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Evolution Model of Coal Failure Using Energy Dissipation under Cyclic Loading/Unloading

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Abstract: The damage and fracture of coal is accompanied by a complex energy conversion process, and these different stages of energy evolution are closely related to coal failure. In this paper, an evolution model describing the behavior of coal failure was proposed using the energy dissipation under cyclic loading/unloading. The energy growth pattern and energy consumption characteristics of the coal fracture were analyzed under cyclic loading/unloading. An evolution model of the energy behavior of coal fracture was established. The damage variables of energy dissipation were defined, and a theoretical model was established. The parameters included the relationship between the energy state, damage state, and strength state according to the uniaxial cyclic loading/unloading test. The results show that there are energy excitation and inhibition effects in the process of coal fracture; that is, the accumulation rate and level of energy are affected by the energy storage state, and the energy storage rate changes in the mode of "low promotion and high inhibition". The abrupt increase in dissipated energy can be regarded as the precursor of coal fracture. Based on the analysis of the characteristics of the damage and failure state and dissipated energy, the discriminant equation for the stability of the coal energy state was constructed; it is a meaningful discovery for predicting and evaluating coal failure.

Keywords: coal failure; uniaxial cyclic loading/unloading test; energy dissipation; energy evolution

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1. Introduction

In recent years, the mining depth of mineral resources has continued to increase. Under the influence of high geo-stress and complex geological conditions, the dynamic phenomenon of instability failure often occurs through external forces in the coal and rock mass. Micro-seismic and acoustic emission monitoring is often used for the damage monitoring of coal rock bodies; on this basis, a series of studies were carried out [1–5]. From the energy point of view, the sudden release of a large amount of energy seriously threatens the construction and production of underground engineering [6]. According to the law of thermodynamics, energy conversion is the essential characteristic of the physical process of matter. The deformation and failure of coal, in the final analysis, is a kind of energy-driven unstable state [7]. Therefore, it is more accessible to the nature of its failure by studying the energy evolution characteristics of the coal instability fracture process from the perspective of energy [8,9].

The damage evolution of coal is essentially a process of energy dissipation and energy release. Building a coal energy evolution model is an effective means to analyze and reveal the mechanical nature of coal deformation and failure process [10,11]. In the field of geotechnical engineering, many experts and scholars have conducted a lot of beneficial research on the energy evolution model of the coal damage and fracture process and obtained fruitful research results.

Through theoretical research, the essential characteristics of energy evolution can be described more precisely. Based on fractal rock mechanics and fracture mechanics theory, a new model of energy consumption during rock fragmentation was proposed [12].

Then, through finite element modeling [13], the seismic energy release rate and strain energy storage rate, which result from mining, were analyzed, and a new, energy-based, burst potential index (BPI) was proposed [14]. A method based on the energy release rate to identify and characterize the energy evolution characteristics of brittle fracture under uniaxial compression was proposed. An isotropic damage model for concrete was presented, and Comi et al. obtained the damage and failure conditions of concrete based on fracture energy [15]. Based on a nonlinear relationship analysis of various energy conversion mechanisms, a self-repression model of energy transformation as a function of axial stress was proposed [16].

The dynamic fracture of materials is essentially a process of energy consumption. Energy dissipation analysis based on experimental research is essential to elucidate the dynamic crushing mechanism of materials [12]. Three-point bending fatigue tests were carried out in order to study the relationships of the dissipated strain energy (DSE) to the stress–strength ratio, temperature, and loading. According to the change laws of the DSE, mathematical models of the DSE and loading rate were established [17]. From the perspective of energy dissipation, the damage model of the fractured rocks was established, which revealed the evolution of the elastic modulus and residual strain of the fractured rocks based on the initial damage [18]. After this, according to the dissipative energy ratio evolution characteristics of rock failure, the peak strength and peak strain damage evolution models of frozen-thawed rocks were established to reveal the energy dissipation mechanism of the rock deformation and the destruction process under the influence of freeze–thaw cycles and the essence of rock freeze–thaw damage. By analyzing the energy accumulation, the dissipation and release characteristics of marble throughout the deformation process under loading/unloading stress path, a rock damage mechanics model was established [19,20]. Numerical modeling to calculate the energy release rate (ERR), energy storage rate (ESR), and burst potential index were used for rock burst prediction [21].

The above research results enrich our knowledge of the energy evolution mechanism of materials under different damage conditions. However, most of the research have focused on the failure stages of material or concentrated on a specific energy. Few studies have emphasized the characteristics of energy evolution and dissipation during the complete materials deformation and failure process [22]. Furthermore, the relationship between dissipation energy and coal damage needs to be further explored. In this paper, the energy evolution process of coal fracture was further revealed through cyclic loading/unloading tests. The internal energy driving mechanism of coal was analyzed using the energy tangential factor, which indicates the existence of a certain degree of energy accumulation–inhibition effects in the energy evolution process. Based on the evolution characteristics and laws of dissipated energy behavior, an improved theoretical model was established with the relationship between the energy state, damage state, and strength state in order to distinguish the damage stages and stability degree of coal. The reliability and accuracy of the model was further verified by the fitting analysis of the experimental data. Therefore, the theoretical model has important theoretical significance for the early warning and stability evaluation of coal fracture and instability.

2. Materials and Methods

2.1. Experiments Materials

Coal specimens were collected from the working face 2201 in the Ordos coal mine of YanKuang Group, and the depth of collection was approximately 169.29 m. Before the experiments, these specimens were processed into standard cylindrical coal specimens of 50 mm in diameter and 100 mm in length; the ends of the specimens were polished to obtain parallelism less than 0.05 mm. The physical and mechanical properties of some coal specimens under uniaxial loading are shown in Table 1.

Table 1. Physical and mechanical parameters of coal samples under uniaxial cyclic compression.

Coal	High (mm)	Diameter (mm)	Quality (g)	Uniaxial Compressive Strength (MPa)	Elasticity Modulus (GPa)	Poisson's Ratio	Fracture Load (kN)
C1	100.24	49.68	259.02	19.410	4.373	0.392	24.58
C2	100.06	49.67	233.56	20.378	5.067	0.373	25.12
C3	100.32	49.73	265.80	21.308	4.974	0.351	24.97
C4	100.18	49.66	250.62	19.680	4.571	0.372	25.60
C5	100.02	49.56	243.56	20.638	4.343	0.361	24.96
C6	100.56	49.78	251.80	21.006	4.814	0.336	24.89

Physical properties of coal specimens were analyzed with SEM (scanning electron microscope), and SEM scanning images of coal specimens with different magnification were obtained, as shown in Figure 1. Depressions represent microcracked coal. The transverse joints and primary microcracks of coal specimens were well developed with more materials filling inside. Figure 1c,d show SEM scans of coal samples at 1000× and 2000× magnification. In Figure 1c, not only can the fracture development of coal samples at larger magnification can be seen, but also the distribution of nucleation defects inside the coal samples can be observed, at which time a certain degree of bifurcation and non-uniform distribution of smaller microfractures is observable, thus indicating the smaller level of internal fracture non-uniformity of distribution. At the same time, it can be clearly observed that the surface of the coal sample also has some micro defects and holes distributed arbitrarily, and the filled material in the large cracks on the surface is fuller; the fine structure characteristics of the fracture of the coal sample can be clearly observed in Figure 1d, and a large amount of coal sample debris and filled material debris remained on the surface of the fracture. The fracture pattern at the fracture was uneven, which indicates that the brittle characteristics of the coal sample are obvious, and the fracture mostly occurred at the brittle fracture.

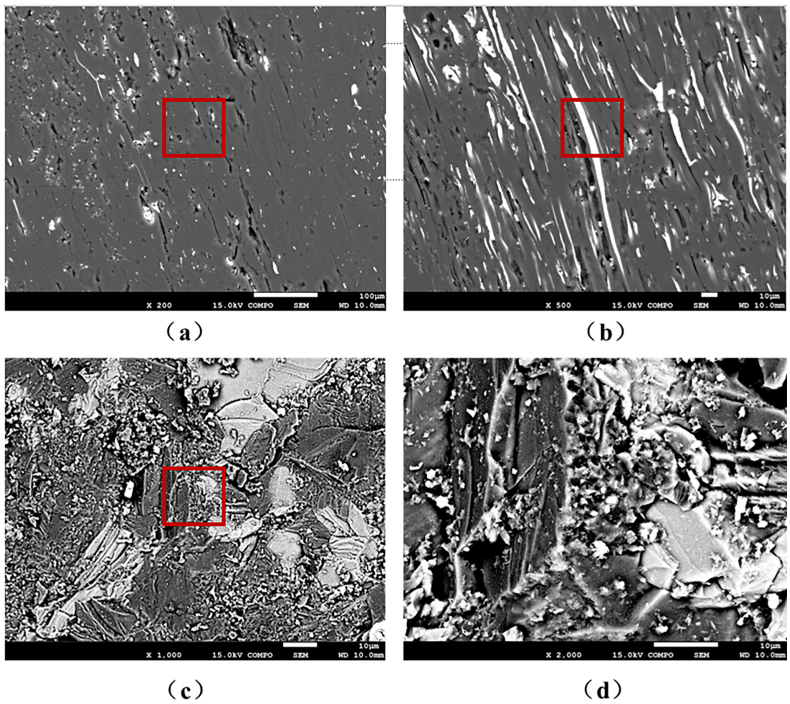

Figure 1. SEM images of coal specimen at different magnifications. (**a**) for specimens magnified by 200 times (**b**) for specimens magnified by 500 times (**c**) for specimens magnified by 1000 times (**d**) for specimens magnified by 2000 times. The red area represents the magnified area.

2.2. Experimental Equipment

In this paper, the loading equipment was a microcomputer-controlled electro-hydraulic servo testing machine (YDSZ-2000). The maximum axial loading force was 2000 kN, and the measuring precision was no more than ±1%. The layout of the test equipment is shown in Figure 2, and the stiffness of the loading equipment used for the experiment could be guaranteed.

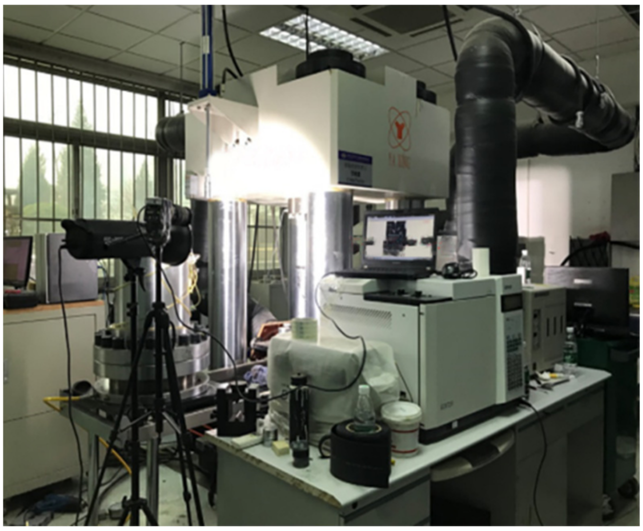

Figure 2. Site layout of loading equipment.

2.3. Experimental Methods

The coal specimens were tested to experience cycle loading/unloading with two different stress loading paths under axial equivalent stress in the lab. Mode 1 is a graded cyclic loading/unloading test, which is mainly used to analyze the influence of loading/unloading behavior on the rupture behavior and rupture mode of coal samples under different loading conditions, as well as the mechanical response characteristics and energy distribution state of coal samples under different stress gradients. Mode 2 is the square equal amplitude cyclic loading and unloading test, which is mainly used to explore the influence of continuous cyclic loading and unloading on the damage state of coal samples under different stress levels, as well as to analyze the difference of the influence of different cyclic loading and unloading times on the fatigue damage state of coal samples under the same stress level. During the test, the loading/unloading rates were both kept at 0.05 MPa/s to reduce the impact of rate differences. The stress loading gradient of the graded cyclic loading/unloading tests was 2 MPa, and the stress loading/unloading path is shown in Figure 3a. In order to prevent the coal sample from separating from the pressure head of the testing machine, the unloading lower limit stress was set to 2 MPa, and the sample was loaded until it was damaged. Test 2 entailed equal amplitudes cyclic loading/unloading tests. According to the uniaxial compressive strength of the coal specimen, 3 cyclic loading/unloading phases with different stress levels were set, and 10 cyclic loading/unloading processes were carried out in each phase. The stress loading/unloading path is shown in Figure 3b.

In the following part, two groups of specimens, which were typical in each group, were selected for analysis.

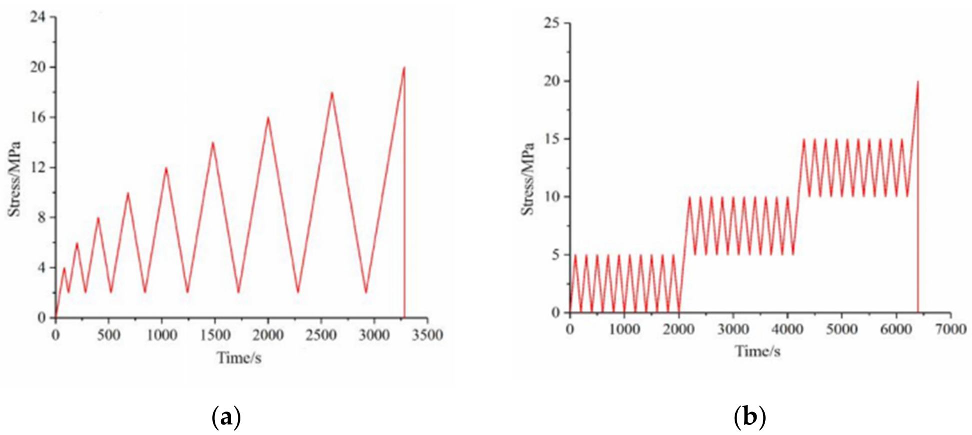

Figure 3. Schematic diagram of coal specimen cyclic loading/unloading stress path. (**a**) the stress loading/unloading path of the graded cyclic loading/unloading test, (**b**) the stress loading/unloading path of the square equal amplitude cyclic loading/unloading test.

3. Results

3.1. Uniaxial Cyclic Loading/Unloading Experiments

3.1.1. Mechanical Characteristics of Coal under Graded Cyclic Loading/Unloading

As shown in Figure 4, the stress–strain curves of coal samples in the process of graded cyclic loading/unloading were characterized by a gradual increase; that is, every increase in the strain gradient generated a corresponding increase in the stress gradient of the coal sample. Compared with the uniaxial compression test, the uniaxial compressive strength of coal under this loading/unloading mode decreased to a certain extent. When the stress gradient reached about 18 MPa, the coal sample failed in strength and was destroyed. This indicates that repeated loading/unloading of coal will cause fatigue damage and energy dissipation.

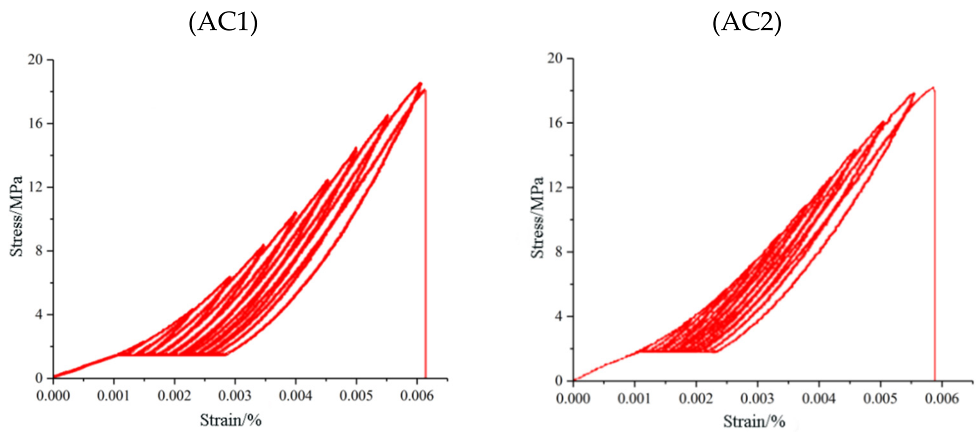

Figure 4. Stress–strain curves of graded cyclic loading/unloading.

3.1.2. Mechanical Characteristics of Coal under Constant Amplitude Cyclic Loading/Unloading

In the process of equi-amplitude cyclic loading/unloading, the stress–strain curves at the same stress level converged to a small hysteresis area, as shown in Figure 5. The reason for this result is that, for AC1 and AC2, the unloading modulus of elasticity was not the same as the loading process. In contrast, for BC1 and BC2, the unloading process was almost the same as the loading process. The repeated cyclic loading/unloading at the same stress level induced a relatively small damage degree to the coal sample, and the energy dissipation behavior was relatively weak. The curve increased linearly, thus indicating that the coal sample had obvious elastic brittleness characteristics.

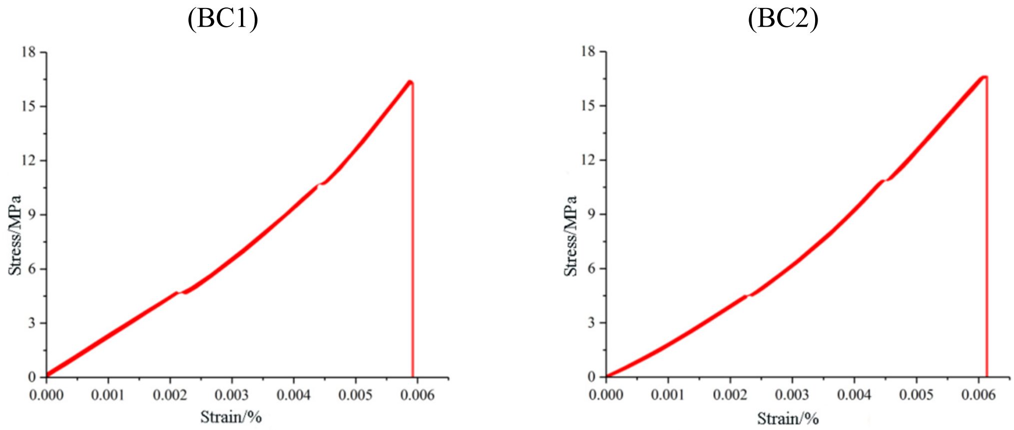

Figure 5. Stress–strain curves of constant amplitude cyclic loading/unloading.

3.2. Characteristics of Energy Consumption Growth in Coal Fracture Evolution

The proportional relationship between dissipated energy and elastic energy in coal, to some degree, can reflect the energy state and stability degree under loading. The accumulation of dissipated energy will cause coal to enter the critical equilibrium state from the original stable state, while the elastic energy stored in the coal can improve its stability. Therefore, the more elastic energy is stored, the more stable the coal will be.

The calculation formula of the energy consumption ratio (ECR) is defined as:

$$K_{GF} = \frac{U_d}{U_e} \tag{1}$$

where K_{GF} is the energy consumption ratio, which reflects the ratio of dissipated energy and elastic energy measured as a %; U_d is the dissipated energy in the process of coal failure measured in J/mm^3; and U_e is the elastic energy in the process of coal failure measured in J/mm^3. The relevant data were concluded using numerical software.

3.2.1. Evolution Characteristics of Energy Consumption Ratio under Graded Cyclic Loading/Unloading

When the loading stress was within the range of 0–16 MPa, the evolution characteristics of the coal energy consumption ratio had a small decrease, but the overall change remained stable, as shown in Figure 6. After the pressure and sealing stage of the initial fissure, the energy accumulation and elastic deformation of the coal were developing steadily. The rate of crack formation was stable, and the energy dissipation behavior did not increase significantly. However, when the loading stress reached 16 MPa, the energy consumption ratio of the coal increased significantly. At this stage, the coal produced a large amount of plastic deformation and was in a critical failure state. Under the action of a small stress, the coal would be completely destabilized and destroyed while releasing a large amount of energy.

3.2.2. Evolution Characteristics of Energy Consumption Ratio under Constant Amplitude Cyclic Loading/Unloading

As shown in Figure 7, under the action of a constant amplitude cyclic load, the energy consumption ratio of the coal in the third stage was much higher than the other two stages. It shows that the energy consumption ratio of the coal at different stress levels was quite different, especially at the critical failure stage, which was characterized by “sudden increase”. However, for the same stress level stage, the energy consumption ratio of the coal at the initial loading was slightly higher. After the initial loading was completed, it was in a stable state with small fluctuations, and the overall change was not significant.

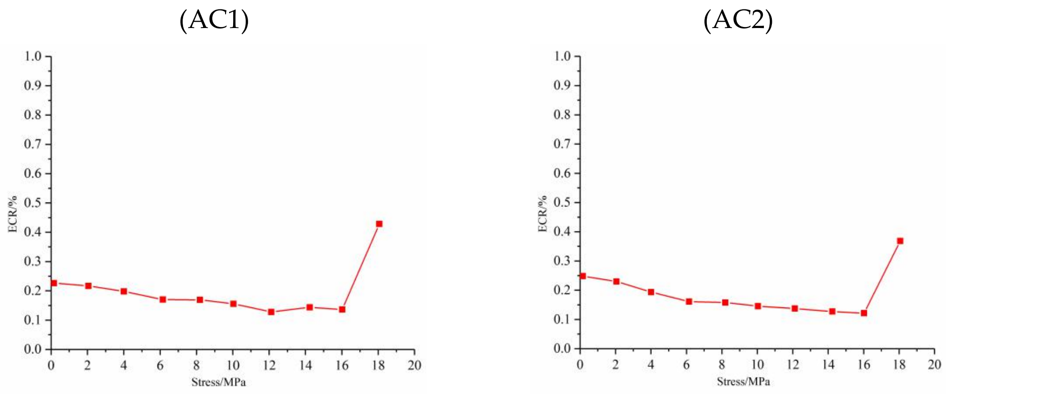

Figure 6. Energy consumption ratio evolution characteristics of coal failure process under graded cyclic loading/unloading.

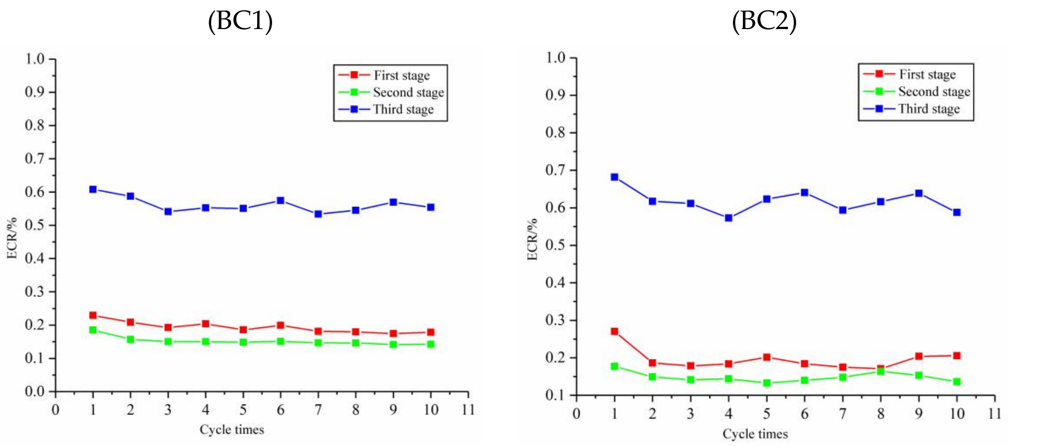

Figure 7. Energy consumption ratio evolution characteristics of coal failure process under constant amplitude cyclic loading/unloading.

4. Analysis and Discussion

4.1. Energy State Analysis of Coal Fracture Process

Throughout the process of deformation and damage from energy input, the loaded coal underwent accumulation, dissipation, and the release of energy [23]. Its energy evolution was a dynamic process of constant transfer and conversion [24]. The deformation and failure state of the coal at any time corresponded to the specific energy. Therefore, the study of deformation failure law from the perspective of energy is closer to its failure nature and can better reveal the failure mechanism under impact loading [25].

Many studies show that the energy state of coal is closely related to the damage evolution process. By studying the fracture mechanism and energy dissipation characteristics of the rock under impact and dynamic loads, scholars have established the relationship between the rock damage degree and energy release rate, and they have analyzed the energy mechanism of rock damage evolution [26–29]. The above studies have described the relationship between the energy state and failure of sample, but they have neglected the interaction between the internal energy. Based on thermodynamic theory and the characteristics of stress–strain curves, the input energy density, elastic energy density, and dissipated energy density of coal under different loads were obtained by Origin software in this paper, as shown in Figure 8. By characterizing the evolution law of different energy, the influence of internal energy interactions on the coal failure could be well analyzed.

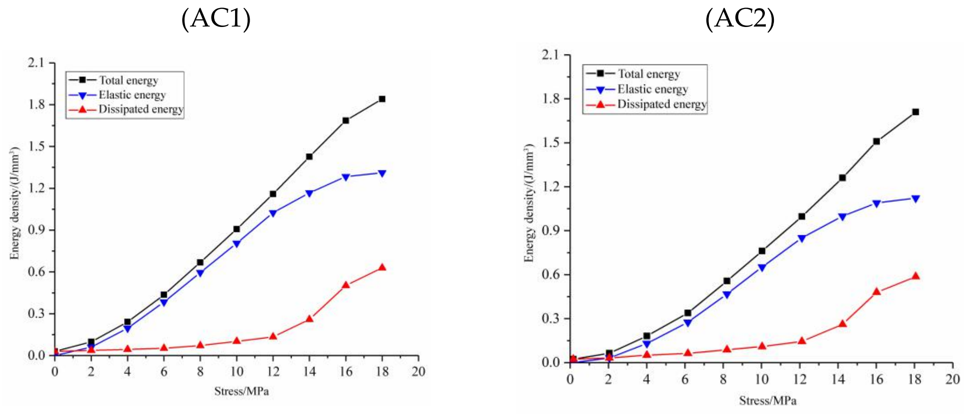

Figure 8. Evolution of coal energy under graded cyclic loading/unloading.

Before the coal failure, the total energy, elastic energy, and dissipated energy of the coal all increased with the increase in stress, and the evolution curves demonstrated significant stage features. The total energy increased the fastest, followed by the elastic energy and the dissipated energy. As shown in Figure 8, the accumulation rate of the elastic strain energy and total energy in coal had the same trend, which was contrary to the change in dissipated energy. In the initial loading stage, the total energy accumulated inside the coal was relatively less, and the energy accumulation mechanism was activated under the action of load, thereby promoting the accumulation rate of the elastic strain energy. With the continuous input of external energy, the energy density of the coal increased linearly and steadily. This stage was dominated by elastic deformation, which improved the energy accumulating efficiency and inhibited the energy releasing degree under the increase in leading stress. The higher the energy density inside the coal, the lower its capacity to store elastic energy, which is reflected in the decrease in the growth rate of total energy and elastic energy. Meanwhile, as the input energy kept increasing, the higher the elastic energy was, the worse the state of its stored elastic energy. The more unstable the coal sample is, the worse is the coal's ability to resist damage and the easier it is to destroy. Therefore, the energy state of the coal affects the internal energy accumulation and dissipation rate, and the energy evolution process also has a certain degree of energy excitation–inhibition effect.

4.2. Energy Driving Mechanism of Coal Fracture Process

According to the laws of thermodynamics, the deformation and damage process of materials is essentially a state instability phenomenon driven by energy dissipation and release [9]. The rate of energy change becomes an important parameter to characterize the stable state of coal, which is of great significance to judge the energy evolution stage and damage mechanism [20]. As shown in Figure 9, to characterize the change in the energy growth rate of coal under axial loading, the concept of tangential factor of energy growth is proposed. The calculation formula is as follows:

$$K_{TF} = \frac{dE_i}{d\sigma_i} = \frac{\Delta E}{\Delta \sigma} \quad (2)$$

where K_{TF} is the energy tangent factor of coal, representing the energy change rate under the loading state in Figure 9, $K_{TF} = K_{MP}$, the energy factor increment is $\Delta E = E_{i+1} - E_i$, and the stress increment is $\Delta\sigma = \sigma_{i+1} - \sigma_i$.

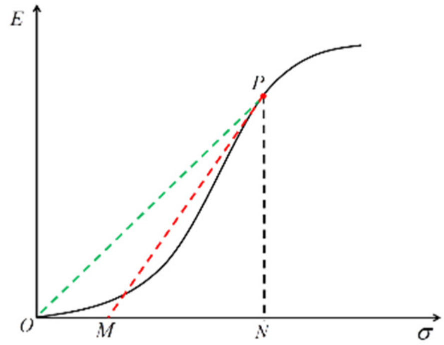

Figure 9. Schematic diagram of energy growth rate calculation of coal failure.

As shown in Figure 10, based on the change curve of the energy tangent factor, different stages of coal deformation and failure could be better distinguished, and the energy growth state of coal at different stress loadings could be obtained. In the I stage (0–4 MPa), the tangent factor of coal energy kept increasing. The low energy storage state of coal promoted the rapid increase of the elastic energy storage rate, which indicated the gradual transition of the coal from the compaction stage to the elastic deformation stage. In the II stage (4–16 MPa), the rate of coal energy growth changed steadily to a higher level. As the loading stress increased linearly, the internal crack of coal expanded stably, while its elastic energy and dissipated energy increased stably and greatly. In the III stage (16–18 MPa). When the energy storage mechanism and energy consumption mechanism of coal reached a dynamic balance, it became unstable and was destroyed.

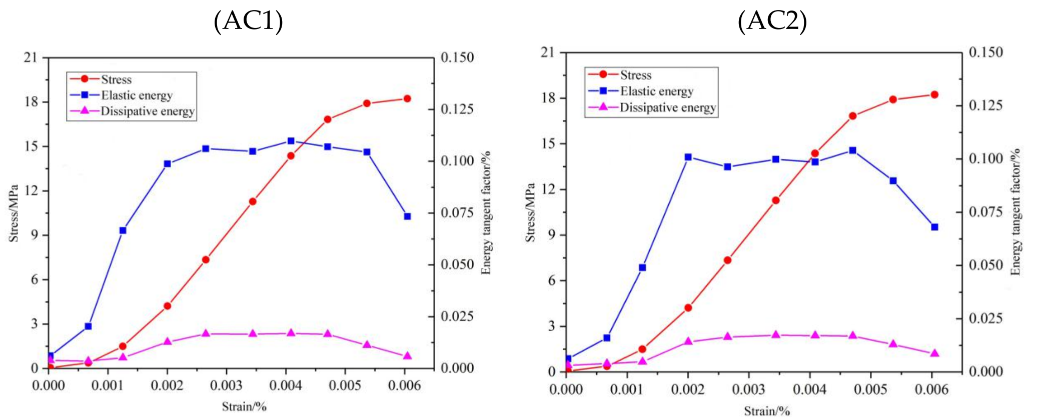

Figure 10. Elastic energy–dissipation energy tangent factor curve of coal under graded cyclic loading/unloading.

The rapid growth rate of the elastic strain energy indicates that elastic deformation is the principal part of coal failure process. As shown in Figure 10, the energy growth rate of coal is nonlinear with the change in axial stress. The tangential factor of energy growth increased rapidly in the initial loading stage, developed steadily after 20% of the ultimate strength, and decreased obviously after 90%. Through quantitative analysis, the corresponding relationship between the energy growth rate and stress–strain can be better described, and the energy storage level of coal at different deformation stages can be characterized.

4.3. Evolution Law of Coal Dissipated Energy

4.3.1. Verification of Evolution Law of Coal Dissipated Energy

Some scholars have considerably studied the stress–strain relationship and the strength characteristics of the material from the perspective of mechanics [30–32]. However, the stress–strain relationship cannot fully describe the failure processes of rocks. The stress–strain curves of rocks may be almost the same, but show different failure patterns and energy release characteristics [33]. From the perspective of thermodynamics, energy dissipation and release are the essence of rock deformation and destruction, which constitute an irreversible process [8]. Therefore, strengthening the research of energy dissipation characteristics can better verify the evolution law of coal rock damage.

According to the analysis in Section 3.2 above, the energy consumption ratio of coal was characterized by a sharp increase. Therefore, based on experimental research, a cusp mutation model of the coal failure process was constructed using the catastrophe theory [34,35], and the theoretical derivation of the evolution law of dissipated energy was obtained.

During the cyclic loading–unloading process, the dissipated energy of the coal accumulated gradually. Its dissipative energy state can be represented by $U_d = f(\varepsilon)$, which can be expanded by the Taylor series:

$$U_d(\varepsilon) = a_0 + a_1\,\varepsilon + a_2\,\varepsilon^2 + a_3\,\varepsilon^3 + \cdots + a_m\,\varepsilon^m \tag{3}$$

According to Equation (3), the first five items are taken to construct the cusp mutation model:

$$U_d(\varepsilon) = a_0 + a_1\,\varepsilon + a_2\,\varepsilon^2 + a_3\,\varepsilon^3 + a_4\,\varepsilon^4 \tag{4}$$

Set $\varepsilon = x - n$; since the state variable x is still a function of the time ε in the cusp mutation model, so

$$U_d(x) = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 \tag{5}$$

In the formula:

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} n^4 & -n^3 & n^2 & -n & 1 \\ -4n^3 & 3n^2 & -2n & 1 & 0 \\ 6n^2 & -3n & 1 & 0 & 0 \\ 1 & -3n & 1 & 0 & -3n \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{pmatrix}$$

If $b_3 = 0$, that is, $a_3 - 4na_4 = 0$, then $n = \frac{a_3}{4a_4}$.

Make $U_d(x) = \frac{U_d(x)}{4b_4}$, $p = \frac{b_2}{2b_4}$, $q = \frac{b_1}{4b_4}$, and $c = \frac{b_0}{4b_4}$; then, we obtain:

$$U_d(x) = \frac{1}{4}x^4 + \frac{1}{2}px^2 + qx + c \tag{6}$$

According to the mutation theory, Equation (7) satisfies the basic form of the mutation function. This is consistent with the abrupt growth of dissipated energy in the process of coal failure. Therefore, the evolution law of dissipated energy can be used as the judgment basis for the precursor characteristics of the coal fracture.

4.3.2. Criterion of Energy Tangent Factor

By analyzing the variation law of the tangent factor of dissipated energy under axial loading, the increasing state of energy under axial loading and the stability degree of the coal sample under different energy states were better characterized. Accordingly, a coal stability criterion based on the tangent factor of dissipated energy was proposed in this paper. When $K_{TF} \leq 0.05 \pm 0.002$, it was in a stable state, and the dissipated energy increased slowly. When $0.05 \pm 0.002 \leq K_{TF} \leq 0.09 \pm 0.003$, the tangent factor of dissipated energy increased by leaps and bounds, and the coal was in a critical failure state. When $K_{TF} \geq 0.09 \pm 0.003$, the growth rate of dissipated energy was slightly reduced compared to the previous one, which was mainly influenced by the self-inhibiting effect of energy growth. In the critical case, there was a process of energy aggregation before the coal sample rupture, in order to aggregate the energy and cause the final damage, thus slowing down the growth rate of the dissipated energy. Therefore, the coal specimen will break and become unstable in this stage. The development curve of the energy tangent factor in the process of coal failure and instability is shown in Figure 11. It can be seen that the energy tangent factor criterion can better understand the energy state of the coal at different deformation stages and make stability discriminations.

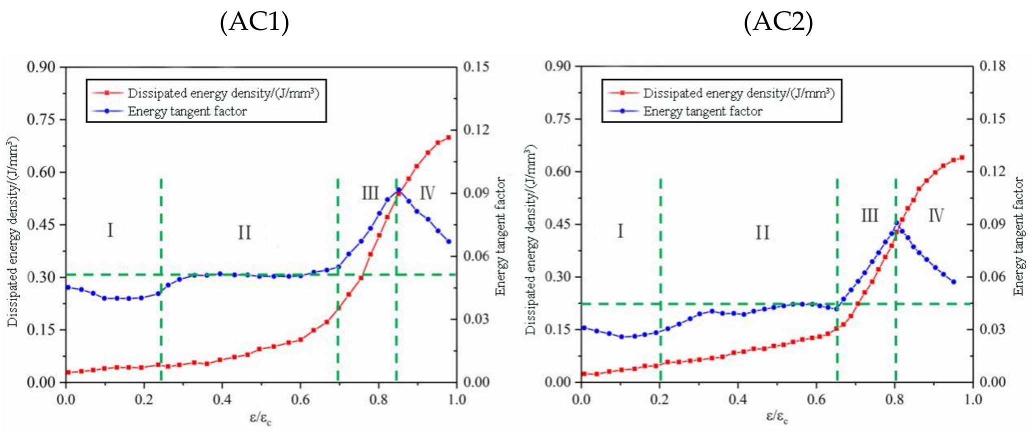

Figure 11. Evolution diagram of energy tangent factor in coal specimen failure process.

4.4. Energy Evolution Model of Coal Fracture

4.4.1. State Equation of Energy Damage

It can be seen from the stress–strain curve that the cyclic loading/unloading process had a non-negligible influence on the damage and deformation characteristics of the coal, which then affected the energy state and stability. In recent years, the damage threshold value and damage evolution law under different confining pressures have been obtained based on the damage model [36]. Considering the initial damage and introducing the concept of equivalent modulus to make the model more suitable to the actual damage [17,37] expressed the damage model as a power exponential function of dissipated energy and provided an effective method for evaluating fatigue damage. The concept of the damage variable has been introduced into the existing research, but the relationship between it and cyclic loading/unloading times has been rarely studied.

Based on the nonlinear characteristics of coal, this paper tried to analyze the influence of constant amplitude cyclic loading/unloading times on coal damage from the perspective of energy dissipation, and established the corresponding equation of the energy damage state. However, under the equal amplitude cyclic loading and unloading, the coal sample was subjected to repeated cyclic loading and unloading at the same stress level, which had a non-negligible effect on the damage characteristics and deformation characteristics of the coal sample and then affected the energy state and stability degree of the coal sample. From the previous analysis, there was a closer relationship between the dissipative energy state inside the coal sample and the stability degree of the coal sample. Therefore, we explored the relationship between the damage change patterns of the coal samples under the same stress levels and the number of cycles based only on the dissipative energy change characteristics of the coal samples under repeated cyclic loading conditions. By referring to the research results of the injury variable theory [19,36,37], the damage variable of coal at the same stress level is determined as follows:

$$D = \frac{U_{d_i} - U_{d_j}}{U_{\max} - U_{d_0}} \tag{7}$$

where D is the damage variable of coal under a constant amplitude cyclic loading; U_{d_i} and U_{d_j} are the dissipated energy of cycle i and cycle j at the same stress level, respectively; and $U_{\max}$ and U_{d_0} are, respectively, the maximum dissipated energy and the initial dissipated energy at the same stress level.

Data fitting was performed between the damage variables and cycle times in the process of constant amplitude cyclic loading to obtain the following variation relationship:

$$D = 1 - [1 - (\frac{N}{N_f})^{\frac{1}{1-\alpha}}]^{\frac{1}{1+\beta}} \tag{8}$$

where N and N_f are, respectively, the cycle times and maximum cycle times at the same stress level; β is a constant; and α is the stress index of axial load, which can be calculated by

$$\alpha = 1 - \gamma(\frac{\sigma_{\max} - \sigma_f}{\sigma - \sigma_{\max}}) \tag{9}$$

where $\sigma_{\max}$ is the maximum axial stress; σ_f is the yield stress threshold; and σ is the stress under any strain state of coal.

As shown in Figure 12, the fitting results show that this model had a good correlation with the experimental data, which could accurately describe the decreasing trend of the damage variable with cycle times.

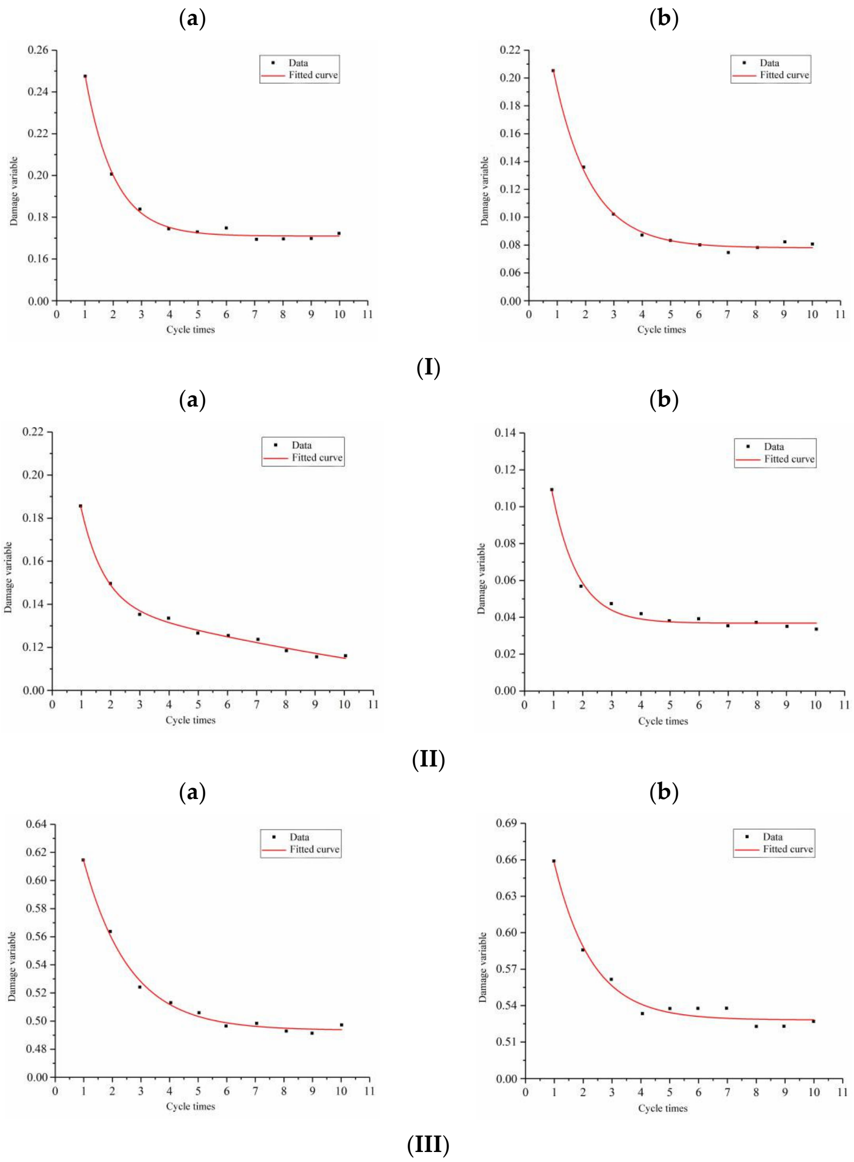

Figure 12. Relationship between damage variable and times of cyclic loading/unloading. (**a**,**b**) are the fitted curves under cyclic loading/unloading with equal amplitude at different (**I**–**III**) stages.

4.4.2. Evolution Equation of Dissipated Energy

By analyzing the relationship between the stress–strain, uniaxial compressive strength, loading rate, and energy dissipation rate [38,39], the dissipated energy state is taken as an important index to judge coal failure. In this paper, the evolution law of dissipated energy was verified by analyzing the evolution characteristics of the energy consumption ratio in the process of coal fracture. The results show that the energy damage evolution equation of the dissipated energy parameter could better describe the energy state and stability of the coal under cyclic loading [40].

Under the cyclic loading/unloading, the energy state and damage characteristics of coal are closely related to the stress level. Coal is composed of micro-structural units, and their state determines the stability of coal. Assuming that the micro-structure units and their strain state inside the coal follow the Weibull distribution, the energy damage parameter can be expressed as:

$$D = 1 - k_N e^{[-(\frac{\varepsilon}{\alpha})^m]} \tag{10}$$

where k_N is a parameter related to the cycle times of coal at different stress levels; α is a constant related to the nature of coal; and m is the shape coefficient of the Weibull distribution, which is related to the micro-structural state inside the coal.

According to the previous analysis, the energy accumulation of coal under an axial load has a certain degree of self-inhibition, and its cumulative inhibition effect can be expressed as:

$$U_e = vE_0 \frac{1}{1 + e^{-\varepsilon + \Phi}} \tag{11}$$

where, U_e is the elastic strain energy accumulated inside the coal; and E_0 is the initial elastic modulus of the coal.

Then, the energy state evolution equation of the dissipated energy in the evolution process of the coal fracture can be deduced as follows:

$$U_d = (1 - v)E_0 \frac{1}{1 + e^{-(\varepsilon - \varepsilon_c)^2} + \zeta} \tag{12}$$

where ε_c is the peak strain variable of coal; and ζ is a constant related to the nature of coal.

Based on the data fitting verification, Equation (12) can fit the evolution law of coal dissipation energy well and describe its abrupt evolution characteristics.

4.4.3. State Equation of Coal Failure Strength

According to the law of thermodynamics, energy conversion is the essential characteristic of the physical process of matter [8]. Based on the analysis of damage evolution, the relationships of the dissipated strain energy to the stress–strength ratio, loading rate, and temperature were studied [18,41], which better predicted the fatigue life of the material. Based on the theoretical analysis of energy dissipation, we defined the concepts of unit dissipated energy, releasable strain energy, strength loss, and overall damage, and we proposed a rock strength criterion [7]. However, coal is a heterogeneous material with high voidage. Under the action of load, the change in external factors directly affects the formation rate of internal micro-defects and further causes the instability failure of coal. Therefore, from the perspective of internal micro-defects to explore the relationship between the strength characteristics and energy state, the energy failure criterion of coal failure can be obtained more directly.

Under the action of equal axial load, the generation rate of micro-defects in coal can be expressed as

$$v_\varepsilon = Ae\left(-\frac{U_0 - \gamma U_d(\varepsilon)}{\alpha}\right) \tag{13}$$

where A, γ, and α are the parameters related to the nature of coal; U_0 is the lowest activation energy required for the generation of internal micro-defects; and $U_d(\varepsilon)$ is the dissipated energy in the coal.

By integrating Equation (13), the cumulative total amount of internal micro-defects is

$$V(\varepsilon) = A\frac{\alpha}{\gamma}e^{(-\frac{U_0}{\alpha})}[e^{(\frac{\gamma\varepsilon U_d(\varepsilon)}{\alpha})} - 1] \tag{14}$$

Therefore, when the coal reaches its failure load σ_c, the relationship between σ_c and internal energy of coal is as follows:

$$\sigma_c = \frac{1}{\gamma}[\alpha\ln(\frac{\gamma\varepsilon}{\alpha}U_d(\varepsilon) + e^{-\frac{U_0}{\alpha}} + U_0)] \tag{15}$$

In fact, when the coal reaches the yield stress threshold σ_f, it has already entered the critical failure state. Therefore, the dissipated energy $U_{fd}(\varepsilon)$ is taken as the key warning point of coal failure when the stress reaches σ_f, and, similarly, the dissipated energy $U_{cd}(\varepsilon)$ is taken as the fracture point of coal when the stress reaches σ_c.

Based on the relationships among the equations of the energy state, damage state, and strength state, an early warning model of coal failure based on energy dissipation was established:

$$\begin{cases} D = \frac{U_{d_i} - U_{d_j}}{U_{\max} - U_{d_0}} \\ U_d = (1-v)E_0\frac{1}{1+e^{-(\varepsilon-\varepsilon_c)^2}+\zeta} \\ \sigma_c = \frac{1}{\gamma}[\alpha\ln(\frac{\gamma\varepsilon}{\alpha}U_d(\varepsilon) + e^{-\frac{U_0}{\alpha}}) + U_0] \end{cases} \tag{16}$$

Additionally, some problems will require further in-depth investigation: (1) The test results in this paper are only at the laboratory scale, so the applicability of dynamic disasters on site such as rock burst should be further verified. (2) According to the compressive strength of the sample, it was necessary to further refine the stress loading gradient and increase the cycle number to improve the accuracy of the tests.

5. Conclusions

Based on experimental investigation and theoretical analysis, the energy evolution laws of coal cyclic loading–unloading processes were obtained, and an energy evolution model of coal fracture based on energy dissipation was established. The following conclusions were obtained:

(1) The concept of the energy tangential factor was proposed to characterize the energy storage level of coal at different deformation stages. The relationship between the energy growth rate and the stress–strain could be better described by quantitative analysis. The results show that the energy excitation–inhibition effect existed in the energy evolution process of coal, which showed that the evolution curves of the total strain energy, elastic strain energy, and dissipative strain energy exhibited significant stage features.

(2) Combined with the mutation theory, the sudden-increase effect of the coal dissipated energy was deduced theoretically, which better verified the damage evolution rule of coal. Dissipated energy had an obvious burgeoning trend that could be used as a precursor to coal fracture and instability. According to the analysis of energy consumption characteristics, the energy dissipation behavior was closely related to different stages of stress loading.

(3) An improved theoretical model that included the relationships among the energy state, damage state, and strength state was established to judge the damage stages and stability degrees of coal. By introducing the damage variable, the relationship between the energy dissipation behavior and coal damage was better described, which realized the early warning of coal failure and instability.

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