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Energy-Efficient Lane Change Trajectory Planning for Highway Traffic Scenarios Considering Different Driving Needs

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Abstract: This paper proposes an energy-saving lane-changing trajectory model for intelligent electric vehicles at high speeds that considers different driving needs under multiple constraints to address the issues of simple lane-changing considerations and poor safety. An economic index is added to construct a multi-objective optimization function based on the comfort, safety, and efficiency of lane changing. The particle swarm optimization algorithm is used to solve the optimal lane-changing time. The weight of the index is obtained by analyzing different driving needs using the Analytic Hierarchy Process in different scenarios. The multi-objective function is adjusted to plan the optimal lane-changing trajectory that meets driving needs. The simulation shows that the proposed model can generate smooth and feasible lane-changing trajectories that meet different driving needs. The energy consumption analysis results indicate that the construction of economic indicators can effectively reduce the energy consumption of vehicles driving on highways. The tracking analysis results indicate that the target vehicle can smoothly and safely change lanes on the planned trajectory, verifying the effectiveness and rationality of the planned trajectory.

Keywords: intelligent vehicle; trajectory planning; energy saving

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1. Introduction

According to traffic reports in recent years, most traffic accidents are caused by unreasonable lane-changing behavior of vehicles, which are human-made [1]. Reasonable lane-changing behavior is one of the effective ways to ensure traffic safety and solve traffic congestion [2]. With the emergence of smart cars and the maturity of autonomous driving technology, traffic accidents caused by human factors have been greatly reduced. The lane-changing system of intelligent vehicles mainly includes the identification of the surrounding traffic environment, the corresponding decision-making, trajectory planning, and control [3].

For a lane change problem, the current research usually uses the rule method, utility theory method, Game theory method, and artificial intelligence method to make decisions [4]. The rule method was first proposed by Gipps [5], who made lane-changing decisions based on the minimum safe distance and comprehensive lane-changing intention. Hidas [6] considered the relationship between lane-changing interactive vehicles to design safe distances and improve the Gipps model. Wang et al. [7] designed a Decision model for lane change with minimum safety distance considering the impact of self-owned vehicle safety and traffic flow. The utility theory method is widely studied in lane-changing problems. Ahmed [8] incorporated the driver's personality into the utility theory and established a Decision model for lane changing. Toledo et al. [9] classified lane-changing issues into mandatory lane changing and autonomous lane changing and analyzed them for different scenarios. The Game theory method is often used in current research. Kita [10] first used non-zero and non-cooperative games to study vehicle confluence. Ali et al. [11] designed a more accurate and effective Decision model for lane changing based on utility theory and Game theory. Artificial intelligence methods have emerged with current developments, utilizing learning from data to obtain the best decisions in the current state. Liu et al. [12] established a support vector machine Decision model for lane changing and conducted a comparative study with the rule method. Ye et al. [13] utilized the proximal strategy optimization algorithm to simulate and learn the optimal strategy from the traffic environment.

Lane-changing trajectory planning, as an indispensable condition for lane-changing, has always been one of the research focus and hotspots of scholars in related fields [14]. For lane-changing trajectory planning, the current mainstream research methods mainly include specific function-based trajectory planning methods, search-based trajectory planning methods, and some intelligent algorithms. Algorithms based on function curves include lane-changing trajectory of sine and cosine algorithms, arc lane-changing trajectory, positive and negative trapezoidal acceleration lane-changing trajectory, and polynomial-based lane-changing trajectory. Among the commonly used geometric curves, the polynomial curve is the most widely used in trajectory planning, which can plan a smooth curve with low computational cost and optimize the trajectory by adjusting the order of the polynomial. Nelson et al. [15] proposed the polynomial method for the first time; they used polynomial curves instead of arc curves to plan continuous curvature trajectories, which ensured the traceability of the trajectory. Yang et al. [16] proposed a trajectory planning model based on cubic polynomial curves; they focused on the problem of trajectory re-planning in dynamic driving environments. Wei et al. [17] increased the polynomial function to the quintic and solved the optimal trajectory by considering the travel time and travel distance as free variables. Reference [18] improves the quintic polynomial by dynamically planning the lane-changing time and adding comfort constraints, calculates the transit state by considering the surrounding traffic conditions during the lane-changing process, and uses two improved quintic polynomials to avoid obstacles to improve security. Zhang Ronghui et al. [19] analyzed the unmanned lane-changing entry criteria and safety assessment, designed an index function that takes into account vehicle characteristics and driving comfort, and obtained the trajectory of the unmanned vehicle lane-changing. Wu Shufan et al. [20] proposed a path planning algorithm based on extension goodness evaluation for changing lanes and tracked it through model predictive control. Zhang Zhiyong et al. [21] considered the longitudinal speed and lateral overtaking path of lane changing to ensure the stability and comfort of driving on high-speed turning lanes. In order to improve the safety of intelligent vehicles in high-speed driving, avoiding obstacles and changing lanes, the literature [22] constructed anti-collision constraints and combined the six-degree polynomial path planning for planning.

Based on the selected curve, Yang et al. [23] used an improved artificial potential field method to consider the driver's style and reaction time and comprehensively considered lane-changing efficiency and comfort to select the optimal path. Zhang et al. [24] compared three polynomial lane-changing models; selected the fifth polynomial; and used the TOPSIS algorithm to solve multi-objective optimization problems for lane-changing comfort, smoothness, and efficiency to obtain the optimal trajectory. Zhao et al. [25] planned the horizontal track and vertical track according to the target state for autonomous vehicle lane change track planning under the multi-lane traffic scenario. Secondly, a cost evaluation function for the trajectory was established, and the optimal trajectory was selected from three aspects: safety, comfort, and lane change time. Li Ya et al. [26] studied the situation of vehicles returning to their original lane when they failed to change lanes. In the study, a quintic polynomial path planning method was used, and a comprehensive benefit function considering comfort, lateral displacement, longitudinal displacement, and time was set up to evaluate the model.

Based on the lane change trajectory research conducted by the above scholars, the following deficiencies still exist in the research and application of lane change trajectory of autonomous vehicles:

1. In most studies on lane change trajectory planning, the lane change time and longitudinal distance are artificially given, which is arbitrary and results in insufficient trajectory performance planning.
2. There are a few factors considered in the trajectory solution. Even if the comfort and safety factors are taken into account during the lane-changing process, the economic factor is hardly considered.
3. In the study of lane-changing trajectory optimization, there is a lack of consideration for the needs of different drivers in completing driving tasks, and there is a lack of optimization for trajectory optimization algorithms, resulting in a long computational time.

In response to the above issues, this article conducts a polynomial lane change trajectory planning based on electric intelligent vehicles, considering different driving needs. For the issue of insufficient trajectory planning performance in current research, we decouple the vertical and horizontal lane-changing trajectories, with lane-changing time as a variable, and select horizontal quintic polynomials and vertical quartic polynomials. At the same time, economic indicators are added to the trajectory planning objective function to achieve energy saving by changing lanes to fill the gap in insufficient consideration of energy consumption issues in research. Considering different driving needs, the Analytic Hierarchy Process is used to adjust the weight coefficients of each indicator to cope with different lane-changing scenarios. The optimal lane-changing time is obtained through particle swarm optimization. The experimental and simulation results show that considering economic factors can effectively reduce energy consumption, improve the algorithm's ability to plan lane change trajectories in different scenarios, and plan suitable lane change trajectories according to driving needs, improving the applicability of lane change planning. The trajectory has strong traceability and good real-time performance. The research framework of this paper is shown in Figure 1:

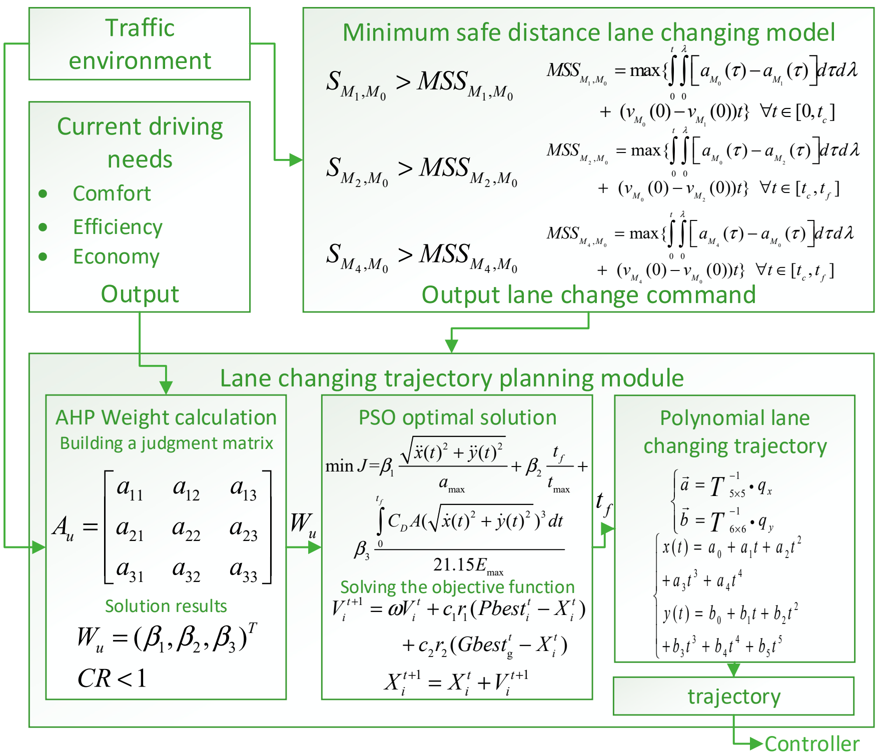


Figure 1. Algorithm framework diagram of the paper.

2. Description of Lane Change Problem

During the driving process of the vehicle, when the speed of the vehicle ahead of the same lane is slow, which affects the driving efficiency, or in order to avoid collision with the obstacle that appears ahead, the vehicle needs to implement the lane-changing behavior. Lane changing falls into two types: mandatory lane changing and free lane changing [27]. In the process of changing lanes, it is necessary to consider not only the running state of the vehicle but also the distance to other vehicles in the lane or the target lane to avoid collision. Whether a collision occurs requires the introduction of a minimum safe distance for judgment. At the same time, on the basis of satisfying the feasibility of changing lanes, it is necessary to plan the trajectory of changing lanes, control the vehicle to follow the trajectory stably, and complete the lane change. For the description of the scene around the lane change, as shown in Figure 2, M_0 is the target vehicle, M_1 and M_3 are the vehicles before and after the lane change, and M_2 and M_4 are the vehicles before and after the target lane. During the lane change process, it is assumed that the M_1 vehicle will adjust the speed by itself with a small probability of colliding with the target vehicle, so M_1, M_2, and M_4 vehicles are mainly considered when considering the longitudinal safety distance.

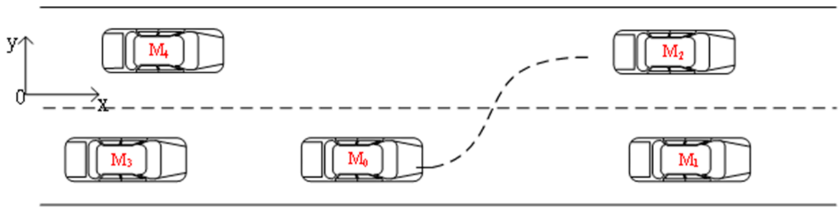


Figure 2. Car lane change diagram.

In the lane-changing scenario shown in Figure 3, due to differences in speed and acceleration, the instantaneous maximum distance between the initial moment and the moment of collision with M_1 is the minimum safe distance between M_0 and M_1 at this time. When the initial distance is not enough, a collision will occur at this maximum value. The minimum safe distance [28] for M_0 and M_1 not to collide is described as Equation (1):

$$MSS_{M_1,M_0} = max\{\int_0^t \int_0^\lambda [a_{M_0}(\tau) - a_{M_1}(\tau)]d\tau d\lambda + (v_{M_0}(0) - v_{M_1}(0))\,t\}\forall t \in [0, t_c] \tag{1}$$

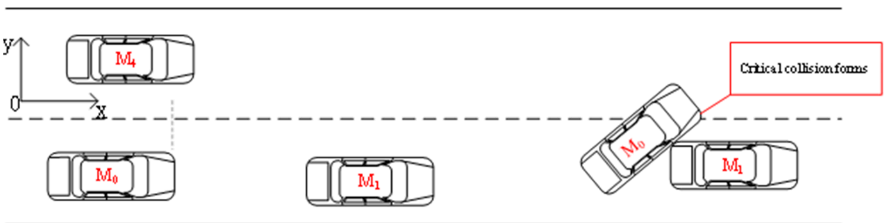


Figure 3. Schematic diagram of M_0 and M_1 collision.

Similarly, the minimum safe distance at this point depends on the maximum distance between the moment of imminent collision with M_2 and the end of the lane change. The minimum safe distance for M_0 and M_2 not to collide is shown in Equation (2):

$$MSS_{M_2,M_0} = max\{\int_0^t \int_0^\lambda [a_{M_0}(\tau) - a_{M_2}(\tau)]d\tau d\lambda + (v_{M_0}(0) - v_{M_2}(0))\,t\}\forall t \in \left[t_c, t_f\right] \tag{2}$$

The minimum safe distance for M_0 and M_4 not to collide can be seen in Equation (3):

$$MSS_{M_4,M_0} = max\{\int_0^t \int_0^\lambda \left[a_{M_4}(\tau) - a_{M_0}(\tau)\right] d\tau d\lambda + \left(v_{M_4}(0) - v_{M_0}(0)\right) t\} \forall t \in \left[t_c, t_f\right] \tag{3}$$

In the above Equations (1)–(3), v_{M_0}, v_{M_1}, v_{M_2}, and v_{M_4} are the speeds of the vehicle, the speed of the vehicle in front of this lane, and the speed of the vehicles in front of and (or) behind the target lane, respectively; a_{M_0}, a_{M_1}, a_{M_2} and a_{M_4} represent the acceleration of the vehicle, the acceleration of the vehicle ahead of this lane, and the acceleration of the vehicles ahead of and behind the target lane; t_c is the collision time between the vehicle and other vehicles, and t_f is the end time of the lane change. Assuming that S is the initial distance between this vehicle and the other, the initial distance should be greater than the minimum lane-changing safety distance.

3. Lane Change Trajectory Planning Subheadings

When the smart car makes a lane-changing decision, it needs to complete the lane-changing trajectory planning according to the current conditions. Because of the simplicity and mall amount of calculation of polynomial curve fitting, the continuous third-order differentiability of the fifth-order polynomial, and the smoothness of curvature, we make the longitudinal and lateral decoupling of the lane-changing according to the lane-changing characteristics and select a quartic polynomial and a quintic polynomial in the x-direction and y-direction, respectively, for planning.

The lane-changing trajectory function is shown in Equation (4).

$$\begin{cases} x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 \\ y(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \end{cases} \tag{4}$$

where $a_0 \sim a_4$ and $b_0 \sim b_5$ are the required coefficients for the polynomials $x(t)$ and $y(t)$. The solution of the polynomial needs to rely on the initial and end conditions of the vehicle lane change. Let the initial time of vehicle lane change be t_0, and the end time of lane change be t_f. The initial state can be expressed by $S_{in} = \left[x_0, \dot{x}_0, \ddot{x}_0, y_0, \dot{y}_0, \ddot{y}_0\right]$, where the six components represent the longitudinal and lateral displacement, velocity, and acceleration of the car at the initial moment, respectively. Here, the speed at the beginning of the lane change is v_0, the speed at the end of the lane change is v_f, and the longitudinal position w is one lane width with a value of 3.75 m. The boundary states of the car are shown in Equations (5) and (6).

$$\begin{cases} x(t_0) = 0, \dot{x}(t_0) = v_0, \ddot{x}(t_0) = 0 \\ y(t_0) = 0, \dot{y}(t_0) = 0, \ddot{y}(t_0) = 0 \end{cases} \tag{5}$$

$$\begin{cases} \dot{x}\left(t_f\right) = v_f, \ddot{x}\left(t_f\right) = 0 \\ y\left(t_f\right) = w, \dot{y}\left(t_f\right) = 0, \ddot{y}\left(t_f\right) = 0 \end{cases} \tag{6}$$

We define the time parameter matrix as follows; see Equations (7) and (8).

$$T_{5\times5} = \begin{bmatrix} 1 & t_0 & {t_0}^2 & {t_0}^3 & {t_0}^4 \\ 0 & 1 & 2t_0 & 3{t_0}^2 & 4{t_0}^3 \\ 0 & 0 & 0 & 6t_0 & 12{t_0}^3 \\ 0 & 1 & 2t_f & 3{t_f}^2 & 4{t_f}^3 \\ 0 & 0 & 2 & 6t_f & 12{t_f}^2 \end{bmatrix} \tag{7}$$

$$T_{6\times 6} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 & t_0^4 & t_0^5 \\ 0 & 1 & 2t_0 & 3t_0^2 & 4t_0^3 & 5t_0^4 \\ 0 & 0 & 2 & 6t_0 & 12t_0^2 & 30t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 & t_f^4 & t_f^5 \\ 0 & 1 & 2t_f & 3t_f^2 & 4t_f^3 & 5t_f^4 \\ 0 & 0 & 2 & 6t_f & 12t_f^2 & 20t_f^3 \end{bmatrix} \tag{8}$$

We define the following Equations (9) and (10) as the coefficient matrix and the boundary matrix, respectively.

$$\begin{cases} \vec{a} = [a_4, a_3, a_2, a_1, a_0] \\ \vec{b} = [b_5, b_4, b_3, b_2, b_1, b_0] \end{cases} \tag{9}$$

$$\begin{cases} q_x = [0, v_0, 0, v_d, 0] \\ q_y = [0, 0, 0, w, 0, 0] \end{cases} \tag{10}$$

Then, the system of equations can be expressed as below; see Equation (11).

$$\begin{cases} \vec{a} = T_{5\times 5}^{-1} \cdot q_x \\ \vec{b} = T_{6\times 6}^{-1} \cdot q_y \end{cases} \tag{11}$$

We can obtain the general form of Equation (11) by using the lane-changing boundary conditions, obtain the optimal lane-changing time in different lane-changing scenarios, and then determine the final lane-changing trajectory.

4. Construction of Multi-Objective Function of Lane-Changing Trajectory

For the car to change lanes, it is necessary and very important to plan a trajectory that matches the actual situation. Different forms of lane-changing trajectories will affect the effect of lane-changing, and unreasonable lane-changing trajectories may even cause dangerous conditions such as instability and sideslip of the car. In order to realize the stable and safe lane changing of the vehicle, we established a multi-objective evaluation function in this paper and planned the optimal trajectory considering the factors of driving safety, comfort, economy, and lane-changing efficiency of the vehicle. In the process of changing lanes, the car first obtains the status of surrounding vehicles through the information of the Internet of Vehicles and judges whether it is feasible to change lanes at the moment according to the minimum safe distance of lane changing and whether there will be a collision during the process of changing lanes. At the same time, during the lane change process, try to ensure that the lane change curvature is small and cannot exceed the maximum value. If possible, in order to ensure the comfort of the occupants during the lane change process, the lane change acceleration and jerk should also be as small as possible. In addition, we take the electric vehicle lane changing as the research object, use the energy consumed by the motor drive to judge the economic index, and use the lane-changing time to reflect the lane-changing efficiency. Obviously, we can see that the shorter the lane-changing time, the faster the lane-changing and the higher the lane-changing efficiency, but this will affect the comfort of the occupants when changing lanes. These constraints in the lane change process interact with each other. Therefore, in order to reasonably plan the lane-changing trajectory, we established a multi-objective function and optimized the solution by combining the different objectives in different lane-changing scenarios.

4.1. Safety

In the process of changing lanes, the lateral position of the vehicle should be within the planned displacement; the longitudinal restraint should be minimal to meet the safety distance requirements. At the same time, in order to ensure the stability of lane changing, the curvature should also be limited.

$$0 < y(t) < w \tag{12}$$

$$\begin{cases} 0 < \dot{x}(t) < v_{x,max} \\ 0 < \dot{y}(t) < v_{y,max} \end{cases} \tag{13}$$

$$K(t) = \frac{y''(x(t))}{(1 + y'[x(t)]^2)^{1.5}} < K_{max} \tag{14}$$

$$\begin{cases} MSS_{M_1,M_0} + d_0 < d_{M_1,M_0} \\ MSS_{M_2,M_0} + d_0 < d_{M_2,M_0} \\ MSS_{M_4,M_0} + d_0 < d_{M_4,M_0} \end{cases} \tag{15}$$

In the above Equations (12)–(15), $y(x)$ is the lateral displacement of the vehicle; $v_{x,max}$ and $v_{y,max}$ are the thresholds of longitudinal and lateral speeds, respectively; K_{max} is the threshold of curvature; d_0 is the minimum longitudinal distance that the vehicle should maintain with other vehicles when cruising [29]; d_{M_1,M_0}, d_{M_2,M_0}, and d_{M_4,M_0} are the longitudinal distances between the vehicle and other vehicles, respectively.

4.2. Comfort

The comfort evaluation of the vehicle is mainly based on the instantaneous acceleration during the lane-changing process. In order to prevent the vehicle from slipping and becoming unstable, the lateral acceleration should be less than 0.4 g [30]. In order to improve the comfort as much as possible and ensure that the impact of the lane-changing process is small, the following constraints can be applied to the lateral acceleration and longitudinal acceleration, respectively, if there are no obstacles when changing lanes or if the distance from the surrounding vehicles is large enough when changing lanes [31].

$$\begin{cases} |\ddot{x}| < 2.5 \\ |\ddot{y}| < 2 \end{cases} \tag{16}$$

4.3. Lane Change Efficiency

During the lane-changing process, under the premise of ensuring safety, if the lane-changing time is shorter, then the lane-changing efficiency will be higher. Therefore, we regard the lane-changing efficiency as an optimization index of the lane-changing time so as to make the lane-changing time as small as possible, as shown in Equation (17) below.

$$min J(t_f) = t_f \tag{17}$$

4.4. Economy

Here, the so-called economy refers to the energy consumption of the entire car. For electric vehicles, electrical energy is mainly consumed in terms of efficiency of power devices, loss of transmission mechanisms, overcoming driving resistance, and on-board electrical equipment. Due to the fact that losses and other factors are only related to the efficiency coefficient, and the power consumption of on-board appliances and other devices is not closely related to lane changing, research will not be considered in lane changing. The driving resistance closely related to vehicle movement is related to the process of lane-changing behavior. Therefore, this article focuses on optimizing the energy consumption of lane-changing resistance. We will use the energy consumed to complete the entire lane-changing task as an economic indicator. Driving resistance energy consumption is mainly used to overcome driving resistance, including rolling resistance, air resistance, slope resistance, and acceleration resistance. Among them, rolling resistance and slope resistance are mainly related to road conditions and generally do not change during lane change, which can be regarded as fixed values. The generation of acceleration resistance is mainly used to increase vehicle speed. Due to the high-speed driving conditions, the driving resistance is mainly air resistance. So, we only consider the energy consumed by air resistance here.

$$E \approx F \cdot S = \frac{C_D A v^3 t_f}{21.15} \tag{18}$$

In the above Equation (18), E is the energy consumed in the process of changing lanes, F is the air resistance, S is the displacement of the vehicle in the process of changing lanes; C_D is the air resistance coefficient, which is generally between 0.30 and 0.41 for cars [32]; A is the windward area of the car, which is generally 1.7~2.1 m^2 [32]; v is the speed of the car, and t_f is the lane change time.

In summary, we establish the final multi-objective function as follows, see Equation (19).

$$\begin{aligned} \min J = \beta_1 \frac{\sqrt{\ddot{x}(t)^2 + \ddot{y}(t)^2}}{a_{max}} + \beta_2 \frac{t_f}{t_{max}} + \beta_3 \frac{C_D A \left(\sqrt{\ddot{x}(t)^2 + \ddot{y}(t)^2}\right)^3 dt}{21.15 E_{max}} \\ \text{s.t.} \quad & 0 < y(t) < w \\ & 0 < \dot{x}(t) < v_{x,\max} \\ & 0 < \dot{y}(t) < v_{y,\max} \\ & \left|\ddot{x}\right| < a_{x,\max} \\ & \left|\ddot{y}\right| < a_{y,\max} \\ & MSS(i,M) + d_0 < d(i,M) \\ & \beta_1 + \beta_2 + \beta_3 = 1 \end{aligned} \tag{19}$$

The three items in Equation (19) represent the acceleration, the lane-changing time, and the energy consumption during the lane-changing, respectively, where β_1, β_2, and β_3 are the weight coefficients of each objective in the multi-objective function with the values all between 0 and 1, and the sum of them is 1. As mentioned above, t_f is the lane change time, $\dot{x}(t)$ and $\dot{y}(t)$ are the speeds in the longitudinal and lateral directions of the lane change, respectively; $\ddot{x}(t)$ and $\ddot{y}(t)$ are the longitudinal and lateral accelerations of lane changing, respectively, a_{max}, t_{max}, and E_{max} are the maximum acceleration, the maximum lane-changing time, and the maximum energy consumption constraint value, respectively, which will be used to normalize the three items on the right side of the objective function. a_{max} can be obtained from the lateral and longitudinal acceleration thresholds, and t_{max} is the maximum time for changing lanes, which is generally taken as 6 s; E_{max} can be obtained from the maximum lane change time and the end speed of the lane change. In different lane-changing scenarios and conditions, we can adjust the three weight coefficients to find the optimal lane-changing time and complete the lane-changing.

5. Weight Setting Considering Different Driving Needs

In fact, it is difficult to define a completely optimal lane change trajectory curve. Usually, when making choices under different circumstances and by different drivers, multiple factors are considered for balance. For example, efficiency seekers tend to accelerate lane changes with shorter lane changing times in order to shorten the overall driving time. The design of the lane-changing trajectory in this article mainly considers efficiency, comfort, and economy. In order to adapt to the needs of various drivers in this article, the Analytic Hierarchy Process (AHP) is used to calculate the weight of the objective function for efficiency, comfort, and economy under each driving demand so as to make the optimal trajectory obtained from the objective function more adaptive.

As early as the 1970s, the Analytic Hierarchy Process (AHP) was proposed by Satty et al. as a method for determining indicator weights; it reduces human subjectivity by comparing the importance of indicators, thereby improving weight accuracy.

The weights to be established in this article include comfort β_1, efficiency weight β_2, and economy weight β_3. Analytic Hierarchy Process is applied to establish the weights.

Firstly, the lane-changing scenarios that require weight setting are classified as follows (Table 1):

Table 1. Classification of lane-changing situations.

Lane Changing Scene *u*	**Driving Needs *v***
1: Vehicles without surrounding obstacles	1: Comfort demand
	2: Efficiency demand
	3: Economic demand
2: Vehicles with surrounding obstacles	1: Comfort demand
	2: Efficiency demand
	3: Economic demand

In both cases, the selection of the optimal trajectory for lane changing should also be different. The analysis is divided into two categories:

5.1. Vehicles without Surrounding Obstacles

At this point, there is no risk of collision caused by surrounding vehicles for lane-changing tasks, and the needs of the driver should be fully considered. For efficiency demand, it is believed that the pursuit of lane change completion time is short. Therefore, the importance of efficiency indicators is the highest. Secondly, the importance of comfort indicators and economic indicators is equivalent. According to the table below (Table 2), assign values to element a_{ij} in the judgment matrix A_{uv}. At this point, the most important item is considered extremely important. Similarly, other needs should also be considered accordingly.

Table 2. Judgment matrix assignment table.

a_{ij} Value	**Meaning**
3	Indicator *i* is extremely important compared to indicator *j*
2	Indicator *i* is important compared to indicator *j*
1	Indicator *i* and indicator *j* are equally important
reciprocal	If the indicator *i* is not significant compared to the indicator *j*, the reciprocal is taken

Therefore, their judgment matrices can be obtained as follows:

Among them, A_{uv} represents the judgment matrix of the demand v in the scenario u, $u = 1,\ 2$ $v = 1, 2, 3$. After calculating the above judgment matrix using the Analytic Hierarchy Process, the corresponding feature vectors W_{uv} and consistency ratios CR_{uv} can be obtained:

$$A_{11} = \begin{bmatrix} 1 & 3 & 3 \\ 0.33 & 1 & 1 \\ 0.33 & 1 & 1 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 1 & 0.33 & 1 \\ 3 & 1 & 3 \\ 1 & 0.33 & 1 \end{bmatrix} \quad A_{13} = \begin{bmatrix} 1 & 1 & 0.33 \\ 1 & 1 & 0.33 \\ 1 & 3 & 1 \end{bmatrix} \tag{20}$$

Among them, A_{uv} represents the judgment matrix of the demand v in the scenario u, $u = 1,\ 2$ $v = 1, 2, 3$. After calculating the above judgment matrix using the Analytic

Hierarchy Process, the corresponding feature vectors W_{uv} and consistency ratios CR_{uv} can be obtained:

$$\begin{gathered} W_{11} = (0.6, 0.2, 0.2)^T \\ CR_{11} = 0 \\ W_{12} = (0.2, 0.6, 0.2)^T \\ CR_{12} = 0 \\ W_{13} = (0.2, 0.2, 0.6)^T \\ CR_{13} = 0 \end{gathered} \tag{21}$$

The values within the feature vector W_{uv} are the weights of the objective function $(\beta_1, \beta_2, \beta_3)$, so a more accurate weight value was obtained through the Analytic Hierarchy Process based on a preliminary comparison of various indicators. And the consistency ratio CR_{uv} is less than 1, all of which pass the consistency test.

5.2. Vehicles with Surrounding Obstacles

In this case, changing lanes too late can lead to insufficient distance between the vehicle and the obstacle vehicle, resulting in safety risks due to the loss of lane-changing conditions. Therefore, in order to ensure the safety of the autonomous driving process, efficiency indicators should have extremely high importance. This article sets the same efficiency demand as before, with efficiency indicators being extremely important while the other two indicators are equally important. For the other two needs, efficiency indicators should still be extremely important, while their respective demand indicators are more important than the remaining indicators. Therefore, the judgment matrix for this is as follows:

$$A_{21} = \begin{bmatrix} 1 & 0.33 & 2 \\ 3 & 1 & 3 \\ 0.50 & 0.33 & 1 \end{bmatrix} \; A_{22} = \begin{bmatrix} 1 & 0.33 & 1 \\ 3 & 1 & 3 \\ 1 & 0.33 & 1 \end{bmatrix} \; A_{13} = \begin{bmatrix} 1 & 0.33 & 0.50 \\ 3 & 1 & 3 \\ 2 & 0.33 & 1 \end{bmatrix} \tag{22}$$

After calculating the above judgment matrix using the Analytic Hierarchy Process, the eigenvectors W_{uv} and consistency ratio CR_{uv} can be obtained:

$$\begin{gathered} W_{21} = (0.252, 0.589, 0.159)^T \\ CR_{21} = 0.051 \\ W_{22} = (0.2, 0.6, 0.2)^T \\ CR_{12} = 0 \\ W_{23} = (0.159, 0.589, 0.252)^T \\ CR_{13} = 0.051 \end{gathered} \tag{23}$$

It can be seen that the weights have undergone corresponding changes according to comparative analysis, and the consistency ratios CR_{uv} are all less than 1, passing the consistency test.

For the different driving needs mentioned above, the corresponding weight values are substituted into the objective function to obtain the optimal lane-changing time that meets the driving needs.

6. Simulation Solution

6.1. Free Lane Change under Barrier-Free Traffic Vehicle Scenarios

When intelligent electric vehicles are driving on highways, they generate the intention to change lanes, and if there are no other vehicles around, they can freely change lanes. The vehicle used in the simulation experiment has a length of 4.2 m and a lane width of 3.75 m. The speed of the lane is 25 m/s, and the speed at the end of the lane change is 30 m/s. According to the aforementioned weight design section, the allocation of weight coefficients for each indicator is provided. When solving multi-objective functions, the obtained comfort demand weight vector W_{11}, efficiency demand weight vector W_{12}, and economic demand weight vector W_{13} can be replaced with the objective function weight $(\beta_1, \beta_2, \beta_3)$ to calculate the optimal lane change time in each case, the value obtained from the AHP algorithm used in Section 5 is Equation (21).

In order to obtain the accurate lane-changing time, here, we select the particle swarm optimization algorithm (PSO), the basic idea of which is to find the optimal solution of the function through the individual extreme value and the global optimal value in the group, and this function is the fitness function. Equations (24) and (25) below are the solution formulas of particle swarm optimization (PSO).

$$V_i^{t+1} = \omega V_i^t + c_1 r_1 \left(Pbest_i^t - X_i^t\right) + c_2 r_2 \left(Gbest_g^t - X_i^t\right) \tag{24}$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{25}$$

where ω is the inertia weight; $Pbest_i^t$ is the historical optimal solution searched from the ith particle to the tth generation; $Gbest_g^t$ is the optimal solution currently searched by the entire particle swarm; X_i^t, and V_i^t are the current position and flight speed of the ith particle, respectively; c_1 and c_2 are acceleration factors; r_1 and r_2 are random numbers between [0, 1].

We set the population size to 10 particles; the maximum velocity is 0.5, c_1 and c_2 both are 2. The fitness function is a multi-objective function. As shown in Figures 4 and 5, the solution method achieved stability after six iterations. The objective function J curve in Figure 5 shows that the PSO algorithm has successfully converged to the optimal value. The PSO algorithm provides the optimal solution and value of the objective function in the form of particle position and fitness function value. The optimal solutions given are the optimal lane changing time under comfort demand is 5.2 s, the optimal lane changing time under efficiency demand is 2.8 s, and the optimal lane-changing time under economic demand is 2.9 s. From Figure 5, it can be seen that the objective function value at this time has indeed reached its lowest point under the three demands.

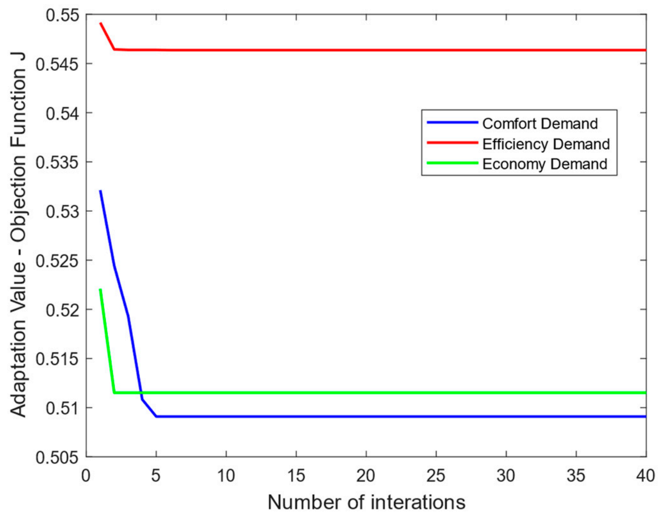


Figure 4. Particle swarm optimization (PSO) iteration.

In the obstacle-free traffic scenario, based on the optimal lane change time and Equation (11), the optimal lane change trajectory and lane change speed of the lane change vehicle under various demands are obtained by simulating the theoretical curve using MATLAB, as shown in Figure 6a,b. The yaw rate and lateral acceleration of lane-changing vehicles are shown in Figure 6c,d.

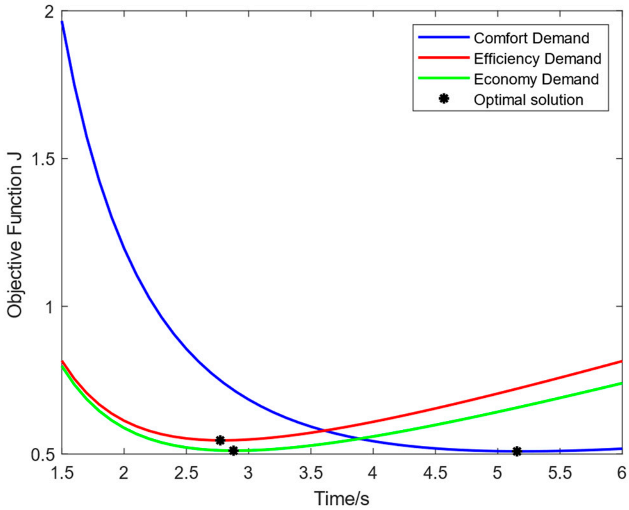


Figure 5. Lane change time solution in barrier-free scenarios.

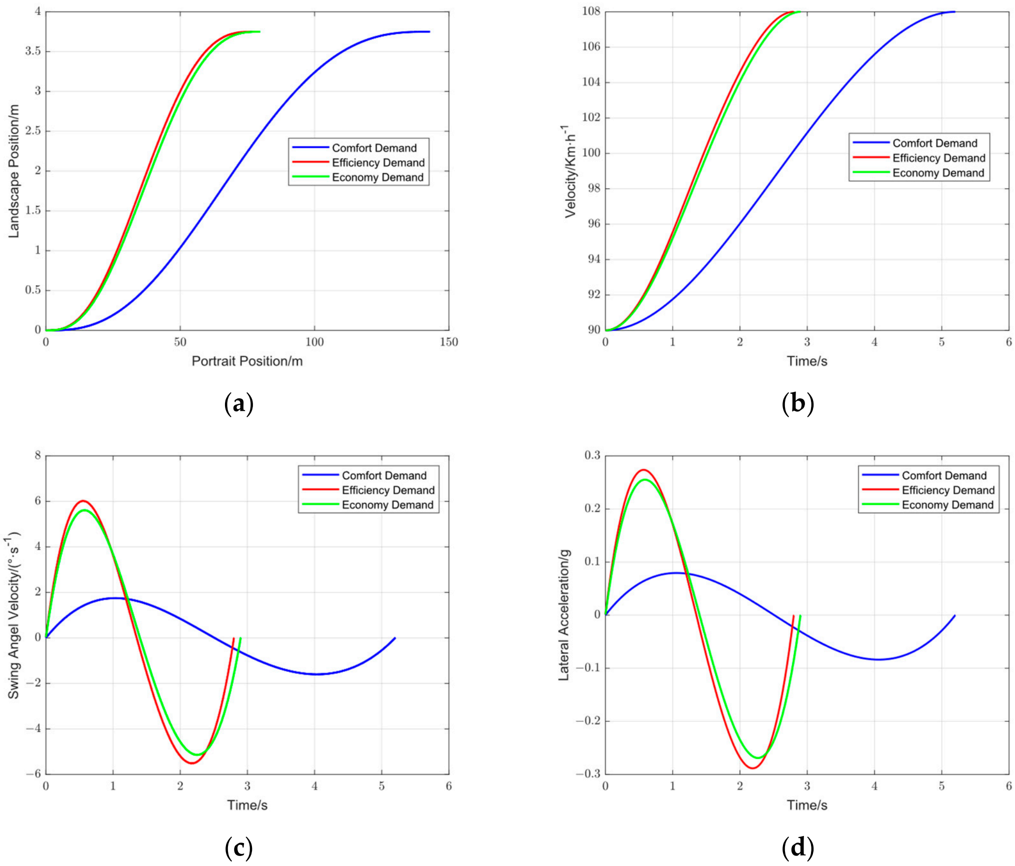


Figure 6. Lane change simulation in barrier-free scene; they should be listed as (**a**) track trace curve, (**b**) lane change speed curve, (**c**) yaw angular velocity curve, (**d**) lateral acceleration curve.

The simulation results in Figure 5 indicate that the planned trajectory and speed are smooth and consistent with the actual lane-changing situation. The efficiency demand and economic demand result in shorter lane-changing times and more rapid lane changing. The efficiency demand aims to shorten the lane-changing time and achieve driving needs through faster acceleration. The economic demand is also achieved by shortening the lane change time to reduce energy consumption during the lane change period. The trajectory obtained under comfort demand has a longer lane changing time and smoother lane changing. During the lane-changing process, the lateral acceleration is not greater than 0.1 g, meeting the comfort demand. Under the above demands, the lateral acceleration of the trajectory is far less than 0.4 g, which is within the stable driving range. In the process of solving the lane change time in obstacle-free lane change scenarios, the requirements of multi-objective planning were met.

6.2. Changing Lanes in Obstructed Traffic Scenarios

When there are traffic vehicles around, lane changing requires more consideration for safety. Assuming that the self-driving vehicle is traveling at a speed of 25 m/s in the self-driving lane, and the vehicles in the target lane and other vehicles in the self-driving lane are traveling at a constant speed, that is, the speed of vehicle M_0 in the self-driving lane is 25 m/s, and the speed of all vehicles in the target lane is 30 m/s. When the self-owned vehicle intends to change lanes, the feasibility of changing lanes will be determined based on the minimum safe distance. Assuming that at the beginning of the lane change, the longitudinal distance between the self-owned vehicle and the vehicle M_1 in front of the target lane is 20 m, the longitudinal distance between the self-owned vehicle and the vehicle M_2 behind the target lane is 30 m, and the speed of the vehicle M_1 in front of the target lane is higher than that of the vehicle M_0, there is generally no collision. When vehicle M_0 cruises with other vehicles, the safe distance between vehicle M_0 and vehicle M_2 is set to a minimum longitudinal distance of 3 m [29]. Based on the judgment of the safe distance for the previous lane change, after calculation, it is determined that the lane change can be carried out. In this case, due to the presence of traffic vehicles around, lane-changing efficiency is the primary consideration for safety reasons. By substituting the comfort demand weight vector W_{21}, efficiency demand weight vector W_{22}, and economic demand weight W_{23} vector in Equation (23) with the objective function weight $(\beta_1, \beta_2, \beta_3)$ into Equation (19), and applying the particle swarm optimization algorithm, the optimal lane-change time in this scenario can be obtained. After three iterations, the objective function reaches convergence, and the optimal lane change time for comfort demand is 3.1 s, for efficiency demand is 2.8 s, and for economic demand is 2.5 s. Figure 7 is the simulation Scene graph diagram of the target lane change process, and Figures 8 and 9 are the iteration diagram of the particle swarm optimization algorithm and the solution diagram of the lane change time, respectively. Through Figure 8, it is proven that the PSO algorithm is effective at this time.

Figure 7. Lane changing scene based on Carsim environment.

In this case, except for the efficiency demand that remains unchanged from the previous situation, it can be seen from Figure 10a,b that the trajectory tracking and speed planning during the lane change process under other indicator demands are in line with expectations. From Figure 10c,d, it can be seen that the yaw rate and lateral acceleration are also within the expected range but generally increase compared to lane changes in obstacle-free scenarios. This is because in lane-changing scenarios with obstacles, we consider safety and lane-changing efficiency more. As a result, the lane change time becomes shorter and comfort decreases.

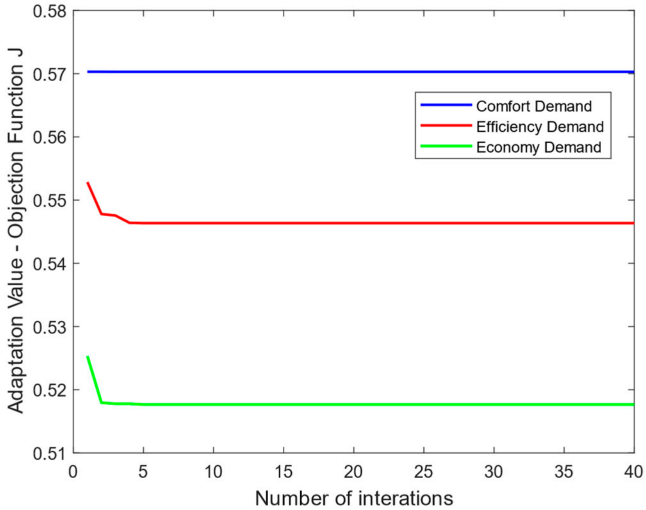


Figure 8. Iteration graph of Particle Swarm Algorithm.

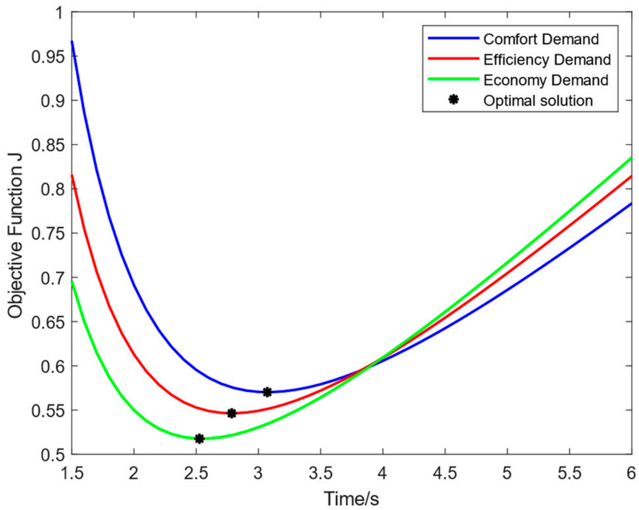


Figure 9. The solution of the lane-changing time of vehicles with obstacles.

Figure 10e shows the longitudinal distance between vehicle M_0 and the vehicle in front of the lane during the lane change time for each indicator demand. They did not collide. Within approximately 6 s, the M_0 in the target lane has exceeded the vehicle M1 in front of the original lane. From Figure 10f, it can be seen that after the vehicle stably changed lanes to the target lane, there was no collision with the vehicle before and after the target lane, and both were within the safe driving range. In summary, we can conclude that in the scene of traffic vehicles changing lanes with obstacles, the planned trajectory meets our expectations and can achieve safe, comfortable, and energy-saving lane changes that meet driving needs.

In order to verify the effect of economic indicators studied in this article on lane change trajectory planning, the energy consumption during the lane change process was calculated for the first two cases. Figure 11 shows the energy consumption to overcome air resistance under different lane change conditions. In an obstacle-free scenario, the energy consumption of the comfort demand trajectory is about $4.231 \cdot 10^4$ N·m, the energy consumption of the efficiency demand trajectory is about $2.287 \cdot 10^4$ N·m, and the energy consumption of the economic demand trajectory is about $2.367 \cdot 10^4$ N·m. However, in traffic vehicle scenarios with obstacles, the comfort demand trajectory energy consumption is about $2.529 \cdot 10^4$ N·m, the efficiency demand trajectory energy consumption is about $2.287 \cdot 10^4$ N·m, and the economic demand trajectory energy consumption is about $2.044 \cdot 10^4$ N·m. From the above data, it can be seen that under the demands of efficiency and economy, the energy consumption during the lane-changing process will be greatly reduced. However, due to the low importance of economic indicators given to the trajectory under comfort demands, the energy consumption during the lane-changing process will greatly increase. From the above analysis, it can be seen that under the condition of meeting the constraints of safe lane changing, when considering economic factors during lane changing, the energy consumption during the lane changing process can be significantly reduced.

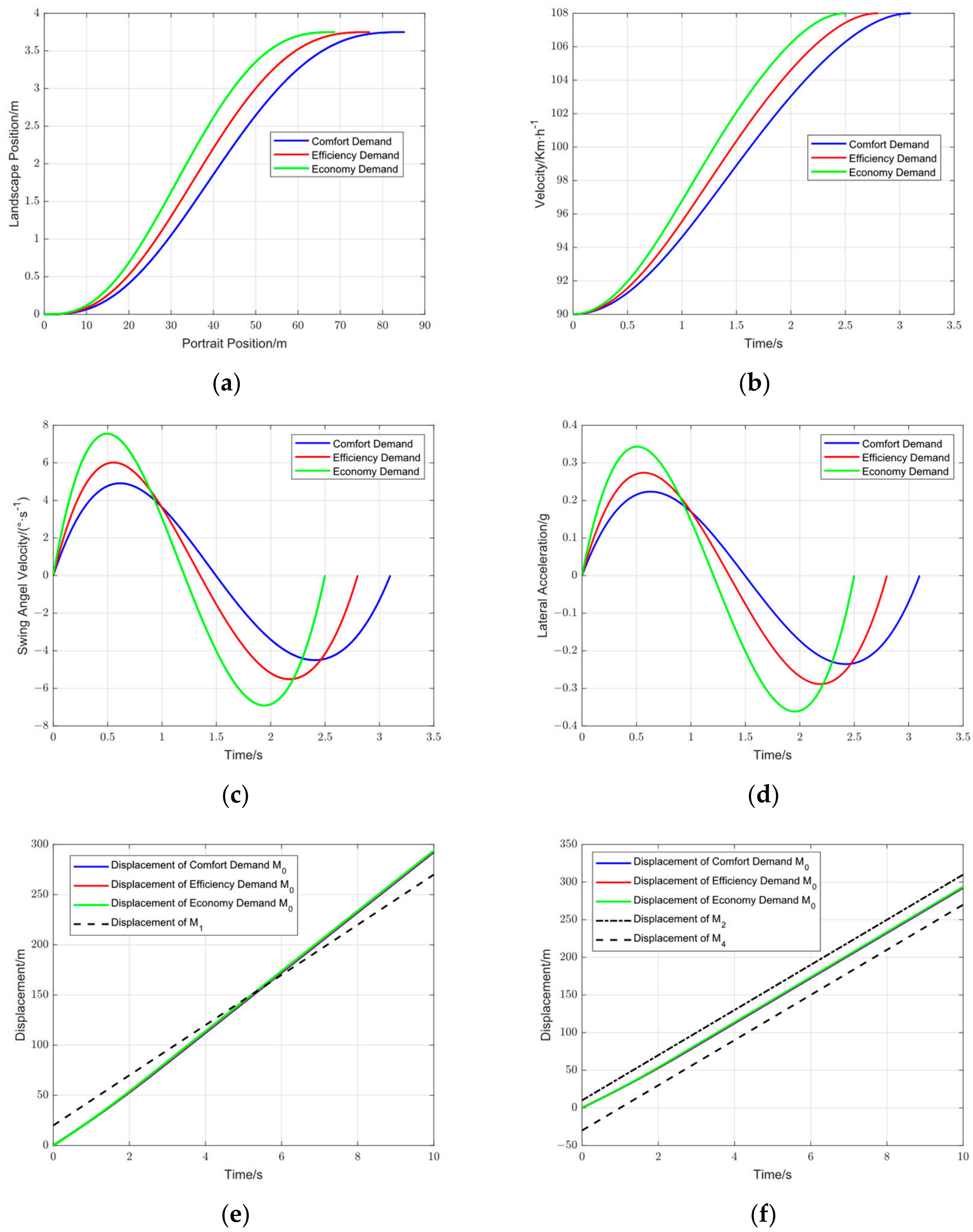
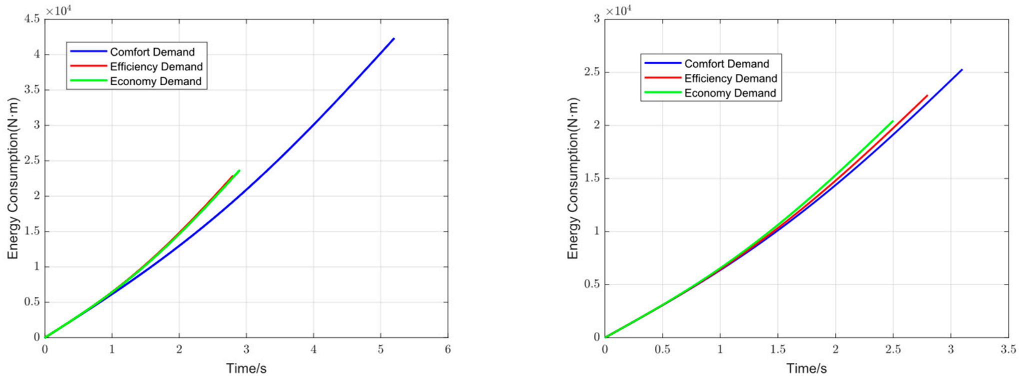


Figure 10. Simulation of lane changing for vehicles with obstacles; they should be listed as (**a**) trajectory tracking curve; (**b**) lane change speed curve; (**c**) yaw angular velocity curve; (**d**) lateral acceleration curve; (**e**) longitudinal displacement with the preceding vehicle in the original lane; (**f**) the longitudinal displacement of the vehicle with the target lane.



(**a**) (**b**)

Figure 11. Energy consumption in different lane-changing scenarios; they should be listed as (**a**) without surrounding obstacle vehicles and (**b**) with surrounding obstacle vehicles.

Due to the efficiency and economic demands, energy consumption during lane changing is reduced by shortening the lane-changing time. For a vehicle that needs to complete a fixed-length driving task, the total energy consumption not only considers the lane-changing process but also includes the energy consumption required to supplement the longitudinal distance difference caused by different lane-changing processes. As the lane change process studied in this article is a variable speed process (90–108 Km/h), the straight section can be completed by 90 Km/h (first straight, then lane change) or 108 Km/h (first lane change then straight). Here, for the lane-changing behavior in the above two situations, the following table is established to analyze the total energy consumption.

Table 3 above provides a more reasonable comparison of the energy consumption throughout the entire process. It can be seen that in each lane changing scenario, if we first go straight at 90 Km/h and then change lanes, the energy consumption situation is as follows: comfort demand energy consumption is the highest, followed by economic demand energy consumption, and finally efficiency demand energy consumption, where economic demand energy consumption is close to efficiency demand energy consumption. If we change lanes first and then proceed straight at 108 Km/h, the energy consumption will be greater than the comfort demand for efficiency and economy.

This is mainly because energy consumption is mainly used to overcome air resistance, which is closely related to vehicle speed at high speeds. During the lane change process, the lateral speed can be ignored, mainly due to the forward speed of the vehicle. Therefore, the speed during the lane change process is between 90 and 108 Km/h. The efficiency demand for completing the lane change quickly and the economic demand for vehicles need to compensate for the longitudinal displacement before or after the lane change. The speed before the lane change is less than after the lane change, so if the lane change is performed first, it will actually increase energy consumption.

Table 3. Overall energy consumption analysis during driving process.

Lane Changing Scene	Driving Needs	longitudinal Displacement/m	Direct Travel Time/s		Energy Consumption during Lane Changing Process 10^4/(N·m)	Direct Energy Consumption 10^4/(N·m)		Total Energy Consumption 10^4/(N·m)	
			90 Km/h	108 Km/h		90 Km/h	108 Km/h	90 Km/h	108 Km/h
Vehicles without surrounding obstacles	Comfort demand	143	0	0	4.231	0	0	4.231	4.231
	Efficiency demand	77	2.64	2.2	2.287	1.592	2.293	3.879	4.580
	Economic demand	79.75	2.53	2.11	2.367	1.526	2.199	3.893	4.566
Vehicles with surrounding obstacles	Comfort demand	85.25	0	0	2.529	0	0	2.529	2.529
	Efficiency demand	77	0.33	0.275	2.287	0.199	0.287	2.486	2.574
	Economic demand	68.75	0.66	0.55	2.044	0.398	0.573	2.442	2.617

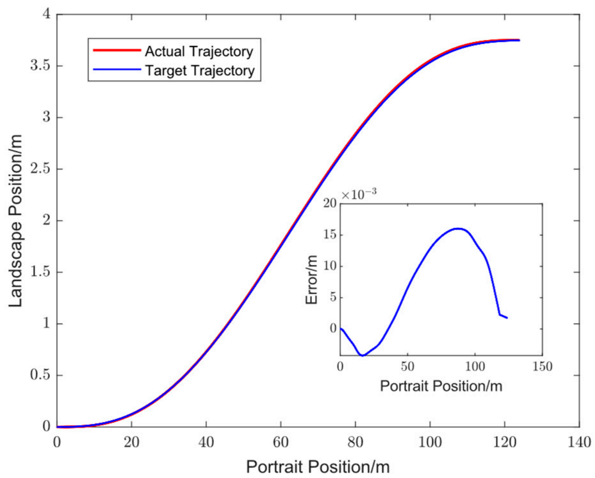
In general, vehicles with efficiency demand tend to complete the lane change task as soon as possible. Therefore, changing lanes first and then going straight is beneficial for them. They can fully utilize the speed after accelerating the lane change to complete their remaining tasks. Therefore, the energy consumption of vehicles with efficiency demand is more inclined towards the total energy consumption (108 Km/h) value in Table 3 above. For vehicles with economy demand, in order to reduce their energy consumption, they can choose to use low-speed straight ahead and then quickly change lanes. Its energy consumption is closer to the total energy consumption (90 Km/h) value in Table 3 above. Therefore, the economic demand of a vehicle can greatly reduce energy consumption. From the above table, it can be seen that when the vehicle is driven in a predetermined manner, the economic demand vehicle can reduce energy consumption by about 7.99% compared to the comfort demand vehicle without surrounding obstacles and reduce energy consumption by about 15.0% compared to the efficiency demand vehicle. In the presence of surrounding obstacles, the energy consumption of vehicles with comfort demand is reduced by about 3.4%, and the energy consumption of vehicles with efficiency demand is reduced by about 5.13%, demonstrating the effect of increasing economic indicators on the objective function. Therefore, the trajectory planned in this article has energy-saving characteristics.

6.3. Traceability Analysis

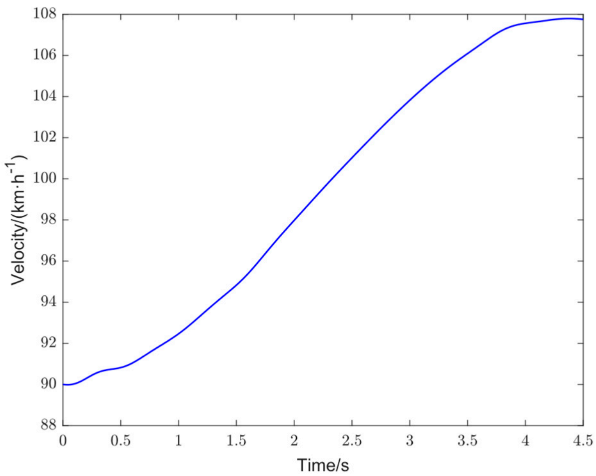
To test the traceability performance of the lane change trajectory designed in this article under low-level control, lane change trajectories with lane change times of 4.5 s and 3.4 s were selected, and tracking control experiments were conducted using Carsim software 2019.0.

Here, we use Carsim software for simulation processing, import the required trajectory into it, and select a C-class vehicle to drive on a one-way, two-lane highway with an adhesion coefficient of 0.85 to change lanes. The other dynamic model parameters of the vehicle are shown in Table 4.

Firstly, the trajectory tracking results obtained when the lane change time is 4.5 s in Figure 12:

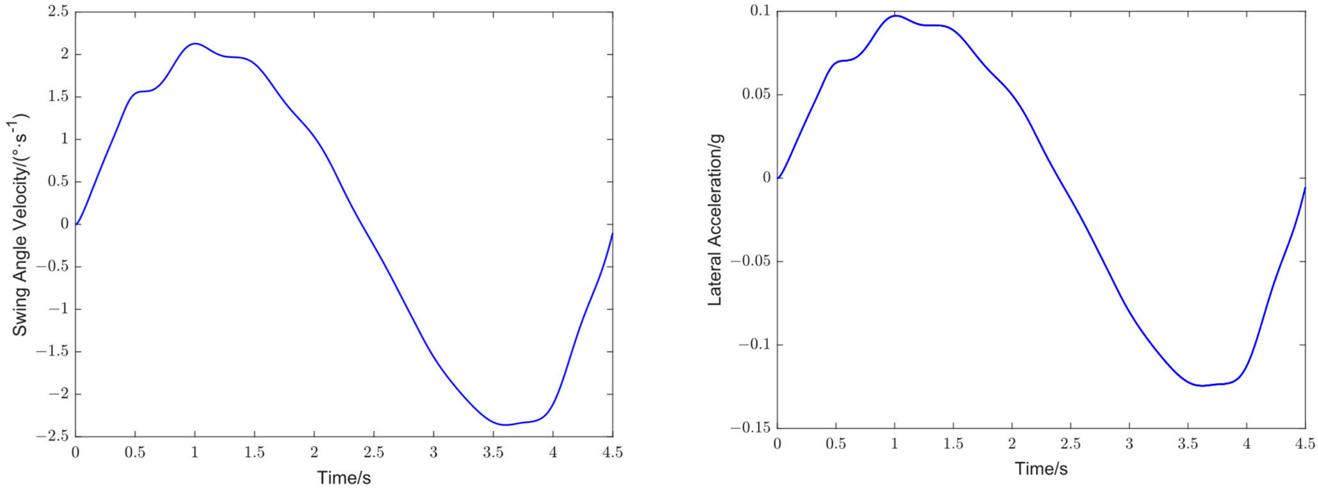


(a)



(b)

Figure 12. *Cont.*



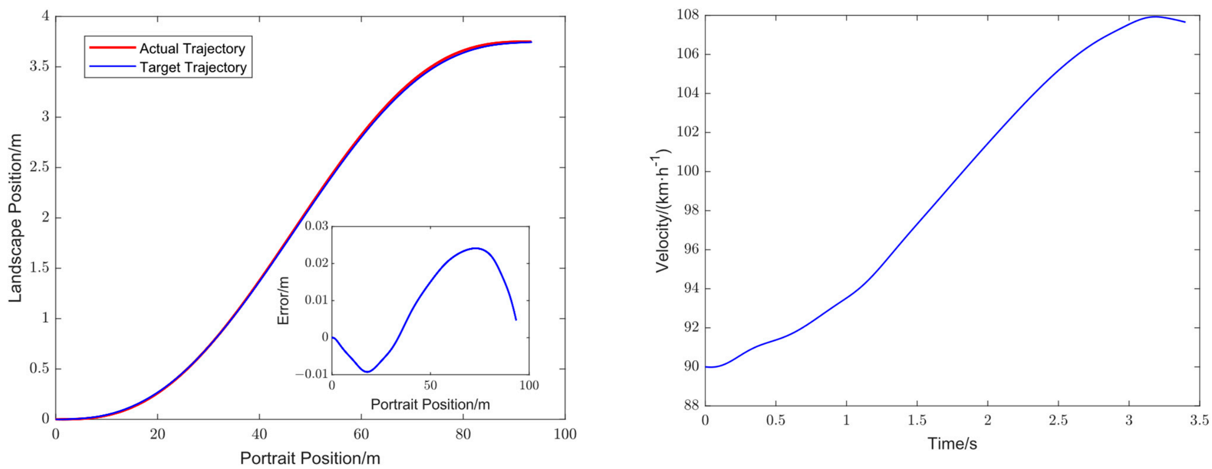
(**c**) (**d**)

Figure 12. Lane change simulation in barrier-free scene; they should be listed as (**a**) track trace curve; (**b**) lane change speed curve; (**c**) yaw angular velocity curve; (**d**) lateral acceleration curve.

Table 4. Kinetic model parameters.

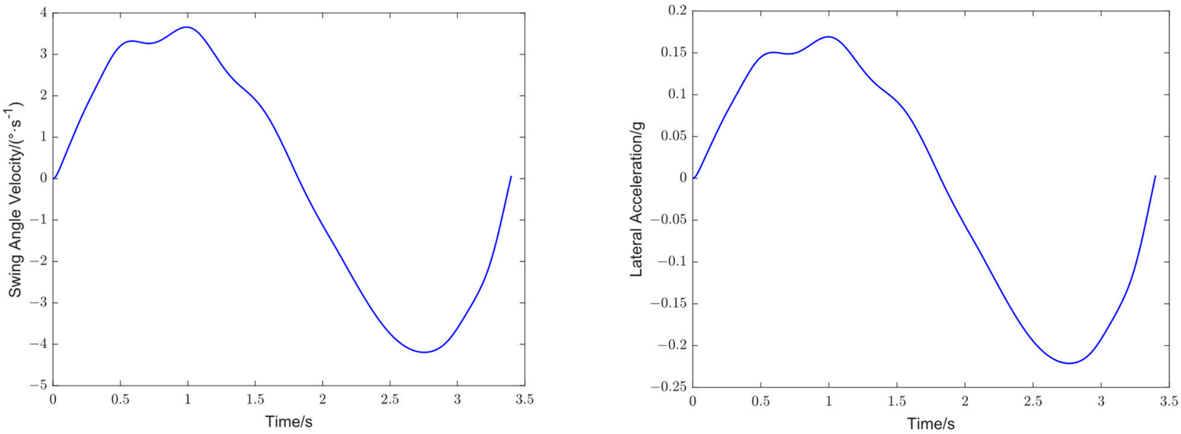
Model Parameters	Symbol	Value	Unit
Sprung mass	m	1592	kg
Distance from center of mass to front axle	a	1180	mm
Distance from center of mass to rear axle	b	1770	mm
Wheelbase	d	2950	mm
Centroid height	h	540	mm
Moment of inertia around the Z axis	I_Z	2488	kg·m^2
Tire radius	R	325	mm

In the case of a shorter lane change time of 3.4 s, the trajectory tracking results are shown in Figure 13:



(**a**) (**b**)

Figure 13. *Cont.*



(**c**) (**d**)

Figure 13. Simulation of lane changing for vehicles with obstacles; they should be listed as (**a**) trajectory tracking curve; (**b**) lane change speed curve; (**c**) yaw angular velocity curve; (**d**) lateral acceleration curve.

Based on the Carsim tracking test results above, it can be seen that the lane change trajectory planned in this paper has excellent tracking performance and minimal tracking error. During the lane change, all parameters of the vehicle change smoothly, the driving speed meets the set changes, and the trajectory meets the design requirements. The lateral acceleration is much less than 0.4 g, and the vehicle can maintain stable driving. Therefore, the lane change trajectory planning algorithm in this article is effective.

6.4. Real-Time Analysis

In order to verify the real-time performance of the PSO algorithm in this article, a comparison was made with the nonlinear programming function fmincon commonly used in MATLAB in research. By solving the objective function set in this article 500 times, the solving time was recorded as shown in the following Figure 14:

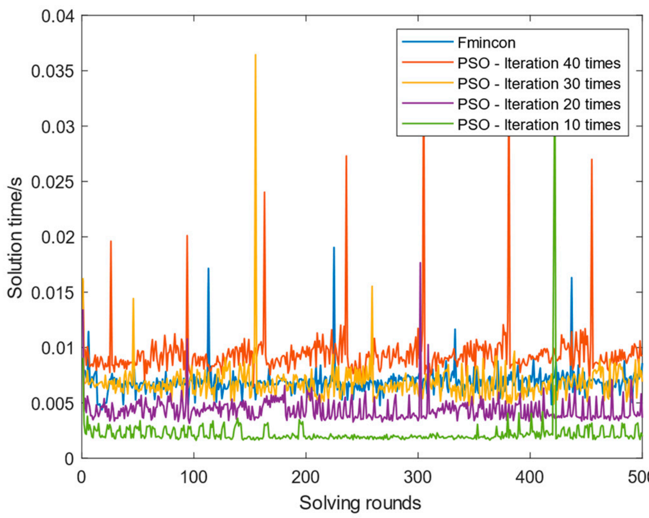


Figure 14. Comparison of real-time performance of solving functions.

From Figure 14, it can be seen that the PSO algorithm has a computational efficiency comparable to the MATLAB fmincon function in 30 iterations. Due to the fact that the variable to be optimized for the objective function involved in this article is only the lane change time t, the difficulty of solving is low. The PSO algorithm can converge in less than

10 iterations. Therefore, the PSO algorithm can greatly improve the real-time performance of the algorithm designed in this article.

7. Conclusions

Based on the research on vehicle lane changing, this paper decouples the longitudinal and lateral trajectories of intelligent vehicle lane changing, adds constraints, and performs polynomial planning. From the above research, we can draw the following conclusions:

(1) The polynomial coefficients are determined by the lane-changing boundary conditions. The economy is introduced into a multi-objective function that considers comfort, safety, and lane-changing efficiency. This can more perfectly describe the lane-changing behavior and obtain a safe, comfortable, and energy-saving lane-changing trajectory. In addition, considering economic indicators while meeting lane change constraints can reduce vehicle energy consumption, which is beneficial for energy conservation.

(2) The weight coefficients of each indicator in the multi-objective function are adjusted using the Analytic Hierarchy Process for different lane-changing scenarios and driving needs. The particle swarm optimization algorithm is then introduced to optimize the solution. This makes the lane-changing model simple, practical, and advantageous. At the same time, the planned trajectory is the optimal trajectory in the current environment, which is adaptive.

(3) The simulation is carried out in Carsim, and the results show that the planned trajectory curve is gentle, satisfies the constraints, and has no collision risk, whether in the obstacle-free lane-change scene or in the obstacle-changed lane-change scene. In addition, this paper fully considers the safety, comfort, economy, and efficiency of lane changing, which reflects the rationality of lane-changing trajectory planning.

In the future, this article will continue to study how to complete various driving demand planning in the entire driving task scenario using V2X technology, consider incorporating uncertainty factors to analyze the impact on the algorithm in this article, and improve the algorithm's anti-interference ability in real environments. In addition, better planning for speed during lane changing is also being considered in future research.

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Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zhao, D.; Lam, H.; Peng, H.; Bao, S.; LeBlanc, D.J.; Nobukawa, K.; Pan, C.S. Accelerated evaluation of automated vehicles safety in lane-change scenarios based on importance sampling techniques. *IEEE Trans. Intell. Transp. Syst.* **2016**, *18*, 595–607. [CrossRef] [PubMed]
2. Kerner, B.S. Failure of classical traffic flow theories: Stochastic highway capacity and automatic driving. *Phys. A Stat. Mech. Its Appl.* **2016**, *450*, 700–747. [CrossRef]
3. Wang, Y.; Wei, C.H.; Liu, E.; Li, S.R. Lane-changing trajectory planning method for automated vehicles under various road line-types. *IET Smart Cities* **2020**, *2*, 14–23. [CrossRef]

4. Yavas, U.; Kumbasar, T.; Ure, N.K. A New Approach for Tactical Decision Making in Lane Changing: Sample Efficient Deep Q Learning with a Safety Feedback Reward. In Proceedings of the 2020 IEEE Intelligent Vehicles Symposium (IV), Las Vegas, NV, USA, 19 October–13 November 2020; pp. 1156–1161.
5. Gipps, P.G. A model for the structure of lane-changing decisions. *Transp. Res. Part B Methodol.* **1986**, *20*, 403–414. [CrossRef]
6. Hidas, P. Modelling lane changing and merging in microscopic traffic simulation. *Transp. Res. Part C Emerg. Technol.* **2002**, *10*, 351–371. [CrossRef]
7. Wang, Z.; Cui, S.; Yu, T. Automatic Lane Change Control for Intelligent Connected Vehicles. In Proceedings of the 2019 4th International Conference on Electromechanical Control Technology and Transportation (ICECTT), Guilin, China, 26–28 April 2019; pp. 286–289.
8. Ahmed, K.I. Modeling Drivers' Acceleration and Lane Changing Behavior. Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 1999.
9. Toledo, T.; Koutsopoulos, H.N.; Ben-Akiva, M.E. Modeling Integrated Lane-Changing Behavior. *Transp. Res. Rec. J. Transp. Res. Board* **2003**, *1857*, 30–38. [CrossRef]
10. Kita, H. A merging-giveway interaction model of cars in a merging section: A game theoretic analysis. *Transp. Res. Part A Policy Pract.* **1999**, *33*, 305–312. [CrossRef]
11. Alia, C.; Gilles, T.; Reine, T.; Ali, C. Local Trajectory Planning and Tracking of Autonomous Vehicles, Using Clothoid Tentacles Method. In Proceedings of the 2015 IEEE intelligent vehicles symposium (IV), Seoul, Republic of Korea, 28 June–1 July 2015; pp. 674–679.
12. Liu, Y.; Wang, X.; Li, L.; Cheng, S.; Chen, Z. A Novel Lane Change Decision-Making Model of Autonomous Vehicle Based on Support Vector Machine. *IEEE Access* **2019**, *7*, 26543–26550. [CrossRef]
13. Ye, F.; Cheng, X.; Wang, P.; Chan, C.Y.; Zhang, J. Automated Lane Change Strategy Using Proximal Policy Optimization-Based Deep Reinforcement Learning. In Proceedings of the 2020 IEEE Intelligent Vehicles Symposium (IV), Las Vegas, NV, USA, 19 October–13 November 2020.
14. Li, L.; Xu, Z.H.G.; Zhao, X.M.; Wang, G.P. Review of Motion Planning Methods of Intelligent Connected Vehicles. *China J. Highw. Transp.* **2019**, *32*, 20–33.
15. Nelson, W. Continuous-curvature paths for autonomous vehicles. In Proceedings of the 1989 International Conference on Robotics and Automation, Scottsdale, AZ, USA, 14–19 May 1989; IEEE: Scottsdale, AZ, USA, 1989; pp. 1260–1264.
16. Wei, C.; Wang, Y.; Asakura, Y.; Ma, L. A nonlinear programing model for collision-free lane-change trajectory planning based on vehicle-to-vehicle communication. *J. Transp. Saf. Secur.* **2019**, *13*, 936–956. [CrossRef]
17. Ammoun, S.; Nashashibi, F.; Laurgeau, C. An Analysis of the Lane Changing Maneuvers on Roads: The Contribution of Inter-vehicle Cooperation Via Communication. In Proceedings of the 2007 IEEE Intelligent Vehicles Symposium, Istanbul, Turkey, 13–15 June 2007; IEEE: Istanbul, Turkey, 2007; pp. 1095–1100.
18. Niu, G.C.; Li, W.S.; Wei, H.X. Intelligent Vehicle Lane Changing Trajectory Planning Based on Double Quintic Polynomials. *Automot. Eng.* **2021**, *43*, 978–986+1004.
19. Zhang, R.H.; You, F.; Chu, X.N.; Guo, L.; He, Z.C.; Wang, R.B. Lane Change Merging Control Method for Unmanned Vehicle Under V2V Cooperative Environment. *China J. Highw. Transp.* **2018**, *31*, 180–191.
20. Wu, S.F.; Wei, M.X.; Teng, D.C.; Wang, Y.; Xing, D.X.; Ren, S.T. Lane Changing Path Planning Algorithm and Tracking Control of Intelligent Vehicles on Curved Roads. *J. Chongqing Univ. Technol. Nat. Sci.* **2021**, *35*, 18–28.
21. Zhang, Z.Y.; Long, K.; Du, R.H.; Huang, C.X. Trajectory Tracking Coordinated Control for Autonomous Vehicle in High-speed Overtaking. *Automot. Eng.* **2021**, *43*, 995–1004.
22. Yang, B.; Zhang, H.H.; Jiang, Z.S. Simulation Analysis of Obstacle Avoidance Path Planning and Tracking Control for Intelligent Vehicles. *China Meas. Test* **2021**, *47*, 71–78.
23. Yang, W.; Li, C.; Zhou, Y.P. A Path Planning Method for Autonomous Vehicles Based on Risk Assessment. *World Electr. Veh. J.* **2022**, *13*, 234. [CrossRef]
24. Zhang, S.L.; Deng, G.H.; Yang, E.C.; Ou, J. Optimal Vehicle Lane Change Trajectory Planning in Multi-Vehicle Traffic Environments. *Appl. Sci.* **2022**, *12*, 9662. [CrossRef]
25. Zhao, Y.; Yu, T.; Zhang, Q.; Wang, Y. *Multi-Lane Lane Change Decision and Trajectory Planning of Vehicle Based on Logic Regression Algorithm and Gaussian Probability Density Model*; SAE International: Warrendale, PA, USA, 2022.
26. Li, Y.; Bai, H.J.; Zhao, Y.N.; Liu, H.W. Micro-Path Planning for Lane-Changing Return of Autonomous Vehicles. In *CICTP 2022: Intelligent, Green, and Connected Transportation, Proceedings of the 22nd COTA International Conference of Transportation Professionals, Changsha, China, 8–11 July 2022*; American Society of Civil Engineers: Reston, VA, USA, 2022; pp. 2152–2163.
27. Qu, D.Y.; Chen, W.J.; Yang, W.S.; Wan, M.F.; Cao, J.Y. Analysis and Modeling of Vehicle Interactions during Lane-changing Process. *J. Highw. Transp. Res. Dev.* **2016**, *33*, 88–94.
28. Jula, H.; Kosmatopoulos, E.B.; Ioannou, P.A. Collision avoidance analysis for lane changing and merging. *IEEE Trans. Veh. Technol.* **2000**, *49*, 2295–2308. [CrossRef]
29. Xiang, Y.; Luo, Y.G.; Cao, K.; Li, K.Q. Automatic Lane Change Control Based on Vehicle-to-vehicle Communications. *J. Highw. Transp. Res. Dev.* **2016**, *33*, 120–126+145.
30. Sun, C.; Wu, C.Z.; Chu, D.F.; Fu, Y.H.; Cui, H.L.; Yu, Y. Safety Evaluation of Driving on Curves Based on Analysis of Vehicle Lateral Stability. *J. Transp. Inf. Saf.* **2014**, *32*, 95–100.

31. Li, W.; Wang, J.; Duan, J.M. Lane changing trajectory planning of intelligent vehicles based on polynomials. *Comput. Eng. Appl.* **2012**, *48*, 242–245.
32. Yu, Z.S. *Automobile Theory*, 5th ed.; China Machinery Industry Press: Beijing, China, 2009.