Article

# Flow-Induced Vibration of Cantilever Type Elastic Material in Straight Tricylinder 

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Featured Application: The findings of this study can be utilized to establish a rational foundation for the arrangement of fluid-structure coupling low-frequency energy harvesting devices, particularly in aquatic environments.


#### Abstract

The hydrodynamic performance and wake evaluation results of a cantilever flexible cylindrical triangular array at $\mathrm{Re}=9000$ and with $\mathrm{L} / \mathrm{D}$ spacing ratios ranging from 3 to 7 are presented in this paper. The improved delayed separation eddy current simulation (IDDES) SST k- $\omega$ turbulence model and tetrahedral mesh are employed to solve the three-dimensional instantaneous flow field. The objective is to find the L/D ratio that the upstream cylinder has the most pronounced effect on the downstream cylinders, to quantify whether a flexible upstream cylinder is more favorable than the rigid upstream cylinder, and to know the flexible cylinder wake distribution. The findings indicate that, at $\operatorname{Re}=9000$, when the spacing ratio is $3 \sim 7$ and the spacing ratio $L / D$ is 6 , the amplitude of the downstream cylinder is most affected by the wake of the upstream cylinder; the amplitude of downstream cylinders of the rigid upstream cylinder exerted a $1 \%$ or $2 \%$ greater effect than the flexible cylinder, with vortices predominantly concentrated in the middle and upper parts of the flexible cylinder rather than at its top.


Keywords: IDDES; fluid-structure coupling (FSI); vortex-induced vibration (VIV); cantilever; flexible cylinder

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## 1. Introduction

The fluid-structure interaction of slender flexible cylinders in a cross-flow is pivotal in various engineering applications, particularly in nuclear reactors, low-frequency energy harvesting, and offshore engineering involving cantilever slender flexible structures [1,2]. The elongated offshore structures are prone to VIV when exposed to ocean currents. Sarpkaya [3] highlights that VIV is characterized by nonlinearity, self-excitation, self-limitation, and multiple degrees of freedom. Even the simplest form of VIV, such as the single-cylinder vortex-induced vibration in two-dimensional uniform flow, remains incompletely understood. In the case of slender and flexible cylinders, it has been observed that the dynamics of VIV response are chaotic, complex, and consist of unpredictable motions [4]. A vibrating flexible cylinder subjected to cross-flow generates a complex vortex wake that significantly influences the trajectory of response motion, phase synchronization between fluid force structural motion, as well as the interaction between the in-line (IL) and cross-flow (CF) responses. For cantilevered flexible cylinders with fewer constraints imposed on them, predicting the characteristics of wake flow becomes more challenging.

At present, the study of vortex-induced vibration of flexible cylinders has attracted the attention of many scholars [5,6]. Chaplin et al. [7] checked various early empirical models (VIVA, VIVANA, VI-Co Mo, SHEAR7, and ABAVIV) according to the experimental results of flexible cylinder vortex-induced vibration and found that there were some errors between the model prediction results and the experiment. He et al. [8] studied three
relatively new models, SST-PANS, SST-SAS, and SST-IDDES, and the results showed that SST-IDDES had the best simulation results. The numerical results from SST-IDDES show its comparable capabilities for the simulation of massively separated hydrodynamic flows and its potential application in the prediction of industrial turbulent flows for vortex-induced motions (VIM). Wu [9] studied the coupling mechanism of the cross-flow direction and forward-flow direction of flexible cylindrical VIV and found that the fluid force coefficient was affected by the phase between CF and IL displacement. In addition, when the rigid cylinder VIV occurs, the structure does not deform, and the fluid force at different section locations is roughly the same, while the flexible cylinder has a large length-diameter ratio (the ratio of length to diameter), and the form of vortex shedding and the fluid force change irregularly along the axis. VIV has typical three-dimensional characteristics, and the spatial distribution of fluid force needs further study.

In research on three cylinders, Yang [10] studied the flow past three circular cylinders in equilateral triangular arrangements at $\operatorname{Re}=50-200$ from the viewpoint of vortex-shedding suppression. Their results showed that for one upstream cylinder and the other two side-by-side downstream, the vortex shedding from the upstream cylinder is suppressed for $\operatorname{Re}=100-200$. However, for two upstream cylinders side-by-side and one downstream cylinder, the suppression occurs on the downstream cylinder at $\operatorname{Re} \leq 175$. At a high $\operatorname{Re}=1.1 \times 10^{4} \sim 6 \times 10^{4}$, Dahl [11] studied the vortex-induced vibration of a cylinder with two degrees of freedom via experiments and found that the amplitude of the lateral displacement response could reach a maximum of 1.35 times the diameter. Gao et al. [12] simulated the flow past three circular cylinders with the incidence angle of $30^{\circ}$ for $200 \leq \operatorname{Re} \leq 3900$ and $1.25 \leq \mathrm{D} / \mathrm{L} \leq 4.0$, and five flow patterns were identified. The boundary spacing ratios between the five regimes are affected by the Reynolds number. With the increase in the spacing ratio or the Reynolds number, the three-dimensionality of the wake flow becomes stronger. Zhang [13] used three-dimensional numerical simulations to investigate the flow around three cylinders in this arrangement at the super-critical Reynolds number $\operatorname{Re}=3 \times 10^{6}$, concentrating on the influence of the spacing ratio among cylinders. When $\mathrm{L} / \mathrm{D}$ is greater than 3.5 , the separation point of the cylinder in the upper stream is close to that of a single cylinder, indicating that the minimum $\mathrm{L} / \mathrm{D}$ for negligible interaction among the cylinders is 3.5.

Learning about flexible slender risers, Wang [14] studied the VIV of a vertical riser subject to uniform and linearly sheared currents. A standing wave response is observed for the single-mode IL and CF vibrations. Dual resonance is found to occur at most of the locations along the riser. Han [15] developed a bidirectional fluid-structure coupling method. By comparing and analyzing the vortex forms of riser wake under different flow rates, it is found that the riser has a slight 3D effect at low flow rates, while the riser has a strong 3D effect at high flow rates. Lin [16] used a Q3D FSI numerical method that couples the strip theory-based DVM and the FEM of structural dynamics developed to simulate the FIV of the flexible cylinder. The study answered two questions: What is the difference between the flexible cylinder FIV response with an upstream wake interference and the isolated flexible cylinder VIV response? How does the difference between the FIV response with the upstream wake interference and the isolated cylinder VIV response come about? Deng [17] used numerical investigations into the VIV of a flexible cylinder with different length-to-diameter ratios, named the aspect ratio. Comparisons among simulations with different aspect ratios were analyzed in detail. The maximum vibration amplitude, the dominant vibration mode, and the non-dimensional dominant vibration frequency in both directions were apparently enlarged with the increase in the cylinder aspect ratio. The typical vibration trajectory changes from the butterfly type to the stripe type with L/D increases. Wang [18] also used the two-dimensional method to study the flexible slender rod and made a comparison with the experiment to prove the feasibility of this method, but these methods were not a true three-dimensional simulation.

This study focuses on the numerical development and analysis of the VIV response of cantilever flexible cylindrical triangular arrays at subcritical Reynolds numbers. To enhance
the investigation into the VIV characteristics of such arrays, we analyze the distribution of flexible cylindrical structures with triangular rigid connections using a combination of the SST-IDDES model and the fluid-structure coupling model.

## 2. Materials and Methods

### 2.1. The Governing Equations

When calculating the flow field, it is essential to ensure the fulfillment of both the continuity equation and momentum equation as a prerequisite, with the governing equation presented as follows.

$$
\begin{gather*}
\frac{\partial \rho_{f}}{\partial t}+\frac{\partial\left(\rho_{f} v_{j}\right)}{\partial x_{i}}=0  \tag{1}\\
\frac{\partial\left(\rho_{f} v_{i}\right)}{\partial t}+\frac{\partial\left(\rho_{f} v_{i} v_{j}\right)}{\partial x_{j}}=\frac{\partial p}{\partial x_{i}}+\frac{\partial}{\partial x_{i}}\left(\mu \frac{\partial v_{i}}{\partial x_{j}}\right)+S_{i} \tag{2}
\end{gather*}
$$

The fluid will cause the vibration of the flexible cylinder, and its governing equation is as follows:

$$
\begin{equation*}
M_{s} \cdot \frac{d^{2} r}{d t^{2}}+C_{s} \cdot \frac{d r}{d t}+K_{s} \cdot r+\tau_{s}=0 \tag{3}
\end{equation*}
$$

In order to ensure that the fluid-structure coupling follows the most basic conservation principle at the fluid-structure coupling interface, the following requirements should be met:

$$
\left\{\begin{align*}
\tau_{f} \cdot n_{f} & =\tau_{s} \cdot n_{s}  \tag{4}\\
d_{f} & =d_{s} \\
q_{f} & =q_{s} \\
T_{f} & =T_{s}
\end{align*}\right.
$$

To solve the fluid-structure coupling problem, it is necessary to list the equations of motion in parallel. Based on the continuity equation and momentum equation of the fluid, the motion equation of the fluid is as follows:

$$
\begin{equation*}
M_{e f} \cdot \ddot{P}_{e}+C_{e f} \cdot \dot{P}_{e}+K_{e f} \cdot P_{e}+\rho R_{e}^{T} \cdot \ddot{U}_{e}=0 \tag{5}
\end{equation*}
$$

When the flexible cylinder is subjected to the pressure exerted by the fluid, its vibration equation is as follows:

$$
\begin{equation*}
M_{s} \cdot \ddot{U}_{e}+C_{i} \dot{U}_{e}+K_{s} \cdot U_{e}-R_{e} \cdot P_{e}=F_{e} \tag{6}
\end{equation*}
$$

By connecting Equations (5) and (6), the equation required to solve the bidirectional fluid-structure coupling is obtained as follows:

$$
\left[\begin{array}{cc}
M_{s} & 0  \tag{7}\\
M_{f s} & M_{e f}
\end{array}\right]\left[\begin{array}{l}
\ddot{U}_{e} \\
\ddot{P}_{e}
\end{array}\right]+\left[\begin{array}{cc}
C_{s} & 0 \\
0 & C_{e f}
\end{array}\right]\left[\begin{array}{c}
\dot{U}_{e} \\
\dot{P}_{e}
\end{array}\right]+\left[\begin{array}{cc}
K_{s} & K_{f s} \\
0 & K_{e f}
\end{array}\right]\left[\begin{array}{c}
P_{e} \\
U_{e}
\end{array}\right]=\left[\begin{array}{c}
F_{e} \\
0
\end{array}\right]
$$

On the fluid-solid coupling surface, each node has the same degree of freedom of displacement and pressure. Then, combined with the known boundary conditions, initial conditions, and given relevant calculation parameters, the coupling equation is solved, and the solution of the equation is obtained. After determining the solution of the contact point of the fluid-solid coupling surface, the solution vector on the coupling surface can be obtained via Equation (7) to solve the fluid-solid coupling dynamics problem.

### 2.2. Numerical Simulation

The computational fluid dynamics solver ANSYS Fluent version 2021 R1 is based on the finite volume method and is utilized for solving the three-dimensional unsteady incompressible Navier-Stokes equation with IDDES turbulent closure.

The mechanical solver ANSYS Transient Structural and the computational fluid dynamics solver ANSYS Fluent via ANSYS System Coupling to solve the problem of the interaction between the pressure generated by the fluid and the displacement generated by the solid, as well as the deformation of the flexible cylinder. Figure 1 is the flow chart of the coupling algorithm.


Figure 1. Flow chart of the coupling algorithm.

### 2.3. Computational Domain

The top view of the flow field is constructed based on the cylindrical array structure, with the origin of coordinates designated as O. Figure 2 illustrates the arrangement of cylinders in a cylindrical configuration. The flow field model consists primarily of two components: the near-wall flow of the cylinder and the outflow field on the opposite wall. The overall dimensions of the flow field are 100 D in length, 40 D in width, and height. The near-wall flow of the cylinder and the outflow field on the opposite wall, with a distance of 2 D separating it from adjacent walls along its length and width directions. The wake direction encryption area is up to 50 D , which is convenient to study. For boundary
conditions, pressure inlet and outlet are employed, while standard non-slip wall boundary conditions are applied to both the flexible cylinder and solid wall.



Figure 2. The schematics of the computational domain for three cylinders in an equilateral-triangle arrangement: (a) 3D view of the computation domain; (b) top view of the computation domain.

To enhance computational efficiency and precision in capturing the vibration and complex turbulence characteristics of the cylinder, a grid-based encryption technique is employed for the fluid-structure coupling surface of the flexible cylinder. In order to minimize DES gray space error, this study utilizes near-wall encrypted flow field to transition the fluid-structure coupling surface into the outflow field grid. The outflow field grid represents a turbulent region dominated by large cavities, with LES cutoff scale primarily dependent on local grid scale and vortex system structure size, determining appropriate grid size selection.

### 2.4. Parameters Setting

A time step $\Delta t$ of 0.5 is chosen to maintain Courant number $\mathrm{Co},|U| \Delta t / \Delta_{L}<1, \Delta_{L}$ in the LES area of the hybrid grid, where $\Delta_{L}$ represents maximum mesh element size. The model employs liquid water as jet viscosity of $1.003 \times 10^{-3} \mathrm{~Pa} \cdot \mathrm{~s}$, density of $998.2 \mathrm{~kg} / \mathrm{m}^{3}$, inlet flow rate of $1.5 \mathrm{~m} / \mathrm{s}, \operatorname{Re}=9000, \operatorname{Re}=\rho U D / \mu$, and performs 20 iteration steps per unit time.

Flexible cylinder parameters $\mathrm{D}=6 \mathrm{~mm}$, density $=950 \mathrm{~kg} / \mathrm{m}^{3}$, Young's modulus $=1100 \mathrm{MPa}$, Poisson's ratio $=0.42$, tensile yield strength $=25 \mathrm{MPa}$, and compressive yield strength $=33 \mathrm{MPa}$.

### 2.5. The Grid Dependence Study

The investigation of grid dependence is a pivotal step in numerical computation as the accuracy of the solution can be influenced by the resolution of the grid. To conduct a prescribed computational fluid dynamics (CFD) simulation, it is imperative to assess both the accuracy and time required for obtaining a solution. Therefore, four distinct types of grids will be generated with increasing grid resolution. For $\operatorname{Re}=10,000$, flow simulations were performed at these different mesh densities and compared against Fujarra's (1997) [19] experiment and C.T. Yamamoto's [20].

Table 1 reports the results of the grid convergence study. Strouhal number $S_{t}$ (dimensionless desvorticity frequency), drag coefficient $C_{D}$ and dimensionless number $\mathrm{Y}^{+}$

$$
\begin{equation*}
s_{t}=\frac{D f}{U} \tag{8}
\end{equation*}
$$

$$
\begin{align*}
C_{D} & =\frac{F_{D}}{\frac{1}{2} \rho U^{2} A}  \tag{9}\\
C_{L} & =\frac{F_{L}}{\frac{1}{2} \rho U^{2} A}  \tag{10}\\
C_{P} & =\frac{P}{\frac{1}{2} \rho U^{2}}  \tag{11}\\
Y^{+} & =\frac{u * y}{v} \tag{12}
\end{align*}
$$

Table 1. Mesh parameter.

| Mesh | Elements | $C_{D}$ | $\mathbf{Y}^{+}$ | $S_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| coarse | 626,726 | 1.6723 | 200 | 0.2238 |
| medium | $1,273,191$ | 1.3137 | 120 | 0.2035 |
| fine | $3,729,179$ | 1.2829 | 75 | 0.1925 |
| finer | $5,806,552$ | 1.2818 | 50 | 0.1924 |
| Fujarra's and C.T. et al [20] | $/$ | 1.28 | $/$ | 0.19 |

The grid dependence testing involved the use of four different mesh, with the total time average resistance coefficient $C_{D}$ and Strouhal number $s_{t}$ parameters. As shown in Table 1, resistance coefficient is less than $1 \%$, while the Strouhal number remains within a $1 \%$ range, even with gradual grid refinement. This indicates that the grid independence study has reached convergence. The fine grid is deemed sufficient for subsequent calculations, as shown in Figure 3.


Figure 3. Meshing: (a) horizontal screenshot. (b) Mesh around the three cylinders.

## 3. Results and Discussion

### 3.1. Lift Drag Coefficient

The drag coefficient $C_{D}$ and lift coefficient $C_{L}$ are more effective in reflecting the primary stress on the cylinder for VIV. The lift and drag coefficient quantify the resistance experienced by the flexible cylinder in the direction of incoming flow (x-axis) and the lift force exerted on it in the lateral direction (z-axis). Figure 4 illustrates the time history curve of $t C_{D}$ and $C_{L}$ at an $\mathrm{L} / \mathrm{D}$ distance ratio ranging from 3 to 7 . At $\mathrm{L} / \mathrm{D}$ ratios of $3,4,5$, and 6 , upstream cylinder one exhibits a lower to $C_{D}$ by approximately 16-27\% compared to downstream cylinders two and three. Moreover, there is a drag difference between downstream cylinders two and three, amounting to around $4-8 \%$. This phenomenon occurs due to the influence of a wake
generated during VIV migration on downstream cylinders as they deform towards the middle O point. Consequently, upstream cylinder one becomes locked with a downstream cylinder, resulting in continuous deviation towards this particular cylinder while experiencing reduced resistance. This effect is particularly evident when considering spacing ratios L/D of either five or six. The coefficient $C_{L}$ generally ranges between -0.1 and +0.1 without significant amplitude differences observed between upstream and downstream cylinders; however, occasional interference from downstream cylinders may cause fluctuations around zero for upstream cylinder motion primarily influenced by wake effects from upstream cylinder, it causing negative deformation along Z-axis direction by negative forces for downstream cylinder two, whereas an opposite trend can be observed for downstream cylinder three instead. When considering a spacing ratio L/D equal to seven, these interference phenomena between cylinders are significantly reduced, as clearly depicted in Figure 5.


Figure 4. Lift and drag coefficients with $\mathrm{L} / \mathrm{D}=3-7$.


Figure 5. Comparisons of the distribution of force coefficients on the cylinder surface at H/D = 30: (a) cylinder 1; (b) cylinder 2; (c) cylinder 3 .

Note that some profiles exhibit a noticeable oscillation in Figure 4. In current transient structure-fluid coupling simulations, we use a relatively large time step yet still achieve
convergent solutions for each case due to the computational resource required for such three-dimensional bidirectional interactions.

The surface pressure coefficient of a cylinder with a spacing ratio of $D / L=3-7$ is illustrated in Figure 5. It can be observed that the differences in pressure coefficients among different spacing ratios are relatively small; Figure 5 shows the pressure at the top section of the cylinder. After its integration, the component in the X direction is the cross-section resistance $F_{D}$, and the component in the Z direction is the cross-section lift $F_{L}$. The resistance is mainly obtained by subtracting the pressure near $0^{\circ}$ from the pressure near $\pm 180^{\circ}$. The lift force is mainly obtained by subtracting the pressure around $-90^{\circ}$ from the pressure around $90^{\circ}$. Of course, this is just the force on the cross-section, and the resistance to lifting on the cylinder has to be integrated into the Y direction. We can briefly judge the force on the section according to the difference in pressure at different angles.

### 3.2. Vorticity Field

Figure 6 illustrates typical vorticity results. One can see that the shedding vortex in the lower region of the cylinder, where the height ratio $H / D$ is smaller, remains relatively unchanged irrespective of the period. Only a minor vortex shedding occurs in this area; the presence of the cylinder significantly influences its trajectory by inducing a persistent skewing effect. The phenomenon of vortex shedding becomes more pronounced in the middle section of the cylinder and exhibits noticeable variations with changes in the period. The influence exerted by the cylinder on these vortices gradually diminishes. Towards the upper part of the cylinder, there is a weakening in vortex shedding phenomena, and overall minimal alterations are observed, regarding the full development of vortices. Additionally, from flow diagrams, it becomes apparent that a distinct vortex structure exists when H/D $=15$ at mid-cylinder, which greatly affects convection-induced vibration phenomena.


Figure 6. Vorticity diagram with $\mathrm{L} / \mathrm{D}=3-7$ and $\mathrm{H} / \mathrm{D}=7.5-30$.

### 3.3. Streamline Field

Based on the observations from Figures 5 and 7a, it can be inferred that the wake of the upstream cylinder has an approximate influence range of 2 D along the Z -axis. In Figure 7 b , it is evident that for $\mathrm{L} / \mathrm{D}=3$ and $\mathrm{L} / \mathrm{D}=4$, the wake exhibits a highly disordered pattern, indicating its impact on the downstream cylinder's wake. For $\mathrm{L} / \mathrm{D}=5$ and $\mathrm{L} / \mathrm{D}=6$, vortices are still discernible; however, some flow lines remain relatively straight and unaffected by vortices. The vortex from upstream and downstream mixed to produce a bigger scale vortex, which indicates that the upstream cylinder wake has a continuous influence on the downstream cylinder wake. When $\mathrm{L} / \mathrm{D}=7$, there is a significant reduction in vortex activity within the wake region; numerous flow lines align closely with straight trajectories, while only minimal mixing occurs between vortices generated by both cylinders. Notably absent are any large-scale vortices affecting all three cylinders simultaneously, thus
indicating a sharp decrease in the influence exerted by an open wake originating from an upstream source.


Figure 7. Main view (a) and right view (b) of a flow chart with $\mathrm{L} / \mathrm{D}=3-7$. The purple bead flow line originates from the cylinder, and the green streamer flow line passes between the cylinders.

It is evident in Figure 7 that the longitudinal vorticity primarily is concentrated in the middle and lower regions. Combining with the vorticity shown in Figure 6 further reveals that the intensity of vorticity is higher near $H / D=15$ and $H / D=22.5$ in the middle region. The development of eddy currents at $\mathrm{H} / \mathrm{D}=30$ on top is predominantly horizontal due to its location above the cylinder's top. As a result, it remains relatively unaffected by the upstream cylinder, leading to a stable flow with reduced Y -axis direction vorticity and dominance in the horizontal direction. In the area near the middle of the column, the influence of the upstream column is greater so that the vorticity formed by it changes obviously in the vertical direction. With an increase in spacing ratio $L / D$, the wake tends to flatten both horizontally and vertically.

The Z-axis flow velocity diagram of different cylinders along the X-axis section when $\mathrm{D} / \mathrm{L}=6$ is presented in Figure 8. The Z-axis flow diagram allows for a quick visualization of the distribution of the flexible cylinder in the Y -axis vortex. For instance, when $H / D=7.5$, numerous regions with opposite flow directions can be observed near cylinder 1 in Figure 8a, indicating the formation of vortices and indirectly reflecting their intensity. In Figure 8 a , concentrated regions opposing the flow direction can be seen around $\mathrm{H} / \mathrm{D}=7.5$, with lengths reaching up to 10 D , suggesting significant vorticity in these areas. In Figure $8 \mathbf{b}$, parts within the same region exhibiting opposite flow directions appear more dispersed, indicating lower vorticity levels here compared to Figure 8a. In Figure 8c, parts within the same region displaying opposite flow directions are concentrated; however, the length of this region is only 5 D , implying smaller vorticity than that observed in Figure 8a but larger than that in Figure 8b. From Figure 5, it can be observed that at this point in time, wake streams from cylinder 1 and cylinder 2 as well as from cylinder 3 mutually influence each other; furthermore, when $\mathrm{H} / \mathrm{D}=7.5$, it is evident that there is relatively high vortex intensity associated with cylinder 3, while relatively low vortex intensity corresponds to cylinder 2 at this point in time, which supports this conclusion.

### 3.4. Amplitude

For a slender and flexible cylinder, it has been observed that the response dynamics of VIV exhibit chaotic and complex behavior, characterized by non-repeatable motions [4]. Consequently, unlike a rigid cylinder, the vibration of a flexible cylinder does not display obvious periodic changes. However, it does exhibit distinct fluctuation intervals which can provide insights into its movement trends. Figure 9 illustrates the Z-axis vibration curve of three cylinders with an L/D ratio of 3 and an H/D ratio of 30. The Z-axis
vibration of the three cylinders tends to remain generally stable within an amplitude interval of approximately 0.1 D. In particular, for the upstream cylinder 1, its Z-axis motion is symmetric along $Z=0$; whereas for cylinder 2 , under the influence of flow field effects, its motion path inclines towards the positive direction on the Z-axis with a deviation around 0.3 D. On the other hand, inflow from cylinder 3 causes its motion trajectory to deviate in the negative direction on the Z -axis with an approximate deviation magnitude also at about -0.3 D . Overall, both cylinders' deviation trajectories are symmetrical along $\mathrm{Z}=0$.


Figure 8. Flow through the center of the cylinder, Ut/D $=3000$, Velocity diagram in the Z-axis direction: (a) cylinder 1, (b) cylinder 2, and (c) cylinder 3.


Figure 9. The Z-axis vibration curve of the cylinder with L/D ratio of 3 and H/D ratio of 30 .
The deformation of flexible cylinder A at an L/D ratio of 3 and an H/D ratio of 30 is illustrated in Figure 10. In the figure, cylinder 2 and cylinder 3 indicate that the upstream cylinder 1 exhibits flexibility in the flow field. Cylinder $2^{*}$ and cylinder $3^{*}$ denote that in the flow field, the upstream cylinder 1 is rigid. The simulation results demonstrate that, in comparison with the rigid cylinder, the upstream flexible cylinder exhibits an overall decrease of approximately 0.02 D in terms of downstream stream-induced vibration amplitude A. Moreover, the amplitude range of cylinder 2 and cylinder 3 is roughly 0.01 D smaller than that of cylinder $2^{*}$ and cylinder $3^{*}$. Regardless of whether it is cylinder 2 , cylinder 3 , or their counterparts denoted as cylinders $2^{*}$ and $3^{*}$, all exhibit maximum amplitudes exceeding 1 D . Considering a spacing ratio $\mathrm{L} / \mathrm{D}=2$ may lead to collision risks; hence, this study commences from a spacing ratio $\mathrm{L} / \mathrm{D}=3$.

The variation in amplitude A of L/D with different spacing ratios at an H/D ratio of 30 is illustrated in Figure 11. It can be observed from the figure that, within the range of the L/D ratio between 3 and 6, both the average values of amplitude $A / D$ and the amplitude interval increase as the spacing ratio $L / D$ increases. The maximum occurs at $L / D=6$, where the maximum amplitude A/D reaches 1.23 , and the amplitude interval reaches 0.1 D . When L/D exceeds 7, although the average relative amplitude A/D approaches it, with a spacing ratio of $\mathrm{L} / \mathrm{D}=3$ exists a larger fluctuation interval of 0.1 D compared to when $\mathrm{L} / \mathrm{D}=3$. By combining Figures $9-11$ together, it can be concluded that the interference
between the cylinders with a spacing ratio of $L / D=7$ has been significantly reduced, and the flow-induced vibration expansion phenomenon has disappeared.


Figure 10. Where the L/D ratio is 3 and the H/D ratio is 30, the deformation of flexible cylinder is A.


Figure 11. The amplitude $A$ of different $L / D$ ratios at an $H / D$ ratio of 30 .

## 4. Conclusions

The IDDES turbulent flow model SST $k$ - $\omega$ method is employed in this study to conduct three-dimensional CFD simulations of flexible cylinders arranged in various positions. The obtained data and wake characteristics of the flexible cylinder are analyzed, leading to the following conclusions:

1. The vibration amplitude of downstream flexible cylinders also increases with the increase in the spacing ratio $L / D$. When the spacing ratio $L / D=6$, flow-induced vibration becomes most pronounced, while for $\mathrm{L} / \mathrm{D}=7$, the phenomenon of flowinduced vibration enlargement is significantly reduced.
2. When the H/D ratio is $7.5,15$, and 22.5 , the vortex intensity generated by the flexible cylinder exhibits significant magnitudes, with distinct three-dimensional characteristics of the vortex being observable. However, in proximity to the free end at the top of the cylinder, a comparatively gentle wake is formed where predominantly horizontal vortex structures prevail.
3. The upstream cylinder was simulated as both flexible and rigid. When $\mathrm{L} / \mathrm{D}=3$, it was found that it has a significant effect on the wake. Considering the amplitude of downstream cylinders, the rigid upstream cylinder exerted a $1 \%$ or $2 \%$ greater effect than the flexible cylinder.

Supplementary Materials: The following supporting information can be downloaded at: https:/ / www.mdpi.com/article/10.3390/app132312724/s1.

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## Glossary

A is the projected area in the direction of lift or resistance of the cylinder, $\mathrm{m}^{2}$;
$C_{e f} \quad$ is damping matrix of fluid, $\mathrm{N} \cdot \mathrm{s} / \mathrm{m}$;
$C_{s} \quad$ is damping matrix of solid, $\mathrm{N} \cdot \mathrm{s} / \mathrm{m}$;
$d_{f} \quad$ is fluid-state boundary displacement at the fluid-structure coupling interface, m ;
$d_{s} \quad$ is solid-state boundary displacement at the fluid-structure coupling interface, m ;
$\mathrm{D} \quad$ is cylindrical diameter, m ;
$f \quad$ is vortex departure frequency, Hz ;
$f_{D} \quad$ is the resistance of the cylinder along the $x$-axis;
$f_{L} \quad$ is the resistance of the cylinder along the z -axis;
$F_{\mathrm{e}} \quad$ is external load matrix, N ;
$\mathrm{i}, \mathrm{j} \quad$ is the Cartesian coordinate system in different directions, the value $1,2,3$;
$\mathrm{k} \quad$ is turbulent kinetic energy, $\mathrm{m}^{2} / \mathrm{s}^{2}$;
$K_{e f} \quad$ is stiffness matrix of fluid, $\mathrm{N} / \mathrm{m}$;
$K_{f s} \quad$ is $\rho \mathrm{R}_{\mathrm{e}}^{\mathrm{T}}$;
$K_{s} \quad$ is stiffness matrix of solid, $\mathrm{N} / \mathrm{m}$;
$n_{f} \quad$ is fluid-state boundary number at the fluid-structure coupling interface, no dimension;
$n_{s} \quad$ is solid-state boundary number at the fluid-structure coupling interface, no dimension;
$M_{e f} \quad$ is mass matrix of fluid, kg ;
$M_{f s} \quad$ is $\mathrm{R}_{\mathrm{e}}$;
$M_{s} \quad$ is mass matrix of solid, kg ;
$\mathrm{p} \quad$ is fluid pressure, Pa ;
$\mathrm{P} \quad$ is pressure on the surface of the cylinder, Pa ;
$P_{e} \quad$ is fluid pressure, Pa ;
$q_{f} \quad$ is fluid-state boundary heat at the fluid-structure coupling interface, $\mathrm{W} / \mathrm{m}^{2}$;
$q_{s} \quad$ is solid-state boundary heat at the fluid-structure coupling interface, $\mathrm{W} / \mathrm{m}^{2}$;
$r$ is solid displacement;
$R_{e} \cdot P_{e} \quad$ is fluid pressure load matrix, Pa ;
$\mathrm{S} \quad$ is the strain rate tensor, $\mathrm{s}^{-1}$;
$S_{i} \quad$ is the source term of the momentum conservation equation, $\mathrm{kg} /\left(\mathrm{m}^{2} \cdot \mathrm{~s}^{2}\right)$;
t is time, s ;
$T_{f} \quad$ is fluid-state boundary temperature at the fluid-structure coupling interface, K ;
$T_{s} \quad$ is solid-state boundary temperature at the fluid-structure coupling interface, K ;
$\tau_{f} \quad$ is fluid-state boundary stress at the fluid-structure coupling interface, N ;
$\tau_{s} \quad$ is solid-state boundary stress at the fluid-structure coupling interface, N ;
$\mathrm{U} \quad$ is inflow velocity, $\mathrm{m} / \mathrm{s}$;

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\(U_{e} \quad\) is structural displacement, m ;
\(u^{*} \quad\) is near-wall friction velocity, \(\mathrm{m} / \mathrm{s}\);
v is the fluid velocity at the corresponding position in the flow field, \(\mathrm{m} / \mathrm{s}\);
\(v \quad\) is kinematic viscosity of fluid, \(\mathrm{m}^{2} / \mathrm{s}\);
\(y \quad\) is the spacing between the first layer grid node and the wall surface, \(m\);
\(\rho_{f} \quad\) is fluid density, \(\mathrm{kg} / \mathrm{m}^{3}\);
\(\rho R_{e} \quad\) is coupled mass matrix, kg ;
\(\Delta_{L} \quad\) is maximum mesh element size, \(m\);
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## References

1. Wang, J.; Geng, L.; Ding, L.; Zhu, H.; Yurchenko, D. The state-of-the-art review on energy harvesting from flow-induced vibrations. Appl. Energy 2020, 267, 114902. [CrossRef]
2. Li, X.; Qi, P.Y. Visualization experimental study on axial flow-induced vibration of slender cantilever rod for reactor. Ann. Nuclear Energy 2021, 160, 108357. [CrossRef]
3. Sarpkaya, T. A critical review of the intrinsic nature of vortex-induced vibrations. J. Fluids Struct. 2004, 19, 389-447. [CrossRef]
4. Modarres-Sadeghi, Y.; Chasparis, F.; Triantafyllou, M.; Tognarelli, M.; Beynet, P. Chaotic response is a generic feature of vortex-induced vibrations of flexible risers. J. Sound Vib. 2011, 330, 2565-2579. [CrossRef]
5. Zhao, M.; Kaja, K.; Xiang, Y.; Yan, G. Vortex-induced vibration (VIV) of a circular cylinder in combined steady and oscil-latory flow. Ocean Eng. 2013, 73, 83-95. [CrossRef]
6. Cao, S.G.; Huang, W.P.; Gu, E.K. Vortex-induced vibration of an elastic cylinder considering fluid structure interaction. J. Vib. Sock 2015, 34, 58-62.
7. Chaplin, J.R.; Bearman, P.W.; Cheng, Y.; Fontaine, E.; Graham, J.M.R.; Herfjord, K.; Huera Huarte, F.J.; Isherwood, M.; Lambrakos, K.; Larsen, C.M.; et al. Blind predictions of laboratory measurements of vortex-induced vibrations of a tension riser. J. Fluids Struct. 2005, 21, 25-40. [CrossRef]
8. He, J.W.; Wang, W.T.; Zhao, W.W.; Wan, D.C. Hybrid turbulence models for flows around a stationary smooth circular cylinder. Ocean Eng. 2022, 262, 112312. [CrossRef]
9. Wu, J.; Lie, H.; Larsen, C.M.; Liapis, S.; Baarholm, R. Vortex-induced vibration of a flexible cylinder: Interaction of the in-line and cross-flow responses. J. Fluids Struct. 2016, 63, 238-258. [CrossRef]
10. Yang, S.C.; Yan, W.W.; Wu, J.W.; Tu, C.; Luo, D. Numerical investigation of vortex suppression regions for three staggered circular cylinders. Eur. J. Mech.-B/Fluids 2016, 55, 207-214. [CrossRef]
11. Dahl, J.; Hover, F.; Triantafyllou, M. Two-degree-of-freedom vortex-induced vibrations using a force assisted apparatus. J. Fluids Struct. 2006, 22, 807-818. [CrossRef]
12. Gao, Y.; Qu, X.; Zhao, M.; Wang, L. Three-dimensional numerical simulation on flow past three circular cylinders in an equilateraltriangular arrangement. Ocean Eng. 2019, 189, 106375. [CrossRef]
13. Zhang, M.; Yin, B.; Guo, D.; Ji, Z.; Yang, G. Numerical Study on the Flow Past Three Cylinders in Equilateral-Triangular Arrangement at $\mathrm{Re}=3 \times 10^{6}$. Appl. Sci. 2022, 12, 11835. [CrossRef]
14. Wang, E.H.; Xiao, Q. Numerical simulation of vortex-induced vibration of a vertical riser in uniform and linearly sheared currents. Ocean Eng. 2016, 121, 492-515. [CrossRef]
15. Han, X.; Lin, W.; Qiu, A.; Feng, Z.; Wu, J.; Tang, Y.; Zhao, C. Understanding vortex-induced vibration characteristics of a long flexible marine riser by a bidi-rectional fluid-structure coupling method. J. Mar. Sci. Technol. 2020, 25, 620-639. [CrossRef]
16. Lin, K.; Wang, J.; Fan, D.; Triantafyllou, M.S. Flow-induced cross-flow vibrations of long flexible cylinder with an upstream wake interference. Phys. Fluids 2021, 33, 065104. [CrossRef]
17. Deng, D.; Zhao, W.; Wan, D. Numerical study of vortex-induced vibration of a flexible cylinder with large aspect ratios in oscillatory flows. Ocean Eng. 2021, 238, 109730. [CrossRef]
18. Wang, Z.; Fan, D.; Triantafyllou, M.S.; Karniadakis, G.E. A large-eddy simulation study on the similarity between free vibrations of a flexible cylinder and forced vibrations of a rigid cylinder. J. Fluids Struct. 2021, 101, 103223. [CrossRef]
19. Gritskevich, M.S.; Garbaruk, A.V.; Schütze, J.; Menter, F.R. Development of DDES and IDDES Formulations for the k- $\omega$ Shear Stress Transport Model. Flow Turbul. Combust. 2012, 88, 431-449. [CrossRef]
20. Gritskevich, M.S.; Garbaruk, A.V.; Menter, F.R. Fine-tuning of DDES and IDDES formulations to the k-w shear stress transport model. EUCASS Proc. Ser. Adv. AeroSpace Sci. 2013, 5, 23-42.

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