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The Application of Fuzzy Mathematics in the Optimization of the Recipe of Filling Paste for Coal Mine Backfill

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Abstract: Backfill is a very important technology that can be used to reduce the environmental footprints resulting from coal mining. The selection of proper filling materials is of great significance to the operation cost and the stability of the goaf. This paper investigated the feasibility of using the coal gangue as the main component of the filling paste so as to reuse the byproducts in coal mining to the maximum extent. The filling pastes were composed of coal gangue as the aggregates, cement or gypsum as cementitious materials, and some additives. In order to determine the optimal recipe, the performances of filling pastes were first comprehensively evaluated according to their fluidity, mechanical properties, shrinkage, and permeability. The results showed that cement content was the most influential factor, while the fly ash addition was the weakest factor for the performance of filling pastes. Moreover, the appropriate use of a water reducer and expansion agent improved the working performance of the paste. Based on the performances of filling pastes, the fuzzy mathematics evaluation method was then used to establish the weight vector and index vector. The principle of maximum membership degree and the principle of maximum closeness were used to identify the identified objects and find the best recipe for the filling paste. The results showed that this evaluation method could fully reflect the influence of various factors and provide accurate evaluation results.

Keywords: coal mine paste; filling material; mechanical properties; fuzzy optimization; fuzzy mathematics

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1. Introduction

With the rapid development of the Chinese economy, the demand for coal resources is growing higher and higher. At present, the amount of buried coal under construction in China is 1.379×10^{10} t, including 9.468×10^{9} t under buildings, 2.417×10^{9} t under railways, and 1.905×10^{9} t under water bodies. With the development of the national economy and the further acceleration of urbanization, the actual amount of buried coal will increase significantly. Therefore, in order to take full advantage of coal resources, it is imperative to use effective technical methods to maximize the exploitation of coal under buildings (structures) while ensuring the safety of the surface buildings (structures) [1,2].

Due to the exploitation of underground resources, the stress equilibrium state in the original rock is changed, and the stress is redistributed, which causes the movement and deformation of the overlying strata and the surface, and damages the stability of the buildings and structures on the surface. Under the influence of mining subsidence, a series of damages occur to surface buildings and structures. The horizontal deformation of the surface can cause tensile cracks in buildings. The tilt deformation of the surface leads to uneven settlement of the surface. For buildings and structures, uneven settlement of the foundation will occur, resulting in a tilt of the building itself, threatening its stability and normal function.

In view of the urgency of this situation, Qian M. proposed the concept of green mining [3], which focused on the green mining technology system. Eker H [4] and Mishra [5] studied the physical and mechanical properties of coal mine waste filling mining to promote the innovation and progress of the coal industry. Zhang F. [6] studied the slump and

strength characteristics of paste filling materials and determined the factors affecting the fluidity of paste filling materials using an orthogonal test. Zheng Q. et al. [7] proved that fly ash and marine clay could be used as materials to prepare the coal mine filling paste. Hu N. [8] and Qiao Y. [9] emphasized the significance of the fuzzy mathematics theory in mining.

The backfill quality directly affects the economic benefits of the mine and the safety of life and property of underground workers. Up to now, many mines have suffered from poor filling quality, which leads to practical problems, such as the collapse of the filling body and an increase in filling cost. The optimization of filling paste can effectively improve the filling effect, accelerate the progress of underground mining, increase the economic benefits of the mine, and provide an important theoretical basis for underground filling [10]. According to the mechanical properties and flow properties of the filling slurry, the factors that affect the filling effect of the coal mine paste include the early and late slump, compressive strength, creep resistance, shrinkage resistance, and permeability resistance of the filling body. Therefore, using the indoor test, the above test data should be analyzed to optimize the recipe of the filling body [11].

The qualitative evaluation method for a qualified filling paste greatly relies on the knowledge, experience, observation, understanding, and changing rules of the evaluators. It is highly subjective, and the accuracy of the evaluation may not be high. Quantitative evaluation methods must have sufficient data support [12]. Simple qualitative analysis is relatively superficial due to insufficient data, so quantitative evaluation is difficult to effectively use and test. Therefore, a combination of qualitative and quantitative methods is required to analyze and evaluate them to make up for the shortcomings of the two. Fuzzy mathematics evaluation is a good method for judging multi-factor and complex, multi-level problems [13].

In this paper, combined experimental research and fuzzy mathematical calculation were therefore used to analyze the work performance of pastes for coal mine backfill, and then a fuzzy mathematical evaluation model was established accordingly. Finally, the recipe for filling material was optimized through fuzzy optimization. This paper provides a basis for selecting a reasonable filling mining method in the future.

2. Experimental Design

2.1. Preparation of Paste Filling Material

At present, there is no relevant experimental procedure standard for coal mine filling materials. Therefore, the experimental mainly referred to the 'Mortar basic performance test method standard' (JGJ/T70-2009) [14] and the 'Ordinary concrete mixture performance test method standard' (GB/T50080-2002) [15].

Coal mine filling paste is an ideal material with good fluidity, uniformity, and low water consumption. In the experiment, coal gangue was used as the main aggregate, which is rich in source, convenient for drawing materials, and low in price while reducing the harm caused by the stacking of gangue on the ground. Cement and gypsum are cementitious materials that can improve stability. They have a very important impact on the physical and mechanical properties of the paste before and after solidification. Water reducer, expansive agents, and fly ash are usually used as additives in experiments. In the experiment, coal gangue was used as the main aggregate, and cement and gypsum were used as cementitious materials to improve stability. Additives included the water reducer, expansion agent, and fly ash. The details of raw materials are as follows:

(1) The natural coal gangue of a coal mine was sampled on site, and the filling aggregate with particle size less than 25 mm was prepared by hammer crushing, jaw crusher crushing, and vibrating screen screening. Coal gangue with particle size less than 25 mm acts as coarse aggregate. The chemical composition of coal gangue included SiO_2, Al_2O_3, Fe_2O_3, CaO, MgO, Na_2O, K_2O, SO_3, and P_2O_5 [16].

(2) Ordinary Portland cement (P-O) was used as cement; its main components are clinker, gypsum (80–95%), and granulated blast furnace slag (5–20%). The bulk density is

300 kg/m^3, and the specific surface area is generally 300 m^2/kg. The fineness requirement of cement is that the sieve residue of 80 μm square hole sieve is not more than 10%, and the sieve residue of 45 μm square hole sieve is not more than 30%. The initial setting time of ordinary Portland cement should not be less than 45 min, the final setting time should not be more than 600 min, and the water–cement ratio is 0.5. The main chemical compositions in cement included 3CaO·SiO_2 (C3S), 2CaO·SiO_2 (C2S), 3CaO·Al_2O_3 (C3A), and 4CaO·Al_2O_3·Fe_2O_3 (C4AF) [17]. The measured 3 d strength was 18.96 MPa, and the 28 d strength was 44.59 MPa. The strength was higher than the theoretical strength.

(3) The relative density of gypsum was 2.96 g/cm^3. The main component was $CaSO_4$. The main parameters of gypsum are shown in Table 1 [17].

Table 1. Performance parameter table of gypsum.

Fineness (0.2 mm) Sieve Residue Percentage (%)	Break Off Strength (MPa)	Compressive Strength (MPa)	The Initial Setting Time (min)	Final Setting Time (min)
10.0	2.1	3.9	≥6	≤30

(4) Water reducer was an extremely necessary and important additive for the filling material. The main parameters are shown in Table 2.

Table 2. Parameter table of water reducer.

Water Reduction	Improved Strength	Slump Loss	Alkali Content	Reduced Cement Usage	Dosage
30%	30%	6%	0.15%	16%	0.4–2.0%

(5) The expansion agent was used to limit the formation and development of cracks, effectively improve the impermeability of the paste, and improve the stability of the entire goaf. The parameters are shown in Table 3.

Table 3. Expansion agent parameter table.

Dosage	Alkali Content	Particle-Size
6–8%	0.3–0.4%	<200 mesh

(6) The main chemical compositions of fly ash were SiO_2, Al_2O_3, FeO, Fe_2O_3, CaO, TiO_2, MgO, K_2O, Na_2O, SO_3, MnO_2, etc. [17]. In this experiment, fly ash was used as additive, and the physical parameters are shown in Table 4.

Table 4. Physical parameters of fly ash.

Density (g/cm^3)	Bulk Density (g/cm^3)	Specific Surface Area (cm^2/g)	Standard Consistency of Raw Ash (%)	Water Absorption Capacity (%)	28 d Compressive Strength Ratio (%)
1.9–2.9	0.531–1.261	800–19,500	27.3–66.7	89–130	37–85

In this experiment, five groups of coal mine filling materials with different proportions were tested. The content of expansive agent in the filling materials was used as an invariant. Using the method of single variable control contrast test, the contents of coal gangue were the same in each group, but the contents of expansive agent, water, fly ash, gypsum, cement, and water-reducing agent were appropriately increased. The filling paste was prepared by manual mixing. The compositions of qualified filling materials are shown in Table 5.

Table 5. Paste filling body experimental material ratio table.

Factor Level / Peer Group	Flyash: Coal Gangue	Gypsum: Coal Gangue	Cement: Coal Gangue	Water Reducing Admixture: Cement, Gypsum	Expansion Agent: Cement
1	3:5	1:10	4:25	1:13	7:8
2	5:5	1:10	4:25	1:13	7:8
3	3:5	2:10	4:25	1:13	7:8
4	3:5	1:10	6:25	1:13	7:8
5	3:5	1:10	4:25	2:13	7:8

2.2. Basic Mechanical Properties and Analysis Results of Filling Materials

Taking the slump, uniaxial compressive strength, creep resistance, shrinkage resistance, and permeability as the main technical indicators, the performance of coal mine filling pasted was evaluated to determine the optimized coal mine filling paste in goaf, accumulate experience in the development of coal mine paste filling technology, and promote the development of coal mine filling technology industry.

(1) The slump is the main index used to evaluate the fluidity, water retention, and cohesiveness of the paste. The fluidity of the filling body is very different. Some have almost no deformation under the action of self-weight, and some show very high fluidity, like fluid; that is, they have self-flow characteristics. The slump experiment was carried out after a setting time of 15 min, 30 min, 45 min, and 60 min under room temperature (20 °C). The curing time of the filling body was 6–8 h in order to reflect its change rule conveniently and intuitively, as shown in Figure 1.

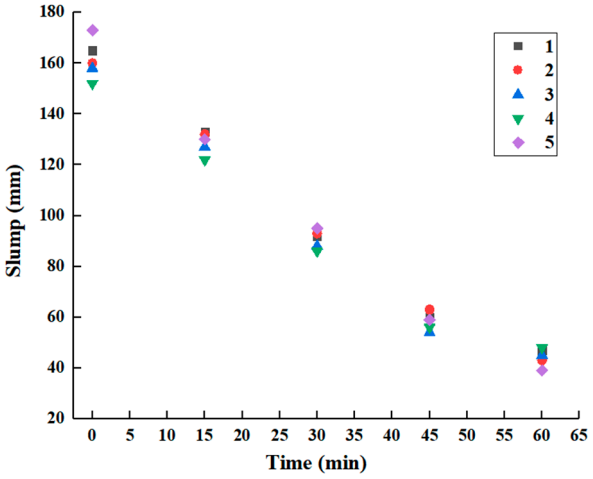


Figure 1. The change in slump with time.

According to the change trend of Figure 1, the slump of filling pastes with different ratios decreases significantly with the extension in setting time. The minimum value of slump of filling pastes is 152 mm, which meets the requirement that the slump of the paste should be greater than 150 mm [18]. The slump experimental values of the first, second, and fifth groups are larger in the early stage and change faster than other groups over time. After 30 min, there was no significant difference in the slump of the five groups, and the development of slump of each group as the extension of static time was basically the same. The slump values of groups 1, 2, and 4 in the later period are ideal, and the change in slump of the third group was slighter than that of the other groups. The analysis shows that the water-reducing agent has a great influence on the slump. With the increase in content of cementitious materials (cement and gypsum), the fluidity of the paste is improved. The greater the amount of cement, the thicker the slurry layer wrapped on the surface of the aggregate particles. This brings about better lubrication effect and reduces the friction

between the aggregates, and thus the fluidity of paste becomes better. Gypsum is mainly used to adjust the setting time of paste. With the increase in gypsum, the fluidity of paste will decrease [19]. However, increasing the addition of fly ash has little effect on the slump of the paste.

(2) Since the goaf paste mainly bears the pressure of the overlying strata, the compressive strength is an important performance. The compressive strength of the filling paste was determined on a [20] 150 mm × 150 mm cube. The test was carried out on the NYL-300 universal pressure tester with a maximum load of 300 kN and a precision of 0.1%. The loading speed was 0.3–0.5 MPa per second. The experimental results are shown in Figure 2.

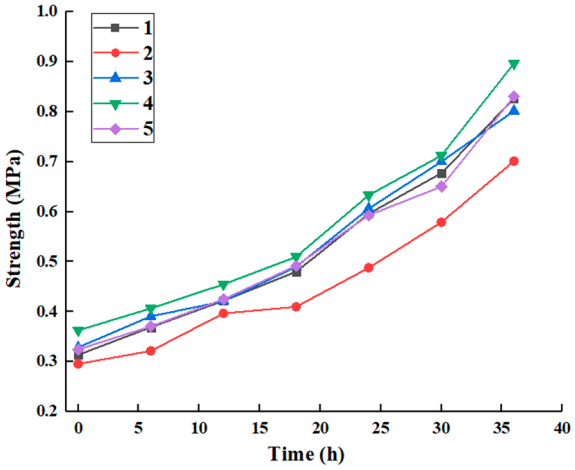


Figure 2. Early strength comparison curve.

According to the results, the strength of the filling body will gradually increase with time, and the later strength will change more obviously than the early strength. The excessive fly ash will reduce the strength of the paste. Cement is the most important factor affecting the strength of the paste. Regardless of early strength or later strength, the strengths of group 4 were the largest, while the strengths of group 2 were the smallest. The analysis shows that the fineness of fly ash is small, which is close to the fineness of slurry aggregate. The promotion effect of fly ash on cementitious materials is limited, which is the main reason for its failure to improve the strength index of slurry. The strengths of groups 3 and 4 are relatively higher. Therefore, the most important factor affecting the strength of the paste is the content of cementitious material [21]. Due to the particle size of the aggregate, the contribution of the aggregate to the strength is limited, and the strength of the paste is mainly determined by the characteristics of the cementitious material.

(3) Creep refers to the trend of slow and permanent movement or deformation of solid materials under the influence of stress [22]. The creep of filling paste was tested on the electro-hydraulic servo rock triaxial testing machine XTR01-01 with a maximum load of 2000 KN. The applied loading method was uniaxial creep loading, and the loading strength was 1.2 MPa according to the actual engineering situation. The results are shown in Figure 3.

The experimental results of the five groups of different components did not show unstable creep in the creep test, indicating that the five kinds of filling pastes can ensure the stability of the goaf after filling. Among them, the creep value of group 5 is the smallest, and the creep value of group 3 is the largest. In terms of its creep resistance, group 5 has excellent creep resistance. Compared with other groups, it is a superior material for meeting the filling performance of paste in goaf. Further analysis shows that cement has a great effect on the creep resistance of the paste. The addition of fly ash and gypsum will reduce the creep resistance of the paste, and the water-reducing agent has no effect on the creep resistance of the paste [23].

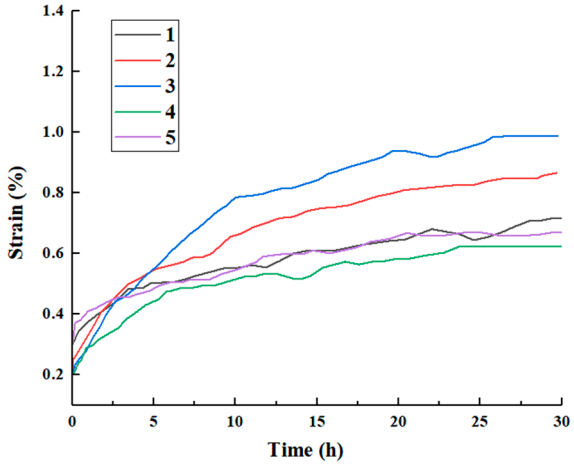


Figure 3. Creep curve diagram.

(4) Like concrete, the filling material of coal mine will produce certain shrinkage [24]. Shrinkage cracks not only affect the bearing capacity but also provide channels for the flow and infiltration of surface water. The shrinkage of the filling paste was thus tested by the concrete strain gauge with the data acquisition instrument. During the experiment, the heat source and cold source around the experimental environment were removed to reduce the slight change in the strain gauge caused by the vibration interference. The purpose of this experiment is to provide favorable suggestions and improvements for the selection of coal gangue backfill paste. The results are shown in Figure 4.

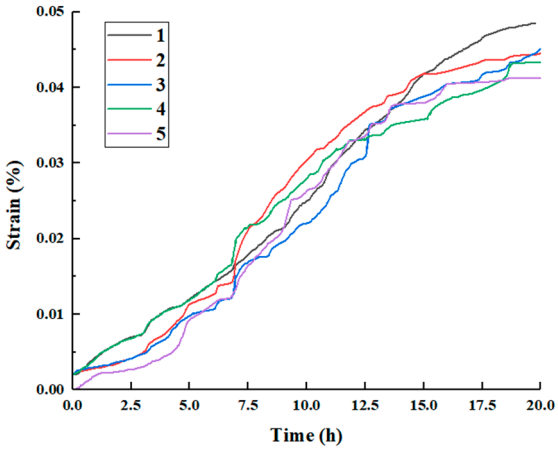


Figure 4. Shrinkage diagram of paste material with time.

The experimental results show that after 24 h, the shrinkage reaches equilibrium. Group 5 presents minimum shrinkage in the early and late stages. Group 1 has the greatest shrinkage and therefore has the highest possibility of cracking in practical engineering applications. The shrinkage of group 2 in the medium term is obviously larger than those of the other groups, which indicates that the contribution of fly ash to the shrinkage is not significant. This may be related to the fact that water-reducing agent in group 5 reduced water usage. The largest shrinkage of group 1 is the result of high content of coal gangue, which leads to serious water loss. The shrinkage performance of ratio 3 and ratio 4 is general. In comparison, the cement content of ratio 4 is slightly greater, and its shrinkage performance is more stable than that of ratio 3.

(5) The existence of groundwater is a threat to the goaf. If the filling paste is highly permeable, the invasion of groundwater may pose a threat to the stability of the goaf after filling. Therefore, the permeability of the filling paste should be investigated to provide a

guarantee for the stability of the goaf [25]. The experiment was carried out on a permeation test machine with a pressure of 1.2 MPa for 8 h. The experimental results were plotted, as shown in Figure 5.

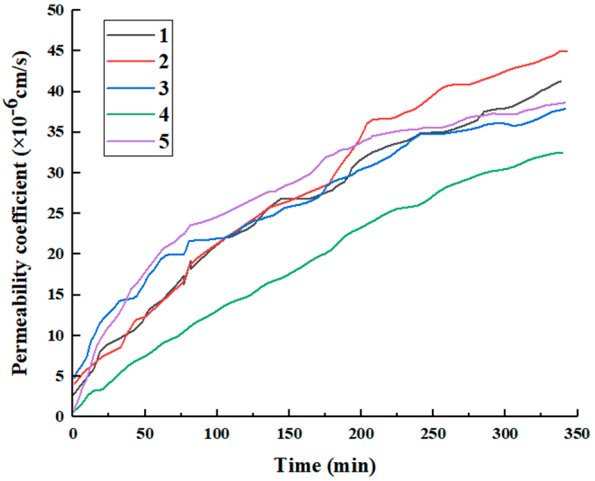


Figure 5. Curve of permeability test results of paste filling body.

During the permeability test, the permeability coefficient of the five groups changed with time, and their ability to resist the deformation caused by water flow infiltration under the influence of groundwater surface water was analyzed. The permeation test shows that the permeation law of the filling paste meets the Darcy permeation law [26], and its impermeability gradually increases with time. The analysis of the results shows that group 4 has the lowest permeability coefficient, and the permeability coefficients of group 1, group 3, and group 5 are basically at approximate level. The permeability coefficient of group 2 is much larger than those of other groups. Specifically, the permeability of group 2 becomes significantly larger after 170 min. Further analysis of the results shows that the content of cementitious material is the most important factor affecting the permeability of filling paste. The larger the content of cementitious material, the smaller the permeability of filling paste. Group 4 has the best permeability performance and is a superior material for meeting engineering needs.

3. Optimization of Paste Filling Material Based on Modulus Mathematical Theory

3.1. Fuzzy Mathematics Theory

According to the existing experimental data, optimizing a filling formula with superior performance is an urgent problem for engineers. People usually use experience and subjective judgment to solve this key problem. At present, the most common optimization method is to rely on mathematical methods for comprehensive analysis and evaluation. It has strong practicability, strong recognition and judgment ability, and accurate calculation results. It is an evaluation method with a high adoption rate in the evaluation system. The establishment of a perfect evaluation system is a huge task. The selection of evaluation factors also necessitates high requirements for the establishment of the system. How to establish a perfect and realistic evaluation system is a difficult task.

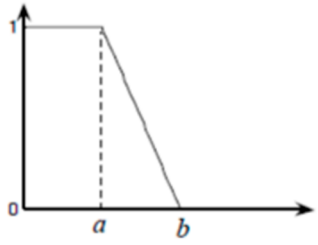
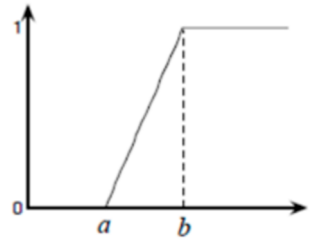
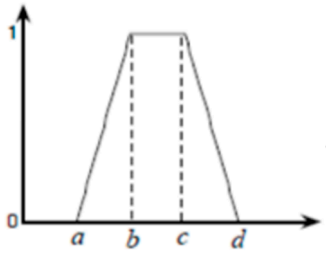
Fuzzy mathematics theory is a good method for solving multi-factor and complex, multi-level problems. It is a process of using fuzzy thinking to mathematically express the cognition of things [27]. It is feasible to use a fuzzy comprehensive evaluation method to optimize coal mine paste filling materials. Fuzzy sets are the most commonly used means to identify fuzzy relations. The functional relationship is mainly based on the membership function relationship, and the weight index is also extremely important. The fuzzy recognition steps are as follows:

(1) Select the characteristic factor set U of modulus, and obtain several different factor subsets:

$$U = \{U_1, U_2, \cdots U_m\} \tag{1}$$

(2) Establish fuzzy recognition and membership relationship, and use the trapezoidal function to express membership function (Table 6):

Table 6. Trapezoidal membership function expression.

Smaller type		$A(x) = \begin{cases} 1 & x < a \\ \frac{b-x}{b-a} & a \leqslant x \leqslant b \\ 0 & b < x \end{cases}$ (2)
Larger type		$A(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leqslant x \leqslant b \\ 1 & b < x \end{cases}$ (3)
Intermediate type		$A(x) = \begin{cases} \frac{x-a}{b-a} & a \leqslant x < b \\ 1 & b \leqslant x < c \\ \frac{d-x}{d-c} & c \leqslant x \leqslant d \\ 0 & x < aord < x \end{cases}$ (4)

(1) Determine the index weight of each factor $U_n(n = 1, 2, \cdots m)$ and evaluate the classification of each factor subset:

$$A_n \times R_n = B_n \tag{5}$$

The fuzzy evaluation valve limit refers to an artificially set value to limit the evaluation of whether an action plan can meet the predetermined expected effect. The provisions below the valve limit do not meet the requirements, and vice versa. In this paper, a multi-valve limit value was adopted [28,29], and the valve limit value $\alpha = 0.35$, $\beta = 0.55$ was set through expert demonstration. It is stipulated that $A \times B < \alpha$ is not satisfied with the filling requirements, $\alpha < A \times B < \beta$ is further verified to meet the filling requirements, and $A \times B > \beta$ is satisfied with the filling requirements.

(2) Use the principle of maximum membership degree and the principle of maximum closeness to identify the identified objects [30].

The selection of factors in fuzzy mathematics theory directly affects the evaluation of results. In order to ensure the accuracy, objectivity, and practicability of the evaluation results, the basic principles of systematicness and universality must be followed when selecting the influencing factors. The indexes include early slump U_1, late slump U_2, early strength U_3, late strength U_4, creep resistance U_5, shrinkage resistance U_6, and permeability resistance U_7.

3.2. Establishment of a Fuzzy Rating System

(1) Determination of indicators

The evaluation interval of each factor can be divided into four levels: excellent (0.9), good (0.7), medium (0.5), and poor (0.3) [31]. Because the later slump value has an important contribution to the flow of the paste in the goaf after pipeline transportation, filling the special-shaped voids and repelling bubbles, the slump is divided into two indexes: early slump and later slump. The uniaxial compressive strength is the compressive strength of a material under unconfined conditions. Since the paste-filling process is a continuous process, the index is divided into early strength and late strength. The anti-creep is to describe the ability of the paste to resist the deformation or failure of the long-term load. It is an indicator of the long-term stability of the goaf. The anti-shrinkage characteristic is the property whereby the paste filling body resists volume shrinkage in the process of hardening. The mechanism of crack formation of paste is studied by using the later safety observation, pathological prevention, and hazard evaluation of the paste filling body. Anti-permeability is a good way to consider the influence of groundwater and surface water in the process of paste filling. Therefore, these seven evaluation indexes will be selected, which have certain engineering practical significance. The scores of each evaluation index are detailed in Table 7.

Table 7. Summary of scores of each evaluation index.

Index Factor Score	Matching				
	1	**2**	**3**	**4**	**5**
Early slump	0.7	0.6	0.5	0.4	0.8
Late slump	0.7	0.5	0.6	0.8	0.4
Early strength	0.6	0.4	0.7	0.8	0.5
Late strength	0.5	0.4	0.7	0.8	0.6
Anti-creep characteristics	0.6	0.7	0.8	0.4	0.5
Anti-shrinkage characteristics	0.7	0.4	0.5	0.8	0.6
Anti-permeability characteristics	0.5	0.4	0.7	0.8	0.6

(2) Construction of vector

A fuzzy vector is a mathematical tool for fuzzy mathematics operation. In the process of fuzzy recognition, fuzzy clustering analysis, and fuzzy reasoning, the same operator is used to perform the same operator operation on a set. Therefore, the expression of the following vector is adopted:

$$B = (\mu_B(\mu_1), \mu_B(\mu_2), \mu_B(\mu_3), \mu_B(\mu_4), \mu_B(\mu_5), \mu_B(\mu_6), \mu_B(\mu_7)) \quad (6)$$

B represents different proportions; $\mu_B(\mu_i)(i = 1, 2, \cdots, 7)$ represents the score of an index. To include the following:

$$\begin{aligned} B_1 &= (0.7, 0.7, 0.6, 0.5, 0.6, 0.7, 0.5) \\ B_2 &= (0.6, 0.5, 0.4, 0.4, 0.7, 0.4, 0.4) \\ B_3 &= (0.5, 0.6, 0.7, 0.7, 0.8, 0.5, 0.7) \\ B_4 &= (0.4, 0.8, 0.8, 0.8, 0.4, 0.8, 0.8) \\ B_5 &= (0.8, 0.4, 0.5, 0.6, 0.5, 0.6, 0.6) \end{aligned} \quad (7)$$

where B_1, B_2, B_3, B_4, B_5 are the scores corresponding to the ratio of 1–5, respectively.

The most critical step in the fuzzy evaluation is the setting of the fuzzy weight A. The setting of the weight is a reflection of the system's extremely high requirements for the professional level. In the process of setting the weight, the system builder needs to have extremely high requirements for professional knowledge. It is also necessary to dabble in the relevant professions. To design an evaluation system by oneself, it is necessary to consult relevant experts and scholars and experienced engineering practitioners. According to the suggestions they give, the weight value of the index is considered comprehensively,

and the average value of the weight value of the opinions of all parties is adopted. The value of the group's weight is processed. Those who do not understand or do not know the opinion should contact the person who expressed the opinion to analyze and discuss the problem and the optimal solution to this problem. The established system is better, the operator uses the weighted average operator, and the results are shown in Table 8.

Table 8. Evaluation weight.

Project	Slump		Single Axis Compressive Strength		Anti-Creep a_5	Anti-Shrinkage a_6	Anti-Permeability a_7
	Early a_1	Late a_2	Early a_3	Late a_4			
Weight	0.28	0.09	0.22	0.31	0.02	0.03	0.05

The average value in the table reflects the important index of each index, and the weight vector is constructed according to the above table:

$$A = (0.28, 0.09, 0.22, 0.31, 0.02, 0.03, 0.05) \tag{8}$$

3.3. Filling Paste Optimization

A fuzzy optimization is a form of fuzzy evaluation [32]. It is a way to calculate the fuzzy matrix and fuzzy vector according to the non-classical mathematical method so as to obtain an optimal scheme. The evaluation is carried out according to the weighted value accumulation:

$$A \times B_n = \mu_B(\mu_1) \times a_1 + \mu_B(\mu_2) \times a_2 + \mu_B(\mu_3) \times a_3 + \mu_B(\mu_4) \times a_4 + \mu_B(\mu_5) \times a_5 + \mu_B(\mu_6) \times a_6 + \mu_B(\mu_7) \times a_7 \tag{9}$$

The evaluation results of each ratio are as follows:

$$\begin{aligned} A \times B_1 &= (0.7 \times 0.28 + 0.7 \times 0.09 + 0.6 \times 0.22 + 0.5 \times 0.31 + 0.6 \times 0.02 + 0.7 \times 0.03 + 0.5 \times 0.05) = 0.604 \\ A \times B_2 &= (0.6 \times 0.28 + 0.5 \times 0.09 + 0.4 \times 0.22 + 0.4 \times 0.31 + 0.7 \times 0.02 + 0.4 \times 0.03 + 0.4 \times 0.05) = 0.471 \\ A \times B_3 &= (0.5 \times 0.28 + 0.6 \times 0.09 + 0.7 \times 0.22 + 0.7 \times 0.31 + 0.8 \times 0.02 + 0.5 \times 0.03 + 0.7 \times 0.05) = 0.631 \\ A \times B_4 &= (0.4 \times 0.28 + 0.8 \times 0.09 + 0.8 \times 0.22 + 0.8 \times 0.31 + 0.4 \times 0.02 + 0.8 \times 0.03 + 0.8 \times 0.05) = 0.68 \\ A \times B_5 &= (0.8 \times 0.28 + 0.4 \times 0.09 + 0.5 \times 0.22 + 0.6 \times 0.31 + 0.5 \times 0.02 + 0.6 \times 0.03 + 0.6 \times 0.05) = 0.614 \end{aligned} \tag{10}$$

The fuzzy comprehensive scores of groups 1, 3, 4, and 5 are greater than the valve limit $\beta = 0.55$, indicating that all of them meet the filling requirements. The score of group 2 $\alpha = 0.35 < 0.471 < \beta = 0.55$, and it is not recommended to be used as filling paste. Group 4 had an excellent performance in strength and impermeability, and it also obtained the highest score in fuzzy evaluation.

4. Discussions

In this study, the physical and mechanical properties of common cementitious materials and aggregates used in filling mining engineering are introduced in detail. Through experiments, the law of change of five groups of formulations was obtained. Most of these studies regard the cemented body after solidification as an elastic material or elastic–plastic material. Although many scholars have studied the physical and mechanical properties of the filling material, they do not consider the slump loss in the paste flow process, the creep characteristics after solidification, the anti-permeability characteristics, the strength formation change law, and the comprehensive shrinkage characteristics. This is not conducive to the analysis of the long-term characteristics of the filling body in filling mining. Therefore, the research results are different from the engineering practice.

Although there are some pieces of literature on the selection and preparation requirements of paste filling materials, as well as the study of paste workability by adding mate-

rials in different formulations, these studies lack a systematic system. A comprehensive evaluation of the quality of paste workability does not have a good optimization system.

Based on the interdisciplinary theory, this paper studies the optimization of the filling body ratio by means of experimental research and fuzzy mathematical calculation. The above preliminary results show that the established fuzzy mathematical evaluation method can fully reflect the influence of various factors on the optimization of paste material ratio, and the evaluation conclusion is clear and accurate. However, there are many factors affecting the ratio of paste filling body materials. This paper only selected five influencing factors for consideration. There are many limitations in the selection of index factors and scoring standards, so it is necessary to study the various factors affecting the ratio of filling materials and analyze their weights to make them more scientific and reasonable. A greater experimental basis is needed to further verify the effectiveness of the method for ratio optimization so as to apply it to practical fields.

5. Conclusions

This paper investigated the optimal recipe for filling paste for coal mine backfill. The filling pastes were composed of coal gauge as the aggregates, cement or gypsum as the cementitious materials, and other additives. This paper first evaluated the influences of these ingredients on the performance of filling paste based on the slump test, uniaxial compressive strength test, uniaxial creep test, shrinkage test, and permeability test. Using fuzzy mathematics theory, the influencing factors were then constructed into evaluation vectors and fuzzy evaluation to further optimize the recipe of the filling paste. The main conclusions that can be drawn from this paper are as follows:

(1) The contributions of various ingredients to the working performance of the paste were analyzed. Cement had a great influence on the working performance of the paste, while fly ash showed little improvement in the working performance of the paste. The proper use of additives improved the working performance of the paste filling body.

(2) Based on the slump test, creep test, permeability test, comprehensive shrinkage test, and strength test, the evaluation index for fuzzy mathematics analysis was established. The performances of paste filling were proposed as evaluation indexes, and a systematic evaluation system was established through the average weight value of expert opinions.

(3) Based on the analysis of mechanical factors, the evaluation index and system evaluation system were established. The fuzzy mathematical evaluation of the five groups of ratio indicators shows that group 4 has good practical application value. The evaluation method can fully reflect the influence of various factors, and the evaluation conclusion is clear and accurate.

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References

1. Cheng, Y.; Pan, Z. Reservoir properties of Chinese tectonic coal: A review. *Fuel* **2020**, *260*, 116350. [CrossRef]
2. Chen, S.; Song, Y.; Zhang, M. Study on the sustainability evaluation and development path selection of China's coal base from the perspective of spatial field. *Energy* **2020**, *215*, 119143. [CrossRef]
3. Zhang, J.; Zhang, Q.; Spearing, A.S.; Miao, X.; Guo, S.; Sun, Q. Green coal mining technique integrating mining-dressing-gas drain-ing-backfilling-mining. *Int. J. Min. Sci. Technol.* **2017**, *27*, 17–27. [CrossRef]
4. Eker, H.; Bascetin, A. Influence of silica fume on mechanical property of cemented paste backfill. *Constr. Build. Mater.* **2021**, *317*, 126089. [CrossRef]
5. Mishra, M.K.; Karanam, U.M.R. Geotechnical characterization of fly ash composites for backfilling mine voids. *Geotech. Geol. Eng.* **2006**, *24*, 1749–1765. [CrossRef]
6. Zhang, F.; Liu, J.; Ni, H.; Li, W.; Liu, Y. Development of Coal Mine Filling Paste with Certain Early Strength and Its Flow Characteristics. *Geofluids* **2021**, *2021*, 1–14. [CrossRef]
7. Cheng, Q.; Guo, Y.; Dong, C.; Xu, J.; Lai, W.; Du, B. Mechanical Properties of Clay Based Cemented Paste Backfill for Coal Recovery from Deep Mines. *Energies* **2021**, *14*, 5764. [CrossRef]
8. Hu, N.; Li, C.; Liu, Y.; Wu, Y. Fuzzy Mathematics Comprehensive Forecasting Analysis of Metal Mine Rockburst Based on Multiple Criteriahere. *IOP Conf. Ser. Earth Environ. Sci.* **2021**, *632*, 022080. [CrossRef]
9. Qiao, Y. The Application of Fuzzy Mathematics on Classification of Producing Dust Condition in Mining Working Face Different Processes and Selection of Nozzles with Dust Removal Function. *IOP Conf. Ser. Earth Environ. Sci.* **2018**, *170*, 032094. [CrossRef]
10. Chang, G.; Hua, X.; Liu, X.; Li, C.; Wang, E.; Sun, B. Fluidity Influencing Factor Analysis and Ratio Optimization of New Filling Slurry Based on the Response Surface Method. *J. Renew. Mater.* **2022**, *10*, 1439–1458. [CrossRef]
11. Jiang, Z.; Liu, X. Calculation of filling body strength and materials ratio optimization in Chambishi Copper Mine. *Nonferrous Met.* **2017**, *96*, 84–97. [CrossRef]
12. Wang, W.Q.; Xin, X.L. Distance measure between intuitionistic fuzzy sets. *Pattern Recognit. Lett.* **2005**, *26*, 2063–2069. [CrossRef]
13. Konwar, N.; Debnath, P. Some new contractive conditions and related fixed point theorems in intuitionistic fuzzy n-Banach spaces. *J. Intell. Fuzzy Syst.* **2018**, *34*, 361–372. [CrossRef]
14. Approval Department Ministry of Construction of the People's Republic of China. *Standard of Test Methods for Basic Properties of Mortar*; China Construction Industry Press: Beijing, China, 2009.
15. Approval Department Ministry of Construction of the People's Republic of China. *Standard for Performance Test Method of Ordinary Concrete Mixture*; China Construction Industry Press: Beijing, China, 2002.
16. Chuncai, Z.; Guijian, L.; Dun, W.; Ting, F.; Ruwei, W.; Xiang, F. Mobility behavior and environmental implications of trace elements associated with coal gangue: A case study at the Huainan Coalfield in China. *Chemosphere* **2014**, *95*, 193–199. [CrossRef] [PubMed]
17. Ministry of Water Resources of the People's Republic of China. *Technical Specification for Application of Fly Ash Concrete*; China Plan Press: Beijing, China, 1991.
18. Creber, K.J.; McGuinness, M.; Kermani, M.F.; Hassani, F.P. Investigation into changes in pastefill properties during pipeline transport. *Int. J. Miner. Process.* **2017**, *163*, 35–44. [CrossRef]
19. Wang, Z.; Zhao, W. Microscopic Pore and Filling Performance of Coal Gangue Cementitious Paste. *J. Wuhan Univ. Technol. Mater. Sci. Ed.* **2018**, *33*, 427–430. [CrossRef]
20. Wang, Z.; Zou, D.; Liu, T.; Zhou, A. Influence of paste coating thickness on the compressive strength, permeability, and mesostructure of permeable concrete. *Constr. Build. Mater.* **2021**, *299*, 123994. [CrossRef]
21. Walsh, R.M.; Woodmansey, K.F.; Glickman, G.N.; He, J. Evaluation of Compressive Strength of Hydraulic Silicate-Based Root-End Filling Materials. *J. Endod.* **2014**, *40*, 969–972. [CrossRef] [PubMed]
22. Sun, Q.; Li, B.; Tian, S.; Cai, C.; Xia, Y. Creep properties of geopolymer cemented coal gangue-fly ash backfill under dynamic disturbance. *Constr. Build. Mater.* **2018**, *191*, 644–654. [CrossRef]
23. Königsberger, M.; Irfan-Ul-Hassan, M.; Pichler, B.; Hellmich, C. Downscaling Based Identification of Nonaging Power-Law Creep of Cement Hydrates. *J. Eng. Mech.* **2016**, *142*, 04016106-1–04016106-11. [CrossRef]
24. Fall, G.M. Coupled thermo-hydro-mechanical-chemical behaviour of cemented paste backfill in column experiments. Part I: Physical, hydraulic and thermal processes and characteristics. *Eng. Geol.* **2013**, *164*, 36–43.
25. Kondraivendhan, B.; Bhattacharjee, B. Prediction of Strength, Permeability, and Hydraulic Diffusivity of Ordinary Portland Cement Paste. *ACI Mater. J.* **2014**, *111*, 171–178. [CrossRef]
26. Zhou, F.; Hu, X.Y.; Meng, Q.X.; Hu, X.D.; Liu, Z.Y. Model and methods for evaluation reservoir permeability using mud invasion effect. *Appl. Geophys.* **2015**, *4*, 482–492. [CrossRef]
27. Anastassiou, G.A. Fuzzy Mathematics: Approximation Theory. *Stud. Fuzziness Soft Comput.* **2010**, *251*, 77–86.
28. Wang, H.L.; Zhang, Z.; Huang, C.L.; Shi, L. A fuzzy optimum model to assess bridge design options. *Bridge Eng.* **2006**, *159*, 9–15. [CrossRef]
29. Zhang, X.Y.; Fan, Y.R.; Yang, J.L. Feature selection based on fuzzy-neighborhood relative decision entropy. *Pattern Recognit. Lett.* **2021**, *146*, 100–107. [CrossRef]
30. Zhao, X.F.; Liu, Y.B.; He, X. Fault Diagnosis of Gas Turbine based on Fuzzy Matrix and the Principle of Maximum Membership Degree. *Energy Procedia* **2012**, *16*, 1448–1454. [CrossRef]

31. Li, T.G.; Yang, B. Study on green logistics operation system of port based on AHP-fuzzy comprehensive evaluation. *IEEE* **2010**, *1*, 175–178.
32. Wang, H.L.; Qin, S.F.; Zhang, Z.; Huang, C.L. Fuzzy optimum model of semi-structural decision for bridge lectotype. In Proceedings of the International Conference on Fuzzy Systems & Knowledge Discovery, Yantai, China, 10–12 August 2010.

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