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Time-Varying Reliability Analysis of Multi-Cracked Beams Considering Maintenance Dependence

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Abstract: Time-varying reliability models of multi-cracked beam structures are established in this paper, which provide a theoretical method for the safety evaluation of multi-cracked beam structures. The reliability models proposed in this paper consider the interaction between the complex statistical correlations between system parameters during system operation and maintenance correlations, which is a difficult problem in the time-varying reliability modeling that takes into account work mechanisms and maintenance behavior. In the proposed models, multiple cracked elements are regarded as a dependent series system. The stresses, crack extensions, and multiple failure modes between each element constitute the complex failure dependence of the system. The time-varying reliability models of a multi-cracked beam structure are established via the neural network method and failure dependence analysis. Moreover, the failure dependence coefficient is proposed to quantify the time-varying failure dependence. Based on the working principle of the beam structures and the maintenance mechanism for the cracked state of the beams, a time-varying system reliability mode considering the maintenance dependence is proposed. Furthermore, the maintenance dependence coefficient index is proposed to quantitatively measure the interaction between the maintenance dependence and the failure dependence. Finally, the validity of the model is verified through the Monte Carlo simulation method. In the numerical examples, the relationship between maintenance dependence and failure dependence is illustrated and the influences of the statistical characteristics of the maintenance characteristic parameters on the maintainability and failure dependence are analyzed.

Keywords: reliability; maintenance dependence; failure dependence; multi-cracked beam; failure mode dependence

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1. Introduction

Due to the defects generated in the processing and manufacturing process, and the metal fatigue resulting from alternating stress during operation and overload conditions, cracks may exist in the components of mechanical systems. These cracks may result in a reduction in the mechanical properties of mechanical components, causing the components to be more likely to fail far earlier than under the design load, resulting in economic losses and threatening personal safety [1–3]. Although cracks are common defects in mechanical parts, it is important to take appropriate measures to suppress or improve the impact of crack defects on mechanical components and mechanical systems [4–6]. In this paper, time-varying reliability models for multi-cracked beam structure will be established. In the models, not only is the complex statistical correlation mechanism between system parameters during system operation considered, but also the maintenance dependence generated by the complex maintenance behavior. The interaction mechanism between the failure dependence during system operation and the maintenance dependence are also considered.

In mechanical systems, many mechanical components can be viewed as beam structures from the perspective of mechanical analysis [7]. Therefore, the multi-cracked beam structure will be studied in this paper. A reliability and maintenance quantitative evaluation model will be developed in this paper. Some scholars have conducted in-depth research on the analysis of beam structures with multiple cracks [8–10]. Skrinar presented a simplified computational model for the calculation of multi-cracked beams with linearly varying heights [11]. The model extended the utilization of the principle of virtual work to obtain a stiffness matrix and a load vector for a uniform load over the whole element. The model can be applied to analyze the response of multi-cracked beams that include an arbitrary number of transverse cracks. A novel technique for the vibration and stability analyses of axially loaded beams with multiple cracks was proposed by Kisa [12], which combines mode synthesis methods and the finite element method. The technique can be used to predict the crack status in the defected structures. A justification of the localized flexibility model of an open crack in a beam in bending deformation was proposed by Caddemi [13]. The results showed that the formulation and solution of the bending problem for multi-cracked beams can be included in the classical formalism of the theory of distributions. Kisa proposed a modal analysis method of beams with a circular cross-section and containing multiple non-propagating open cracks by combining the finite element and component mode synthesis methods [14]. The method can be used to calculate the natural frequencies and analyze the mode shapes of a beam with an arbitrary number of cracks.

During the process of system operation, the crack state parameters will show random characteristics. Uncertainty in the maintenance effect can also lead to new uncertainties in the crack state and the system's operating state after repair. Regarding the uncertainty analysis of multi-cracked beam structures, some in-depth research has been carried out. An explicit treatment of dynamic problems of damaged structures in the presence of cracks with variable intensities was developed by Cannizzaro, exploiting the generalised function approach [15]. The proposed models were applied to an analysis of structures with uncertain cracks. A method based on interval analysis to assess the dynamic response of damaged beams was presented by Cannizzaro [16]. This method is capable of handling uncertainties in beam structures and inferring the upper and lower bounds of response parameters without introducing any probability content. By introducing uncertainty in the model, the proposed method effectively evaluates the dynamic response range of damaged beams while significantly reducing the computational burden. Santoro proposed an approach to compute the bounds of the response for multi-cracked beams with uncertain parameters [17]. According to the response function for each uncertain parameter, two different models were adopted to calculate the response bounds. This approach provided accurate bounds even for large uncertainties. Moreover, a non-probabilistic approach to evaluate the frequency response of multi-cracked beams was presented by Santoro [18]. The interval variables were used to describe the parameters of each crack, instead of the traditional probabilistic approach. Furthermore, a two-step method to evaluate the bounds of all response variables was presented in the proposed models. Time-varying reliability evaluation methods for crack-containing beam structures, considering maintainability, need further study.

Although studies have been conducted on the performance and uncertainty analysis of multi-cracked structures, the following difficulties often occur during the evaluation of the time-varying reliability and maintainability of multi-cracked beam structures:

1. Multiple cracks in a beam structure interact with each other. The stresses in cracks are statistically dependent under common operating loads. Furthermore, the stress dependence results in an interaction between the expansion rates of multiple cracks, which again results in a statistical correlation between the stresses in the vicinity of each crack at different moments. This complex correlation is a key challenge that needs to be addressed to ensure a quantitative assessment of the time-varying reliability and maintainability of beam structures. Although the stress-strength interference model and its extension can reflect the relationship between stress and strength, the working

mechanism of cracked parts and the complex effects of crack defects on these parts cannot be considered in the model.
2. Cracks and their extension have a large impact on the system stiffness, and also generate large stresses, which affect the system strength degradation. Therefore, there is a statistical correlation between the two failure modes of stiffness degradation and the strength degradation of crack-containing beam structures. The complex correlation of the stresses mentioned above makes a correlation analysis of the two failure modes even more complex.
3. When repairing mechanical parts, the randomness of crack repair leads to varying operational states in multi-cracked parts after repair. Therefore, maintainability is closely related to the failure dependence (FD) of components. Maintainability and FD during work tend to be relatively independent in traditional models. Reliability modeling that considers the complex statistical correlations and mechanical mechanisms of factors such as stresses and cracks, and that is capable of quantitatively evaluating the relationship between maintainability dependence (MD) and system FD, faces great challenges.

In order to solve the above problems, the complex mechanism of FD and MD is analyzed in detail in this paper. Moreover, quantitative time-varying reliability models for multi-cracked beam structures are developed considering MD and FD. In Section 2, the multi-cracked structure is regarded as a dependent series system. The complex statistical correlation mechanism among the working load, crack depth, crack expansion rate, stress, stiffness and strength is discussed. In Section 3, a time-varying reliability model for multi-cracked beam structures is developed, which considers the interaction of FD and MD. In Section 4, the correctness and validity of the model are verified through Monte Carlo simulations (MCS). Furthermore, the interaction between the FD and MD is illustrated through the examples. Finally, the conclusions are given in Section 5.

2. Failure-Related and Repair-Related Mechanism Analysis of Multi-Cracked Beams

2.1. Stress Analysis

In this paper, the distributions of load and the material parameters should be known. In a multi-cracked beam structure, as shown in Figure 1, individual cracks may lead to the overall failure of that beam structure. According to the definition of reliability, these cracks constitute a series system in the logical structure of reliability analysis. These cracks interact with each other under common operating loads. In order to facilitate the analysis, the beam structure is divided into a separate element for each crack according to the location in which the crack is located. For a beam structure with N cracks, the effect of N elements on the failure of the whole system needs to be considered. These N elements constitute an N-dimensional series system. In addition, the cracks in each element generate higher stresses in the vicinity of the cracks under the action of F. These stresses are the intrinsic cause of transient failure or crack extension. The stresses in the vicinity of different crack tips under the action of working load F are calculated using the finite element method as follows [19]:

$$M_{\mathrm{B}}\ddot{X} + C_{\mathrm{B}}\dot{X} + K_{\mathrm{B}}X = F \tag{1}$$

where M_{B} is the overall mass matrix of the multi-cracked beam structure, C_{B} is the damping matrix of the multi-cracked beam structure, and K_{B} is the overall stiffness matrix of the multi-cracked beam structure. X is the overall displacement matrix, and F is the external force matrix. In order to easily characterize the effect of multiple cracks on the time-varying reliability and maintainability of the system, a combination of planar solid elements and contact elements is used to characterize the contact effects and response calculations of the cracked surfaces. This method is also an important method in the current structural mechanical analysis of cracked beams. The contact force at the cracked surface can be expressed as follows:

$$F_n = \begin{cases} 0 & \varepsilon > 0 \\ K_n \varepsilon^{\,q} + \Gamma\dot{\varepsilon} & \varepsilon \leqslant 0 \end{cases} \tag{2}$$

where K_n is the contact normal stiffness, ε is the penetration distance between contact nodes, and Γ is the damping coefficient. In the finite element calculation, using 1/4 node element, the crack tip can be simulated and the crack tip singularity problem can be solved to obtain the approximate stress value near the crack tip under different accuracy requirements [20].

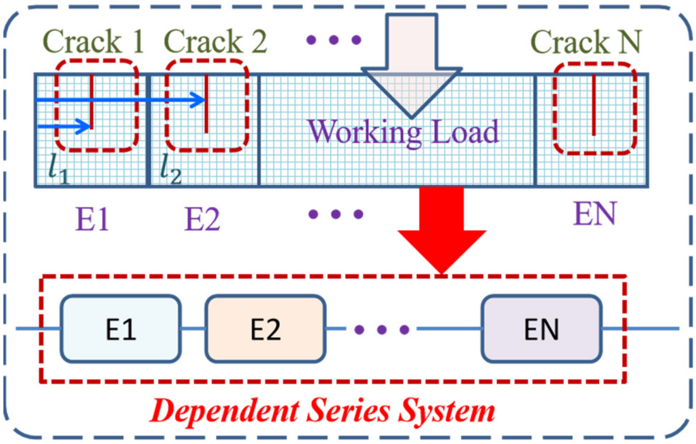

Figure 1. Reliability logic block diagram of a multi-cracked beam.

2.2. FD Analysis

The above method enables the calculation of the stresses in different elements in a multi-cracked beam structure. Stress is the main cause of FD in the system. Hence, in this section, an FD mechanism analysis will be performed based on stress analysis. The stress in the jth element can be expressed as:

$$s_j = \tau_j(F) \tag{3}$$

The state of a crack (crack depth, crack location, etc.) has an effect on the state of other cracks. Thus, the above equation can be further expressed as:

$$s_j = \tau_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N) \tag{4}$$

The extension of the crack in the jth element is generally determined by a combination of factors such as material properties and stress. The crack depth can be expressed as:

$$a_j = \lambda_j(s_j) = \lambda_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N) \tag{5}$$

From the relationship between crack depth and stress in each element, the above equation can be further expressed as:

$$a_j = \Theta_j(F, a_1, a_2, a_{j-1}, a_{j+1}, \cdots, a_N) \tag{6}$$

From the above derivations, it can be seen that the stresses between each element, the crack extension of the element cracks, and the depth of the element cracks are statistically correlated throughout the working process of the beam structure when the randomness of the working loads is considered. This correlation is time-varying and dynamically interacts over the whole working cycle. These phenomena will result in complex time-varying FD for crack-containing beam structures, which will make time-varying reliability modeling more difficult.

Furthermore, the repeated action of the working loads and the combination of factors such as corrosion and stresses will lead to a continuous decrease in the residual strength of the beam structure. The residual strength of each element is calculated as follows:

$$r_j = \epsilon_j(s_j) = \epsilon_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N) \tag{7}$$

The continuous expansion of cracks in each cracked element causes a time-varying degradation in the stiffness of the cracked beam element. The degradation is related to each crack, as well as to the stresses. The stiffness of each element is:

$$k_j = \omega_j(a_j) = \omega_j(F, a_1, a_2, a_{j-1}, a_{j+1}, \cdots, a_N) \quad (8)$$

Therefore, the above statistical correlations of stress, crack extension rate and crack depth among different crack elements directly lead to the statistical correlations of the cracks in the two failure modes of stiffness degradation and strength degradation. The statistical correlation of failure modes and the above statistical correlation between crack elements complicate the time-varying FD of the system.

2.3. Interaction between MD and FD

The maintenance of a cracked beam structure is required when it is operated for a specified period of time or when the crack depth reaches a certain threshold. Although mechanical repairability models provide more analytical methods and quantitative calculation models, the metrics to measure repair are often vague and not easy to directly quantify based on the operating principles and design parameters of the mechanical structure system. Characterizing repair effectiveness in terms of crack depth and crack expansion rate has a clear physical meaning and can be used for maintainability and reliability analyses of crack-containing structures.

In the process of cracked beam repair, the repair of cracks and material parameters tends to be more random due to the working conditions, repair techniques, repair tools and other factors. The ideal repair is the maximum repair, which can restore the beam structural performance to the original level. However, this kind of repair is more idealized for cracked structures, and the reality is a more imperfect repair, that is, the crack characteristics and material parameters of cracked elements are restored to a certain level, and the degree of this restoration has a certain degree of randomness. In this case, the degree of recovery of the properties of different crack elements is highly statistically correlated, i.e., MD, due to the working habits of the repairers, the use of the same repair equipment for group repairs, etc. Specifically, if repairs are performed by different maintenance personnel and different repair tools are used on different parts, the randomness of the degree of recovery of each part of the system after repair is independent. However, if the same equipment or the same maintenance personnel are used to perform uniform repairs on a group of parts, the randomness of the degree of recovery of different parts may be statistically correlated. Therefore, the maintainability model under the traditional independence assumption is not applicable to this MD analysis. MD is also highly correlated with the FD between different cracked elements after repair through crack parameters, which, in turn, affects the reliability assessment during operation as shown in Figure 2. It can be seen that the MD and the system time-varying FD affect each other. An in-depth analysis of the two forms of dependence based on the system working mechanism is needed, as well as the further development of a quantitative reliability model that considers the effects of the two forms of dependence at the same time. In order to quantify the above statistical correlations, failure dependence coefficients and maintenance dependence coefficients will be proposed for quantitative characterization. MD analysis will help maintenance personnel to more accurately assess the actual time-varying reliability of the repaired system based on the actual maintenance equipment, maintenance experience and maintenance level. Furthermore, the models provide an analytical basis for maintenance strategies based on quantitative calculations.

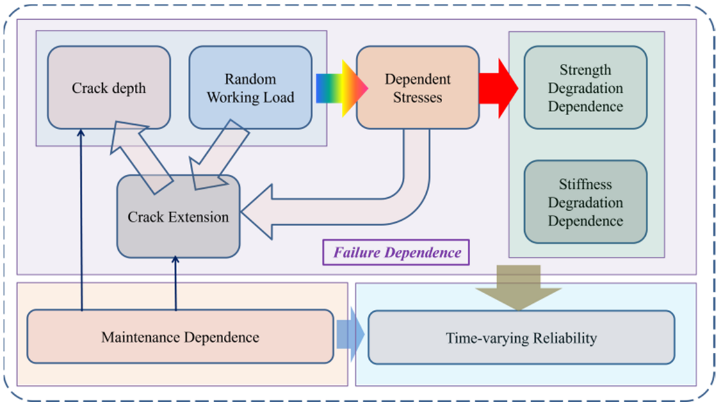

Figure 2. Failure dependence and maintenance dependence.

3. Time-Varying Reliability Model of the Systems Considering MD

3.1. FD and System Reliability Assessment during the Working Period

3.1.1. Reliability for Independent System

The classical series system reliability models need to be obtained by synthesizing the structure function and the reliability of each component, expressed in the following form:

$$R = R_1 R_2 \cdots R_n \tag{9}$$

In order to use each component strength and stress as inputs to the component reliability function, the component reliability can be calculated as follows:

$$R_j = P(s_j < r_j)$$

$$= \int_0^{\infty} f_{s_j}(\tau_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N)) \int_{s_j}^{\infty} f_{r_j}(\epsilon_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N)) dr_j ds_j \tag{10}$$

Hence, the reliability of the system can be expressed as:

$$R = \prod_{j=1}^{\mathrm{N}} \int_0^{\infty} f_{s_j}(\tau_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N)) \int_{s_j}^{\infty} f_{r_j}(\epsilon_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N)) dr_j ds_j \tag{11}$$

The above model can only be considered for static system reliability calculations. If the time effects and the time-varying effects of stresses and residual strengths of individual cracked elements during different operational cycles are taken into account, the probability that the system will not fail within n operational cycles, i.e., the reliability, can be calculated as follows:

$$R_1(n) = \prod_{i=1}^{\mathrm{n}} \prod_{j=1}^{\mathrm{N}} \int_0^{\infty} f_{s_j}(\tau_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N, i)) \int_{s_j}^{\infty} f_{r_j}(\epsilon_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N, i)) dr_j ds_j \tag{12}$$

where $\tau_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N, i)$ and $\epsilon_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N, i)$ denote the stress and residual strength of the jth element at the ith operational cycle, respectively. The corresponding system failure rate is:

$$\psi(n) = \left(\prod_{i=1}^{n}\prod_{j=1}^{N}\int_0^{\infty} f_{s_j}(\tau_j(F,s_1,s_2,s_{j-1},s_{j+1},\cdots,s_N,i))\int_{s_j}^{\infty} f_{r_j}(\epsilon_j(F,s_1,s_2,s_{j-1},s_{j+1},\cdots,s_N,i))dr_j ds_j - \prod_{i=1}^{n+1}\prod_{j=1}^{N}\int_0^{\infty} f_{s_j}(\tau_j(F,s_1,s_2,s_{j-1},s_{j+1},\cdots,s_N,i))\int_{s_j}^{\infty} f_{r_j}(\epsilon_j(F,s_1,s_2,s_{j-1},s_{j+1},\cdots,s_N,i))dr_j ds_j\right) / \left(\prod_{i=1}^{n}\prod_{j=1}^{N}\int_0^{\infty} f_{s_j}(\tau_j(F,s_1,s_2,s_{j-1},s_{j+1},\cdots,s_N,i))\int_{s_j}^{\infty} f_{r_j}(\epsilon_j(F,s_1,s_2,s_{j-1},s_{j+1},\cdots,s_N,i))dr_j ds_j\right) \quad (13)$$

The above equation is a model used to calculate the reliability index obtained according to the classical system reliability theory. However, it should be pointed out that, as shown by the analysis in Section 2, the stresses of each element in the system are statistically correlated and receive the joint influence of crack depth and working load. Moreover, the changes in the stresses in different operational cycles also affect each other. Therefore, it is necessary to develop system reliability models that can take into account a complex FD.

Before establishing a system reliability model considering a complex FD, it is necessary to solve the stress solution problem. Since the stresses in each element are influenced by each other, it is unrealistic to try to directly obtain the common probability density function for different operating cycles and different elements. Meanwhile, by separately counting the probability distribution function of the stresses in each element, the correlation between the stresses in each element is neglected, which may result in a large error in the reliability assessment. Due to the need to consider the material parameters, crack characteristics and the randomness of the working load, the analytical expression of the element stresses is difficult, and a particularly large sample size is required in the statistical analysis, which is impossible to afford in practice.

3.1.2. Stress Calculation

In order to solve the above problem, the effect of working load and different crack states on the internal stresses in different operational cycles is considered in this paper by means of a BP neural network (BPNN). The BPNN consists of a multi-layer structure including an input layer, a hidden layer and an output layer. The BPNN has more advantages [21]. Firstly, it has very good nonlinear approximation ability. The BPNN hidden layer is capable of nonlinear transformation and of mapping input data, while using nonlinear activation functions in each neuron, and using the back propagation algorithm to adjust the parameters of the neural network efficiently, which means that the BP neural network is able to learn and represent complex nonlinear functional relationships. Secondly, BPNN can automatically adjust the weights through training to adapt to different data distributions and relationships and flexibly adapt to different fitting tasks. Finally, BPNN are tolerant to noise and incomplete information in the input data, which can help to deal with a certain degree of data interference and missing.

In order to fit the stresses at different crack depths and working loads using BPNN, it is necessary to fit the stresses at different combinations of working loads and different crack depths. The crack depth and working load in each crack element and the individual element stress calculation can be obtained from the finite element analysis method in Section 2. In general, crack extension is a slow-changing process and the extension process can be characterized by fitting the average stress state of different operational cycles, as shown in Figure 3. Through neural network fitting, the relationship based on the working load and the stress of each element can be obtained as follows:

$$s_j = \eta_j(F,a_1,a_2,\cdots,a_N) \quad (14)$$

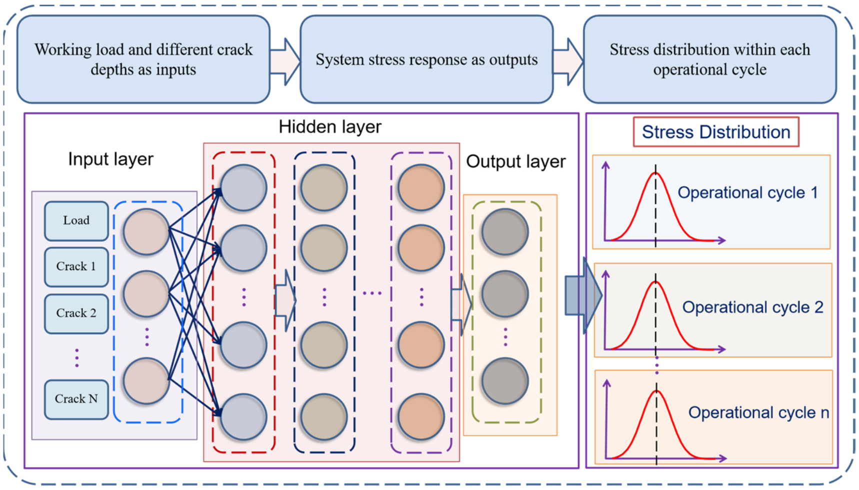

Figure 3. BPNN for stress analysis.

The trained neural network is capable of calculating the stresses in each element. Hence, when cracks in the elements are in different states, the trained neural network fit can be used directly for the calculation of the element stresses in each operational cycle.

$$s_j(i) = \eta_j(F(i), a_1(i), a_2(i), \cdots, a_N(i), i) \quad i = 1, 2, \cdots, n \tag{15}$$

This avoids the requirement for a large number of fits for the stresses in each operational cycle and it is not necessary to obtain the stress distribution in each operational cycle through a large number of data statistics. Thus, the above method is able to reduce the sample size for the statistical characterization of stresses, substantially reducing the computational burden and the cost of physical experiments.

3.1.3. Reliability for Dependent System

When the BPNN fitting results of the element stresses under different crack states and operating loads are obtained using the above method, the reliability of the system considering the FD can be calculated based on the distribution of random operating loads over different operating cycles, as follows:

$$R_2(n) = \prod_{i=1}^{n} \int_{-\infty}^{\infty} f_i(F(i)) \prod_{j=1}^{N} \int_{\eta_j(F(i), a_1(i), a_2(i), \cdots, a_N(i), i)}^{\infty} f_{r_j}(\epsilon_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N, i)) dr_j dF(i) \tag{16}$$

The corresponding system failure rate can be expressed as:

$$\begin{aligned} \psi_1(n) = \Bigg(& \prod_{i=1}^{n} \int_{-\infty}^{\infty} f_i(F(i)) \prod_{j=1}^{N} \int_{\eta_j(F(i), a_1(i), a_2(i), \cdots, a_N(i), i)}^{\infty} f_{r_j}(\epsilon_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N, i)) dr_j dF(i) \\ & - \prod_{i=1}^{n+1} \int_{-\infty}^{\infty} f_i(F(i)) \prod_{j=1}^{N} \int_{\eta_j(F(i), a_1(i), a_2(i), \cdots, a_N(i), i)}^{\infty} f_{r_j}(\epsilon_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N, i)) dr_j dF(i) \Bigg) \\ & / \left(\prod_{i=1}^{n} \int_{-\infty}^{\infty} f_i(F(i)) \prod_{j=1}^{N} \int_{\eta_j(F(i), a_1(i), a_2(i), \cdots, a_N(i), i)}^{\infty} f_{r_j}(\epsilon_j(F, s_1, s_2, s_{j-1}, s_{j+1}, \cdots, s_N, i)) dr_j dF(i) \right) \end{aligned} \tag{17}$$

The above reliability and failure rate calculations consider the FD between the working load, each unit stress and crack extension, avoiding the traditional assumption of element independence. However, there are fewer quantitative measures of FD in the existing literature, which can directly reveal the influence of different materials and design parameters on the FD and can guide the reliability design of mechanical components. In order to quantify the FD and analyze the impact of FD on reliability, the failure correlation coefficient (FDC) is proposed as follows:

$$\beta(n) = \left(\prod_{i=1}^{n}\int_{-\infty}^{\infty} f_i(F(i))\prod_{j=1}^{N}\int_{\eta_j(F(i),a_1(i),a_2(i),\cdots,a_N(i),i)}^{\infty} f_{r_j}(\epsilon_j(F,s_1,s_2,s_{j-1},s_{j+1},\cdots,s_N,i))dr_j dF(i)\right) / \left(\prod_{i=1}^{n}\prod_{j=1}^{N}\int_0^{\infty} f_{s_j}(\tau_j(F,s_1,s_2,s_{j-1},s_{j+1},\cdots,s_N,i))\int_{s_j}^{\infty} f_{r_j}(\epsilon_j(F,s_1,s_2,s_{j-1},s_{j+1},\cdots,s_N,i))dr_j ds_j\right) \quad (18)$$

FDC is an important indicator that can quantitatively characterize the FD of a system, and it can characterize the degree of system correlation and its impact on the time-varying reliability of the system. By setting different input parameters to calculate FDC, it can reflect the influence of different parameters on the FD. When the value of FDC tends toward 1, the classical reliability independent hypothesis model is consistent with the actual reliability, indicating that the FD effect is not obvious and the traditional independent hypothesis model can be used for the evaluation of the reliability of the system. The larger the value of FDC, the more obvious the FD effect. Therefore, it is necessary to pay attention to the factors that may cause the FD. The elements with a strong correlation should be analyzed in groups and modeled independently as FD subsystems in the system reliability logic block diagram. In addition, FDC can also quantitatively reflect the FD effects of a system in different operational cycles and during different tasks, so that the elements with a strong FD can be paid attention to during the specific time periods where the FDC value is larger. The proposed reliability model overcomes the shortcomings of traditional mechanical system reliability models in which independent assumptions are made about the components or empirical FD parameters that are used.

3.2. System Time-Varying Reliability Models Based on MD and FD Analyses

The models presented above are used for calculating the reliability of the multi-cracked beam structures over the working period. As mentioned in Section 2, cracked beam structures need to be repaired periodically and there is often uncertainty regarding the repair effect, which is very closely related to the state of the beam structure and the experience of the repairers, as well as the repair tools. Due to the maintenance habits, the use of the same maintenance equipment and group maintenance, the MD often has a very important influence on the operational status of the structure after maintenance. Time-varying reliability models that take into account the joint influence of maintenance uncertainty and MD should be developed. In addition, the MD has a close relationship with the FD. FD affects the state of each crack element of the beam structure, which, in turn, affects the repair effect. The repair effect will have a new impact on the FD between individual crack elements during the working period after repair, thus affecting the time-varying reliability assessment. Therefore, analytical models that can quantitatively characterize the relationship between MD and FD are proposed. In Section 3.1, FDC is able to characterize the degree of system FD and its impact on reliability. In this section, the relationship between MD and FDC will be established through the maintenance correlation coefficient (MDC) to quantitatively analyze the effects of both on the time-varying reliability of the system. In this paper, it is assumed that the strength is restored to its original extent after repair. When the structure is repaired for the mth time, the correction factor for each element crack relative to the initial crack depth $a_0 = [a_{01}, a_{02}, \cdots, a_{0N}]$ is:

$$\varphi_m = [\varphi_{1m}, \varphi_{2m}, \cdots, \varphi_{Nm}] \quad (19)$$

Then, the initial crack depth in the next working period after the mth repair of the crack can be expressed as:

$$a_{0m} = [\varphi_{1m} * a_{01}, \varphi_{2m} * a_{02}, \cdots, \varphi_{Nm} * a_{0N}] \quad (20)$$

In addition, the correction factor for the crack expansion rate of each element relative to the initial crack expansion rate $v_0 = [v_{01}, v_{02}, \cdots, v_{0N}]$ is expressed as:

$$\delta_m = [\delta_{1m}, \delta_{2m}, \cdots, \delta_{Nm}] \tag{21}$$

Then, the initial crack expansion rate in the next working period after the mth repair of the crack can be expressed as:

$$v_{0m} = [\delta_{1m} * v_{01}, \delta_{2m} * v_{02}, \cdots, \delta_{Nm} * v_{0N}] \tag{22}$$

When the randomness of the repair effect is considered, from the probability density function (PDF) of φ_m and δ_m, denoted by $f_{\varphi_m}(\varphi_m)$ and $f_{\delta_m}(\delta_m)$, respectively, the time-varying reliability considering the FD in the next operating period after the mth repair is obtained as follows:

$$R_{2\mathrm{m}}(n) = \int_{-\infty}^{\infty} f_{\varphi_m}(\varphi_m) \int_{-\infty}^{\infty} f_{\delta_m}(\delta_m) \prod_{i=1}^{\mathrm{n}} \int_{-\infty}^{\infty} f_{im}(F_m(i)) *$$

$$\left(\prod_{j=1}^{\mathrm{N}} \int_{\eta_j(F_m(i), a_{1m}(i), a_{2m}(i), \cdots, a_{Nm}(i), i)}^{\infty} f_{r_{jm}}(\epsilon_j(F_m(i), s_{1m}, s_{2m}, s_{j-1m}, s_{j+1m}, \cdots, s_{Nm}, i)) dr_{jm} dF_m(i) \right) d\varphi_m d\delta_m \tag{23}$$

The time-varying reliability under the independence assumption in the next working period after the mth repair is:

$$R_{1\mathrm{m}}(n) = \int_{-\infty}^{\infty} f_{\varphi_m}(\varphi_m) \int_{-\infty}^{\infty} f_{\delta_m}(\delta_m) \prod_{i=1}^{\mathrm{n}} \prod_{j=1}^{\mathrm{N}} \int_{0}^{\infty} f_{s_{jm}}(\tau_{jm}(F_m, s_{1m}, s_{2m}, s_{j-1m}, s_{j+1m}, \cdots, s_{Nm}, i)) *$$
$$\left(\int_{s_{jm}}^{\infty} f_{r_{jm}}(\epsilon_{jm}(F_m, s_{1m}, s_{2m}, s_{j-1m}, s_{j+1m}, \cdots, s_{Nm}, i)) dr_{jm} ds_{jm} \right) d\varphi_m d\delta_m \tag{24}$$

The FDC of the system in the next working period after the mth repair can be expressed as:

$$\beta_m(n) = \int_{-\infty}^{\infty} f_{\varphi_m}(\varphi_m) \int_{-\infty}^{\infty} f_{\delta_m}(\delta_m) \prod_{i=1}^{\mathrm{n}} \int_{-\infty}^{\infty} f_{im}(F_m(i)) *$$
$$\left(\prod_{j=1}^{\mathrm{N}} \int_{\eta_j(F_m(i), a_{1m}(i), a_{2m}(i), \cdots, a_{Nm}(i), i)}^{\infty} f_{r_{jm}}(\epsilon_j(F_m(i), s_{1m}, s_{2m}, s_{j-1m}, s_{j+1m}, \cdots, s_{Nm}, i)) dr_{jm} dF_m(i) \right) d\varphi_m d\delta_m$$
$$/ \int_{-\infty}^{\infty} f_{\varphi_m}(\varphi_m) \int_{-\infty}^{\infty} f_{\delta_m}(\delta_m) \prod_{i=1}^{\mathrm{n}} \prod_{j=1}^{\mathrm{N}} \int_{0}^{\infty} f_{s_{jm}}(\tau_{jm}(F_m, s_{1m}, s_{2m}, s_{j-1m}, s_{j+1m}, \cdots, s_{Nm}, i)) *$$
$$\left(\int_{s_{jm}}^{\infty} f_{r_{jm}}(\epsilon_{jm}(F_m, s_{1m}, s_{2m}, s_{j-1m}, s_{j+1m}, \cdots, s_{Nm}, i)) dr_{jm} ds_{jm} \right) d\varphi_m d\delta_m \tag{25}$$

When considering the MD, φ_m and δ_m will no longer be independent variables. Both will receive the common influence of maintenance personnel and maintenance equipment at the same time, which have a corresponding relationship. In this case, the initial crack depth and the initial crack expansion rate variables are expressed as a matrix with statistically correlative column elements, as follows:

$$A_0 = \begin{bmatrix} a_{01} & a_{02} & \cdots & a_{0N} \\ v_{01} & v_{02} & \cdots & v_{0N} \end{bmatrix} \tag{26}$$

The matrix of repair coefficients after the mth repair is expressed by:

$$A_1 = \begin{bmatrix} \varphi_{1m}(a_{01}, v_{01}) & \varphi_{1m}(a_{02}, v_{02}) & \cdots \varphi_{1m}(a_{0N}, v_{0N}) \\ \delta_{1m}(a_{01}, v_{01}) & \delta_{1m}(a_{02}, v_{02}) & \cdots \delta_{1m}(a_{0N}, v_{0N}) \end{bmatrix} \tag{27}$$

Then, the crack depth and the initial crack expansion rate after the mth repair become:

$$A_2 = \begin{bmatrix} \varphi_{1m}(a_{01},\ v_{01}) * a_{01} & \varphi_{1m}(a_{02},\ v_{02}) * a_{02} & \cdots \varphi_{1m}(a_{0N},\ v_{0N}) * a_{0N} \\ \delta_{1m}(a_{01},\ v_{01}) * v_{01} & \delta_{1m}(a_{02},\ v_{02}) * v_{02} & \cdots \delta_{1m}(a_{0N},\ v_{0N}) * v_{0N} \end{bmatrix} \quad (28)$$

Thus, considering the case of MD, after the mth repair, the initial crack depth and the crack expansion rate are determined by A_2, which are brought into Equation (16), the time-varying reliability considering the failure correlation in the next working period after the repair can be obtained as:

$$R_{3\mathrm{m}}(n) = \int_{-\infty}^{\infty} f_{A_1}(A_1) \prod_{i=1}^{\mathrm{n}} \int_{-\infty}^{\infty} f_{im}(F_m(i)) *$$

$$\left(\prod_{j=1}^{\mathrm{N}} \int_{\eta_{Aj}(F_m(i), a_{1Am}(i), a_{2Am}(i), \cdots, a_{NAm}(i), i)}^{\infty} f_{r_{jAm}}(\epsilon_j(F_m(i), s_{1Am}, s_{2Am}, s_{j-1Am}, s_{j+1Am}, \cdots, s_{NAm}, i)) dr_{jAm} dF_m(i) \right) \mathrm{d}A_1 \quad (29)$$

where $f_{A_1}(A_1)$ is the PDF of A_1. Similarly, the time-varying reliability under the independence assumption of the elements in the next working period after the mth repair is:

$$R_{4\mathrm{m}}(n) = \int_{-\infty}^{\infty} f_{A_1}(A_1) \prod_{i=1}^{\mathrm{n}} \prod_{j=1}^{\mathrm{N}} \int_{0}^{\infty} f_{s_{jAm}}(\tau_{jAm}(F_m, s_{1Am}, s_{2Am}, s_{j-1Am}, s_{j+1Am}, \cdots, s_{NAm}, i))^*$$

$$\left(\int_{s_{jAm}}^{\infty} f_{r_{jAm}}(\epsilon_{jAm}(F_m, s_{1Am}, s_{2Am}, s_{j-1Am}, s_{j+1Am}, \cdots, s_{NAm}, i)) dr_{jAm} ds_{jAm} \right) \mathrm{d}A_1 \quad (30)$$

The FDC of the system in the next working period after the mth repair can be expressed as:

$$\beta_{1m}(n) = \int_{-\infty}^{\infty} f_{A_1}(A_1) \prod_{i=1}^{\mathrm{n}} \int_{-\infty}^{\infty} f_{im}(F_m(i)) *$$

$$\left(\prod_{j=1}^{\mathrm{N}} \int_{\eta_{Aj}(F_m(i), a_{1Am}(i), a_{2Am}(i), \cdots, a_{NAm}(i), i)}^{\infty} f_{r_{jAm}}(\epsilon_j(F_m(i), s_{1Am}, s_{2Am}, s_{j-1Am}, s_{j+1Am}, \cdots, s_{NAm}, i)) dr_{jAm} dF_m(i) \right) \mathrm{d}A_1$$

$$/ \int_{-\infty}^{\infty} f_{A_1}(A_1) \prod_{i=1}^{\mathrm{n}} \prod_{j=1}^{\mathrm{N}} \int_{0}^{\infty} f_{s_{jAm}}(\tau_{jAm}(F_m, s_{1Am}, s_{2Am}, s_{j-1Am}, s_{j+1Am}, \cdots, s_{NAm}, i))^*$$

$$\left(\int_{s_{jAm}}^{\infty} f_{r_{jAm}}(\epsilon_{jAm}(F_m, s_{1Am}, s_{2Am}, s_{j-1Am}, s_{j+1Am}, \cdots, s_{NAm}, i)) dr_{jAm} ds_{jAm} \right) \mathrm{d}A_1 \quad (31)$$

In order to characterize the effect of MD on FD, the maintenance correlation coefficient (MDC) $\alpha_m(n)$ for the mth maintenance can be defined as follows:

$$\alpha_m(n) = \left\{ \int_{-\infty}^{\infty} f_{\varphi_m}(\varphi_m) \int_{-\infty}^{\infty} f_{\delta_m}(\delta_m) \prod_{i=1}^{\mathrm{n}} \int_{-\infty}^{\infty} f_{im}(F_m(i)) * \right.$$

$$\left(\prod_{j=1}^{\mathrm{N}} \int_{\eta_j(F_m(i), a_{1m}(i), a_{2m}(i), \cdots, a_{Nm}(i), i)}^{\infty} f_{r_{jm}}(\epsilon_j(F_m(i), s_{1m}, s_{2m}, s_{j-1m}, s_{j+1m}, \cdots, s_{Nm}, i)) dr_{jm} dF_m(i) \right) d\varphi_m d\delta_m$$

$$/ \int_{-\infty}^{\infty} f_{\varphi_m}(\varphi_m) \int_{-\infty}^{\infty} f_{\delta_m}(\delta_m) \prod_{i=1}^{\mathrm{n}} \prod_{j=1}^{\mathrm{N}} \int_{0}^{\infty} f_{s_{jm}}(\tau_{jm}(F_m, s_{1m}, s_{2m}, s_{j-1m}, s_{j+1m}, \cdots, s_{Nm}, i))^*$$

$$\left. \left(\int_{s_{jm}}^{\infty} f_{r_{jm}}(\epsilon_{jm}(F_m, s_{1m}, s_{2m}, s_{j-1m}, s_{j+1m}, \cdots, s_{Nm}, i)) dr_{jm} ds_{jm} \right) d\varphi_m d\delta_m \right\} /$$

$$\left\{\int_{-\infty}^{\infty} f_{A_1}(A_1)\prod_{i=1}^{n}\int_{-\infty}^{\infty} f_{im}(F_m(i)) *\right.$$

$$\left(\prod_{j=1}^{N}\int_{\eta_{Aj}(F_m(i),a_{1Am}(i),a_{2Am}(i),\cdots,a_{NAm}(i),i)}^{\infty} f_{r_{jAm}}(\epsilon_j(F_m(i),s_{1Am},s_{2Am},s_{j-1Am},s_{j+1Am},\cdots,s_{NAm},i))dr_{jAm}dF_m(i)\right)\mathrm{d}A_1$$

$$/\int_{-\infty}^{\infty} f_{A_1}(A_1)\prod_{i=1}^{n}\prod_{j=1}^{N}\int_{0}^{\infty} f_{s_{jAm}}(\tau_{jAm}(F_m,s_{1Am},s_{2Am},s_{j-1Am},s_{j+1Am},\cdots,s_{NAm},i))^{*}$$

$$\left.\left(\int_{s_{jAm}}^{\infty} f_{r_{jAm}}(\epsilon_{jAm}(F_m,s_{1Am},s_{2Am},s_{j-1Am},s_{j+1Am},\cdots,s_{NAm},i))dr_{jAm}ds_{jAm}\right)\mathrm{d}A_1\right\} \quad (32)$$

MDC is able to quantitatively characterize the MD. More importantly, MDC establishes a quantitative analytical relationship with FDC, which is able to reflect the relationship between FD and MD in the working process of the system. MDC describes the influences of maintenance on system reliability according to the system working principle and system performance parameters. When the value of MDC tends toward 1, this indicates that the MD effect is not obvious and the MD can be ignored. The larger the value of MDC, the more obvious the MD effect. Furthermore, it is necessary to pay attention to the important factors that may cause the MD. For ease of understanding, a flowchart of the system reliability solution is shown in Figure 4.

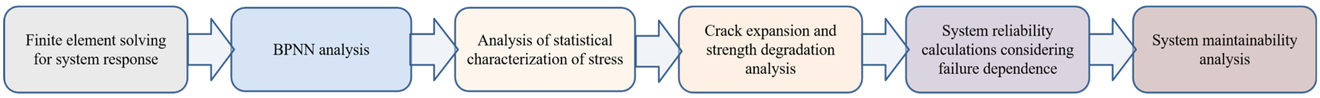

Figure 4. System reliability calculation flowchart.

4. Numerical Examples

Case 1: Consider a cantilever beam structure containing two cracked elements, as shown in Figure 2, with the left end of the cantilever beam being fixed. The material and dimensional parameters of the beam structure are listed in Table 1. The rightmost end of the beam structure is subjected to a random working load, *F*. The stochastic statistical characteristics of *F* are shown in Table 1. The mean values of the variables are indicated in Table 1. The residual strength of each crack element is expressed as [22]:

$$\mathrm{r(n)} = \mathrm{r_0}(1 - \frac{\mathrm{n}\int_0^{\infty} \mathrm{s^e f_s(s)ds}}{\mathrm{C}})^{g} \quad (33)$$

where r_0 is the initial strength, *a* and *C* are material parameters. The relationship between the crack depth at the *j* + 1st operational cycle and the *j*th operational cycle is expressed as the function of the stress *s* and the expansion rate *v*, as follows:

$$a_{j+1} = a_j + s^{*}v \quad (34)$$

The reliability of the Monte Carlo simulation (MCS) is compared with the time-varying reliability calculated by the models proposed in this paper, as shown in Figure 5. The MCS simulates the system working process without being affected by any analytic reliability model. Thus, it can be used to verify the validity and correctness of the proposed models. The time-consuming problem of MCS has now been widely raised in the field of reliability engineering, which is the main reason the method is heavily used to validate analytical models in reliability analyses. The variability between analytical models and MCS in terms of computation time is affected by many factors, including the number of random variables, the statistical characteristics of random variables, the system operating principles and the system operating life, and is sensitive to these parameters. Overall, the analytic model is able to provide accurate computational results while saving computational time. In addition, a comparison between the reliability of the dependent system and that of the independent system is shown in Figure 6.

Table 1. Material parameters.

Parameter	Value	Unit
l_1	0.1	m
l_1	0.3	m
Mean value of working load $\mu(F)$	3000	N
Standard deviation of working load $\sigma(F)$	200	N
Initial crack depth in Element 1	0.0231	m
Initial crack depth in Element 2	0.0255	m
Crack expansion rate in Element 1	3×10^{-8}	
Crack expansion rate in Element 2	3×10^{-8}	
Modulus of elasticity E	2×10^{11}	Pa
Density ρ	7800	kg/m^3
Length of beams	3	m
Height of beams	0.1	m
Mean value of initial residual strength	1.12×10^8	Pa
Standard deviation of initial residual strength	1×10^6	Pa
C	10^{21}	Pa^2
G	1	
E	2	

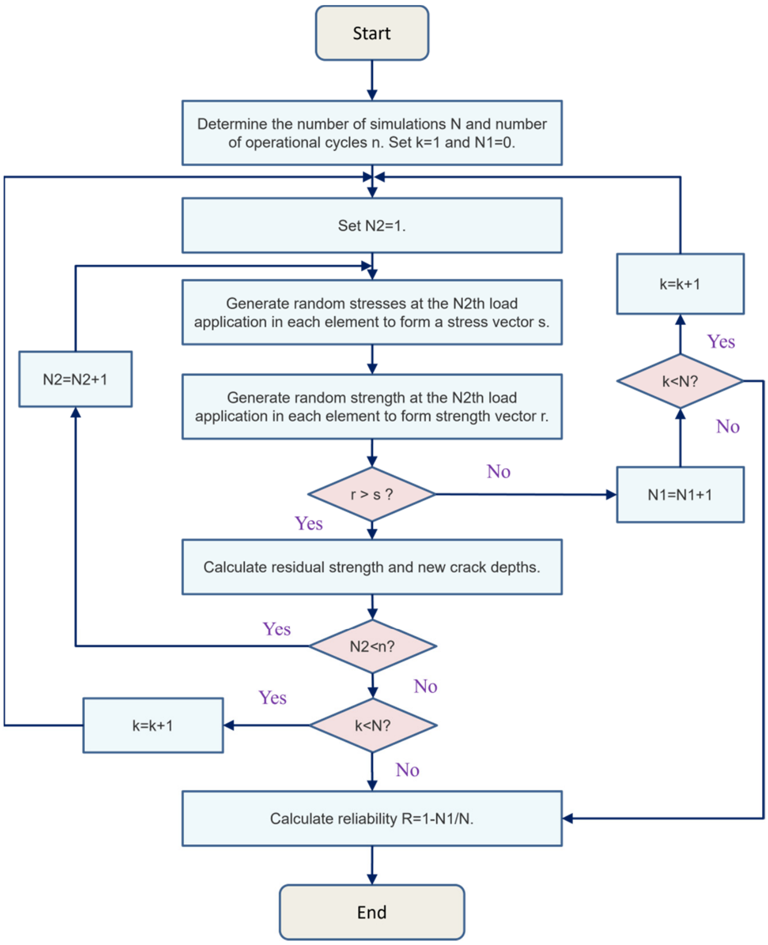

Figure 5. MCS flowchart.

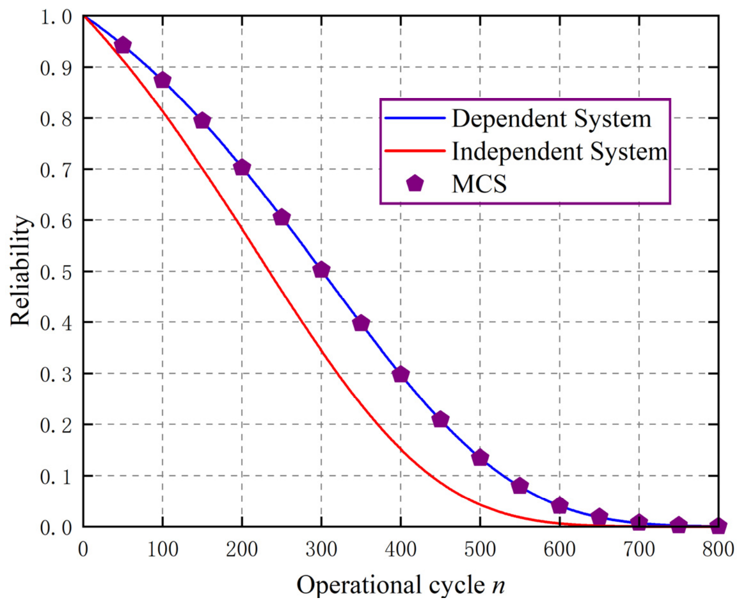

Figure 6. Time-varying reliability of the multi-cracked beam system.

As can be seen in Figure 6, the MCS results are in good agreement with the results of the reliability models proposed in this paper. In addition, the FD has a large impact on the multi-cracked beam structure. The reliability model, when used under the traditional independence assumption, can result in a underestimation of the time-varying reliability. This error may cause excessive margin design in the system reliability design, resulting in wasted costs, such as material costs. Meanwhile, the effect of FD on the beam structure is mainly concentrated in the middle stage of the working period. The reliability error caused by FD increases continuously with time and disappears when the beam structure is close to complete failure. In addition, the FD causes an underestimation of the life of the beam structure, which results in the early maintenance of the structure and affects the system's fault diagnosis and maintenance strategy development.

Case 2: In order to analyze the effect of maintenance behavior on the time-varying system reliability, the discrete PDFs of φ_m and δ_m in the case of considering MD, and those in the case without MD, are shown in Tables 2 and 3, respectively. The time-varying reliability of the system after repair is shown in Figure 7 and the time-varying system FDC is shown in Figure 8.

Table 2. Discrete PDF of φ_m and δ_m considering MD.

[φ_m, δ_m]	**[1, 1]**	**[1.015, 1.3]**	**[1.03, 1.6]**
Probability	1/3	1/3	1/3

Table 3. Discrete PDF of φ_m and δ_m without considering MD.

Possible value of φ_m	1	1.015	1.03
Probability of φ_m	1/3	1/3	1/3
Possible value of δ_m	1	1.3	1.6
Probability of δ_m	1/3	1/3	1/3

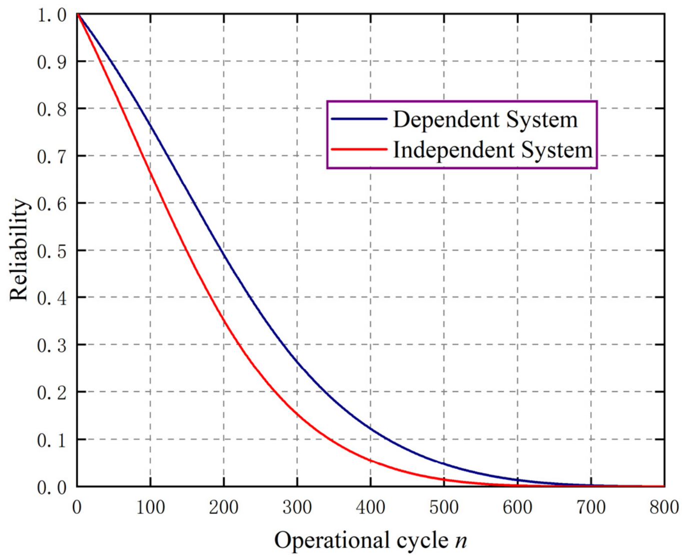

Figure 7. Comparison of time-varying system reliability.

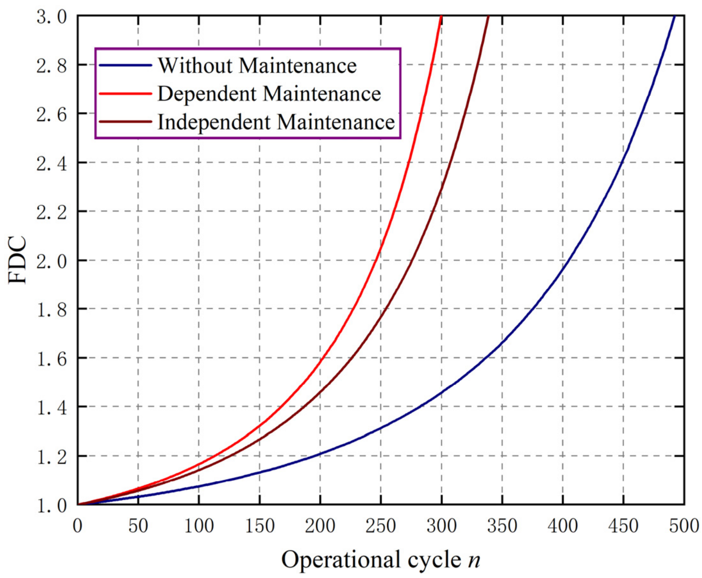

Figure 8. System FDC curve.

As can be seen from Figures 7 and 8, due to the imperfect maintenance of the system, the system performance still cannot be restored to the original statistics. The time-varying reliability of the system under both the independent and FD assumptions decreases compared to that before maintenance. However, the effect of FD is similar to that before repair, which results in an underestimation of the time-varying system reliability, affecting the reliability-based optimal design and the formulation of maintenance strategies. In addition, the FDC enhancement after repair is more obvious, indicating that the repair behavior increases the degree of FD. Moreover, the MD will make the FD more obvious. In the whole life-cycle assessment of beam structures, attention should be paid to the influence of repair behavior, especially the MD effect, on the time-varying reliability of the system.

Case 3: In order to analyze the effect of the dispersion of φ_m on the FD and MD, the discrete PDFs of φ_m and δ_m are shown in Tables 4 and 5, and the time-varying FDC and the time-varying MDC of the system are shown in Figures 9 and 10, respectively.

Table 4. Discrete PDF of φ_m and δ_m considering MD.

[φ_m, δ_m]	**[1, 1]**	**[1.01, 1.3]**	**[1.02, 1.6]**
Probability	1/3	1/3	1/3

Table 5. Discrete PDF of φ_m and δ_m without considering MD.

Possible value of φ_m	1	1.01	1.02
Probability of φ_m	1/3	1/3	1/3
Possible value of δ_m	1	1.3	1.6
Probability of δ_m	1/3	1/3	1/3

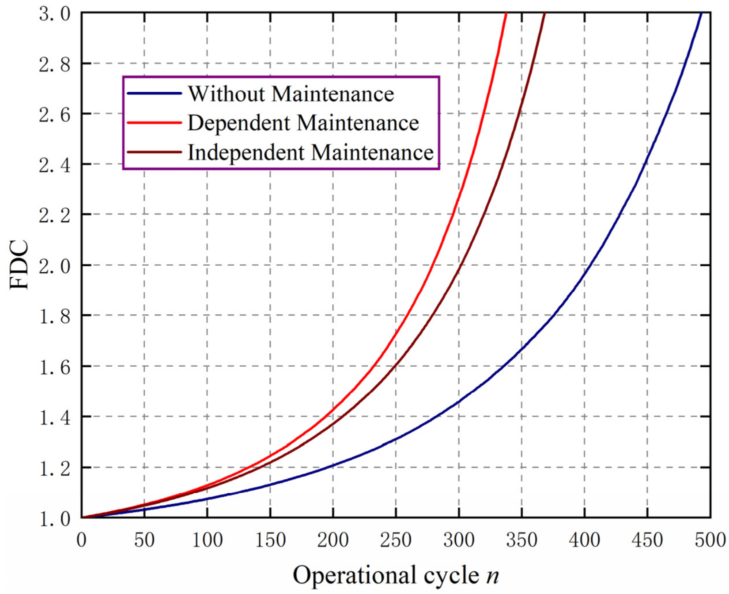

Figure 9. System FDCs with different dispersion of φ_m.

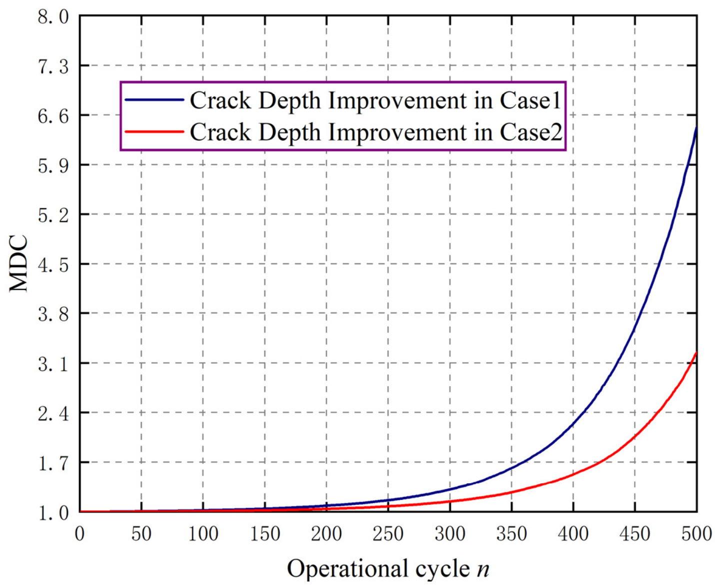

Figure 10. System MDCs with different dispersion of φ_m.

As can be seen from Figures 9 and 10, the trend of FD with and without considering MD is similar to that in case 2. However, as the dispersion of φ_m decreases, the FDC significantly decreases, which reduces the effect of the FD on the time-varying reliability of the system to a larger extent. In addition, the decrease in the dispersion of φ_m leads to a more pronounced decrease in MDC, which attenuates the MD effect and the extent to which the MD influences the FD.

Case 4: In order to analyze the effect of the dispersion of δ_m on the FD and MD, the discrete PDFs of φ_m and δ_m are shown in Tables 6 and 7, and the time-varying FDC and the time-varying MDC of the system are shown in Figures 11 and 12, respectively.

Table 6. Discrete PDF of φ_m and δ_m considering MD.

[φ_m, δ_m]	**[1, 1]**	**[1.015, 1.2]**	**[1.03, 1.4]**
Probability	1/3	1/3	1/3

Table 7. Discrete PDF of φ_m and δ_m without considering MD.

Possible value of φ_m	1	1.015	1.03
Probability of φ_m	1/3	1/3	1/3
Possible value of δ_m	1	1.2	1.4
Probability of δ_m	1/3	1/3	1/3

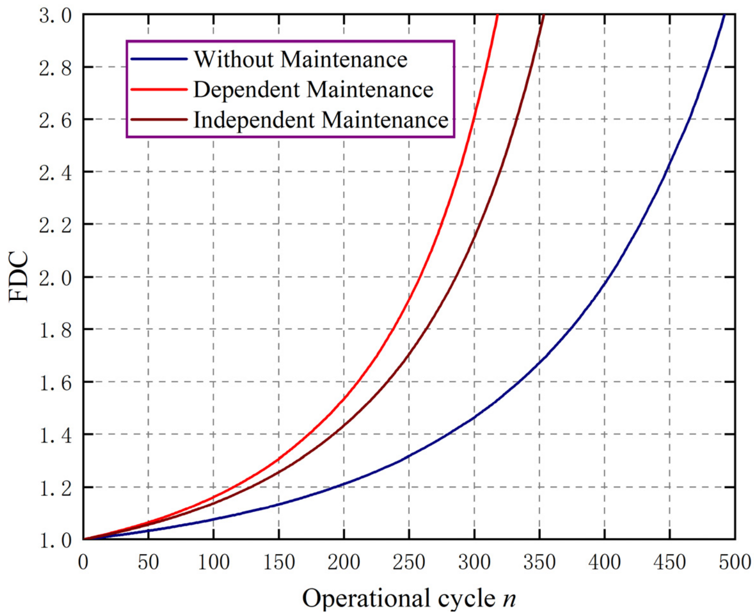

Figure 11. System FDCs with different dispersion of δ_m.

As can be seen from Figures 11 and 12, the dispersion of δ_m has a pronounced effect on FDC and MDC. As the dispersion of δ_m decreases, the influences of FD on the time-varying reliability of the system decreases, weakening the FD effect. In addition, the system MD effect also decreases when the dispersion of δ_m decreases, weakening the influence of MD on FD as well as the system reliability. The results indicate that the proposed model can quantitatively evaluate the relationship between MD, FD and system time-varying reliability, which provide a theoretical basis for system fault diagnosis, reliability optimization design and maintenance strategy formulation.

It should be noted that the data in Tables 2–7 provide discrete distributions empirically derived maintenance parameters that are appropriately assumed to characterize different maintenance effects. However, the methodology presented in this paper is not limited to specific distribution data, and any actual distribution data that are obtained can be used in the reliability calculation model presented in this paper to assess reliability.

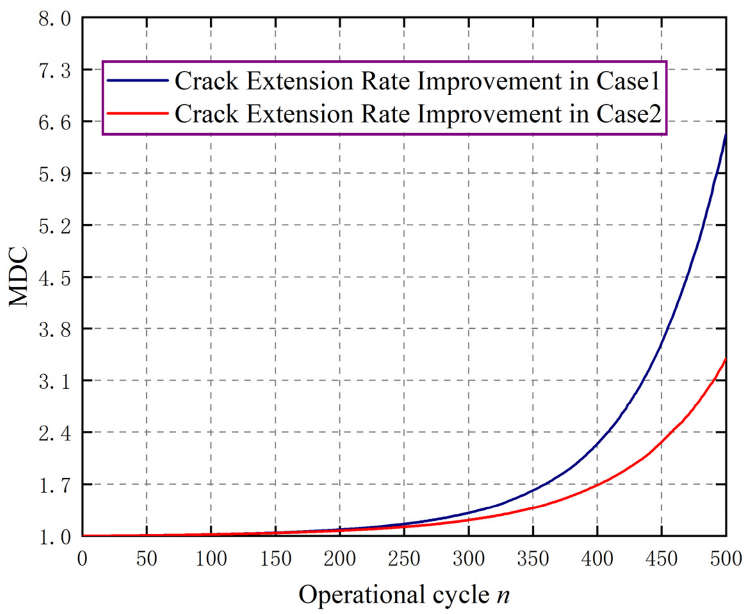

Figure 12. System MDCs with different dispersion of δ_m.

5. Conclusions

In this paper, the time-varying reliability models of multi-cracked beam structures considering MD and FD are established. As the multiple cracks are subjected to common working loads, there are complex statistical correlations between different cracks and between the stresses of the crack elements. The multiple crack elements are regarded as a series system with FD. The time-varying reliability models of multi-cracked beam structures are further developed by the neural network method considering the complex FD which results from the stress dependence, crack extension dependence, and multi-failure mode dependence. In order to characterize the system FD effect, the system FDC index is proposed to measure the influence of FD on the time-varying reliability assessment of the system. On this basis, according to the working principle of the beam structure and the maintenance mechanism for the crack defects, the time-varying system reliability models considering the MD is proposed and the MDC index is further proposed. By establishing the relationship between the MDC and FDC, a method is proposed to quantitatively measure the interaction between the MD and the FD. In addition, the validity and correctness of the model are verified by the MCS method.

A numerical example is used to point at that the reliability models under the traditional independence assumption can result in an underestimation of time-varying reliability and service life. This error may cause excessive margin design in system reliability design, increasing design and maintenance costs. In addition, the MD will make the FD more pronounced. As the dispersion of φ_m and φ_m decrease, the FDC and MDC significantly decrease, weakening the MD effect and the influences of MD on the FD. The proposed models are capable of quantitatively evaluating the relationship between MD, FD and the time-varying reliability of the system, which provides a theoretical basis for the optimal design of reliability and the formulation of maintainability strategies.

In this paper, time-varying reliability models for crack-containing structural systems considering complex statistical correlations are proposed and the proposed models were validated using the MCS methodology. However, the reliability evaluation methodology based on the analysis of physical experiments is also an important value for their practical application in engineering, which is an important next step that will be carried out in future work. Moreover, different design and maintenance parameters, as well as BPNN parameters, have an impact on the system reliability calculations, which will be investigated in future work.

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Informed Consent Statement: The study did not involve humans.

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