

Article

Numerical Modeling Technique of Damage Behavior of MaSonry-Infilled RC Frames

Bo Liu ¹, Chunhui Liu ^{1,*}, Xiaomin Wang ^{2,3,*}, Jingchang Kong ¹  and Zhiwang Chang ⁴

¹ School of Civil Engineering, Yantai University, Yantai 264005, China

² Key Laboratory of Earthquake Engineering and Engineering Vibration, Institute of Engineering Mechanics, China Earthquake Administration, Harbin 150080, China

³ Key Laboratory of Earthquake Disaster Mitigation, Ministry of Emergency Management, Harbin 150080, China

⁴ School of Civil Engineering, Southwest Jiaotong University, Chengdu 610031, China

* Correspondence: lch_hit@163.com (C.L.); xiaomin_wang@iem.ac.cn (X.W.)

Abstract: The damage pattern of masonry-infilled reinforced concrete (RC) frame structures in earthquake events is complicated, and understanding the detailed failure behavior of these structures and modeling it accurately has been a challenging task. In this paper, the extended finite element method (XFEM) is introduced to reproduce arbitrary cracks initiating and propagating in concrete frame and masonry units, combined with interface elements to model various behaviors of masonry-infilled RC frames. Within the finite element analysis program FEAP, a user element subroutine is adopted for the incorporation of XFEM and two types of extended finite elements with and without crack tip enrichments are built to simulate the behavior of concrete material for frame members and masonry blocks for the infill panel, respectively. In addition, a macro command is created to check the crack-propagation criterion and update crack and enrichment information. Furthermore, numerical examples are performed with existing test data, which reveal the efficiency of the implementation procedure. A comparison of the analytical and experimental results show that the proposed modeling can be used to predict the crack and failure process and the load-bearing capacity curves of the structures and reflect accurately the interaction of masonry infill and RC frames.

Keywords: masonry-infilled RC frames; implementation procedure; extended finite element method (XFEM); user element subroutine; macro command



Citation: Liu, B.; Liu, C.; Wang, X.; Kong, J.; Chang, Z. Numerical Modeling Technique of Damage Behavior of MaSonry-Infilled RC Frames. *Appl. Sci.* **2023**, *13*, 1521. <https://doi.org/10.3390/app13031521>

Academic Editors: Linsheng Huo and Dongdong Chen

Received: 14 December 2022

Revised: 20 January 2023

Accepted: 21 January 2023

Published: 24 January 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Masonry-infilled RC frame structures are widely built in seismically active areas around the world [1]. The observation of post-earthquake damage has shown that the interaction between the infill and the bounding frame has a significant impact on the seismic performance of these structures that may or may not be beneficial [2,3]. Therefore, it is always important to consider the behavior of the frame–infill wall interaction in seismic regions.

A large number of numerical methods have been proposed in the literature to consider the nonlinear behavior of masonry-infilled frames. Generally, two main modeling approaches are used for the cracking and damage behavior of the masonry material, these being the continuum approach and the discretization approach. In the continuum approach, the infill is usually modeled as a homogenized continuum and the weakness of mortar joints is smeared out in the entire wall. It has been widely adopted in the past due to its efficiency of calculation and convenience of implementation [4–12], which allows it to be used for the seismic performance analysis of actual large-scale structures [13,14]. In this approach, the crack and damage propagation path is represented by a zone in which the load-bearing capacity is gradually lost. The best-known model of this approach is the smeared cracking model, which was pioneered with the fixed smeared cracking

model [15,16], then extended to the rotating smeared cracking model [17–19]. Furthermore, in order to consider the anisotropy and post-peak softening behavior caused by the presence of mortar in masonry walls, some heterogeneous continuum models with the features of orthotropic elastic-plastic behavior with softening have been proposed [20–22].

In the discretization approach, continuum elements are combined with interface elements to separately represent masonry units, mortar joints and unit–mortar interfaces. In fact, mortar joints and unit–mortar interfaces are usually reduced to interfaces for computational efficiency. With this approach, different failure modes can be captured in an accurate manner. Therefore, it is also widely used in a lot of literature to explore the detailed structural behavior of masonry-infilled RC frames [23–30].

In addition, some novel simulation methods have been proposed for the comprehensive consideration of computational efficiency and simulation accuracy. Milani [31] proposed a homogenization model of masonry walls at a structural level by uniformly discretizing the wall into rigid triangular elements and nonlinear interface elements. Pirsaheb et al. [32,33] proposed a novel multi-pier (MP) method to analyze the in-plane behavior of masonry walls, especially for walls with openings. With this MP method, Khaleghi et al. [34,35] predicted the load-bearing capacity and stiffness of masonry walls by the application of machine learning, which greatly improves calculation efficiency.

In the analysis of masonry-infilled RC frames, the choice of numerical method depends on the accuracy and detail required. The uncertainties of the frame–infill wall interaction may result in different failure modes, which leads to complexity in the numerical analysis. The discretization approach is usually preferred to describe the failure behavior of masonry-infilled RC frames in detail. In this approach, however, interface elements are required to be preset in the modeling process before the analysis to represent the potential cracking behavior. Shing [36,37] tried to add interface elements into the modeling of columns to model the diagonal shear failure in the columns according to the experimental observation of cracking behavior. However, in frame members and masonry blocks, the diagonal cracks and many distributed cracks may develop in an arbitrary manner; it is therefore difficult to preset interface elements in the modeling procedure. For this reason, Stavridis and Shing [25] introduced a great many of horizontal, vertical and diagonal interface elements in columns to cover potential crack-propagation paths.

Considering the inherent mesh dependence in the interface element and the complication of a finite element discretization scheme, this approach is still not satisfactory for dealing with the arbitrary crack-propagation problem [38]. Therefore, an extended finite element method (XFEM) is introduced and combined with interface elements to predict the response of masonry-infilled RC frame structures. XFEM is an effective numerical method for solving discontinuous problems. This paper focuses on the implementation aspects of a numerical simulation of masonry-infilled RC frame structures using XFEM, and presents a feasible and effective means to incorporate XFEM into the finite element analysis program FEAP [39]. Some critical and difficult parts in the implementation of XFEM are also explained in detail.

Research Significance

The failure of masonry-infilled RC frame structures has been very common in past earthquakes. The existence of the infill wall affects the load-bearing capacity and failure mode of such structures. Due to the complexity of the interaction between the infill wall and the bounding frame, and the numerous factors affecting the structural behavior, it is not easy to determine the failure behavior of infilled RC frames. Therefore, it is desirable and challenging to explore this failure behavior in detail using accurate numerical methods. To this end, the discretization approach combining continuous elements and interface elements is a widely used approach. To overcome the difficulty of arbitrary crack propagation in this approach, XFEM is introduced into this study to simulate the cracking behavior of frame concrete and masonry units, and the key elements in the implementation of XFEM are elaborated. Finally, numerical examples show the effectiveness of the proposed model.

The study will help academics better understand the performance of masonry-infilled RC frame structures, and help readers to extend its application to a broader class of problems.

2. Introduction of XFEM

In this section, a review of XFEM is concisely introduced. The kernel idea of XFEM is to incorporate special enrichment functions into a standard finite element space to represent a discontinuous displacement field. The implementation of XFEM is similar to the conventional finite element method, which allows XFEM to be easily introduced within a general finite element program with a few modifications. The detailed description for XFEM can be found in [40–45].

2.1. Basic Principle

The displacement field for the XFEM approximation can be expressed as:

$$\mathbf{u}(x) = \sum_I N_I(x) \mathbf{u}_I + \sum_J N_J(x) (H(\phi(x)) - H(\phi(x_J))) \mathbf{b}_J + \sum_K N_K(x) \left(\sum_{l=1}^4 (\Phi_l(r, \theta) - \Phi_l(x_K)) \mathbf{c}_K^l \right) \quad (1)$$

where the three terms on the right side represent standard finite element approximation, crack discontinuity enrichment and crack tip enrichment, respectively. A geometric enrichment strategy [46] is adopted in this paper for crack tip enrichments, which ensures that the enrichment domain is independent of the mesh size. Since degrees of freedom are enriched on nodes of existing elements, the modification of topology can be avoided.

The modified Heaviside step function $H(\phi(x))$ is given by:

$$H(\phi(x)) = \begin{cases} +1 & \text{if } \phi(x) > 0 \\ -1 & \text{if } \phi(x) < 0 \end{cases} \quad (2)$$

This function implies that the discontinuity occurs at the location of the crack surface.

For a cohesive crack, the crack tip enrichment functions $\Phi_l(r, \theta)$ can take the following formats [47]:

$$\Phi_l(r, \theta) = \left\{ r \sin \frac{\theta}{2}, r \cos \frac{\theta}{2}, r \sin \frac{\theta}{2} \sin \theta, r \cos \frac{\theta}{2} \sin \theta \right\} \quad (3)$$

where (r, θ) is a local polar coordinate system at the crack tip. The functions are used to describe the displacement field around a crack tip. The first function $r \sin \theta/2$ in Equation (3) is discontinuous across the crack surface, whereas the last three functions are continuous.

In fact, these functions are allowed to be omitted, and the modified Heaviside step function is suitable for the entire crack, including the crack tip, since near-tip fields are no longer singular for the cohesive crack [48–50]. Nevertheless, the use of crack tip enrichment functions allows the crack tips to be inside a finite element, rather than being located on the element boundaries. In this work, step and crack tip enrichment functions are both retained for modeling concrete columns and beams, whereas only a step enrichment function is adopted for masonry blocks, considering the fact that masonry blocks in the infill panel are isolated with each other.

2.2. Incremental Equilibrium Equation

Consider a two-dimensional domain Ω bounded by Γ , as shown in Figure 1. The boundary Γ includes three complementary sets: Γ_t , Γ_u and Γ_c . The traction is prescribed on Γ_u , the displacement is prescribed on Γ_c , and the internal discontinuity surface is represented by Γ_c , which consists of Γ_c^+ and Γ_c^- .

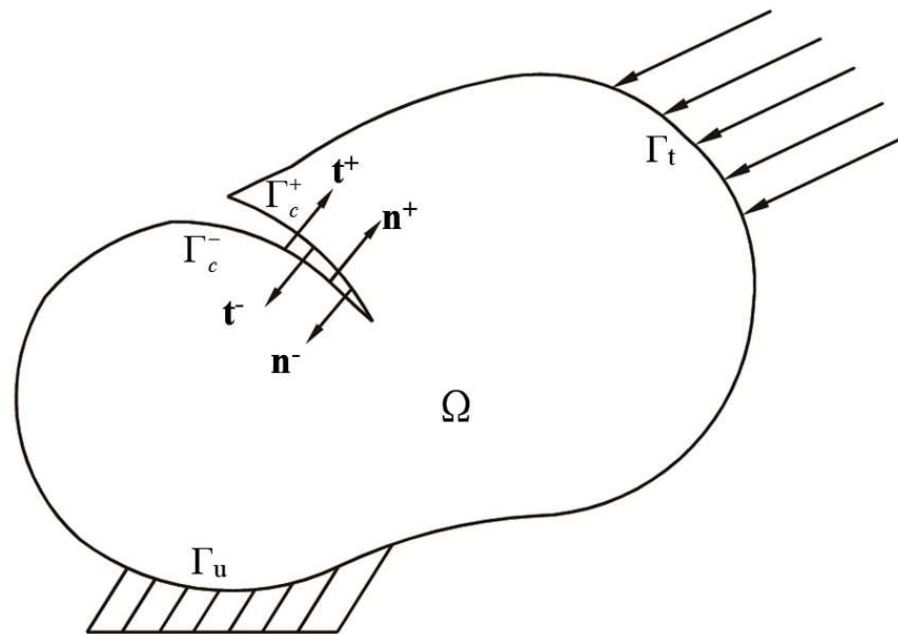


Figure 1. A cracked domain with boundary conditions.

The equilibrium equation in the strong form and corresponding boundary conditions can be written as:

$$\nabla \cdot \sigma + b = 0 \quad \text{in } \Omega \quad (4a)$$

$$\sigma \cdot n = \bar{t} \quad \text{on } \Gamma_t \quad (4b)$$

$$u = \bar{u} \quad \text{on } \Gamma_u \quad (4c)$$

$$\sigma \cdot n^+ = -\sigma \cdot n^- = t^+ = -t^- = t \quad \text{on } \Gamma_c \quad (4d)$$

where σ is the Cauchy stress tensor in the bulk material, b is body force per unit volume, and $\bar{t}$ and $\bar{u}$ are prescribed tractions and displacements, respectively. t^+ and t^- are interface tractions acting upon the two sides of Γ_c . n is the unit outward normal.

It is assumed that the displacement remains small and the strain field can be expressed as the derivation of the displacement field:

$$\varepsilon = \varepsilon(u) = \nabla_s u \quad (5)$$

The magnitude of displacement discontinuity w across Γ_c can be computed as the displacement separation on two sides of the crack surface:

$$w = u^- - u^+ \quad (6)$$

To describe the nonlinearity involved in solving the problem, the incremental equilibrium equation in weak form is necessary. With the admissible variational displacement fields, it can be given as:

$$\int_{\Omega} \dot{\sigma}(\dot{u}) : \varepsilon(v) dx + \int_{\Gamma_c} \dot{t}(\dot{u}) \cdot w(v) ds = \int_{\Omega} \dot{b} \cdot v d\Omega + \int_{\Gamma_t} \dot{\bar{t}} \cdot v d\Gamma \quad \forall v \in U_0 \quad (7)$$

where $w(v) = v^- - v^+$.

2.3. Discrete Equation

Following this, we briefly introduce the discrete equation for the implementation of XFEM. The discretized strain field in terms of nodal displacements can be described as:

$$\varepsilon = Bu = \begin{bmatrix} B_I^u & B_J^b & B_K^{c1} & B_K^{c2} & B_K^{c3} & B_K^{c4} \end{bmatrix} \begin{bmatrix} u_I \\ b_J \\ c_K^1 \\ c_K^2 \\ c_K^3 \\ c_K^4 \end{bmatrix} \quad (8)$$

where

$$B_I^u = \begin{bmatrix} N_{I,x} & 0 \\ 0 & N_{I,y} \\ N_{I,y}N_{I,x} \end{bmatrix} \quad (9)$$

$$B_J^b = \begin{bmatrix} (N_J(H - H(f(x_j))))_{,x} & 0 \\ 0 & (N_J(H - H(f(x_j))))_{,y} \\ (N_J(H - H(f(x_j))))_{,y} & (N_J(H - H(f(x_j))))_{,x} \end{bmatrix} \quad (10)$$

$$B_K^{cl} \Big|_{l=1,2,3,4} = \begin{bmatrix} (N_K(\Phi_1 - \Phi_1(f(x_K))))_{,x} & 0 \\ 0 & (N_K(\Phi_1 - \Phi_1(f(x_K))))_{,y} \\ (N_K(\Phi_1 - \Phi_1(f(x_K))))_{,y} & (N_K(\Phi_1 - \Phi_1(f(x_K))))_{,x} \end{bmatrix} \quad (11)$$

To implement a nonlinear finite element solution procedure, the constitutive relations in a rate form are necessary. The stress rate in the bulk of the continuum material can be expressed in terms of the corresponding strain rate as:

$$\dot{\sigma} = D\dot{\varepsilon} = DB\dot{u} \quad (12)$$

where D being the stiffness matrix of the bulk material.

Similarly, the traction rate at the crack surface can be expressed in terms of the corresponding separation rate on two sides of the crack surface as:

$$\dot{t} = T\dot{w} = TN_{coh}\dot{u} \quad (13)$$

where T is the stiffness matrix at the crack surface; the shape function matrix N_{coh} for the crack surface can be written as:

$$N_{coh} = 2 \begin{bmatrix} N_{coh,I}^u & N_{coh,J}^b & N_{coh,K}^{c1} & N_{coh,K}^{c2} & N_{coh,K}^{c3} & N_{coh,K}^{c4} \end{bmatrix} \quad (14)$$

where

$$N_{coh,I}^u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (15)$$

$$N_{coh,J}^b = \begin{bmatrix} N_J H & 0 \\ 0 & N_J H \end{bmatrix} \quad (16)$$

$$N_{coh,K}^{cl} \Big|_{l=1,2,3,4} = \begin{bmatrix} N_K \Phi_1 & 0 \\ 0 & N_K \Phi_1 \end{bmatrix} \quad (17)$$

Substituting the rate form of constitutive relations for the bulk material and the cohesive discontinuity interface into the incremental equilibrium Equation (7) leads to:

$$[K + K_{coh}]\dot{u} = f^{ext} - f^{int} \quad (18)$$

The stiffness matrix for the continuous part can be written as:

$$\mathbf{K} = \int_{\Omega} \mathbf{B}^T \mathbf{D} \mathbf{B} d\Omega \quad (19)$$

The stiffness contribution of the cohesive discontinuity interface is:

$$\mathbf{K}_{\text{coh}} = - \int_{\Gamma_c} \mathbf{N}_{\text{coh}}^T \mathbf{M}^T \mathbf{T} \mathbf{M} \mathbf{N}_{\text{coh}} d\Gamma \quad (20)$$

where $\mathbf{M}$ is the orthogonal coordinate transformation matrix from global coordinate system to local coordinate system, and it is given by:

$$\mathbf{M} = \begin{bmatrix} \sin\theta & -\cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix} \quad (21)$$

The expression for the external force is:

$$\mathbf{f}^{\text{ext}} = \left\{ f_I^u \quad f_J^b \quad f_K^{c1} \quad f_K^{c2} \quad f_K^{c3} \quad f_K^{c4} \right\}^T \quad (22)$$

where the terms of the external force are:

$$f_I^u = \int_{\Gamma_t} \mathbf{N}_I \bar{\mathbf{t}} d\Gamma + \int_{\Omega} \mathbf{N}_I \mathbf{b} d\Omega \quad (23)$$

$$f_J^b = \int_{\Gamma_t} \mathbf{N}_J (\mathbf{H} - \mathbf{H}(f(x_J))) \bar{\mathbf{t}} d\Gamma + \int_{\Omega} \mathbf{N}_J (\mathbf{H} - \mathbf{H}(f(x_J))) \mathbf{b} d\Omega \quad (24)$$

$$f_K^{cl} \Big|_{l=1,2,3,4} = \int_{\Gamma_t} \mathbf{N}_K (\Phi_l - \Phi_l(f(x_K))) \bar{\mathbf{t}} d\Gamma + \int_{\Omega} \mathbf{N}_K (\Phi_l - \Phi_l(f(x_K))) \mathbf{b} d\Omega \quad (25)$$

Finally, the internal force is given by:

$$\mathbf{f}^{\text{int}} = \int_{\Omega} \mathbf{B}^T \boldsymbol{\sigma} d\Omega + \int_{\Gamma_c} \mathbf{N}_{\text{coh}}^T \mathbf{M}^T \mathbf{t} d\Gamma \quad (26)$$

3. Numerical Implementation of XFEM

FEAP is an open source structural analysis program for solving linear and nonlinear problems in solid mechanics and other subject areas. It allows users to add new elements, material modules or command language statements to meet some specific application requirements.

In this paper, the operation of incorporating the XFEM capabilities within FEAP is broken down into the following subcategories: 1. nodal degrees of freedom; 2. input material parameters; 3. information passing; 4. numerical integration strategy; 5. constitutive relations; 6. assembly procedure; 7. output for post-processing; 8. crack-propagation strategy. Of all these tasks, Task 8 is implemented by adding a macro command with the name “UMACR1”, and the incorporation of all the others is implemented in a user-developed element. Different tasks are carried out according to the values of the parameter ISW passed as the 11th parameter in the user element subprogram “ELMTnn”. The implementation frameworks of extended finite elements for concrete and masonry block are very similar, except for some differences in implementation details due to the different enrichment types. Detailed descriptions for the implementation of these eight tasks are elaborated in the following sections.

3.1. Nodal Degrees of Freedom

In a classical finite element modeling for a two-dimensional problem, two degrees of freedom at each node are usually needed in solid elements to perform the continuum mechanics analysis, which correspond to nodal displacements in two coordinate directions.

In XFEM, as described in Equation (1), apart from the classical degrees of freedom, there are several additional enriched degrees of freedom that should be introduced for the displacement field description, and these enriched degrees of freedom are also associated with the corresponding element nodes [40]. To be more specific, two enriched degrees of freedom at each node are needed for the modified Heaviside enrichment, and eight enriched degrees of freedom at each node are built for the crack tip enrichment.

In this study, two types of extended finite element modules are built in FEAP using a user subprogram called “ELMTnn”: one is programmed to simulate the behavior of concrete for columns and beams, and the other is used for the masonry blocks in the infill panel. The difference in enrichment type for two modules leads to different numbers of required nodal degrees of freedom. For masonry elements, two enriched degrees of freedom per node associated with the modified Heaviside enrichment are added. For concrete elements, ten enriched degrees of freedom per node associated with both the modified Heaviside and crack tip enrichments are used. It should be pointed out that multiple cracks are allowed in the modeling. Specifically, if either the modified Heaviside or crack tip enrichment is activated, degrees of freedom numbered 3, 4 or 5–12 are correspondingly used to perform the subsequent work. If two modified Heaviside enrichments for two cracks are activated at the same node, the degrees of freedom 3–6 are used. If modified Heaviside and crack tip enrichments associated with two different cracks are activated, the degrees of freedom 3–12 are all adopted. Two crack tip enrichments at the same node and more than one crack intersecting in the same element are not considered in this paper. If needed, the number of degrees of freedom should be appropriately set according to the desired analysis type.

Furthermore, there are usually several element types used to perform the analysis, and these elements have different numbers of degrees of freedom. In the first part of an input file for FEAP, the value of NDF (maximum number of unknowns per node) is required to show the problem size information. For an element whose NSD (the number of specified degrees of freedom) is less than NDF, it may not be necessary to have the additional degrees of freedom activated. In FEAP, therefore, a convenient method is provided to automatically eliminate the degrees of freedom with values greater than NSD, by inserting the following codes into the user element subroutine for the ISW = 1 task.

```
DO i = NSD + 1, NDF
  IX(i) = 0
END DO
```

For concrete and masonry-block elements, the NSD values are specified as 12 and 4, respectively. This avoids the need to specify appropriate fixed boundary conditions for the unused values.

Additionally, some element types in FEAP may have more than two degrees of freedom at each node. For instance, apart from two degrees of freedom, the frame element has an additional degree of freedom, numbered ‘3’, to represent the bending behavior. The use of the same location for additional degrees of freedom in the XFEM introduces a limitation in application. However, users might define alternative degree-of-freedom assignments to deal with this conflict.

3.2. Input Material Parameters

In FEAP, the program allows users to define their own parameters that are stored in the array `ud(*)`. In this study, a subroutine “`uimate(ud)`” is built under the ISW = 1 task to define various user parameters, and a sample module is shown as follows:

```

subroutine uimate(ud)
c    Purpose: Input material parameters for material models
implicit none
c    Common blocks
character text(2)*15
logical  pcomp, errck, tinput, inputs
real*8   ev(10),ud(*)
inputs = .true.
do while(inputs)
errck = .true.
do while(errck)
errck = TINPUT(text,1,ev,9)
end do ! while
c    1 Youngs.Modulus    2 Poissons.Ratio
if (PCOMP(text(1),'EEnu',4)) then
ud(1) = ev(1)
ud(2) = ev(2)
c    Initial state vector of interface  s0 r0 u0
elseif (PCOMP (text(1),'q0q0',4)) then
ud(3) = ev(1)
ud(4) = ev(2)
ud(5) = ev(3)
c    Stiffnesses of interface    Dnn Dtt
elseif (PCOMP(text(1),'DnDt',4)) then
ud(6) = ev(1)
ud(7) = ev(2)
c    Check end of data
elseif (PCOMP (text(1),'    ',4)) then
inputs = .false.
endif
end do ! while
end

```

The subroutine “TINPUT” is a standard input subprogram in FEAP. The subroutine “PCOMP” is used to compare character strings for a match and upper/lower case differences are ignored. “EEnu”, “q0q0” and “DnDt” are command names set by users. A conflict between user-set command names and standard FEAP command names should be prevented.

To input the user parameters, the mesh “MATERial” command should include associated statements. A sample command sequence in an input file can adopt the following format:

```

MATERial ma

USER n

EEnu E nu

q0q0 s r miu

DnDt Dn Dt

```

where ma represents the material number, n represents the user element number (1 for a concrete element and 2 for a masonry-block element in this study).

3.3. Information Passing

In FEAP, conventional information such as material parameters, element nodal solution parameters and element nodal coordinates is provided to the user element subroutine through data passed as arguments. For user-defined information such as crack geometry

and enrichment types, FEAP also provides options for each user element to create and manage user data arrays through the common block.

In XFEM, the crack propagation is completely independent of the finite element mesh. Naturally, we can represent cracks as contiguous piecewise linear segments, with new crack segments added as cracks propagate. That means a user array containing a series of vertex coordinates is needed to describe the crack geometry information. FEAP allows users to add allocation for their own arrays by calling a subprogram "UALLOC". However, it is not easy to determine the length of the data array since the number of vertices for each crack and number of cracks are constantly changing as cracks propagate and new cracks are initiated. In addition, some extra attention must be paid to the management of user-defined arrays. Therefore, an alternative approach is proposed in this study to describe the crack geometry information by utilizing element-history variables.

In FEAP, element-history variables are provided for each element to manage additional variables that constantly change during the solving process. It is a user's choice to decide what is retained in each history-variable type. Generally, element history variables are set to store internal variables for constitutive relations. In this study, the coordinates of intersections between the crack and element edges as well as the vertices of the crack lying inside the current element are also recorded in the corresponding element history variables. With these data, plotting piecewise linear crack segments of each element will automatically form the distribution of all crack curves. In this method, it is easy to specify the amount of history variables associated with the crack geometry information, although it may waste some storage space due to the duplicate records of some intersects in adjacent elements. Meanwhile, this method also eases the implementation of element partitioning, the determination of integration points and the calculation of enrichment functions for integration points, since the intersection coordinates of the crack and element edges have been stored.

In addition, nodal enrichment types for each element should be stored in element history variables, too. Because more than one enrichment may be included at the same node to deal with a multiple-cracks situation, five numbers (i.e., 0 for no enrichment, 1 for modified Heaviside enrichment, 2 for crack tip enrichment, 3 for two modified Heaviside enrichments, 6 for one modified Heaviside enrichment and one crack tip enrichment) have been defined to represent crack-enrichment information. It should be noted that the appropriate enrichment type indicates the associated enriched degrees of freedom, which further ensures the implementation of an appropriate assembly.

3.4. Numerical Integration Strategy

In this study, an efficient numerical integration strategy is adopted to accurately assemble the contribution to the weak form on both sides of the crack surface. This strategy is simple yet effective, and it is also capable of adapting to much more complex crack-propagation problems.

The elements cut by the crack are firstly subdivided into 16 sub-quadrilaterals, as shown in Figure 2. Following this, two sets of subdivision strategies are adopted to remedy the integration error due to the incompatibility between the sub-quadrilateral edges and the crack surface [51]. To be specific, one set with sub-elements compatible with the crack surface is used to compute the global stiffness matrix, as shown in Figure 2a,b, and the other set with sub-elements independent of the crack surface is used for residual vector and plastic flow, as denoted in Figure 2c,d. For each sub-element, four-order standard Gauss–Legendre quadrature is used to determine the location of integration points.

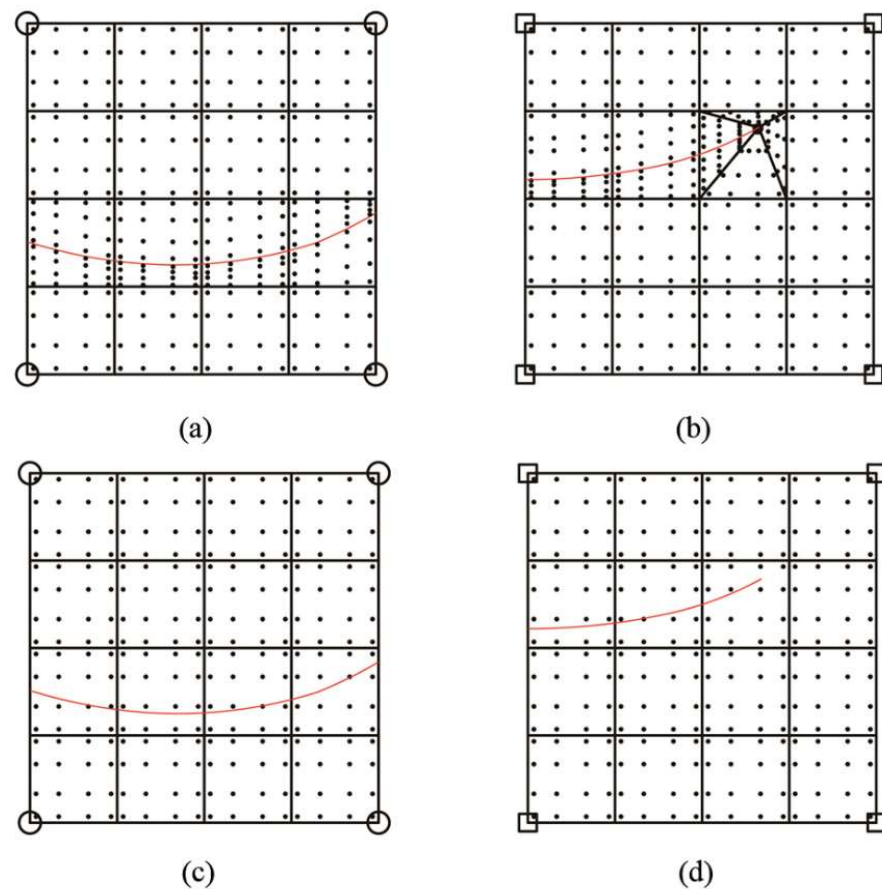


Figure 2. Element partitioning strategy.

In the first set of the sub-element division strategy (as shown in Figure 2a,b), sub-quadrilaterals that cracks pass through are further divided into several sub-blocks according to the intersection of the crack surface with the sub-quadrilateral. Considering that the crack propagation occurs in an arbitrary manner in a finite element solution domain during the analysis, there should be a series of intersection situations that may occur. In the following, the possible intersection situations are summarized in detail, and the division strategy of sub-quadrilaterals for each case is elaborated.

In Figure 3, the subdivision strategy of the sub-quadrilaterals with a crack tip is presented. First of all, the subdivision for three simple intersection situations (the crack tip touches the boundary of a sub-quadrilateral but not penetrate it) is shown in Figure 3a–c.

A sub-quadrilateral is subdivided into four to six triangles with the crack tip as a common vertex if the crack tip lies in the sub-quadrilateral but not on the boundary, as illustrated in Figure 3d–g. In order to integrate the function on a triangle within a prescribed accuracy, a mapping scheme with a two-step coordinate transform is adopted in this study, as depicted in Figure 4. Each triangle is firstly mapped on a reference isosceles right-angled triangle; then, this reference triangle is further mapped on a reference square with a side of length 2 by Duffy coordinate transform [52].

A sub-quadrilateral is further divided into a set of quadrilaterals; if it is penetrated by a straight crack and the crack tip lies on the boundary, see Figure 3h–l. That is, a triangle is further divided into three sub-quadrilaterals by connecting the midpoint of each side and the centroid of the triangle.

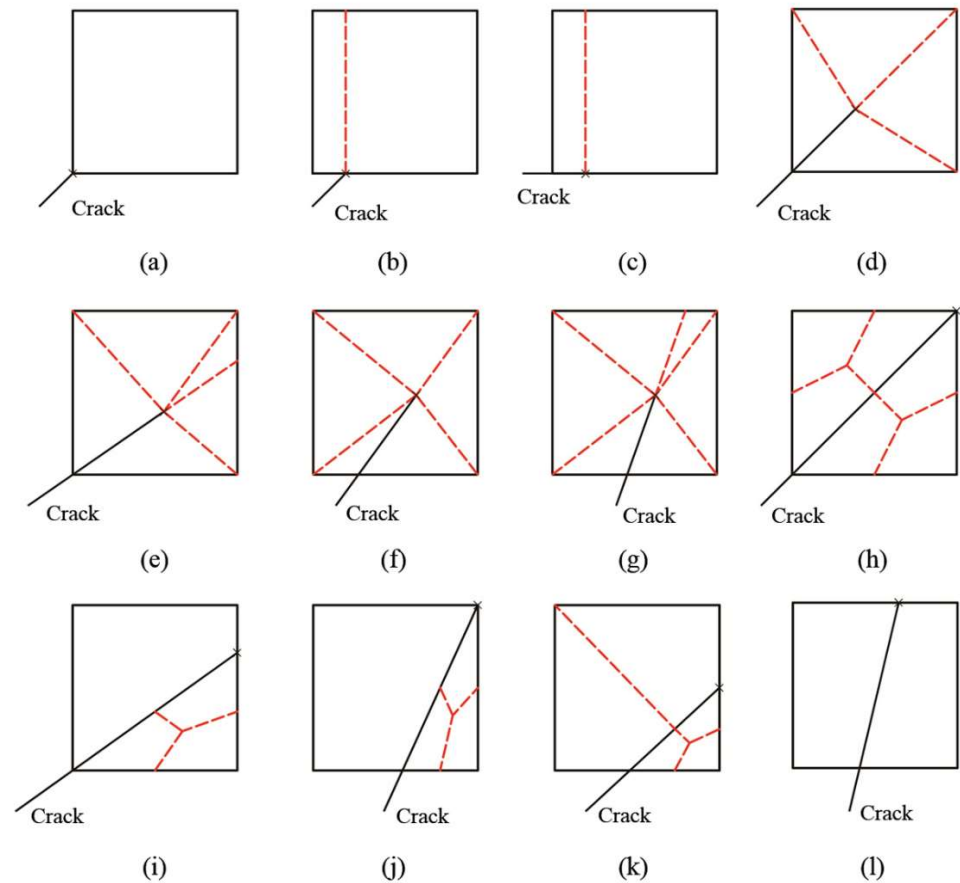


Figure 3. Subdivision of sub-quadrilaterals with a crack tip.

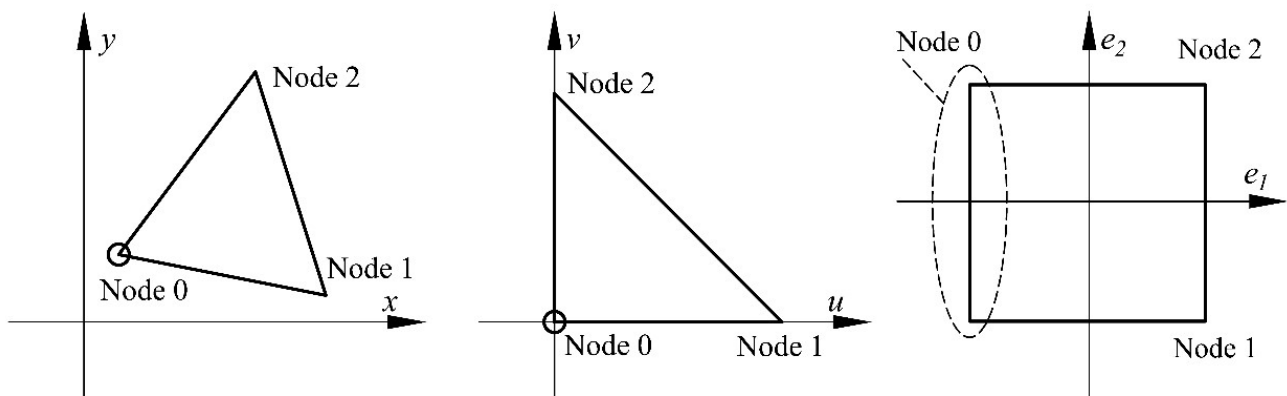


Figure 4. Mapping scheme.

For a sub-quadrilateral that is penetrated by the crack but does not contain the crack tip, the subdivision strategy is illustrated in Figure 5. Similar to crack tip cases, two simple division situations without an actual division needed are firstly presented in Figure 5a,b. In Figure 5c–e, the sub-quadrilaterals containing a crack tip before (Figure 3d–g) have been cut through by the propagating crack. Similar to the division strategy shown in Figure 3d–g, these sub-quadrilaterals are also divided into a set of triangles for consistency, and the same mapping scheme is also adopted for each triangle. Following this, a sub-quadrilateral penetrated by a straight crack is divided into several quadrilaterals (see Figure 5f–i) in the same manner as the division strategy shown in Figure 3h–l.

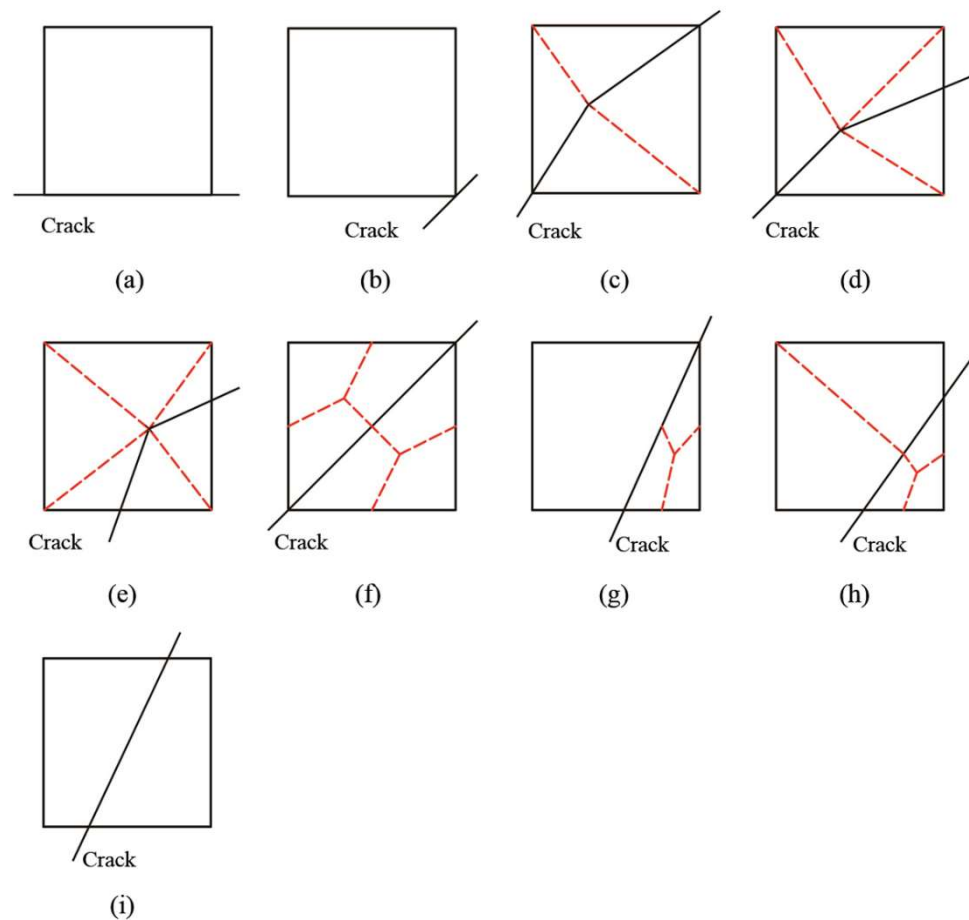


Figure 5. Subdivision of sub-quadrilaterals penetrated by the crack.

With the above strategy, sub-quadrilaterals cut by the cracks are divided into several simple geometric figures (triangles and quadrilaterals), and the location and number of integration points are then determined. It should be mentioned that the subdivision information is not stored in the procedure since the intersection situations for the subdivision may vary during the numerical analysis; it is calculated when required.

3.5. Constitutive Relations

In order to precisely model the nonlinear behavior of masonry-infilled RC frame structures, the rational bulk material constitutive relation (Equation (12)) and discrete interface constitutive relation (Equation (13)) are required to properly consider the distributed crushing and cracking behavior of the concrete and masonry blocks. For this purpose, two of constitutive relation subroutines called “umatmises” and “umatinterface” are built.

Firstly, an elastic-plastic constitutive model called J2-plasticity model [53] is employed in this study to model the crushing process of concrete and masonry blocks, as depicted in Figure 6. In this model, a yield criterion combining the von Mises yield criterion and tension cutoff criterion are adopted, as displayed in Figure 6a. The von Mises yield criterion can be written in the following format:

$$J_2 - \frac{1}{3}\sigma_e^2(\epsilon_p) = \frac{1}{2}s_{ij}s_{ij} - \frac{1}{3}\sigma_e^2(\epsilon_p) = 0 \quad (27)$$

where J_2 is second invariant of the deviatoric stress. For plane stress state, $\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$. Therefore, the second invariant of the deviatoric stress tensor can be expressed as [54]:

$$J_2 = s_{ij}s_{ij} = \sigma^T P \sigma \quad (28)$$

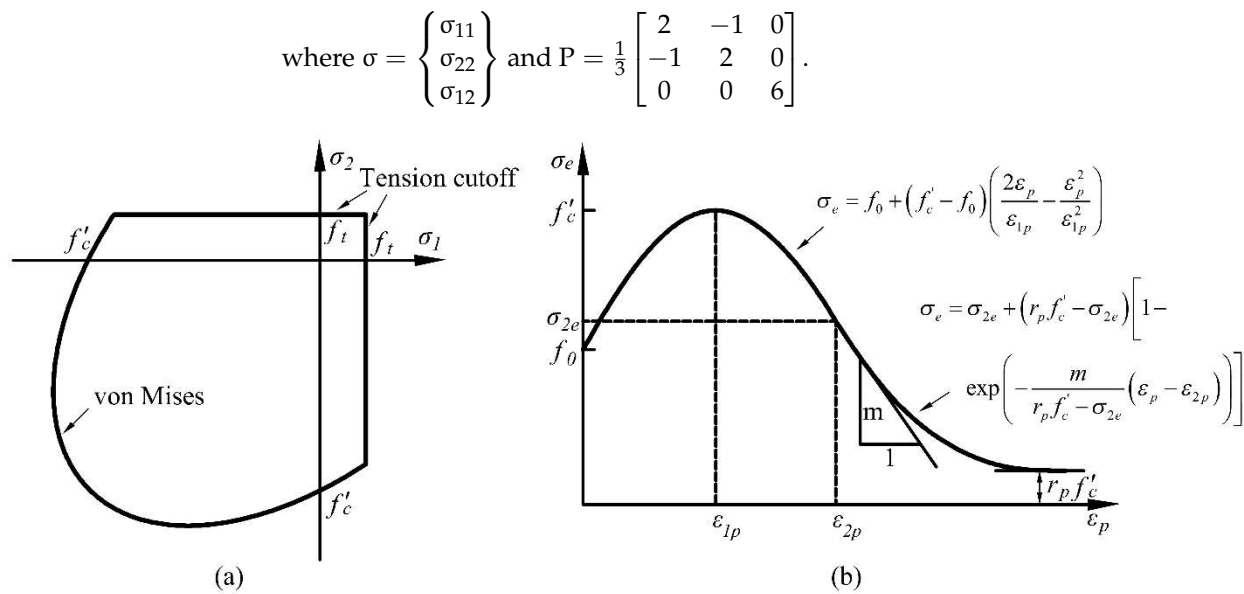


Figure 6. J_2 -plasticity model. (a) yield criterion (b) effective stress-plastic strain relationship.

Moreover, σ_e is effective stress and ϵ_p is effective plastic strain. The effective stress is a function of the effective plastic strain, which governs the isotropic strain hardening/softening law. The function is described as a parabolic function followed by an exponential decay function, which is determined by the material parameters f_0, f_c, ϵ_{1p} and ϵ_{2p} , as shown in Figure 6b.

In this analysis, the implementation of the constitutive relation is in the rate form, and it can be described as follows: upon discretization in time, the stress vector and effective plastic strain in the current step are given; then in the next step, a new stress vector and effective plastic strain are updated in a prescribed strain rate. To be specific, elastic predictor stress vector is firstly obtained through the current stress vector, stiffness matrix and strain rate. Together with the current effective plastic strain, the value of the yield function is then calculated. If the value is negative, the elastic predictor stress vector is the new admissible stress vector for the next step; or an iteration procedure is preformed to correct the predictor stress vector and effective plastic strain, until the yield function is equal to zero within a permissible tolerance. Finally, the stress vector and effective plastic strain at the next step are updated.

The discrete interface constitutive relation in Equation (13) usually plays an important role in modeling the failure behavior of masonry-infilled frames due to the weak performance of cracks. In this study, a precise discrete constitutive relation [28,36,38,55] is employed to model the cracking behavior of the concrete and masonry blocks, and it is able to simulate the degradation of tensile strength and tensile stiffness, normal compaction, shear dilatation and decreases in friction surface roughness under combined normal and shear tractions.

The interface displacement discontinuity w is decomposed into three displacement modes, i.e., the conventional elastic and plastic displacements, as well as a geometric part to account for the reversible dilatation caused by the wedging effect of asperity surfaces. A hyperbolic yield criterion [28] is adopted to deal with the yielding of the crack surface in both tension-shear and compression-shear regions. It can be expressed in the following format:

$$F(\sigma, q) = \tau^2 - \mu^2(\sigma - s)^2 + 2r(\sigma - s) = 0 \quad (29)$$

where s is material tensile strength, r is the radius of curvature at the vertex, and μ is the slope of asymptotes of the hyperbola, which represents the frictional coefficient of the crack. These three parameters govern the state of yield surfaces, and can be combined into a state vector q . In addition, a non-associated flow rule, in which two different forms are adopted

according to the normal tensile or compressive stress state, is used in this study to improve the numerical stability [56].

Similarly, an elastic predictor-plastic corrector method is also adopted here to update the new stress vector and state vector q for a given stress vector, state vector q at the current step, and interface displacement increment dw for the next step. Some details of the discrete interface constitutive relation can be found in [28,56,57].

3.6. Assembly Procedures

When implementing the user element program in the environment of XFEM, the element partitioning discussed in Section 3.4 is executed once the ISW = 3 and 6 tasks are activated. After the location of integration points is obtained according to the appropriate subdivision, a loop over all the integration points of each element is specified to compute the element tangent matrix and residual vector of the continuous part. The standard shape functions and their derivatives at each integration point are firstly calculated. If the modified Heaviside enrichment is activated for at least one node, the value of $H(f(x))$ function at each integration point is computed. In addition, the values of $H(f(x))$ at element nodes are also computed since the shifted-based enrichment is used. Analogously, for the activated crack tip enrichment, the values of crack tip functions $\Phi_1(x)$ and their derivatives at each integration point, along with the nodal values of $\Phi_1(x)$ are calculated. With these values, the strain-displacement element matrix B is subsequently constructed. The stiffness matrix and residual vector for the continuous part are then calculated.

Moreover, the stiffness and residual contributions from the discontinuity surface are also needed, once a crack has occurred in the element. For each segment of crack surfaces, the shape function matrix N_{coh} and transformation matrix M at each Gauss integration point are calculated according to Equations (14) and (21). Subsequently, the stiffness matrix and residual vector for the discontinuity surface are obtained.

3.7. Output for Post-Processing

During the analysis, it is necessary to output the element variables, nodal projections and the crack information into an output file for post-processing, and this can be realized under the ISW = 4 and 8 tasks in the user element subroutine of FEAP. In this study, the subdivision with 16 uniform sub-quadrilaterals is used for storing element variables, and command “Stress” is used to output the corresponding information. Meanwhile, commands “Displacement” and “Reaction” are also adopted to store the nodal displacements and nodal reactions into the output file. With these data, a post-processing program for load capacity and damage patterns is built to investigate the failure process of masonry-infilled RC frames.

Alternatively, some post-processing software can be used to obtain the same failure results. For this, however, the instructions for the specific adopted software must be strictly followed when writing the output file, which also requires an additional program to modify the format of the output file.

In addition, it should be mentioned that the output commands do not have to be executed in each solution step. In fact, these commands are executed every ten steps in this study to prevent an oversized output file.

3.8. Crack-Propagation Strategy

FEAP allows users to add their own macro commands to realize some specific functions, which are performed by adding a subroutine called “UMACRn”. In this study, a macro command with name “CKPG” is built to check the element crack-propagation criterion and to update the crack and enrichment information, and its implementation process is shown in Figure 7.

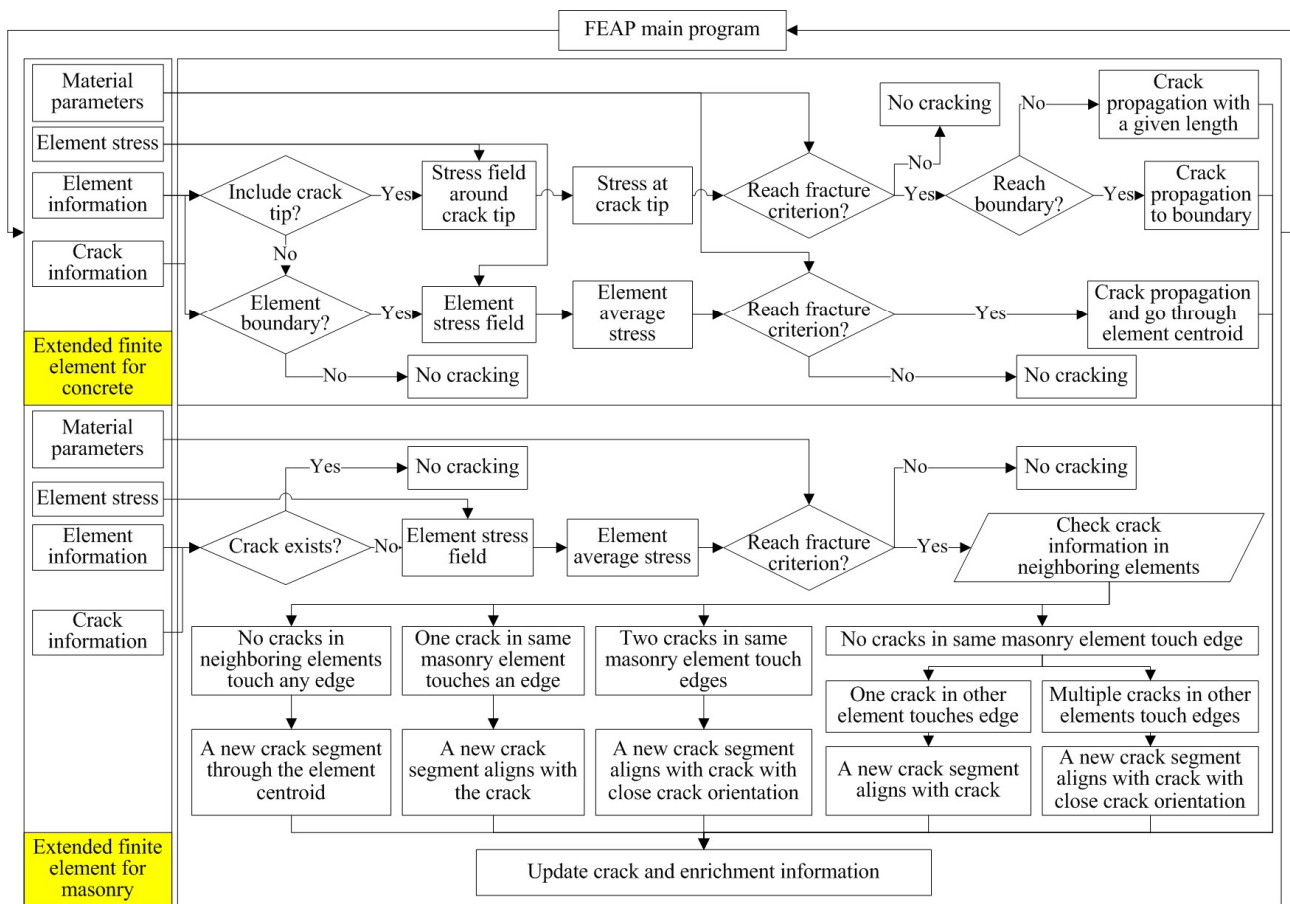


Figure 7. Implementation process of macro command “CKPG”.

In previous literature, only one or a few cracks are allowed to occur, and the starting position of cracks is usually prescribed before carrying out the numerical analysis. Some good results have been achieved in these numerical analyses; however, it will become much more difficult to ensure cracks propagate properly in real structures, since a large number of cracks may occur and affect each other. Therefore, an appropriate crack-propagation strategy is proposed in this study, so that the crack can propagate in a rational way.

In this study, the maximum principal stress criterion is adopted as the fracture criteria, which means a new crack segment normal to the major principal stress direction is specified if the major principal stress reaches the tensile strength. For a real structure, however, if a new crack segment occurs once the fracture criterion is satisfied, there could be numerous distributed cracks occurring. This may lead to a locking problem if these cracks cannot be properly connected. In this study, a new crack segment is allowed to occur only if it starts at the free edge of a finite element mesh, or it is an extension of the existing cracks. It should be mentioned that the propagation strategies used for concrete and blocks are different due to the difference in the enrichment of the two materials.

For concrete members, two steps are included. Firstly, the elements containing the crack tip are checked. A new crack segment with a given length is added if the stress around the crack tip reaches the fracture criterion. Next, the elements located on the boundary of the mesh and without enriched degrees of freedom activated are checked. A new crack segment that passes through the element centroid is inserted if the average element stress is beyond the tensile strength of concrete.

For masonry blocks, one crack segment can pass through one element at one time. At each loading step, the fracture criterion is checked within every finite element to determine if a crack segment may initiate. If so, the direction of this potential crack segment is also calculated. Following this, the crack information of neighboring elements is checked. If

there are no cracks that touch any edges of the current element, the crack segment is simply placed through the centroid of the current element. For elements in the same block with the current element, if only one crack in the neighboring elements touches an edge of the current element, the new crack segment is forced to align with the crack of the neighboring element; if there are two cracks in two neighboring elements that could potentially be aligned with the new crack segment, the new crack is then aligned with the one of the neighboring element whose angle of orientation is closest to that of the new crack. If no cracks in the neighboring elements could be aligned with the crack in the current element of the same block, a similar method is used to align the crack of the current element with the crack of those elements in the neighboring block; please see Figure 7 for the implementation process.

As the crack propagates, the information for the cracks, enrichment and the crack tips is updated and stored in the corresponding locations of the element history variables, as described in Section 3.3.

4. Numerical Verification

Two full-scale one-story one-bay RC frame specimens with and without infill walls are utilized to demonstrate the effectiveness of the proposed modeling [58]. The geometry and design information for test specimens is displayed in Figure 8. In the test, the concrete hollow block of $390 \times 190 \times 190 \text{ mm}^3$ is used to construct the infill wall. The thickness of the bed and head joints is 10 mm. A vertical compressive load of 700 kN is firstly applied on the top of two columns and is kept constant during the test. Following this, a series of lateral reversed cyclic loads are applied on the two ends of the beam.

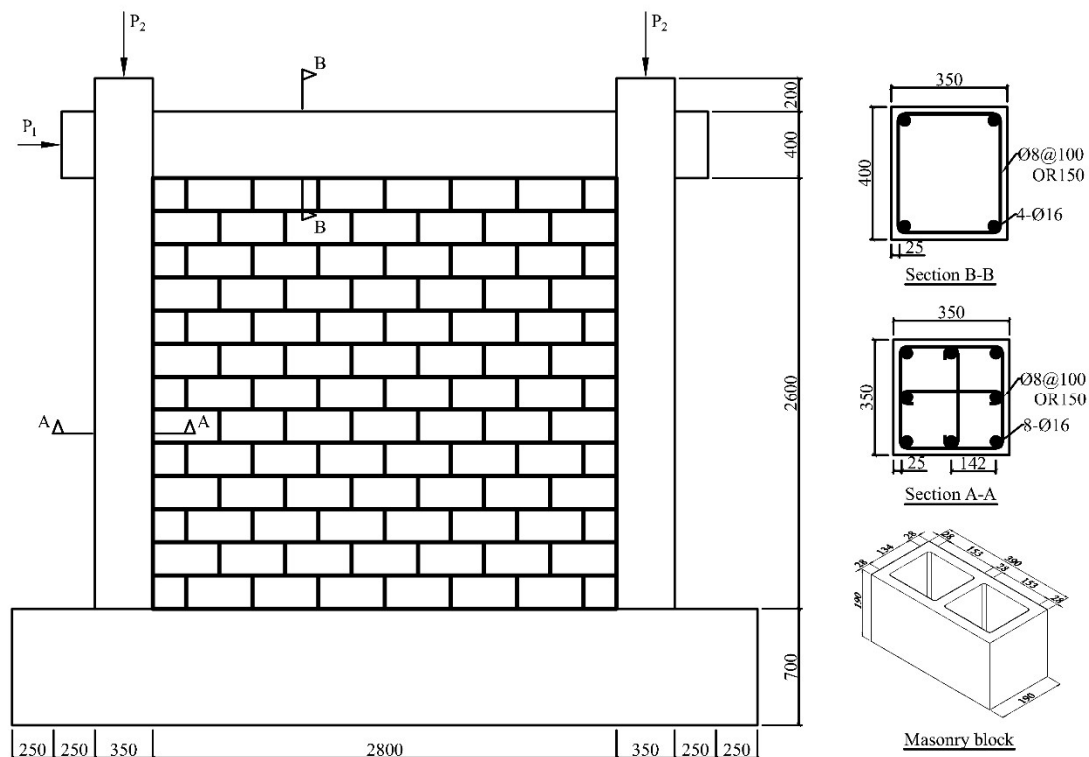


Figure 8. Geometry and design information of lightweight masonry-infilled frames (unit: mm).

Finite element modeling for two specimens are created in FEAP by inputting a series of parameters, such as dimensions, numbers of the reinforcement and material properties, into the programed modeling routine. The thickness of the infill panel is chosen to be two face shells of 56 mm thickness. The base of the model is fixed in all directions. The material parameters for the compressive behavior of the concrete members and masonry blocks are

summarized in Table 1. The input parameters of the discrete cracks' constitutive relations used for concrete cracks, masonry cracks and mortar joints are listed in Table 2.

Table 1. Parameters for compressive behavior of lightweight masonry-infilled frames.

	Specimen	E , GPa	f_c , MPa	ε_{1p}	ε_{2p}
Concrete members	Bare frame	20.5	27.53	0.0019	0.005
	Infilled frame				
Masonry blocks	Infilled frame	0.568	1.90	0.002	0.0025

Table 2. Parameters for discrete constitutive relations of lightweight masonry-infilled frames.

	Specimen	D_{nn} , GPa/m	D_{tt} , GPa/m	$s_0(f_t)$, MPa	μ_0	μ_r	r_0 , MPa	r_r , MPa
Concrete cracks	Bare frame	2100	2700	2.75	0.95	0.75	0.14	0.035
	Infilled frame							
Bed joints	Infilled frame	85	80	0.15	0.8	0.7	0.14	0.035
Head joints	Infilled frame	85	80	0.05	0.7	0.6	0.12	0.025
Frame/infill interface	Infilled frame	85	80	0.13	0.7	0.6	0.12	0.025
Masonry cracks	Infilled frame	180	220	0.3	0.95	0.8	0.14	0.035
	Specimen	G_f^I , N/m	η	α , m/kN	β , m/kN	$\zeta_{dil,0}$	$\zeta_{dil,r}$	d_0 , m
Concrete cracks	Bare frame	190	300	11.5	12.5	0.63	0.1	0.002
	Infilled frame							
Bed joints	Infilled frame	35	300	11.5	12.5	0.1	0.001	0.001
Head joints	Infilled frame	23	300	11.5	12.5	0.1	0.001	0.001
Frame/infill interface	Infilled frame	23	300	11.5	12.5	0.1	0.001	0.001
Masonry cracks	Infilled frame	105	350	11.5	12.5	0.1	0.001	0.001

To properly model the behavior of masonry-infilled RC frames, there is need of a systematic calibration process to define various parameters for the proposed finite element modeling. The compressive parameters of concrete material are determined with the data from compression cylinder tests. The tensile strength, f_t , can be obtained from the tensile tests, or estimated from the compressive strength according to the suggestion of design codes. For the cohesion-softening behavior, the mode-I fracture energy G_f^I can be obtained from mode-I fracture tests, and the mode-II fracture energy G_f^{II} is assumed to be ten times G_f^I . The material parameters controlling the evolution of the shape of the failure surface, as well as the dilatation parameters, are determined according to the recommendations in [38,56].

The parameters of the interface elements for mortar joints in the infill wall are usually calibrated through a series of shear tests of mortar joints under different constant compressive stresses. The state parameters of the initial yielding and final failure surfaces can be determined by nonlinear fitting analyses using several groups of compressive stress and shear stress values. The mode-II fracture energy is obtained by continuously adjusting its value until the numerical shear stress displacement curves coincide with the experimental curves, and the mode-I fracture energy is then evaluated. Moreover, the dilatation parameters are calibrated by adjusting the analytical normal displacement-shear displacement curves to match the experimental results. In addition, the material properties of head joints and the frame-to-wall joints are assumed to be weaker than that of the bed joints due to the partial filling and the general poorer condition of these mortar joints.

The compressive parameters of masonry unit elements are calibrated with masonry prism tests to reflect the composite properties of a masonry assembly. The parameters for tensile and shear behaviors of the masonry units are calibrated in the same way as those for the concrete material.

The bare frame is firstly analyzed to verify the capability of the proposed modeling with regard to bare RC frames, and the analytical lateral load-bearing capacity curve for the bare frame, are compared with the envelope curve of hysteresis loops in the experiment, as shown in Figure 9. It is concluded that the analysis predicts the experimental curve very well, with only the initial stiffness being overestimated.

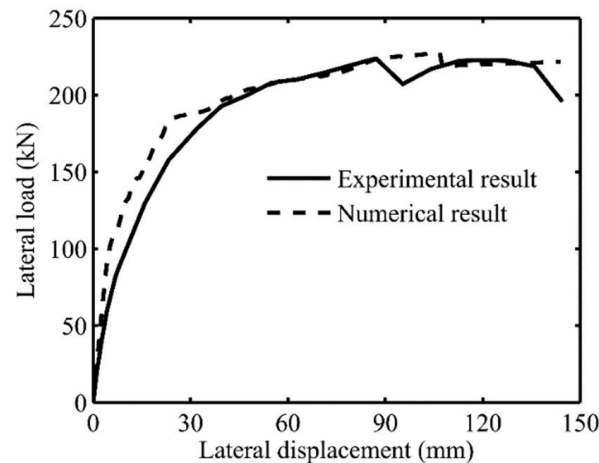


Figure 9. Lateral load-bearing capacity curves for the bare frame.

Figure 10 displays the lateral load-bearing capacity curve of the infilled frame for the analytical and experimental results. It is shown that the relation at the ascend stage till the ultimate lateral load is well captured by the analysis, whereas a rapid decline from the post-peak point is not precisely predicted. This is because an out-of-plane failure at the upper part of the infill panel was observed at the post peak stage in the experiment, while it is not considered in the 2D finite element analysis.

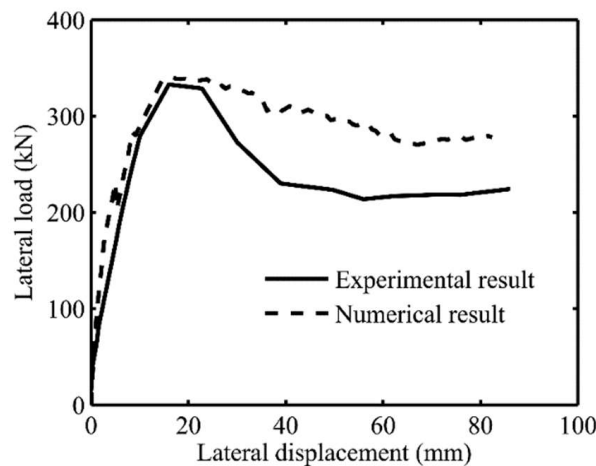


Figure 10. Lateral load-bearing capacity curves for the infilled frame.

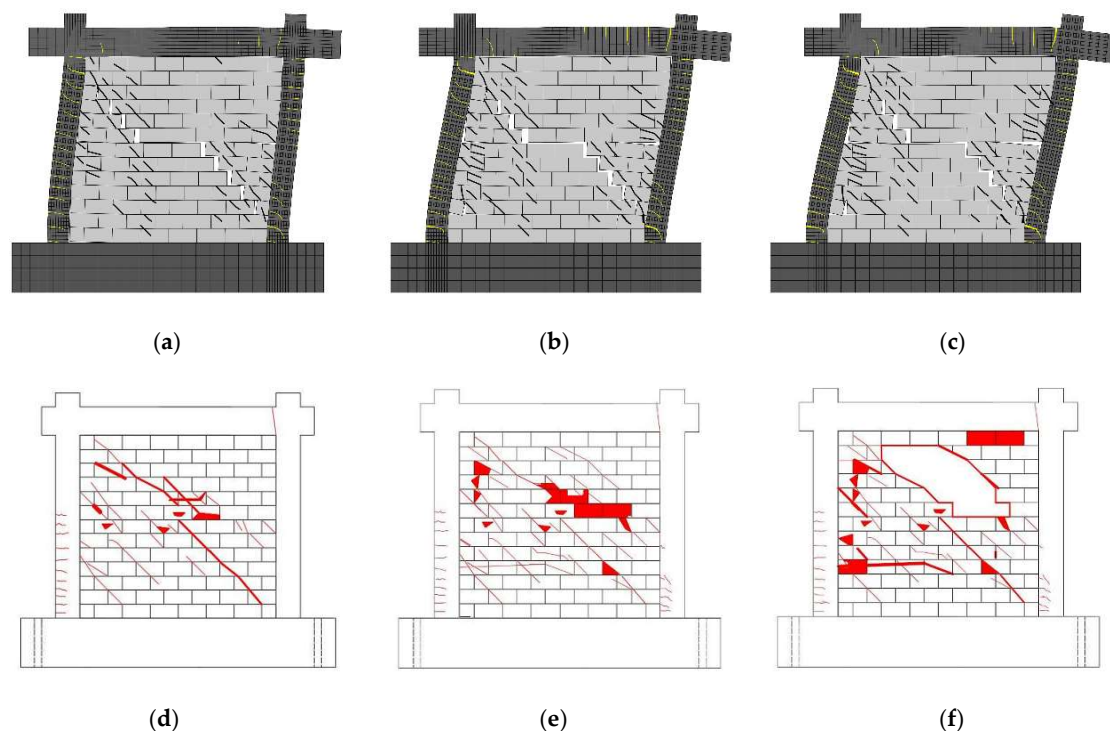
More specifically, the differences of peak load, initial stiffness, and displacement corresponding to the peak load for the numerical and experimental results of two specimens are presented in Table 3. It can be seen that the comparison of three parameters also confirms the results observed in the curves.

Table 3. Comparison of experimental and numerical results. (unit: kN and mm).

	Bare Frame			Infilled Frame		
	Peak Load	Peak Dis- placement	Initial Stiffness	Peak Load	Peak Dis- placement	Initial Stiffness
Experimental result	223.72	87.10	16.80	332.88	15.99	77.18
Numerical result	227.56	106.70	22.07	340.05	16.10	90.78
Relative error (%)	1.72	22.50	31.37	2.15	0.69	17.62

Next, the damage patterns for two specimens are discussed. For the bare frame specimen, the numerical result exhibits the damage pattern very well, successfully showing the flexural yielding at the ends of columns and beam, as well as the plastic hinges at the bottom of columns and at the two ends of the beam in the late stage of the test. For conciseness, the deformed mesh is not exhibited.

The damage patterns of the infilled frame for analytical and experimental results are compared in Figure 11, and a reasonable prediction result is obtained by the numerical analysis. In both results, the damage to the columns and the beam is characterized by a series of flexural cracks, and the inflection point of columns derived from the crack distribution is located at the middle height. In the infill panel, some inclined cracks that start from the top-left corner and bottom-right corner are well captured by the analysis. As the load increases, the cracks and deformation gradually increase, and expand to the bottom-left corner and top-right corner. Finally, a diagonal compressive failure is formed, and a large number of cracks and larger deformation of masonry blocks are observed, which shows the serious damage that has occurred in the infill panel. However, the crack distribution at the top of columns predicted by the analysis is not clearly observed in the experiment, which may be due to the out-of-plane failure, which dramatically reduces the damage to columns.

**Figure 11.** Numerical and experimental damage patterns for the infilled frame. Lateral displacement: (a,d) 2.4 cm, (b,e) 4.0 cm, (c,f) 4.8 cm.

5. Conclusions

In this paper, we presented the implementation procedure of finite element modeling within FEAP for the numerical analysis of masonry-infilled RC frames. In this modeling, XFEM is combined with interface elements to predict the complicated failure process of masonry-infilled RC frames, such as diagonal cracking, sliding and crushing in the infill wall, as well as the flexural and shear failure in RC frame members. The implementation is based on a user element subroutine; two user element subroutines with and without crack tip enrichment functions included within XFEM are built to model the nonlinear behavior of the concrete in RC frame members and the masonry blocks in the infill panel, respectively. Additionally, a macro command used to check the crack-propagation criterion and update the crack, and enrichment information is added to assist in the implementation of XFEM. Some critical and difficult issues related to the selection of enriched nodal degrees of freedom, input material parameters, information passing, numerical integration strategy, constitutive relations, assembly procedure of stiffness matrix and force vector, output for post-processing, and crack-propagation strategy are addressed.

A quasi-static test of two RC frames with and without infill walls is analyzed to verify the capability of the proposed finite element modeling. It revealed that the relative errors of peak load, displacement corresponding to peak load and initial stiffness for the two specimens are relatively small, and the load-bearing capacity curves are accurately predicted. Meanwhile, the failure process of the test specimens is well represented, especially for the cracking propagation in the concrete material and masonry units. These findings sufficiently demonstrate the efficacy of the XFEM implementation in FEAP and the effectiveness of the proposed modeling for representing the complicated behavior of masonry-infilled RC frames.

Further parametric studies using the proposed modeling can better understand the behavior of masonry-infilled RC frame buildings, which will contribute to the development of valuable seismic design advice and simplified approaches. Moreover, the proposed modeling is an effort to solve the fracture problems of practical structures using XFEM. The description of the implementation procedure will help readers to apply XFEM for a broader class of problems. The uncertainties in the mechanical properties of masonry units and mortar joints has a great impact on the performance of the infilled wall RC frame, and it is not considered in this study. In future research, the spatial variation in material properties in infill walls can be considered in the proposed finite element model to better understand the failure behavior of infilled wall RC frame structures.

Author Contributions: Conceptualization, B.L., C.L. and X.W.; methodology, B.L., X.W. and J.K.; software, B.L., X.W., J.K. and Z.C.; validation, B.L., C.L. and J.K.; formal analysis, B.L., C.L. and X.W.; investigation, Z.C.; resources, C.L., X.W., J.K. and Z.C.; data curation, X.W. and Z.C.; writing—original draft preparation, B.L., C.L. and X.W.; writing—review and editing, B.L., C.L., J.K. and Z.C.; visualization, B.L., C.L. and J.K.; supervision, Z.C.; project administration, J.K.; funding acquisition, X.W., C.L. and J.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by National Nature Science Foundation of China (Nos. 51808478), and Natural Science Foundation of Heilongjiang Province of China (Nos. LH2022E122). These supports are greatly appreciated.

Data Availability Statement: Not applicable.

Acknowledgments: Grateful acknowledgment is given to Maosheng Gong and Baofeng Zhou at Institute of Engineering Mechanics, China Earthquake Administration for the guidance and suggestions on theoretical analyses.

Conflicts of Interest: The authors declare no conflict of interest.

References

- Shah, S.A.A.; Shahzada, K.; Samiullah, Q. Influence of Brick Masonry Infilled Wall on Seismic Performance of Reinforced Concrete Frame. *NED Univ. J. Res.* **2020**, *7*, 15–29. [\[CrossRef\]](#)
- Khan, N.; Monti, G.; Nuti, C.; Vailati, M. Effects of Infills in the Seismic Performance of an RC Factory Building in Pakistan. *Buildings* **2021**, *11*, 276. [\[CrossRef\]](#)
- Mazza, F.; Donnici, A. In-plane-out-of-plane single and mutual interaction of masonry infills in the nonlinear seismic analysis of RC framed structures. *Eng. Struct.* **2022**, *257*, 114076. [\[CrossRef\]](#)
- Liauw, T.; Lo, C. Multibay infilled frames without shear connectors. *Struct. J.* **1988**, *85*, 423–428. [\[CrossRef\]](#)
- Gambarotta, L.; Lagomarsino, S. Damage Models for the Seismic Response of Brick Masonry Shear Walls. Part II: The Continuum Model and Its Applications. *Earthq. Eng. Struct. Dyn.* **1997**, *26*, 441–462. [\[CrossRef\]](#)
- Asteris, P.G. Lateral Stiffness of Brick Masonry Infilled Plane Frames. *J. Struct. Eng.* **2003**, *129*, 1071–1079. [\[CrossRef\]](#)
- Asteris, P.G. Finite Element Micro-Modeling of Infilled Frames. *Electron. J. Struct. Eng.* **2008**, *8*, 11. [\[CrossRef\]](#)
- Noor-E-Khuda, S.; Dhanasekar, M.; Thambiratnam, D.P. An explicit finite element modelling method for masonry walls under out-of-plane loading. *Eng. Struct.* **2016**, *113*, 103–120. [\[CrossRef\]](#)
- Milanesi, R.R.; Morandi, P.; Magenes, G. Local effects on RC frames induced by AAC masonry infills through FEM simulation of in-plane tests. *Bull. Earthq. Eng.* **2018**, *16*, 4053–4080. [\[CrossRef\]](#)
- Georgiou, E.S.; Kyriakides, N.C.; Chrysostomou, C.Z.; Kotronis, P.; Filippou, C.A. Numerical simulation of the experimental results of the seismic strengthening of existing structures. In Proceedings of the 16ECEE Conference 2018, Thessaloniki, Greece, 18–21 June 2018.
- Albu-Jasim, Q.; Medina-Cetina, Z.; Muliana, A. Calibration of a concrete damage plasticity model used to simulate the material components of unreinforced masonry reinforced concrete infill frames. *Mater. Struct.* **2022**, *55*, 36. [\[CrossRef\]](#)
- Georgiou, E.; Kyriakides, N.; Chrysostomou, C.Z. Numerical simulation of RC frames infilled with RC walls for seismic strengthening of existing structures. *Bull. Earthq. Eng.* **2022**, *20*, 2369–2398. [\[CrossRef\]](#)
- Franchi, A.; Napoli, P.; Crespi, P.; Giordano, N.; Zucca, M. Unloading and Reloading Process for the Earthquake Damage Repair of Ancient Masonry Columns: The Case of the Basilica di Collemaggio. *Int. J. Archit. Herit.* **2021**, *16*, 1683–1698. [\[CrossRef\]](#)
- Micelli, F.; Cascardi, A. Structural assessment and seismic analysis of a 14th century masonry tower. *Eng. Fail. Anal.* **2020**, *107*, 104198. [\[CrossRef\]](#)
- Ngo, D.; Scordelis, A.C. Finite Element Analysis of Reinforced Concrete Beams. *ACI J. Proc.* **1967**, *64*, 152–163. [\[CrossRef\]](#)
- Rashid, Y. Analysis of prestressed concrete pressure vessels. *Nucl. Eng. Des.* **1968**, *7*, 265–286. [\[CrossRef\]](#)
- Cope, R.J.; Rao, P.V.; Clark, L.A.; Norris, P. Modelling of reinforced concrete behaviour for finite element analysis of bridge slabs. *Numer. Method Nonlinear Probl.* **1980**, *1*, 457–470.
- Gupta, A.K.; Akbar, H. Cracking in Reinforced Concrete Analysis. *J. Struct. Eng.* **1984**, *110*, 1735–1746. [\[CrossRef\]](#)
- Willam, K.; Pramono, E.; Sture, S. Fundamental Issues of Smeared Crack Models. In Proceedings of the Fracture of Concrete and Rock, New York, NY, USA, 1989; pp. 142–157.
- Berto, L.; Saetta, A.; Scotta, R.; Vitaliani, R. An orthotropic damage model for masonry structures. *Int. J. Numer. Methods Eng.* **2002**, *55*, 127–157. [\[CrossRef\]](#)
- Casolo, S. Rigid element model for non-linear analysis of masonry façades subjected to out-of-plane loading. *Commun. Numer. Methods Eng.* **1999**, *15*, 457–468. [\[CrossRef\]](#)
- Lourêço, P.B.; De Borst, R.; Rots, J.G. A plane stress softening plasticity model for orthotropic materials. *Int. J. Numer. Methods Eng.* **1997**, *40*, 4033–4057. [\[CrossRef\]](#)
- Lourêço, P.B.; Rots, J.G. Multisurface Interface Model for Analysis of Masonry Structures. *J. Eng. Mech.* **1997**, *123*, 660–668. [\[CrossRef\]](#)
- Attard, M.M.; Nappi, A.; Tin-Loi, F. Modeling Fracture in Masonry. *J. Struct. Eng.* **2007**, *133*, 1385–1392. [\[CrossRef\]](#)
- Stavridis, A.; Shing, P.B. Finite-Element Modeling of Nonlinear Behavior of Masonry-Infilled RC Frames. *J. Struct. Eng.* **2010**, *136*, 285–296. [\[CrossRef\]](#)
- Mohyeddin, A.; Goldsworthy, H.M.; Gad, E.F. FE modelling of RC frames with masonry infill panels under in-plane and out-of-plane loading. *Eng. Struct.* **2013**, *51*, 73–87. [\[CrossRef\]](#)
- Pashaie, M.R.; Mohammadi, M. Estimating the local and global effects of infills on steel frames by an improved macro-model. *Eng. Struct.* **2019**, *187*, 120–132. [\[CrossRef\]](#)
- Lotfi, H.R.; Shing, P.B. An appraisal of smeared crack models for masonry shear wall analysis. *Comput. Struct.* **1991**, *41*, 413–425. [\[CrossRef\]](#)
- Zeng, B.; Li, Y.; Cruz Noguez, C. Modeling and parameter importance investigation for simulating in-plane and out-of-plane behaviors of un-reinforced masonry walls. *Eng. Struct.* **2021**, *248*, 113233. [\[CrossRef\]](#)
- Wambacq, J.; Ulloa, J.; Lombaert, G.; François, S. A variationally coupled phase field and interface model for fracture in masonry. *Comput. Struct.* **2022**, *264*, 106744. [\[CrossRef\]](#)
- Milani, G. Simple homogenization model for the non-linear analysis of in-plane loaded masonry walls. *Comput. Struct.* **2011**, *89*, 1586–1601. [\[CrossRef\]](#)
- Pirsaheb, H.; Javad Moradi, M.; Milani, G. A Multi-Pier MP procedure for the non-linear analysis of in-plane loaded masonry walls. *Eng. Struct.* **2020**, *212*, 110534. [\[CrossRef\]](#)

33. Pirsaeheb, H.; Javad Moradi, M.; Milani, G. A Multi-Pier MP method for the non-linear static analysis of out-of-plane loaded masonry walls. *Eng. Struct.* **2020**, *223*, 111040. [\[CrossRef\]](#)
34. Khaleghi, M.; Salimi, J.; Farhangi, V.; Moradi, M.J.; Karakouzian, M. Application of Artificial Neural Network to Predict Load Bearing Capacity and Stiffness of Perforated Masonry Walls. *CivilEng* **2021**, *2*, 48–67. [\[CrossRef\]](#)
35. Khaleghi, M.; Salimi, J.; Farhangi, V.; Moradi, M.J.; Karakouzian, M. Evaluating the behaviour of centrally perforated unreinforced masonry walls: Applications of numerical analysis, machine learning, and stochastic methods. *Ain Shams Eng. J.* **2022**, *13*, 101631. [\[CrossRef\]](#)
36. Mehrabi, A.B.; Shing, P.B. Finite Element Modeling of Masonry-Infilled RC Frames. *J. Struct. Eng.* **1997**, *123*, 604–613. [\[CrossRef\]](#)
37. Shing, P.B.; Spencer, B. Modeling of Shear Behavior of RC Bridge Structures. In *Modeling of Inelastic Behavior of RC Structures Under Seismic Loads*; ASCE Publications: Reston, VA, USA, 1999; pp. 315–333.
38. Zhai, C.; Wang, X.; Kong, J.; Li, S.; Xie, L. Numerical Simulation of Masonry-Infilled RC Frames Using XFEM. *J. Struct. Eng.* **2017**, *143*, 04017144. [\[CrossRef\]](#)
39. Zienkiewicz, O.C.; Taylor, R.L.; Taylor, R.L.; Taylor, R.L. *The Finite Element Method: Solid Mechanics*; Butterworth-heinemann: Oxford, UK, 2000; Volume 2.
40. Sukumar, N.; Prévost, J.H. Modeling quasi-static crack growth with the extended finite element method Part I: Computer implementation. *Int. J. Solids Struct.* **2003**, *40*, 7513–7537. [\[CrossRef\]](#)
41. Bordas, S.; Nguyen, P.V.; Dunant, C.; Guidoum, A.; Nguyen-Dang, H. An extended finite element library. *Int. J. Numer. Methods Eng.* **2007**, *71*, 703–732. [\[CrossRef\]](#)
42. Abdelaziz, Y.; Hamouine, A. A survey of the extended finite element. *Comput. Struct.* **2008**, *86*, 1141–1151. [\[CrossRef\]](#)
43. Fries, T.-P.; Belytschko, T. The extended/generalized finite element method: An overview of the method and its applications. *Int. J. Numer. Methods Eng.* **2010**, *84*, 253–304. [\[CrossRef\]](#)
44. Jaśkowiec, J.; van der Meer, F.P. A consistent iterative scheme for 2D and 3D cohesive crack analysis in XFEM. *Comput. Struct.* **2014**, *136*, 98–107. [\[CrossRef\]](#)
45. Khoei, A.R. *Extended Finite Element Method: Theory and Applications*; Wiley: New York, NY, USA, 2015.
46. Béchet, E.; Minnebo, H.; Moës, N.; Burgardt, B. Improved implementation and robustness study of the X-FEM for stress analysis around cracks. *Int. J. Numer. Methods Eng.* **2005**, *64*, 1033–1056. [\[CrossRef\]](#)
47. Moës, N.; Belytschko, T. Extended finite element method for cohesive crack growth. *Eng. Fract. Mech.* **2002**, *69*, 813–833. [\[CrossRef\]](#)
48. Duarte, C.A.; Babuška, I.; Oden, J.T. Generalized finite element methods for three-dimensional structural mechanics problems. *Comput. Struct.* **2000**, *77*, 215–232. [\[CrossRef\]](#)
49. Wells, G.N.; Sluys, L.J. A new method for modelling cohesive cracks using finite elements. *Int. J. Numer. Methods Eng.* **2001**, *50*, 2667–2682. [\[CrossRef\]](#)
50. Zi, G.; Belytschko, T. New crack-tip elements for XFEM and applications to cohesive cracks. *Int. J. Numer. Methods Eng.* **2003**, *57*, 2221–2240. [\[CrossRef\]](#)
51. Elguedj, T.; Gravouil, A.; Combescure, A. Appropriate extended functions for X-FEM simulation of plastic fracture mechanics. *Comput. Methods Appl. Mech. Eng.* **2006**, *195*, 501–515. [\[CrossRef\]](#)
52. Duffy, M.G. Quadrature Over a Pyramid or Cube of Integrands with a Singularity at a Vertex. *SIAM J. Numer. Anal.* **1982**, *19*, 1260–1262. [\[CrossRef\]](#)
53. Lotfi, H.R.; Shing, P.B. Interface Model Applied to Fracture of Masonry Structures. *J. Struct. Eng. ASCE* **1994**, *120*, 63–80. [\[CrossRef\]](#)
54. Simo, J.C.; Taylor, R.L. A return mapping algorithm for plane stress elastoplasticity. *Int. J. Numer. Methods Eng.* **1986**, *22*, 649–670. [\[CrossRef\]](#)
55. Koutromanos, I.; Shing, P.B. Cohesive Crack Model to Simulate Cyclic Response of Concrete and Masonry Structures. *ACI Struct. J.* **2012**, *109*, 349–358. [\[CrossRef\]](#)
56. Koutromanos, I.; Stavridis, A.; Shing, P.B.; Willam, K. Numerical modeling of masonry-infilled RC frames subjected to seismic loads. *Comput. Struct.* **2011**, *89*, 1026–1037. [\[CrossRef\]](#)
57. Wang, X. Research on Numerical Simulation of Masonry-Infilled RC Frames Using XFEM. Ph.D. Thesis, Harbin Institute of Technology, Harbin, China, 2017.
58. Zhai, C.; Kong, J.; Wang, X.; Chen, Z. Experimental and Finite Element Analytical Investigation of Seismic Behavior of Full-Scale Masonry Infilled RC Frames. *J. Earthq. Eng.* **2016**, *20*, 1171–1198. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.