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Using System Reliability Concepts to Derive Partial Safety Factors for Punching Shear with Shear Reinforcement: An Explorative Analysis

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Abstract: Punching shear-reinforced flat slabs can fail in three modes: (1) inside the shear-reinforced zone, (2) outside the shear-reinforced zone, or (3) at the maximum punching shear capacity. By using the theory of structural systems reliability, this study investigates how these failure modes combined affect the safety level of reinforced concrete flat slabs with punching shear reinforcement, and ultimately, the variation of partial factors. Based on DIN EN 1992-1-1 together with the German National Annex DIN EN 1992-1-1+NA(D), and by using distinct reliability-based methods (i.e., levels II and III), the variation in the system failure probabilities and, consequently, partial factors were analysed. The results indicated that partial factors can fluctuate significantly depending on the governing ultimate limit state function and the scatter of basic variables. Furthermore, they indicated that the chosen reliability-based method had a significant influence on the resulting safety level. The sensitivity analysis confirmed the premise that partial factors can be reduced without compromising socially accepted safety levels (e.g., recommended in DIN EN 1990). Ultimate benefits include material and cost savings and CO_2 emission reductions. Finally, the approach addressed in this paper offers a new perspective on the derivation of partial factors that can be considered by engineering practitioners.

Keywords: systems reliability; RC flat slabs; punching shear; punching shear reinforcement; partial factors; reliability-based analysis

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1. Introduction

Reinforced concrete (RC) flat slabs are well-accepted structural solutions because they enable easy and flexible partitions of space compared with slabs with beams, leading to an easy way to simplify and speed up site operations (e.g., [1]). However, it is also recognised that in slab-column connections of RC flat slabs, shear stress concentrations can occur and lead to a sudden brittle punching failure. A punching shear failure may occur in regions surrounding the columns of RC flat slabs with a much lower load than that which could cause flexure failure [1]. Nonetheless, when addressed during the design phase, punching shear failure can be prevented by using drop panels, increasing the column head dimensions and/or using punching shear reinforcement (for example, through stirrups, bent-up bars or studs). It is generally accepted that adequate use of punching shear reinforcement can either improve the punching strength capacity or reach the flexural capacity of RC flat slabs.

Yet, providing that RC flat slabs have punching shear reinforcement, failure modes (or mechanisms) can still occur in any of the following situations (e.g., [1–3]):

(1) Failure inside the zone of punching shear reinforcement (i.e., failure in the surface crossing one or more rows of punching shear reinforcement);
(2) Failure outside the zone of punching shear reinforcement (i.e., failure in the surface located in the region outside the punching shear reinforcement elements);
(3) Failure at the maximum punching shear capacity (i.e., failure in the compression zone directly at the face of the column).

The typical crack formations of these punching shear failure modes, with and without punching shear reinforcement, are illustrated in Figure 1.

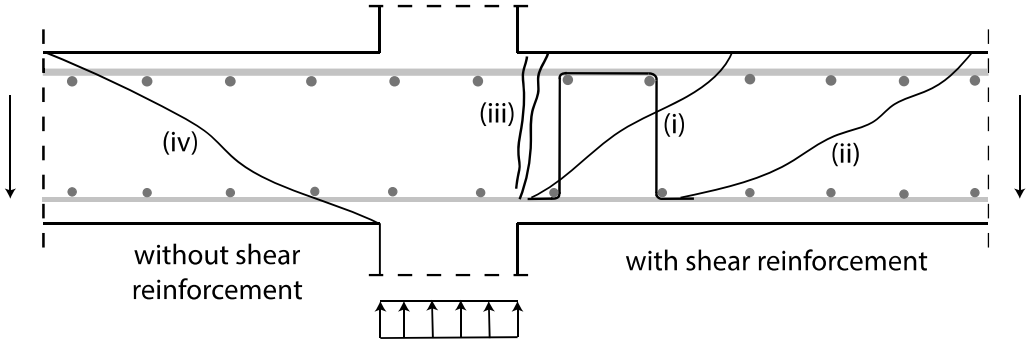


Figure 1. Scheme of the typical crack formations of punching shear failure modes: (**i**) inside and (**ii**) outside the zones of punching shear reinforcement, (**iii**) at maximum punching shear capacity and (**iv**) without punching shear reinforcement (adapted from [2]).

In Germany, the punching shear design provisions (with and without reinforcement) included in the European structural code DIN EN 1992-1-1 [4,5] are supplemented by the National Annex NA(D) [6,7]. In the following sections, this combination is abbreviated as DIN EN 1992-1-1+NA(D) [4–7].

If a low amount of punching shear reinforcement is installed and a shear crack appears, the punching shear reinforcement will be activated. Due to the low amount of reinforcement, it will start to yield immediately and the crack width cannot be controlled. This lack of control can lead to immediate failure. This should be avoided through a minimum amount of punching shear reinforcement, which is established by the structural code. For larger amounts of punching shear reinforcement, a failure can occur at the maximum punching shear capacity, inside the shear-reinforced zone, or even outside the shear-reinforced zone, depending on the number of reinforcement rows. Despite the design being normally governed by the smallest punching shear resistance, the extent to which the above-described failure modes influence partial factors for building materials (i.e., concrete and reinforcing steel) can vary. When it is unclear which type of failure is decisive due to the scatter of basic variables (namely, material and geometric properties and model uncertainties), the use of reliability-based methods enables the assessment of failure probabilities and the corresponding reliability indices β (as a measure of structural safety level) for a structural system or component.

The theory of structural systems reliability offers means to assess these failure probabilities and reliability indices β when a system is subjected to multiple failure modes (e.g., [8–13]). In principle, a structural system can be idealised and modelled in two ways (e.g., [8–13]): (1) a series system, also known as the weakest link system, which is said to fail if at least one component (e.g., structural element, section, or point in the structure where yielding occurs) fails or the conditions are met to fail in one of several mutually exclusive modes [10], and (2) a parallel system, which is said to fail if all components have failed or all originally redundant structural configurations or paths to carry the load have been eliminated [10]. In theory, a third type of structural system, a so-called hybrid or mixed system, is also possible to mechanically idealise and model by combining the concepts of series and parallel systems (e.g., [11]).

By using the theory of structural systems reliability and focusing on the structural code DIN EN 1992-1-1+NA(D) [4–7], this study explores how the punching shear failure modes affect the safety level of a RC flat slab with punching shear reinforcement, and ultimately, the variation of partial factors for concrete. This study was undertaken in the context of a research initiative promoted by the German Federal Ministry for Economic Affairs and Energy (in German: *Bundesministerium für Wirtschaft und Energie*). One of the goals of this research project was the development of an open-source software package (developed in the software language *R* [14]) for structural reliability computations. Named as *TesiproV* (Hochschule Biberach/RWTH Aachen) [15], the ultimate goal of this package is to facilitate and promote the use of reliability-based approaches among structural engineering practitioners and scientific communities [16,17]. It should be also noted that this investigation is in line with previous reliability assessments of punching shear resistance models for flat slabs (e.g., [18]). Yet, the parametric study addressed in this paper is original and not addressed in any other publication.

This paper is organised in the following manner: Section 2 presents the design provisions for punching shear with shear reinforcement as described in DIN EN 1992-1-1+NA(D) [4–7]. Section 3 offers a brief overview of the theory of systems reliability and Section 4 describes the considerations adopted for the design and the reliability-based analyses of the structural system addressed in this investigation. Then, the respective results are presented in Section 5 and discussed in Section 6. Finally, Section 7 summarises the main conclusions of this investigation, alongside the main limitations and recommendations for further research developments.

2. Designing Flat Slabs against Punching Shear Failures with Shear Reinforcement

2.1. General Considerations

As mentioned above, the main design provisions for punching shear with and without shear reinforcement are given in the European structural code DIN EN 1992-1-1 [4,5], i.e., the "base" Eurocode 2. As explained, for example, in [19,20], such provisions are grounded on the design concept given in CEB/FIP Model Code 1990 [21], which is mostly based on empirical models (i.e., those that were fitted to experimental evidence). Yet, in Germany, the punching shear design provisions partly deviate from the original provisions in DIN EN 1992-1-1 [4,5], mainly to guarantee that the safety levels recommended in DIN EN 1990 [22] and DIN 1055-100 [23] can be reached (e.g., [2,3]). As a result, complementary design provisions were added to the National Annex NA(D) [6,7]. For the sake of this investigation, the design provisions for punching shear with and without shear reinforcement in the combined code DIN EN 1992-1-1+NA(D) [4–7] were adopted.

2.2. Design Provisions for Punching Shear with Shear Reinforcement according to DIN EN 1992-1-1 (Eurocode 2)

For RC flat slabs without shear reinforcement, the punching shear resistance is evaluated along different control perimeters. For slabs without shear reinforcement, DIN EN 1992-1-1 [4,5] also demands that the shear stress v_{Ed} at the periphery of the loaded area u_0 does not exceed the web-crushing limit (i.e., crushing of the compressive strut) for beams $\nu_{Rd,max}$:

$$v_{Ed} = \frac{\beta \cdot V_{Ed}}{u_0 \cdot d} \leq \nu_{Rd,max} \text{ with } \nu_{Rd,max} = 0.4 \cdot \nu \cdot f_{cd} \quad (1)$$

with d being the distance from the extreme compression fibre to the centroid of the longitudinal tensile reinforcement (also known as the effective depth) and ν being a factor accounting for the strength reduction of concrete compression struts in cracked concrete due to lateral tension stresses and given by $\nu = 0.6 \cdot (1 - f_{ck}/250)$, with f_{ck} referring to the characteristic compressive strength of concrete (for cylinders). The term β refers to a load increase factor to account for load eccentricities (i.e., the non-uniform shear distribution along the column perimeter).

The second control perimeter u_1 applies at a distance of $2.0d$ from the periphery of the loaded area (Figure 2). The shear stress v_{Ed} along this perimeter should not exceed the punching shear resistance without shear reinforcement $\nu_{Rd,c}$:

$$v_{Ed} = \frac{\beta \cdot V_{Ed}}{u_1 \cdot d} \leq \nu_{Rd,c} \tag{2}$$

Note that DIN EN 1992-1-1 [4,5] provides three methods to allow for the effect of combined shear and an unbalanced moment [24]: (1) the assumption of a fully plastic distribution of shear stresses along the critical perimeter, (2) a reduction in the basic control perimeter, or (3) the use of constant shear force multipliers. For non-sway systems, and where the adjacent spans do not differ in length by more than 25%, the load increase factors may be taken as β equal to 1.5, 1.4 and 1.15 for the corner, edge and interior columns, respectively. Two other methods are described in [24].

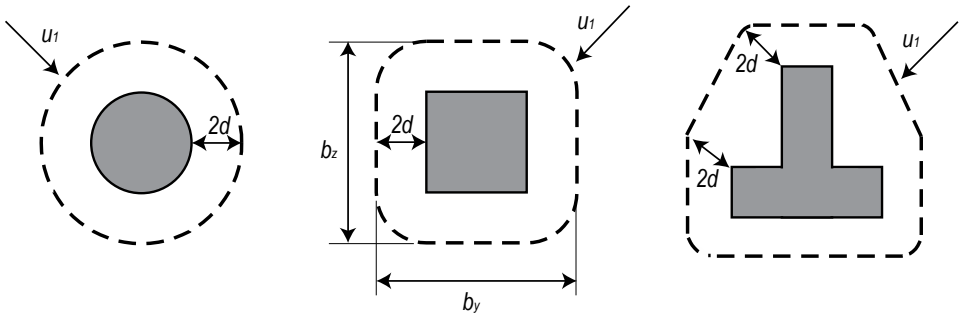


Figure 2. Typical critical control sections around load introduction areas according to DIN EN 1992-1-1 [4,5].

The punching shear resistance without punching shear reinforcement is determined by using

$$v_{Rd,c} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \geq \nu_{min} + k_1 \cdot \sigma_{cp} \tag{3}$$

with $C_{Rd,c}$ being the empirical factor for flat slabs (=$0.18/\gamma_C$ recommended in DIN EN 1992-1-1 [4,5]), with γ_C being the partial factor for concrete (=1.5 recommended in DIN EN 1992-1-1 [4,5]); ρ_l referring to the mean flexural reinforcement ratio considering a slab width equal to the column width plus $3d$ on each side and given by $\left(\rho_{lx} \cdot \rho_{ly}\right)^{1/2} \leq 0.02$. As referred above, the term f_{ck} refers to the characteristic compressive strength of concrete (for cylinders). The term k is given by $1 + (200/d)^{1/2} \leq 2.0$ and refers to a size effect factor for the effective depth d; the parameter $\nu_{min} = 0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2}$ refers to the minimum punching shear capacity, which is especially critical for small flexural reinforcement amounts in combination with a high concrete compressive strength. The term k_1 refers to the empirical factor that defines the amount of normal stress to be considered (=0.10 is recommended in DIN EN 1992-1-1 [4,5]) and σ_{cp} refers to the normal concrete stress in the critical section (e.g., due to pre-stressing).

The punching shear design of prestressed interior column-slab connections is discussed in detail in [25].

The punching shear resistance inside the shear-reinforced zone is calculated by using a strut-and-tie model with a compression strut inclination of 33° in combination with a constant concrete contribution:

$$v_{Rd,cs} = 0.75 \cdot v_{Rd,c} + 1.5 \cdot \left(\frac{d}{s_r}\right) \cdot A_{sw} \cdot f_{ywd,ef} \cdot \left(\frac{1}{u_1 \cdot d}\right) \cdot \sin \alpha \leq k_{max} \cdot v_{Rd,c} \tag{4}$$

with $v_{Rd,c}$ being the punching shear resistance without shear reinforcement (calculated with Equation (3)), k_{max} being a factor that limits the punching shear capacity (=1.5 recommended for stirrups as punching shear reinforcements), A_{sw} being the amount of shear

reinforcement provided in each row and s_r being the radial spacing between adjacent rows of shear reinforcement. The term $f_{ywd,ef}$ refers to the effective design strength of the punching shear reinforcement, which considers the worse anchorage of the punching shear reinforcement for small slab thicknesses and is defined as $250 + 0.25d \leq f_{ywd}$. The term α refers to the angle between the punching shear reinforcement and the plane of the slab.

The amount of punching shear reinforcement that is calculated must be provided in several rows. The precise number of rows required is determined by the shear strength without shear reinforcement $v_{Rd,c}$ (calculated with Equation (3)) checked along the outer perimeter, which is located at a distance of $1.5d$ from the outermost row.

At least two rows of shear reinforcement are needed. The first row must be placed at a distance of $0.30d \leq s_0 \leq 0.50d$ from the boundary of the loaded area. The radial spacing between additional rows of shear reinforcement s_r should be limited to $0.75d$. For inclined bars, the effective cross-section must be determined using $A_{sw} = A_s \cdot \sin\alpha$. Note that the punching shear capacity with punching shear reinforcement is limited by the maximum capacity $v_{Rd,max}$ (Equation (1)), which must be checked along the column perimeter u_0.

2.3. Complementary Design Provisions for Punching Shear with Shear Reinforcement in the German National Annex NA(D)

The design value of the applied shear stresses along the critical perimeter at a distance $2.0d$ from the boundary of the loaded area can be calculated in accordance with DIN EN 1992-1-1+NA(D) [4–7] using

$$v_{Ed} = \frac{\beta \cdot V_{Ed}}{u_1 \cdot d} \tag{5}$$

For interior column-slab connections, the load increase factor β is slightly reduced in comparison to DIN EN 1992-1-1 [4,5] to 1.1. It should be noted here that limiting the shear stresses directly at the perimeter of the load application area u_0 to a value equal to the web-crushing limit of beams $v_{Rd,max}$ is, according to DIN EN 1992-1-1+NA(D) [4–7], obsolete since a previous comparison with test results clearly showed a dependency on the concrete compressive strength and the specific column perimeter u_0/d [2,26,27]. In addition, limiting the shear stresses at the face of the column to $v_{Rd,max}$ means that it is not possible to consider a potential increase in the punching shear resistance due to improved punching shear elements, such as double-headed studs or lattice girders [28–30].

As in DIN EN 1992-1-1 [4,5], the punching shear capacity without shear reinforcement is calculated according to Equation (3). For interior column–slab connections and u_0/d ratios < 4.0, a reduced empirical pre-factor $C_{Rd,c}$ is given by

$$C_{Rd,c} = \frac{0.18}{\gamma_C} \cdot \left(0.1 \cdot \frac{u_0}{d} + 0.6\right) \quad \text{for } \frac{u_0}{d} < 4.0 \tag{6}$$

To avoid compression reinforcement, DIN EN 1992-1-1+NA(D) [4–7] limits the flexural reinforcement ratio to $\rho_l \leq \min\left(0.02;\ 0.5 \cdot f_{cd}/f_{yd}\right)$. For effective depths > 600 mm, the minimum punching shear capacity is reduced in comparison to DIN EN 1992-1-1 [4,5] and is equal to the minimum shear capacity of the German bridge-building provisions "DIN Fachbericht 102" [31]:

$$v_{min} = (0.0525/\gamma_C) \cdot k^{3/2} \cdot f_{ck}^{1/2} \quad \text{for } d \leq 600 \text{ mm} \tag{7}$$

$$v_{min} = (0.0375/\gamma_C) \cdot k^{3/2} \cdot f_{ck}^{1/2} \quad \text{for } d > 800 \text{ mm} \tag{8}$$

For effective depths between 600 mm and 800 mm, the pre-factor $C_{Rd,c}$ may be interpolated linearly.

Limiting the shear stresses to $\nu_{Rd,max}$ (i.e., web-crushing limit) at the column perimeter was not adopted in Germany. The maximum punching shear capacity was defined as 1.4 times the punching shear capacity without shear reinforcement [2,3,26,27]:

$$v_{Ed} \leq \nu_{Rd,max} = 1.4\,\nu_{Rd,c}, \text{ for stirrups as punching shear reinforcement} \quad (9)$$

Furthermore, DIN EN 1992-1-1+NA(D) [4–7] demands that the amount of punching shear reinforcement is increased by a factor of $\kappa_{sw,1} = 2.5$ for the first row (at a distance of $0.3d$ to $0.5d$) and by a factor of $\kappa_{sw,2} = 1.4$ for the second row. From the third row onwards, a factor of $\kappa_{sw} = 1.0$ is assumed. The distance between adjacent rows needs to be adjusted to a maximum of $0.75d$.

In principle, as many rows of punching shear reinforcement are to be arranged as it is required until the verification of the punching shear capacity of a linearly supported slab (one-way shear) is fulfilled along an outer control perimeter at the distance of $1.5d$ from the outermost punching shear reinforcement row. The required length of the outer circular section u_{out} is determined using

$$u_{out} = \frac{\beta \cdot V_{Ed}}{v_{Rd,c} \cdot d} \quad (10)$$

with the $v_{Rd,c}$ corresponding to the punching shear capacity without shear reinforcement (Equation (3)). Note that for the verification of the punching shear capacity outside the punching shear reinforced area according to NA(D) [6,7], the term $v_{Rd,c}$ in Equation (3) shall have an empirical pre-factor $C_{Rd,c} = 0.15/\gamma_C$ (with $\gamma_C = 1.5$) and a factor k_1 equal to 0.10.

3. Foundations of the Theory of Structural Systems Reliability

Normally, failures in structural systems are dependent events since the same uncertain variables are embedded in the functions describing the passage into the failure modes of different components; this dependency makes the probabilistic evaluation of a system's reliability rather complex [10]. During the 1960s, Cornell [8] proposed a simple approach to assess the reliability of a structural system, the so-called simple bounds. Simple bounds are a first-order technique to evaluate the reliability of a structural system when it is affected by multiple failure modes. The outcome is a range of values (or, as the name implies, bounds) of failure probabilities $P(F_i)$ and corresponding reliability indices β_i containing the "precise" failure probability. These bounds are defined by the extreme cases of correlation between failure modes: perfect or full correlation ($\rho = 1$) and no correlation ($\rho = 0$).

For the sake of this investigation, it was assumed that the failure modes described in Section 1 are idealised in a structural series system, which, as described above, represents the failure of a system if any of the failure modes is activated (i.e., the weakest link of the system governs the failure). The simple bounds of a series system are defined as (e.g., [8–13]):

$$\max_{i=1}^{n}\{P(F_i)\} \leq P_F^S \leq 1 - \prod_{i=1}^{n}(1 - P(F_i)) \quad (11)$$

with the lower bound being the failure probability when all elements are fully correlated (i.e., the failure probability of the system is equal to the failure probability of the failure mode with the largest failure probability, i.e., a system where the weakest linl may be cleary identified). The upper bound being the failure probability when all the elements are uncorrelated (i.e., the system will not necessarily fail in the failure mode with the largest failure probability; there is a probability that failure will take place in any of the links of the system). The corresponding reliability index of a system β^S (i.e., a system safety level) is calculated as follows (e.g., [8–13]):

$$-\Phi^{-1}\left(\sum_{i=1}^{n}\Phi(-\beta_i)\right) \leq \beta^S \leq \min_{i=1}^{n}\beta_i \quad (12)$$

with β_i being the reliability indices of the individual failure modes F_i.

4. Considerations for the Reliability Analyses of the Structural System

4.1. Database of Experimental Tests and Data Filtering Criteria

To set up the resistance models for punching shear with shear reinforcement, the database compiled at the Institute of Structural Concrete (IMB) of RWTH Aachen University (Germany) was used. Initiated by Beutel [32], the database has been continuously maintained and updated by multiple researchers (e.g., [33–37]). The version considered in this investigation comprises 308 tests on RC flat slabs with punching shear reinforcement. According to the database, these tests are divided into different failure modes. Since the tests could not always be allocated to a single failure mode, some tests were assigned to several failure modes. The sum of the tests evaluated for the different failure modes is, therefore, greater than the total number of tests.

For a proper statistical evaluation of the model uncertainties (Section 4.2), the tests were filtered according to the following criteria:

(1) The tests should contain at least one row of punching shear reinforcement.
(2) Only star-like punching shear reinforcement was used.
(3) Only stirrup reinforcement (or comparable reinforcement) was considered (note that this criterion does not apply to failure mode (ii) referring to failure outside the punching shear reinforcement zone).
(4) The tests should have complete documentation of all the parameters needed for the design calculation according to DIN EN1992-1-1+NA(D) [4–7].

In addition to these filters defined on the basis of DIN EN1992-1-1+NA(D) [4–7], individual test series were excluded from the evaluation after reviewing the corresponding literature. This applied to the cases where the results were not comprehensible, or the data parameters were not properly described. The application of these filters together with this individual analysis reduced the database to 38 tests regarding failure inside the punching shear reinforced zone (failure mode (i)), 44 tests regarding failure outside the punching shear reinforced zone (failure mode (ii)) and 41 tests regarding failure at the maximum punching shear capacity (failure mode (iii)).

4.2. Stochastic Models for the Basic Variables

The stochastic models for the basic variables that affect the design equations described in Section 2 were defined based on the provisions of the JCSS Probabilistic Model Code [38]. These models are presented and described throughout Tables 1–3. For the sake of this investigation, all the basic variables are considered uncorrelated.

The stochastic models for the concrete compressive strength f_c described in Table 1 are particularly relevant. As a material property, concrete compressive strength is typically described using a logarithmic normal distribution (i.e., lognormal distribution), as it does not allow for negative values (e.g., [39]). Since concrete compressive strength is affected by multiple factors, such as limited sample sizes, the production site and the time of year, it is generally accepted that its mean and its standard deviation are also considered random variables.

In statistics, the conjugate prior of a normal or lognormal distribution with an unknown mean and standard deviation is a normal-gamma distribution (e.g., [39]), which is a bivariate four-parameter family of a continuous probability distribution. These four parameters are commonly denoted as m, n, s and ν, with m referring to the mean value of a sample with size n and s being the empirical standard deviation of a sample of size $\nu + 1$. To describe the stochastic model for the concrete compressive strength, a four-parameter distribution (log-Student's-t distribution) was used, whose parameters are given in the JCSS Probabilistic Model Code [38] for different concrete strength classes (Table 1). Note that a log-Student's-t distribution arises as the marginal distribution, which results from the integration of a normal-gamma posterior distribution and is the required distribution density for concrete compressive strength (e.g., [39]).

Table 1. Stochastic values for the concrete compressive strength f_c variable.

Concrete Class	Prior Parameters (*) [38]				Parameters for a Lognormal Distribution		f_{cm}	σ_{fcm}	f_{ck}
	m'	n'	s'	v'	Mean Value μ_{LND}	Standard Deviation σ_{LND}			
C25	3.65	3.0	0.12	10	3.65	0.164	39.00	6.45	28.36
C35	3.85	3.0	0.09	10	3.85	0.123	47.35	5.86	38.37
C45	3.98	3.0	0.07	10	3.98	0.096	53.76	5.17	45.71

(*) Parameters for ready-mixed concrete. f_{cm}: mean value of concrete comp. strength (cylinder) $\rightarrow f_{cm} = E(X) = e^{\mu_{LND}+0.5\sigma^2_{LND}}$. $\sigma_{f_{cm}}$: standard deviation of concrete comp. strength (cylinder) $\rightarrow \sigma_{f_{cm}} = SD(X) = f_{cm}\sqrt{e^{\sigma^2_{LND}}-1}$. f_{ck}: characteristic concrete comp. strength at 28 days (cylinder) $\rightarrow f_{ck} = \exp(\mu_{LND} - 1.645\sigma_{LND})$

Another relevant basic variable is the model uncertainty. The model uncertainty generally accounts for effects that are neglected in model simplifications and mathematical relationships (e.g., [40,41]). As previous studies already demonstrated (e.g., [18,40,41]), uncertainties in resistance models can play a significant role in structural safety evaluations and the calibration of partial factors in semi-probabilistic designs and, therefore, their thorough consideration is critical for the feasibility of any reliability-based investigation. According to the JCSS Probabilistic Model Code [38], the model uncertainty variable is typically characterised by a lognormal distribution. Nonetheless, in the context of this investigation, this assumption needed to be properly evaluated. Thus, the statistical parameters of the distribution function describing the model uncertainty were estimated from the data stored in the experimental database described in Section 4.1. To this end, the theoretical (i.e., predicted) punching shear resistance (according to DIN EN 1992-1-1+NA(D) [4–7]) was compared with the experimental failure loads. Note that for the comparison, the punching shear $V_{R,c}$ was used, all the partial factors in the design equations were taken as unity and the mean value of concrete compressive strength (for cylinders) was adopted. Then, the statistical parameters of the related resistances were determined under the assumption of both the normal and lognormal distributions. By doing this, the quotients of the experimental failure loads and the theoretical punching shear resistances were determined.

In addition to the statistical characterisation of the model uncertainty, three consecutive analyses were carried out. First, a goodness-of-fit test–in this case, the Kolmogorov–Smirnov test (e.g., [42–45])—was adopted to evaluate whether the data followed the assumed distribution function (i.e., a lognormal distribution). The test hypothesis evaluates whether the data sample fits this assumed distribution function by comparing the deviation values with a target value, which depends on the significance level α adopted and the sample size. The assumed lognormal distribution for the model uncertainty was confirmed for all the investigated resistance models (in this case, a resistance model for each of the failure modes identified in Section 1).

Second, the David–Hartley–Pearson test was used to detect potential outliers in the dataset [46]. The test checks whether it is likely that an observed extreme value belongs to a normally distributed population or whether it is an extreme observation. If an outlier is detected, the test is repeated, excluding the test result with the greatest distance from the mean value. This process is repeated until no further outliers are identified. Due to the lognormality of the data, before applying the David–Hartley–Pearson test, the data values were previously transformed to normality: $y_i = \ln(x_i)$. The test statistic was then determined for the transformed data values y_i. Once this evaluation was concluded, the database was limited to 37 tests for failure inside the punching shear reinforced zone (failure mode (i)), 44 tests for failure outside the punching shear reinforced zone (failure mode (ii)) and 41 tests for failure at the maximum punching shear capacity (failure mode (iii)).

Third, the estimators for the real parameters were determined by means of point estimators and interval estimators (Table 2). As the name implies, a point estimator is a procedure used to produce an estimate of a parameter of interest and it is usually a

function of only sample data values when these are available [47]. For the estimation of the expected value, the arithmetic mean of the population was used and for the estimation of the standard deviation, the unbiased empirical standard deviation of the population was adopted. The point estimators were then contrasted with the interval estimators; in this case, a confidence level of 95% was considered due to the limited size of the dataset. The lower estimates of the expected values and the upper estimates of the standard deviations were initially determined for a normal distribution. Then, the lognormal distributed parameters (i.e., expected value, standard deviation and coefficient of variation) were also determined for the point estimators and interval estimators.

Table 2. Statistical parameters for the model uncertainty.

Failure Mode (Design Equation)	Parameter	Normal Distribution (ND)		Lognormal Distribution (LND)				
				Point Estimators		Interval Estimators (95%)		
		Point Estimators	Interval Estimators (IE) (95%)	LND Values	LND Values Converted	IE Values	IE Values Converted	Corrected Values
(i) Inside shear reinf. zone (Equation (4))	μ	1.3927	[1.3397; +∞]	0.3219	1.3927 [(a)]	[0.2841; +∞]	[**1.3478**; +∞] [(d)]	1.3478
	σ	0.1910	[−∞; 0.2375]	0.1362	0.1906 [(b)]	[−∞; 0.1694]	[−∞; 0.2300] [(e)]	0.2199 [(g)]
	CoV	0.1371	[−∞; 0.1773]	0.4231	0.1368 [(c)]	[−∞; 0.5962]	[−∞; 0.1706] [(f)]	**0.1631**
(ii) Outside shear reinf. zone (Equation (10))	μ	1.1072	[1.0654; +∞]	0.0903	1.1075 [(a)]	[0.0514; +∞]	[**1.0713**; +∞] [(d)]	1.0713
	σ	0.1648	[−∞; 0.2008]	0.1536	0.1712 [(b)]	[−∞; 0.1871]	[−∞; 0.2023] [(e)]	0.1951 [(g)]
	CoV	0.1489	[−∞; 0.1885]	1.7013	0.1545 [(c)]	[−∞; 3.6442]	[−∞; 0.1888] [(f)]	**0.1821**
(iii) At max. punching shear capacity (Equation (9))	μ	1.0855	[1.0567; +∞]	0.0769	1.0855 [(a)]	[0.0500; +∞]	[**1.0596**; +∞] [(d)]	1.0596
	σ	0.1095	[−∞; 0.1345]	0.1021	0.1111 [(b)]	[−∞; 0.1254]	[−∞; 0.1334] [(e)]	0.1224 [(g)]
	CoV	0.1009	[−∞; 0.1273]	1.3277	0.1023 [(c)]	[−∞; 2.5058]	[−∞; 0.1259] [(f)]	**0.1155**

μ: expected value; σ: standard deviation; CoV: coefficient of variation. Equations for the calculation of the parameters of a lognormally distributed variable X (i.e., conversions [(a) to (f)] and the correction of σ_θ(g)): Point estimators—[(a)] $\mu_{LND,conv} = e^{\mu_{LN}+0.5\sigma^2_{LN}}$; [(b)] $\sigma_{LND,conv} = \mu_{LN,conv} \cdot \sqrt{e^{\sigma^2_{LN}} - 1}$; [(c)] $CoV_{LND,conv} = \sqrt{e^{\sigma^2_{LN}} - 1}$. Interval estimators—[(d)] $\mu_{IE,LND,conv} = e^{\mu_{IE,LN}+0.5\sigma^2_{IE,LN}}$; [(e)] $\sigma_{IE,LND,conv} = \mu_{IE,LND,conv} \cdot \sqrt{e^{\sigma^2_{IE,LN}} - 1}$; [(f)] $CoV_{IE,LND,conv} = \sqrt{e^{\sigma^2_{IE,LN}} - 1}$. Correction of σ_θ using the coefficient of variation based on [48]: [(g)] $\sigma_\theta = \mu_{LND,conv} \cdot \sqrt{CoV^2_{\theta LND,conv} - 0.05^2}$

Table 3. Probabilistic models of the basic variables according to [38].

Basic Variables		Distr.	Unit	Nominal Value	Mean Value μ_X	Stand. Deviation σ_X
Concrete compressive strength f_c (also see Table 1)	C25	LSD	N/mm²	25	39.00	6.45
	C35			35	47.35	5.86
	C45			45	53.76	5.17
Yield strength	fy	ND	N/mm²	500	$500 + 2\sigma_{fy}$	30
Effective depth	d	ND	mm	d_{nom}	d_{nom} +10	10
Column dimensions	c, c∅	ND	mm	c_{nom}	c_{nom} + 0.003c_{nom}	4 + 0.006c_{nom}
Slab width	h	ND	mm	h_{nom}	h_{nom} + 10	10
Reinf. area	As	ND	mm²	As,nom	As,nom	0.02As,nom
Radial spacing of perimeter of shear reinforcement	s_r	ND	mm	$s_{r,nom}$	$s_{r,nom}$	$s_{r,nom} \pm 20$
Model uncertainty:						
(i) Inside reinf. zone	θ_{cs}	LND	-	1.3478	$\theta_{cs,nom}$	CoV: 0.1631
(ii) Outside reinf. zone	θ_{out}	LND	-	1.0713	$\theta_{out,nom}$	CoV: 0.1821
(iii) At max. punching shear capacity	θ_{max}	LND	-	1.0596	$\theta_{max,nom}$	CoV: 0.1155

LSD: log-Student's-t distribution (parameters m, n, s and ν described in Table 1). ND: normal distribution; LND: lognormal distribution; CoV: coefficient of variation.

Finally, by following the recommendation of Taerwe [48], the coefficient of variation was corrected with a value of 0.05. It should be clarified here that the geometric and strength variables included in the tests showed some scatter, which was included in the model uncertainty. If the model uncertainty is combined with such variables, the overall scatter can be overestimated. Therefore, the coefficient of variation of the model uncertainty is recommended to be reduced by 0.05. The entire set of test results cannot be presented in this paper due to space limitations. However, for the purpose of illustration, Figure 3 shows the model uncertainties of the failure mode at maximum punching shear capacity

(failure mode (iii)). All the values attained for the model uncertainties are described in Table 2 and summarised in Table 3.

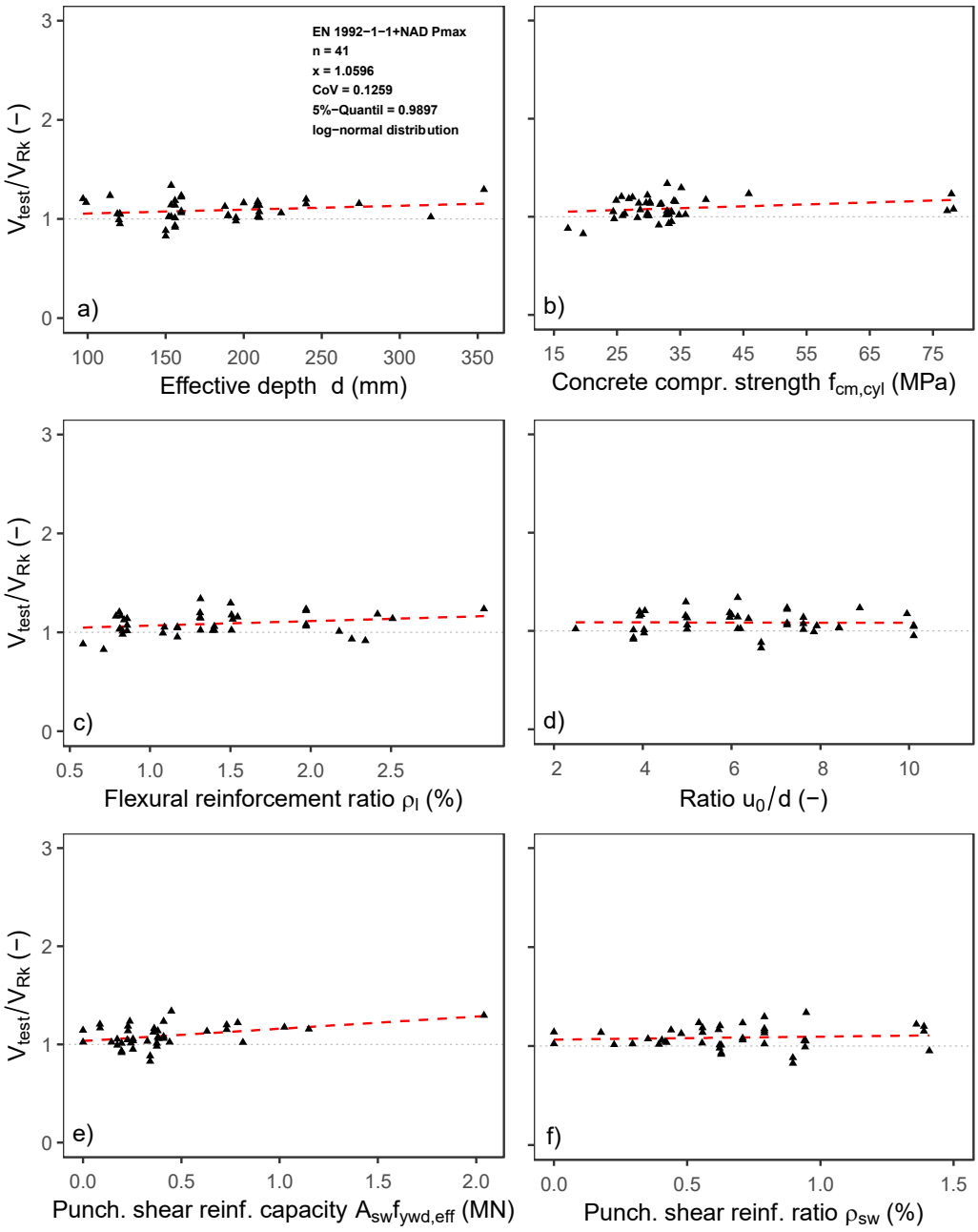


Figure 3. Model uncertainties at the maximum punching shear capacity (i.e., failure mode (iii) described in Section 1) according to DIN EN 1992-1-1+NA(D) [4–7] with $f_{ck} = f_{cm,cyl}$ as a function of the (**a**) effective depth d (mm), (**b**) concrete compressive strength (cylinder) $f_{cm,cyl}$ (MPa), (**c**) flexural reinforcement ratio ρ_l (%), (**d**) ratio u_0/d (–), (**e**) punching shear reinforcement capacity $A_{sw} \cdot f_{ymw,eff}$ (MN) and (**f**) punching shear reinforcement ratio ρ_{sw} (%).

4.3. Computational Considerations

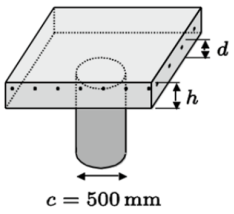
4.3.1. Design of the Structural System

For the design of the structural system, the parameters described in Table 4 were considered. Note that for the sake of this investigation, the load side was neglected. It should be noted here that to decouple the resistance and load sides, a constant sensitive value α_R equal to 0.80 was applied based on the provisions of DIN EN 1990 [22]. The

resulting reliability indices β_i were, therefore, divided by 0.8, which enabled comparisons with the target value of 3.8 recommended in DIN EN 1990 [22].

Table 4. Assumptions for the design of the structural system.

Design Parameters	Values
Concrete compressive strength f_{ck}	35 N/mm^2
Yield strength fy	500 N/mm^2
Effective depth d	280 mm
Column dimensions c∅	500 mm
Slab width h	320 mm
Cross-sectional reinforcement area of the first and second rows	Varying between 1334 mm^2 (min.) and 6000 mm^2 (max.)
Flexural reinforcement ratio ρ_l (*)	0.8%
u_0/d	5.61



(*) It was considered an even distribution of the flexural reinforcement (i.e., equally distributed) over a distance equal to the column width plus 3d on each side.

4.3.2. Reliability-Based Methods (Levels II and III Methods)

For the system reliability analyses, the so-called level II and level III methods were used. For the level II methods, the simple bounds approach of Cornell [8] was adopted (see Section 3). This approach is supported by a first-order reliability algorithm, which uses a first-order Taylor series approximation of a limit state function at the so-called design point and uses second-moment statistics (i.e., means and variances) of the respective basic variables. In this case, the Rackwitz–Fiessler [49] procedure was adopted. Although it is one of the pioneering procedures of first-order second-moment methods, the Rackwitz–Fiessler approach is still accepted as an efficient, simple and easy way to solve the uncorrelated non-normal reliability problems by transforming original non-normal variables into equivalent normal variables based on the equivalent normal conditions [50].

For the level III methods, direct (or crude) Monte Carlo simulations (MCC) and Monte Carlo simulations with variance reduction were considered. Here, the variance reduction methods refer to Monte Carlo simulation with importance sampling (MC-IS) (e.g., [18,51–53]) and Monte Carlo simulation with subset sampling (MC-SuS) (e.g., [53,54]). It is relevant to highlight that the MC-IS method follows the approach proposed by Melchers [52,55]. The approach uses a multi-modal sampling function, where each component of the function is selected for the ith limit state function in the same way as for an individual function, with the most interesting regions being those that contribute the greatest probability density for the functions. For the calculation with MC-SuS, the following symbolic equation for a series system reliability problem was defined:

$$G = \min\{F_1, F_2, \ldots, F_i\} \quad (13)$$

where G is the performance function expressed as the minimum of all the significantly dominant failure modes F_i affecting the structural system. The reliability-based calculations were carried out in the abovementioned software package *TesiproV* [15].

5. Results

5.1. System Reliability Analyses for Fixed Partial Factors

Figures 4–7 show the results of the system reliability analyses for varying amounts of punching shear reinforcement. Each system reliability was calculated according to the methods described in Section 4.3.2 for different partial factors for concrete γ_C. The grey columns of Figures 4–7 respectively refer to the three failure modes discussed in Section 1: $V_{Rd,cs}$ (failure mode (i)), $V_{Rd,out}$ (failure mode (ii)) and $V_{Rd,max}$ (failure mode (iii)). The variation of the amount of punching shear reinforcement considered in the analysis range between a minimum of 824 mm^2, which was determined according to DIN EN

1992-1-1+NA(D) [4–7], and 6000 mm^2. The β target value of 3.8 was taken from DIN EN 1990 [22] and refers to the recommended value for a reliability class 2 (RC2) and a 50-year reference period. This RC2 is associated with the consequence class 2 (CC2), which is mostly related to common structures where the consequences of failure are medium (e.g., residential and office buildings, public buildings).

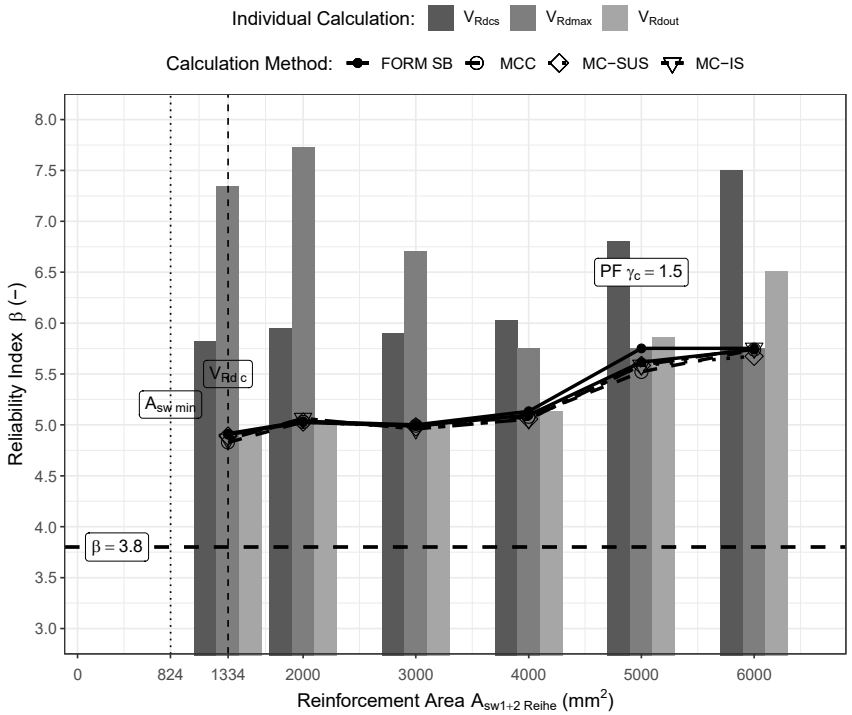


Figure 4. Systems reliability analysis for a partial factor γ_C of 1.5.

Figure 4 shows the results of the system reliability analysis that considered the partial factor γ_C equal to 1.5. Here, it can be observed that for all the reinforcement amounts, the calculated β values varied between 4.8 and 5.7 (approximately), meaning that the safety level of the system was always above the β value of 3.8 recommended in DIN EN 1990 [22]. Figure 4 also shows that the safety levels increase quite sharply for the reinforcement amounts of 5000 mm^2 and 6000 mm^2. Additionally, the results highlight that the β values generated through simple bounds and the simulation methods were quite similar for small amounts of reinforcement. This similarity could be an indication that the first-order methods already produced quite accurate results for the failure modes addressed in this investigation. For higher amounts of reinforcement, the β values generated through simple bounds were slightly higher than those produced through simulation methods. It was particularly noticeable that for the punching shear reinforcement with a cross-section of 5000 mm^2 in the first two rows, the simple bounds were only marginally wider than the bounds obtained for small amounts of reinforcement. Additionally, it can be observed that for reinforcement amounts lower than 4000 mm^2, the failure outside the zone of punching shear reinforcement $V_{Rd,out}$ was governing the system failure. However, for higher amounts of reinforcement (i.e., 5000 mm^2 and 6000 mm^2), the failure at the maximum punching shear capacity $V_{Rd,max}$ became dominant. The failure mode inside the zone of punching shear reinforcement $V_{Rd,cs}$ never became dominant across the range of reinforcement amounts considered.

By reducing the partial factor γ_C to 1.35 (Figure 5), it was noticeable that the overall system safety level decreased despite still being above the 3.8 β threshold. Figure 5 shows that the lowest reliability index (β = 4.3) was attained for a rather low amount of reinforcement (i.e., 1334 mm^2). It is also visible that for reinforcement amounts higher than 2000 mm^2, the variations in the β values were relatively small. For reinforcement

cross-sections of 4000 mm^2 and 5000 mm^2, the variations remained marginal and the maximum β value (β = 4.7) was found for the highest amounts of reinforcement considered (i.e., 5000 mm^2 and 6000 mm^2). In fact, Figure 5 shows that the different reliability-based methods generated very similar results for the entire spectrum of reinforcement cross-sections. As seen in the results illustrated in Figure 4 (for a partial factor γ_C of 1.5), the failure outside the zone of punching shear reinforcement $V_{Rd,out}$ was governing the system failure for reinforcement amounts lower than 4000 mm^2. For higher amounts of reinforcement (i.e., 5000 mm^2 and 6000 mm^2), the failure at the maximum punching shear capacity $V_{Rd,max}$ became dominant. Similar to the previous analysis, the failure mode inside the zone of punching shear reinforcement $V_{Rd,cs}$ never became dominant. Note that here, only the MC-SuS was used for reasons of computational efficiency.

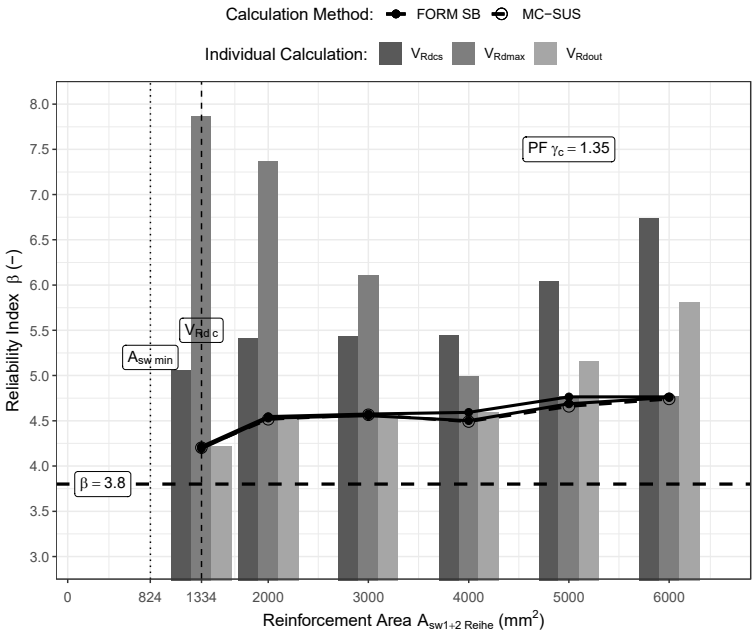


Figure 5. Systems reliability analysis for a partial factor γ_C of 1.35.

Figure 6 shows that a further reduction of the partial factor γ_C to 1.3 also led to an overall reduction of the system safety levels for all the reinforcement cross-sections considered. To some extent, it can be argued that this reduction was marginal since the maximum β values decreased to values around 4.3–4.5. However, an exception was visible for the safety level corresponding to the reinforcement amount of 1334 mm^2, where the β value was only marginally above the 3.8 threshold. These results seemed to indicate that a reduction of partial factors was not proportional (i.e., linear) to a reduction in the safety level across all the reinforcement amounts. As with the use of higher partial factors, here the failure outside the zone of punching shear reinforcement $V_{Rd,out}$ also governed for reinforcement amounts lower than 4000 mm^2. For higher amounts of reinforcement (i.e., 5000 mm^2 and 6000 mm^2), the failure at the maximum punching shear capacity $V_{Rd,max}$ was again dominant. The failure mode inside the zone of punching shear reinforcement $V_{Rd,cs}$ is never dominant.

The results of a further reduction of the partial factor γ_C to 1.2 are plotted in Figure 7. This figure shows that the system safety levels are now below the threshold value (β = 3.8) for reinforcement amounts smaller than 2000 mm^2 and for reinforcement amounts of 5000 mm^2 and 6000 mm^2. By comparing the β. values produced by the different reliability-based methods, it was visible that for the reinforcement amount of 4000 mm^2, the simple bounds seemed to be slightly larger than for the remaining reinforcement amounts. Note that it is normally accepted that when one failure mode is not dominant in relation to the other failure modes, the simple bounds are generally too wide, which could be less inter-

esting for practical use (e.g., [56]). Nonetheless, since these bounds were only marginally wider and the results of the lower bounds were practically the same as the values generated using the simulation methods, the variation in the safety level for the reinforcement amount of 4000 mm^2 in the first two rows was further analysed in detail. It should also be noted that here, the above-described trends for the governing failure modes are maintained: The failure mode inside the zone of punching shear reinforcement $V_{\mathrm{Rd,cs}}$ never became dominant. For reinforcement amounts lower than 4000 mm^2, the failure outside the zone of punching shear reinforcement $V_{\mathrm{Rd,out}}$ governed the system failure, while for higher amounts of reinforcement (i.e., 5000 mm^2 and 6000 mm^2), the failure at the maximum punching shear capacity $V_{\mathrm{Rd,max}}$ is governing instead.

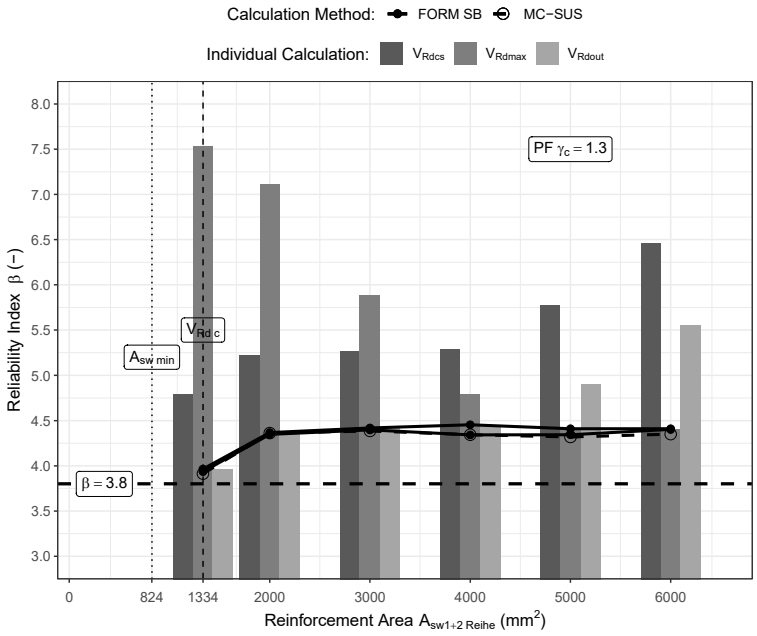


Figure 6. Systems reliability analysis for a partial factor γ_C of 1.3.

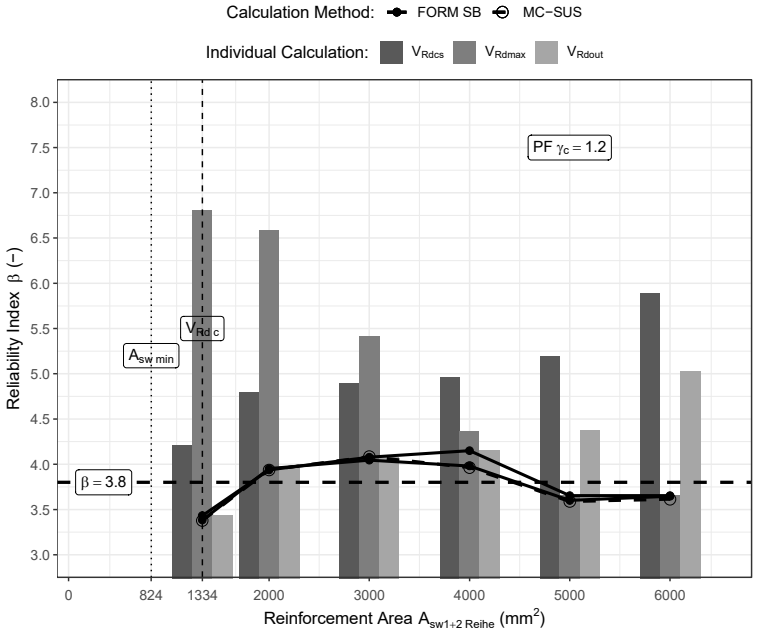


Figure 7. Systems reliability analysis for a partial factor γ_C of 1.2.

5.2. System Reliability Analyses for Variable Partial Factors (Sensitivity Analysis)

Focusing on the system reliability for a reinforcement area of 4000 mm^2, Figure 8 shows the impact of varying the partial factors on the system safety level. Here it was visible that to ensure a safety level above the 3.8 threshold, the simple bounds (Figure 8, grey area) gave a relatively wide spectrum of possible "minimum" partial factors, which ranged between 1.10 and 1.17 (approximately). Furthermore, for a β value of 3.8, the value obtained with the MC-SuS method seemed to be close to the lower bound of the simple bounds approach with a partial factor close to 1.17. The relatively wide spectrum of the simple bounds did not enable drawing a precise conclusion about the accuracy of these "minimum" partial factors. However, it should be highlighted that the interpretation of this value is solely based on a "medium" reinforcement amount (i.e., 4000 mm^2). Nonetheless, addressing other reinforcement amounts and investigating additional variations is not only relatively easy with the use of advanced computation tools but also vital to avoid unsafe decisions regarding the selection of a partial factor. Furthermore, the results of this analysis also suggested that with slightly additional computational power, the use of a variance reduction technique (i.e., level III method) might lead to even more precise results.

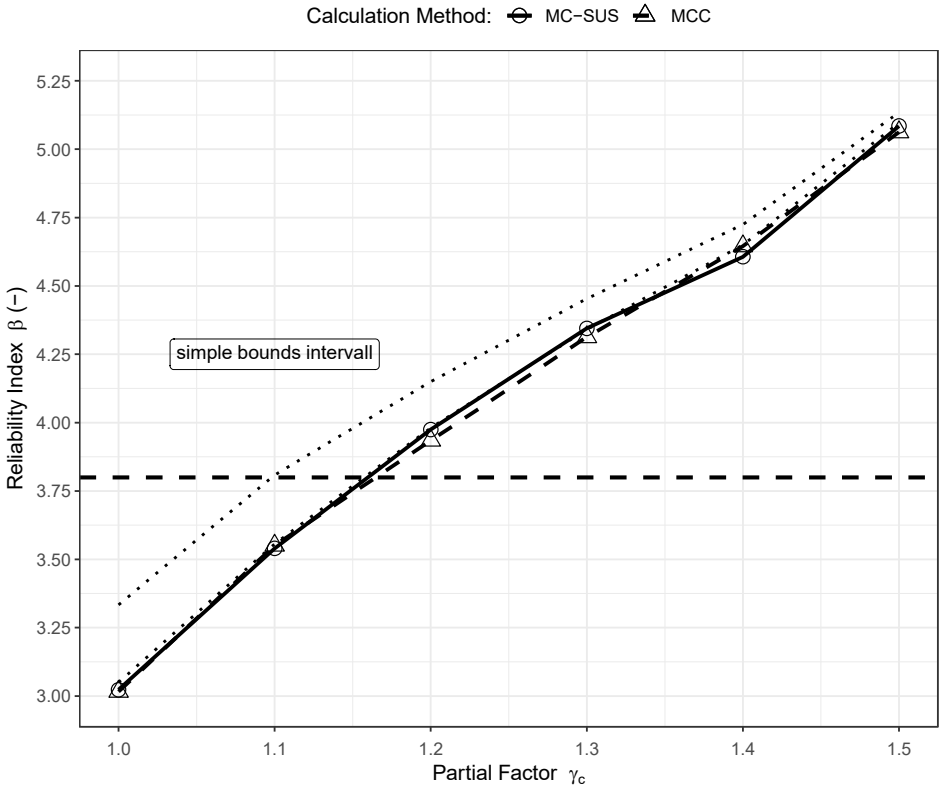


Figure 8. Sensitivity analysis: variation in the safety level in relation to the partial factor (reinforcement area of 4000 mm^2).

6. Discussion

The above-described results for the system safety level (Figures 4–7) showed that its reliability indices β_i varied according to the limit state functions. The results indicated that to ensure satisfactory system reliability (i.e., above a pre-defined threshold or code recommendation), no simple linear dependencies can be derived, which increases the complexity associated to the derivation of partial factors. It is important to stress here that the scatter of the basic variables affecting the design equations of the system failure modes also plays a relevant role on the resulting system safety level, and ultimately, on the resulting partial factors. Likewise, the chosen reliability-based method used to calculate the system failure probabilities also had a significant influence on the derived partial

factors. Nonetheless, this investigation also showed that the use of the simple bounds in combination with a first-order reliability method could give a reasonable first indication of a possible system safety level. However, the results should be carefully interpreted to avoid deriving unsafe partial factors. If sufficient computing power is available—which should be relatively possible with modern computational tools and software packages, such as the *TesiproV*—simulation methods should be used to calculate a more precise operational failure probability and the corresponding reliability index for the structural system.

A possible alternative is to use the simple bounds approach in combination with the values of a second-order reliability method (SORM). In theory, this could generate more accurate results compared with those produced by a first-order method that requires only marginal additional computational power; this is particularly valid for strongly non-linear functions. If the simple bounds limit the results of a system failure probability and simulation methods are too computationally costly, second-order bounds, such as the Ditlevsen bounds [57], can also be considered, as these tend to be slightly more accurate than first-order approximation methods. With the approximation method developed by Ditlevsen [57], tighter bounds for the system failure probabilities can in theory be calculated by determining a correlation (i.e., a value between 0 and 1) between the safety levels of the individual failure modes. This is especially valid when it is not clear which failure mode is dominant due to similar failure probabilities when multiple failure modes occur. Overall, the results of this investigation confirm the premise that a more precise analysis of the system reliability can reduce the partial factors for concrete in the case of punching shear with shear reinforcement. As a result, potential benefits, such as material and cost savings can be achieved without falling below a socially accepted safety level (e.g., the β value of 3.8 recommended in DIN EN 1990 [22]).

7. Conclusions

Based on the results of this investigation, the following conclusions can be drawn:

- The concepts related to systems reliability can be applied to derive partial factors for design equations. The above-described results indicated that the partial factors can vary considerably depending on the governing limit state function and the scatter of the basic variables. This confirmed the importance of a thorough definition of stochastic models for the basic variables and the importance of a careful selection of the reliability-based method (i.e., level II or level III method).
- In this investigation, it was seen that the use of the simple bounds in combination with a first-order reliability method (i.e., level II methods) can give a decent preliminary indication of the system safety level. Yet, the results should be carefully interpreted to avoid deriving unsafe partial factors. If sufficient computing power is available, simulation methods (i.e., level III methods) should be used since they enable the calculation of more precise system failure probabilities.
- The results of the sensitivity analysis confirmed the premise that a thorough analysis of the reliability may support the reduction of partial factors γ_C. This could be a possible approach for practitioners to save material and costs and reduce CO_2 emissions during the structural design process without compromising socially accepted safety levels (e.g., the β value of 3.8 recommended in DIN EN 1990 [22]).

The investigated approach seems to have potential for further applications and developments. For example, instead of changing a partial factor, other parameters of the design equations could instead vary, such as the pre-empirical factor $C_{Rd,c}$. This would enable the reduction in $C_{Rd,c}$ while maintaining the code-recommended value for a partial factor. Further investigations could include the evaluation of the impact of different basic variables on the derivation of partial factors. Another possibility is to consider correlated variables in the analysis. A more precise analysis that involves using second-order bounds (e.g., Ditlevsen bounds) should be also conducted. Finally, distinct failure mechanisms could be considered to evaluate the transferability of the proposed concept, not only regarding other structural components but also concerning a broader scope of Eurocode 2.

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