

Article

A Sector-Matching Probability Hypothesis Density Filter for Radar Multiple Target Tracking

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Abstract: The development of high-tech, dim, small targets, such as drones and cruise missiles, brings great challenges to radar multi-target tracking (MTT), making it necessary to extend the beam dwell time to obtain a high signal-to-noise ratio (SNR). In order to solve the problem of radar sampling time variation exacerbated by extending the beam dwell time when detecting weak targets, a sector-matching (SM) PHD filter is proposed, which combines the actual radar system with a PHD filter and quantifies the relationship between the beam dwell time, the false alarm rate and the detection probability. The proposed filter divides the scanning area into small sectors to obtain actual multi-target measurement times and rederives the prediction and update steps based on the actual sampling time. Furthermore, a state correction step is added before state extraction. Applying the SM structure to the basic Gaussian mixture PHD (GM-PHD) filter and labeled GM-PHD filter, the simulation results demonstrate that the proposed structure can improve the accuracy of multi-weak-target state estimation in the dense clutter and can continuously generate explicit trajectories. The overall real-time performance of the proposed filter is similar to that of the PHD filter.

Keywords: multi-target tracking; radar; random finite sets; sampling time variety; weak targets



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1. Introduction

Radar has been widely used in the field of military and civilian applications since World War II [1–4], and has been applied to complicated scenarios [5,6]. The development of stealth technology, small aircraft, drones, cruise missiles, and other modern high-tech devices brings weak echo energy, resulting in a low signal-to-noise ratio, which places higher demands on radar detection and tracking performance. Multi-target tracking (MTT) is one of the important functions of radar systems and the estimation of a weak target state and trajectory has become a key, hot issue in the field of MTT. In traditional MTT methods, joint probabilistic data association (JPDA) [7,8], multiple hypothesis tracking (MHT) [9], and probabilistic multiple hypothesis tracking [10–12] have mostly been used. In recent years, random finite set (RFS) [1,13] methods have been widely used in sonar [14,15], autonomous vehicles, robotics [16], and computer vision [17,18] due to them not using data association procedures. Probabilistic hypothesis density (PHD) filters [19] have become a hot research topic due to their low computational complexity by propagating the first moment of multi-target posteriors. While the PHD filter provides an estimation of target states, it provides no target trajectory information.

In order to obtain the trajectory of each target, the concept of labeled RFS was introduced in the generalized labeled multiple Bernoulli (GLMB) filter [20,21], which improves the accuracy of ground multi-target state extraction significantly and is suitable for the estimation of explicit target trajectories. Based on the GLMB filter, a joint GLMB filter [22] was

proposed which combines prediction and updating into one step and reduces the computational complexity. The GLMB filter [20–22] has difficulty generating the correct multi-target trajectories in good real-time performance. The trajectory Poisson multi-Bernoulli filter and the trajectory Poisson multi-Bernoulli hybrid filter proposed in [23] have better filtering accuracy and real-time performance than the joint GLMB filter. Ref. [24] proposed message-passing-based multitarget tracking methods which can cope with clutter, missed detections, and unknown associations between targets and measurements. Based on the general coordinate ascent variational filtering framework, an online VB-AbNHPP tracker was proposed in [25], which is superior to other competing methods in terms of implementation efficiency and tracking accuracy. Ref. [26] provided an algorithm for approximating the inference in this model using a Markov chain Monte Carlo (MCMC)-based auxiliary variable particle filter, which significantly reduced the computational cost per iteration of the Markov chain. Ref. [27] introduced two new methods (the peak-fusion method and the group-target track-before-detect algorithm) for OTHR tracking, which significantly improved the ability to follow maneuvers. Ref. [28] brought different schools of the tracking community together by demonstrating that PGFLs are very precise and succinct models of the combinatorial probability structures are involved in multitarget tracking. All results of these filters [23–28] have shortcomings in terms of real-time performance.

How to construct a PHD filter suitable for trajectory generation has become the focus of MTT in recent years, as PHD filters have lower computational complexities. Based on the output estimates of a PHD filter, a data association step was performed in [29,30] to generate multi-target trajectories. In [31,32], labels were added to the state estimation of the PHD filter. In [33], a trajectory probability hypothesis density (TPHD) filter was proposed which used the trajectory set as the state for MTT. In [34], a Gaussian mixture trajectory PHD filter was proposed, which allowed trajectory estimation of surviving targets without adding labels; however, it has difficulty in shielding against the interference from clutter nearby the targets. In [35], a labeled Gaussian mixture PHD (LGM-PHD) filter was proposed to shield against the interference and achieve trajectory maintenance by attaching labels to Gaussian components and introducing an update matrix to obtain target tracking information.

Sampling time variety due to radar beam scanning has not been taken into account in PHD filters [36]. In contrast to an optical imaging system, where all measurements within the monitoring space are simultaneously generated at the moment when photos are taken, radar obtains measurements when the beam illuminates the target. It takes a long time for the beam to scan the entire airspace and this causes targets at different locations to be detected at different times. The sampling time difference cannot be ignored, for it leads to a mismatch between multi-target state prediction and measurement, resulting in additional measurement errors, especially when the beam scans large surveillance areas or searches for weak targets and illuminated targets with a long dwell time to obtain a better signal-to-noise ratio (SNR) [13,37]. Liu previously proposed a time-labeled RFS which has been applied to a number of problems, such as the Bayesian framework [13] and the ET-PHD filter [37]. However, specific measurement time information is generally not included in radar echoes and Liu's work considered the detection probability as a constant and did not consider the variation in the detection probability with beam dwell time in actual radar operation.

In this paper, a novel sector-matching PHD filter is proposed. In order to make the sector-matching framework more suitable for radar real-time MTT in a complex scenario, we combine a real radar system with the LGM-PHD filter [35] and quantify the relationship between beam dwell time, signal-to-noise ratio (SNR), false alarm rate and detection probability. To simplify the problem, we consider a radar one-way electro-scan scenario, in which the target position (in particular, the bearing) and target measurement time correspond one-to-one. The main contributions of this article are as follows:

- The proposed filter divides the detection space into several small sectors and assumes that the different sectors are independent of each other;

- Based on these sectors and measurement states, the actual measurement time can be obtained and used to split and derive the prediction and update equations of the PHD filter;
- The quantitative relationship between beam dwell time, false alarm rate, detection probability and SNR is combined with PHD filters;
- Since we interested in the target state at the end of each scan, a state correction step is added before extracting the target state;
- We evaluate the effectiveness of the proposed method using the constant velocity (CV) and constant acceleration (CA) dynamic models, and demonstrate that the proposed filter can solve the problem of high clutter rate and radar sampling time variety when detecting weak targets well, and enables the track maintenance function.

The rest of the article is organized as follows. Section 2 presents the technical background. Section 3 presents the specific design of the improved SM PHD multi-filter using its Gaussian mixture implementations. Quantitative simulation experiments are described in Section 4. Section 5 presents our concluding remarks.

2. Background

2.1. GM-PHD Filter

In RFS-based filters, the multi-target states and multi-target measurements at the k th time step are defined as finite sets:

$$\mathbf{X}_k = \{\mathbf{x}_k^1, \mathbf{x}_k^2, \dots, \mathbf{x}_k^{N_{x,k}}\} \in F(\mathbb{X}), \quad (1)$$

$$\mathbf{Z}_k = \{\mathbf{z}_k^1, \mathbf{z}_k^2, \dots, \mathbf{z}_k^{N_{z,k}}\} \in F(\mathbb{Z}), \quad (2)$$

where $\mathbf{x}_k^1, \mathbf{x}_k^2, \dots, \mathbf{x}_k^{N_{x,k}}$ represent the states of $N_{x,k}$ targets, $F(\mathbb{X})$ is the set of multi-target state sets $\mathbb{X}$, $\mathbf{z}_k^1, \mathbf{z}_k^2, \dots, \mathbf{z}_k^{N_{z,k}}$ represent the measurements of $N_{z,k}$ targets and $F(\mathbb{Z})$ is the set of multi-target measurement sets $\mathbb{Z}$. The single target motion model and measurement model are defined as:

$$\begin{cases} \mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k-1} \\ \mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k \end{cases}, \quad (3)$$

where $\mathbf{x}_k \in \mathbb{R}^n$ and $\mathbf{z}_k \in \mathbb{R}^m$ are state vector and measurement vector, respectively, $\mathbf{F}_{k-1}$ represents the target state transition function, $\mathbf{H}_k$ is the measurement equation of targets, $\mathbf{w}_{k-1}$ and $\mathbf{v}_k$ are the system process noise and measurement noise, distributed as a Gaussian with variance $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$, respectively.

The posterior hypothesis posterior PHD at the $k-1$ moment can be expressed in the form of a Gaussian sum as:

$$D_{k-1|k-1}(\mathbf{x}) = \sum_{i=1}^{J_{k-1}} \omega_{k-1}^i N(\mathbf{x}; \mathbf{m}_{k-1}^i, \mathbf{P}_{k-1}^i), \quad (4)$$

where J_{k-1} is the total number of components and the mean and variance of the i th Gaussian component are $\mathbf{m}_{k-1}^i$ and $\mathbf{P}_{k-1}^i$ respectively.

The prediction of PHD can be expressed in a Gaussian mixed form as:

$$\begin{aligned} D_{k|k-1}(\mathbf{x}) &= D_{S,k|k-1}(\mathbf{x}) + D_{\beta,k|k-1}(\mathbf{x}) + \gamma_k(\mathbf{x}) \\ &= \sum_{i=1}^{J_{k|k-1}} \omega_{k|k-1}^i N(\mathbf{x}; \mathbf{m}_{k|k-1}^i, \mathbf{P}_{k|k-1}^i), \end{aligned} \quad (5)$$

where $D_{S,k|k-1}(\mathbf{x})$ is the probability hypothesis density of survival targets, $D_{\beta,k|k-1}(\mathbf{x})$ is the spawned target probability hypothesis density and $\gamma_k(\mathbf{x})$ is the newborn target probability hypothesis density [19].

The predicted intensity of the survival target, $D_{S,k|k-1}(\mathbf{x})$, can be derived from the target survival rate as well as the prediction equation:

$$D_{S,k|k-1}(\mathbf{x}) = p_{S,k} \sum_{i=1}^{J_{k-1}} \omega_{k-1}^i N(\mathbf{x}; \mathbf{m}_{S,k|k-1}^i, \mathbf{P}_{S,k|k-1}^i), \quad (6)$$

$$\mathbf{m}_{S,k|k-1}^i = \mathbf{F}_{k-1} \mathbf{m}_{k-1}^i, \quad (7)$$

$$\mathbf{P}_{S,k|k-1}^i = \mathbf{F}_{k-1} \mathbf{P}_{k-1}^i \mathbf{F}_{k-1}' + \mathbf{Q}_{k-1}, \quad (8)$$

where $p_{S,k}$ denotes the target survival probability and $\omega_{k|k-1}^i = p_{S,k} \omega_{k-1}^i$. The spawned targets at time k are generated from surviving targets, so that $D_{\beta,k|k-1}(\mathbf{x})$ can be expressed as:

$$D_{\beta,k|k-1}(\mathbf{x}) = \sum_{i=1}^{J_{k-1}} \sum_{j=1}^{J_{\beta,k}} \omega_{k-1}^i \omega_{\beta,k}^j N(\mathbf{x}; \mathbf{m}_{\beta,k|k-1}^{i,j}, \mathbf{P}_{\beta,k|k-1}^{i,j}), \quad (9)$$

$$\mathbf{m}_{\beta,k|k-1}^{i,j} = \mathbf{F}_{\beta,k-1}^j \mathbf{m}_{k-1}^i + \mathbf{d}_{\beta,k-1}^j, \quad (10)$$

$$\mathbf{P}_{\beta,k|k-1}^{i,j} = \mathbf{F}_{\beta,k-1}^j \mathbf{P}_{k-1}^i \mathbf{F}_{\beta,k-1}^{j'} + \mathbf{Q}_{\beta,k-1}^j, \quad (11)$$

where $\omega_{\beta,k}^j, \mathbf{F}_{\beta,k-1}^j, \mathbf{d}_{\beta,k-1}^j, \mathbf{Q}_{\beta,k-1}^j, j = 1, \dots, J_{\beta,k}$ are all spawned target parameters.

The PHD of the newborn targets can also be expressed in a Gaussian mixed form as:

$$\gamma_k(\mathbf{x}) = \sum_{i=1}^{J_{\gamma,k}} \omega_{\gamma,k}^i N(\mathbf{x}; \mathbf{m}_{\gamma,k}^i, \mathbf{P}_{\gamma,k}^i), \quad (12)$$

where $\omega_{\gamma,k}^i, \mathbf{m}_{\gamma,k}^i, \mathbf{P}_{\gamma,k}^i, i = 1, \dots, J_{\gamma,k}$ are newborn targets parameters.

If the measurement equation can also satisfy the linear Gaussian condition, the updated PHD at time k can also be expressed in the form of a Gaussian mixture:

$$D_{k|k}(\mathbf{x}) = (1 - P_{D,k}) D_{k|k-1}(\mathbf{x}) + \sum_{i=1}^{J_{k|k-1}} \omega_{k|k}^i(z) N(\mathbf{x}; \mathbf{m}_{k|k}^i(z), \mathbf{P}_{k|k}^i), \quad (13)$$

$$\omega_{k|k}^i(z) = \frac{P_{D,k} \omega_{k|k-1}^i N(z; \mathbf{H}_k \mathbf{m}_{k|k-1}^i, \mathbf{S}_k^i)}{\lambda_k(z) + P_{D,k} \sum_{j=1}^{J_{k|k-1}} \omega_{k|k-1}^j N(z; \mathbf{H}_k \mathbf{m}_{k|k-1}^j, \mathbf{S}_k^j)}, \quad (14)$$

$$\mathbf{m}_{k|k}^i(z) = \mathbf{m}_{k|k-1}^i(z) + \mathbf{K}_k^i (z - \mathbf{H}_k \mathbf{m}_{k|k-1}^i), \quad (15)$$

$$\mathbf{P}_{k|k}^i = [\mathbf{I} - \mathbf{K}_k^i \mathbf{H}_k] \mathbf{P}_{k|k-1}^i, \quad (16)$$

$$\mathbf{S}_k^i = \mathbf{R}_k + \mathbf{H}_k \mathbf{P}_{k|k-1}^i \mathbf{H}_k', \quad (17)$$

$$\mathbf{K}_k^i = \mathbf{P}_{k|k-1}^i \mathbf{H}_k' (\mathbf{H}_k \mathbf{P}_{k|k-1}^i \mathbf{H}_k' + \mathbf{R}_k)^{-1}, \quad (18)$$

where λ_k is the clutter or false measurements received by the radar and $P_{D,k}$ is the detection probability.

In order to reduce the computational effort, the increasing Gaussian components must be pruned and merged [19]. Then the posterior intensity, $D_{k|k}$, is expressed as:

$$D_{k|k}(\mathbf{x}) = \sum_{i=1}^{J_k} \omega_k^i(z) N(\mathbf{x}; \mathbf{m}_k^i, \mathbf{P}_k^i), \quad (19)$$

where J_k is the number of components of $D_{k|k}$.

2.2. Radar Detection

We generally use the method of increasing the beam dwell time (generally considered to be the accumulation time) to improve the signal-to-noise ratio and target detection probability, as the pulse-echo signal power, A_i^2 , received by the radar from an individual target is very low for dim, small targets.

Consider a pulsed radar with pulse width τ , pulse repetition frequency (PRF) f_r and peak transmit power P_t . N pulses are accumulated in one frame, then the radar equation can be written as:

$$SNR = \frac{P_t G^2 \lambda^2 \sigma T_i f_r \tau}{(4\pi)^3 R^4 k T_0 F L}, \quad (20)$$

where G is the radar antenna gain, λ is the radar operating wavelength, σ is the radar reflection cross section (RCS) of the target, R is the distance between the target and radar, $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant, $T_0 = 290$ K, $T_i = N \cdot \tau$ is the radar beam dwell time, F is the receiver noise factor and L is the loss of radar. As the accumulated echo signal power

$$A^2 = (N \cdot A_i)^2, \quad (21)$$

it can be noted that the SNR is directly proportional to beam dwell time, T_i , and the amplitude, A , of the accumulated echo signal is also in direct proportion to T_i .

The radar false alarm probability is defined as the probability that the sampling r of radar echo signal $r(t)$ exceeds the threshold value U_T when only noise is present in $r(t)$. The probability density function of the output voltage of Gaussian white noise after passing through the envelope detector obeys the Rayleigh distribution as follows:

$$p_{fa}(r) = r \exp\left(-\frac{r^2}{2}\right), \quad r \geq 0, \quad (22)$$

by integrating the above equation, the false alarm probability, P_{fa} , can be written as:

$$P_{fa} = \int_{U_T}^{\infty} p_{fa}(r) dr = \exp\left(-\frac{U_T^2}{2}\right), \quad (23)$$

then, with a given P_{fa} , the threshold voltage can be found as:

$$U_T = \sqrt{2 \ln \frac{1}{P_{fa}}} \quad (24)$$

The detection probability, P_D , is the probability that a sampling value r of $r(t)$ exceeds the threshold voltage when the radar echo signal, $r(t)$, contains both noise and signal. After the Gaussian noise and radar echo signal with amplitude A pass through the envelope detector, the probability density function of the envelope output by the detector is:

$$p_d(r) = r \exp\left(-\frac{r^2 + A^2}{2}\right) I_0(rA), \quad r \geq 0, \quad (25)$$

where $I_0(\cdot)$ is the first class zero-order modified Bessel function.

By integrating Equation (25), the detection probability, P_D , can be obtained as follows:

$$P_D = \int_{U_T}^{\infty} p_d(r) dr = \int_{U_T}^{\infty} r \exp\left(-\frac{r^2 + A^2}{2}\right) I_0(rA) dr, \quad (26)$$

The relationship between detection probability and false alarm probability is shown in Figure 1. The area in red represents the false alarm probability, P_{fa} , the area in purple represents the detection probability, P_D , and the horizontal coordinate of the intersection of

the noise envelope probability curve and the envelope curve of the noise plus signal is the voltage threshold, U_T .

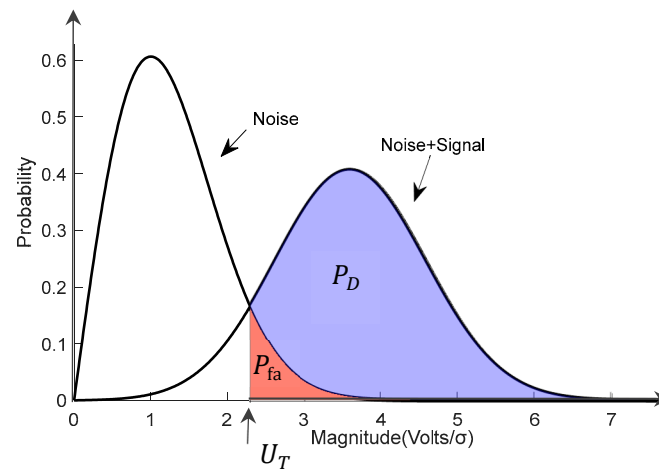


Figure 1. This is the probability density of a filter output for a representative signal-to-noise ratio. The area under the curve to the right of U_T is the probability of detection, P_D . Note that while increasing U_T decreases P_{fa} , it also decreases P_D .

For a single threshold detector, the average time between false alarms for the system is calculated as follows:

$$t_{fa} = \frac{T_i}{P_{fa} N_{TD}}, \quad (27)$$

where N_{TD} is the number of threshold detectors. The false alarm rate is the frequency of false alarm occurrence per unit time, which is the reciprocal of the false alarm time, that is:

$$\lambda = \frac{1}{t_{fa}} = \frac{P_{fa} N_{TD}}{T_i}, \quad (28)$$

2.3. Problem Formulation

The basic GM-PHD filter described in Section 2.1 does not adapt to the radar dynamic scanning model and trajectory estimation, especially for detecting weak targets. The method we proposed earlier in Sections 2.2 and 2.3 solves the problem of explicit tracking maintenance and is able to solve the problem of misdetection, false state estimation and adverse interference caused by close targets.

In radar systems, the antenna beam width is limited, and it takes a long time to scan the entire airspace, which causes the targets at different locations to be detected at different times during a scan, this leads to a variety of sampling times. However, in a basic PHD filter, targets are considered to be sampled at the same time, and the measurements generated by different targets are used to update the prediction prior to the same moment, which leads to a mismatch between the measurement time and the prediction time.

As shown in Figure 2, the k th scan of a radar is clockwise, as indicated by θ_1 , and the $k + 1$ th scan is in the counterclockwise direction, indicated by θ_2 .

It can be seen that the sampling times corresponding to the measurements of z'_k and z'_{k+1} during the k th and $k + 1$ th scans are t'_k and t'_{k+1} instead of t_k and t_{k+1} .

If time t_k and t_{k+1} are used in the prediction and update step, the clutter near positions x_k and x_{k+1} will obtain larger weights and is more likely to be considered as a target measurement, while the actual target measurements x'_k and x'_{k+1} will be considered as false alarms, resulting in additional estimation errors and affecting the accuracy of target state estimation and trajectory generation.

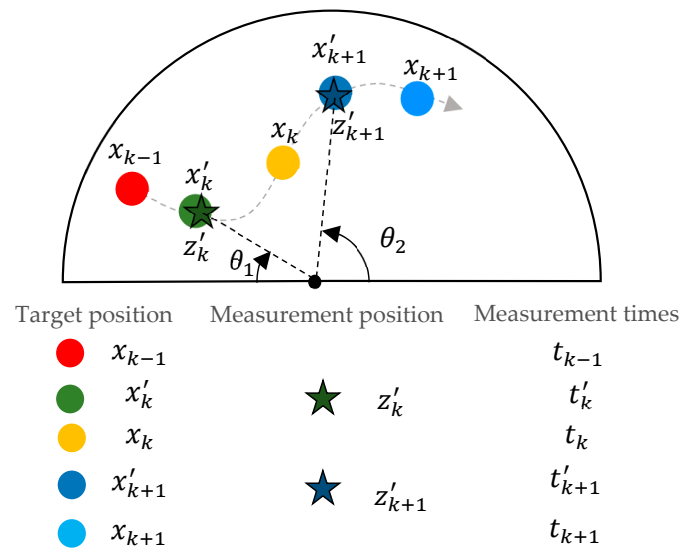


Figure 2. Diagram of the relationship between the actual position and the measured position of the target during radar two-dimensional scanning.

In order to solve the above problem and implement a PHD filter-based radar multi-target tracker, a sector-matching PHD filter framework is designed in this paper, which involves the measurement time as part of the multi-target state information to participate in prediction, update and state extraction operations.

3. Sector-Matching PHD Filter

A sector-matching PHD filter is proposed in this section. We first propose the sector-matching and sector division issues, and rederive the prediction and update steps based on the actual measurement time. Then, a state correction step is given.

3.1. Sector-Matching and Sector Division Issues

In the process of radar dynamic scanning, the direction that the beam is scanned in and the sampling time correspond one-to-one [13].

For radars in tracking-while-scanning (TWS) mode, the antenna scans at a constant angular velocity, so we can divide the surveillance area into a number of sectors equally. As shown in Figure 3, where $z_k^1, z_k^2, \dots, z_k^7$ are the seven measurements obtained in the k th scan, the surveillance area is divided into n sectors ($n = 18$ in Figure 2) and the following assumptions are given.

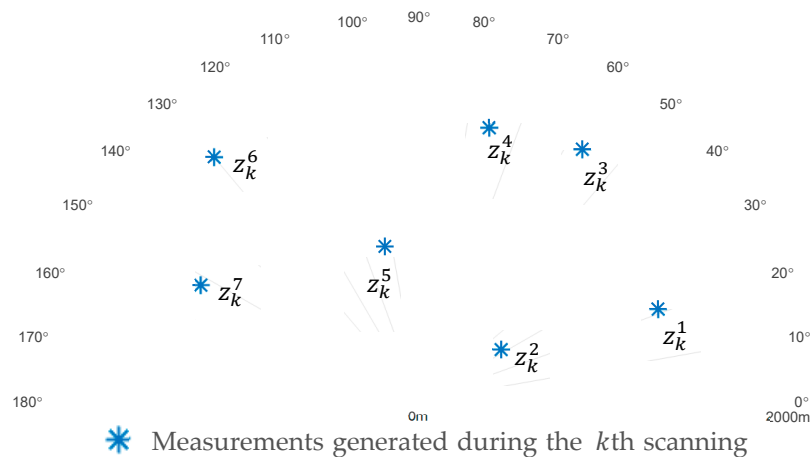


Figure 3. Radar measurements generated in the k th scanning cycle and sector division according to the sampling time.

Assumption 1. The true state of each target and the generated measurements are independent of each other [13].

Assumption 2. Each sector is scanned once at most in each scan.

Assumption 3. Any target can only appear in one sector during a scanning cycle.

Assumption 1 is very common in MTT applications. Assumptions 2 and 3 hold in the case of the TWS operating mode and where the targets do not appear to migrate through resolution cells (MTRC). In tracking-and-searching (TAS) mode, Assumptions 2 and 3 do not always hold, which will be developed in later studies.

To simplify the issue, we consider the case of a radar counterclockwise electronic scan at a uniform angular velocity. The radar scanning period is T and the left and right boundaries of the detected airspace are θ_{max} and θ_{min} . The detected airspace is divided equally into n sectors S_m , $m = 1, \dots, n$, then the time interval between neighboring sectors is $d_t = n/T$, the width of the sector is $\Delta\theta = (\theta_{max} - \theta_{min})/n$ and the boundary of sector S_m is denoted as $[\theta_{min} + (m-1)\Delta\theta, \theta_{min} + m\Delta\theta]$, $m = 1, \dots, n$. The angle θ_m of sector S_m is approximated by the average of the left and right boundaries, i.e., $\theta_m = \theta_{min} + (m-1/2)\Delta\theta$.

Radar measurements, $z_k^i = (r_k^i, b_k^i)$, $i = 1, \dots, |Z_k|$, in the k th scanning cycle, contain the range r_k^i and bearing b_k^i . If $|b_k^i - \theta_m|$ takes the minimum value when $m = m_i'$, $i = 1, \dots, |Z_k|$, measurement z_k^i falls into the m_i' th sector, and the measurement time is:

$$t_k^i = (k-1)T + m_i' \times d_t - \frac{d_t}{2}, \quad (29)$$

Thus, the multi-target measurement model in Equation (2) is rewritten as:

$$Z_k = \left\{ \left(z_k^1, t_k^1 \right), \dots, \left(z_k^{J_{t,k}}, t_k^{J_{t,k}} \right) \right\}, \quad (30)$$

where $t_k^1, \dots, t_k^{J_{t,k}}$ is the set of times when different targets are detected in the k th scanning cycle, correspondingly, then we can rewrite Equation (1) as:

$$X_k = \left\{ \left(x_k^1, t_k^1 \right), \dots, \left(x_k^{J_{t,k}}, t_k^{J_{t,k}} \right), \left(x_k^{J_{t,k}+1}, t_k \right) \right\}, \quad (31)$$

where t_k is the time at the end of k th scanning cycle and is used to represent the measurement time of the undetected target.

3.2. Initialization and Prediction

Within the k th scanning cycle, the RFSs of the newborn target is expressed in Gaussian mixture form as shown in Equation (35):

$$\gamma_{\gamma,k}^{t_i}(x) = \sum_{i=1}^{J_{\gamma,k}} \omega_{\gamma,k}^i \mathcal{N}\left(x; \left(m_{\gamma,k}^i, t_{\gamma,k}^i \right), P_{\gamma,k}^i \right) \quad (32)$$

Each newborn Gaussian component's sampling time $t_{\gamma,k}^i$ is obtained according to Equation (32). According to Equation (35), the initial intensity, $D_{0|0}$, is described as $\left\{ \left(m_{\gamma,0}^i, t_{\gamma,0}^i \right), \omega_{\gamma,0}^i, P_{\gamma,0}^i \right\}_{i=1}^{J_{\gamma,0}}$.

The posterior probability density, $D_{k-1|k-1}(x)$, at the end of the $k-1$ th scan can be approximated as: $\left\{ \left(m_{k-1}^j, t_{k-1}^j \right), \omega_{k-1}^j, P_{k-1}^j \right\}_{j=1}^{J_{k-1}}$, where $t_{k-1} = (k-1)T$ is the timestamp of the surviving Gaussian component at time $k-1$. The predicted density at the end of k th scan is:

$$D_{k|k-1}(x) = \bigcup_{i=1}^{|Z_k|} D_{t_k^i|k-1}(x) \quad (33)$$

$$D_{t_k^i|k-1}(\mathbf{x}) = \left\{ (m_{t_k^i|k-1}^j, t_{k-1}^j), \omega_{t_k^i|k-1}^j, P_{t_k^i|k-1}^j \right\}_{j=1}^{J_{t_k^i|k-1}} \quad (34)$$

where $J_{t_k^i|k-1} = J_{k-1} + J_{\gamma, t_k^i}$, J_{k-1} is the number of components at the $k-1$ th scanning cycle and J_{γ, t_k^i} is the number of newborn components at time t_k^i .

Without considering spawned targets, Equations (5)–(8) are rewritten as:

$$D_{t_k^i|k-1}(\mathbf{x}) = D_{S, t_k^i|k-1}(\mathbf{x}) + \gamma_{t_k^i}(\mathbf{x}) = \sum_{j=1}^{J_{t_k^i|k-1}} \omega_{t_k^i|k-1}^j \mathcal{N}\left(\mathbf{x}; \left(m_{t_k^i|k-1}^j, t_{k-1}^j\right), P_{t_k^i|k-1}^j\right) \quad (35)$$

$$D_{S, t_k^i|k-1}(\mathbf{x}) \& = p_{S, t_k^i} \times \sum_{j=1}^{J_{k-1}} \omega_{t_k^i|k-1}^j \mathcal{N}\left(\mathbf{x}; \left(m_{S, t_k^i|k-1}^j, t_{k-1}^j\right), P_{S, t_k^i|k-1}^j\right) \quad (36)$$

$$m_{S, t_k^i|k-1}^j \& = F_{t_k^i|k-1} m_{k-1}^j \quad (37)$$

$$P_{S, t_k^i|k-1}^j \& = F_{t_k^i|k-1} P_{k-1}^j F_{t_k^i|k-1}' + Q_{t_k^i|k-1} \quad (38)$$

$$\omega_{t_k^i|k-1}^j = p_{S, t_k^i} \omega_{k-1}^j \quad (39)$$

where the parameters in $F_{t_k^i|k-1}$ indicate the time interval is $t_k^i - t_{k-1}^j$.

3.3. Observation Selection

Considering the real-time performance of the filter, it is most appropriate to remove the clutter in the low priori probability density region by using the wave gate before updating the Gaussian component. Knowing the zero-mean measurement noise covariance matrix, $R_k = \text{diag}\left(\left[\sigma_x^2 \sigma_y^2\right]^T\right)$, the threshold of the wave gate is $d(a) = a \|\left[\sigma_x \sigma_y\right]^T - [0 \ 0]^T\|$, where $\|\cdot\|$ is the Euclidean distance and a is the confidence coefficient. Then, $d^{(i,j)} = \|z_k^{(i)} - H_k m_{k|k-1}^{(j)}\|$, $i = 1, \dots, |Z_k|$, $j = 1, \dots, J_{k|k-1}$ is computed. The values of $z_k^{(i)}$ that satisfies $d^{(i,j)} \leq d(a)$ are selected and denoted as efficient observations $z_{k,ef}^i$. Then, the efficient observations set $Z_{k,ef} = \left\{z_{k,ef}^i, t_{k,ef}^i\right\}_{i=1}^{|Z_{k,ef}|}$ is obtained for the k th scanning cycle.

3.4. Update

Considering the sector-matching issues and combining the PHD filter with actual radar, the posterior density, $D_{k|k}$, at step k can be described as:

$$D_{k|k}(\mathbf{x}) \& = \bigcup_{i=1}^{|Z_{k,ef}|} D_{t_k^i|t_k^i}(\mathbf{x}), \quad (40)$$

$$D_{t_k^i|t_k^i} \mathbf{x} \& = (1 - P_{D,k}) D_{t_k^i|k-1} \mathbf{x} + \sum_{z_{k,ef}^{(i)} \in Z_{k,ef}} D_{D, t_k^i}(\mathbf{x}; \{z_{k,ef}^i, t_{k,ef}^i\}) \quad (41)$$

$$D_{D, t_k^i}(\mathbf{x}; \{z_{k,ef}^i, t_{k,ef}^i\}) = \sum_{j=1}^{J_{k|k-1}} \omega_{t_k^i|t_k^i}^j \left(\{z_{k,ef}^i, t_{k,ef}^i\} \right) \times \mathcal{N}\left(\mathbf{x}; m_{t_k^i|t_k^i}^j \left(\{z_{k,ef}^i, t_{k,ef}^i\} \right), P_{t_k^i|t_k^i}^j\right), \quad (42)$$

$$\omega_{t_k^i|t_k^i}^j(z_k^i) = \frac{P_{D, t_k^i} \omega_{t_k^i|k-1}^j N\left(z_k^i; H_{t_k^i} m_{t_k^i|k-1}^j, S_{t_k^i}^j\right)}{\lambda_{t_k^i}(z_k^i) + P_{D, t_k^i} \sum_{j=1}^{J_{k|k-1}} \omega_{t_k^i|k-1}^j N\left(z_k^i; H_{t_k^i} m_{t_k^i|k-1}^j, S_{t_k^i}^j\right)}, \quad (43)$$

$$m_{t_k^i|t_k^i}^j(z_k^i) = m_{t_k^i|k-1}^j(z_k^i) + K_{t_k^i}^j \left(z_k^i - H_{t_k^i} m_{t_k^i|k-1}^j \right), \quad (44)$$

$$\mathbf{P}_{t_k^i|t_k^i}^j = \left[\mathbf{I} - \mathbf{K}_{t_k^i}^j \mathbf{H}_{t_k^i}^j \right] \mathbf{P}_{t_k^i|t_{k-1}^i}^j, \quad (45)$$

$$\mathbf{S}_{t_k^i}^j = \mathbf{R}_{t_k^i} + \mathbf{H}_{t_k^i}^j \mathbf{P}_{t_k^i|t_{k-1}^i}^j \mathbf{H}_{t_k^i}^{j'} \quad (46)$$

$$\mathbf{K}_{t_k^i}^j = \mathbf{P}_{t_k^i|t_{k-1}^i}^j \mathbf{H}_{t_k^i}^{j'} \left(\mathbf{H}_{t_k^i}^j \mathbf{P}_{t_k^i|t_{k-1}^i}^j \mathbf{H}_{t_k^i}^{j'} + \mathbf{R}_{t_k^i} \right)^{-1}, \quad (47)$$

The parameter set $\left\{ \mathbf{m}_{k|k}^j(z_{k,ef}^i), \omega_{k|k}^j(z_{k,ef}^i), t_{k,ef}^i, \mathbf{P}_{k|k}^j \right\}$ of the updated Gaussian components is obtained, and $P_{D,k}$, P_{D,t_k^i} and $\lambda_{t_k^i}^j(\mathbf{z})$ are written, respectively, as:

$$P_{D,k} \& = \int_{U_T}^{\infty} p_{d,k}(r) dr = \int_{U_T}^{\infty} r \exp\left(-\frac{r^2 + A_k^2}{2}\right) I_0(r A_k) dr, \quad (48)$$

$$P_{D,t_k^i} \& = \int_{U_T}^{\infty} p_{d,t_k^i}(r) dr = \int_{U_T}^{\infty} r \exp\left(-\frac{r^2 + A_{t_k^i}^2}{2}\right) I_0(r A_{t_k^i}) dr, \quad (49)$$

$$\lambda_{t_k^i}^j = \frac{1}{t_{fa}} = \frac{P_{fa} N_{TD}}{t_{int}}, \quad (50)$$

The undetected targets corresponding to $(1 - p_{D,k})D_{k|k-1}(\mathbf{x})$ are approximated by the parameter set $\left\{ \mathbf{m}_{k|k-1}^i, (1 - P_{D,k})\omega_{k|k-1}^i, t_k, \mathbf{P}_{k|k-1}^i \right\}_{i=1}^{J_{k|k-1}}$.

3.5. Undetected Targets Filtering

In contrast to the detected targets that have a definite sampling time, since we do not know which sector the missed target corresponds to, it is most appropriate to uniformly assume that the measurement time of the missed target is at the end of the k th scan. The parameters of the Gaussian components are:

$$D_{U,k}(\mathbf{x}) = D_{k|k-1}(\mathbf{x}), \quad (51)$$

$$\mathbf{m}_{U,k|k}^j = \mathbf{m}_{U,k|k-1}^j, \quad (52)$$

$$\mathbf{P}_{U,k|k}^j = \mathbf{P}_{U,k|k-1}^j, \quad (53)$$

$$\omega_{k|k}^j = \omega_{k|k-1}^j, \quad (54)$$

where U represents the undetected targets.

3.6. Pruning and Merging

Then, the Gaussian components are pruned and merged. This procedure also requires pruning and merging the sector-matching information together. The new Gaussian component after merging is:

$$\tilde{\omega}_{t_k^i|t_k^i}^l = \sum_{j=1}^n \omega_{t_k^i|t_k^i}^j, \quad (55)$$

$$\tilde{\mathbf{m}}_{t_k^i|t_k^i}^l = \frac{1}{\tilde{\omega}_{t_k^i|t_k^i}^l} \sum_{j=1}^n \omega_{t_k^i|t_k^i}^j \mathbf{m}_{t_k^i|t_k^i}^j, \quad (56)$$

$$\tilde{\mathbf{P}}_{t_k^i|t_k^i}^l \& = \frac{1}{\tilde{\omega}_{t_k^i|t_k^i}^l} \sum_{j=1}^n \omega_{t_k^i|t_k^i}^j \times \left[\mathbf{P}_{t_k^i|t_k^i}^j + \left(\tilde{\mathbf{m}}_{t_k^i|t_k^i}^l - \mathbf{m}_{t_k^i|t_k^i}^j \right) \left(\tilde{\mathbf{m}}_{t_k^i|t_k^i}^l - \mathbf{m}_{t_k^i|t_k^i}^j \right)^T \right], \quad (57)$$

and the actual measurement time corresponding to the new Gaussian component after merging is:

$$\tilde{t}_{k|k}^l = \frac{1}{n} \sum_{j=1}^n t_{k|k}^{l,j}, \quad (58)$$

3.7. State Correction

In the state extraction step, since we aim to obtain the target state at the end of each scan, a correction to the posterior is required:

$$\hat{m}_{k|k}^l \& = F_{k|t_k} \tilde{m}_{k|k}^l, \quad (59)$$

$$\hat{\omega}_{k|k}^l \& = \omega_{k|k}^l, \quad (60)$$

note that $\hat{m}_{k|k}^l$ is only used for state extraction and not for prediction and updating operations. Then, $\hat{m}_{k|k}^l$ is used to carry out subsequent state extraction operations.

4. Simulations

In this section, we first simulate and analyze the variation in the false alarm rate and detection probability with accumulation time for several specific false alarm probabilities in radar operation. Then, the results acquired using the proposed method in multi-target tracking simulations are presented. We apply the proposed method to the PHD filter and LGM-PHD filter proposed by Gao in [35], and compare them with the basic GM-PHD filter [19] and the LGM-PHD filter [35]. Simulations are designed using the multi-target constant velocity (CV) motion model and a constant acceleration (CA) motion model in dense scenarios.

4.1. False Alarm Rate and Detection Probability

With other radar parameters kept constant, we simulated and analyzed the variation in the false alarm rate and detection probability with accumulation time for several specific false alarm probabilities. For radar MTT, a lower false alarm rate and higher detection probability is desired.

As shown in Figure 4, reducing the constant false alarm detection threshold (i.e., increasing the false alarm probability) can raise the detection probability, but it also leads to an increase in the false alarm rate. As the beam dwell time increases, the detection probability increases and the false alarm rate decreases. Therefore, to improve the performance of radar MTT, a suitable choice is to extend the accumulation time.

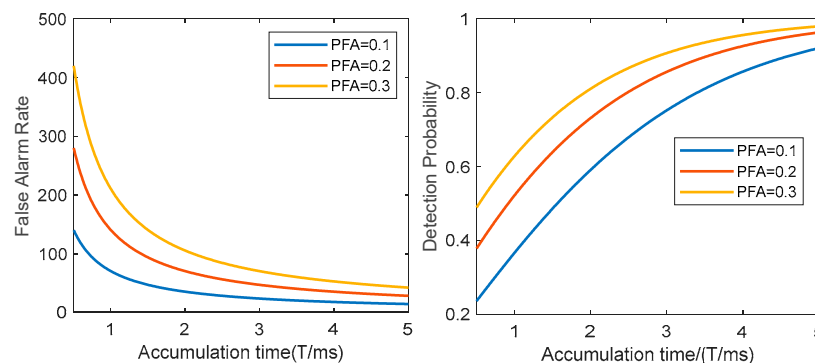


Figure 4. Variation in false alarm rate and detection probability with accumulation time with different detection probabilities for the given radar in the same scenario.

In the following simulation presented in this paper, the false alarm probability of the radar is set to $P_{fa} = 0.3$, and it can be seen from Figure 4 that $\lambda = 104$ and $p_{D,k} = 0.8$ at $T_i = 2$ ms, and $\lambda = 52$ and $p_{D,k} = 0.96$ at $T_i = 4$ ms.

4.2. Example 1: CV Motion Model

In this scenario, each target moves according to the following Gaussian linear model:

$$\mathbf{x}_k = \left(\begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} \otimes \mathbf{I}_d \right) \mathbf{x}_{k-1} + \sigma_w^2 \left(\begin{bmatrix} T_s^2/4 & T_s^3/2 \\ T_s^3/2 & T_s^2 \end{bmatrix} \otimes \mathbf{I}_d \right) \quad (61)$$

where $\mathbf{I}_d$ is the identity matrix of dimension d , where $d = 2$ in this scenario. $\mathbf{x}_k = [\mathbf{x}_{1,k}, \mathbf{x}_{2,k}, \mathbf{x}_{3,k}, \mathbf{x}_{4,k}]^T$, $[\mathbf{x}_{1,k}, \mathbf{x}_{3,k}]^T$ represents the target position at the end of the k th scan and $[\mathbf{x}_{2,k}, \mathbf{x}_{4,k}]^T$ is the target velocity. T_s is the sampling period and $\sigma_w = 5 \text{ m/s}^2$.

The actual target trajectories and the initial states of the targets are given in Figure 5 and Table 1, targets are created from the black dot •, and disappear at the end of their respective tracks. The multi-target measurement model is given by Equation (60):

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{r}((\mathbf{H}_k \otimes \mathbf{I}_d) \mathbf{x}_k) \\ \boldsymbol{\theta}((\mathbf{H}_k \otimes \mathbf{I}_d) \mathbf{x}_k) \end{bmatrix} + \begin{bmatrix} \mathbf{r}(\mathbf{e}_k) \\ \boldsymbol{\theta}(\mathbf{e}_k) \end{bmatrix} \quad (62)$$

where $\mathbf{r}((\mathbf{H}_k \otimes \mathbf{I}_d) \mathbf{x}_k)$ and $\boldsymbol{\theta}((\mathbf{H}_k \otimes \mathbf{I}_d) \mathbf{x}_k)$ denote the range and bearing of state $\mathbf{x}_k$, respectively. $\mathbf{H}_k = [1 \ 0]$ is the observation matrix and $\mathbf{e}_k$ is a Gaussian white noise with covariance $\text{diag}([10, 0.1])$.

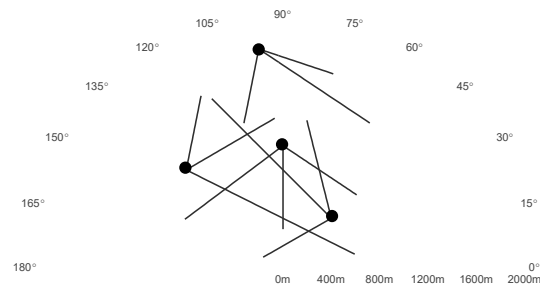


Figure 5. Target trajectories in example 1 (CV motion model), targets are created from the black dot •.

Table 1. List of initial target states in example 1 (CV model).

Target Index	Lifetime (s)	Initial States (m,m/s,m,m/s)
No. 1	(1,70)	[0; 0; 0; −10]
No. 2	(1,100)	[400; −10; −600; 10]
No. 3	(1,70)	[−800; 20; −200; −10]
No. 4	(20,100)	[400; −7; −600; −4]
No. 5	(20,100)	[400; −2.5; −600; 10]
No. 6	(20,100)	[0; 7.5; 0; −5]
No. 7	(40,100)	[−800; 12; −200; 7]
No. 8	(40,100)	[−200; 15; 800; −10]
No. 9	(60,100)	[−800; 3; −200; 15]
No. 10	(60,100)	[−200; −3; 800; −15]
No. 11	(80,100)	[0; −40; 0; −30]
No. 12	(80,100)	[−200; 30; 800; −10]

As shown in Figures 6 and 7, the sector-matching method proposed in this paper can achieve radar MTT when considering radar measurement time diversity. Applying the method to the LGM-PHD filter can achieve tracking maintenance even in the presence of strong clutter interference, dense targets in the surveillance area and long radar scan periods. In Figure 6b,d, it can be seen that a low detection probability of $p_{D,k} = 0.8$ causes the trajectory to be broken, and the reason why the trajectory can be identified as the same target's trajectory instead of a new-born target trajectory after the trajectory is broken is that in the simulation made in this paper, it is assumed that the newborn

targets can only appear at the known new-born points, i.e., it is assumed that no newborn target will appear at the trajectory breaking points. In Figure 7b,d, due to the extension of the accumulation time, the detection probability $p_{D,k} = 0.96$ and the probability of track breakage is significantly reduced.

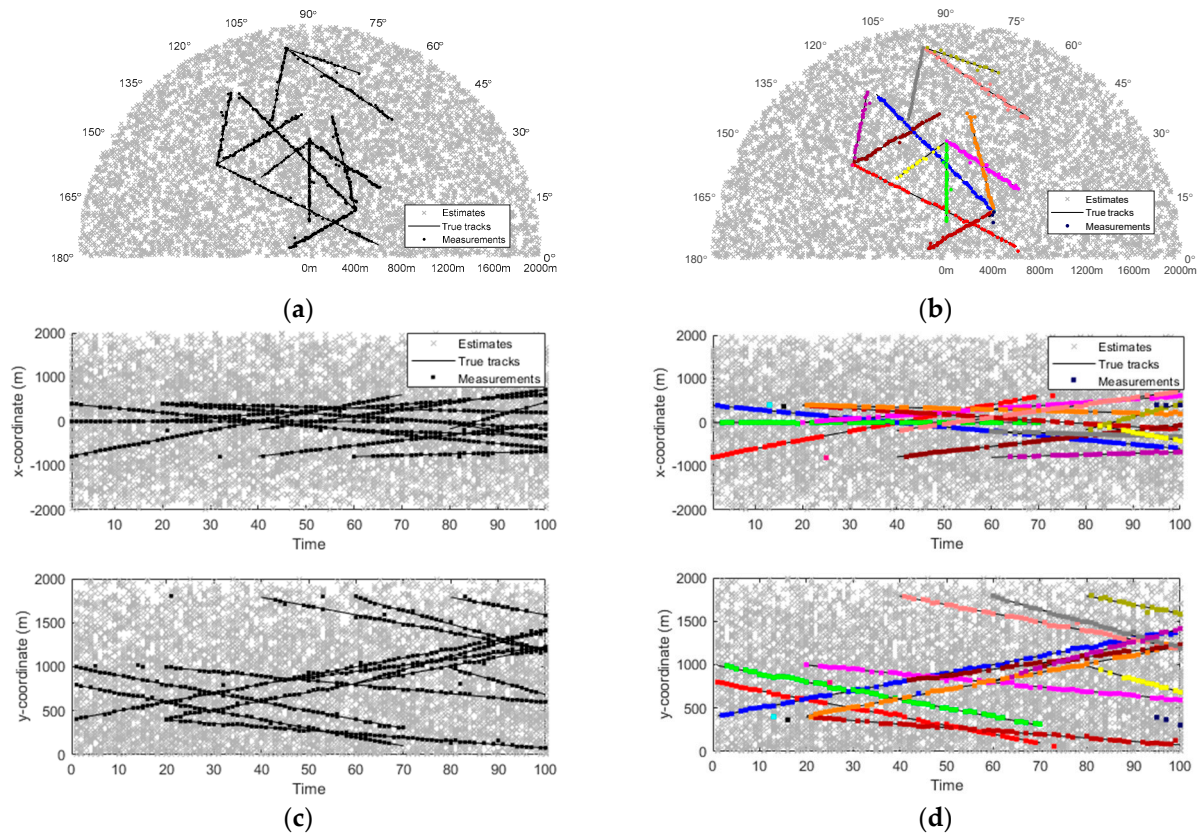


Figure 6. The estimations of the multi-target trajectories and the actual measurements in example 1 (CV motion model) for a sample run. (a) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-GM-PHD filter for $T_s = 1$ s ($T_i = 2$ ms, $\lambda = 104$, $p_{D,k} = 0.8$). (b) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-LGM-PHD filter for $T_s = 1$ s, and the different trajectories are denoted by different color dots. (c) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-GM-PHD filter in x and y coordinates for $T_s = 1$ s. (d) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-LGM-PHD filter in x and y coordinates for $T_s = 1$ s.

As shown in Figure 8, the computation time (CT) of the MATLAB simulation for 100 sample runs is given and the average CT is shown in Table 2. The basic GM-PHD filter and the modified SM-GM-PHD filter have relatively smooth computation times. However, for the LGM-PHD filter, as there is an extra step in which the matrix U is used to determine trajectories, the computation speed of this process depends on the distance between the Gaussian components and the weight of the components. If a large number of measurements are generated in a given scan cycle, this also makes the computation time higher. Therefore, the computation times of the LGM-PHD and the SM-LGM-PHD filters vary randomly. The results show that the real-time performance of the proposed method for both the LGM-PHD filter and the basic GM-PHD filter is similar to that of the basic PHD filter and the LGM-PHD filter, and much better than that of the joint GLMB filter. In addition, extending the accumulation time can significantly reduce the calculation time.

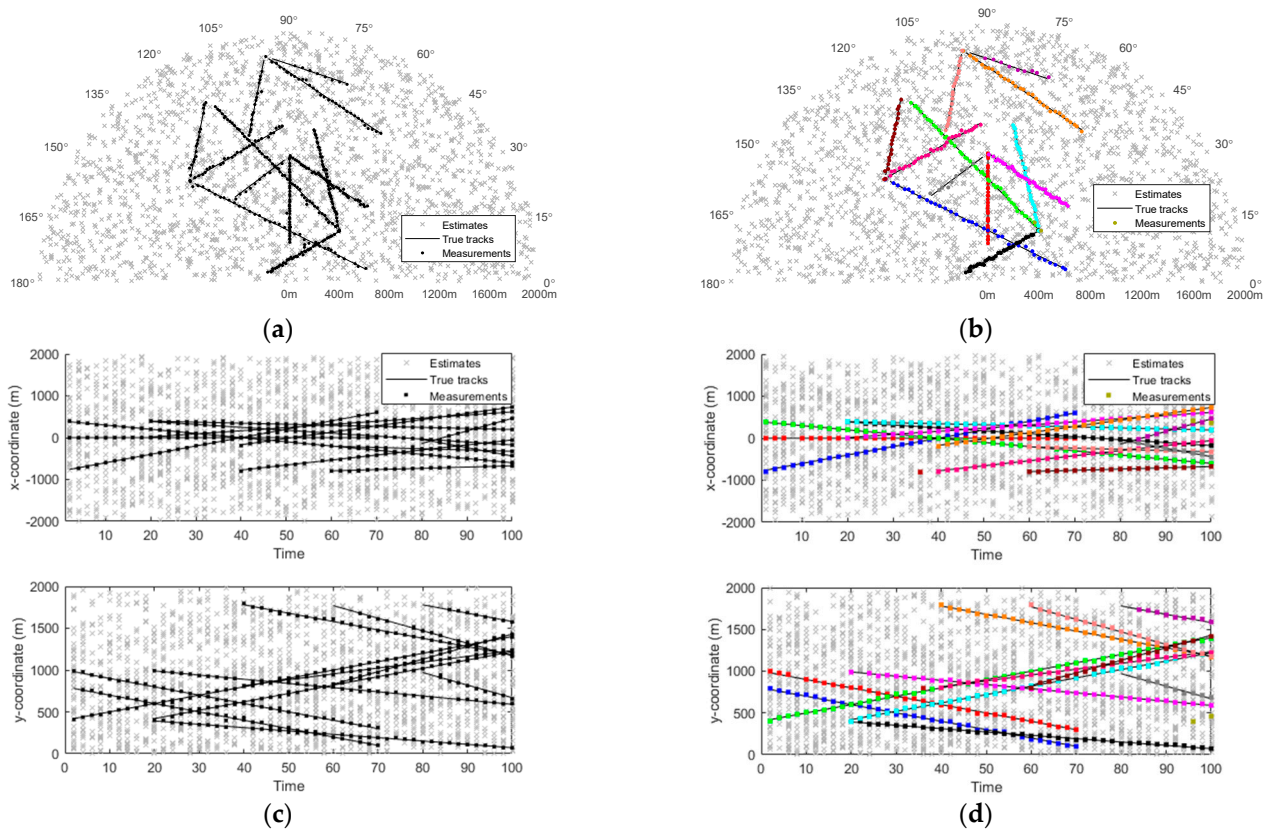


Figure 7. The estimations of the multi-target trajectories and the actual measurements in example 1 (CV motion model) for a sample run. (a) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-GM-PHD filter for $T_s = 2$ s ($T_i = 4$ ms, $\lambda = 52$, $p_{D,k} = 0.96$). (b) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-LGM-PHD filter for $T_s = 2$ s, and the different trajectories are denoted by different color dots. (c) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-GM-PHD filter in x and y coordinates for $T_s = 2$ s. (d) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-LGM-PHD filter in x and y coordinates for $T_s = 2$ s.

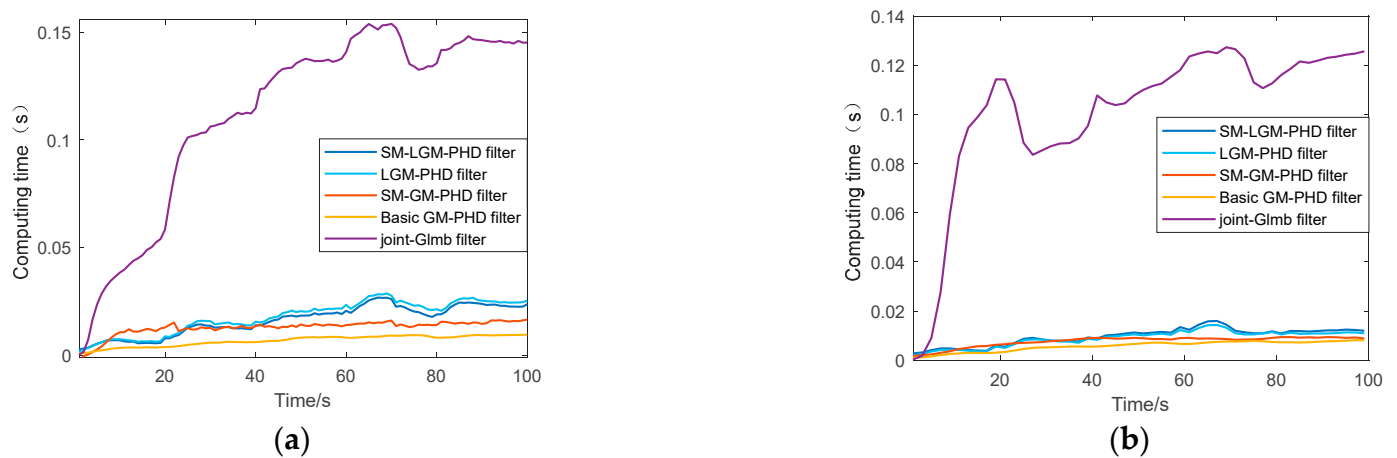


Figure 8. The computation time of different filters for 100 sample runs at $T_s = 1$ s ($T_i = 2$ ms, $\lambda = 104$, $p_{D,k} = 0.8$) (a) and $T_s = 2$ s ($T_i = 4$ ms, $\lambda = 52$, $p_{D,k} = 0.96$) (b) in example 1 (CV motion model).

Table 2. List of the average CT and average OSPA of each filter in example 1 (CV model).

	SM-LGM-PHD Filter	LGM-PHD Filter	SM-GM-PHD Filter	Basic GM-PHD Filter	Joint-GLMB Filter
CT ($T_s = 1$ s)	0.0164 s	0.0180 s	0.0130 s	0.0069 s	0.1125 s
CT ($T_s = 2$ s)	0.0097 s	0.0090 s	0.0078 s	0.0058 s	0.1009 s
OSPA ($T_s = 1$ s)	25.66 m	28.71 m	30.60 m	33.18 m	12.51 m
OSPA ($T_s = 2$ s)	16.62 m	21.61 m	18.20 m	28.17 m	14.80 m

In Figures 9 and 10, the OSPA distances of the MATLAB simulation for 100 sample runs is given and the average OSPA distances are given in Table 2. Compared with the basic GM-PHD filter, the average tracking performance of the SM-LGM-PHD filter, the LGM-PHD filter, the SM-GM-PHD filter and the joint-GLMB filter in this scenario is improved by 22.66%, 13.47%, 7.78% and 62.29%, respectively, at $T_s = 1$ s ($T_i = 2$ ms, $\lambda = 104$, $p_{D,k} = 0.8$), and 41%, 13.47%, 35.39% and 47.46%, respectively, at $T_s = 2$ s ($T_i = 4$ ms, $\lambda = 52$, $p_{D,k} = 0.96$). Compared with the $T_s = 1$ s scenario, the performance of the SM-LGM-PHD filter and the SM-GM-PHD filter has been significantly improved. The LGM-PHD filter, the basic GM-PHD filter and the joint-GLMB filter are not time-matched, which is why the performance of these three filters is not that significantly improved, and the performance of the joint-GLMB filter even decreases. In addition, it can be seen in Figures 9b and 10b that the values of the OSPA location of the LGM-PHD filter, the basic GM-PHD filter and the joint-GLMB filter all increase with the accumulation of time. After 80 s, with the appearance of two targets, No.11 and No.12, at a higher speed, the targets' position estimation errors of the LGM-PHD filter, the basic GM-PHD filter and the joint-GLMB filter increase at a faster rate, while the SM-LGM-PHD filter and the SM-GM-PHD filter are not significantly affected. This is mainly caused by the additional measurement error caused by the sampling time diversity mentioned earlier.

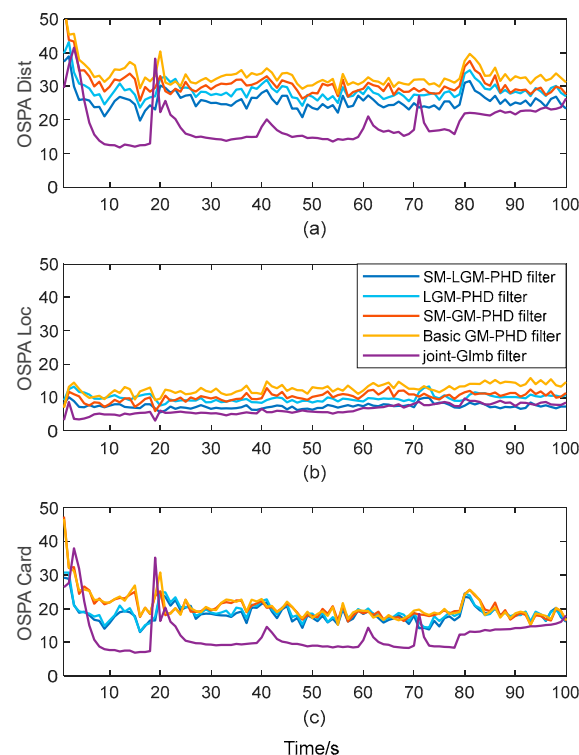


Figure 9. The average OSPA of different filters in example 1 (CV motion model) for 100 sample runs when $T_s = 1$ s ($T_i = 2$ ms, $\lambda = 104$, $p_{D,k} = 0.8$). (a) Average OSPA distances of different filters. (b) OSPA locations of different filters. (c) OSPA cardinalities of different filters.

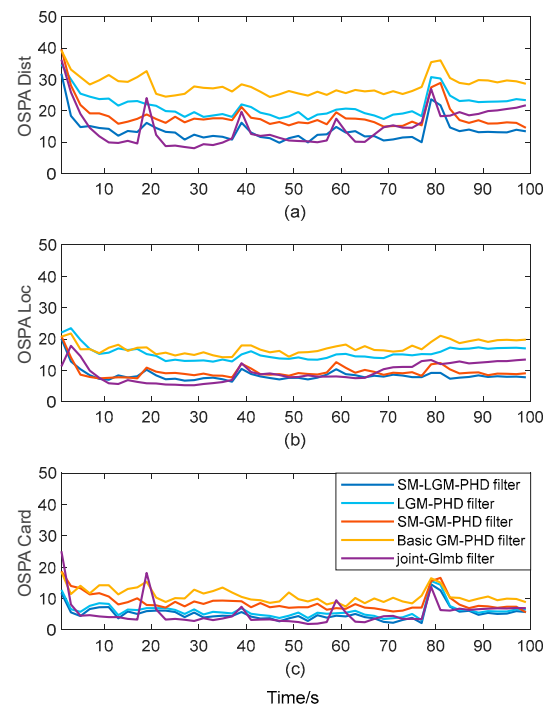


Figure 10. The average OSPA of different filters in example 1 (CV motion model) for 100 sample runs, when $T_s = 2$ s ($T_i = 4$ ms, $\lambda = 52$, $p_{D,k} = 0.96$). (a) Average OSPA distances of different filters. (b) OSPA locations of different filters. (c) OSPA cardinalities of different filters.

Comparing the results in Figures 9 and 10, it can be found that the accuracy of multi-target state estimation is consistent with what we expected. The estimation accuracy of the filter deteriorates as the scan period increases and also deteriorates with the increase in target velocity.

4.3. Example 2: CA Motion Model

In this scenario, each target moves according to the following Gaussian linear model:

$$\mathbf{x}_k = \left(\begin{bmatrix} 1 & T_s & T_s^2/2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix} \otimes \mathbf{I}_d \right) \mathbf{x}_{k-1} + \mathbf{w}_k \quad (63)$$

where $\mathbf{I}_d$ is the identity matrix of dimension d , where $d = 2$. $\mathbf{x}_k = [\mathbf{x}_{1,k}, \mathbf{x}_{2,k}, \mathbf{x}_{3,k}, \mathbf{x}_{4,k}, \mathbf{x}_{5,k}, \mathbf{x}_{6,k}]^T$, $[\mathbf{x}_{1,k}, \mathbf{x}_{4,k}]^T$ represents the target position at the end of the k th scan, $[\mathbf{x}_{2,k}, \mathbf{x}_{5,k}]^T$ is the target velocity and $[\mathbf{x}_{3,k}, \mathbf{x}_{6,k}]^T$ is the target acceleration. T_s is the sampling period. The actual target trajectories and the initial states of the targets are given in Figure 11 and Table 3, targets are born from the black dot •, and disappear at the end of their respective tracks. The multi-target measurement model is given by Equation (62):

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{r}((\mathbf{H}_k \otimes \mathbf{I}_d)\mathbf{x}_k) \\ \boldsymbol{\theta}((\mathbf{H}_k \otimes \mathbf{I}_d)\mathbf{x}_k) \end{bmatrix} + \begin{bmatrix} \mathbf{r}(\mathbf{e}_k) \\ \boldsymbol{\theta}(\mathbf{e}_k) \end{bmatrix} \quad (64)$$

where $\mathbf{r}((\mathbf{H}_k \otimes \mathbf{I}_d)\mathbf{x}_k)$ and $\boldsymbol{\theta}((\mathbf{H}_k \otimes \mathbf{I}_d)\mathbf{x}_k)$ denote the range and bearing of state $\mathbf{x}_k$, respectively. $\mathbf{H}_k = [1 \ 0 \ 0]$ is the observation matrix and $\mathbf{e}_k$ is the Gaussian white noise with covariance $\text{diag}([10, 0.1])$.

The radar divides the detection airspace into 500 parts. A detailed comparison of the performance of the individual filters at $T_s = 1$ s, $T_i = 2$ ms, $\lambda = 104$, $p_{D,k} = 0.8$ and $T_s = 2$ s, $T_i = 4$ ms, $\lambda = 52$, $p_{D,k} = 0.96$, and the effect of different sampling times on the filter performance are given in Figures 12–16.

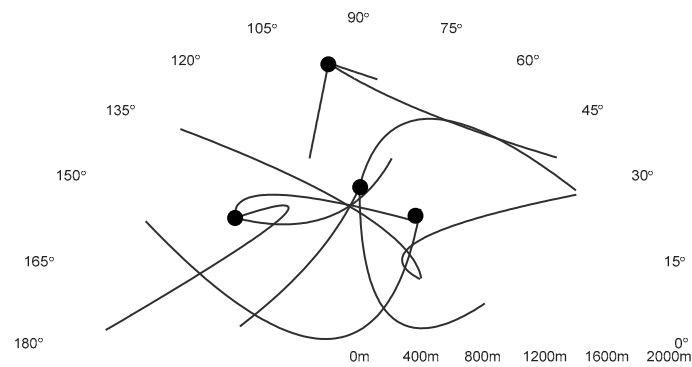


Figure 11. Target trajectories in example 2 (CA motion model), where targets are born from the black dot •.

Table 3. List of initial target states in the CA motion model.

Target Index	Lifetime (s)	Initial States (m,m/s,m/s ² ,m,m/s,m/s ²)
No. 1	(1,70)	[0; 0; 0.03; 0; -37;0.75]
No. 2	(1,100)	[400; -10; 0.4; -600; 5;0.01]
No. 3	(1,70)	[-800; 20; -0.16; -200; -5;0.3]
No. 4	(20,100)	[400; -7; -0.37; -100; -42;1]
No. 5	(20,100)	[400; -1.4; -0.44; -600; 10; 0.05]
No. 6	(20,100)	[0; 5.5; 0.29; 0; 22; -0.55]
No. 7	(40,100)	[-800; 32; -1.5; -200; 11; -0.75]
No. 8	(40,100)	[-200; 15; 0.3; 800; -10; 0]
No. 9	(60,100)	[-800; -3; 1.5; -200; 15; -0.75]
No. 10	(60,100)	[-200; -3; 0; 800; -15; 0]
No. 11	(80,100)	[0; -20; -1.6; 0; -45; 0.2]
No. 12	(80,100)	[-200; 15; 0; 800; -5; 0]

As shown in Figures 12 and 13, the sector-matching framework proposed in this paper can achieve radar MTT when considering radar measurement time diversity. Applying the method to the LGM-PHD filter can achieve track maintenance even in the presence of strong clutter interference, dense targets in the surveillance area and long radar scan periods. Similar to example 1, in Figure 12, a low detection probability of $p_{D,k} = 0.8$ causes the trajectory to be broken, and in Figure 12b,d, the reason why the trajectory can be identified as the same target's trajectory instead of a new-born target trajectory after the trajectory is broken is that in the simulation made in this paper, it is assumed that the new-born targets can only appear at the known new born points, i.e., it is assumed that no new-born target will appear at the trajectory breaking points. In Figure 13, due to the extension of accumulation time, the detection probability ($p_{D,k} = 0.96$) and the probability of track breakage is significantly reduced.

In Figure 14, the computation time (CT) of the MATLAB simulation for 100 sample runs is given and the average computation is shown in Table 4. The results in Figure 14 are the same as in Figure 8, which show that the real-time performances of the proposed method for both LGM-PHD filter and the basic GM-PHD filter are similar to that of the basic PHD filter and the LGM-PHD filter, and much better than that of the joint GLMB filter. In addition, extending the accumulation time can reduce the calculation time.

In Figures 15 and 16, the average OSPA distance of the MATLAB simulation for 100 sample runs is given and the average OSPA distances are given in Table 4. Compared with the basic GM-PHD filter, the average tracking performance of the SM-LGM-PHD filter, the LGM-PHD filter, the SM-GM-PHD filter and the joint-GLMB filter in this scenario is improved by 41.57%, 16.40%, 28.78% and 47.67%, respectively, at $T_s = 1$ s ($T_i = 2$ ms, $\lambda = 104$, $p_{D,k} = 0.8$), and 57.32%, 29.11%, 34.66% and 37.69%, respectively, at $T_s = 2$ s ($T_i = 4$ ms, $\lambda = 52$, $p_{D,k} = 0.96$). Compared with the $T_s = 1$ s scenario, the performance

of the SM-LGM-PHD filter and the SM-GM-PHD filter is highly improved in the $T_s = 2$ s scenario. The LGM-PHD filter, the basic GM-PHD filter and the joint-GLMB filter are not time-matched, which is why the performance of these three filters is not particularly improved, and the performance of the joint-GLMB filter even decreases. In addition, it can be seen in Figures 15b and 16b that the values of OSPA location of the LGM-PHD filter, the basic GM-PHD filter and the joint-GLMB filter all increase with the accumulation of time. After 80s, with the appearance of two targets, No.11 and No.12, at a higher speed, the targets' position estimation errors of the LGM-PHD filter, the basic GM-PHD filter and the joint-GLMB filter increase at a faster rate, while the SM-LGM-PHD filter and the SM-GM-PHD filter are not significantly affected. This is mainly caused by the additional measurement error caused by sampling time diversity mentioned earlier.

Compared with the CV model in example 1, the CA motion model has a quadratic term in time, so the increases in the estimation error due to the time accumulation of the basic PHD filter, the LGM-PHD filter and the GLMB filter are more significant.

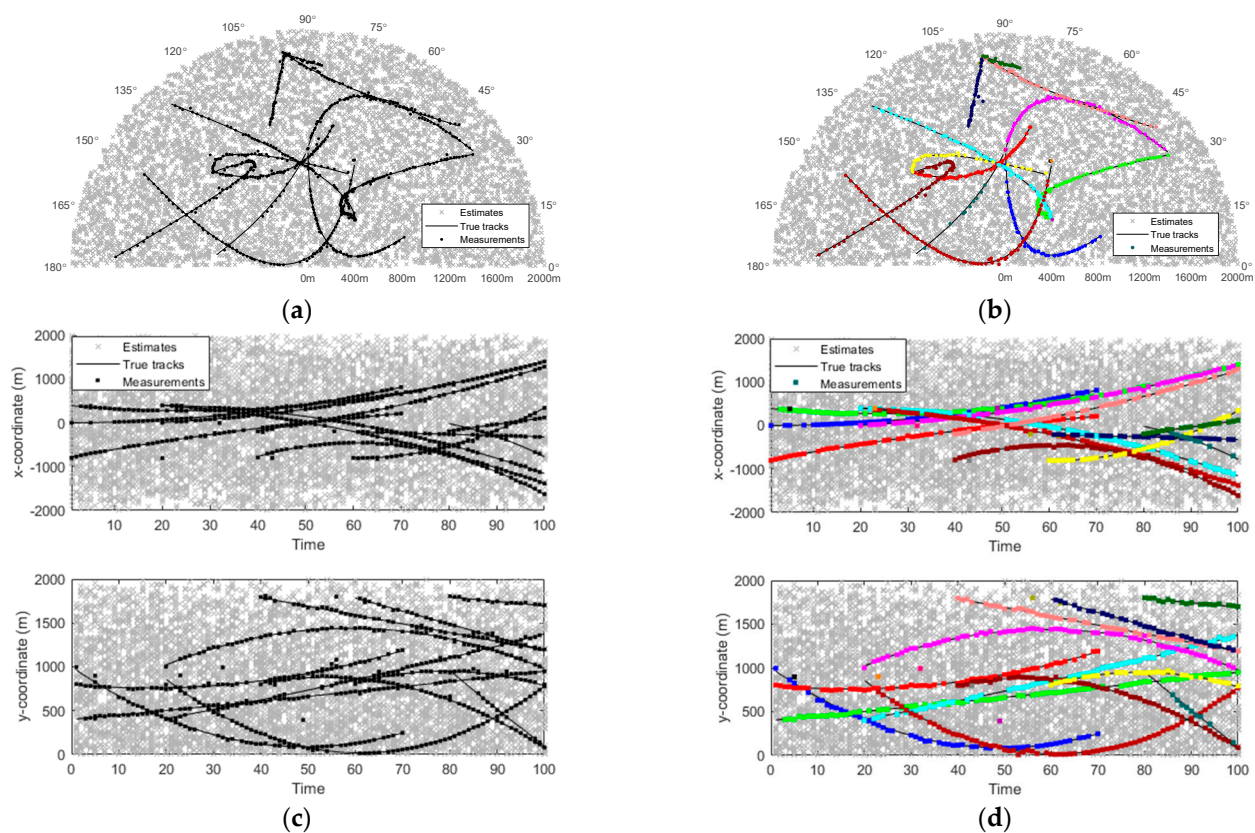


Figure 12. The estimations of the multi-target trajectories and the actual measurements in example 2 (CA motion model) for a sample run. (a) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-GM-PHD filter for $T_s = 1$ s ($T_i = 2$ ms, $\lambda = 104$, $p_{D,k} = 0.8$). (b) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-LGM-PHD filter for $T_s = 1$ s, and the different trajectories are denoted by different color dots. (c) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-GM-PHD filter in x and y coordinates for $T_s = 1$ s. (d) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-LGM-PHD filter in x and y coordinates for $T_s = 1$ s.

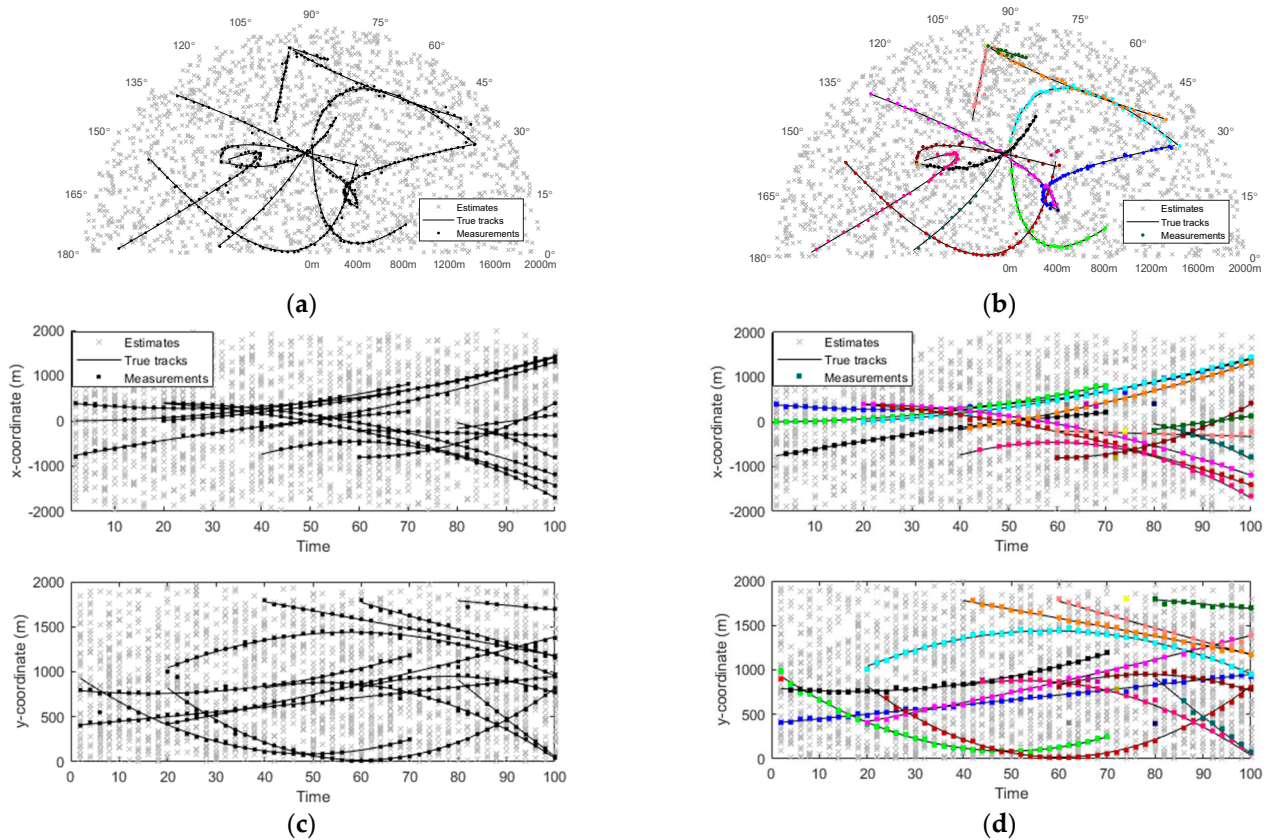


Figure 13. The estimations of the multi-target trajectories and the actual measurements in example 2 (CA motion model) for a sample run. (a) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-GM-PHD filter for $T_s = 2$ s ($T_i = 4$ ms, $\lambda = 52$, $p_{D,k} = 0.96$). (b) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-LGM-PHD filter for $T_s = 2$ s, and the different trajectories are denoted by different color dots. (c) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-GM-PHD filter in x and y coordinates for $T_s = 2$ s. (d) The actual measurements, actual trajectories and estimated trajectories obtained after filtering by the SM-LGM-PHD filter in x and y coordinates for $T_s = 2$ s.

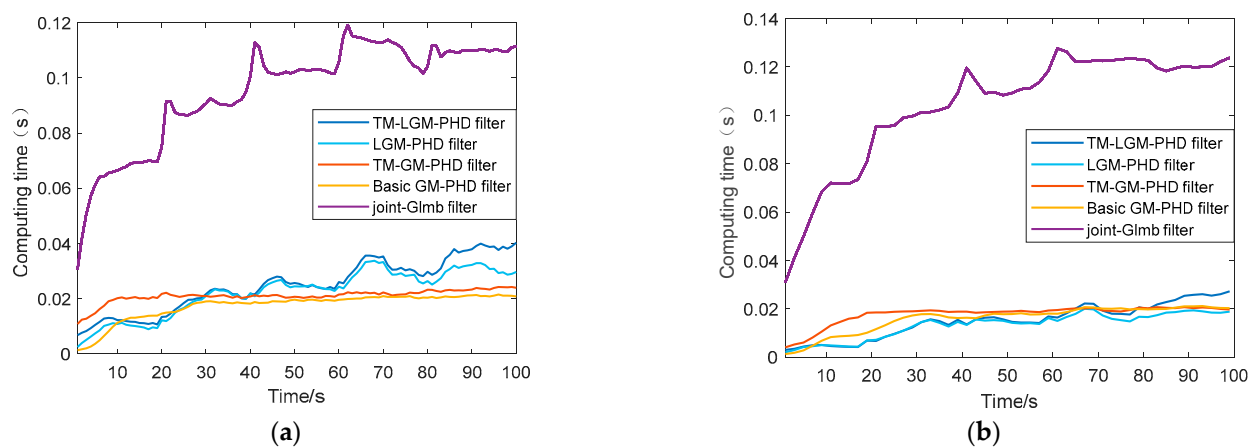


Figure 14. Computation time of different filters for 100 sample runs at $T_s = 1$ s ($T_i = 2$ ms, $\lambda = 104$, $p_{D,k} = 0.8$) (a) and $T_s = 2$ s ($T_i = 4$ ms, $\lambda = 52$, $p_{D,k} = 0.96$) (b) in example 2 (CA motion model).

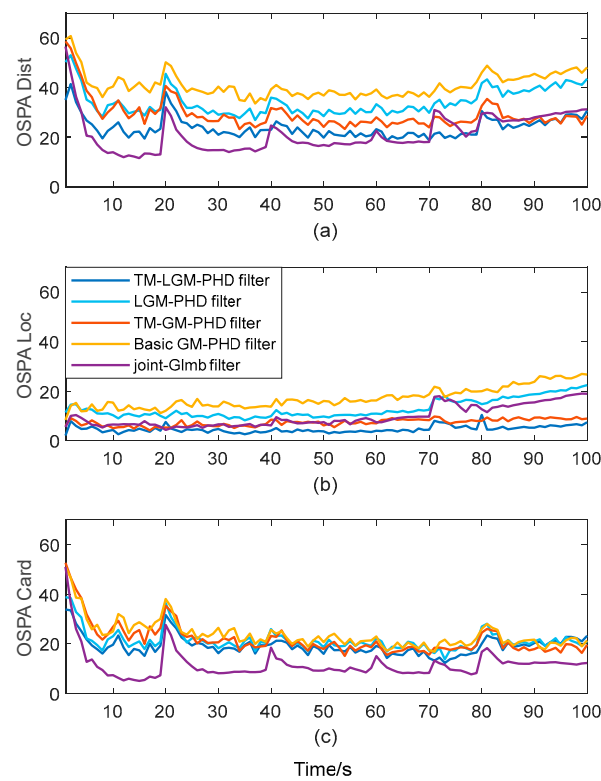


Figure 15. The average OSPA of different filters in example 2 (CA motion model) for 100 sample runs when $T_s = 1$ s ($T_i = 2$ ms, $\lambda = 104$, $p_{D,k} = 0.8$). (a) Average OSPA distances of different filters. (b) OSPA locations of different filters. (c) OSPA cardinalities of different filters.

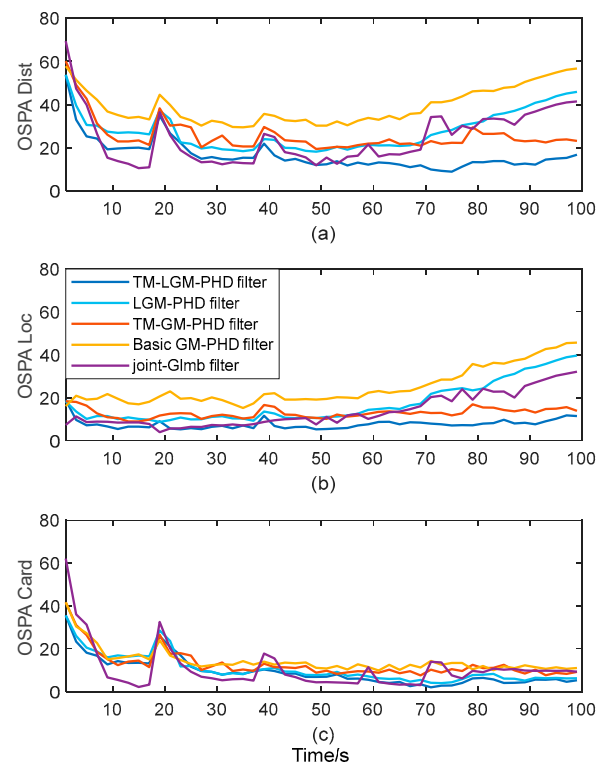


Figure 16. The average OSPA of different filters in example 2 (CA motion model) for 100 sample runs, when $T_s = 2$ s ($T_i = 4$ ms, $\lambda = 52$, $p_{D,k} = 0.96$). (a) Average OSPA distances of different filters. (b) OSPA locations of different filters. (c) OSPA cardinalities of different filters.

Table 4. List of the average CT and average OSPA of each filter in example 2 (CA model).

	SM-LGM- PHD Filter	LGM- PHD Filter	SM-GM- PHD Filter	Basic GM- PHD Filter	Joint- GLMB Filter
CT ($T_s = 1$ s)	0.0248 s	0.0223 s	0.0210 s	0.0174 s	0.0956 s
CT ($T_s = 2$ s)	0.0151 s	0.0133 s	0.0178 s	0.0160 s	0.1038 s
OSPA ($T_s = 1$ s)	23.98 m	34.31 m	29.23 m	41.04 m	21.47 m
OSPA ($T_s = 2$ s)	16.76 m	27.84 m	25.66 m	39.27 m	24.47 m

5. Conclusions

A computationally efficient sector-matching PHD filter is proposed in this paper. With the radar sampling time diversity considered, the proposed filter is well adapted to the MTT problem during radar dynamic scanning. The proposed sector-matching structure quantifies the relationship between beam dwell time, false alarm rate and detection probability, and divides the scanning area into small sectors to obtain actual multi-target measurement times. The proposed filter rederives the prediction and updating steps based on the actual sampling time, and a state correction step is added. The performance is evaluated by simulations using the Gaussian linear model, and the results demonstrate that the proposed filter provides enhanced filtering performance in multi-target tracking. There are still some challenges with the proposed filter, including radars operating in TAS mode and target migration through resolution cells. We will address these issues in future research. In addition, as the performance of the TPHD [33] and TPMB [23] filters is much better than the basic PHD filter, future work will investigate the combination of “sector matching” structures with trajectory filters.

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References

1. Mahler, R.P.S. *Statistical Multisource-Multitarget Information Fusion*; Artech House: Norwood, MA, USA, 2007.
2. Mahler, R.P.S. *Advances in Statistical Multisource-Multitarget Information Fusion*; Artech House: Norwood, MA, USA, 2014.
3. Pace, P.E. *Detecting and Classifying Low Probability of Intercept Radar*; Artech House: Norwood, MA, USA, 2008.
4. Fu, W.; Jiang, D.; Su, Y.; Qian, R.; Gao, Y. Implementation of wideband digital transmitting beamformer based on LFM waveforms. *IET Signal Process.* **2017**, *11*, 205–212. [\[CrossRef\]](#)
5. Jiang, Q.; Wang, R.; Zhou, C.; Zhang, T.; Hu, C. Modified Bayesian Group Target Track Initiation Algorithm Based on Algebraic Graph Theory. *J. Electron. Inf. Technol.* **2021**, *43*, 531–538.
6. Granstrom, K.; Lundquist, C.; Gustafsson, F.; Orguner, U. Random Set Methods Estimation of Multiple Extended Objects. *IEEE Robot. Autom. Mag.* **2014**, *21*, 73–82. [\[CrossRef\]](#)
7. Fortmann, T.; Bar-Shalom, Y.; Scheffe, M. Sonar tracking of multiple targets using joint probabilistic data association. *IEEE J. Ocean. Eng.* **1983**, *8*, 173–184. [\[CrossRef\]](#)
8. Chong, C.Y. Tracking and data fusion: A handbook of algorithms (bar-shalom, y. et al.; 2011)[bookshelf]. *IEEE Control. Syst. Mag.* **2012**, *32*, 114–116. [\[CrossRef\]](#)
9. Blackman, S. Multiple hypothesis tracking for multiple target tracking. *IEEE Aerosp. Electron. Syst. Mag.* **2004**, *19*, 5–18. [\[CrossRef\]](#)
10. Wang, S.; Bao, Q.; Chen, Z. Refined PHD Filter for Multi-Target Tracking under Low Detection Probability. *Sensors* **2019**, *19*, 2842. [\[CrossRef\]](#)

11. Chai, L.; Yi, W.; Jiang, X.; Kong, L. A Distributed PHD Filter for On-line Joint Sensor Registration and Multi-target Tracking. In Proceedings of the International Conference on Information Fusion, Ottawa, ON, Canada, 2–5 July 2019; pp. 1–6.
12. Li, T.; Mallick, M.; Pan, Q. A Parallel Filtering-Communication-Based Cardinality Consensus Approach for Real-Time Distributed PHD Filtering. *IEEE Sens. J.* **2020**, *20*, 13824–13832. [[CrossRef](#)]
13. Jiang, D.; Liu, M.; Gao, Y.; Gao, Y.; Fu, W.; Han, Y. Time-Matching Random Finite Set-Based Filter for Radar Multi-Target Tracking. *Sensors* **2018**, *18*, 4416. [[CrossRef](#)]
14. Clark, D.; Vo, B.T.; Bell, J. GM-PHD filter multi-target tracking in sonar images. In Proceedings of the Conference on Signal Processing, Sensor Fusion, and Target Recognition XV, Kissimmee, FL, USA, 17–19 April 2006. [[CrossRef](#)]
15. Clark, D.; Ruiz, I.; Petillot, Y.; Bell, J. Particle PHD filter multiple target tracking in sonar image. *IEEE Trans. Aerosp. Electron. Syst.* **2007**, *43*, 409–416. [[CrossRef](#)]
16. Moratuwage, D.; Vo, B.N.; Wang, D.W. Collaborative Multi-Vehicle SLAM with Moving Object Tracking. In Proceedings of the IEEE International Conference on Robotics and Automation (ICRA), Karlsruhe, Germany, 6–10 May 2013.
17. Hoseinnezhad, R.; Vo, B.N.; Vo, B.T.; Suter, D. Visual tracking of numerous targets via multi-Bernoulli filtering of image data. *Pattern Recognit.* **2012**, *45*, 3625–3635. [[CrossRef](#)]
18. Pollard, E.; Plyer, A.; Pannetier, B.; Champagnat, F.; Le Besnerais, G. Gm-phd filters for multi-object tracking in uncalibrated aerial videos. In Proceedings of the 12th International Conference on Information Fusion, Seattle, WA, USA, 6–9 July 2009; IEEE: New York, NY, USA, 2009; pp. 1171–1178.
19. Vo, B.N.; Ma, W.K. The Gaussian Mixture Probability Hypothesis Density Filter. *IEEE Trans. Signal Process.* **2006**, *54*, 4091–4104. [[CrossRef](#)]
20. Vo, B.T.; Vo, B.N. Labeled Random Finite Sets and Multi-Object Conjugate Priors. *IEEE Trans. Signal Process.* **2013**, *61*, 3460–3475. [[CrossRef](#)]
21. Reuter, S.; Vo, B.T.; Vo, B.N.; Dietmayer, K. The Labeled Multi-Bernoulli Filter. *IEEE Trans. Signal Process.* **2014**, *62*, 3246–3260.
22. Vo, B.N.; Vo, B.T.; Hoang, H. An Efficient Implementation of the Generalized Labeled Multi-Bernoulli Filter. *IEEE Trans. Signal Process.* **2017**, *65*, 1975–1987. [[CrossRef](#)]
23. García-Fernández, Á.F.; Svensson, L.; Williams, J.L.; Xia, Y.; Granström, K. Trajectory Poisson Multi-Bernoulli Filters. *IEEE Trans. Signal Process.* **2020**, *68*, 4933–4945. [[CrossRef](#)]
24. Meyer, F.; Kropfreiter, T.; Williams, J.L.; Lau, R.; Hlawatsch, F.; Braca, P.; Win, M.Z. Message passing algorithms for scalable multitarget tracking. *Proc. IEEE* **2018**, *106*, 221–259. [[CrossRef](#)]
25. Gan, R.Z.; Li, Q.; Godsill, S. A Variational Bayes Association-based Multi-object Tracker under the Non-homogeneous Poisson Measurement Process. In Proceedings of the 2022 25th International Conference on Information Fusion (FUSION), Linköping, Sweden, 4–7 July 2022; IEEE: New York, NY, USA, 2022; pp. 1–8.
26. Khan, Z.; Balch, T.; Dellaert, F. Mcmc Data Association and Sparse Factorization Updating for Real Time Multitarget Tracking with Merged and Multiple Measurements. *IEEE Trans. Pattern Anal. Mach. Intell.* **2006**, *28*, 1960–1972. [[CrossRef](#)]
27. Davey, S.J.; Fabrizio, G.A.; Rutten, M.G. Detection and tracking of multipath targets in over-the-horizon radar. *IEEE Trans. Aerosp. Electron. Syst.* **2019**, *55*, 2277–2295. [[CrossRef](#)]
28. Streit, R.; Degen, C.; Koch, W. The pointillist family of multitarget tracking filters. *arXiv* **2015**, arXiv:1505.08000.
29. Lin, L.; Bar-shalom, Y.; Kirubarajan, T. Tracking labeling and PHD filter for multitarget tracking. *IEEE Trans. Aerosp. Electron. Syst.* **2006**, *42*, 778–795. [[CrossRef](#)]
30. Yang, J.; Ji, H. A novel track maintenance algorithm for PHD/CPHD filter. *Signal Process.* **2012**, *92*, 2371–2380. [[CrossRef](#)]
31. Li, Y.; Xiao, H.; Wu, H.; Fu, Q.; Hu, R. Modified Labeled Particle Probability Hypothesis Density Filter for Joint Multi-target Tracking and Classification. In Proceedings of the International Conference on Signal Processing and Communication Systems, Cairns, QLD, Australia, 14–16 December 2015; pp. 1–5.
32. Li, T.; Sun, S.; Corchado, J.M.; Siyan, M.F. A particle dyeing approach for track continuity for the SMC-PHD filter. In Proceedings of the International Conference on Information Fusion, Salamanca, Spain, 7–10 July 2014; pp. 1–8.
33. García-Fernández, Á.F.; Svensson, L. Trajectory probability hypothesis density filter. In Proceedings of the 2018 21st International Conference on Information Fusion (FUSION), Cambridge, UK, 10–13 July 2018; pp. 1430–1437.
34. García-Fernández, Á.F.; Svensson, L. Trajectory PHD and CPHD Filters. *IEEE Trans. Signal Process.* **2019**, *67*, 5702–5714. [[CrossRef](#)]
35. Gao, Y.; Jiang, D.; Zhang, C.; Guo, S. A Labeled GM-PHD Filter for Explicitly Tracking Multiple Targets. *Sensors* **2021**, *21*, 3932. [[CrossRef](#)]
36. Zhang, Y.; Liu, M.; Zhao, K.; Gao, Y.; Zhang, T. Time-matching Recursive Extended Target Probability Hypothesis Density Filter for High Resolution Phased Array Radar. In Proceedings of the 2021 33rd Chinese Control and Decision Conference (CCDC), Kunming, China, 22–24 May 2021; IEEE: New York, NY, USA, 2021; pp. 7218–7223.
37. Jiang, D.F.; Liu, M.; Gao, Y.Y.; Gao, Y. Time-matching extended target probability hypothesis density filter for multi-target tracking of high resolution radar. *Signal Process.* **2019**, *157*, 151–160. [[CrossRef](#)]

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