

Academic Editors: Mingzhen Gui, Jin Liu, Chengxi Zhang and Mingzhe Dai

Received: 9 June 2024

Revised: 29 June 2024

Accepted: 3 July 2024

Published: 8 July 2024



Copyright: © 2024 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

This paper analyzes the optimal control law of a spacecraft equipped with a Solar Wind Ion Focusing Thruster (SWIFT) [1,2] in a heliocentric, three-dimensional (3D) orbit transfer. The SWIFT is an interesting propellantless propulsion system concept, which has been proposed as a possible alternative in a typical interplanetary orbit transfer to more conventional propellantless thrusters such as, for example, classical solar sails [3,4] or the more recent E-sail [5,6].

From a conceptual viewpoint, a SWIFT-based spacecraft uses the solar wind stream to produce a net thrust in the interplanetary space, as in the case of the E-sail propulsion system proposed by Dr. Janhunen two decades ago [7,8]. However, different from the more studied E-sail, a SWIFT propulsion system captures the solar wind charged particles through a large conical space structure formed by a mesh of conducting (thin) cables, which are maintained at a high positive electric potential, as sketched in Figure 1. The ion stream is then collected by a suitable spacecraft subsystem and accelerated out of the spacecraft in a manner that resembles the acceleration process of a classical ion electric thruster. Accordingly, a part of the SWIFT-induced thrust vector can be steered within a prescribed range without varying the axis direction of the mesh-formed conical structure. The latter, in fact, nominally remains aligned with the Sun–spacecraft line during the interplanetary flight, as indicated in Figure 1.

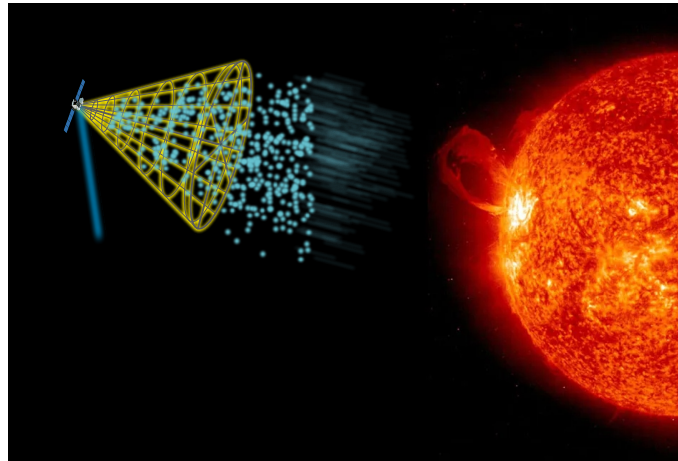


Figure 1. Artistic concept of a spacecraft equipped with a SWIFT. The axis of the conical structure coincides with the Sun–vehicle line during the flight.

This paper considers a 3D heliocentric mission scenario, wherein the mathematical results discussed in this work extend the two-dimensional (2D) model recently proposed by the authors in Ref. [9]. In particular, considering a SWIFT-propelled spacecraft in the context of a 3D heliocentric orbit transfer, the optimal control law is obtained in analytical form by solving an optimization problem in which the performance index is the total flight time. To this end, the problem is analyzed by using the calculus of variations, which allows for obtaining the (optimal) direction of the SWIFT-induced thrust vector as a function of both the propulsion system design characteristics and the components of the costate vector.

The obtained optimal guidance laws are used to simulate the SWIFT-propelled spacecraft's optimal performance in some typical 3D orbit-to-orbit transfers within a heliocentric framework. In this regard, a classic Earth–Venus and Earth–Mars interplanetary transfer is analyzed in detail, with the aim of comparing the results of the literature with those obtained using the mathematical model proposed in this work. Starting from the thrust model used to study a 2D scenario, which is briefly described in Appendix A, the next section discusses a simplified thrust model for the analysis of a general 3D heliocentric transfer. The latter is used in Section 3 to obtain the optimal guidance law with the aid of Pontryagin's maximum principle. In particular, a number of different forms of the spacecraft state vector are considered in the trajectory optimization process, and the validity of the obtained optimal control law is discussed in Section 4 by comparing the outputs of the (3D) numerical simulations with the results of the recent literature [9]. More precisely, Section 4 compares the Earth–Venus and Earth–Mars minimum transfer times obtained in a 3D mission scenario with the results obtained with the 2D simplified approach proposed in Ref. [9].

2. Thrust Vector Model for Trajectory Design in a 3D Mission Scenario

In this section, we discuss a simple thrust vector analytical model for the study and optimization of the SWIFT-based trajectories in a heliocentric 3D mission scenario. The proposed mathematical model will be employed in the next section to obtain the optimal guidance law during a typical interplanetary transfer between two assigned Keplerian orbits, without ephemeris constraints. In particular, the SWIFT-induced thrust vector illustrated in this section originates from the mathematical model, proposed by the authors in Ref. [9], for the more simple case of a 2D heliocentric orbit transfer. Note that the latter mathematical model (i.e., the model for 2D scenarios) has been briefly summarized in Appendix A for the sake of completeness and for convenience, because the approach proposed in this section begins from the expression of the propulsive acceleration vector $\mathbf{a}$ given by Equation (A1). In fact, bearing in mind the model illustrated in Appendix A and

according to Equation (A1), at a given distance r from the Sun, the maximum value of the magnitude of $\mathbf{a}$ is given by

$$\max(\|\mathbf{a}\|) = a_{D_{\oplus}} \left(\frac{r_{\oplus}}{r} \right)^2 (1 + K) \quad (1)$$

where the terms $a_{D_{\oplus}}$, $r_{\oplus}$, and K are defined in Appendix A. In particular, Equation (1) is obtained from Equation (A1) by enforcing the condition $\hat{\mathbf{t}} = \hat{\mathbf{r}}$, that is, by assuming a configuration in which the direction of the steerable part of the SWIFT-induced propulsive acceleration (such that the direction is indicated by the unit vector $\hat{\mathbf{t}}$) coincides with the Sun–spacecraft direction. The latter is indicated by the radial unit vector $\hat{\mathbf{r}}$, as discussed in Appendix A.

Paralleling the terminology used in the literature concerning the trajectory design of spacecraft equipped with photonic solar sails [3,4,10,11], such as the E-Sail [5,6] or Zubrin’s Magnetic Sail [12–14], for a SWIFT-propelled spacecraft, we introduce the concept of characteristic acceleration a_c , that is, the maximum magnitude of $\mathbf{a}$ when the distance of the spacecraft from the Sun is equal to 1 astronomical unit. Bearing in mind that, by definition, $r_{\oplus} = 1$ au, Equation (1) gives the expression of the characteristic acceleration for a SWIFT-propelled vehicle. In particular, a_c is a function of the two propulsion system design parameters $\{a_{D_{\oplus}}, K\}$, and it is written in a compact form as

$$a_c = a_{D_{\oplus}} (1 + K) \quad (2)$$

Accordingly, using the previous equation to express $a_{D_{\oplus}}$ as a function of a_c and K , the vector $\mathbf{a}$ given by Equation (A1) becomes

$$\mathbf{a} = \frac{a_c}{1 + K} \left(\frac{r_{\oplus}}{r} \right)^2 (\hat{\mathbf{r}} + K \hat{\mathbf{t}}) \quad (3)$$

in which, at a given solar distance r , the propulsion system performance parameters are collected in the pair $\{a_c, K\}$, while the control term is given by the unit vector $\hat{\mathbf{t}}$. Therefore, the thrust vector of the SWIFT-based spacecraft has two scalar control parameters, i.e., the two independent (scalar) terms that define the components of the unit vector $\hat{\mathbf{t}}$. In this context, from the point of view of trajectory design and paralleling the classical solar sail-related literature [15–17], it is more convenient to introduce the two typical control angles α and δ . To this end, we introduce a classical, right-handed, Radial–Transverse–Normal (RTN) orbital reference frame [18], in which $\hat{\mathbf{i}}_R$ is the radial unit vector, $\hat{\mathbf{i}}_T$ is the transverse unit vector, and $\hat{\mathbf{i}}_N$ is the normal unit vector; see the scheme of Figure 2, which has been adapted from Ref. [19].

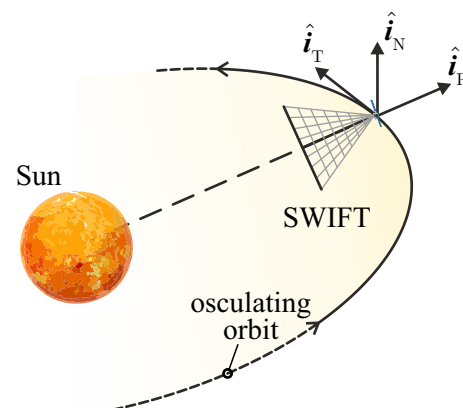


Figure 2. Sketch of the RTN reference frame of unit vectors $\hat{\mathbf{i}}_R$, $\hat{\mathbf{i}}_T$, and $\hat{\mathbf{i}}_N$.

In particular, the unit vectors of the RTN reference frame are defined as

$$\hat{\mathbf{i}}_R \equiv \hat{\mathbf{r}} \quad , \quad \hat{\mathbf{i}}_N = \frac{\mathbf{h}}{\|\mathbf{h}\|} \quad , \quad \hat{\mathbf{i}}_T = \hat{\mathbf{i}}_N \times \hat{\mathbf{i}}_R \quad (4)$$

where $\mathbf{h}$ is the spacecraft specific angular momentum vector. Note that the plane $(\hat{\mathbf{i}}_R, \hat{\mathbf{i}}_T)$ coincides with the plane of the spacecraft osculating orbit, see Figure 2, while the spacecraft inertial velocity $\mathbf{v}$ has a positive component along the direction of the transverse unit vector, that is, $\mathbf{v} \cdot \hat{\mathbf{i}}_T > 0$.

Consider now the scheme of Figure 3 that shows the RTN reference frame and a generic configuration of the unit vector $\hat{\mathbf{t}}$, which indicates the direction of the steerable part of the SWIFT-induced acceleration; see the discussion in Appendix A about the SWIFT thrust model.

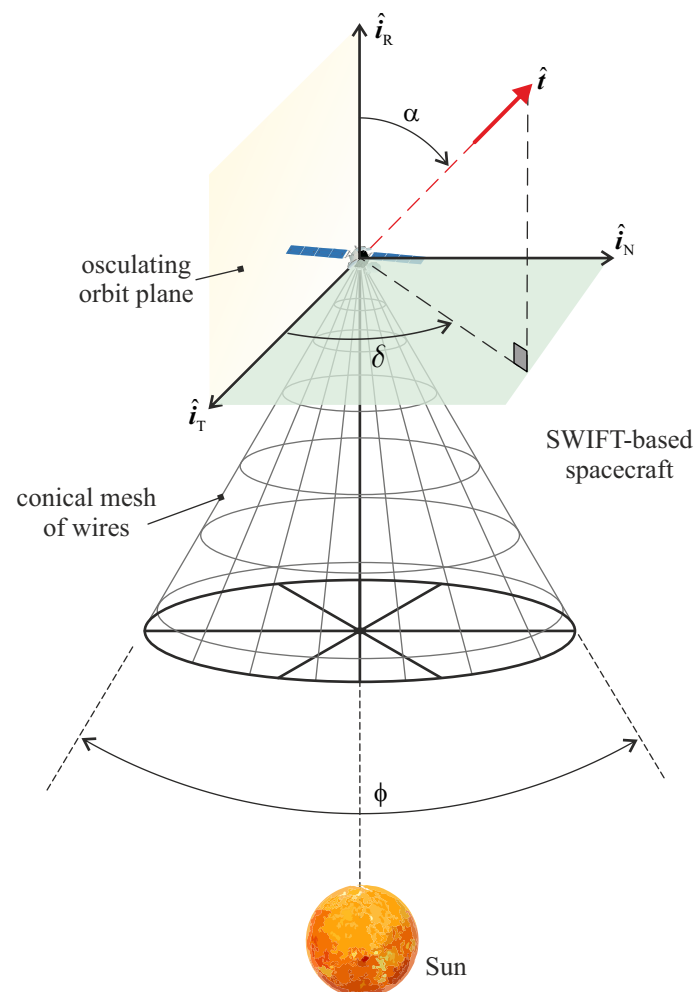


Figure 3. Unit vector $\hat{\mathbf{t}}$ in the RTN reference frame, definition of the SWIFT cone α , and clock δ angle. The scheme also shows the aperture angle ϕ of the conical mesh of conducting wires, which constitutes the large part of the propulsion system structure.

Figure 3 also shows the aperture angle $\phi \in (0, 180)$ deg of the conical mesh of (thin) conducting wires of the SWIFT propulsion system. The latter conical structure should be considered a sort of geometric constraint on the direction of $\hat{\mathbf{t}}$, because the stream of charged particles accelerated out of the spacecraft cannot be directed inside the (conical) mesh of wires. From a mathematical viewpoint, that constraint can be expressed through the following inequality:

$$\hat{\mathbf{t}} \cdot \hat{\mathbf{i}}_R > -\cos(\phi/2) \quad (5)$$

or

$$\alpha \leq \alpha_{\max} \quad \text{with} \quad \alpha_{\max} < 180 \text{ deg} - \phi/2 \quad (6)$$

where ϕ has an assigned value. According to Figure 3, in the RTN reference frame, the unit vector $\hat{\mathbf{t}}$ can be written as a function of the cone angle $\alpha \in [0, \alpha_{\max}]$ and the clock angle $\delta \in [0, 2\pi]$ rad as

$$\hat{\mathbf{t}} = \cos \alpha \hat{\mathbf{i}}_R + \sin \alpha \cos \delta \hat{\mathbf{i}}_T + \sin \alpha \sin \delta \hat{\mathbf{i}}_N \quad (7)$$

Therefore, bearing in mind that $\hat{\mathbf{r}} \equiv \hat{\mathbf{i}}_R$, the vector $\mathbf{a}$ given by Equation (3) can be rewritten in a more convenient form as

$$\mathbf{a} = \frac{a_c}{1+K} \left(\frac{r_{\oplus}}{r} \right)^2 [(1+K \cos \alpha) \hat{\mathbf{i}}_R + K \sin \alpha \cos \delta \hat{\mathbf{i}}_T + K \sin \alpha \sin \delta \hat{\mathbf{i}}_N] \quad (8)$$

Note that, in this case, the two angles α and δ are not defined by considering the direction of the propulsive acceleration (or the total thrust) vector. In fact, in a SWIFT-based scenario, the pair $\{\alpha, \delta\}$ indicates the direction of $\hat{\mathbf{t}}$, that is, the direction of only a part of the SWIFT-induced acceleration, as discussed in Appendix A.

The expression of vector $\mathbf{a}$, as a function of both the two control angles $\{\alpha, \delta\}$ (or, equivalently, the unit vector $\hat{\mathbf{t}}$) and the SWIFT design characteristics $\{a_c, K\}$, will be employed in the next section to determine the 3D optimal guidance law. To this end, a companion (geometric) problem is firstly solved in the last part of this section, because its analytical result allows for the study and the discussion of the optimal control law to be considerably simplified. Such a companion (geometric) problem is illustrated in the next subsection.

Maximization of the Projection of Vector $\mathbf{a}$ along a Given Direction

Consider a given direction in the RTN reference frame, which is indicated by the unit vector $\hat{\mathbf{d}}$; see the scheme of Figure 4. According to that figure, the unit vector $\hat{\mathbf{d}}$ is defined by the two angles $\alpha_d \in [0, \pi]$ rad and $\delta_d \in [0, 2\pi]$ rad, which play the same role as the two control angles $\{\alpha, \delta\}$ in the case of the unit vector $\hat{\mathbf{t}}$; compare Figure 4 with the sketch in Figure 3.

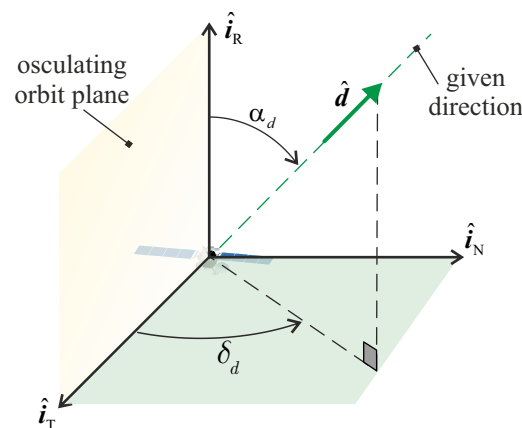


Figure 4. Generic assigned direction in the RTN reference frame. That assigned direction is indicated by the unit vector $\hat{\mathbf{d}}$, which is given by the two angles $\{\alpha_d, \delta_d\}$.

Bearing in mind the form of Equation (7), in the RTN frame, one has

$$\hat{\mathbf{d}} = \cos \alpha_d \hat{\mathbf{i}}_R + \sin \alpha_d \cos \delta_d \hat{\mathbf{i}}_T + \sin \alpha_d \sin \delta_d \hat{\mathbf{i}}_N \quad (9)$$

The problem addressed here is to find the values of the two control angles, α^* and δ^* , that maximize the projection of $\mathbf{a}$ along the (assigned) direction of $\hat{\mathbf{d}}$. This corresponds to maximize the scalar index P defined as the dot product of the two vectors $\mathbf{a}$ and $\hat{\mathbf{d}}$, viz.

$$P \triangleq \mathbf{a} \cdot \hat{\mathbf{d}} \quad (10)$$

In this context, bearing in mind Equations (8) and (9), the scalar index P to be maximized becomes

$$P = \frac{a_c}{1+K} \left(\frac{r_{\oplus}}{r} \right)^2 [(1+K \cos \alpha) \cos \alpha_d + K \sin \alpha \cos \delta \sin \alpha_d \cos \delta_d + K \sin \alpha \sin \delta \sin \alpha_d \sin \delta_d] \quad (11)$$

Accordingly, the previous equation and the necessary condition $\partial P / \partial \delta = 0$ give the value $\delta = \delta^*$ of the clock angle that maximizes the scalar index P as

$$\delta^* = \delta_d \quad (12)$$

which indicates that the three vectors $\hat{\mathbf{i}}_R$, $\hat{\mathbf{d}}$, and $\mathbf{a}$ always belong to the same plane. Bearing in mind that the maximum value of the cone angle α is constrained by $\alpha_{\max}$, Equation (11) and the necessary condition $\partial P / \partial \alpha = 0$ give the value of $\alpha = \alpha^*$ that maximizes P , viz.

$$\alpha^* = \begin{cases} \alpha_d & \text{if } \alpha_d \leq \alpha_{\max} \\ \alpha_{\max} & \text{otherwise} \end{cases} \quad (13)$$

Note that Equations (12) and (13) indicate that the unit vector $\hat{\mathbf{t}}$ coincides with $\hat{\mathbf{d}}$ when $\alpha_d \leq \alpha_{\max}$. Moreover, Equations (12) and (13) can be used to obtain a compact expression of the unit vector $\hat{\mathbf{t}}$ (indicated with the superscript $\star$), which maximizes the scalar index P . In fact, recalling that $\hat{\mathbf{i}}_R \equiv \hat{\mathbf{r}}$, one has

$$\hat{\mathbf{t}}^* = \begin{cases} \hat{\mathbf{d}} & \text{if } \alpha_d \leq \alpha_{\max} \\ \frac{\sin(\alpha_d - \alpha_{\max})}{\sin \alpha_d} \hat{\mathbf{r}} + \frac{\sin \alpha_{\max}}{\sin \alpha_d} \hat{\mathbf{d}} & \text{if } \alpha_d \in (\alpha_{\max}, 180 \text{ deg}) \end{cases} \quad (14)$$

where $\alpha_d \in [0, 180) \text{ deg}$ is given by

$$\alpha_d = \arccos(\hat{\mathbf{d}} \cdot \hat{\mathbf{r}}) \quad (15)$$

In other terms, when $\alpha_d > \alpha_{\max}$, the direction of the unit vector $\hat{\mathbf{d}}$ belongs to a cone of an aperture equal to $2(180 \text{ deg} - \alpha_{\max})$, whose axis coincides with the radial direction. The situation is illustrated in Figure 5, where the red cone indicates the forbidden zone related to the presence of the SWIFT conical mesh of conducting wires. In the special case of $\alpha_d = 180 \text{ deg}$, that is, when the direction of $\hat{\mathbf{d}}$ is purely radial with $\hat{\mathbf{d}} \cdot \hat{\mathbf{i}}_R = -1$, we assume that $\hat{\mathbf{t}}^*$ belongs to the $(\hat{\mathbf{i}}_R, \hat{\mathbf{i}}_T)$ plane; therefore, using Equations (4) and (7), the result is

$$\hat{\mathbf{t}}^* = \cos \alpha_{\max} \hat{\mathbf{r}} + \frac{\sin \alpha_{\max}}{\|\mathbf{h}\|} (\mathbf{h} \times \hat{\mathbf{r}}) \quad \text{if } \alpha_d = 180 \text{ deg} \quad (16)$$

The vector Equation (14) is quite general, and it can be used to obtain the optimal guidance law in a typical heliocentric orbit transfer by using a set of different forms of the spacecraft state vector, as discussed in the next section.

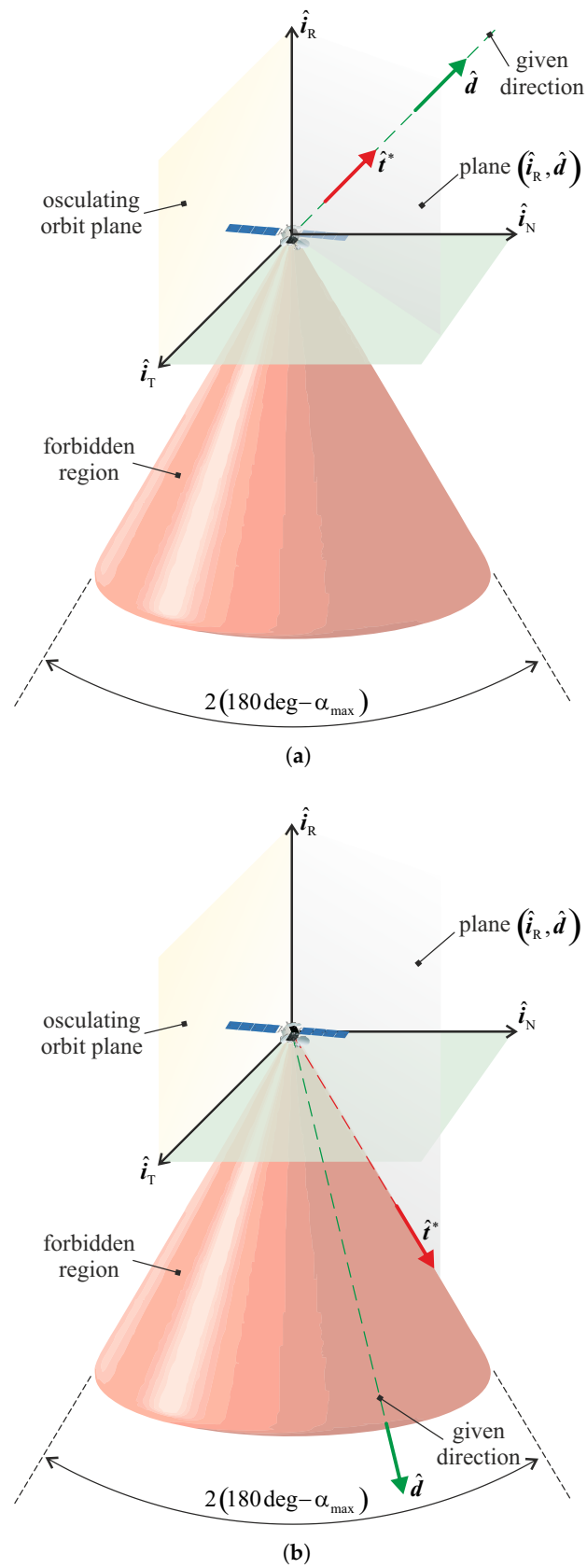


Figure 5. Configuration of the unit vector $\hat{i}^*$ that maximizes the scalar index P defined in Equation (10); see also Equation (14). The red cone indicates the forbidden zone related to the presence of the SWIFT conical mesh of conducting wires. (a) Case of $\alpha_d \leq \alpha_{\max}$. (b) Case of $\alpha_d > \alpha_{\max}$.

3. Optimal Guidance Laws

In this section, the optimal control law is analyzed by considering a heliocentric orbit transfer. In particular, the spacecraft trajectory minimizes the total flight time so that the optimal guidance laws give a rapid transfer between two assigned heliocentric orbits. In this context, the spacecraft position along the initial and the final Keplerian orbit is left free in order to obtain a typical orbit-to-orbit (minimum time) interplanetary transfer. The optimization process is solved using an indirect approach [20–22], in which the optimal control law is obtained with the aid of the classical Pontryagin’s maximum principle [23,24].

More precisely, the optimal control law is studied considering a spacecraft dynamics in terms of (1) the Cartesian position and velocity vector components $\{r_x, r_y, r_z, v_x, v_y, v_z\}$; (2) the state variables $\{r, \theta, \gamma, v_r, v_\theta, v_\gamma\}$ in a heliocentric spherical reference frame; (3) the Walker’s modified equinoctial orbital elements $\{p, f, g, h, k, L\}$ [25]; and (4) the classical elements $\{a, e, i, \Omega, \omega, \nu\}$ [26,27]. In the latter two cases, the modified or the classical orbital elements define the characteristics of the spacecraft osculating orbit during the interplanetary transfer between the two Keplerian orbits.

3.1. Spacecraft State in Terms of Cartesian Components of Position and Velocity Vector

Consider the typical case in which the spacecraft state is defined in terms of the three scalar components of the position vector $\mathbf{r} = [r_x, r_y, r_z]^T$ and the three scalar components of the velocity vector $\mathbf{v} = [v_x, v_y, v_z]^T$ in a Cartesian, heliocentric ecliptic, 3D, right-handed, reference frame $\mathcal{T}_C(O; x, y, z)$. The latter reference frame, which is accurately described in Section 2.2.1 of Ref. [26], is sketched in Figure 6, where the origin O coincides with the Sun’s center of mass.

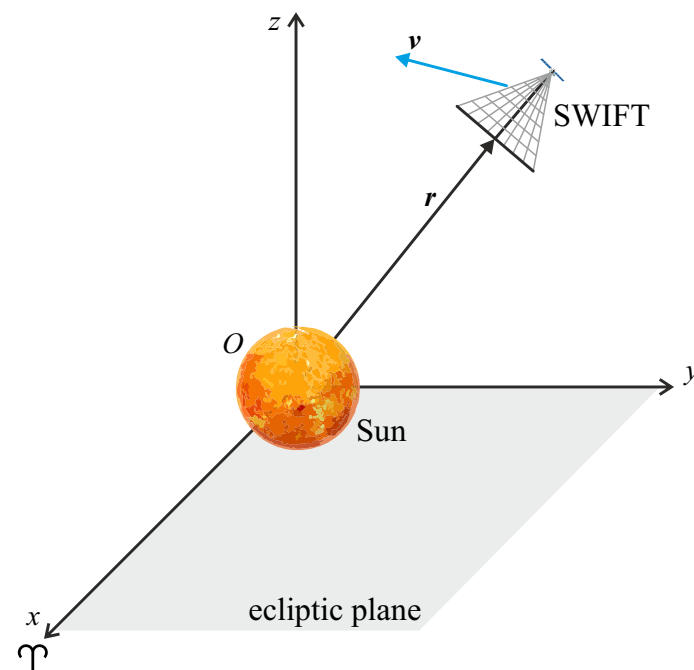


Figure 6. SWIFT-propelled spacecraft in a Cartesian, heliocentric ecliptic, reference frame $\mathcal{T}_C(O; x, y, z)$.

In this case, the vector form of the SWIFT-propelled spacecraft equations of motion (EoMs) is given by

$$\dot{\mathbf{r}} = \mathbf{v} \quad (17)$$

$$\dot{\mathbf{v}} = -\left(\frac{\mu_\odot}{r^3}\right)\mathbf{r} + \mathbf{a} \quad (18)$$

where $\mu_{\odot}$ is the gravitational parameter of the Sun, and $\mathbf{a}$ is the propulsive acceleration vector given by Equation (3) as a function of both the propulsion system design characteristics $\{a_c, K\}$ and the control (vector) term $\hat{\mathbf{t}}$.

Introducing the vector adjoint to the position $\lambda_r \in \mathbb{R}^3$ and the vector adjoint to the velocity $\lambda_v \in \mathbb{R}^3$, the Hamiltonian function $\mathcal{H} \in \mathbb{R}$ is

$$\mathcal{H} = \lambda_r \cdot \mathbf{v} - \left(\frac{\mu_{\odot}}{r^3} \right) \lambda_v \cdot \mathbf{v} + \lambda_v \cdot \mathbf{a} \quad (19)$$

Taking the SWIFT thrust vector model into account and rewriting the Lawden's primer vector λ_v [28–31] as

$$\lambda_v = \lambda_v \hat{\lambda}_v \quad (20)$$

with $\lambda_v \triangleq \|\lambda_v\| \neq 0$ and $\hat{\lambda}_v \triangleq \lambda_v / \lambda_v$, it is possible to enucleate the part of $\mathcal{H}$ which explicitly depends on $\mathbf{a}$, i.e., on the control term. In fact, substituting Equation (20) into Equation (19), one obtains the scalar function $\mathcal{H}_c$, which coincides with the part of $\mathcal{H}$ that depends on vector $\mathbf{a}$, as

$$\mathcal{H}_c = \mathbf{a} \cdot \hat{\lambda}_v \quad (21)$$

According to Refs. [23,24], the control term $\hat{\mathbf{t}}$ is selected to maximize, at any time instant, the value of $\mathcal{H}$. Bearing in mind that $\hat{\mathbf{t}}$ appears in the expression of $\mathbf{a}$ given by Equation (3), the Pontryagin's maximum principle is more conveniently applied to the (reduced) function $\mathcal{H}_c$ defined in Equation (21). In this case, comparing Equation (21) with (10), one concludes that $\mathcal{H}_c$ substantially coincides with P if the unit vector $\hat{\mathbf{d}}$ is formally substituted by the unit vector $\hat{\lambda}_v$. Therefore, the optimal control law, i.e., the value of $\hat{\mathbf{t}}$ that maximizes $\mathcal{H}$ (or, equivalently, $\mathcal{H}_c$), is directly obtained from Equations (14) and (15) as a function of the adjoint unit vector $\hat{\lambda}_v$ as

$$\hat{\mathbf{t}}^* = \begin{cases} \hat{\lambda}_v & \text{if } \alpha_{\lambda_v} \leq \alpha_{\max} \\ \frac{\sin(\alpha_{\lambda_v} - \alpha_{\max})}{\sin \alpha_{\lambda_v}} \hat{\mathbf{r}} + \frac{\sin \alpha_{\max}}{\sin \alpha_{\lambda_v}} \hat{\lambda}_v & \text{if } \alpha_{\lambda_v} \in (\alpha_{\max}, 180 \text{ deg}) \end{cases} \quad (22)$$

where $\alpha_{\lambda_v} \in [0, 180) \text{ deg}$ is an auxiliary angle given by

$$\alpha_{\lambda_v} = \arccos(\hat{\lambda}_v \cdot \hat{\mathbf{r}}) \quad (23)$$

In particular, the time variation of the adjoint vectors λ_r and λ_v (and, thus, the time variation of $\hat{\lambda}_v$, which gives the optimal control law) is obtained through the Euler–Lagrange equations

$$\dot{\lambda}_r = -\frac{\partial \mathcal{H}}{\partial \mathbf{r}}, \quad \dot{\lambda}_v = -\frac{\partial \mathcal{H}}{\partial \mathbf{v}} \quad (24)$$

whose analytical expressions can be easily obtained using the approach described in Ref. [32]. The procedure illustrated in this scenario, where the spacecraft dynamics is studied in a Cartesian heliocentric ecliptic reference system, will be replicated in the next three subsections, where the vehicle dynamics will be described using different forms of the state vector.

3.2. Spacecraft Dynamics in a Heliocentric Spherical Reference Frame

Introduce a heliocentric spherical reference frame $\mathcal{T}_S(O; r, \theta, \gamma)$, illustrated in Figure 7, where θ is the azimuthal angle measured (counterclockwise) into the ecliptic plane from the direction of the x axis of the Cartesian reference frame $\mathcal{T}_C$, and γ is the elevation angle, which gives the spacecraft angular position with respect to the ecliptic plane (x, y) . The unit vectors of the spherical reference frame $\mathcal{T}_S$ are $\hat{\mathbf{i}}_r$, $\hat{\mathbf{i}}_{\theta}$, and $\hat{\mathbf{i}}_{\gamma}$, that is, the radial unit vector, the transverse (or azimuthal) unit vector, and the normal unit vector, respectively. In particular, $\hat{\mathbf{i}}_{\gamma}$ points towards the north celestial pole, the direction of $\hat{\mathbf{i}}_r$ is radial (i.e., $\hat{\mathbf{i}}_r \equiv \hat{\mathbf{r}}$), and the plane $(\hat{\mathbf{i}}_{\theta}, \hat{\mathbf{i}}_{\gamma})$ coincides with the local horizontal plane.

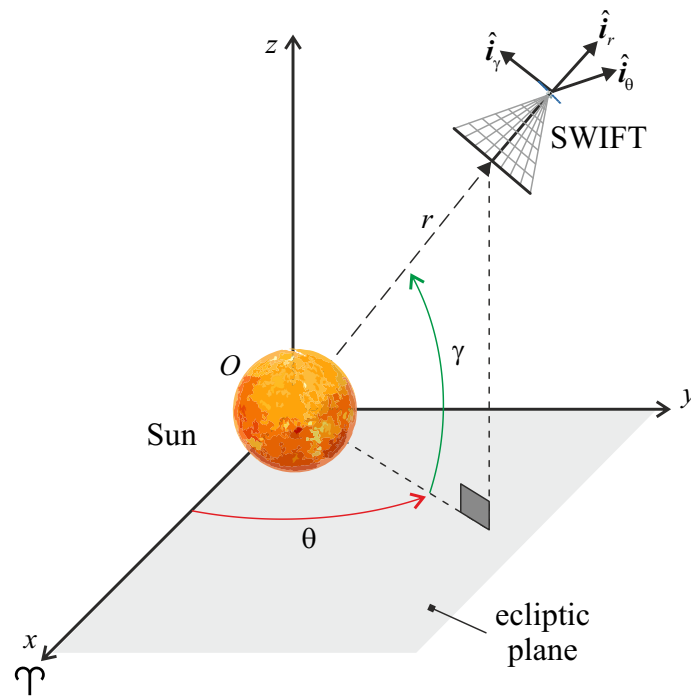


Figure 7. SWIFT-propelled spacecraft in a heliocentric spherical reference frame $\mathcal{T}_S(O; r, \theta, \gamma)$.

In the spherical reference frame $\mathcal{T}_S(O; r, \theta, \gamma)$, the components of the spacecraft state vector are $\{r, \theta, \gamma, v_r, v_\theta, v_\gamma\}$, where $\{v_r, v_\theta, v_\gamma\}$ are the three components of the spacecraft inertial velocity, i.e., $\mathbf{v} = v_r \hat{\mathbf{i}}_r + v_\theta \hat{\mathbf{i}}_\theta + v_\gamma \hat{\mathbf{i}}_\gamma$, while the position vector is simply given by $\mathbf{r} = r \hat{\mathbf{i}}_r$. Using the procedure detailed in Ref. [33], the six scalar EoMs of the SWIFT-propelled spacecraft are

$$\begin{aligned} \dot{r} &= v_r, \quad \dot{\theta} = \frac{v_\theta}{r \cos \gamma}, \quad \dot{\gamma} = \frac{v_\gamma}{r}, \quad \dot{v}_r = \frac{v_\theta^2 + v_\gamma^2}{r} - \frac{\mu_\odot}{r^2} + \mathbf{a} \cdot \hat{\mathbf{i}}_r, \\ \dot{v}_\theta &= \frac{v_\theta v_\gamma \tan \gamma - v_r v_\theta}{r} + \mathbf{a} \cdot \hat{\mathbf{i}}_\theta, \quad \dot{v}_\gamma = -\frac{v_\theta^2 \tan \gamma + v_r v_\gamma}{r} + \mathbf{a} \cdot \hat{\mathbf{i}}_\gamma \end{aligned} \quad (25)$$

where the vector $\mathbf{a}$ is given by Equation (3). Introducing the six adjoint variables $\{\lambda_r, \lambda_\theta, \lambda_\gamma, \lambda_{v_r}, \lambda_{v_\theta}, \lambda_{v_\gamma}\}$ in this case, the expression of the scalar Hamiltonian function [34] is

$$\begin{aligned} \mathcal{H} &= \lambda_r v_r + \frac{\lambda_\theta v_\theta}{r \cos \gamma} + \frac{\lambda_\gamma v_\gamma}{r} + \lambda_{v_r} \left(\frac{v_\theta^2 + v_\gamma^2}{r} - \frac{\mu_\odot}{r^2} + \mathbf{a} \cdot \hat{\mathbf{i}}_r \right) + \\ &+ \lambda_{v_\theta} \left(\frac{v_\theta v_\gamma \tan \gamma - v_r v_\theta}{r} + \mathbf{a} \cdot \hat{\mathbf{i}}_\theta \right) + \lambda_{v_\gamma} \left(-\frac{v_\theta^2 \tan \gamma + v_r v_\gamma}{r} + \mathbf{a} \cdot \hat{\mathbf{i}}_\gamma \right) \end{aligned} \quad (26)$$

while the Euler–Lagrange equations, which give the time variation of the generic adjoint variable, are obtained through the following relationships:

$$\begin{aligned} \dot{\lambda}_r &= -\frac{\partial \mathcal{H}}{\partial r}, \quad \dot{\lambda}_\theta = -\frac{\partial \mathcal{H}}{\partial \theta}, \quad \dot{\lambda}_\gamma = -\frac{\partial \mathcal{H}}{\partial \gamma}, \\ \dot{\lambda}_{v_r} &= -\frac{\partial \mathcal{H}}{\partial v_r}, \quad \dot{\lambda}_{v_\theta} = -\frac{\partial \mathcal{H}}{\partial v_\theta}, \quad \dot{\lambda}_{v_\gamma} = -\frac{\partial \mathcal{H}}{\partial v_\gamma}, \end{aligned} \quad (27)$$

Bearing in mind Equations (3) and (26) and introducing the auxiliary vector $\lambda_S \in \mathbb{R}^3$ defined as

$$\lambda_S \triangleq [\lambda_{v_r}, \lambda_{v_\theta}, \lambda_{v_\gamma}]^T \quad (28)$$

with

$$\lambda_S \triangleq \sqrt{\lambda_{v_r}^2 + \lambda_{v_\theta}^2 + \lambda_{v_\gamma}^2} \quad \text{and} \quad \hat{\lambda}_S \triangleq \frac{\lambda_S}{\lambda_S} \quad (29)$$

one obtains that the function $\mathcal{H}_c$ can be written as

$$\mathcal{H}_c = \mathbf{a} \cdot \hat{\lambda}_S \quad (30)$$

which is substantially coincident with Equation (21) with the formal substitution $\hat{\lambda}_S \rightarrow \hat{\lambda}_v$. Therefore, according to Equations (22) and (23), the optimal guidance law is given by

$$\hat{\mathbf{t}}^* = \begin{cases} \hat{\lambda}_S & \text{if } \alpha_{\lambda_S} \leq \alpha_{\max} \\ \frac{\sin(\alpha_{\lambda_S} - \alpha_{\max})}{\sin \alpha_{\lambda_S}} \hat{\mathbf{r}} + \frac{\sin \alpha_{\max}}{\sin \alpha_{\lambda_S}} \hat{\lambda}_S & \text{if } \alpha_{\lambda_S} \in (\alpha_{\max}, 180 \text{ deg}) \end{cases} \quad (31)$$

where $\alpha_{\lambda_S} \in [0, 180) \text{ deg}$ is another auxiliary angle defined as

$$\alpha_{\lambda_S} = \arccos(\hat{\lambda}_S \cdot \hat{\mathbf{r}}) \quad (32)$$

3.3. Spacecraft Dynamics in Terms of Variation of the Walker's Modified Equinoctial Orbital Elements

The use of nonsingular orbital elements [35] is a common choice in the design and optimization of the spacecraft transfer trajectory both in a heliocentric and in a planetocentric scenario [36,37]. In this context, we describe the spacecraft dynamics in terms of the six equinoctial elements $\{p, f, g, h, k, L\}$ proposed by Walker et al. [25]. In particular, the geometric interpretation of the Walker's elements, together with the calculation of both the classical orbital elements and the Cartesian state vector components, are summarized in the last part of Ref. [38]. From the spacecraft dynamics point of view and using the nomenclature described in a recent author's work [39], the EoMs in the heliocentric mission scenario are given by the following (vector) nonlinear differential equation:

$$[\dot{p}, \dot{f}, \dot{g}, \dot{h}, \dot{k}, \dot{L}]^T = \mathbb{M} \mathbf{a} + \left[0, 0, 0, 0, 0, \sqrt{\mu_\odot} p \left(\frac{1 + f \cos L + g \sin L}{p} \right)^2 \right]^T \quad (33)$$

where the components of the vector $\mathbf{a}$ are expressed in the RTN reference frame illustrated in Section 2, that is, $\mathbf{a} = [a_R, a_T, a_N]^T$, while $\mathbb{M} \in \mathbb{R}^{6 \times 3}$ is a matrix defined as [40]

$$\mathbb{M} \triangleq \begin{bmatrix} 0 & \frac{2p}{1+f \cos L + g \sin L} \sqrt{\frac{p}{\mu_\odot}} & 0 \\ \sin L \sqrt{\frac{p}{\mu_\odot}} & \frac{(2+f \cos L + g \sin L) \cos L + f}{1+f \cos L + g \sin L} \sqrt{\frac{p}{\mu_\odot}} & -\frac{g(h \sin L - k \cos L)}{1+f \cos L + g \sin L} \sqrt{\frac{p}{\mu_\odot}} \\ -\cos L \sqrt{\frac{p}{\mu_\odot}} & \frac{(2+f \cos L + g \sin L) \sin L + g}{1+f \cos L + g \sin L} \sqrt{\frac{p}{\mu_\odot}} & \frac{f(h \sin L - k \cos L)}{1+f \cos L + g \sin L} \sqrt{\frac{p}{\mu_\odot}} \\ 0 & 0 & \frac{(1+h^2+k^2) \cos L}{2(1+f \cos L + g \sin L)} \sqrt{\frac{p}{\mu_\odot}} \\ 0 & 0 & \frac{(1+h^2+k^2) \sin L}{2(1+f \cos L + g \sin L)} \sqrt{\frac{p}{\mu_\odot}} \\ 0 & 0 & \frac{h \sin L - k \cos L}{1+f \cos L + g \sin L} \sqrt{\frac{p}{\mu_\odot}} \end{bmatrix} \quad (34)$$

In this scenario, the (scalar) Hamiltonian function $\mathcal{H}$ can be easily expressed by introducing the six costates $\{\lambda_p, \lambda_f, \lambda_g, \lambda_h, \lambda_k, \lambda_L\}$ so that, taking Equations (33) and (34) into account, one has

$$\mathcal{H} = \begin{bmatrix} \lambda_p & \lambda_f & \lambda_g & \lambda_h & \lambda_k & \lambda_L \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{f} \\ \dot{g} \\ \dot{h} \\ \dot{k} \\ \dot{L} \end{bmatrix} \equiv \begin{bmatrix} \lambda_p & \lambda_f & \lambda_g & \lambda_h & \lambda_k & \lambda_L \end{bmatrix} (\mathbb{M} \mathbf{a}) + \lambda_L \sqrt{\mu_\odot} p \left(\frac{1 + f \cos L + g \sin L}{p} \right)^2 \quad (35)$$

According to the previous equation, the part $\mathcal{H}_c$ of the Hamiltonian function can be obtained by introducing the auxiliary vector $\lambda_W \in \mathbb{R}^3$ defined as

$$\lambda_W \triangleq \mathbb{M}^T \begin{bmatrix} \lambda_p \\ \lambda_f \\ \lambda_g \\ \lambda_h \\ \lambda_k \\ \lambda_L \end{bmatrix} \quad (36)$$

with

$$\lambda_W \triangleq \|\lambda_W\| \quad \text{and} \quad \hat{\lambda}_W = \frac{\lambda_W}{\lambda_W} \quad (37)$$

In particular, the expressions of the components of λ_W in the RTN reference frame are

$$\lambda_W = \begin{bmatrix} M_{21} \lambda_f + M_{31} \lambda_g \\ M_{22} \lambda_f + M_{32} \lambda_g + M_{12} \lambda_p \\ M_{63} \lambda_L + M_{23} \lambda_f + M_{33} \lambda_g + M_{43} \lambda_h + M_{53} \lambda_k \end{bmatrix} \quad (38)$$

where M_{ij} is the entry of matrix $\mathbb{M}$ at the i th row and j th column. Therefore, using Equation (35), one obtains the part of the reduced Hamiltonian function $\mathcal{H}_c$ as

$$\mathcal{H}_c = \mathbf{a} \cdot \hat{\lambda}_W \quad (39)$$

whose right side is (again) of the same form of the right side of Equation (21) when $\hat{\lambda}_v \rightarrow \hat{\lambda}_W$. Accordingly, upon also examining Equations (22) and (23), the optimal control law is

$$\hat{\mathbf{t}}^* = \begin{cases} \hat{\lambda}_W & \text{if } \alpha_{\lambda_W} \leq \alpha_{\max} \\ \frac{\sin(\alpha_{\lambda_W} - \alpha_{\max})}{\sin \alpha_{\lambda_W}} \hat{\mathbf{r}} + \frac{\sin \alpha_{\max}}{\sin \alpha_{\lambda_W}} \hat{\lambda}_W & \text{if } \alpha_{\lambda_W} \in (\alpha_{\max}, 180 \text{ deg}) \end{cases} \quad (40)$$

where $\alpha_{\lambda_W} \in [0, 180) \text{ deg}$ is now defined as

$$\alpha_{\lambda_W} = \arccos(\hat{\lambda}_W \cdot \hat{\mathbf{r}}) \quad (41)$$

3.4. Spacecraft Dynamics in Terms of Classical Orbital Elements

In this last part of Section 3, we analyze the optimal control law in the typical case in which the spacecraft dynamics are studied by using the classical orbital elements. The latter set, in fact, describes the characteristics of the spacecraft osculating orbit during the interplanetary transfer in terms of the semimajor axis a , eccentricity e , inclination i with respect to the ecliptic plane, longitude of the ascending node Ω , argument of perihelion

ω , and true anomaly ν , which gives the spacecraft its angular position with respect to the osculating orbit apse line.

Paralleling the procedure described in the previous section, we begin the discussion of the optimal control law by writing the time derivative of the spacecraft state variables $\{a, e, i, \Omega, \omega, \nu\}$. In this context, according to Ref. [41] (see Equation (10.41) at p. 488 of [41]), one has

$$\dot{a} = \frac{2a^2}{h} \left(e a_R \sin \nu + \frac{p}{r} a_T \right) \quad (42)$$

$$\dot{e} = \frac{1}{h} \{ p a_R \sin \nu + [(p+r) \cos \nu + r e] a_T \} \quad (43)$$

$$\dot{i} = \frac{r \cos(\omega + \nu)}{h} a_N \quad (44)$$

$$\dot{\Omega} = \frac{r \sin(\omega + \nu)}{h \sin i} a_N \quad (45)$$

$$\dot{\omega} = \frac{1}{h e} [-p a_R \cos \nu + (p+r) a_T \sin \nu] - \frac{r \sin(\omega + \nu) \cos i}{h \sin i} a_N \quad (46)$$

$$\dot{\nu} = \frac{h}{r^2} + \frac{1}{e h} [p a_R \cos \nu - (p+r) a_T \sin \nu] \quad (47)$$

where $\{a_R, a_T, a_N\}$ are the components of $\mathbf{a}$ in the RTN reference frame, $h = \sqrt{\mu_\odot p}$ is the magnitude of the angular momentum vector with $p = a(1 - e^2)$, while the radial distance r can be expressed through the usual (polar) equation as $r = p/(1 + e \cos \nu)$. The time derivatives of $\{a, e, i, \Omega, \omega, \nu\}$ given by Equations (42)–(47) are rewritten, in a more convenient compact form, as

$$[\dot{a}, \dot{e}, \dot{i}, \dot{\Omega}, \dot{\omega}, \dot{\nu}]^T = \mathbb{N} \mathbf{a} + \left[0, 0, 0, 0, 0, \frac{h}{r^2} \right]^T \quad (48)$$

where $\mathbb{N} \in \mathbb{R}^{6 \times 3}$ is a matrix defined as

$$\mathbb{N} = \begin{bmatrix} \frac{2a^2 e \sin \nu}{h} & \frac{2a^2 p}{r h} & 0 \\ \frac{p \sin \nu}{h} & \frac{(p+r) \cos \nu + r e}{h} & 0 \\ 0 & 0 & \frac{r \cos(\omega + \nu)}{h} \\ 0 & 0 & \frac{r \sin(\omega + \nu)}{h \sin i} \\ \frac{-p \cos \nu}{h e} & \frac{(p+r) \sin \nu}{h e} & -\frac{r \sin(\omega + \nu) \cos i}{h \sin i} \\ \frac{p \cos \nu}{e h} & \frac{-(p+r) \sin \nu}{e h} & 0 \end{bmatrix} \quad (49)$$

Introducing the adjoint variables $\{\lambda_a, \lambda_e, \lambda_i, \lambda_\Omega, \lambda_\omega, \lambda_\nu\} \in \mathbb{R}$, the scalar Hamiltonian function in this case is written as

$$\mathcal{H} = [\lambda_a, \lambda_e, \lambda_i, \lambda_\Omega, \lambda_\omega, \lambda_\nu] (\mathbb{N} \mathbf{a}) + \frac{\lambda_\nu h}{r^2} \quad (50)$$

so that the Euler–Lagrange equations are

$$\dot{\lambda}_a = -\frac{\partial \mathcal{H}}{\partial a}, \quad \dot{\lambda}_e = -\frac{\partial \mathcal{H}}{\partial e}, \quad \dot{\lambda}_i = -\frac{\partial \mathcal{H}}{\partial i}, \quad \dot{\lambda}_\Omega = -\frac{\partial \mathcal{H}}{\partial \Omega}, \quad \dot{\lambda}_\omega = -\frac{\partial \mathcal{H}}{\partial \omega}, \quad \dot{\lambda}_v = -\frac{\partial \mathcal{H}}{\partial v} \quad (51)$$

Therefore, as discussed in the previous section, one can define an auxiliary vector $\lambda_o \in \mathbb{R}^3$ as

$$\lambda_o \triangleq \mathbb{N}^T \begin{bmatrix} \lambda_a \\ \lambda_e \\ \lambda_i \\ \lambda_\omega \\ \lambda_\Omega \\ \lambda_v \end{bmatrix} \equiv \begin{bmatrix} N_{11}\lambda_a + N_{21}\lambda_e + N_{51}\lambda_\omega + N_{61}\lambda_v \\ N_{12}\lambda_a + N_{22}\lambda_e + N_{52}\lambda_\omega + N_{62}\lambda_v \\ N_{43}\lambda_\Omega + N_{33}\lambda_i + N_{53}\lambda_\omega + N_{63}\lambda_v \end{bmatrix} \quad (52)$$

where N_{ij} is the entry of matrix $\mathbb{N}$ at the i th row and j th column, with

$$\lambda_o \triangleq \|\lambda_o\| \quad \text{and} \quad \hat{\lambda}_o = \frac{\lambda_o}{\lambda_o} \quad (53)$$

Using Equations (50) and (52), the function $\mathcal{H}_c$ is

$$\mathcal{H}_c = a \cdot \hat{\lambda}_o \quad (54)$$

so that, according again to Equations (21)–(23), the optimal guidance law is

$$\hat{\mathbf{t}}^* = \begin{cases} \hat{\lambda}_o & \text{if } \alpha_{\lambda_o} \leq \alpha_{\max} \\ \frac{\sin(\alpha_{\lambda_o} - \alpha_{\max})}{\sin \alpha_{\lambda_o}} \hat{\mathbf{r}} + \frac{\sin \alpha_{\max}}{\sin \alpha_{\lambda_o}} \hat{\lambda}_o & \text{if } \alpha_{\lambda_o} \in (\alpha_{\max}, 180 \text{ deg}) \end{cases} \quad (55)$$

where $\alpha_{\lambda_o} \in [0, 180) \text{ deg}$ is given by

$$\alpha_{\lambda_o} = \arccos(\hat{\lambda}_o \cdot \hat{\mathbf{r}}) \quad (56)$$

4. Potential Transfer Trajectories and Case Study

The optimal control law described in the previous section is now employed to analyze the minimum time transfer of a SWIFT-propelled spacecraft in two typical, heliocentric, 3D mission scenarios. In this regard, and with the aim also to validate the mathematical model discussed in this paper, we consider the propulsion system characteristics and the two interplanetary mission scenarios analyzed in Ref. [9]. More precisely, Section 4 of Ref. [9] reports the results of a 2D optimization problem in which a spacecraft equipped with a SWIFT propulsion system of performance parameters $a_{D\oplus} = 0.035 \text{ mm/s}^2$ and $K = 1$ [so that the characteristic acceleration is $a_c = 0.07 \text{ mm/s}^2$; see Equation (2)], moves from a circular (ecliptic) orbit of radius $r = 1 \text{ au}$ to a target, coplanar, circular orbit of radius $r \in \{0.723, 1.524\} \text{ au}$. In particular, the scenario in which the radius of the target circular orbit is 0.723 au (or 1.524 au) is consistent with a simplified, 2D Earth–Venus (or Earth–Mars) interplanetary transfer in which both the eccentricity and the inclination of the planetary orbits are neglected. In those mission scenarios, the 2D analysis discussed in Ref. [9] gives a minimum value of the (circle-to-circle) flight time of roughly 3.62 years for the Earth–Venus mission and 8.09 years for the Earth–Mars mission.

In this section, instead, we consider the actual 3D shape of the heliocentric orbits of the three planets involved in the transfer by using the ephemerides data obtained on July 1st 2024 obtained by the well known JPL Horizon system. In particular, the characteristics of the planet orbits used in the trajectory optimization process are summarized in terms of classical orbital elements $\{a, e, i, \Omega, \omega\}$ in Table 1. Recall that the angular position of the spacecraft along the two Keplerian orbits, both at the beginning and the end of the transfer, is left free.

Table 1. Classical orbital elements, which give the characteristics of the Keplerian orbits of Earth, Mars, and Venus, used in the numerical simulations. Data obtained by the JPL Horizon system on 1 July 2024.

Orbital Parameter	Earth	Venus	Mars
semimajor axis a [au]	1.0004	0.7233	1.5236
eccentricity e	1.6335×10^{-2}	6.73308×10^{-3}	9.3289×10^{-2}
inclination i [deg]	4.1705×10^{-3}	3.3944	1.8478
long. of the asc. node Ω [deg]	146.5136	76.6118	49.4893
arg. of perihelion ω [deg]	317.9011	55.1947	286.6906

The boundary value problem connected to the optimization process is solved using a numerical method based on a classical shooting procedure [42] with a tolerance of 10^{-8} . The initial guess of the boundary value problem solution is obtained by adapting the semianalytical procedure described in Refs. [43,44]. Finally, the differential equations, that is, the EoMs and the Euler–Lagrange equations, are integrated using the Adams–Bashforth methods [45] with a relative tolerance of 10^{-10} .

In the case of an Earth–Venus orbit-to-orbit 3D transfer, the solution of the optimization problem gives a minimum flight time of about 3.93 years (+8.5% with respect to the value obtained in a rapid 2D circle-to-circle transfer [9]). The time variation of the components of the control unit vector $\hat{t}$ in the RTN reference frame and the optimal transfer trajectory in the Cartesian heliocentric ecliptic reference frame are reported in Figure 8. In particular, the scale of the z axis in Figure 8b is different from that of the other two axes to highlight the three-dimensionality of the transfer.

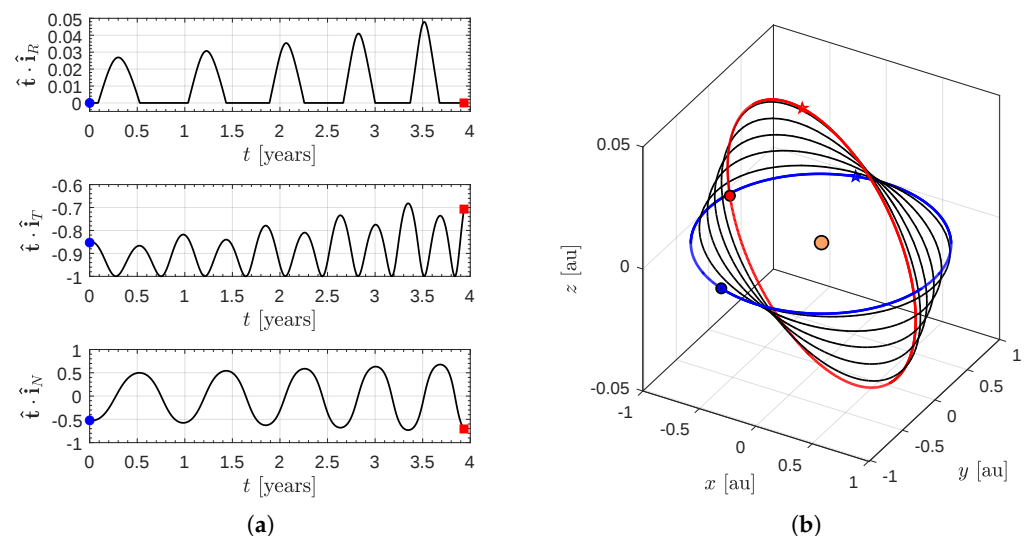


Figure 8. Results of the optimization process in a 3D Earth–Venus mission scenario when $a_c = 0.07 \text{ mm/s}^2$ and $K = 1$. (a) Time variation of the components of $\hat{t}$ in the RTN reference frame. (b) Optimal transfer trajectory (black line). Blue line $\rightarrow$ is Earth's orbit; red line $\rightarrow$ is Venus' orbit; blue circle $\rightarrow$ is start point; red circle $\rightarrow$ is arrival point; blue star $\rightarrow$ is Earth's perihelion; red star $\rightarrow$ is Venus' perihelion; orange circle $\rightarrow$ is the Sun.

Finally, in the Earth–Mars case, the flight time is 8.76 years (+8.3% with respect to the 2D case [9]), while the optimal guidance law and the transfer orbit are shown in Figure 9.

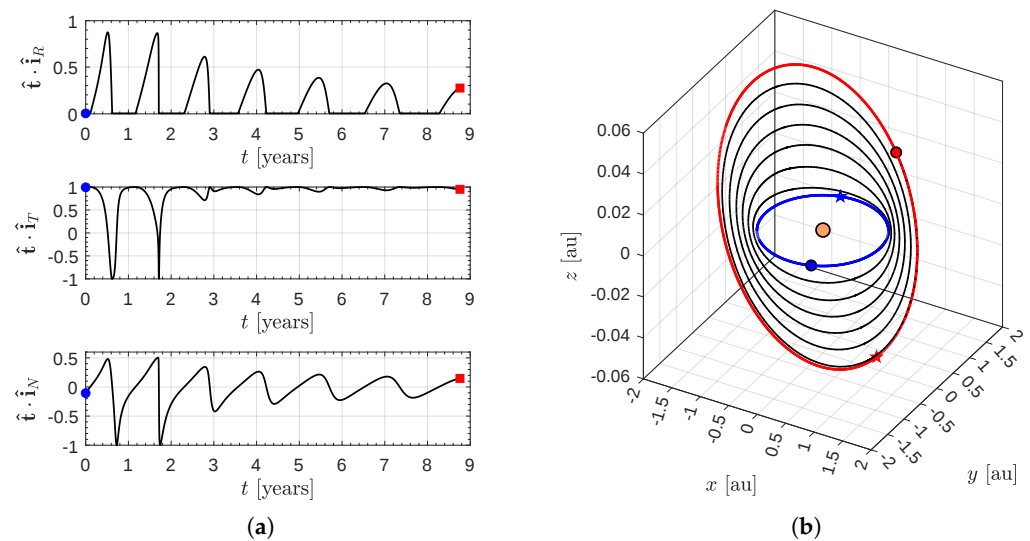


Figure 9. Results of the optimization process in a 3D Earth–Mars mission scenario when $a_c = 0.07 \text{ mm/s}^2$ and $K = 1$. (a) Time variation of the components of $\hat{\mathbf{f}}$ in RTN. (b) Optimal transfer trajectory (black line). Blue line $\rightarrow$ is Earth's orbit; red line $\rightarrow$ is Mars' orbit; blue circle $\rightarrow$ is start point; red circle $\rightarrow$ is arrival point; blue star $\rightarrow$ is Earth's perihelion; red star $\rightarrow$ is Mars' perihelion; orange circle $\rightarrow$ is the Sun.

5. Conclusions

This paper presented the optimal control law of a spacecraft equipped with a SWIFT in a 3D heliocentric transfer between two assigned Keplerian orbits. The proposed mathematical model extends the recent literature results on this advanced, rather exotic, propulsion system concept that allows for achieving interplanetary orbit transfer without the expenditure of a propellant and with a continuous propulsive acceleration. The thrust vector model discussed in this paper can be used to obtain the optimal control law in a wide range of mission applications. The thrust model has been used in this work to study the rapid orbit-to-orbit transfer by describing the spacecraft dynamics in terms of Cartesian vectors or orbital (both classic and equinoctial) elements. In particular, the heliocentric dynamics of the spacecraft has been modeled assuming only the effects of SWIFT-induced acceleration and the gravitational pull of the Sun. In this regard, a potential limitation of the proposed approach is represented by the absence of the inevitable orbital perturbations that can influence the shape of the optimal transfer trajectory. In this sense, a more refined model of orbital dynamics (which perhaps also includes possible uncertainties on the performance characteristics of the propulsion system) would allow for a more precise transfer trajectory.

The simulation results show that the transfer performance of a SWIFT-based spacecraft in a 3D scenario strictly depends on the value of the design parameter K . In fact, the transverse (i.e., perpendicular to the radial direction) component of the vector $\mathbf{a}$ depends on K so that a small value of that design parameter gives a small transverse component of $\mathbf{a}$, with the characteristic acceleration being fixed. In a 3D orbit transfer, the value of the transverse component of the vector $\mathbf{a}$ plays an important role in achieving a short flight time, especially in mission scenarios requiring a large variation in the orbital inclination. A possible extension of this work is related to the analysis of displaced, heliocentric, non-Keplerian orbits for planetary observation using a SWIFT-propelled spacecraft. In fact, the proposed thrust vector model allows the geometric characteristics of the heliocentric displaced orbit to be obtained as a function of the two design parameters of the SWIFT system. In that context, the optimal control law obtained in the case of a spherical reference frame can be used to study the rapid transfer trajectories from the Earth's parking orbit and the target (heliocentric) displaced orbit.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Conflicts of Interest: The author declares no conflicts of interest.

Appendix A. SWIFT Thrust Vector Model and Spacecraft Dynamics for 2D Mission Scenarios

In this appendix, we briefly summarize the thrust vector model proposed in Ref. [9] for the study of a 2D heliocentric mission scenario. That mathematical model has been used to design the optimal transfer trajectory between two coplanar, Keplerian orbits of assigned characteristics by using a classical indirect approach [46,47] and the well-known Pontryagin’s maximum principle [24,29,48]. In particular, the thrust model illustrated in Ref. [9] originates from the recent seminal work by Gemmer et al. [1] that, in turn, refines the mathematical model of the SWIFT propulsion system concept, which was proposed in an interesting set of conference papers by the same authors [2,49–51]. More precisely, the mathematical approach used in Ref. [9] to describe the SWIFT-induced propulsive acceleration vector is focused on the trajectory analysis and optimization. In this sense, the thrust model described in Ref. [9] was designed, according to the physical results illustrated in Ref. [1], with the aim of obtaining a set of analytical relations simple enough to be used in a trajectory optimization software [52–54], which can be used for a preliminary mission analysis and design.

Bearing in mind that the spacecraft total mass is constant, according to Gemmer et al. [1] and using the results of Ref. [9], in a 2D heliocentric mission scenario, the vector $\mathbf{a}$ can be written as the (vector) sum of two contributions: (1) the part due to the radial component of the SWIFT-induced force (i.e., a sort of “drag” originated by focusing the solar wind stream [1]), which is directed along the Sun–spacecraft unit vector $\hat{\mathbf{r}}$ and (2) the part due to the acceleration of the collected charged particles out of the vehicle (i.e., a sort of “thrust” term, thus borrowing the terminology used to describe the performance of a typical electric thruster), which can be steered within a wider field and is directed along the unit vector $\hat{\mathbf{t}}$. From a mathematical point of view, and using the nomenclature introduced in Ref. [9], the vector $\mathbf{a}$ can be written in a compact form as

$$\mathbf{a} = a_{D\oplus} \left(\frac{r_{\oplus}}{r} \right)^2 (\hat{\mathbf{r}} + K \hat{\mathbf{t}}) \quad (\text{A1})$$

where r is the Sun–spacecraft distance, $r_{\oplus} = 1 \text{ au}$ is a reference distance that roughly coincides with the semimajor axis of the Earth’s orbit around the Sun, $a_{D\oplus}$ is the reference acceleration defined as the value of the magnitude of the “drag” part of the propulsive acceleration vector when $r = r_{\oplus}$, and $K > 0$ is a dimensionless parameter, which coincides with the ratio of the exhaust velocity of the charged particles stream over the local velocity of the solar wind stream. In particular, the two terms $\{a_{D\oplus}, K\}$ should be considered as propulsive system performance parameters [1] so that their values depend on both the characteristics of the SWIFT and the spacecraft mass breakdown. In this regard, Ref. [9] contains a mathematical model that is useful to estimate the values of the pair $\{a_{D\oplus}, K\}$ as a function of the geometric, inertial, and propulsive characteristics of the SWIFT-based spacecraft.

In the case of a 2D heliocentric mission scenario as that studied in Ref. [9], during the flight, the unit vector $\hat{\mathbf{t}}$ is assumed to belong to the plane of the initial parking orbit so that the propelled trajectory of the spacecraft is 2D. In this context, the spacecraft dynamics can be studied by using a classical polar reference frame with the origin in the Sun’s center of mass, in which $\hat{\boldsymbol{\theta}}$ is the transverse unit vector, while the radial unit vector coincides with $\hat{\mathbf{r}}$. Recall that the spacecraft inertial velocity vector has a positive component along the

direction of $\hat{\theta}$. Introducing the pitch angle $\alpha_n \in [-\alpha_{\max}, \alpha_{\max}]$ defined as the angle between the Sun–spacecraft (radial) line and the direction of the steerable part of the vector $\mathbf{a}$ [1,9], where $\alpha_{\max} \in (90, 180)$ deg is an angle that accounts for the geometric constraints due to the conical shape of the SWIFT tethers grid—see the red cone in the conceptual sketch of Figure 5—the unit vector $\hat{\mathbf{t}}$ is written as

$$\hat{\mathbf{t}} = \cos \alpha_n \hat{\mathbf{r}} + \sin \alpha_n \hat{\theta} \quad (\text{A2})$$

so that Equation (A1) becomes

$$\mathbf{a} = a_{D\oplus} \left(\frac{r_{\oplus}}{r} \right)^2 [(1 + K \cos \alpha_n) \hat{\mathbf{r}} + K \sin \alpha_n \hat{\theta}] \quad (\text{A3})$$

Accordingly, the heliocentric dynamics of the SWIFT-based spacecraft can be analyzed, as discussed in Ref. [9], by using the classical set of EoMs in the polar reference frame of unit vectors $\{\hat{\mathbf{r}}, \hat{\theta}\}$

$$\dot{r} = u, \quad \dot{\theta} = \frac{v}{r}, \quad \dot{u} = -\frac{\mu_{\odot}}{r^2} + \frac{v^2}{r} + a_{D\oplus} \left(\frac{r_{\oplus}}{r} \right)^2 (1 + K \cos \alpha_n), \quad \dot{v} = -\frac{uv}{r} + a_{D\oplus} \left(\frac{r_{\oplus}}{r} \right)^2 K \sin \alpha_n \quad (\text{A4})$$

where θ is the polar angle measured from the Sun–spacecraft line at the initial time instant, $\mu_{\odot}$ is the Sun’s gravitational parameter, u is the radial component of the spacecraft inertial velocity vector, and v is the transverse component of the spacecraft inertial velocity vector. The polar reference frame is sketched in Figure A1.

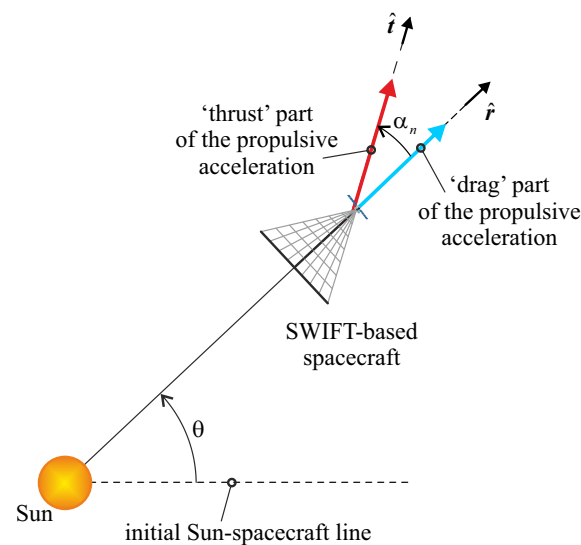


Figure A1. Sketch of the polar reference frame and the sail pitch angle α_n in a 2D heliocentric mission scenario of a SWIFT-propelled spacecraft, such as that used in Ref. [9].

In particular for a 2D scenario, the pitch angle α_n is the single scalar control parameter in Equation (A4), and its local value has been evaluated in Ref. [9] by using an optimization procedure based on the calculus of variations.

References

1. Gemmer, T.R.; Yoder, C.D.; Mazzoleni, A.P. Performance Analysis and Parametric Studies of the Solar Wind Ion Focusing Thruster (SWIFT) for Interplanetary Travel. *J. Br. Interplanet. Soc.* **2021**, *74*, 30–40.
2. Gemmer, T.R.; Mazzoleni, A.P. Introduction and Performance Analysis of the Solar Wind Ion Focusing Thruster (SWIFT). In Proceedings of the 65th International Astronautical Congress, Toronto, ON, Canada, 29 September–3 October 2014.
3. Gong, S.; Macdonald, M. Review on solar sail technology. *Astrodynamics* **2019**, *3*, 93–125. [\[CrossRef\]](#)
4. Fu, B.; Sperber, E.; Eke, F. Solar sail technology—A state of the art review. *Prog. Aerosp. Sci.* **2016**, *86*, 1–19. [\[CrossRef\]](#)
5. Janhunen, P.; Toivanen, P.K.; Polkko, J.; Merikallio, S.; Salminen, P.; Haeggström, E.; Seppänen, H.; Kurppa, R.; Ukkonen, J.; Kiprich, S.; et al. Electric solar wind sail: Toward test missions. *Rev. Sci. Instruments* **2010**, *81*, 111301. [\[CrossRef\]](#)

6. Bassetto, M.; Niccolai, L.; Quarta, A.A.; Mengali, G. A comprehensive review of Electric Solar Wind Sail concept and its applications. *Prog. Aerosp. Sci.* **2022**, *128*, 100768. [\[CrossRef\]](#)
7. Janhunen, P. Electric sail for spacecraft propulsion. *J. Propuls. Power* **2004**, *20*, 763–764. [\[CrossRef\]](#)
8. Janhunen, P.; Sandroos, A. Simulation study of solar wind push on a charged wire: Basis of solar wind electric sail propulsion. *Ann. Geophys.* **2007**, *25*, 755–767. [\[CrossRef\]](#)
9. Quarta, A.A.; Niccolai, L.; Mengali, G.; Bassetto, M. Optimal Interplanetary Transfer of Solar Wind Ion Focusing Thruster-based Spacecraft. *Appl. Sci.* **2023**, *13*, 3820. [\[CrossRef\]](#)
10. Spencer, D.A.; Johnson, L.; Long, A.C. Solar sailing technology challenges. *Aerosp. Sci. Technol.* **2019**, *93*, 105276. [\[CrossRef\]](#)
11. Zhao, P.; Wu, C.; Li, Y. Design and application of solar sailing: A review on key technologies. *Chin. J. Aeronaut.* **2023**, *36*, 125–144. [\[CrossRef\]](#)
12. Zubrin, R.M.; Andrews, D.G. Magnetic sails and interplanetary travel. *J. Spacecr. Rocket.* **1991**, *28*, 197–203. [\[CrossRef\]](#)
13. Andrews, D.G.; Zubrin, R.M. Magnetic sails and interstellar travel. *J. Br. Interplanet. Soc.* **1990**, *43*, 265–272.
14. Perakis, N.; Hein, A.M. Combining magnetic and electric sails for interstellar deceleration. *Acta Astronaut.* **2016**, *128*, 13–20. [\[CrossRef\]](#)
15. Garwin, R. Solar Sailing—A Practical Method of Propulsion Within the Solar System. *Jet Propuls.* **1958**, *28*, 188–190.
16. Wright, J.L. *Space Sailing*; Gordon and Breach Science Publishers: Philadelphia, PA, USA, 1992; pp. 223–233. ISBN 978-2881248429.
17. McInnes, C.R. *Solar Sailing: Technology, Dynamics and Mission Applications*; Springer-Praxis Series in Space Science and Technology; Springer: Berlin, Germany, 1999; pp. 46–54, 119–120. [\[CrossRef\]](#)
18. Hintz, G.R. *Orbital Mechanics and Astrodynamics*; Springer: New York, NY, USA, 2015; Chapter 5, pp. 214–216. [\[CrossRef\]](#)
19. Mengali, G.; Quarta, A.A. Optimal Trajectories of Diffractive Sail to Highly Inclined Heliocentric Orbits. *Appl. Sci.* **2024**, *14*, 2922. [\[CrossRef\]](#)
20. von Stryk, O.; Bulirsch, R. Direct and indirect methods for trajectory optimization. *Ann. Oper. Res.* **1992**, *37*, 357–373. [\[CrossRef\]](#)
21. Liu, H.; Tongue, B.H. Indirect Spacecraft Trajectory Optimization Using Modified Equinoctial Elements. *J. Guid. Control Dyn.* **2010**, *33*, 619–623. [\[CrossRef\]](#)
22. Taheri, E.; Arya, V.; Junkins, J.L. Costate mapping for indirect trajectory optimization. *Astrodynamics* **2021**, *5*, 359–371. [\[CrossRef\]](#)
23. Kopp, R.E. *Optimization Techniques with Applications to Aerospace Systems*; Mathematics in Science and Engineering; Elsevier: New York, NY, USA, 1962; pp. 255–279. [\[CrossRef\]](#)
24. Ross, I.M. *A Primer on Pontryagin's Principle in Optimal Control*; Collegiate Publishers: San Francisco, CA, USA, 2015; Chapter 2, pp. 127–129.
25. Walker, M.J.H. A set of modified equinoctial orbit elements. *Celest. Mech.* **1985**, *36*, 409–419; Erratum in *Celest. Mech.* **1986**, *38*, 391–392. [\[CrossRef\]](#)
26. Bate, R.R.; Mueller, D.D.; White, J.E. *Fundamentals of Astrodynamics*; Dover Publications: New York, NY, USA, 1971; Chapter 2, pp. 54–55, 58–61.
27. Curtis, H.D. *Orbital Mechanics for Engineering Students*; Elsevier: Oxford, UK, 2014; Chapter 4, pp. 187–237. [\[CrossRef\]](#)
28. Lawden, D.F. *Optimal Trajectories for Space Navigation*; Butterworths & Co.: London, UK, 1963; pp. 54–60.
29. Prussing, J.E. *Optimal Spacecraft Trajectories*; Oxford University Press: Oxford, UK, 2018; Chapter 4, pp. 32–40.
30. Zheng, M.; Luo, J.; Dang, Z. Optimal impulsive rendezvous for highly elliptical orbits using linear primer vector theory. *Chin. J. Aeronaut.* **2024**, *37*, 194–207. [\[CrossRef\]](#)
31. Oguri, K.; Lantoine, G. Indirect trajectory optimization via solar sailing primer vector theory: Minimum solar-angle transfers. *Acta Astronaut.* **2023**, *211*, 405–415. [\[CrossRef\]](#)
32. Mengali, G.; Quarta, A.A. Optimal three-dimensional interplanetary rendezvous using nonideal solar sail. *J. Guid. Control Dyn.* **2005**, *28*, 173–177. [\[CrossRef\]](#)
33. Quarta, A.A.; Mengali, G.; Bassetto, M. Optimal Solar Sail Transfers to Circular Earth-Synchronous Displaced Orbits. *Astrodynamics* **2020**, *4*, 193–204. [\[CrossRef\]](#)
34. Bryson, A.E.; Ho, Y.C. *Applied Optimal Control*; Hemisphere Publishing Corporation: New York, NY, USA, 1975; Chapter 2, pp. 71–89. ISBN 0-891-16228-3.
35. Broucke, R.A.; Cefola, P.J. On the equinoctial orbit elements. *Celest. Mech.* **1972**, *5*, 303–310. [\[CrossRef\]](#)
36. Kechichian, J.A. Optimal low-thrust rendezvous using equinoctial orbit elements. *Acta Astronaut.* **1996**, *38*, 1–14. [\[CrossRef\]](#)
37. Lagona, E.; Hilton, S.; Afful, A.; Gardi, A.; Sabatini, R. Autonomous Trajectory Optimisation for Intelligent Satellite Systems and Space Traffic Management. *Acta Astronaut.* **2022**, *194*, 185–201. [\[CrossRef\]](#)
38. Coverstone, V.L.; Prussing, J.E. Technique for Escape from Geosynchronous Transfer Orbit Using a Solar Sail. *J. Guid. Control Dyn.* **2003**, *26*, 628–634. [\[CrossRef\]](#)
39. Quarta, A.A.; Abu Salem, K.; Palaia, G. Solar sail transfer trajectory design for comet 29P/Schwassmann-Wachmann 1 rendezvous. *Appl. Sci.* **2023**, *13*, 9590. [\[CrossRef\]](#)
40. Betts, J.T. Very low-thrust trajectory optimization using a direct SQP method. *J. Comput. Appl. Math.* **2000**, *120*, 27–40. [\[CrossRef\]](#)
41. Battin, R.H. *An Introduction to the Mathematics and Methods of Astrodynamics*, Revised ed.; AIAA: Reston, VA, USA, 1999; Chapter 10, pp. 488–490. [\[CrossRef\]](#)
42. Yang, W.Y.; Cao, W.; Kim, J.; Park, K.W.; Park, H.H.; Joung, J.; Ro, J.S.; Hong, C.H.; Im, T. *Applied Numerical Methods Using MATLAB*; John Wiley & Sons, Inc.: Hoboken, NJ, USA, 2020; Chapters 3 and 6, pp. 166–172, 312.

43. Quarta, A.A. Fast initialization of the indirect optimization problem in the solar sail circle-to-circle orbit transfer. *Aerosp. Sci. Technol.* **2024**, *147*, 109058. [[CrossRef](#)]
44. Quarta, A.A. Initial costate approximation for rapid orbit raising with very low propulsive acceleration. *Appl. Sci.* **2024**, *14*, 1124. [[CrossRef](#)]
45. Shampine, L.F.; Reichelt, M.W. The MATLAB ODE Suite. *SIAM J. Sci. Comput.* **1997**, *18*, 1–22. [[CrossRef](#)]
46. Stengel, R.F. *Optimal Control and Estimation*; Dover Books on Mathematics; Dover Publications, Inc.: New York, NY, USA, 1994; pp. 222–254.
47. Betts, J.T. Survey of Numerical Methods for Trajectory Optimization. *J. Guid. Control Dyn.* **1998**, *21*, 193–207. [[CrossRef](#)]
48. Prussing, J.E. *Spacecraft Trajectory Optimization*; Cambridge University Press: Cambridge, UK, 2010; Chapter 2, pp. 16–36. [[CrossRef](#)]
49. Gemmer, T.R.; Mazzoleni, A.P. Solar wind ion focusing thruster (SWIFT) Orbital Performance Analysis. In Proceedings of the 66th International Astronautical Congress, Jerusalem, Israel, 12–16 October 2015.
50. Gemmer, T.R.; Mazzoleni, A.P. Comparative orbital performance study of a solar wind ion focusing thruster (SWIFT). In Proceedings of the 67th International Astronautical Congress, Guadalajara, Mexico, 26–30 September 2016.
51. Gemmer, T.R.; Yoder, C.; Mazzoleni, A.P. Increasing the accuracy of electric sail mission performance estimates. In Proceedings of the 68th International Astronautical Congress, Adelaide, Australia, 25–29 September 2017; Volume 14, pp. 8992–8997.
52. Rishikof, B.H.; McCormick, B.R.; Pritchard, R.E.; Sponaugle, S.J. SeGRAM: A practical and versatile tool for spacecraft trajectory optimization. *Acta Astronaut.* **1992**, *26*, 599–609. [[CrossRef](#)]
53. Rubinsztein, A.; Sandel, C.G.; Sood, R.; Laipert, F.E. Designing trajectories resilient to missed thrust events using expected thrust fraction. *Aerosp. Sci. Technol.* **2021**, *115*, 106780. [[CrossRef](#)]
54. Morante, D.; Sanjurjo Rivo, M.; Soler, M. A Survey on Low-Thrust Trajectory Optimization Approaches. *Aerospace* **2021**, *8*, 88. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.