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Non-Extensive Statistical Mechanics in Acoustic Emissions: Detection of Upcoming Fracture in Rock Materials

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Abstract: Acoustic emission (AE), recorded during uniaxial compressive loading with constantly increasing stress and stepped stress increments until the fracture of prismatic marble specimens, were analyzed in terms of non-extensive statistical mechanics (NESM). Initially introduced by Tsallis, NESM has proven to be an autonomous robust theoretical framework for studying fracture mechanisms and damage evolution processes during fracture experiments in specimens made of brittle materials. In the current work, the time intervals of the recorded AE data are analyzed in terms of NESM. For each examined specimen, the corresponding q entropic indices and the β_q parameters were calculated, and their variability in terms of the stress applied were studied. Furthermore, a possible linear relationship between the entropic index q and the parameter β_q was examined, and it was investigated whether the observed deviation from monotonicity between q and β_q may signal increased accumulation of damage, eventually leading to the final fracture of the specimens. Through this work, the emergence of an additional pre-failure indicator (i.e., the deviation from monotonicity between q and β_q) alongside well-established ones can provide further insight regarding the underlying crack development mechanisms and damage accumulation processes during the fracture of rock materials.

Keywords: acoustic emissions; non-extensive statistical mechanics; Tsallis entropy; uniaxial loading; pre-failure indicators



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1. Introduction

Fracture, in brittle and heterogeneous materials, is characterized by complex processes. At the laboratory scale, the recording of acoustic emissions (AE) has been used to study the response of mechanically stimulated materials that are gradually driven to fracture. The study of AEs seeks to understand the mechanisms of development and propagation of microcracks within the material, as well as their interactions during the formation of macrocrack networks in mechanically stimulated materials [1].

The acoustic emission technique was founded around the detection of transient elastic waves produced due to a momentary invocation of strain energy during the creation and propagation of cracks. These waves travel spherically through the material towards its boundaries, where they are detected and recorded by special equipment. Although almost a century has passed since the introduction of the AE technique [2,3], it is still undergoing further development [4] and has gradually become one of the most mature and reliable non-destructive testing techniques.

Many parameters and consequent statistical quantities related to acoustic signals have been used to study crack growth mechanisms and fracture processes within the material, providing useful information regarding its proximity to fracture. Properties such as cumulative energy, the cumulative counts, the frequency of occurrence of acoustic hits, the energy release rate, the distribution of acoustic amplitudes, the relation between the rise time per amplitude of the signals and their average frequency, the b-value analysis, etc., are

some of parameters that are studied [5–11]. By utilizing the AE recordings during loading in rock materials, it has been observed that when the stress reaches a critical point, the acoustic activity increases dramatically [12,13]. In addition, the amplitude-hit correlation can be used as one of the criteria for brittle or ductile failure [14].

Although different in scale, the phenomenon of acoustic emissions can be considered a process analogous to seismicity, as it has been found to exhibit similar statistics [15,16]. Consequently, it can be reasonably assumed that seismicity analysis methodologies can be applied to the analysis of AEs after relevant modifications. Following this analogy, non-extensive statistical mechanics (NESM) are used herein to analyze the recorded acoustic data. NESM was firstly introduced by Tsallis almost 30 years ago, and is a generalization of Boltzmann–Gibbs (BG) statistical mechanics [17–19] based on the concept of non-additive entropy. NESM has become a reliable theoretical framework for the study of complex systems in equilibrium and non-equilibrium states, systems exhibiting multifractality, self-similar structures, systems with long-range correlations, systems that show memory effects, etc. [20,21]. Furthermore, NESM has been applied in several scientific fields (for applications, see Paragraph 1 of ref. [22] and the references cited therein). However, in the context of the present work, NESM applications have been used for the analysis of fracture processes at the laboratory scale in fracture experiments [23–33] and in field applications such as seismology [34–38].

In a previous work [32], the authors have examined the use of the entropic index q as a pre-failure indicator. Specifically, the AE timeseries was used for recording during mechanical loading until fracture when Greek Dionysos marble specimens were subjected to various loading schemes (i.e., diametral compression, three-point bending, direct tension), and cement mortar specimens subjected to three-point bending were analyzed under the concept of NESM. The results showed that the index q exhibited a systematic behavior, seemingly related to the different stages of the applied loading protocol and the consequent level of damage.

In the present work, prismatic specimens of Greek Dionysos marble were subjected to two loading schemes of uniaxial compressive stress until fracture, i.e., constantly increasing stress and stepped stress increments (UCSS). The aim was to highlight the behavior of the entropic index q at the various stress levels and to compare the present findings with the respective findings discussed in [32], considering the differences between the applied loading schemes and the specimen's geometry. Another key point is the investigation of a possible relationship between the entropic index q and the parameter β_q , as well as whether the observed deviation from monotonicity between q and β_q may signal significant damage accumulation with consequent final fracture.

2. Theoretical Preliminaries

The “Tsallis entropy”, denoted here as S_q , for a variable X with a probability distribution p_i for the occurrence of the value X_i , is defined as [17]:

$$S_q = k_B \frac{1}{q-1} \left(1 - \sum_{i=1}^w p_i^q \right) \quad (1)$$

where k_B is the Boltzmann's constant, w is the total known multitude of the system's microstates (all possible values of X_i), and q is the entropic index which expresses the non-additivity degree of the system [17–21]. When the index q is greater than unity ($q > 1$) the system is characterized by sub-additive processes (sub-additivity), while when the index q is less than unity ($q < 1$), the system is characterized by super-additive processes. Equation (1), for $q \rightarrow 1$, changes to the familiar expression of Boltzmann–Gibbs entropy formulation (S_{BG}): $S_{BG} = -k_B \sum_{i=1}^w p_i \ln p_i$.

Tsallis entropy constitutes a generalization of Boltzmann–Gibbs entropy; therefore, they have several properties in common, some of which are positivity, concavity, and Lesche experimental stability (for a more comprehensive comparison, readers should refer to Table

3.10 of Reference [21]). A key difference between Tsallis entropy and Boltzmann–Gibbs entropy is that the latter highlights the short-range correlations of a given physical system, with the total system’s entropy depending on the size and the number of subsystems of the total microstates that comprise the system. On the other hand, as indicated by Equation (1), the Tsallis entropy S_q can show long-range correlations and may be more suitable for describing complex dynamical systems [17].

The cumulative distribution function (CDF) of parameter X is described in terms of a q -exponential function as follows [25]:

$$P(> X) = \exp_q(-\beta_q X) \quad (2)$$

The term $\exp_q(X)$ denotes the “ q -exponential function” (q -exponential function), which is defined as:

$$\exp_q(X) = \begin{cases} [1 + (1 - q)X]^{\frac{1}{1-q}} & \text{when } [1 + (1 - q)X] \geq 0 \\ 0 & \text{when } [1 + (1 - q)X] < 0 \end{cases} \quad (3)$$

The inverse form of the q -exponential function is called the “ q -logarithmic function” and is given by the following expression: $\ln_q(X) = \frac{1}{1-q}(X^{1-q} - 1)$. In the case of Boltzmann–Gibbs statistical mechanics, when the index q tends to unity ($q \rightarrow 1$), Equation (3) changes to the usual exponential function (so does the q -log function, which transitions to the usual q -logarithmic function). The q -exponential function is a generalization of the Zipf–Mandelbrot distribution, which is recovered for index values index less than unity. ($q < 1$). However, in the case of sub-additivity ($q > 1$) Equation (3) shows an asymptotic power law tail, while in the case of super-additivity ($0 < q < 1$), it exhibits a cutoff at $X_c = \frac{1}{(q-1)}$ [34,35].

3. Materials and Experimental Apparatus-Loading Protocols

In this work, prismatic specimens with dimensions of $40 \times 40 \times 100 \text{ mm}^3$, made of Greek Dionysos marble, were used. The material in question was chosen because it is used extensively in the restoration projects of the Athenian Acropolis monuments. Its composition, as well as the mechanical characteristics of Greek Dionysos marble, have been extensively studied in the works referenced in the bibliography [39,40]. The recording of AE was achieved through the PCI-2 AE (Physical Acoustics Corp-PAC) acquisition system and the use of a PAC R15a acoustic sensor with a resonant frequency of 75 kHz. The acoustic sensor was placed in the middle of the anterior longitudinal side of the specimen. The specimen–acoustic sensor coupling was achieved via a thin layer of silicone. An Instron 300DX Static Hydraulic Testing Machine loading frame with 300 kN capacity was used in all experimental protocols. Finally, a Kyowa strain gauge attached to the Microlink-770 120 Ω resistor bridge was fitted, diametrically opposite from the acoustic sensor, to the middle of the rear longitudinal side of the specimen to measure the axial strain. Figure 1 depicts the experimental setup.

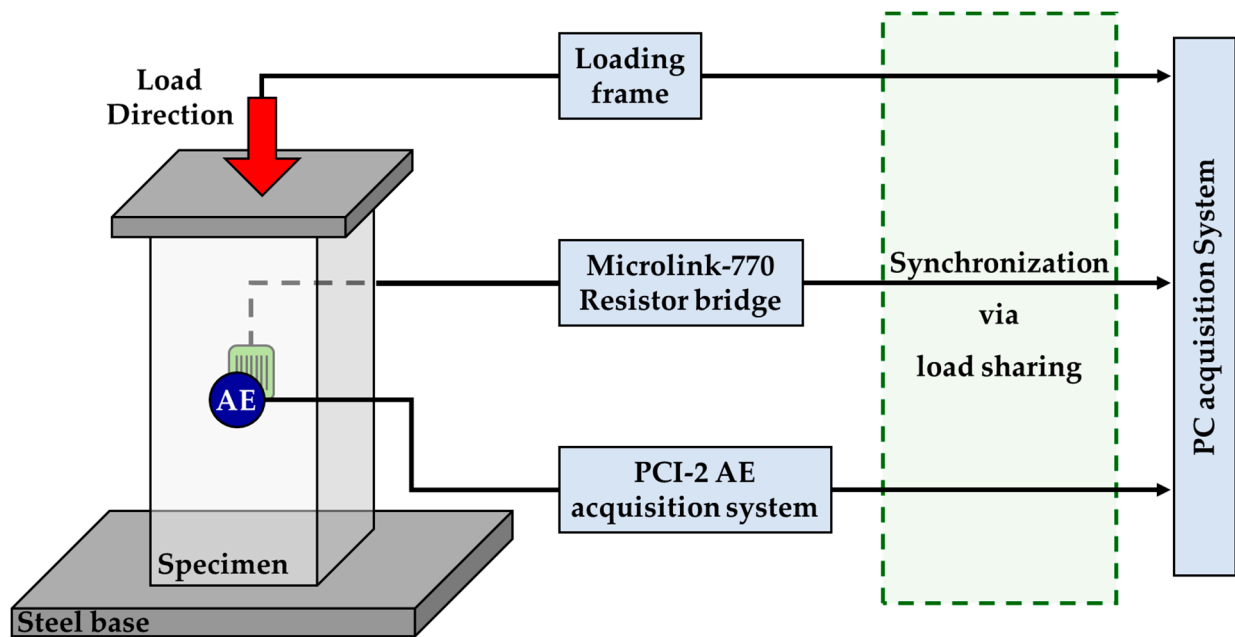


Figure 1. Experimental set-up of the described loading protocols.

Preliminary tests on standardized specimens subjected to uniaxial compression loading protocols showed that the total strength of the specimens under study was located within the range of (60 ± 2) MPa. Figure 2 shows the stress–strain curve for a typical prismatic marble specimen. It can be observed that the linear region of the stress–axial strain curve gives a modulus of elasticity of approximately 75 MPa, compatible with previous experimental protocols [39], and is located in a region that extends from 16% to 88% of the specimens’ fracture limit.

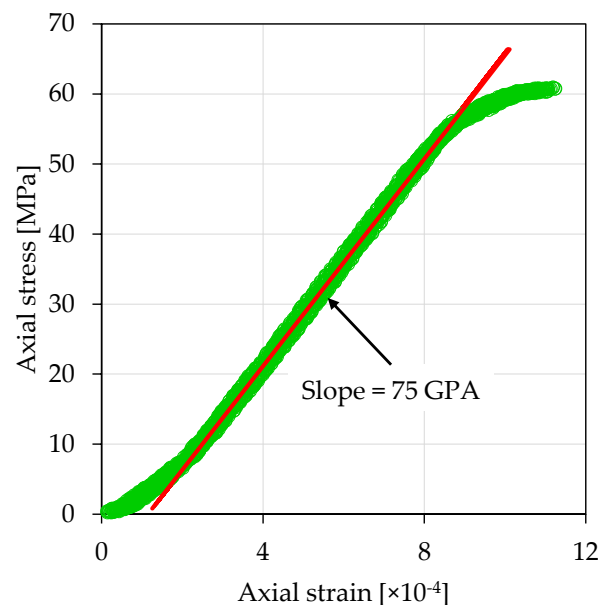


Figure 2. Stress–strain curve of the marble specimens under study.

Loading Protocols and Acoustic Activity

Two loading protocols were implemented on the prismatic marble specimens. The first loading protocol, coded UC, concerned a uniaxial compression at a constant load increase rate until fracture. The stress rate was 0.44 MPa/s, indicating a near-static loading

experiment, with the final fracture of the specimen occurring when the stress reached a value of $S_f = 60.90$ MPa. Figure 3a shows the temporal variation of the recorded acoustic activity during the UC loading process in terms of the occurrence frequency of AE hits (*hits per second*—*hits/s*), in juxtaposition with the stress applied. During the initial loading stages, the acoustic activity was limited, showing a very slightly increasing trend. After the 120th second, a significantly increased AE hit rate was observed, while in the last three seconds before the final fracture, the AE hits rate increased significantly. It should be noted that for $t = 120$ s, the stress rose to 53 MPa, which corresponds to 89% of the specimen's total strength. At this point, considering Figure 2, the specimen under study entered its non-linear region in terms of its mechanical behavior, i.e., a phase in which a large number of new cracks are steadily developing, both in number and size.

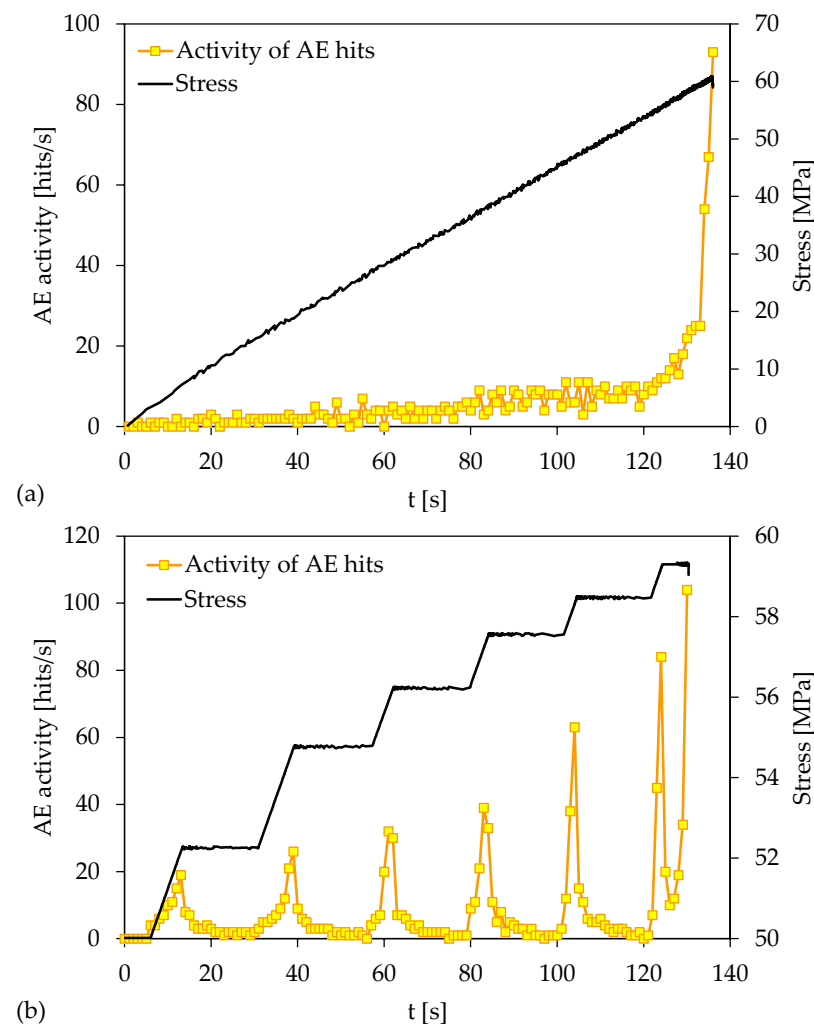


Figure 3. The acoustic activity in terms of hits per second against the applied stress for (a) the UC and (b) the UCSS loading protocol.

The second loading protocol used uniaxial compressive loading with stepped stress increments, and will, herein, be coded as UCSS. In this protocol, a total of six successive stress steps were applied, initially starting from a prestress level of 50 MPa. The temporal variability of the recorded acoustic activity during the UCSS experiment is shown in Figure 3b in juxtaposition with the stress applied. Each stress step is characterized by two stress stages. In the first stage, the stress increases linearly at a rate of about 0.3 MPa/s, while during the second stage, the applied stress remains constant until the acoustic activity reaches a level of approximately 1 AE hit per second. For the first stress step, a transition from a stress level of 50 MPa to about 52.3 MPa was chosen. Using the information retrieved

from the UC experiments (see Figure 3a), the stress initialization was set at 50 MPa so that a significant number of AE hits would be recorded and, thus, a reliable analysis of the AE hits' time intervals distribution in the context of NESM would be possible. As can be seen in Figure 3b, outbursts of acoustic activity can be observed when the stress increase is completed, followed by stress stabilization, during which the acoustic activity declines. The last stress step is particularly interesting. When the stress stabilized at 59.3 MPa, the acoustic activity showed a fluctuation, i.e., while initially showing a declining trend, similarly to the previous steps, a sudden strong increase appeared, and then the specimen collapsed. The stress remained at about 59.3 MPa for about 6 s until failure, and a total of 184 AE hits (86 AE hits during the attenuating of the acoustic activity and 98 AE hits during the increasing) were recorded. This behavior of the acoustic activity can be associated with the coalescence of cracks and the consequent ultimate specimen failure. More details regarding the study of acoustic emissions for damage characterization in corresponding experimental protocols with stepped stress increments in marble specimens have been reported in previous works [41–43].

Figure 4a is indicative of the temporal variation of the AE time intervals, in juxtaposition with the stress applied for the case of the UC loading protocol. Equivalently, Figure 4b also shows the temporal variation of the AE time intervals in juxtaposition with the stress applied, in terms of the $(t_f - t)$ parameter (where t_f is the time instant when the fracture occurred), along a logarithmic scale. The depiction in terms of the $(t_f - t)$ parameter offers greater clarity regarding the evolution of the AE time intervals during the last seconds before fracture.

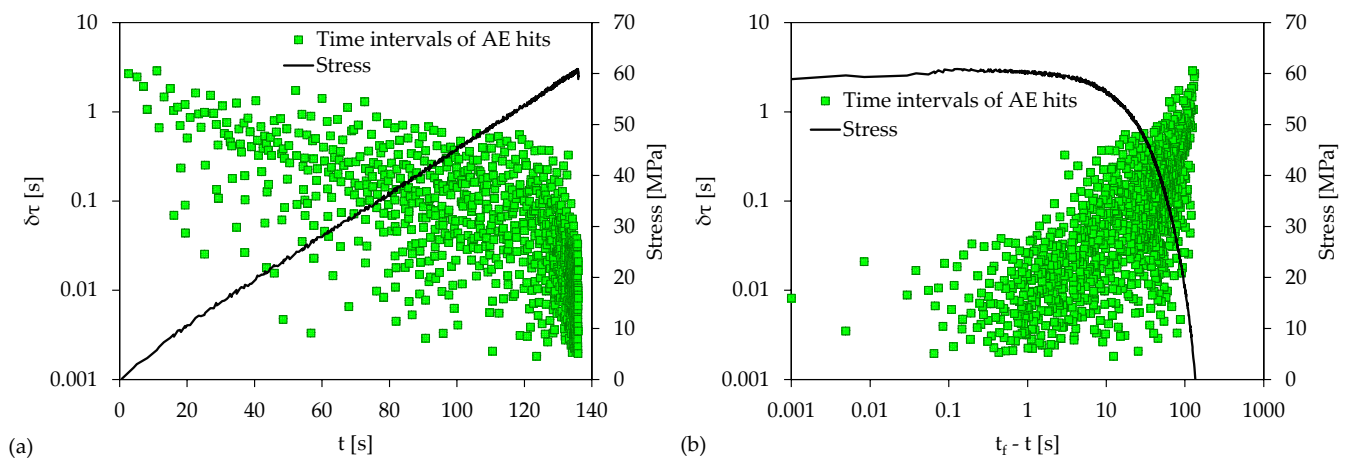


Figure 4. The temporal evolution of the AE time intervals ($\delta\tau$) in juxtaposition with the stress applied, (a) using a conventional time-scale, (b) using the $(t_f - t)$ time-scale parameter for the case of the UC loading protocol.

4. Results and Discussion

4.1. Using AE Time Intervals to Study the Variability of the Entropic Index q

The following analysis aims to determine the variability of the entropic index q in the various loading stages until fracture by utilizing the time intervals of the AE data that were recorded during the two aforementioned loading schemes. The time interval ($\delta\tau$) between two consecutive AE hits is defined as:

$$\delta\tau_i = (t_{i+1} - t_i) \quad (4)$$

where t_i is the recording time of the i^{th} hit; and t_{i+1} that of the consequent one. In previous works [25–33], the time intervals of two consecutive AEs have been exploited in the context

of NESM. It has been confirmed that the CDFs of the time intervals obey q -exponential functions of the form:

$$P(> \delta\tau) = \exp_q(-\beta_q \cdot \delta\tau) = [1 + (q-1)\beta_q \cdot \delta\tau]^{\frac{1}{1-q}} \quad (5)$$

The procedure that was applied to study the temporal variability of the entropic index q and the parameter β_q , contained in the CDF of Equation (5), is described herein. According to the total number n of AE hits recorded from the beginning of the loading protocol until the specimen's fracture, the AE hits are divided into k consecutive groups, with a degree of overlap, as will be described in detail in the sections below. Then, the time intervals ($\delta\tau$) of the AE hits of each group were calculated; the corresponding CDF [$P(> \delta\tau)$] was plotted for each AE hit group; and, through fitting curves, the values of the entropic index q and the parameter β_q were estimated.

4.2. The Variability of Index q

In the case of the UC experiment, a total of $N = 898$ AE hits were recorded, which were divided into 5 groups of $n = 180$ consecutive AE hits. Each group was derived from the previous one by sliding by $n/2 = 90$ AE hits. Thus, $k = 9$ AE hit groups were created, and for each group, the values of the CDFs were calculated after they were rendered graphically [$P(> \delta\tau)$ vs $\delta\tau$]. The fitting based on Equation (5) made it possible to determine the values of the entropic index q and the parameter β_q . For each AE hit group, the mean value of the mechanical stress $\bar{S}$ was also calculated. Table 1 includes the values of the entropic indices q , the parameters β_q and τ_q , and the corresponding average normalized stress $\bar{S}_n = \bar{S}/S_f$, with S_f being the ultimate stress strength for each group of AE hits. In the same table, the quantity F is included, which expresses the average occurrence frequency of the AE hits in each group and is calculated as the inverse of the average value of the time intervals of each group ($F = 1/\bar{\delta\tau}$). An average value is related to both the entropic index q and the time parameter β_q through the relation [29]:

$$\bar{\delta\tau} = \frac{1}{\beta_q} \cdot \frac{B\left(2, \frac{2-q}{q-1}\right)}{(q-1)^2} \quad (6)$$

where $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$ is the so-called beta function [44], with $x = 2$ and $y = \frac{2-q}{q-1}$. If $Q(q)$ denotes the function: $(q-1)^2/B\left(2, \frac{2-q}{q-1}\right)$, it results that:

$$\frac{1}{\beta_q} = \frac{Q(q)}{F} \quad (7)$$

Table 1. The entropic and mechanical parameters during the UC experiment for each consequent AE hit group.

AE Hit Group	$\bar{S}_n$	q	β_q (s ⁻¹)	F (s ⁻¹)
1	0.398	1.15	2.56	2.22
2	0.581	1.27	5.81	4.59
3	0.692	1.26	8.48	6.69
4	0.779	1.28	9.66	7.84
5	0.857	1.32	11.92	9.06
6	0.918	1.39	19.33	12.81
7	0.957	1.47	37.99	22.10
8	0.978	1.36	73.84	41.54
9	0.988	1.16	102.03	77.19

Figure 5 shows the evolution of the entropic index q and the acoustic activity, expressed in terms of the quantity F , as functions of the normalized stress $\bar{S}_n$. For the values of the

index q , the stress limits (asterisks) of the normalized stress $\bar{S}_n$ that correspond to each AE hit group are also shown. During the initial stages of the loading protocol for stress values $\bar{S}_n < 0.8$, the entropic index q attains low values ($1.15 < q < 1.30$), which is indicative of the low degree of organization regarding the system of microcracks created due to the closure and rubbing of the pre-existing microcracks and the discontinuities inside the marble specimen. At the same time, the acoustic activity showed a weakly increasing trend, which intensified progressively with the increase in stress ($\bar{S}_n$). When the normalized stress $\bar{S}_n$ approached levels ranging from 0.8 to 0.96, the values of q showed an increasing trend, reaching a maximum value of 1.47. At this stage, the mechanisms of creation, development, and propagation of microcracks seemed to be characterized by a higher degree of organization, and the formation of a large number of new microcracks was observed as well as an increase in the dimensions of the already existing microcracks, which was also related to the increased acoustic activity. Finally, for $\bar{S}_n > 0.96$, a noticeable decrease in the index q values was observed, and the last group of AE hits corresponded to a q index value of 1.16. Therefore, in the last and most critical loading stage before failure, during which the coalescence of microcracks dominated the specimen, these processes were characterized by a low degree of organization and, according to NESM, the various subsystems that had formed were characterized by a low degree of mutual interaction, approximating a behavior similar to that of the Boltzmann–Gibbs statistics.

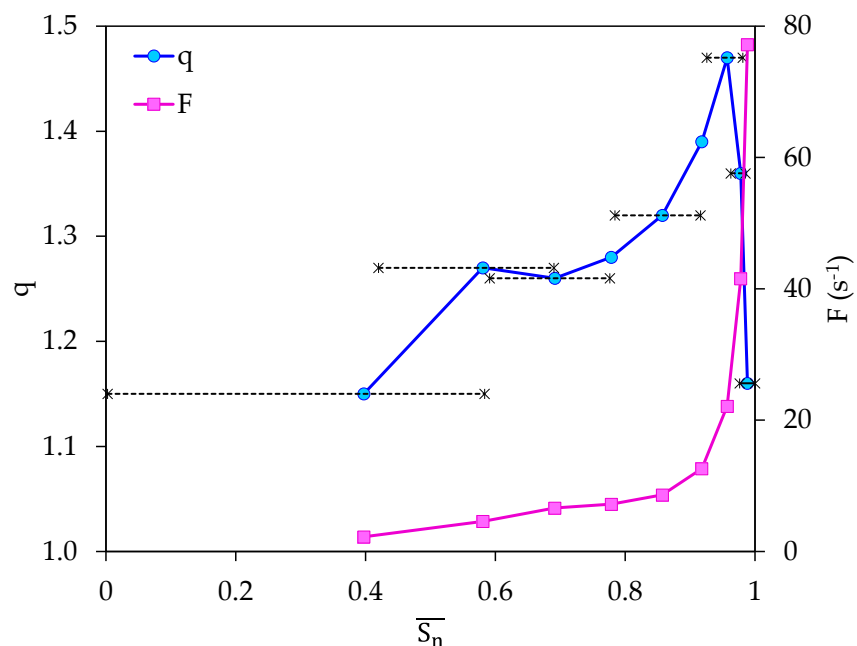


Figure 5. The variability of the entropic index q and the variability of the acoustic activity, expressed in terms of the quantity F as functions of the normalized stress $\bar{S}_n$.

In the case of the UCSS protocol (i.e., uniaxial compressive loading using stepped stress increments), the behavior of the entropic index q was studied during the six stress stages in which the stress increase was implemented at a constant rate. Each q value was assigned to the average normalized stress $\bar{S}_n$ during its increasing stage (see Table 2). Subsequently, the behavior of index q was studied in the stress stages when the applied stress was kept constant. Especially during the last stress step under constant stress of 59.3 MPa, where the final fracture occurred, two values for the index q were calculated. The first value was calculated using the AE time intervals, during which the acoustic activity was attenuated (see Figure 3b), and the second was calculated when it showed a sharp increase before the fracture. The values of the entropic index q , as well as the values of the parameter β_q , are presented in Table 2. Figure 6 shows the evolution of the values of q as a

function of the normalized stress $\bar{S}_n$ during the stages of the linear stress increase, and in the stages where the stress was kept constant.

Table 2. The entropic and mechanical parameters during the UCSS experiment for each consequent AE hit group.

Increasing Stress				Constant Stress			
AE Hit Group	$\bar{S}_n$	q	$\beta_q (s^{-1})$	AE Hit Group	$\bar{S}_n$	q	$\beta_q (s^{-1})$
1	0.868	1.19	9.41	1	0.881	1.14	3.65
2	0.910	1.30	14.14	2	0.924	1.25	4.37
3	0.940	1.37	23.50	3	0.948	1.34	5.60
4	0.964	1.45	34.36	4	0.971	1.46	7.17
5	0.982	1.50	57.50	5	0.986	1.52	10.04
6	0.997	1.37	73.49	6a	1.000	1.36	42.64
				6b	1.000	1.22	47.99

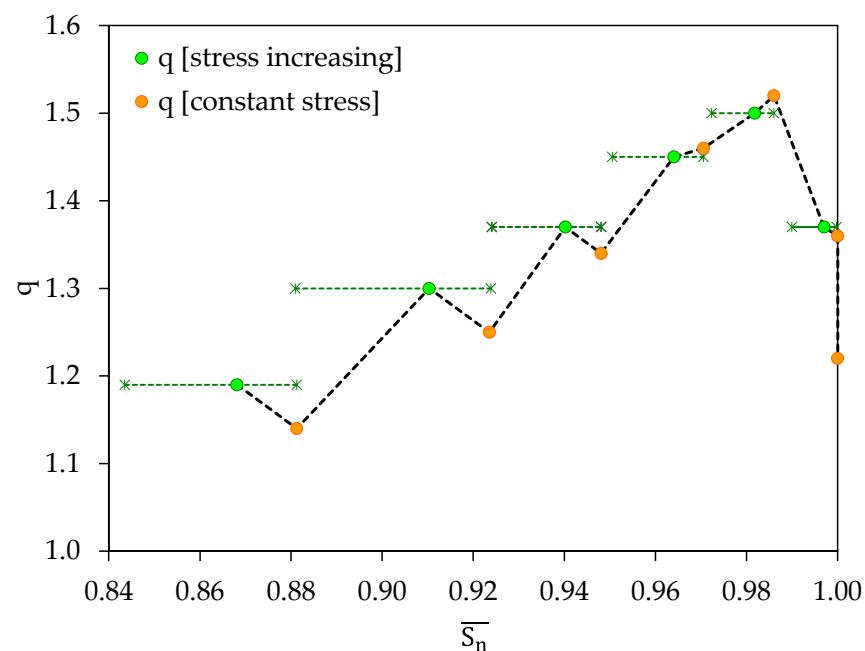


Figure 6. The evolution of the q entropic indices as functions of the normalized stress $\bar{S}_n$ calculated for the UCSS loading protocol.

The result show that the entropic index q for the case of the UCSS loading protocol shows similar behavior to that of the entropic index q observed in the UC protocol. A maximum q value appears again just before the final fracture. The decrease observed in the values of q close to the final fracture (i.e., $0.99 \leq \bar{S}_n \leq 1.00$) observed in UC is also confirmed here.

Comparing the evolution of the q entropic indices of both loading protocols (UC and UCSS) in Figure 7, the following conclusions were drawn: A systematic increase in the q values is observed as the final fracture approaches, indicating that the mechanisms of creation, growth, and propagation of microcracks are characterized by an increasing degree of organization. The highest degree of organization dominated the fracture processes within the stress range $0.96 \leq \bar{S}_n \leq 0.99$ (see the yellow-shaded area in Figure 7). The UCSS protocol provides a better understanding concerning the rapid decrease in the values of q when the applied stress reached the specimen's strength limit. This, as mentioned above, can be attributed to the fact that the coalescence processes of the microcracks that lead to macrocrack failure are characterized by a low degree of organization, as well as the fact

that the failure plane is shaped and the characteristics of the inevitable event of fracture are defined. Through the UCSS protocol, especially during the last loading step, enough time is provided for the specimen to stay in the region of its strength limit, resulting in a significant number of AE hits being recorded so that NESM analysis can be applied and highlight the observed decrease in q values. It should be noted that an experiment, such as the UCSS protocol, that the final fracture occurs during the stress stabilization phase it is difficult to conduct and therefore requires a significant number of experiments to achieve it.

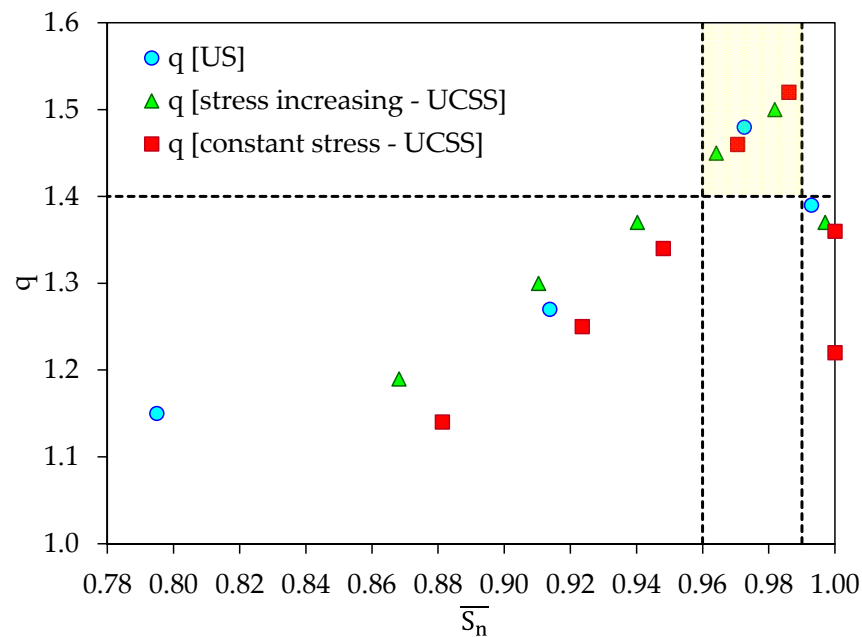


Figure 7. The evolution of the q entropic indices of loading protocols UC (blue circles) and UCSS (green triangles and red squares).

4.3. Correlation between q and β_q

In the work described in Reference [30], a relationship between the entropic index q and the parameter $1/\beta_q$ was investigated by analyzing the AE data recorded during cyclic compression tests on two different types of brittle materials, namely concrete and basalt. Results showed the emergence of a linear correlation between the quantities q and $1/\beta_q$. The investigation of a possible correlation is considered in the context of this work in both loading protocols.

Figure 8 shows the value of the index q as a function of the parameter $1/\beta_q$ for the UC test. As can be seen, a linear relationship emerged, with a slope of -2.28 , that is found only in the stress region where the normalized stress $\bar{\sigma}_n$ ranges between 69% and 96% of the specimen's strength (see green circles in Figure 8). It is characteristic that the initial loading stages (blue circles), as well as those corresponding to the last loading stages during the collapse of the specimen (red circle), deviated from the linear fitting curve. More enlightening is the corresponding behavior that emerged in the UCSS protocol, in which the applied stress initiated at least 84% of the specimen's total strength (see Figure 9). The linear relationship between the index q and the parameter $1/\beta_q$ was evident both when the values were plotted for the regions of stress increases (Figure 9a) and during the regions of constant stress (Figure 9b). It is remarkable that during the last stress step, a significant deviation from the linear relationship (red circles in Figure 9) emerged.

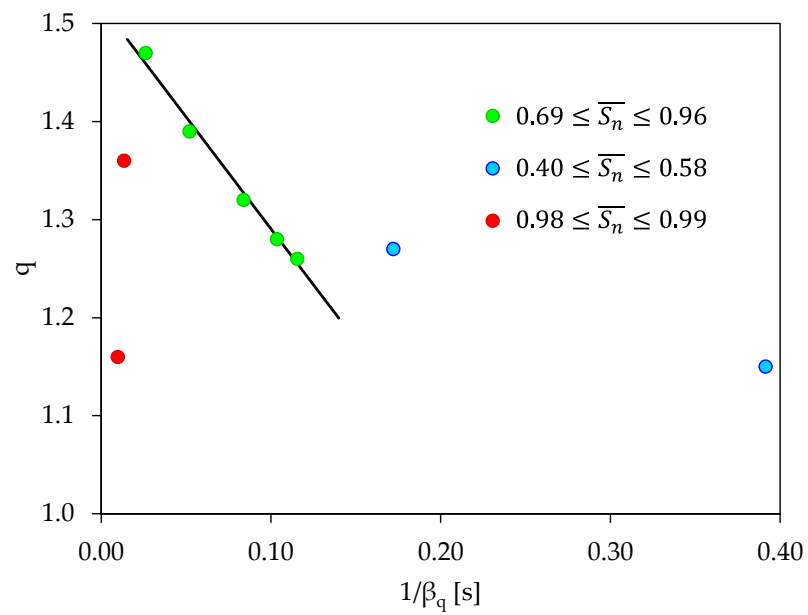


Figure 8. The values of the entropic index q as a function of the parameter $1/\beta_q$ for the UC protocol.

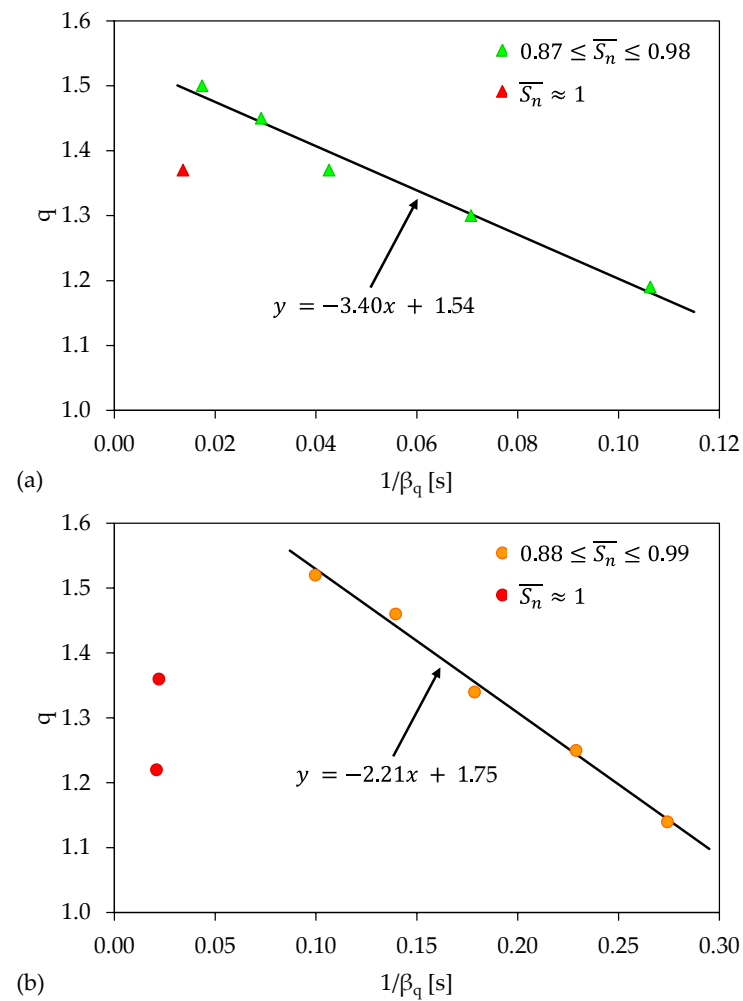


Figure 9. The values of the entropic index q as a function of the parameter $1/\beta_q$ for the UC protocol, (a) for the stages of stress jumps; (b) for the stages when the stress was kept constant.

A computational explanation of the linear relationship between q and $1/\beta_q$, which emerged as a failure approach, was attempted based on Equation (5). Using the values of the entropic index q obtained from the UC loading protocol in the region where q vs. $1/\beta_q$ presented a linear relationship, the values $F(q)$ and $Q(q)$ were calculated and subsequently plotted (see Figure 10a). As the specimen failure approached and the entropic index q value increased due to the organized system of microcracks, $F(q)$ showed a relatively strong increase, while the function $Q(q)$ showed a slight decrease. Then, as in Figure 10b, the resulting values were depicted based on the relationship $1/\beta_q^* = Q(q)/F$, and thus, the linear relationship between q and $1/\beta_q$ became apparent. Finally, in Figure 10c, the $1/\beta_q^*$ values are presented together with the $1/\beta_q$ values resulting from the fitting of Equation (5) which were presented in Figure 8. The small variations between the values of $1/\beta_q^*$ and $1/\beta_q$ were due to the deviations occurring in the long-time interval values ($\delta\tau$) with the fitting based on Equation (5). Indicatively, in Figure 10d, the CDFs and the corresponding fitting are presented; in this way, the q and β_q values for group 3 of the UC protocol were calculated. The deviations in the tail of the distribution, with values indicated by $\delta\tau$, were smaller than those predicted by the fitting, and gave slightly smaller mean values of $\overline{\delta\tau}$ and slightly larger values of F compared to the $1/\beta_q$ value obtained through fitting. Based on the $1/\beta_q^* = Q(q)/F$, $1/\beta_q^*$ relationship, the values were slightly smaller than the results of the corresponding $1/\beta_q$, which is evident in Figure 10d.

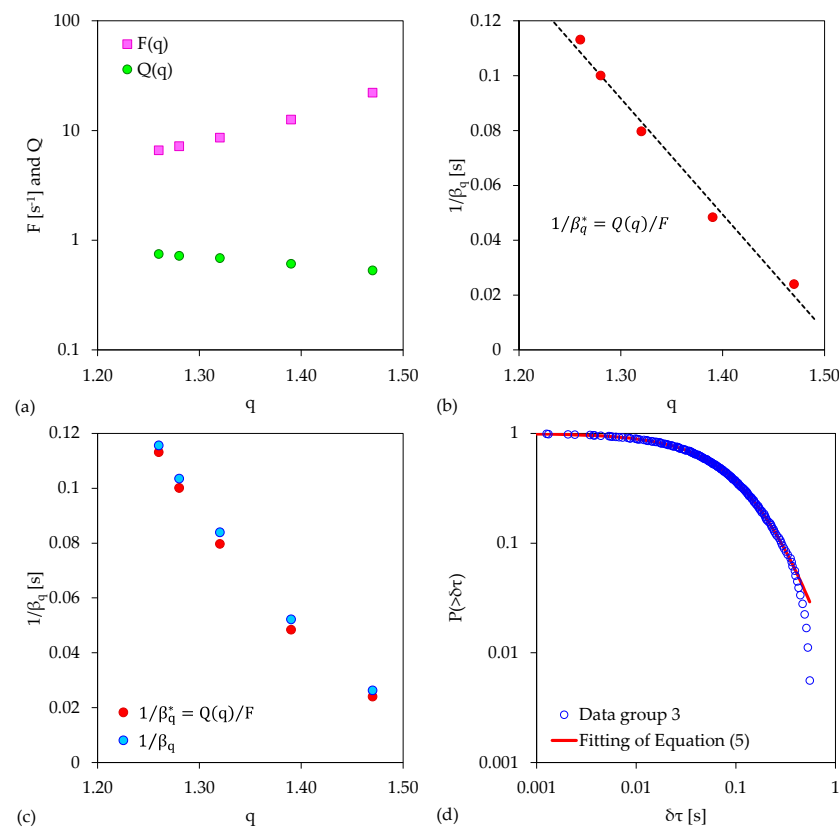


Figure 10. Regarding the UC loading protocol: (a) The variability of the values $F(q)$ and $Q(q)$ as functions of the index q . (b) The linear relationship of the index q and the parameter β_q based on the relationship $1/\beta_q^* = Q(q)/F$. (c) Comparison between the variability of $1/\beta_q^*$ and $1/\beta_q$ as functions of the index q . (d) The CDFs and the corresponding fitting for AE hit group 3.

5. Conclusions

Acoustic emission data, recorded during constant-rate uniaxial compressive loading as well as during uniaxial compressive loading using stepped-stress increments until the fracture of prismatic marble specimens, were analyzed under non-extensive statistical

mechanics (NESM). The AE hit time intervals were studied in terms of a NESM analysis by determining the entropic index q and the parameter β_q at different loading levels of the specimens. The main conclusions are summarized as follows.

Approaching failure, the entropic index q progressively increased, reaching maximum values close to 1.5, which is seemingly related to the development of a fairly organized system of microcracks.

In the last stage, and when the final fracture was imminent, the values of q showed a decrease, suggesting that the processes of microcracks coalescing into macrocracks and, subsequently, to failure are not characterized by mutual interactions. In contrast, the frequency with which the AEs are recorded, as well as the parameter β_q , were the values that increased the most.

At pre-failure loading levels, specifically in the range from 70% to 96% of the specimens' strength, a linear correlation was observed between the entropic index q and the parameter $1/\beta_q$. This behavior can be considered an indication of preliminary damage, providing a warning of entry into the critical stage of impending failure.

When the test specimens were subjected to loading near their ultimate strength, the linear relationship between q and $1/\beta_q$ was reversed due to the decreasing values of q , which signaled imminent failure.

In conclusion, the ability of NESM to describe the transition of dynamical systems from an equilibrium state to a non-equilibrium state has been studied in a series of publications (e.g., see References [22–38]) which touch on various scientific fields, including seismicity and fracture experiments. Parallels can be drawn between the application of NESM in fracture experiment applications and NESM findings in the field of seismicity. Moreover, a further experimental study is needed to transition from small laboratory-scaled specimens to larger and more complicated in situ structures.

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