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Nash Equilibrium and Stackelberg Approach for Traffic Flow Optimization in Road Transportation Networks—A Case Study of Warsaw

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Abstract: The article discusses the issue of modelling traffic flows and the transport network. Faced with an increase in the number of vehicles in road networks, the problem of congestion and the need to optimise traffic and adapt the transport infrastructure to changing demand are growing, especially in large cities. With this in mind, the authors of this publication developed a model of the road network in the north-eastern part of the Warsaw agglomeration based on the proposed algorithm. Two methods were used to optimise the distribution of traffic flows: the Nash equilibrium and the Stackelberg approach. The Nash equilibrium assumes the aim of achieving equal average times on all roads for each origin–destination (O-D) pair. This describes the state pursued by a decentralised system guided by the individual benefits of the traffic users. On the contrary, the Stackelberg approach aims to achieve optimal travel times for the entire system. The study was carried out for three scenarios that differed in the assumed traffic demand on the road network. The basic scenario assumed the average hourly traffic demand during the morning peak hour based on traffic measurements. On the other hand, the two alternative scenarios were developed as a 10% variation in traffic volumes from the baseline scenario. On the basis of the results, it was concluded that an increase in traffic volumes for all O-D pairs could result in a decrease in traffic volumes on some links of the road network. This means that the transport network is a complex system and any change in parameters can cause significant and difficult to predict changes. Therefore, the proposed approach is useful in terms of traffic forecasting for road networks under conditions of changing traffic flow volumes. Additionally, the total travel time for the entire system differed for each scenario by a percentage difference of 0.67–1.07% between the optimal solution according to the Nash equilibrium and the Stackelberg approach.

Keywords: urban road networks; optimization; traffic congestion; traffic flow; Stackelberg approach; Nash equilibrium

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1. Introduction

Transport is a complex process that aims to move people and goods from the point of origin to the destination, within the existing infrastructure. The efficiency of transport processes can be determined by the value of the goal function of the optimization problem [1,2]. In practice, the fundamental indices considered when optimizing a transport process are its costs [3,4] and travel time [5–7].

Road congestion is a widespread global phenomenon resulting from high population density and increased number of motor vehicles [8]. The effects of congested road networks are a significant burden in terms of time loss, pollution, increased industrial costs, and road maintenance costs. The management of the causes and effects of congestion will become more urgent as the population grows, as congestion increases travel times and causes significant delays. From an industrial perspective, delays reduce productivity and consequently increase operating costs [9,10].

Many modern vehicles are equipped with navigation devices that can determine the shortest route from origin to destination taking into account current road conditions. To reduce congestion or even avoid congestion, it is necessary for all users of the system to coordinate the route and for users to follow suggested routes. In the future, with the proliferation of autonomous vehicles, this could become a realistic solution to road congestion problems [11]. The scientific approach to traffic problems focuses on developing complex models that determine the conditions and constraints of processes in transportation systems.

A model is a qualitative and quantitative representation of a certain reality fragment (studied system) presented using a different material basis. It should reflect selected features and system properties, which are important both in terms of the studies conducted and, above all, consistent with the modelling objective(s) [12]. Creating a model reflecting selected functional aspects of a system makes studying the system become easier and more beneficial from the economic perspective. Moreover, in the case of designed systems, this enables its detailed analysis [13].

A mathematical model is a finite set of mathematical symbols and relations and strict rules for their operation, in which the symbols and relations contained in the model have an interpretation related to specific elements of the modelled part of reality [14,15]. It describes a phenomenon studied using variables that can take numerical values. Studying such a model is possible due to the use of a mathematical apparatus, which allows accurate estimation of system behaviour in assumed and allowed scenarios [16–18].

Mathematical models can be classified according to the several different criteria. Taking into consideration time, models are divided into: static—reflecting the system and its surroundings in a given state (not taking time into account) and dynamic (so-called process models)—reflecting system operation as a function of time. Another criterion is the cause-and-effect relationship, which classifies models as deterministic and stochastic. The deterministic models reflect a system in the event of complete certainty of information regarding the analyzed system. Stochastic (probabilistic) models take into account the risk and lack of information on the structure of the characteristics of the system studied, where a mathematical description of the process is reflected by random variables. The form of relationships and relations classifies models as linear or nonlinear. The linear models describe evaluation criteria, conditions, and restrictions through linear equations (inequalities) with respect to decision variables, while the nonlinear models contain decision variable functions in the form of curves of at least the second degree.

Mathematical modelling involves describing phenomena using the language of mathematics and formal logic. It is widely applied in exact sciences (including mathematics and natural sciences) and economics. Mathematical modelling is also used in the field of transport systems when designing and optimizing transport processes and systems [19–21]. The subject matter and the objective of the research are the most important factors in selecting an appropriate tool to create a model. A transport system model should reflect the modelling objective(s) and an appropriate accuracy level adapted to it. Simple models are easy to create and analyse but may provide incomplete information about the studied system and consequently lead to incorrect conclusions. On the other hand, too complex models are very time-consuming (also cost-consuming), and excessive volumes of information can alter the reliable analysis of processes and phenomena [22,23].

The increase in traffic volume in the Warsaw transport network is permanently monitored in the traffic measurements carried out by the General Directorate of National Roads and Motorways. The results of the conducted research have led to the authors' interest in transport optimization. Caban and Drozdziel [24] conducted an analysis of traffic flows in six largest cities in Poland and showed that during the peak of the morning the highest congestion occurs in Warsaw, with an average of approximately 65%. Improving road network and reducing travel time are important components of the sustainable development of large urban agglomerations. The research carried out indicates the complex nature of the problem in both scientific and practical terms.

The aim of this article is to determine the impact of the changing traffic flow on the optimal solutions, taking into consideration the travel time. The Nash equilibrium and the Stackelberg approach have been compared in a real case study. A new algorithm was proposed, on the basis of which a model of the transport system was developed. The model includes a graph of the transport network with link characteristics. Based on traffic measurements, the volume of traffic flows was determined for the three O-D pairs. The study was carried out for three scenarios: a baseline defined on the basis of traffic measurements and alternative scenarios assuming a 10% deviation. Optimal traffic flow distributions according to Nash equilibrium and the Stackelberg approach were calculated using nonlinear programming and advanced IT software. Additionally, the proposed approaches and algorithms were used to optimize the classic Nguyen–Dupuis network, and the results obtained were compared with other approaches. The novelty of this paper is the development of the new algorithm and its implementation. The real case was conducted for the optimization of the existent transport system. Process optimization using two methods (tools) allows estimating the improvement of the efficiency of the transport system operating under current and future technical conditions. Based on the results of the real traffic case study, the Price of Anarchy (PoA) was determined. Furthermore, it has been observed that an increase in the volume of traffic flows in all O-D pairs can cause a decrease in traffic volumes on certain links in the road network. The published papers did not refer to this phenomenon, which may occur during the optimization of real traffic flows. The article is divided into seven sections. In the next section, a literature review has been presented that takes into account the most relevant studies from recent years. Section 3 presents the mathematical model of the transport system and a description of the Nash equilibrium and the Stackelberg approach. Section 4 shows the results of the case study research in the three assumed scenarios. Section 5 contains a discussion of the results obtained, while Section 6 describes the limitations in applying the proposed method. Finally, the conclusions of the study were presented and directions for further research have been indicated. All notations, abbreviations and acronyms used in the text are described in Tables A1 and A2 in Appendix A.

2. Literature Review

Transport systems with a centralized decision-making process use the Stackelberg approach to optimize traffic flow distribution. The authors of [25] analyzed intermodal water–rail–road transport using the Stackelberg approach and an iterative algorithm. The optimization of an enterprise in the rail transport industry is presented in [26], using a two-tier approach to Stackelberg's game theory based on efficiency maximization.

The Nash equilibrium is used to optimize decentralized systems [5]. The authors of [27] suggested a cellular transmission model to find an optimal balance of the traffic flow density of a highway network in a highway network.

In [28], the authors analyzed a transport system through decomposing it into local subsystems managed independently. Such subsystems often cooperate or function under conditions of mutual competition. Using both the Nash equilibrium equation and the Stackelberg approach as a numerical example, this paper proves that locally determined travel prices can be both advantageous and disadvantageous for traffic users within the entire transport system, depending on the structure of the road network and movement routes. However, in the case of parallel intercity networks [29], it was demonstrated that the application of the Stackelberg approach contributes to improving the cost efficiency of a transport process relative to the solutions obtained with the Nash equilibrium.

An important element in studies of the optimization of traffic flow distribution in the transport network is determining the relationship between the value of objective functions obtained with the Nash equilibrium and the Stackelberg approach. The quotient of the objective function value for both approaches was defined as the "Price of Anarchy" (PoA) [30–34]. The authors of [35] present research on road traffic in London, Boston, and New York. The maximum PoA values for the analyzed agglomerations were 1.24–1.28, which translates into a percentage difference in the range of 24–28%. In turn, the authors of [36] analysed the road network in Singapore, followed by determining the PoA values on individual calendar days within the study period (April 2012). The results fell within the range of 1.0–2.4.

Optimization of a single criterion transport process based on the time or cost criterion often leads to a reduction in adverse impacts on the environment [37–39]. The publication [40], based on the example of a perishable product distribution network, demonstrated that achieving an optimal solution for the transport time criterion could lead to a reduction in fuel consumption and CO_2 emissions into the atmosphere, while the authors of [41], using an example of supply chains in France, demonstrated a strong correlation between logistics costs and CO_2 emissions using only road transport.

Djenouri et al. [42] developed a graph convolution neural network to forecast traffic flow in urban transport networks. Hui et al. [43], however, developed a long-term spatial–temporal graph convolutional fusion network model (LSTFGCN), which was validated on factual data from the Caltrans performance measurement system. In [44], a deep learning approach is presented to model traffic congestion. Using traffic data from Google Maps, a hybrid support vector machine Xception was validated. The model achieved an accuracy of 97.16%. Xueting et al. [45] developed a model based on Self-Organising Feature Mapping (SOFM) to categorise urban areas according to road congestion. They conducted their study using the example of the road network in the urban area of Guiyang city.

Vandana et al. [46] proposed an approach based on Ant Colony Optimisation and Genetic Algorithm (GA) to optimise traffic light management at a road junction. In turn, Tay and Osorio [47] developed a model based on Bayesian optimisation (BO) and Gaussian process (GP), which they used to optimise the management of the traffic signal road network in New York. A solution of the problem of stop-and-go waves on a motorway was proposed in [48]. Strnad and Marsetic performed a macroscopic simulation using numerical optimization with a differential evolution algorithm. The results showed a reduction in the queue length and the number of stops.

There are many studies on the modelling and optimization of road networks and traffic flows nowadays. A large part of them focuses on traffic forecasts using a number of tools, including game theory, advanced algorithms, or neural networks. Nash equilibrium and the Stackelberg approach are mainly used for optimizing travel time, costs, and road congestion. However, there is a lack of research on the impact of changes in traffic flow volumes on the congestion that occurs in individual elements of the transport network. Therefore, in the paper a multi-scenario analysis of changes in traffic intensity in the road network was carried out. It was observed that despite the overall increase in traffic on the network under study, there was a reduction in the number of vehicles on some links.

Table 1 contains a synthetic overview of selected publications by other authors, taking into account the case study, applied optimization approaches and adopted criteria.

Table 1. Overview of publications on transport system modelling and optimization.

Paper	Case Study	Approaches	Criteria
[25]	Intermodal transport system	Stackelberg approach and iterative algorithm	Transport time
[28]	Set of local transport systems in in a competitive and cooperative environment	Nash equilibrium and Stackelberg approach	Transport costs
[49]	Road junction	Hybrid of Nash and Stackelberg approaches with Q-learning algorithm	Traffic congestion
[29]	Intercity parallel road network	Nash equilibrium and Stackelberg approach	Transport costs
[50]	Freight forwarding problem	Nash equilibrium	Transport costs
[51]	Transport network	Nash equilibrium and stochastic equilibrium	Transport time
[52]	Transport network	Nash–Cournot equilibrium	Transport costs
[5]	Transport network	Nash equilibrium	Transport costs
[42]	Urban road networks	Graph Convolutional Neural Network	Traffic flow
[43]	Urban road networks (California)	Long-term spatial-temporal graph convolutional fusion network (LSTFGCN)	Traffic flow

Table 1. *Cont.*

Paper	Case Study	Approaches	Criteria
[44]	Selected road networks in India (data from Google Maps)	Deep Learning Technique (hybrid Xception support vector machine)	Traffic congestion
[45]	Road network in urban area of Guiyang city	Self-Organizing Feature Mapping (SOFM)	Traffic congestion
[46]	Traffic light junction (Ranchi, India)	Ant Colony Optimization and Genetic Algorithm (GA)	Traffic flow management
[47]	Road network in New York	Bayesian optimization (BO) and Gaussian process (GP)	Traffic flow management
[48]	Motorway	Numerical optimization with a differential evolution algorithm	Stop-and-go waves

In this paper, an algorithm was proposed and implemented to create a model of the transport system to solve the optimisation problem of traffic flow distribution within the road transport network. The optimization criterion is the transport time of individual road traffic users and the movement time of an entire flow within the network, over adopted origin—destination (O-D) pairs. A road network of the north-eastern part of the Warsaw agglomeration was selected to provide a practical computational example due to the significant traffic volumes therein. Three main sources of traffic flow with a single destination were identified. This was followed by the development of a mathematical transport system model, which involved defining a transport network graph, characteristics of network linear elements and traffic flow volumes together with movement direction. Transport time minimization was adopted as the single-criterion optimization objective function. Two optimization approaches were proposed. These were the Nash equilibrium and the Stackelberg approach. The considerations assumed three scenarios for traffic flow volume, namely, a baseline scenario based on traffic measurements and two alternatives. The baseline scenario corresponds to the most probable traffic situation, while the alternative scenarios assume 10% deviations of the traffic flow volumes occurring within the assumed movement routes. The traffic volume fluctuations adopted by the authors at a level of $\pm$10% correspond to actual measurements presented and analyzed within many transport systems [53–55]. Optimal traffic flow distributions according to both suggested approaches were determined using IT software.

3. Transport System Modelling and Optimization

To analyse systems and optimize traffic flow distribution, transport engineering commonly uses mathematical models of transport systems. They classically focus on one of the following four elements [56], namely:

- structure—covering linear and point elements,
- characteristics—corresponding to structural properties,
- traffic flow—depicting the demand for transport services,
- traffic organization—characterizes the distribution of traffic flow in a given transport network.

The mathematical notation of a transport system model is presented by Equation (1):

$$MST = \{G, F, P, O\}, \tag{1}$$

where: MST—transport system model, G—system structure graph, F—set of structure graph node and (or) link functions (characteristics), P—traffic flow, O—traffic organization of the transport system.

A transport system graph is made up of a set of vertices that represent point elements of the transport infrastructure and a set of links that represent linear elements. In mathematical notation, the structure graph is presented by the relationship (2) [11]:

$$G = \{W, L\}, \tag{2}$$

where: W—set of graph nodes G, L—set of graph links G, which is a Cartesian product of the nodes.

The set of all nodes in the transport network can be presented in the form of a mathematical relationship (3):

$$W = \{w : w = 1, \ldots, \overline{W}\}, \tag{3}$$

where: $\overline{W}$ is the population of the set W.

The location of a demand for transport services determines the decomposition of the node set into three subsets:

- $A = \{a : a \in W\}$—origin nodes,
- $V = \{v : v \in W\}$—intermediate nodes,
- $B = \{b : b \in W\}$—destination nodes.

In mathematical notation, it is represented by the following relationship (4):

$$W = A \cup V \cup B. \tag{4}$$

According to the assumption adopted when modelling transport networks, all three subsets A, V and B are disjoint pairwise, which is expressed as a relationship (5):

$$\begin{cases} A \cap B = \varnothing \\ A \cap V = \varnothing. \\ B \cap V = \varnothing \end{cases} \tag{5}$$

Assuming that the element l_{ij} presents a transport connection between nodes w_i and w_j, it is assigned the following conventional values: 1—when there is a connection or 0—in the absence of a connection. Therefore, a set of relationships between transport network structure graph nodes can be presented as a binary matrix in the following form:

$$L = \begin{bmatrix} 0 & \cdots & l_{1N} \\ \vdots & \ddots & \vdots \\ l_{N1} & \cdots & 0 \end{bmatrix}. \tag{6}$$

Points of the transport infrastructure and linear or nonlinear characteristics are described in the model of the transport system model as node and link functions. They are both determined by properties, i.e., node and link capacity, movement distance, time, and cost of movement over given links, etc. The structure graphs of the system, together with the node and link functions form a transport network (S), described with Equation (7):

$$S = \{G, F_W, F_L\}, \tag{7}$$

where: G—transport system structure graph, F_W—set of functions defined on a set of nodes W, F_L—set of functions defined on a set of links L.

Traffic flow is a representation of people and goods moving through points and linear elements of a transport network [57,58]. The flow appearing on the origin nodes, i.e., at the entrance to a transport system, is usually identified with the market demand for transport services, while in the case of system engineering, it means the environmental impact on the studied system. However, the flow entering the transport system through the destination nodes shows the impact of the system on its surroundings. The manner in which the traffic flow passes through a transport network is called a traffic organization. This phenomenon is subject to the following requirements imposed on traffic flow in the course of executing transport system processes:

- condition of satisfying transport demand,
- condition of non-negativity of traffic flow,
- condition of traffic flow additivity,
- condition of maintained traffic flow.

Figure 1 shows the algorithm for creating a mathematical model of a transport system (TSMA) focused on optimizing the distribution of traffic flow. Defining a transport system is the initial modelling stage. It is followed by the construction of a transport network graph that indicates the origins and destinations of the traffic flow. The next step involves determining a set of links L that join individual transport nodes w_{ij}. A set of movement pairs of O-D E and all possible travel routes P^{ab} for each pair of O-D (a, b) are determined for such a defined structure. Depending on the adopted criterion (set of criteria), each important element of the transport network must have an appropriate characteristic assigned to it. The final stage is to determine the traffic flow volumes for a selected set E of pairs O-D .

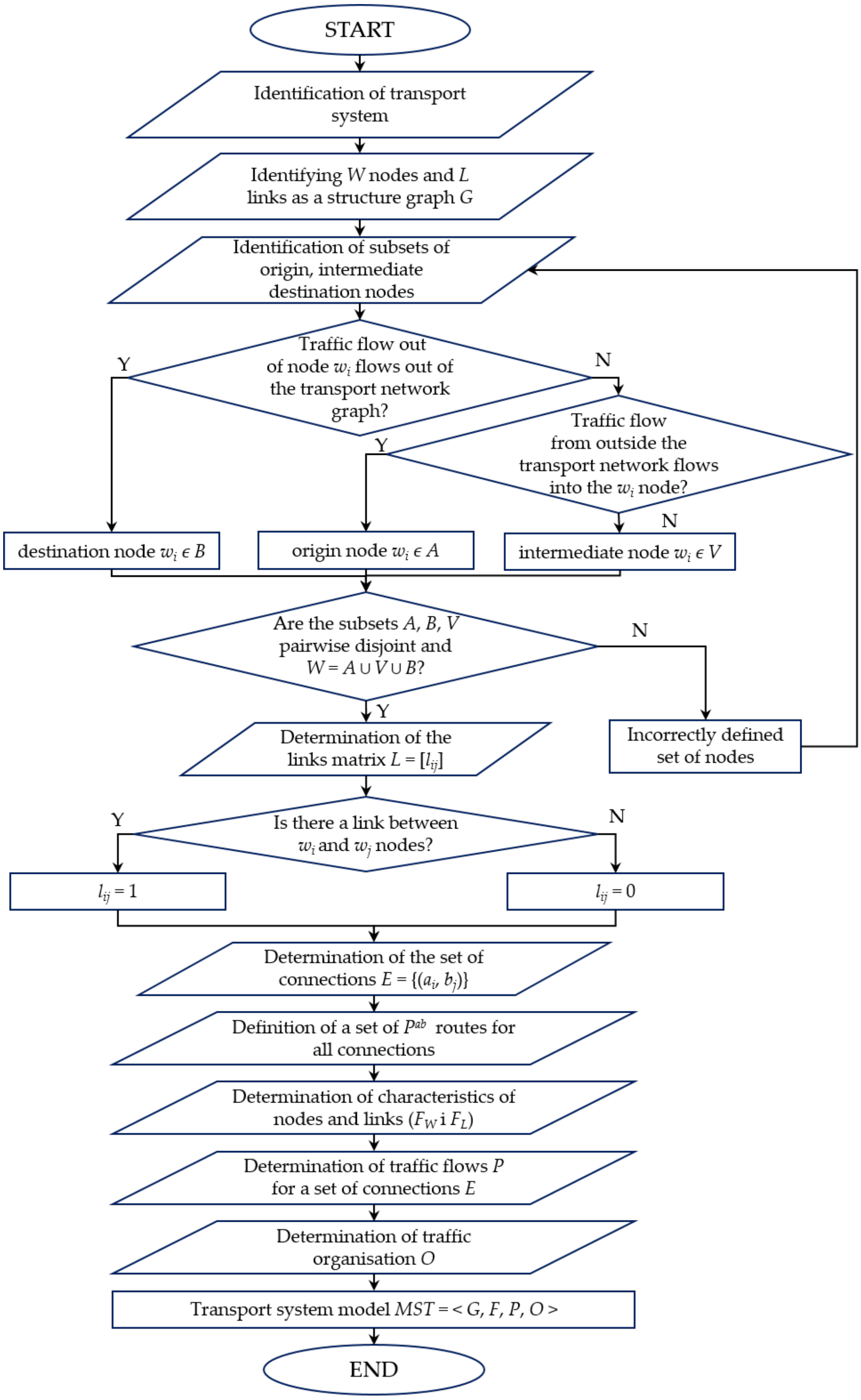

Figure 1. Transport system model creation algorithm (TSMA).

The structure of a model created as a result of the algorithm implemented provides grounds for an analysis of the transport system and enables optimization of the distribution of traffic flow in the transport network according to an assumed criterion (set of criteria).

3.1. Nash Equilibrium

In game theory, a Nash equilibrium refers to a non-cooperative game with two or more participants, assuming that each participant knows the strategy of the other participants. Every static game has at least one Nash equilibrium [59].

The Nash equilibrium assumes that each transport system user independently makes a decision regarding the choice of the transport route, focusing on minimizing travel time or transport costs. This corresponds to a road traffic situation, in which a vehicle driver, having current information on the volume of traffic on the individual link of the transport network (i, j) and the predicted travel time $t^{p,ab}\left(x^{p,ab}\right)$ selects the route of the route of movement p. Road situation can be obtained on an ongoing basis (on-line) from navigation applications [60,61] or intelligent systems [62].

Traffic flows occurring at origin node $a \in A$ are assumed to be displaced to destination node $b \in B$, as described by Equation (8):

$$x^{(a,b)} = \sum_{p \in P^{ab}} x^{p,ab}, \ \forall (a,b) \in E. \tag{8}$$

The sum of traffic flows entering node $j \in W$ is equal to the sum of the traffic flows leaving that node, according to the relation (9):

$$\sum_{i \in W} x_{ij} = \sum_{k \in W} x_{jk}, \ \forall j \in W. \tag{9}$$

The aggregate flow on the link (i, j) is equal to the sum of the flows in all O-D pairs $(a, b) \in E$ on all routs $p \in P^{ab}$ containing link (i, j), according to Equation (10) [11]:

$$x_{ij} = \sum_{(a,b) \in E} \sum_{p \in P^{ab}} a_{ij}^{p,ab} \delta^{p,ab} = \sum_{(a,b) \in E} \sum_{p \in P^{ab}} x^{p,ab}, \ \forall (i,j) \in L, \tag{10}$$

Under the non-negativity condition of the traffic flow (11) [11]:

$$x_{ij} \geq 0, \ \forall (i,j) \in L. \tag{11}$$

The Nash equilibrium (user equilibrium) can also be interpreted as a potential stable point in a dynamic adjustment process in which individuals (road users) adapt their behaviour to that of road users, looking for strategies that give them better results [63].

The system is in a state of equilibrium, if the average transport times for all possible travel routes in relation to individual movement O-D pairs are equal. Assuming that $t^{p,ab}$ is the average movement time of the transport unit within the O-D pair (a, b) on route p, the Nash equilibrium equation takes the following form [27,50,64]:

$$min \sum_{(i,j) \in L} \int_0^{x_{ij}} t_{ij}(x) \, dx, \tag{12}$$

$$\forall (a,b) \in E, \ \forall p \in P_{x>0}^{ab} t^{p,ab}\left(x^{p,ab}\right) = \alpha^{ab} \wedge \alpha^{ab} \to min, \tag{13}$$

where: E—set of movement pairs O-D within a transport system, P^{ab}—set of roads for movement pairs O-D (a, b), $x^{p,ab}$—traffic flow volume on road p within a pair O-D (a, b), $t^{p,ab}$—travel time of a traffic flow unit by route p within a pair O-D (a, b), α^{ab}—minimum travel time within a pair O-D (a, b).

If Nash equilibrium is achieved by the transport system, then any change in the path of movement by any of the traffic participants cannot be individually beneficial to them. In this situation, each traffic participant, guided by individual profit (minimum travel time), remains at his own choice.

3.2. Stackelberg Approach

In the case of transport systems where traffic flow distribution is determined by a central decision-maker (routed traffic), the total transport time may be treated as a decisive criterion for traffic flow distribution within a transport network. In the Stackelberg game, there are restrictions and conditions on the movement of traffic flows determined by the Equations (8)–(11). Therefore, given the above assumptions, the objective function with respect to the optimization of the transport system can be written as Equation (14) [26,65]:

$$T_c(X^*) = min\left\{T_c(X) : X \in D^{dop}\right\}, \quad (14)$$

where:

$$T_c(X) = \sum_{(a,b)\in E} \sum_{p\in P^{ab}} x^{p,ab} \times t^{p,ab} = \sum_{(i,j)\in L} x_{ij} t_{ij}(x_{ij}), \quad (15)$$

where: X—traffic flow distribution matrix, X*—optimal traffic flow distribution matrix, $T_c(X)$—criterion function (total traffic flow movement time), D^{dop}—set of permissible solutions, $x^{p,ab}$—traffic flow volume on road p within a O-D pair (a, b), $t^{p,ab}$–movement time for a traffic flow unit on road p within the O-D pair (a, b).

In road traffic engineering, optimization based on the minimum total time criterion is called the Stackelberg traffic flow distribution or system optimum (SO) solution. Optimization is carried out according to the criterion of equal end times in order to achieve this state of a transport system. For each pair of transport O-D (a, b), the marginal times $m^{p,ab}$ for all movement routes $p \epsilon P$ should be mutually equal, as described by the relationship (16) [25,28,29,66]:

$$\forall(a,b) \in E,\ \forall p,q \in P^{ab}\ m^{p,ab}(x) = m^{q,ab}(x). \quad (16)$$

The marginal time for the route p is calculated as the sum of the marginal times of the links on the route p, as presented in Equation (17):

$$m^{p,ab}(x) = \sum_{(i,j)\in p} m_{ij}(x_{ij}). \quad (17)$$

And the marginal time for the link (i, j) is equal to the derivative of the product of the average travel time through link (i, j) and the size of the traffic flow on link (i, j), which is described by the Equation (18):

$$m_{ij}(x_{ij}) = \frac{d\left[t_{ij}(x_{ij}) \cdot x_{ij}\right]}{dx_{ij}}. \quad (18)$$

If the equality of marginal times in O-D pairs is satisfied, then a change of route by any vehicle would result in a loss for the whole system, moving the value of the objective function away from the optimal value.

3.3. Description of Research

Figure 2 shows a schematic of the methodology of the research conducted. After developing the transport network model, three scenarios were defined for the size of traffic flows. The all the constraints and conditions that the solutions must meet were determined. For the Nash equilibrium and the Stackelberg approach, the corresponding objective functions were established. Using non-linear programming, the optimal solutions for both methods were determined. On the basis of these, the traffic congestion on the individual links in the road network was calculated. Differences in total travel times were used to calculate the Price of Anarchy.

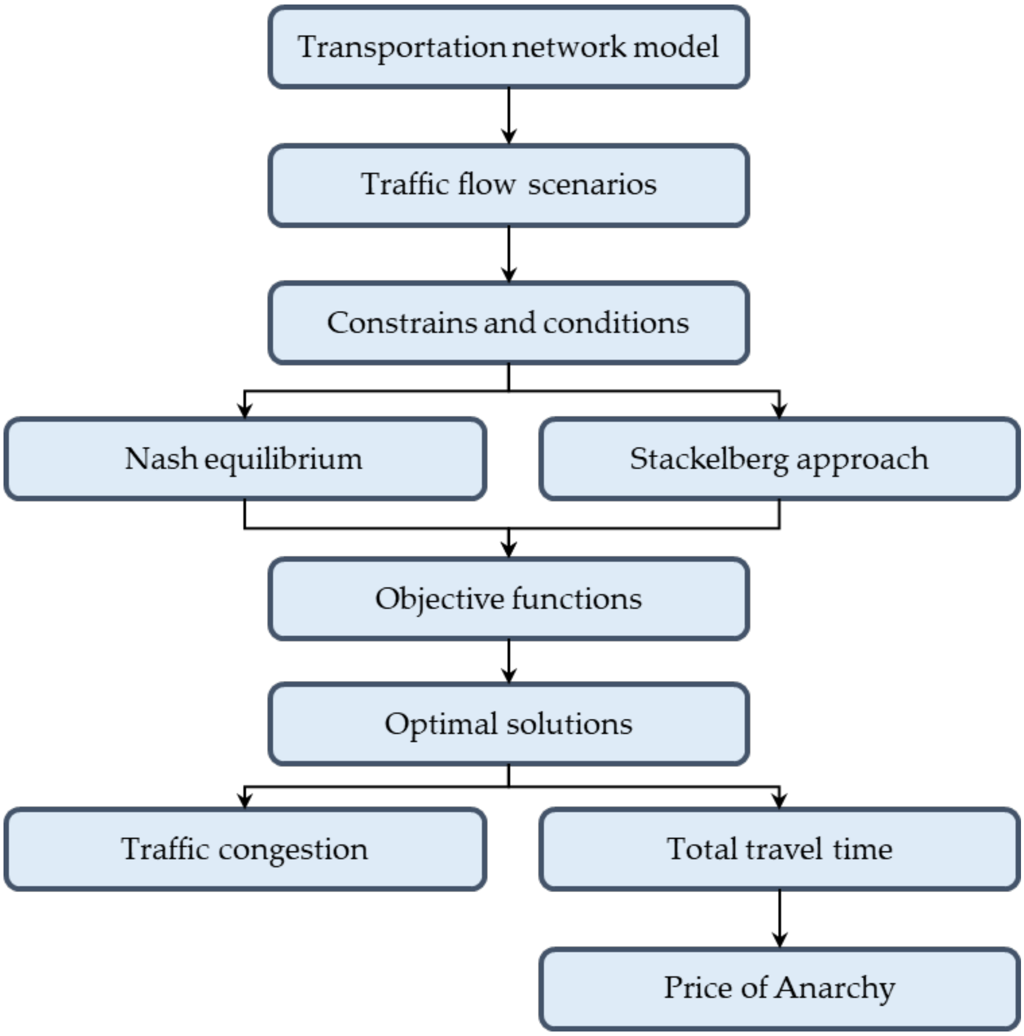

Figure 2. Flowchart of the research.

4. Results

This section presents the results of a study on optimizing traffic flows in relation to a real case study. A road network model of the north-eastern part of the Warsaw agglomeration was developed according to the proposed methodology. The optimal solutions were then determined for three scenarios of traffic flow volume according to Nash equilibrium and the Stackelberg approach.

4.1. Case Study

The presented optimization approaches have been applied to a solution to an actual road transport problem, occurring in the north-eastern part of the Warsaw agglomeration. It is an area that often experiences the phenomenon of traffic congestion caused by heavy traffic flows during the morning and afternoon rush hours. The lack of a well-developed road network in relation to the number of traffic users implies the need to search for new and better solutions, assuming appropriate limiting conditions. Figure 3 shows a road map with the main nodes plotted of the analyzed transport system.

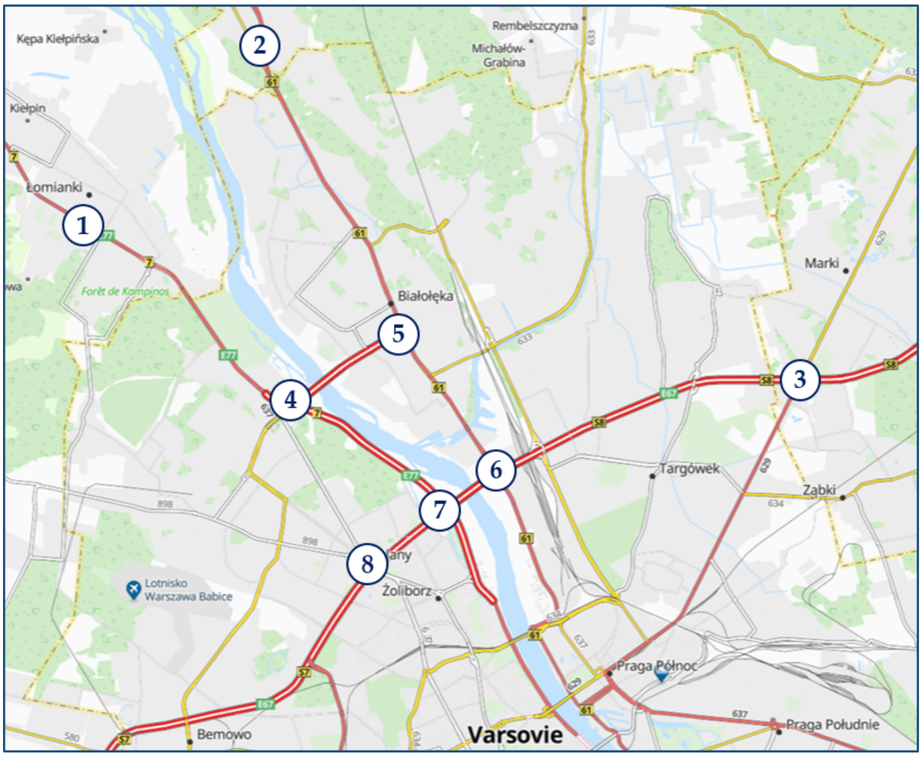

Figure 3. Road map with road nodes plotted.

Figure 4 shows a graph of the analysed transport network. The transport network consists of eight nodes, three of which are the origin nodes, and one is the traffic flow destination node. Three traffic flows marked as x_1, x_2 and x_3 are found at the network entrance. Vectors were used to determine the direction of movement of allowed traffic flows along nine transport links. Traffic flow marked as x_8, which is the sum of flows in all origin nodes, is headed for the destination node.

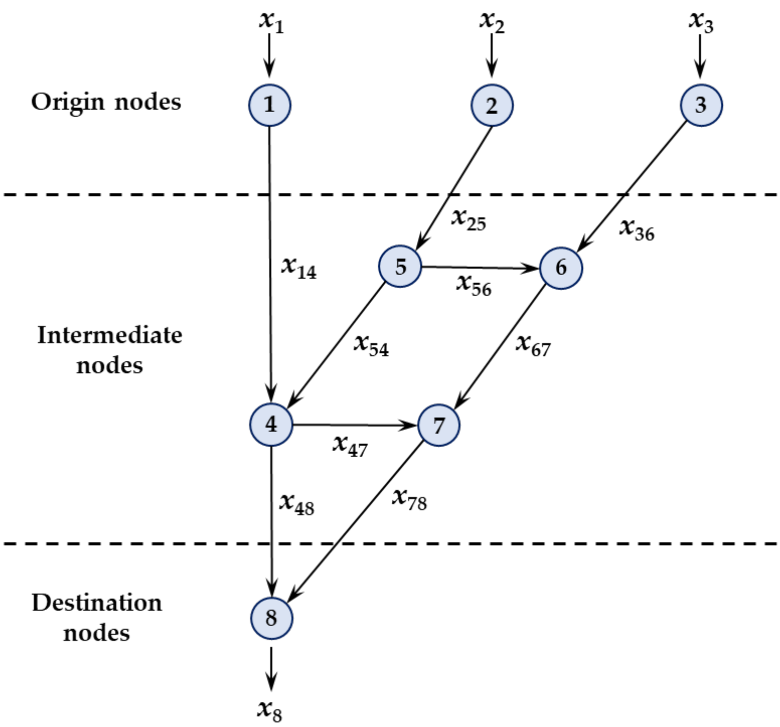

Figure 4. Transport network graph.

A detailed description of the transport network can be found in Table 2. There are three movement O-D pairs within the analyzed system: (1,8), (2,8) and (3,8). The transport network graph for each O-D pair was used to determine the sets of possible routes composed of links L_{ij}.

Table 2. Characteristics of transport network.

Element of Model	Mathematical Notation
Structure	G = <W, L>
Set of nodes	W = {1, 2, 3, 4, 5, 6, 7, 8}
Set of origin nodes	A = {1, 2, 3}
Set of intermediate nodes	V = {4, 5, 6, 7}
Set of destination nodes	B = {8}
Set of O-D pairs	E = {(1,8), (2,8), (3,8)}
Set of links	L = {(1,4), (2,5), (3,6), (4,7), (4,8), (5,4), (5,6), (6,7), (7,8)}
Set of routes for O-D pair (1,8)	P^{18} = {p_1, p_2} = {<(1,4), (4,8)>, <(1,4), (4,7), (7,8)>}
Set of routes for O-D pair (2,8)	P^{28} = {p_3, p_4, p_5} = {<(2,5), (5,4), (4,8)>, <(2,5), (5,4), (4,7), (7,8)>, <(2,5), (5,6), (6,7), (7,8)>}
Set of routes for O-D pair (3,8)	P^{38} = {p_6} = {<(3,6), (6,7), (7,8)>}

Each link in the transport network has been assigned its time characteristics, which determine the travel time through a given section of the road, depending on the flow of traffic. Increased traffic volume leads to an elevated average travel time through a transport link and a significant reduction in the average travel speed (especially in bottlenecks), which in consequence may result in congestion. Equation (19) was used to present the characteristic time matrix of the analyzed road network.

$$T = \begin{bmatrix} - & - & - & t_{14}(x_{14}) & - & - & - & - \\ - & - & - & - & t_{25}(x_{25}) & - & - & - \\ - & - & - & - & - & t_{36}(x_{36}) & - & - \\ - & - & - & - & - & - & t_{47}(x_{47}) & t_{48}(x_{48}) \\ - & - & - & t_{54}(x_{54}) & - & t_{56}(x_{56}) & - & - \\ - & - & - & - & - & - & t_{67}(x_{67}) & - \\ - & - & - & - & - & - & - & t_{78}(x_{78}) \\ - & - & - & - & - & - & - & - \end{bmatrix}. \quad (19)$$

For the traffic flow x_{ij} expressed in thousands of vehicles, the characteristics of individual links in the transport network are shown in the form of a system of Equations (20):

$$\begin{cases} t_{14}(x_{14}) = 5 + 0.05x_{14} + 0.025x_{14}^2 \\ t_{25}(x_{25}) = 10 + 0.04x_{25} + 0.065x_{25}^2 \\ t_{36}(x_{36}) = 6 + 0.03x_{36} + 0.025x_{36}^2 \\ t_{47}(x_{47}) = 4 + 0.04x_{47} + 0.04x_{47}^2 \\ t_{48}(x_{48}) = 5 + 0.08x_{48} + 0.125x_{48}^2 \\ t_{54}(x_{54}) = 4 + 0.035x_{54} + 0.02x_{54}^2 \\ t_{56}(x_{56}) = 5 + 0.035x_{56} + 0.04x_{56}^2 \\ t_{67}(x_{67}) = 1 + 0.035x_{67} + 0.03x_{67}^2 \\ t_{78}(x_{78}) = 2 + 0.025x_{78} + 0.035x_{78}^2 \end{cases}. \quad (20)$$

The set of permissible solutions is determined by the conditions to be satisfied by the distribution of the traffic flow within a transport network, which were presented through systems of equations and inequalities (21)–(24):

- condition of satisfying transport demand (21), assuming that the total traffic flow within a given O-D pair moves entirely along identified transport network roads:

$$\begin{cases} x_1 = x_{1,18} + x_{2,18} \\ x_2 = x_{3,28} + x_{4,28} + x_{5,28}, \\ x_3 = x_{6,38} \end{cases} \tag{21}$$

- condition of traffic flow non-negativity (22):

$$\begin{cases} x_{14} \geq 0 \\ x_{25} \geq 0 \\ x_{36} \geq 0 \\ x_{47} \geq 0 \\ x_{48} \geq 0, \\ x_{54} \geq 0 \\ x_{56} \geq 0 \\ x_{67} \geq 0 \\ x_{78} \geq 0 \end{cases} \tag{22}$$

- condition of traffic flow additivity (23), according to which traffic flows along various O-D pairs and routes add together on common transport links:

$$\begin{cases} x_{14} = x_{1,18} + x_{2,18} \\ x_{25} = x_{3,28} + x_{4,28} + x_{5,28} \\ x_{36} = x_{6,38} \\ x_{47} = x_{2,18} + x_{4,28} \\ x_{48} = x_{1,18} + x_{3,28} \\ x_{54} = x_{3,28} + x_{4,28} \\ x_{56} = x_{5,28} \\ x_{67} = x_{5,28} + x_{6,38} \\ x_{78} = x_{2,18} + x_{4,28} + x_{5,28} + x_{6,38} \end{cases}, \tag{23}$$

- condition of maintained traffic flow (24), which assumes that the sum of all flows entering a node is equal to the sum of flows exiting this node:

$$\begin{cases} x_1 = x_{14} \\ x_2 = x_{25} \\ x_3 = x_{36} \\ x_{14} + x_{54} = x_{47} + x_{48} \\ x_{25} = x_{54} + x_{56} \\ x_{36} + x_{56} = x_{67} \\ x_{47} + x_{67} = x_{78} \\ x_{48} + x_{78} = x_8 \end{cases}. \tag{24}$$

4.2. Optimization with the Nash and Stackelberg Approaches

The Nash equilibrium is based on the assumption of equal average movement times per vehicle during transport operations performed within a transport network. The average travel time through route p is equal to the total of average travel times through the links that make up this link. The average travel times within the analyzed transport network are expressed by the relationship (25):

$$\begin{cases} t_{1,18}(x) = t_{14}(x_{14}) + t_{48}(x_{48}) \\ t_{2,18}(x) = t_{14}(x_{14}) + t_{47}(x_{47}) + t_{78}(x_{78}) \\ t_{3,28}(x) = t_{25}(x_{25}) + t_{54}(x_{54}) + t_{48}(x_{48}) \\ t_{4,28}(x) = t_{25}(x_{25}) + t_{54}(x_{54}) + t_{47}(x_{47}) + t_{78}(x_{78}) \\ t_{5,28}(x) = t_{25}(x_{25}) + t_{56}(x_{56}) + t_{67}(x_{67}) + t_{78}(x_{78}) \\ t_{6,38}(x) = t_{36}(x_{36}) + t_{67}(x_{67}) + t_{78}(x_{78}) \end{cases}. \tag{25}$$

For the travel O-D pair (1,8) there are two routes p_1 and p_2, while for the O-D pair (2,8), three possible routes p_3, p_4 and p_5 were established. The state of equilibrium is achieved if the following conditions are met (26):

$$\begin{cases} t_{1,18}(x) = t_{2,18}(x) \\ t_{3,28}(x) = t_{4,28}(x) = t_{5,28}(x) \end{cases}. \tag{26}$$

The O-D pair (3,8) has only one possible movement route, resulting in the lack of equilibrium for flows moving between nodes 3 and 8. The equilibrium state described by the system of Equations (26) corresponds to a situation in which a change in the route of movement of any participant in the traffic would result in an extended travel time.

To perform the optimization using the Stackelberg approach, the marginal times $m_{ij}(x_{ij})$ for individual links of the road network were calculated. They are presented in the form of relationship (27):

$$\begin{cases} m_{14}(x_{14}) = \frac{\mathrm{d}[t_{14}(x_{14})\cdot x_{14}]}{\mathrm{d}x_{14}} = 5 + 0.1x_{14} + 0.075x_{14}^2 \\ m_{25}(x_{25}) = \frac{\mathrm{d}[t_{25}(x_{25})\cdot x_{25}]}{\mathrm{d}x_{25}} = 10 + 0.08x_{25} + 0.195x_{25}^2 \\ m_{36}(x_{36}) = \frac{\mathrm{d}[t_{36}(x_{36})\cdot x_{36}]}{\mathrm{d}x_{36}} = 6 + 0.06x_{36} + 0.075x_{36}^2 \\ m_{47}(x_{47}) = \frac{\mathrm{d}[t_{47}(x_{47})\cdot x_{47}]}{\mathrm{d}x_{47}} = 4 + 0.08x_{47} + 0.12x_{47}^2 \\ m_{48}(x_{48}) = \frac{\mathrm{d}[t_{48}(x_{48})\cdot x_{48}]}{\mathrm{d}x_{48}} = 5 + 0.16x_{48} + 0.375x_{48}^2 \\ m_{54}(x_{54}) = \frac{\mathrm{d}[t_{54}(x_{54})\cdot x_{54}]}{\mathrm{d}x_{54}} = 4 + 0.07x_{54} + 0.06x_{54}^2 \\ m_{56}(x_{56}) = \frac{\mathrm{d}[t_{56}(x_{56})\cdot x_{56}]}{\mathrm{d}x_{56}} = 5 + 0.07x_{56} + 0.12x_{56}^2 \\ m_{67}(x_{67}) = \frac{\mathrm{d}[t_{67}(x_{67})\cdot x_{67}]}{\mathrm{d}x_{67}} = 1 + 0.07x_{67} + 0.09x_{67}^2 \\ m_{78}(x_{78}) = \frac{\mathrm{d}[t_{78}(x_{78})\cdot x_{78}]}{\mathrm{d}x_{78}} = 2 + 0.05x_{78} + 0.105x_{78}^2 \end{cases} \tag{27}$$

The Stackelberg approach assumes the presence of a minimum total movement time with equal marginal travel times on individual routes in the transport network routes. The marginal travel time through route p is equal to the total marginal travel times through the links comprising route p. Marginal travel times within the analyzed transport network are expressed by the relationship (28):

$$\begin{cases} m_{1,18}(x) = m_{14}(x_{14}) + m_{48}(x_{48}) \\ m_{2,18}(x) = m_{14}(x_{14}) + m_{47}(x_{47}) + m_{78}(x_{78}) \\ m_{3,28}(x) = m_{25}(x_{25}) + m_{54}(x_{54}) + m_{48}(x_{48}) \\ m_{4,28}(x) = m_{25}(x_{25}) + m_{54}(x_{54}) + m_{47}(x_{47}) + m_{78}(x_{78}) \\ m_{5,28}(x) = m_{25}(x_{25}) + m_{56}(x_{56}) + m_{67}(x_{67}) + m_{78}(x_{78}) \\ m_{6,38}(x) = m_{36}(x_{36}) + m_{67}(x_{67}) + m_{78}(x_{78}) \end{cases}. \tag{28}$$

The condition of equal marginal times for the analyzed road network is presented by the system of Equations (29):

$$\begin{cases} m_{1,18}(x) = m_{2,18}(x) \\ m_{3,28}(x) = m_{4,28}(x) = m_{5,28}(x) \end{cases}. \tag{29}$$

4.3. Baseline Scenario #1

The first to be analyzed was the baseline scenario, in which the traffic flow volumes that occur for individual O-D pairs during morning rush hours were estimated based on the results of the General Traffic Measurement (GPR) implemented by the General Directorate of National Roads and Motorways (GDDKiA) in the years 2020–2021. For movement O-D pairs (1,8), (2,8) and (3,8), the transport demand values amount to 4000, 5000 and 7000 veh./h, respectively.

Calculations for both optimization approaches were performed with computer assistance using LINGO 18.0 software (Lindo Systems, Chicago, USA). Table 3 lists the results obtained for the individual routes of the transport network. According to the assumptions of the Nash equilibrium, the travel times for each available route p within each O-D pair (a, b) are equal. A transport system reaches an equilibrium state with the traffic flow distribution shown in the column „Nash" in Table 3. A movement O-D pair (3,8) may be implemented only through one route p_6, therefore the entire traffic flow must be moved through route p_6, regardless of the optimization approach.

Table 3. Results of optimization with the Nash and Stackelberg approaches for movement routes in scenario #1.

O-D Pair	Route	Traffic Flow Volume ($\times 10^3$ veh.)		Average Travel Time (Min)		Marginal Travel Time (Min)
		Nash	Stackelberg	Nash	Stackelberg	Stackelberg
(1,8)	p_1	4.000	4.000	15.54	15.32	28.30
	p_2	0.000	0.000	-	-	-
	p_3	1.973	1.835	26.14	26.19	39.73
(2,8)	p_4	1.610	3.026	26.14	26.83	39.73
	p_5	1.417	0.139	26.14	25.48	39.73
(3,8)	p_6	7.000	7.000	16.62	16.09	29.54

By translating the volume of traffic flow within individual routes of the O-D pairs in question, the authors calculated the volumes of traffic within the links of the road network. The results are listed in Table 4 and presented graphically in Figures 5 and 6.

Table 4. Results of optimization with the Nash and Stackelberg approaches for road network links in scenario #1.

Link	Traffic Flow Volume ($\times 10^3$ veh.)		Average Travel Time (Min)	
	Nash	Stackelberg	Nash	Stackelberg
(1,4)	4.000	4.000	5.60	5.60
(2,5)	5.000	5.000	11.83	11.83
(3,6)	7.000	7.000	7.44	7.44
(4,7)	1.610	3.026	4.17	4.49
(4,8)	5.973	5.835	9.94	9.72
(5,4)	3.583	4.861	4.38	4.64
(5,6)	1.417	0.139	5.13	5.01
(6,7)	8.417	7.139	3.42	2.78
(7,8)	10.027	10.165	5.77	5.87

Traffic flow volume and travel time through links (1,4), (2,5) and (3,6) have the same values in the case of the Nash equilibrium and the Stackelberg approach. This results from the transport network structure, which implies the need to send all the traffic flow originating in nodes 1, 2 and 3, respectively, through these three links. The linear element most loaded of the road network is link (7,8), for which the traffic flow within the analyzed system exceeds 10,000 veh./h. On the other hand, the lowest volume is experienced in link (5,6). In the case of the Nash equilibrium, it is more than ten times higher than in the case of the Stackelberg approach traffic distribution.

Road congestion values were calculated based on travel time through individual road network links obtained for individual solutions under the assumptions of the Nash and Stackelberg approaches. The level of congestion was calculated as a quotient of the travel time through the link (i,j) under specific traffic conditions and the travel time through this link under minimum traffic conditions. The authors calculated the an increase in travel time relative to the conditions of fully free traffic. The least favorable road conditions were observed for link (6,7), in the case of Nash equilibrium, where the transport time was more than 240% longer than under standard conditions. Regardless of the optimization approach, the lowest level of traffic congestion occurred for link (5,6), where the travel time increased by less than 3%. The results for all links in the road network are summarized in Table 5.

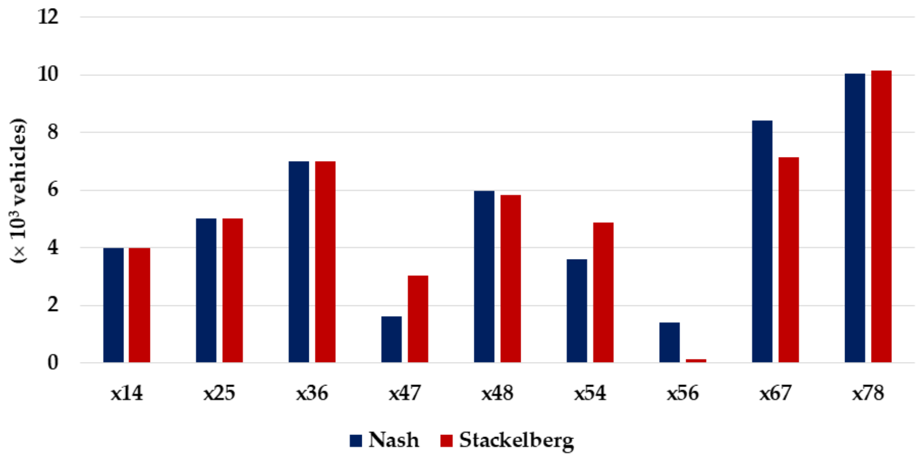

Figure 5. Comparison of traffic volume on individual links to road network for scenario #1.

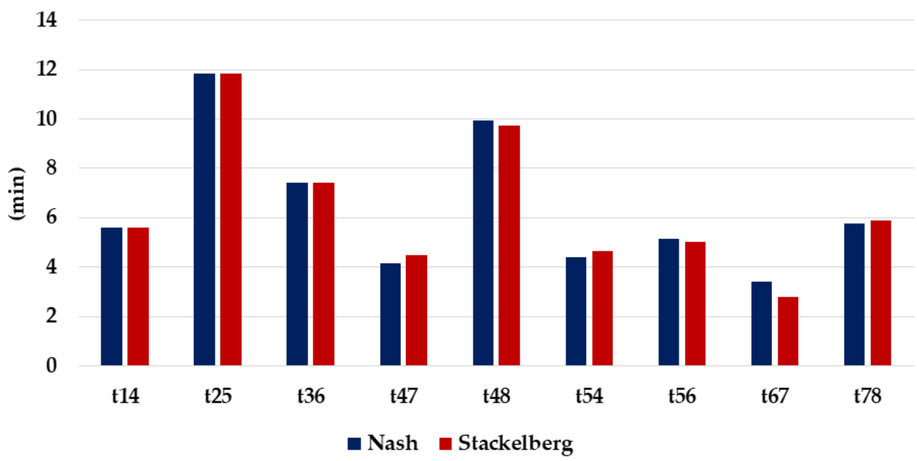

Figure 6. Compared travel times through individual links to road network for scenario #1.

Table 5. Traffic congestion in transport network links for scenario #1.

Link	Congestion		Travel Time Increase (%)	
	Nash	Stackelberg	Nash	Stackelberg
(1,4)	1.12	1.12	12.00	12.00
(2,5)	1.18	1.18	18.25	18.25
(3,6)	1.24	1.24	23.92	23.92
(4,7)	1.04	1.12	4.20	12.18
(4,8)	1.99	1.94	98.75	94.44
(5,4)	1.10	1.16	9.55	16.07
(5,6)	1.03	1.00	2.60	0.11
(6,7)	3.42	2.78	242.02	177.89
(7,8)	2.88	2.94	188.48	193.54

4.4. Alternative Scenario #2

The alternative scenario #2 is a hypothetical road situation, which assumes a decrease in traffic flow volumes for all movement O-D pairs by 10%, relative to the baseline scenario. In this case, the transport demand takes the following respective values: 3600, 4500 and 6300 veh./h. The results of optimization with the Nash and Stackelberg approaches are summarized in Tables 6 and 7 and illustrated in Figures 7 and 8.

Table 6. Results of optimization with the Nash and Stackelberg approaches for movement routes in scenario #2.

O-D Pair	Route	Traffic Flow Volume ($\times 10^3$ veh.)		Average Travel Time (Min)		Marginal Travel Time (Min)
		Nash	Stackelberg	Nash	Stackelberg	Stackelberg
(1,8)	p_1	2.585	2.376	14.63	14.39	25.56
	p_2	1.015	1.224	14.63	15.02	25.56
(2,8)	p_3	2.846	2.886	24.88	24.88	35.90
	p_4	0.000	1.346	-	25.52	35.90
	p_5	1.654	0.268	24.88	24.18	35.90
(3,8)	p_6	6.300	6.300	15.40	14.86	25.92

Table 7. Results of optimization with the Nash and Stackelberg approaches for road network links in scenario #2.

Link	Traffic Flow Volume ($\times 10^3$ veh.)		Average Travel Time (Min)	
	Nash	Stackelberg	Nash	Stackelberg
(1,4)	3.600	3.600	5.50	5.50
(2,5)	4.500	4.500	11.50	11.50
(3,6)	6.300	6.300	7.18	7.18
(4,7)	1.015	2.570	4.08	4.37
(4,8)	5.431	5.262	9.12	8.88
(5,4)	2.846	4.232	4.26	4.51
(5,6)	1.654	0.268	5.17	5.01
(6,7)	7.954	6.568	3.18	2.52
(7,8)	8.969	9.138	5.04	5.15

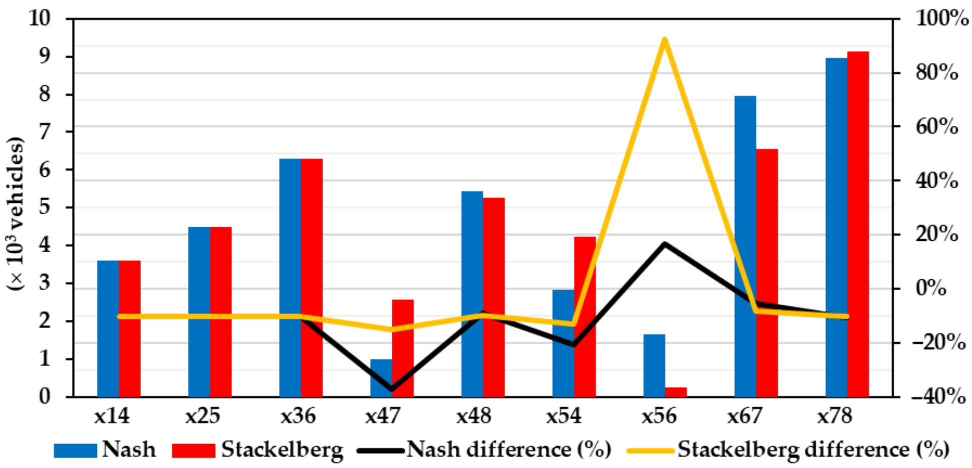

Figure 7. Traffic volume compared in individual links to road network for scenario #2. (Nash and Stackelberg differences (%) represent the percentage increase in traffic volumes on each link compared to the results of scenario #1).

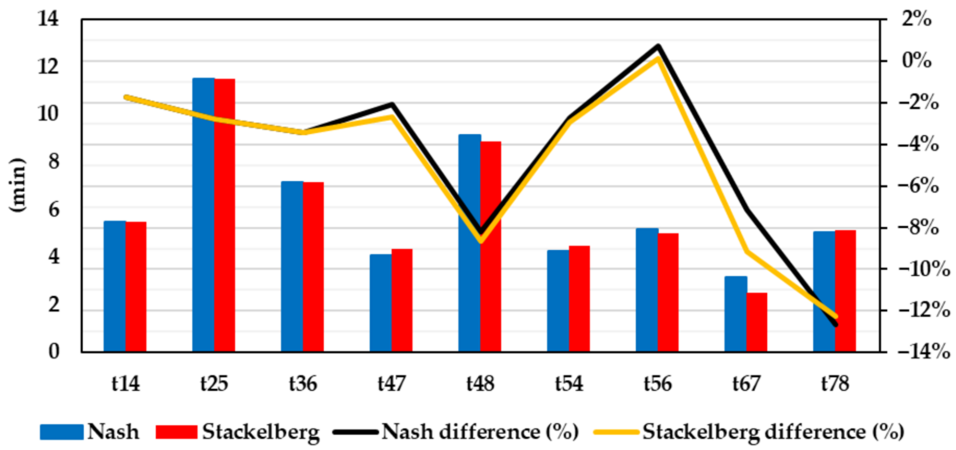

Figure 8. Compared travel times through individual links to road network for scenario #2. (Nash and Stackelberg differences (%) represent the percentage increase in travel time through each link compared to the results of scenario #1).

The number of vehicles traveling on the links to the starting nodes is strictly determined by the value of demand for transportation on each route. It should be stressed that, in the case of the analyzed system, they do not depend on the applied traffic flow distribution approach. Significant differences in traffic flow depended on the applied optimization approach occurring in links (4,7), (5,4), (5,6), and (6,7). Regardless of the applied approach (Nash, Stackelberg), the most loaded road section in both cases was on link (7,8). On the other hand, the least loaded is link (4,7) for the solution obtained with the Nash approach and link (5,6) in the case of the Stackelberg approach.

Despite the reduced transport demand for all O-D pairs by 10% compared to scenario #1, the travel time through link (5,6) for the solution using the Nash approach increased slightly, from a level of 5.13 (min) to 5.17 (min), which translates to a relative difference of 0.73%. This is caused by an increase in link traffic volume by 16.68%. In turn, when it comes to the results obtained with the Stackelberg approach, a vehicle number increased within the link (5,6) by more than 92% resulting in the travel time being extended by only 0.13%. The increase in the volume of traffic within transport links, for relatively low levels of the absolute number of vehicles, leads to minor differences in the average travel times for these links.

Table 8 shows the impact of the traffic flow distribution on the state of traffic congestion for scenario #2. The highest congestion rate was obtained for links (6,7) and (7,8), while in the case of the remaining segment of the road network this value did not exceed 2 for the results obtained regardless of the optimization approach adopted. It should be emphasized the Stackelberg approach allows a better balance of the value of the congestion rate by reducing the difference between its maximum and minimum values within the road network.

Table 8. Traffic congestion on transport network links for scenario #2.

Link	Congestion		Travel Time Increase (%)	
	Nash	Stackelberg	Nash	Stackelberg
(1,4)	1.10	1.10	10.08	10.08
(2,5)	1.15	1.15	14.96	14.96
(3,6)	1.20	1.20	19.69	19.69
(4,7)	1.02	1.09	2.05	9.18
(4,8)	1.82	1.78	82.43	77.63
(5,4)	1.07	1.13	6.54	12.66
(5,6)	1.03	1.00	3.35	0.24
(6,7)	3.18	2.52	217.62	152.40
(7,8)	2.52	2.58	151.98	157.56

4.5. Alternative Scenario #3

The third scenario assumes an increase in transport demand for each analyzed O-D pair by 10%, relative to the baseline scenario. This corresponds to traffic flow values of: 4400, 5500 and 7700 veh./h. The optimization results with the Nash and Stackelberg approaches are summarized in Tables 9 and 10, and illustrated in Figures 9 and 10.

Table 9. Results of optimization with the Nash and Stackelberg approaches for movement routes in scenario #3.

O-D Pair	Route	Traffic Flow Volume ($\times 10^3$ veh.)		Average Travel Time (Min)		Marginal Travel Time (Min)
		Nash	Stackelberg	Nash	Stackelberg	Stackelberg
(1,8)	p_1	4.400	4.400	16.55	16.35	31.32
	p_2	0.000	0.000	-	-	-
	p_3	2.125	2.009	27.55	27.63	43.95
(2,8)	p_4	2.167	3.470	27.55	28.26	43.95
	p_5	1.208	0.021	27.55	26.91	43.95
(3,8)	p_6	7.700	7.700	17.98	17.43	33.52

Table 10. Results of optimization with the Nash and Stackelberg approaches for road network links in scenario #3.

Link	Traffic Flow Volume ($\times 10^3$ veh.)		Average Travel Time (Min)	
	Nash	Stackelberg	Nash	Stackelberg
(1,4)	4.400	4.400	5.70	5.70
(2,5)	5.500	5.500	12.19	12.19
(3,6)	7.700	7.700	7.71	7.71
(4,7)	2.167	3.470	4.27	4.62
(4,8)	6.525	6.409	10.84	10.65
(5,4)	4.292	5.479	4.52	4.79
(5,6)	1.208	0.021	5.10	5.00
(6,7)	8.908	7.721	3.69	3.06
(7,8)	11.075	11.191	6.57	6.66

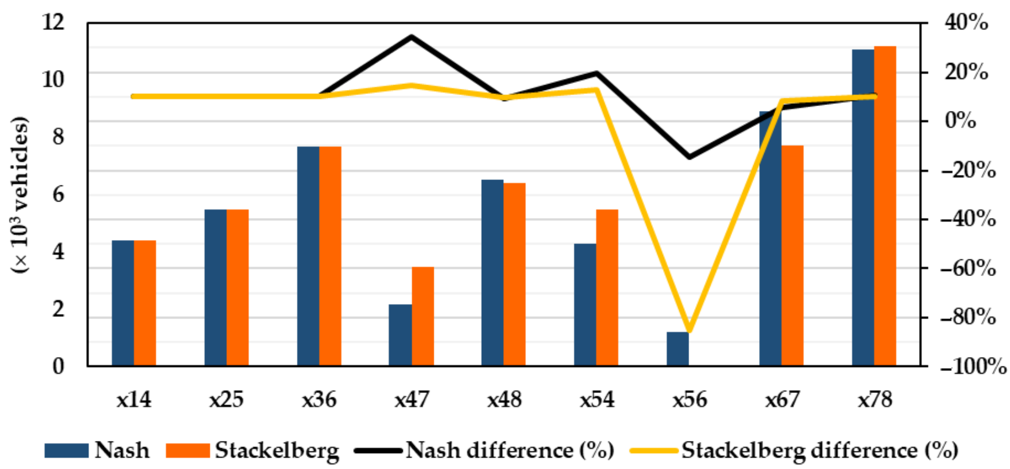

Figure 9. Traffic volume compared on individual links to the road network for scenario #3. (Nash and Stackelberg differences (%) represent the percentage increase in traffic volumes on each link compared to the results of scenario #1).

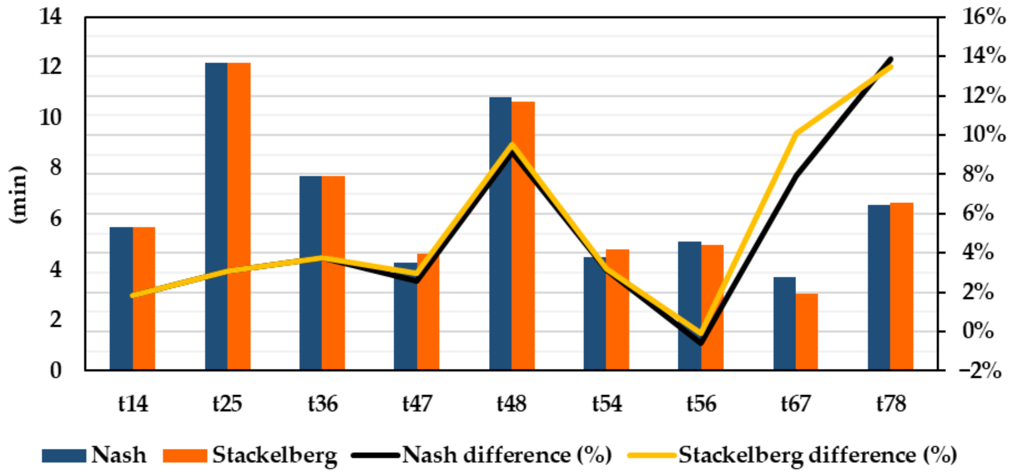

Figure 10. Compared travel times through individual links to the road network for scenario #3. (Nash and Stackelberg differences (%) represent the percentage increase in travel time through each link compared to the results of scenario #1).

In the case of the analyzed system, an increased transport demand for all O-D pairs of movement entails a decreased link load (5,6). This is caused by the increased sensitivity of the travel time through link (6,7) together with the increased volume of traffic. The higher volume of traffic flow results in route p_5 of the O-D pair (2,8) becoming less competitive in terms of travel time relative to routes p_3 and p_4. This is caused by the traffic flow that moves along the O-D pair (3,8), for which there is also a route within the analyzed transport system.

Table 11 shows the results of traffic congestion analysis for scenario #3. Similar to scenarios #1 and #2, the highest congestion rate values were achieved by links (6,7) and (7,8). The average travel times for these sections of the road are greater by more than 200% in a free traffic situation. Congestion with a rate beyond 2 appeared in this scenario also on link (4,8). In turn, despite a growth in transport demand, the situation of the road on other links still did not deteriorate considerably.

Table 11. Traffic congestion in links of transport network for scenario #3.

Link	Congestion		Travel Time Increase (%)	
	Nash	Stackelberg	Nash	Stackelberg
(1,4)	1.14	1.14	14.08	14.08
(2,5)	1.22	1.22	21.86	21.86
(3,6)	1.29	1.29	28.55	28.55
(4,7)	1.07	1.16	6.86	15.51
(4,8)	2.17	2.13	116.88	112.96
(5,4)	1.13	1.20	12.97	19.80
(5,6)	1.02	1.00	2.01	0.01
(6,7)	3.69	3.06	269.24	205.86
(7,8)	3.28	3.33	228.49	233.14

4.6. Compared Optimization Results for Three Scenarios

The last stage of the research was to determine the value of the criterion for the entire transport system. The total travel time T_c for units of all traffic flow within a road network is expressed with the Equation (30):

$$T_c = \sum_{(i,j) \in L} x_{ij} t_{ij}\left(x_{ij}\right). \tag{30}$$

The results of the calculations for the three scenarios of traffic flow volume analyzed in the transport system are shown as a graph in Figure 11. The Stackelberg approach allowed to achieve more favourable solutions for the entire transport system by 0.67–1.07%, while the difference for the baseline scenario #1 was 0.84%, which means that the PoA value was 1.0084. Furthermore, in the case of scenario #2, where traffic volumes were less than 10% relative to the baseline scenario #1, the difference in total times T_c between the solutions obtained using the Stackelberg approach and the Nash equilibrium was the highest.

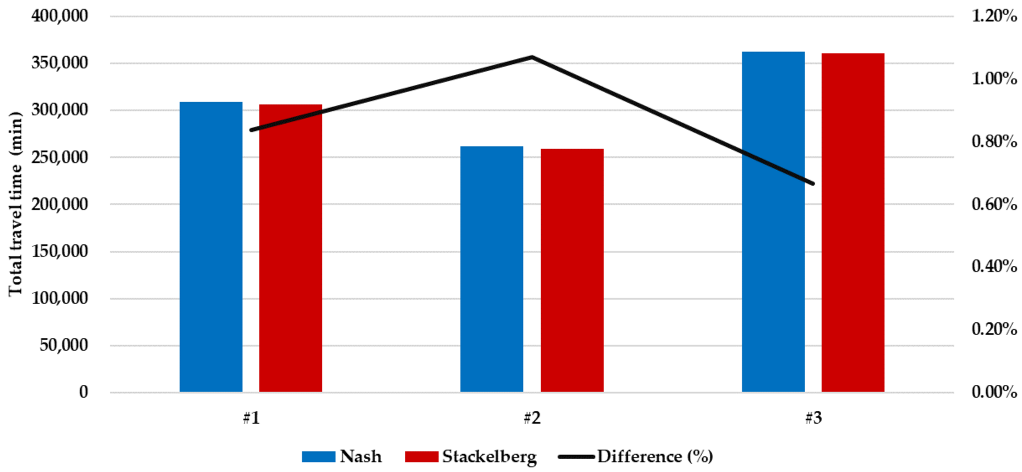

Figure 11. Compared total travel times of the entire traffic flow for the three scenarios. (The difference (%) represents the percentage increase in total travel time between the results of the Stackelberg approach and the Nash equilibrium for each scenario).

5. Discussion

For the case study analyzed for three scenarios, considerations were made for the traffic flow distribution according to the Nash equilibrium and the Stackelberg approach. Computational analyses resulted in the determination of the optimum size of traffic flows on the links of the transport network. The criteria of equal average times and equal marginal times were met for all O-D pairs in the three scenarios.

An increase in traffic flows for each O-D pair in the transport system may result in a reduction in traffic flows on certain links in the road network. For the case study presented, this situation occurred at link (5,6). With an overall increase in traffic flows, it becomes less attractive to travel for vehicles moving from origin node 2 to destination node 8. The selection of node (5,6) for travel in the O-D pair (2,8) results in the need to travel also through node (6,7), which is loaded with a large traffic flow moving in the O-D pair (3,8). Therefore, the p_5 route is more attractive when traffic volumes are low in the transportation network.

Analysis of road congestion as an increase in travel time through transport links identifies the weakest elements of the road network, which have the greatest impact on the decrease in efficiency of transport processes. In the model for the northern part of Warsaw's road network, the greatest road congestion occurs on links (4,8), (6,7) and (7,8). According to the Nash equilibrium for scenario #3, there was a 269.24% increase in travel time on link (6,7). For the baseline scenario, the average congestion levels for the analyzed network were 66.64% and 60.93% for the Nash equilibrium and the Stackelberg approach, respectively. Relating these results to studies by other authors [24], it was found that the congestion for the user equilibrium condition obtained using the developed model corresponds to the level determined by traffic measurements.

Despite focusing on individual benefits for road users, the Nash equilibrium approach turned out to be just slightly less effective compared to minimizing the travel time of the entire traffic flow. The total transport time in the case of the Stackelberg approach was more favourable by approximately 1% compared to the Nash equilibrium approach. It should also be stressed that the time saved can contribute to potential benefits for the environment, due to lower energy consumption and reduced CO_2 emissions to the atmosphere. The aim of optimizing traffic flow distribution should be to minimize travel time and energy consumption. Regarding the findings of other authors [35,36], it should be highlighted that the PoA for the Warsaw case study is at a significantly lower value.

Table 12 summarises the results obtained for the Nash and Stackelberg approaches and the results from the literature for the Nguyen–Dupuis network. The graph of the Nguyen–Dupuis network is shown in Appendix D in Figure 1A. Compared to the algorithm proposed by Nguyen and Dupuis [67], both the Nash equilibrium and the Stackelberg approach obtained better results. The model proposed by Xu et al. [68] achieved shorter travel times. The approaches considered in the articles [69–71] usurped significantly higher travel times for the whole network due to additional restrictions introduced by the authors. In contrast, the decision support model for infrastructure investment issues proposed by Lin [72], in the absence of any investment, was less effective in optimising travel times.

Table 12. Results of optimization on the Nash and Stackelberg approaches in the Nguyen–Dupuis network.

This Paper		Literature	
Results	**Approaches**	**Results**	**Approaches**
83 228.36	Nash equilibrium	85,028.14 [67]	Nguyen–Dupuis algorithm
82 772.50	Stackelberg approach	79,290 [68]	EC-TEP model
		80,269 [68]	EC-TEP model with environmental constrains
		194,025 [69]	Multi-class multibehaviour equilibrium
		~240,000 [70]	Generalized Nash equilibrium problem
		~180,000 [71]	Bush–Mosteller reinforcement learning
		~(75,000–91,000) [72]	Bi-level model for investment decision making

Table 13 compares the Nash equilibrium and the Stackelberg approach in terms of objective function, constraints and conditions, advantages and disadvantages. The Nash equilibrium aims at minimizing travel time for individual users. This approach reflects the current traffic situation and describes the mutual dependencies of the route choices. However, it may result in time losses for the system as a whole and an increase in the level of congestion at certain links in the network. On the contrary, minimizing the system travel time in the Stackelberg approach requires a centralized decision-making system and the compliance of all traffic users. The advantages of this method are not only a reduction in total travel time, but also a reduction in the average level of congestion in the road network.

Table 13. Comparison of Nash equilibrium and Stackelberg approach.

Approach	Objective Function	Constrains and Conditions	Advantages	Disadvantages
Nash equilibrium	Minimization of individual users travel time	• Decentralised system • Condition of satisfying transport demand • Conditions of non-negativity, additivity, maintaining of traffic flow	• Description of present situation in the road networks • Reflects the mutual influence of individual traffic users on route choice	• Causes losses for the entire system (PoA) • May cause high levels of congestion on certain links in the network
Stackelberg approach	Minimization of system total travel time	• Centralised system • Condition of satisfying transport demand • Conditions of non-negativity, additivity, maintaining of traffic flow	• Reduces travel time for the whole system compared to a decentralised system • Reduces the average level of congestion in the road network	• Requires a decision-making system and the compliance of all traffic users • May increase travel time for some traffic users

6. Limitations

The road transport system aims to maintain a state consistent with the Nash equilibrium, provided that all road users have up-to-date and reliable information on the traffic situation and make decisions based on them. On the other hand, achieving an optimal state in line with the Stackelberg approach requires mutual cooperation of individual decision-makers or a centralized IT system that makes decisions with respect to selecting travel routes for the entire traffic flow. In the case of the current organization of the road transport system, implementing optimization based on equal marginal times is impossible. However, in future systems equipped with fully automatic vehicles, the Stackelberg approach can be useful in solving road traffic problems.

Furthermore, the computational complexity of the mathematical model increases significantly with the size of the analyzed road network. In order to program the problem of the nonlinear traffic flow distribution in this case, it is necessary to use advanced computer software to search for optimal solutions according to the criterion of equal mean times for the Nash equilibrium and equal marginal times for the Stackelberg approach.

7. Conclusions

In summary, the proposed TSMA algorithm was used to develop a transport network model. The presented case study covered three main origin nodes and one destination node. Based on the characteristics of the transport links, a numerical analysis of the traffic flow distribution problem was presented using nonlinear programming. The developed deterministic model of the transport system made it possible to optimize the distribution of the traffic flows within the road network. Minimizing travel time along assumed movement routes was adopted as the objective function for both approaches: the Nash equilibrium and the Stackelberg equation. The analysis presented in this paper has highlighted the utility of the proposed TSMA algorithm and nonlinear programming based on Nash equilibrium and the Stackelberg approach in solving optimisation problems related to the road transport networks.

Based on three scenarios, it was found that the proposed approaches make it possible to accurately determine the optimal distribution of traffic flows in the transport network according to the adopted criteria. Increased traffic volume on individual routes in the transport system has been observed to lead to a decrease in traffic volume on some links of the road network. In the analyzed road network, the increase in traffic on the link (6.7) causes congestion and significant travel delays. This means changes in the distribution of traffic flows for the O-D pair (2,8) between the route containing the link (6,7) and the routes composed of other connections. Therefore, the p_5 route is taken by a small number of traffic users, which results in a reduction in traffic on the (5,6) link, which is only part of the p_5 route. This phenomenon is not intuitive and requires examining the elements that make up road networks in a systems approach, taking into account the complex mechanisms of traffic flow.

A drawback of the developed transport system model is its deterministic character and the failure to take into account planned and random events that affect the distinguishing functions of the elements of the road network. Such important events include collisions, road accidents, road infrastructure renovations, and extremely adverse weather conditions. Therefore, in the next stage of the research, we will try to develop a dynamic model corresponding to a real (variable) road situation. Moreover, we hope to apply fuzzy logic to the decision-making problem, which reflects uncertainty and random components. Expanding the developed transport system model onto a larger area (than the one discussed in the paper) of the Warsaw agglomeration might also be also an appropriate direction of research.

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Appendix A

Table A1. Abbreviations and acronyms.

Abbreviation/Acronym	Meaning
EC-TEP	Environmentally constrained traffic equilibrium problem
GDDKiA	General Directorate for National Roads and Motorways
GPR	General Traffic Measurement
MST	Transport system model
O-D	Origin–Destination
PoA	Price of Anarchy
TSMA	Transport system model creation algorithm

Table A2. Notations.

Notation	Definition
A	Set of origin nodes
B	Set of destination nodes
D^{dop}	Set of permissible solutions
E	Set of O-D pairs
F	Set of structure graph node and (or) link functions
F_L	Set of functions defined on a set of links L
F_W	Set of functions defined on a set of nodes W
G	System structure graph
L	Set of graph links G
O	Transport system traffic organization
P	Traffic flow in MST
P^{ab}	Set of routes for (a, b) O-D pair
S	System structure graphs
T	Time characteristic matrix
T_c	Total traffic flow movement time
V	Set of intermediate nodes
W	Set of graph nodes G
$\overline{W}$	Population of the set W
X	Traffic flow distribution matrix
X^*	Traffic flow optimal distribution matrix
α^{ab}	Minimum travel time within a O-D pair (a, b)
$a_{p,ab}^{ij}$	1 if route $p \in \underline{P^{ab}}$ contains link $(i, j) \in L$, 0 otherwise
$\delta^{p,ab}$	Flow rate of O-D pair $(a, b) \in E$ routed on path $p \in \underline{P^{ab}}$
l_{ij}	Link between w_i and w_j nodes
$m_{ij}(x_{ij})$	Marginal travel time by link (i, j)
$m^{p,ab}$	Marginal travel time by route p within a O-D pair (a, b)
p	Route
$t^{p,ab}$	Average travel time by route p within a O-D pair (a, b)
x_{ij}	Traffic flow volume on link (i, j)
$x^{p,ab}$	Traffic flow volume on road p within a O-D pair (a, b)

Appendix B

Code of the Nash equilibrium (scenario #1) for LINGO 18.0

```
t14=5+0.05*x14+0.025*(x14^2);
t25=10+0.04*x25+0.065*(x25^2);
t36=6+0.03*x36+0.025*(x36^2);
t47=4+0.04*x47+0.04*(x47^2);
```

```
t48=5+0.08*x48+0.125*(x48^2);
t54=4+0.035*x54+0.02*(x54^2);
t56=5+0.035*x56+0.04*(x56^2);
t67=1+0.035*x67+0.03*(x67^2);
t78=2+0.025*x78+0.035*(x78^2);

t1=t14+t48;
t2=t14+t47+t78;
t3=t25+t54+t48;
t4=t25+t54+t47+t78;
t5=t25+t56+t67+t78;
t6=t36+t67+t78;

x14>=0;
x25>=0;
x36>=0;
x47>=0;
x48>=0;
x54>=0;
x56>=0;
x67>=0;
x78>=0;

x1=4;
x2=5;
x3=7;

x18=4;
x28=5;
x38=7;

x18=x118+x218;
x28=x328+x428+x528;
x38=x638;

x1=x14;
x2=x25;
x3=x36;
x14+x54=x47+x48;
x25=x54+x56;
x36+x56=x67;
x47+x67=x78;
x48+x78=x14+x25+x36;
x8=x48+x78;

x14=x118+x218;
x25=x328+x428+x528;
x36=x638;
x47=x218+x428;
x48=x118+x328;
x54=x328+x428;
x56=x528;
x67=x528+x638;
x78=x218+x428+x528+x638;

t1=t2;
t3=t4;
t4=t5;
```

Appendix C

Code of the Stackelberg approach (scenario #1) for LINGO 18.0

```
m14=5+2*0.05*x14+3*0.025*x14^2;
m25=10+2*0.04*x25+3*0.065*x25^2;
m36=6+2*0.03*x36+3*0.025*x36^2;
m47=4+2*0.04*x47+3*0.04*x47^2;
m48=5+2*0.08*x48+3*0.125*x48^2;
m54=4+2*0.035*x54+3*0.02*x54^2;
m56=5+2*0.035*x56+3*0.04*x56^2;
m67=1+2*0.035*x67+3*0.03*x67^2;
m78=2+2*0.025*x78+3*0.035*x78^2;

m1=m14+m48;
m2=m14+m47+m78;
m3=m25+m54+m48;
m4=m25+m54+m47+m78;
m5=m25+m56+m67+m78;
m6=m36+m67+m78;

x14>=0;
x25>=0;
x36>=0;
x47>=0;
x48>=0;
x54>=0;
x56>=0;
x67>=0;
x78>=0;

x1=4;
x2=5;
x3=7;

x18=4;
x28=5;
x38=7;

x18=x118+x218;
x28=x328+x428+x528;
x38=x638;

x1=x14;
x2=x25;
x3=x36;
x14+x54=x47+x48;
x25=x54+x56;
x36+x56=x67;
x47+x67=x78;
x48+x78=x14+x25+x36;
x8=x48+x78;

x14=x118+x218;
x14=x118+x218;
x25=x328+x428+x528;
x36=x638;
x47=x218+x428;
x48=x118+x328;
x54=x328+x428;
```

```
x56=x528;
x67=x528+x638;
x78=x218+x428+x528+x638;
```

```
m1=m2;
m3=m4;
m4=m5;
```

Appendix D

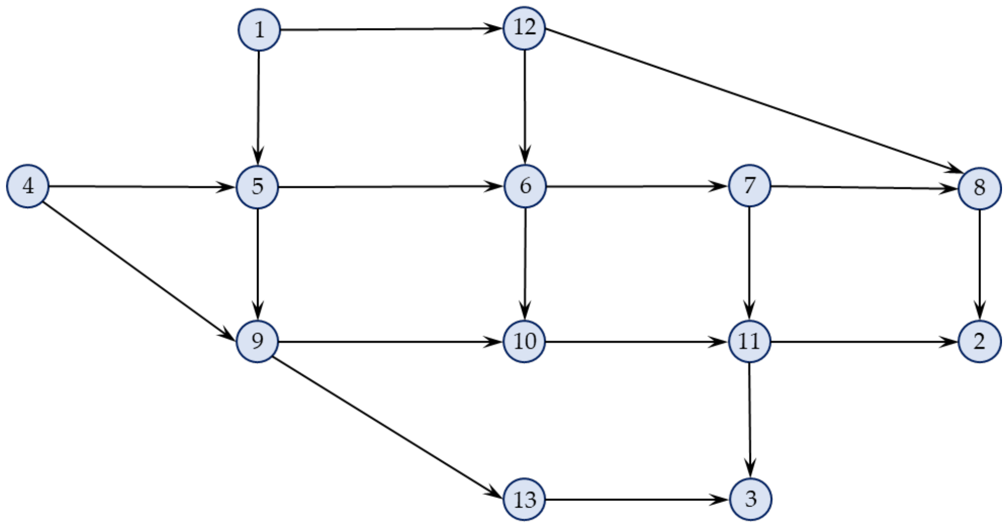

Figure A1. Nguyen–Dupuis network [67].

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