

Vehicle Lane Change Models—A Historical Review

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Abstract: Lane changing is a complex operation that has a significant impact on traffic safety. The accurate identification and assessment of potential risks in the driving environment before lane changing is crucial for the safe and smooth completion of a lane change. In this paper, the research status of vehicle lane change models is reviewed. Firstly, various factors affecting lane change models are analyzed. Different drivers will be affected by vehicle dynamic parameters, vehicle driving states, and driver characteristics under various road environments. Secondly, the vehicle lane change models are divided into four types: the empirical model of lane changing, the physical model of lane changing, the cognitive model of lane changing, and the mixed model of lane changing. The advantages and disadvantages of different types of lane change models are analyzed, and the key problems to be solved by different lane change models are expounded, respectively, from the aspects of input variables and reasoning algorithms. Finally, according to the advantages and disadvantages of different lane change models, a future research direction is proposed.

Keywords: lane change model; empirical model of lane changing; physical model of lane changing; cognitive model of lane changing; mixed model of lane changing



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1. Introduction

Driving vehicles on the road is a mode of daily transport, but driving vehicles on roads with heavy traffic will spawn a variety of unexpected conditions, such as emergency lane changes, emergency braking, and so on, resulting in a series of traffic accidents. Lane change behavior has a significant impact on traffic flow during commuting, as well as driving planning and driving safety. Lane changing is a common traffic behavior in the process of driving and an indispensable part of driving behavior. It mainly refers to the process of the target vehicle moving from the driving lane to another target lane. The correct operation of lane change behavior will enhance the driver's sense of driving experience, improve the efficiency of traffic flow during commutes, and avoid dangerous traffic conditions ahead of time. On the contrary, drivers changing lanes dangerously often causes traffic flow turbulence and may even lead to traffic accidents [1,2]. Traffic accidents caused by lane changes account for 5% of the total number of accidents, including 75% caused by drivers changing lanes dangerously [3]. Statistics from the National Highway Traffic Safety Administration (NHTSA) show that approximately 240,000 to 610,000 traffic accidents are caused by drivers changing lanes each year, and at least 60,000 people are injured in traffic accidents caused by lane changes [4]. In 2019, a total of 10,706 traffic accidents were caused by overtaking and lane changing, accounting for 4.32% of the total traffic accidents in China [5]. Therefore, the prediction of lane change risk is essential to improve road commuting, travel planning, and traffic safety.

The decisions involved in changing lanes on roads with heavy traffic are complex and dynamic. Lane change behavior involves the surrounding vehicles. According to interactions with surrounding vehicles, lane change behavior can be divided into two categories: voluntary and involuntary. Voluntary lane changing is an active lane change behavior based on the driver's own will, such as overtaking and lane changing. Involuntary

lane changing is a behavior in which drivers are forced to change lanes under the influence of external environmental factors, such as the avoidance of obstacles. In addition, lane change behavior is also affected by a variety of factors, such as driver characteristics (gender, driving experience, age, etc.), different traffic environments (urban roads, expressways, township roads, etc.), vehicle dynamic performance, and the differences between morning, afternoon, and evening travel times. Dangerous and emergency lane changes will cause instability for the vehicles behind the target vehicle, indirectly increasing the probability of traffic congestion and accidents [6]. Currently, the index type of traditional lane change prediction model training is relatively simple, and the parameters considered in model training are not comprehensive, meaning there is a need to further improve the accuracy and real-time performance of prediction models. In fact, the relevant indicators of vehicles in the process of lane changing are related, and training models with various reference indicators can effectively improve the detection accuracy of lane change models. Lane change models are usually composed of vehicle dynamic parameters, vehicle driving states, and driver characteristic indicators. The vehicle dynamic parameters include the steering wheel angle, lateral speed, steering speed, and brake pedal position [7–9], and the driver's characteristic indicators include head movement, eye movement, limb movement, and EEG changes [10–13]. The driving state of the vehicle includes speed, acceleration, driving direction, and other factors [14–17]. If the driver wants to display a reasonable lane change behavior at the time of a lane change, they need to carry out accurate lane change judgment and lane change processing. So, the lane change prediction model can be made more accurate if the driver's state and also relevant information about the car and the surrounding vehicles are factored in.

Lane change models are mainly divided into four types: empirical models, physical models, cognitive models, and mixed models. The lane change experience model is built on the basis of the driver's subjective experience and intuition, so it will be affected by the differences between individual drivers, lane change habits, and subjective biases. The physical model of lane changing is mainly modeled based on vehicle dynamics and road conditions, and the accuracy of the model is affected by the vehicle type, road conditions, and driving style. The lane change cognitive model is built on the basis of the driver's cognitive processes and behavior. The accuracy and reliability of the model depend on the influence of the driver's cognitive ability and behavioral habits, while the influences of the driver's emotions, attention, and fatigue state on lane change decisions cannot be considered. A single lane change model can provide effective lane change prediction results under specific scenarios and conditions. However, when a lane change is performed in a more complex traffic environment, it is necessary to combine the empirical model of lane changing, the physical model of lane changing, and the cognitive model of lane changing to form a mixed lane change model, which is more advantageous and effective. The contributions of this study are as follows: In order to further optimize lane change models, we reviewed the different algorithm types in the empirical, physical, cognitive, and mixed models of lane changing; discussed the advantages and disadvantages of each algorithm; and identified the problems that remain to be improved according to the advantages and disadvantages of these algorithm models. The purpose of this study is to provide a variety of algorithm models for the researchers of vehicle lane change models, so that they can be inspired and achieve multi-dimensional thinking in the development process of lane change models, which will promote the further development of vehicle lane change model algorithms.

2. Vehicle Lane Change Models

It is a complex task to understand how different drivers behave in the process of driving. To do so, we can explore the driving behaviors of different drivers, to unpick the consciousness, decision-making, and behaviors of different drivers under various conditions. Researchers model lane change behavior by collecting traffic data and variables

of uncertainty on the road. Lane change behavior prediction models are mainly divided into the following four categories: empirical, physical, cognitive, and mixed.

2.1. Empirical Models of Lane Change

The lane change empirical model is mainly based on actual observed data, taking the form of a large number of real vehicle lane change behavior samples for analysis, from which we can deduce the characteristics of lane change behavior. These types of lane-changing behavior models are mainly based on the driver's driving habits and experience. There are two common empirical models of lane change: the logit model [18] and reinforcement learning model.

2.1.1. Logit Models

Logit models are commonly used empirical models for lane change, mainly used to build probabilistic models for vehicle lane-changing decision-making. The models mainly use the driver's observational information and the estimated parameters to describe the driver's decision process during a lane change. The assumed error terms of the traditional logit model are independent and identically distributed and Gumbel distributed, but this approach incorporates irrelevant properties. Aiming at overcoming the defects of this logit model, researchers have successively proposed the path-size logit (PLS) model [19], the cross-nested logit (CNL) model [20], and the mixed logit model [21]. However, when these algorithms analyze the route and road information of multiple accident locations, the results are not very satisfactory. In order to include more influential factors such as vehicle spacing, speed difference, and traffic flow density to improve the performance of the model, Anastasopoulos et al. [22] studied the logit model with fixed and random parameters and adopted more detailed data such as road shape, road condition, weather conditions, and traffic characteristics. The detailed data improve the accuracy of the model. Farhi et al. [23] introduced a logit lane allocation model to describe multi-lane traffic flow from a macroscopic perspective. Each available lane has a specific utility for the driver in the process of running the vehicle, and the driver will choose the lane with the highest utility. The model is represented using a system of conservation laws with smooth and implicitly defined flux functions. The cited paper discusses how the traffic data support traffic speed as an explanatory variable for lane assignment in the dataset, and then illustrates its convergence through several numerical schemes such as the Lax-Fredrich scheme, the Euler-Lagrange scheme, and the Lagrange scheme when dealing with the Riemannian problem. Lane change around large commercial vehicles is an extremely challenging task, and collisions resulting from dangerous lane changes can lead to serious collision consequences. To better analyze the relationship between collision factors and severity in commercial vehicle lane change crashes, Adanu, E.K. et al. [24] constructed a mixed (random parameter) logit model based on Alabama traffic crash data from 2009 to 2016. According to the analysis, about 4% of traffic accidents are severe, and more than 50% of traffic accidents are caused by driver error. The results of the model evaluation showed that the likelihood of serious crashes increased when lane change crashes occurred in dark and unlit sections and involved older drivers, drivers who caused crashes through negligence, or female drivers. On the other hand, serious lane change accidents are less likely to occur on motorways with more lanes. These findings raise awareness of the dangers of changing lanes near large commercial vehicles or driving in the blind spots of large commercial vehicles. Lane changing requires a relatively high level of driving experience from the driver, and lane changing is a difficult driving task, especially for young, inexperienced drivers and older drivers. Incorrect lane changing can have a significant impact on traffic flow and road safety. Adanu et al. [25] analyzed collision data from the Critical Analysis Reporting Environment (CARE) system developed by the Center for Advanced Public Safety at the University of Alabama. The data analysis showed that 63.1% of the crashes were caused by young drivers. The authors determined the relationship between crash factors and crash outcomes by developing an average model with random

logit and heterogeneity. In a complex urban environment, Ng et al. [26] established a logical lane change prediction model based on macro cells [27], and combined it with the cell transport model (CTM) to improve the accuracy of macro traffic state estimation. Based on the difference in speed and density between lanes, a binary logistic lane change model is proposed to describe the lane change. The binary logistic lane change model is then integrated into the refined CTM, which describes how the vehicle moves vertically from one unit to another. The performance of the model was evaluated by comparing the simulated cell occupancy of the optimized model with the cell occupancy of the US-101 Next Generation Simulation (NGSIM) [28] data. The arbitrary lane change behavior is closely related to the incidence of accidents. Li et al. [29] conducted a random parameter logit method analysis on the mean and variance heterogeneity of distance- and speed-related factors in the micro-vehicle trajectory dataset. This new method was proposed based on three sets of the 1:1 matching logit model [29], which uses the NGSIM dataset and smooths it before use [30]. Researchers have shown that three adjacent vehicles have a significant influence on lane-changing behavior [31–33]. Therefore, four types of vehicles—vehicles requiring lane change (SV), vehicles ahead of lane change (CLV), vehicles in front of target lane (TLV), and vehicles behind target lane (TFV)—are selected, as shown in Figure 1 below.

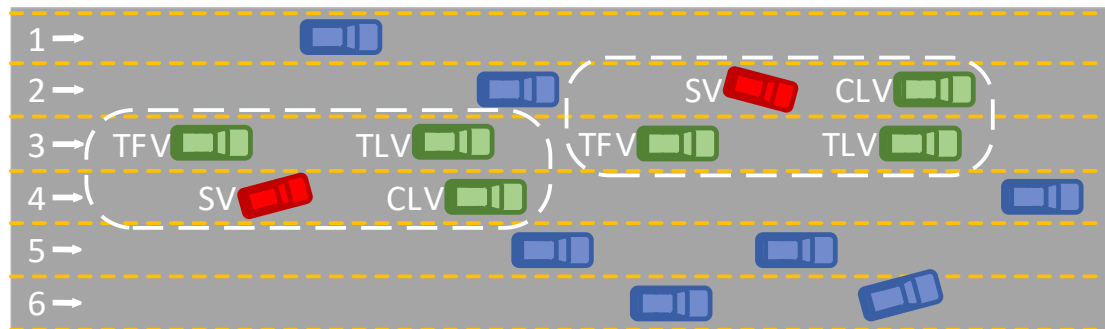


Figure 1. The red vehicle is the lane changing vehicle (SV) in the process of lane change, the green vehicle (CLV) is the position relationship between the vehicle in front of the lane changing vehicle, the vehicle in front of the target lane (TLV), and the vehicle behind the target lane (TFV) and the lane changing vehicle (SV), and the blue vehicle represents the related vehicles in the surrounding lane [29].

The four types of vehicles are analyzed in any combination, and the dynamic factors of the lane change decision process are extracted using the algorithmic model. The results show that the random parameter corresponding to the distance-correlated variable has mean and variance heterogeneity, while only one random parameter corresponding to the speed-correlated variable has mean and variance heterogeneity. The average heterogeneous relationship between the speed of the vehicle preparing to change lane and the distance between the leading vehicle in the same lane and the vehicle about to change lane indicates that the driver considers the distance and speed between the vehicle and the neighboring vehicles when making a lane change. This algorithm can allow us to better understand and quantify the driver's habitual decisions in lane change behavior. Table 1 provides a summary of the logit models.

Table 1. Input parameters of different logit lane change models and key problems to be solved.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[23]	Multi-lane macro traffic flow	Logit	The multi-lane traffic flow is described from a macroscopic perspective
[24]	Traffic accident data	Mixed (random parameters) logit model	Increased awareness of the dangers of changing lanes near large commercial vehicles or driving in the blind area of large commercial vehicles

Table 1. Cont.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[25]	Crash data	Random logit with heterogeneity in means models	Relationship between young, inexperienced and older drivers and crash factors during lane changes
[26]	US-101 NGSIM database	Binary logistic + CTM	Improve the accuracy of macroscopic traffic state estimation
[29]	Distance-dependent factors and velocity-dependent factors	A random parameters logit approach with heterogeneity in means and variances	The dynamic trade-off between safety and efficiency in the decision-making process of autonomous lane change and its influencing factors

2.1.2. Reinforcement Learning Models

Reinforcement learning models are a type of machine learning that can derive optimized strategies from the vehicle's interactions with the environment. In the lane change model, reinforcement learning trains intelligent vehicles to change lanes. Vehicles in lane change models mainly refer to autonomous vehicles and vehicles in traffic simulation systems. Reinforcement learning enables vehicles to realize lane change behavior by learning the optimal lane change strategy. Reinforcement learning includes environment, state, action, reward, strategy, etc. The environment can be a real road or a traffic simulation environment. The state mainly refers to the location, speed, and variable information of adjacent vehicles. The movement mainly consists of lane keeping and lane changing to the left or right. The reward is mainly to teach the lane change model what constitutes safe lane change behavior, direct it away from dangerous lane change behavior, and optimize the lane change strategy through continuous trial and error and learning. The strategy is the lane change mode selected by the intelligent vehicle according to the current environmental state. The performance of the reinforcement learning lane change model heavily depends on the quality and quantity of the training dataset. In the process of practical application, the model needs to be optimized. Sharifzadeh et al. [34] proposed a deep Q-network inverse reinforcement learning (IRL) method to extract rewards with large state spaces. The explosion state space problem is solved, and a highway driving simulator is realized. Mukadam et al. [35] addressed the problem of automatic lane changing of autonomous vehicles in a multi-lane environment with adjacent vehicles. Traditional rule-based approaches are difficult to apply to formulate high-level policies, but this framework demonstrates a more structured and data-efficient end-to-end learning scheme for complete policies on a well-designed low-level controller. Q-masking incorporates prior knowledge, constraints, and information from low-level controllers directly into the learning process, simplifying the reward function and making learning faster and more efficient. Automatic lane changing is an important function of intelligent vehicles. In a complex lane change environment, the uncertainty of adjacent drivers needs to be taken into account. The agents in the simulation environment newly developed by Treiber, M. et al., include vehicle dynamics, lateral controller, adaptive cruise controller [36], and a safe lane change algorithm [37] to optimize performance. Q-learning performs well in planning [38] and autonomous system control [39,40], and it further optimizes the deep reinforcement learning algorithm [41]. This led prior research to train a deep reinforcement learning agent [42]. This agent shows excellent performance for uncertain and random environments, and it can perform well in complex lane-changing environments. The lane change behavior of the driver is based on adapting to the driving situation of the road ahead or based on their driving experience. Unreasonable lane change behavior is the main cause of traffic congestion and accidents. According to the uncertainty and complexity of the driving environment, Ye et al. [43] proposed a lane change strategy based on near-end strategy [44] optimization, which, in turn, is based on deep reinforcement learning. Through training, a smooth, safe, and efficient lane change behavior can be achieved. Due to the variability and complexity of the

traffic environment, it is challenging to model the decision-making about whether to make a vehicle lane change. In a rule-based constrained environment [45], Ghimire et al. [46] used a deep Q-network to make lane change decisions. A safer and more efficient lane change strategy can be obtained by combining high-level lateral decision-making with rule-based trajectory monitoring. The experimental results showed that the rule-based reinforcement learning algorithm performed better than the reinforcement learning algorithm. Baheri [47] proposed an automatic driving reinforcement learning system based on previous studies [48,49], which can improve the safety of multi-mode future trajectory prediction. The safety system consists of two safety components: rule-based and multimodal learning. The rule-based safety system is mainly founded on general driving rules, while the multimodal learning safety system learns safety patterns from historical driving data. Mixed-density recurrent neural networks (MD-RNNs) [50] are used to predict multimodal future trajectories and to accelerate the learning process by simulating the potential behavior of autonomous agents [51]. Experimental simulation results show that the proposed automatic driving reinforcement learning system has excellent performance in terms of average reward and collision frequency. Efficient lane changing is an important task in intelligent transportation systems, and it is challenging to realize efficient lane changing for multiple vehicles at the same time in a crowded traffic environment. Guo et al. [52] proposed a lane change model that discretizes traffic lanes into units and combines forced lane change and free lane change maneuvers [53], and they proposed a request–response mechanism algorithm based on a deep Q-network. The lane change system is divided into an independent group and symmetric group, based on Q-learning and the concept of a “central agent”, to simplify the training process. In the proposed method, Guo et al. [52] outlined a request group and response group. During training, the request group only considers the status of the group to train the agent. During training, the response group not only considers the status of the group but also has to consider the superimposed actions (i.e., request messages) from the request group, as shown in Figure 2 below. The implementation process, like the training process, is handled in a decentralized manner. The proposed algorithm is compared with a rule-based method, game-theory-based decomposition algorithm, and simple deep Q-learning. The experimental results show that the performance of this algorithm is not much different from that of the game-theory-based decomposition algorithm, and the efficiency of lane change is better than the other two algorithms. The calculation time during the execution of this algorithm is less than that of the decomposition algorithm based on game theory [54]. Table 2 gives a summary of the models based on reinforcement learning.

Table 2. Input parameters of different reinforcement learning lane change models and key problems to solve.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[34]	The 208 binary features give rise to 2^{52} possible states	Inverse reinforcement learning (IR-L)	The problem of explosion state space is solved
[35]	Prior knowledge, system constraints, and information about the underlying controller	Q-masking	Simplify the reward function to make learning faster and more efficient
[42]	Different noise environment	Q-learning	Improve the ability to deal with uncertain and random environments
[43]	Self vehicle: longitudinal position, longitudinal speed, acceleration, lateral position, and lateral speed. Surrounding vehicles versus self vehicles: relative distance, longitudinal speed, acceleration, and lateral position	Deep reinforcement learning + P-PO	Perform lane changes smoothly, safely, and efficiently

Table 2. Cont.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[46]	A state matrix for traffic condition coding	Rule-based Deep Q-Network	Obtain safe and efficient lane change behavior
[47]	Driving history data	DDQN + MDRN-N	The safety system has been improved in terms of average rewards and collision frequency
[52]	The request group trains the agent by considering only the state of the group, while the response group evaluates the superimposed actions from the request group in addition to the state of the group	MARL + DQNRR	Multiple vehicles make optimal lane changes at the same time

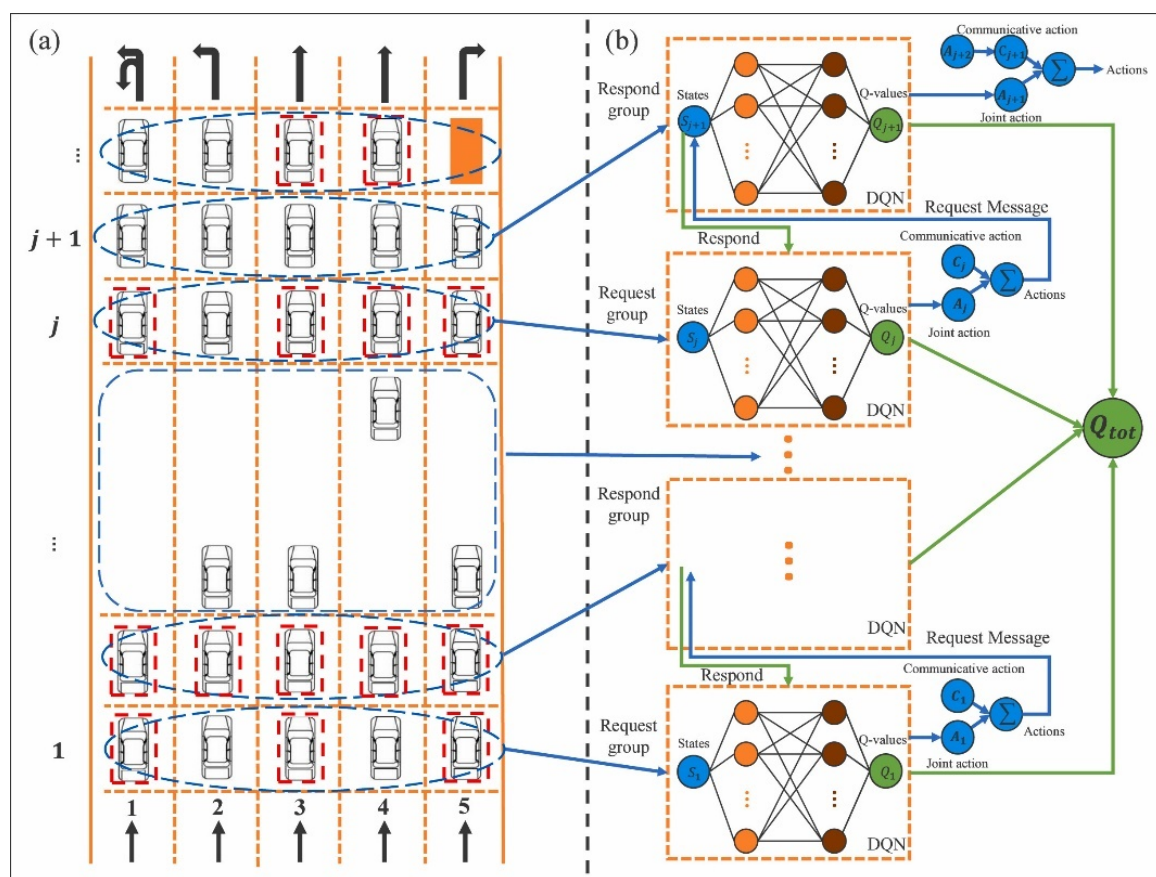


Figure 2. Deep Q-learning schematic of the request and response mechanism. (a) divides the entire lane change system into multiple groups, each row of vehicles is a group, and the vehicles in the same row are regarded as agents in the same group. Agents in the same group perform joint actions just like the central agent. If a group of agents does not reach the target lane after performing the basic joint action generated by the request Q-network, the agent acting as the request group (Group j in (b)) will send a request message to the agent acting as the response group (group $j + 1$ in (b)) in the front. When the request group receives a request message, the response group will take the response action generated by the response Q-network to maximize the Q-value. As the q function converges, a joint action corresponding to the maximum Q-value may be selected based on the current state and the request message. After an action is performed on each agent, the team's reward is calculated and the new state of the environment is updated. The experience is then stored in the replay buffer for the next training [52].

2.2. Physical Models of Lane Change

The physical models of lane change are mainly based on the characteristics of the vehicle kinematics and dynamics to describe lane change behavior. In the physical models of lane change, factors such as vehicle speed, acceleration, and position are considered, and the motion state and trajectory change of the vehicle are described using mathematical equations. The common physical models of lane change are mainly divided into three types: kinematic, dynamic, and vehicle control.

2.2.1. Kinematic Models

A kinematic model mainly involves the position information, velocity, and acceleration information. This model adds certain motion constraints, such as constant acceleration or velocity, to describe the lane change time and lane change distance of the lane change behavior. Lane changing is one of the important functions to improve traffic safety, and it is also one of the avoidance behaviors of the driver under the critical condition of safety. Evasive behavior is mainly divided into three categories: the first type is avoidance by braking, which mainly increases the longitudinal deceleration of the vehicle. The second type is steering evasion, which mainly increases the lateral acceleration of the vehicle. The third type is co-avoidance by braking and steering, and the combination of the two may enable better avoidance [55,56]. Models of avoidance behavior that combine braking and steering are complex and have only been used by a few researchers. Most researchers are more focused on using a categorical approach to predict the choice of evasive behavior [57,58]. Sugimoto [59] proposed an evasive behavior model that includes braking and steering, to measure the effectiveness of collision-mitigation braking systems (CMBSs). Based on ideas from neuroscience, Markkula [60] proposed a method to model steering and pedal behavior as a series of individual control adjustments. Sarkar et al. [58] used facial video and a kinematic response to reduce the occurrence of events, and used a random-forest-based classifier to verify the relationship between the driver's pre-accident behavior and the driver's perception of the event and the criticality of the event as measured by collision time. Since the unexpected lane change of motor vehicles affects the normal running of vehicles behind, if the lane change behavior of vehicles can be predicted in advance, the adverse impact of the lane change will be very low. Bai et al. [61] proposed a collaborative lane change motion planning algorithm based on the influence of lane change on traffic flow in partially connected and automated environments. Collaborative lane change is realized through mutual communication between vehicles and mutual communication between vehicles and infrastructure. The algorithm reduces the vibration and shockwave caused by lane change through cooperative lane change, and it further improves the traffic maneuverability. Model predictive control follows a numerical algorithm based on dynamic programming, which mainly involves a state vector, control vector, system dynamics, and cost function. A dynamic programming method is adopted to solve the problem of cooperative lane change, the predicted visual boundary is discretized into N steps, and the maximum deceleration and acceleration of the vehicle are adjusted through the correlation matrix in the system dynamics. If the time lead is less than the safe time lead or the speed exceeds the maximum speed, the time headway and speed can be corrected to ensure that the given constraint conditions are met. The experimental results show that this optimization method improves the lane change technology compared to the traditional method. Guo et al. [62] consider that a model with assumed constraints is relatively poor in terms of flexibility, and thus extreme driving behaviors occurring at critical moments may not be effectively avoided. Therefore, they proposed a prospective risk-detection method based on SSM, and they used the data-driven method to build the avoidance behavior model. Guo et al. [62] adopted a model-free non-strategy reinforcement learning algorithm from DDPG [39], which is characterized by its ability to capture complex nonlinear relationships. They carried out experimental observation of the state for $S_t = (d_{lon}(t), v_{lon}(t), \Delta v_{lon}(t), d_{lat}(t), v_{lat}(t), \Delta v_{lat}(t))$, using a natural driving dataset of sampling time step t , where $d_{lon}(t)$ and $d_{lat}(t)$ are the longitudinal and lateral distances

between this vehicle and the leading vehicle, $v_{lon}(t)$ and $v_{lat}(t)$ are the longitudinal and lateral velocities between this vehicle and the leading vehicle, and $\Delta v_{lon}(t)$ and $\Delta v_{lat}(t)$ are the relative longitudinal and lateral velocities between this vehicle and the leading vehicle, respectively. The action $A_t = (\hat{a}_{lon}(t), \hat{a}_{lat}(t))$ generated by the agent consists of longitudinal acceleration $\hat{a}_{lon}(t)$ and transverse acceleration $\hat{a}_{lat}(t)$. In order to satisfy the interaction between the agent and the environment, the kinematic model of the state update is constructed as follows:

$$\hat{v}_{lon}(t+1) = v_{lon}(t) + \hat{a}_{lon}(t)\Delta t \quad (1)$$

$$\Delta \hat{v}_{lon}(t+1) = \Delta v_{lon}(t) - \hat{a}_{lon}(t)\Delta t \quad (2)$$

$$\hat{d}_{lon}(t+1) = d_{lon}(t) + (v_{lon}(t) + \Delta v_{lon}(t))\Delta t - \frac{v_{lon}(t) + \hat{v}_{lon}(t+1)}{2}\Delta t \quad (3)$$

$$\hat{v}_{lat}(t+1) = v_{lat}(t) + \hat{a}_{lat}(t)\Delta t \quad (4)$$

$$\Delta \hat{v}_{lat}(t+1) = \Delta v_{lat}(t) - \hat{a}_{lat}(t)\Delta t \quad (5)$$

$$\hat{d}_{lat}(t+1) = d_{lat}(t) + (v_{lat}(t) + \Delta v_{lat}(t))\Delta t - \frac{v_{lat}(t) + \hat{v}_{lat}(t+1)}{2}\Delta t \quad (6)$$

In (1) to (6), $\hat{v}_{lon}(t+1)$, $\Delta \hat{v}_{lon}(t+1)$, $\hat{d}_{lon}(t+1)$, $\hat{v}_{lat}(t+1)$, $\Delta \hat{v}_{lat}(t+1)$, and $\hat{d}_{lat}(t+1)$ are the longitudinal velocity, longitudinal relative velocity, longitudinal distance, transverse velocity, transverse relative velocity, and transverse distance of the vehicle and the preceding vehicle, respectively, at the time step $t+1$. The t update interval is 0.1 s. Table 3 gives a summary of the kinematic models.

Table 3. Input parameters of different kinematic lane change models and key problems to be solved.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[61]	Driving environment, driver behavior, event criticality, and driver response	Random forest	Anticipate driver maneuvers and identify key behavioral and environmental factors that lead to such evasive maneuvers
[62]	State vector, control vector, system dynamics, and cost function	Collaborative lane change motion planning	The motion planner can effectively reduce vibration and has excellent real-time performance
[39]	Longitudinal distance, longitudinal velocity, longitudinal acceleration, lateral distance, lateral velocity, and lateral acceleration	DDPG	Avoid a security emergency

2.2.2. Dynamic Models

Dynamic vehicle models mainly focus on the dynamic performance, such as the acceleration, steering, and braking abilities of the vehicle. They are constructed using Newtonian mechanics and dynamic equations to describe the acceleration, braking, and steering capabilities of the vehicle in the process of lane change. Dynamic models can represent the balance and change in forces and moments during lane changes, as well as the vehicle's interaction with the environment during lane changes. Vehicle following and lane changing are two important components of traffic behavior, which have been widely studied by scholars. As early as several decades ago, the car-following model, which evolved from the car-following theory, was successfully applied to the study of traffic

dynamics [63–76]. Vehicle lane change behavior has also been extensively studied and modeled [77–90]. The main research method of vehicle lane change behavior is to introduce specific lane change rules according to different needs and scenarios. Chowdhury's [79] model expresses the driver's expectation, and it uses an asymmetric model to express that the processes of moving from fast lane to slow lane and from slow lane to fast lane are controlled by different forms. Heuristic lane-changing rules are generally used in the micro-modeling of multi-lane traffic, but the modeling results often cannot meet people's needs. Nagel et al. [80] summarized different lane change methods and results, and used a cellular automaton model for two-lane traffic to meet people's needs, creating density inversion when the density was slightly lower than the maximum flow density. Several different lane change rules are used to test the effectiveness of the scheme. Although there are some differences in the results, similar and real-world-applicable results are obtained. On the real road, drivers may show different lane change behaviors due to the different vehicles in front of them. If the acceleration and top speed of the front car are relatively low, the car with the strong acceleration will generally change lanes more. Li et al. [81] introduced this factor and proposed a symmetric two-lane cellular automata model to study the lane-changing behavior of fast vehicles. The experimental results show that the positive lane-changing behavior of fast vehicles inhibits the speed of slow vehicles and may cause road congestion. Komada and Nagatani [82] derived a basic diagram of the traffic flow of toll stations to represent the queuing and traffic status, representing multi-lane toll stations on expressways, where the traffic flow can be extended to the nearest toll station under the condition of high density. Wei et al. [84] extracted vehicle track data from urban streets and modeled different lane change behaviors. The decision model, condition model, and maneuver model were mainly used. The decision model depends on the change in traffic environment, the condition model describes different types of lane change conditions, and the maneuver model describes the change in speed and lane change duration. Frequent lane changes by vehicles in the diverging, confluence, and intersection areas of expressways may increase the traffic accident rate. Wen-Long [88] proposed a moving wave model to describe the traffic dynamics, and studied the bottleneck effect of lane change traffic and the aggregate traffic dynamics of roads with lane change areas. Lv, W. et al. [91] conducted a study based on Wen-Long's [88] idea to obtain the lateral interaction through the intensity variable of lane change, with the main purpose of describing the process of lane change by constructing a microscopic lane change model. Different from Wen-Long [88], who described lateral interactions at the level of macroscopic polymerization, Lv, W. et al. [91] described microscopic lane change decisions and maneuvers. They proposed a new idea of transforming the lane change process into the vehicle-following process through the simulation environment. In order to verify the effectiveness of the model, traffic flow, lane change frequency, and vehicle speed were obtained from the experiment and compared with the results in the simulation environment. With the maturing of intelligent technology, intelligent cars have begun to be slowly integrated into our travel. In the process of intelligent vehicle driving on the road, the success of lane changing plays a decisive role in driving safety. In order to achieve the ideal driving experience on the road, drivers often choose to avoid slower vehicles, pedestrians, obstacles that interfere with normal driving, or lane closures. In order to cope with mobility bottleneck and lane reduction, Li et al. [92] studied the theory of micro-dynamic vehicle following and lane changing, proposed additional lane-changing rules, and analyzed the impact of lane changing on traffic safety, traffic efficiency, and economy. Through the simulation of different traffic scenarios, some factors affecting traffic were obtained. The results have important implications for the efficiency and safety of transport and commuting. In order to improve the safety, comfort, and energy saving of vehicle lane change, Zhao et al. [93] proposed a multi-objective collaborative optimization lane change trajectory-planning algorithm. The constraints of the trajectory-planning method depend on variables such as vehicle lane change time, longitudinal and lateral velocity, and acceleration. On the basis of vehicle kinematic and dynamic theory, the safe range of vehicle lane change [94] was

analyzed, a sixth-degree polynomial lane change trajectory model was established, and the lane change trajectory cluster was obtained by using the genetic algorithm-BP neural network to predict the lane change completion time and target position. By analyzing performance indicators such as vehicle lane change safety, comfort, and economy, the multi-performance collaborative optimization objective function and constraint conditions were constructed, and the whale optimization algorithm [95] was used to optimize the lane change trajectory cluster. Finally, the multi-performance target collaborative vehicle lane change trajectory-planning algorithm was obtained. In order to verify its accuracy, an experiment was carried out using an L3 intelligent vehicle. The test results showed that the simulation of vehicle lane change trajectory planning is effectively improved. Kou et al. [96] analyzed and summarized the situation of vehicle lane change, and found that vehicle lane change is influenced by surrounding vehicles; based on this finding, the characteristics of low-carbon lane change were further improved. When a vehicle changes lanes, the dynamics of the surrounding vehicles play a decisive role in whether the vehicle successfully changes lanes. By combining the traffic impact of the vehicle change process with low-carbon driving, the objective function is established, and a method of traffic impact and low-carbon trajectory optimization is proposed. The optimization method combines the vehicle dynamic model with the operating condition, and establishes the safety domain according to the position relationship between the vehicle itself and the fore and rear vehicles. The lane change safety zone is the area where the vehicle is not likely to collide with the vehicles behind and in front. In the process of changing lanes, if the rear vehicle does not guarantee a safe distance, the current vehicle will collide with the left rear wheel of the rear vehicle. If a safe distance is maintained between the current vehicle and the rear vehicle in the target lane, the vehicle will not interact with the rear vehicle during the lane change. In the process of changing lanes, if the speed of the current lane is relatively high, a collision between the right front of the car and the left rear of the car in front may occur. If there is sufficient safe distance between the current vehicle and the rear vehicle in the target lane, the vehicle will not interact with the rear vehicle in the process of changing lanes. Vehicles on the road are influenced by the dynamics of the surrounding vehicles. In order to ensure the vehicles involved in the process of lane changing can be realized and rationalized, the quintic polynomial model in the ideal state is improved, and the longitudinal sextic polynomial model and the horizontal quintic polynomial model are established, as shown in Equation (7) below:

$$\begin{cases} x_t = a_6t^6 + a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0 \\ y_t = b_5t^5 + b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0 \end{cases} \quad (7)$$

In order to reasonably predict the lane change process, a BP neural network is established according to the acquired feature points, and the neural network is improved using the particle swarm optimization algorithm. The particle swarm optimization algorithm (PSO)-BP neural network is used to establish the lane change end time, and a relative longitudinal distance model is used to obtain the lane change end time and end position, as shown in Figure 3. By introducing the characteristics of the genetic algorithm, the evaluation results are optimized. Using car no. 689 of the NGSIM dataset as a sample, the trajectory planning model and the optimization algorithm are used to form the final lane change trajectory equation. The trajectory equations of lane change under different states are simulated, and the more suitable trajectories are highlighted to demonstrate that the method can effectively solve the problem of lane change. Table 4 gives a summary of the dynamic models.

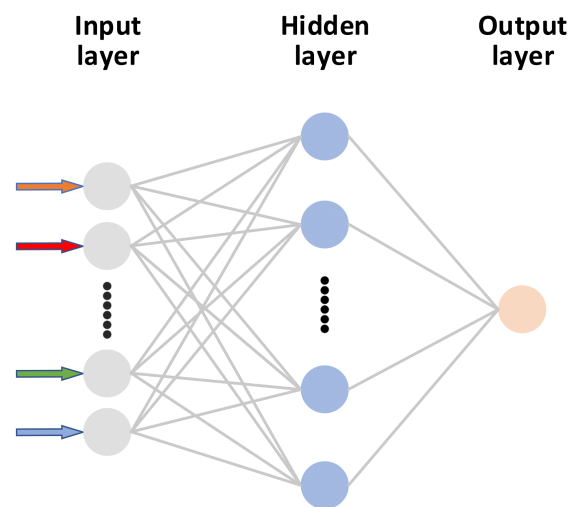


Figure 3. Structure of particle swarm optimization (PSO)-BP neural network [96].

Table 4. Input parameters of different dynamic lane change models and key problems to be solved.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[91]	Traffic flow, lane change frequency, and vehicle speed	Microscopic LCP model	Can handle non-continuous lane change processes
[92]	Traffic efficiency, traffic safety, fuel consumption, including emergency sign distance before lane closure, speed limit, traffic density, etc.	Linear regression model of the logarithmic lane changing	Additional lane change rules are proposed to deal with movement bottleneck and lane reduction, and the impact of lane change on traffic efficiency, traffic safety, and fuel consumption is studied
[93]	Vehicle lane change time, vertical and horizontal speed, and acceleration are key variables	Genetic algorithm-back propagation neural network	It improves the quasi-human and real-time performance of intelligent vehicle lane change trajectory planning
[96]	Vehicle number 689 in the NGSIM data	PSO-BP neural network + genetic algorithm	Safe and efficient lane changes can be planned for vehicles in real-time

2.2.3. Control Models

A vehicle control model mainly refers to the influence of the vehicle control system and control strategy on the lane change process, and describes the connection between control input (such as steering angle, brake, and accelerator pedal) and vehicle lane change, so as to study the influence of the control system on lane change. With the continuous development of vehicle automation, the demand for control technology is also increasing. There are two main directions of vehicle control technology: the first is optimization-based control, which summarizes the driving task as an objective function of multiple constraints' minimization or maximization and then solves it [97,98]. The second is intelligence-based control, which is mainly based on deep learning, reinforcement learning, and other AI-related techniques, and mainly uses the function approximation capabilities of deep neural network models to solve complex decision processes. Intelligent-control-related models have been applied to traffic flow prediction [99–102]. Hou et al. [103] adopted the fusion of random forest and AdaBoost to make lane change decisions via the first lane change model established through NGSIM. The test results show that the integrated learning method of random forest and AdaBoost can obtain higher classification accuracy than Bayesian or decision tree classifiers. Yang et al. [104] proposed that a lane-changing vehicle should constantly adjust its driving speed and position according to the environment, to ensure safety according to the dynamically changing objects around the vehicle. In order to solve this problem, a dynamic lane-changing trajectory planning model was developed that is composed of the

lane-changing starting point determination module, the trajectory decision module, and the trajectory generation module. It avoids making assumptions of lane change speed and acceleration that are unrealistic in the current vehicle lane change model. Suh et al. [105] studied the lane change trajectory-planning model of probabilistic prediction combined with deterministic prediction for automatic driving in a complex traffic environment, and established a stochastic model predictive control problem to obtain the steering angle and longitudinal acceleration of vehicles under constraints. Xie et al. [106] proposed a data-driven lane change model based on deep learning, which a combined deep belief network (DBN) and long short-term memory (LSTM) neural network, and modeled the lane change process data provided by the dataset (NGSIM) with the combined network model. Ali et al. [107], addressing the impact of a connected vehicle environment on forced lane change behavior, used the CARRS-Q advanced driving simulator for testing and analyzed various performance indicators in forced lane change. They did so by applying repeated measurement AN variance analysis and generalized estimation equation of a linear hybrid model, and they concluded that a connected vehicle environment improves the safety of forced lane change. Zhang et al. [108] used a deep learning model long and short memory network (LSTM) to simulate vehicle following and lane change behavior at the same time, proposed a hybrid retraining constraint method to optimize the LSTM network, and used the vehicle time series and memory effect characteristics of the NGSIM dataset to predict the lane change behavior, which improved the prediction accuracy of lane change. Chen et al. [6] proposed a method that could predict the risk level of lane change before it was completed, and proposed an algorithm combining resample and machine learning classifier for key spatial sequence features of vehicles. The verification on the dataset NGSIM found that both feature stability and risk level showed significantly improved performance. Chen et al. [109] proposed that the safety and mobility of intelligent vehicles can be improved within the environmental space of sharing traffic information and issuing control motion instructions. However, due to the combination and volatility of intelligent vehicle networks, there are certain difficulties in the actual control process, which cannot be effectively solved using conventional control technology. Therefore, algorithms based on deep reinforcement learning are proposed, using roadside cloud platforms or other control facilities. To overcome a limited detection range of sensors, the integration of short- and long-distance information in the driving environment can help autonomous vehicles detect the surrounding environmental information more comprehensively and thus ensure appropriate decision-making and safe and effective motion planning during lane changes. Dong et al. [110] summarized the automatic driving control method, which was mainly used for the traffic information perceived in the nearby traffic environment. But how is it possible to integrate the nearby traffic information with the long-distance traffic information, as well as traffic information obtained with different time stamps? Currently, there is relatively little research on how to incorporate this traffic information to enhance the control of autonomous driving. Dong et al. [110] proposed a method based on deep reinforcement learning, which made three main contributions. The first contribution was to propose a DRL-based model (using a modified Deep Sets procedure), which integrates the information obtained locally with the traffic state information obtained from the vehicle connectivity function. The second contribution was the development of an end-to-end network architecture to control lane changes by combining the perceived and distant road information collected by the autopilot, that is, obstacles near the autopilot and details of vehicles at a distance, and combining these data to optimize lane changes of the autopilot vehicle. The third contribution was an evaluation of the impact of traffic density on the lane change process, providing an indication of connectivity thresholds to ensure optimal vehicle performance. The reinforcement learning algorithm was used to verify the safety and efficiency of connected driving vehicles in the simulation environment, as shown in Figure 4 below.

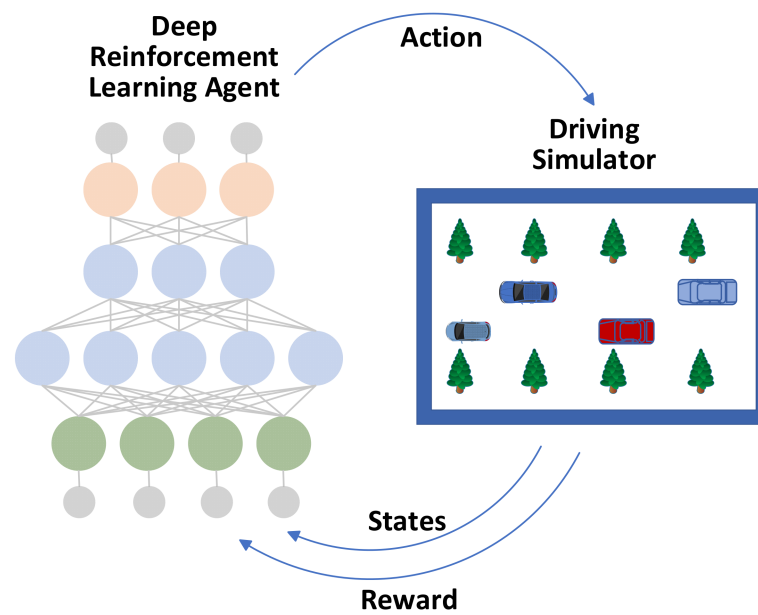


Figure 4. Reinforcement learning in simulation of connected driving vehicles. On the left is the deep reinforcement learning agent, and on the right is the driving simulation environment, where the driving simulation inputs state and reward functions into the deep reinforcement learning agent, and the deep reinforcement learning agent feeds actions back into the driving simulation environment [110].

The agent feeds the state information of the exploration environment into the reinforcement learning algorithm, which promotes desirable and weakens undesirable driving behavior [111]. In the simulation process, the agent drives by observing the current state, receives feedback information from the environment, and reacts to positive and negative information. By evaluating the feedback signals and making adjustments accordingly, a good model of driving behavior is obtained. Table 5 gives a summary of the control models.

Table 5. Input parameters of different control lane change models and key problems to be solved.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[108]	Time series historical location information of the subject's vehicle and six surrounding vehicles	HRC LSTM	Accurate estimation of vehicle following and lane-changing behavior at the same time
[6]	US-101 segments of NGSIM data	P-LRLP	Estimate the collision risk level of lane change events in advance
[109]	Movement and combined lane change data for freeway exit ramp entry sections	GCN + DQN	In the context of sharing traffic information and issuing control movement commands, the safety and mobility of intelligent vehicles can be improved
[110]	Relative distance, relative speed, and relative lane position	DRL + end-to-end framework	Use reinforcement learning to facilitate the safe and efficient movement of connected autonomous vehicles in a simulated environment

2.3. Cognitive Models of Lane Change

In the lane change cognitive models, lane change behavior is dependent on the driver's decisions, which are made according to their cognition, understanding, and judgment. Accordingly, a lane change model is established to describe how drivers perceive environmental information, make lane change decisions, and perform lane change behaviors.

There are three common cognitive models: decision tree, artificial neural network, and information processing.

2.3.1. Decision Tree Model

The decision tree model decomposes the driver's lane change behavior into decision nodes and condition judgment. According to the driver's situation and driving environment, they decide whether to change lanes according to the decision nodes and condition judgment in the decision tree model. Toshiyuki et al. [112] applied the decision tree model and production rule algorithm commonly used in data mining to study the behavior of driving path selection. Compared with the artificial neural network algorithm, this algorithm has an advantage, which can help researchers to determine the correlation between explanatory variables and choice. Park et al. [113] further studied the sensitivity method of decision tree based on the work of Toshiyuki et al. [112]. Soft discretization of continuous values in the optimized fuzzy decision tree can improve the classification of robustness, and the fuzzy decision tree assigns values by deciding on the degree of certainty of proposals generated with fuzzy reasoning. This feature makes it possible to solve multiple proposal problems. The experimental results were compared with the non-fuzzy adaptive path selection, and the prediction accuracy of the fuzzy decision tree model was higher. The decision tree induction method of Park et al. [114] is a type of machine learning algorithm that can build a path selection model. Decision trees can accommodate routing preferences, but "sharp cutoff points" can make the tree too sensitive, leading to misclassification. Park et al. [114] added fuzzy decision tree induction to reduce sensitivity. The fuzzy decision tree optimizes the discretization in the classical decision tree to soft discretization via induction, thus improving the flexibility of new case classification. Hou et al. [115] developed a lane change assistance system to remind the driver of the safe distance when changing lanes safely. The lane change model is modeled by using the decision tree method and Bayesian classifier, and the NGSIM dataset is used to train the model. The model predicts whether drivers will merge as a function of input variables. By using the majority voting principle, the decision tree method and Bayesian classifier are combined into one classifier, and the best result is obtained. The decision-making process of lane changing means the driver easily makes mistakes, so the optimization of collision avoidance system modeling and prediction of lane change maneuvers have become the focuses of research. Networked vehicle data provide the possibility for accurate lane change modeling. Such a system has high requirements for the accuracy of lane change initiation detection, and comprehensive vehicle lane change data can ensure that the lane change initiation accuracy requirements are met. Most lane change models of vehicles fail to achieve reliable accuracy in lane change prediction. Mousa et al. [116] chose the extreme gradient boosting (XGB) algorithm. Extreme gradient boosting (XGB) is a type of machine learning algorithm that can build a relatively strong learner by training multiple weak learners, generally a set of multiple decision trees. XGB uses the gradient lifting algorithm, which can iteratively train the decision tree model and use the connected vehicle data to predict the start of the lane change maneuver. The results obtained with the XGB model were compared with the decision tree, gradient lifting, and random forest, and the results showed that the XGB model performed better than the other algorithms. An advanced driver assistance system (ADAS) can warn drivers whether they are in a safe lane-changing environment according to the detected data. Hou et al. [104] proposed integrating the random forest algorithm with the AdaBoost algorithm to improve the performance of the lane-changing assistance system. The experimental results show that the performance of the two integrated learning methods is better than that of the single learning algorithm model such as Bayes and the decision tree classifier. The traditional vehicle lane change decision-making model cannot accurately and completely describe the lane change process, and relying on experience to formulate the model lacks a certain rationality and cannot reflect the driver's psychological and physiological reactions in the process of a lane change. In the process of a lane change, appropriate variables must be selected as the inputs of the model. Gu et al. [117] used

a random forest algorithm to establish a decision-making model of vehicle lane change behavior. The vehicle data were extracted from the NGSIM dataset, and the errors in the NGSIM data were corrected using the symmetric exponential moving average algorithm, with the test results showing a good degree of prediction and fitting. In order to make the driving process safer and more reliable, it is important for advanced driver assistance systems (ADASs) to reasonably understand and accurately predict human driving behavior. Deng et al. [118] developed a driving assistance system based on prediction of the driver's behavior. They modeled three driving behaviors: lane change to the left, lane change to the right, and lane keeping. The prediction model was trained using machine learning methods such as hidden Markov model (HMM), support vector machine (SVM), convolutional neural network (CNN), and random forest (RF). The model incorporates information about eye movement trajectories. Experimental results showed that the random forest algorithm has the best performance among the four algorithms. The random forest algorithm takes a decision tree as a component unit and combines a large number of decision trees to form a random forest. However, if many factors in the driving environment make the process of lane changing complicated, inappropriate lane-changing behavior will lead to serious traffic accidents. Therefore, Wang et al. [119] proposed a lane change prediction method based on extremely random decision trees, which divides the lane change process into two stages according to the driving environment, as shown in Figure 5 below.

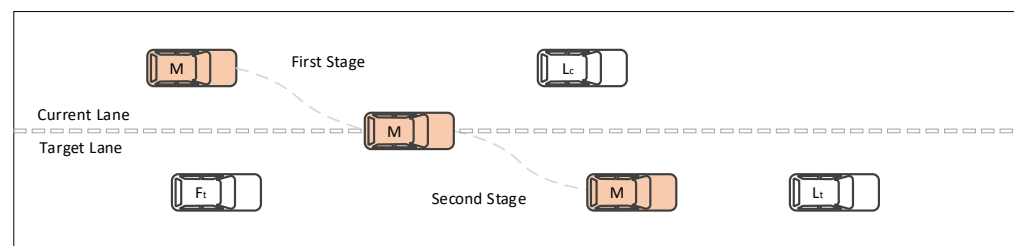


Figure 5. Two-stage lane change process. Red indicates the lane changing vehicles, white vehicles indicate the vehicles around the red vehicle during the lane changing process, and the gray dashed line indicates the path of the red lane changing vehicles [119].

The first stage ends when the front center of the vehicle crosses the lane line, and the second stage begins when the front center of the vehicle crosses the target lane and ends when the vehicle has corrected its direction of travel. On the road, M represents the lane-changing vehicle, Lc represents the leading vehicle in the current lane, Lt represents the leading vehicle in the target lane, and Ft represents the following vehicle in the target lane. The input variables for an extremely random decision tree model are $X = \{\Delta V_{ij}, \Delta a_{ij}, \varnothing_{ij}\}$, where ΔV_{ij} represents the relative speed between the lane-changing vehicle and the surrounding vehicles, Δa_{ij} represents the relative acceleration between the lane-changing vehicle and the surrounding vehicles, and $\varnothing_{ij}$ represents the relative potential between the lane-changing vehicle and surrounding vehicles. The output variable is the driving angle $Y = \alpha_i$, calculated from the transverse and longitudinal positions of the lane-changing vehicles, and the sample is $S = \{X, Y\}$. The calculation process of the extremely random decision tree model is similar to that of the random forest model. The main difference is that the random forest uses the random sampling bootstrap to filter the sampled dataset, while the extremely random decision tree model uses the original dataset for training. The random forest selects the optimal segmentation points according to the information gain, while the extremely random decision tree model randomly selects the segmentation nodes. The calculation process [120] is as follows: when $|S| < n_{min}$, the candidate property is constant or the output variable is constant, and the result returns a leaf marked with the average output. n_{min} represents the minimum number of samples of split nodes, while $|S|$ represents the length of the vector S . In addition, N attributes $\{S_1, \dots, S_N\}$ are selected

from all candidate attributes. The maximum value of $Score_R(s_g, S)$ is s_g , and the fraction of S and split S , are defined as shown in Equation (8):

$$Score_R(s, S) = \frac{var\{y|S\} - \frac{|S_l|}{|S|}var\{y|S_l\} - \frac{|S_r|}{|S|}var\{y|S_r\}}{var\{y|S\}} \quad (8)$$

In Equation (8), $var\{y|S\}$ represents the variance of the output variable of sample S , and S_l and S_r represent two subsets of S . One must construct the left subtree t_l and the right subtree t_r of the split node s_g . Then, one must make a node split into s_g , with t_l and t_r as the left and right subtrees of the node, and output that as a tree, where the result is the average value of the output. The performance of the extreme random decision tree model was compared with the traditional machine learning method, and the detection performance of this method was proven. Table 6 gives a summary of the decision tree models.

Table 6. Input parameters of different decision tree lane change models and key problems to be solved.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[104]	US-101 and I-80 segments of NGSIM data	Random Forest + AdaBoost	Improve the accuracy of safety-critical lane changes and reduce the incidence of traffic accidents
[117]	US-101 and I-80 segments of NGSIM data	Random Forest	It can accurately describe the lane change mode and complete driver behavior characteristics
[118]	Driving behavior + eye-tracking data	HMM + SVM + CN-N + RF	Help the driver judge when it is appropriate to turn left or right or maintain their direction
[119]	Relative velocity, relative acceleration, and potential energy	Extremely randomized decision trees + harmonic potential field method	Prediction of driving angle during lane change

2.3.2. Artificial Neural Network Model

The artificial neural network model uses the neural network to process the driver's lane change behavior. The artificial neural network mainly consists of three parts. The first part is the input layer of the neural network, through which traffic information and road information related to lane changing are transmitted to the network. The second part is the hidden layer. Traffic information and road information related to the lane change are passed through the hidden layer via the input layer. The hidden layer mainly processes the input information to obtain the desired results. The third part is the output layer, through which the results obtained from the hidden layer are output. In the lane change behavior model, each neuron in the output layer represents specific lane change behavior information. The neural network has a strong learning ability, generalization ability, information synthesis ability, and self-adaptability, and it offers a way for researchers to improve the lane change model. The lane-changing process is a relatively important maneuver during driving, which will cause a large number of traffic jams and vehicle collisions. Tomar et al. [121] found that the existing lane change model does not consider the uncertainty and perceptions involved in human behavior during the lane change process. The neural network can learn related uncertainties to improve the accuracy when predicting the lane change trajectory. For instance, a multi-layer perceptron (MLP) was trained on NGSIM data to predict the lane change trajectory of a vehicle. The experimental results showed that the perceptron (MLP) can accurately predict the future path of the discrete trajectory, but cannot predict the future path of the complete trajectory. Ding et al. [122] used a backpropagation (BP) neural network to predict the validity of lane change trajectories. The driving simulator data and NGSIM data were smoothly preprocessed. The experimental results show that

the BP neural network can accurately predict lane change behavior in urban traffic. The accurate prediction of driving behavior is very important for the performance of active safety systems. Peng et al. [123] designed a lane change behavior prediction model based on natural road experiments. The time window of lane change intention is determined by extracting the visual features of the rearview mirror. A backpropagation neural network model was established to integrate the drivers' visual search behavior, driving conditions, vehicle operation behavior, and vehicle motion state. The experimental results showed that the lane change model can accurately predict the driver's lane change behavior at least 1.5 s in advance. In this regard, a real-world classification problem involves multiple variables and analyses in the time domain. Time series data have high dimensionality, large data volume, and constantly updated performance. Gao et al. [124] used three physiological signals of drivers to predict lane changes, namely electrocardiogram (ECG), electrodermal response (GSR), and respiratory rate (RR). On the basis of these, a new model of MTS-GCNN for time series (MTS) pattern classification was proposed. A new structure learning algorithm was proposed in the training phase of the MTS-GCNN model. Different from other classification methods, the MTS-GCNN model generates temporal and spatial features by extracting suitable internal structures through convolution and subsampling. The experimental results showed that the MTS-GCNN model performs better than other advanced models in terms of prediction accuracy. Environmental factors make for uncertain lane changing behavior, which may cause serious traffic accidents. Effectively detecting the driver's intention to change lanes can improve safety. Tang et al. [125] proposed a lane change prediction model based on the fuzzy C-means clustering algorithm and adaptive neural network (FCMNN), and added a new prediction process: an unsupervised learning method classifies the distribution features of the original dataset into different clusters, and then a supervised learning approach optimizes the sub-neural network structure and weight parameters for each cluster or pattern. Compared with some traditional methods, the prediction performance and stability of this model are improved significantly. With the rapid development of automobile automation and intelligence, intelligent driving assistance functions occupy an increasingly important part of the human driving process. The mutual cooperation and understanding between intelligent vehicles and human drivers is the key to achieving collaborative driving and efficiency. Xing et al. [126] designed a two-stage spatiotemporal inference learning framework for a driver's lane change intention, which consists of two parts. The first part is a driving behavior recognition system based on the deep convolutional neural network (CNN), which is mainly used to identify the driver's driving behavior and check the rearview mirror. Driver behavior features based on a deep convolutional neural network (CNN) are used to construct a driver intent inference model composed of recurrent neural network (RNN) and long short-term memory (LSTM) algorithms. Experimental results show that the prediction accuracy of this model is more than 91%. The lane change prediction model, gives its good performance, can help drivers better understand the driving environment and thereby improve road safety. Yuan et al. [127] designed a temporal convolutional network (TCN-ATM) model based on the attention mechanism, to detect a driver's intention to change lanes. The multi-task learning (MTL) framework was used to predict various vehicle status indicators during a lane change. In addition, based on the multi-task learning (MTL) framework, a lane change intention recognition and driving state prediction modeling framework (LC-IR-SP) was proposed, as shown in Figure 6.

The LC-IR-SP is composed of two core modules: lane change intent recognition (LC-IR) and lane change state prediction (LC-SP). The former is a classification model used to identify whether a vehicle has the intention to change lanes to the left or right. When it recognizes a lane change intention, the lane change state prediction (LC-SP) module starts to predict the driving state of the lane-changing vehicle. The LC-SP module consists of independent multi-task and single-task learning models. The multi-task learning model is responsible for predicting relevant variables, while the single-task learning model predicts irrelevant variables. The experimental results show that the classification accuracy of the

driver's lane change intention in the TCN model with the attention mechanism increases from 96.14% to 98.20%. Table 7 gives a summary of the models based on the artificial neural network.

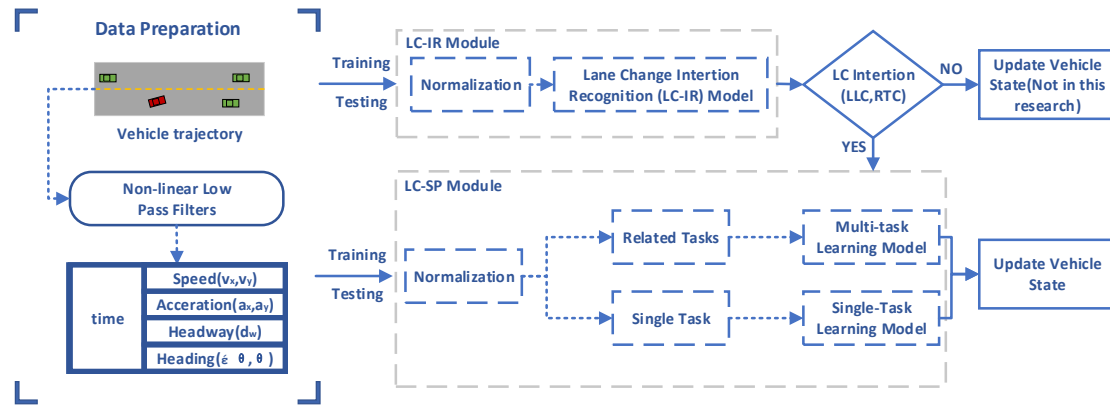


Figure 6. Modeling framework for lane change intention recognition and driving state prediction [127].

Table 7. Input parameters of different artificial neural network lane-changing models and key problems to be solved.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[124]	Electrocardiogram (ECG), g-alvanic skin response (GSR), and respiration rate (RR)	MTS-GCNN	This algorithm uses economical physiological sensors instead of expensive ones to predict the driver's lane change behavior accurately and effectively
[125]	47 drivers participated in the experiment and 141 valid lane change records were collected	Fuzzy C-means clustering algorithm + FCMNN	The intention recognition of driving behavior is deeply analyzed to improve the safety of lane change behavior
[126]	CranData	CNN + RNN + LSTM	The two-stage learning framework can significantly improve the flexibility and accuracy of the model
[127]	Longitudinal velocity, lateral velocity, longitudinal acceleration, lateral acceleration, vehicle heading, and deflection angle	TCN-ATM + LC-IR-SP	The model has great potential to improve the perception and prediction capabilities of autonomous vehicles and improve vehicle control strategies

2.3.3. Information-Processing Model

The information-processing model is a classic cognitive model to describe the decision process of a lane change. The model divides lane changing into three stages, namely perception, judgment, and execution. In the first perception stage, the driver acquires information about the road environment mainly through hearing and seeing. In the second judgment stage, when the driver perceives the environmental information of the relevant road, they analyze and judge it according to their personal driving experience, to decide whether to change lanes. In the third execution stage, when the driver decides to change lanes, they will control the speed and operate the steering wheel of the vehicle to perform the actual lane change behavior. Lane change behavior has received more and more attention, and more and more algorithms have been developed. However, most of the algorithm models are derived and verified using the vehicle trajectory data, without considering the driver's behavioral characteristics. Sun et al. [128] studied relevant information concerning drivers

and classified them according to their background information and verbal description, to combine the drivers' lane change behavior and driving characteristics into the lane change model. Both types of information contained quantitative and qualitative responses and were incorporated into the lane change model.

The driving behavior of the surrounding vehicles is an important factor in driving safety. Wang et al. [129] comprehensively considered the relationship between the two factors and proposed a prediction method based on a fuzzy reasoning system (FIS) and long short-term memory (LSTM) neural network. The fuzzy reasoning system is used to simulate the driver's cognitive process in the driving environment, and the traffic environment information is transformed into the feasibility of lane change. The lane change feasibility and vehicle trajectory are predicted as input variables of the LSTM neural network, and then the decision strategy of path planning is designed based on the experimental results. The experiment used an NGSIM dataset for training and testing, and the results showed that this method can improve the ability to process lane changes. The needs and intentions of road users play an important role in improving the autonomous recognition of driver assistance systems and the assessment of driving situations. In order to effectively predict lane changes, the characteristics of different situations and drivers must be taken into account. Leonhardt et al. [130] proposed an algorithm to predict lane changes by evaluating driving conditions, driver behavior, and vehicle motion. This algorithm makes high demands on the perception of the vehicle's driving environment, combined with the analysis of the driver's observation behavior of the environment before changing lanes. The artificial neural network, parameterized via machine learning, is used to fuse the parameter information and use it for classification. Driving behavior prediction technology is of great significance to the optimization of advanced driver assistance systems (ADASs). Deng et al. [131] proposed a driving behavior prediction method based on the hidden Markov model (HMM). The three different driving actions of left lane changing, right lane changing, and lane keeping were the hidden states of the hidden Markov model (HMM). On the basis of observation (training), the hidden Markov model (HMM) method could calculate the most likely driving behavior from the observed sequence. Observation sequences were also used to train the HMM. In order to improve the performance of the model, the collected signals were quantified into observation sequences with specific features. Compared with other methods, the validity of the driving behavior prediction was proven. The process of autonomous lane changing is mainly divided into three stages: perception, decision, and execution. In the first stage, a lot of environmental information is collected; before changing lanes, the driver must first observe the changes in the surrounding driving environment. In the second stage, the lane change decision is influenced by the driving environment parameters around the vehicle. At this stage, there are some parameters that influence the lane change decision and some parameters that influence the driver's lane change. The study by Bağdatlı, M.E.C. et al. [132] aimed to investigate the parameters that influence drivers' lane changes. In that study, the parameters determined by interviews with drivers were verified, and the significant association feature extraction (SigAFE) algorithm was used to extract the parameters influencing lane change decisions. Using the influence value of lane change decisions, the fuzzy cognition graph (FCM) model of arbitrary lane change decisions was developed. In lane change decision-making, drivers need to observe the traffic conflicts in both the current lane and the target lane. Li et al. [133] used driver physiological data and vehicle dynamic data to predict driving risks during lane changes. The data from the experiment were obtained from a wearable sensor assessing the actual driving process. The hidden Markov model was used to correlate driving risk with driver physiological information and vehicle dynamic data. The effects of eye movement, heart rate variability, and vehicle dynamic parameters on driving risk were compared. The standard deviation of the normal to normal R-R interval of heart rate (SDNN), fixation time, sash-up range, and average speed were selected as inputs to the hidden Markov model. The model uses data from physiological measurement sensors to predict the transition probability between hazardous driving states and normal driving

states. A multi-dimensional information interaction platform improves the identification of specific lane-changing strategies. Intelligent driving systems cannot make human-like decisions just by detecting lane change behavior; in fact, poor environmental cognition is the main reason why intelligent driving systems cannot make correct decisions on the road like humans, so identifying specific lane change strategies can provide support for improving safety performance. The research objective of Sun [134] was to identify subjects' different lane change strategies when the rear vehicle in the target lane approached the lane change node. In the experiment, 42 experienced drivers (37 males and 5 females) were selected to participate in the experiment, to obtain different lane change strategies, and the obtained lane change types were divided into three types: forced, concession, and waiting. The forced lane change strategy is shown in Figure 7. During the forced lane change, the driver must observe the road environment and confirm that the environment can allow the target vehicle to make a safe lane change. When the target vehicle and the rear adjacent vehicle meet the conditions required for a lane change, the participant will quickly change to the target lane. As shown in Figure 7, when the subject drives the vehicle so that it approaches the target lane to change lanes, it is necessary to assess whether there is any risk in the traffic environment around the target lane. When the current lane change is too risky, the lane change cannot be completed, and the participant needs to return the vehicle to the center of the current lane. The waiting lane strategy is shown in Figure 7. When the driver assesses the traffic environment during the process of a lane change, the driver can change lanes when the vehicle behind the target lane no longer poses a threat to the target vehicle. The characteristic parameters such as lane change duration, aisle time, distance from lane line, steering wheel angle, relative distance, and relative speed differ for different lane change types. The random forest classifier is used to construct the lane change recognition model, and the recognition performance of the attention-bidirectional long short-term memory (attenbilstm) model and the genetic algorithm-support vector machine (GA-SVM) model is compared. The experimental results show that the global recognition accuracy and recognition time under different input parameters are better than with the GA-SVM model, and the performance of the attention-bidirectional long short-term memory (attenbilstm) model is comparable. Table 8 gives a summary of the information-processing models.

Table 8. Input parameters of different information-processing lane change models and key problems to be solved.

Paper	Input Variables	Algorithm	Key Issues to Be Resolved
[129]	NGSIM dataset	FIS + LSTM	The influence of the driver and driving environment on lane change behavior is taken into account
[132]	Driver complex behavior and motivation characteristics	FCM	Discover the parameters that prompt the driver to change lanes
[133]	Driver physiological information and vehicle dynamic data	HMM	A method to predict driving risk during lane change using driver physiological measurement data and vehicle dynamic data
[134]	Under different lane change strategies, lane change duration, aisle time, distance from the lane line, steering wheel angle, relative distance, relative speed, and other characteristic parameters	Random forest classifier	Find out the different lane change strategies of the subject vehicle when there is a rear-approaching vehicle in the target lane

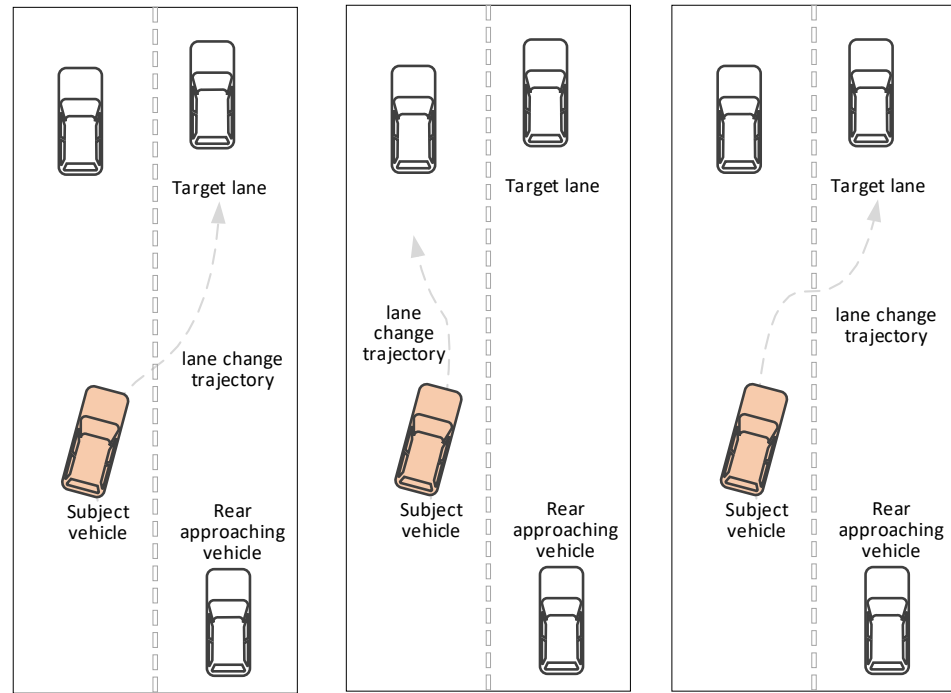


Figure 7. Three typical lane change strategies. Red indicates the target vehicle of the lane change, white vehicle indicates the vehicles around the red vehicle during the lane change process, gray dashed line indicates the path of the red lane change vehicle, and arrow indicates the direction of travel. The left picture indicates that the vehicle successfully completed the lane change, the middle picture indicates that the vehicle did not complete the lane change, and the right picture indicates that the vehicle successfully completed the lane change after waiting in the process of lane change [134].

2.4. Mixed Models of Lane Change

The mixed lane change models can also improve the overall performance by combining the advantages of different models, and thereby obtain a more comprehensive lane change decision. By combining the results of multiple models, the driver's lane change process can be more accurately simulated. In order to improve the safety of vehicle lane change, Jiabin Wu et al. [135] proposed a coevolutionary lane change trajectory-planning algorithm (CLTP), as shown in Figure 8. A GRU algorithm optimized by a recurrent neural network (RNN) collects vehicle movement information from adjacent lanes of lane change vehicles. The algorithm adds reset, update, and input modulation gates to the model to finetune the state flow and input flow of historical data. The GRU algorithm optimizes the problem of gradient disappearance and backpropagation in long-term memory. Compared to the LSTM algorithm, the GRU has a simpler training mode and better training model efficiency and performance. The hidden layer of the GRU includes reset and update gates. The reset gate mainly combines the newly entered information with the historical information, and the update gate mainly stores the previous historical information in the current step. The GRU predicts the vehicle track information of adjacent lanes, and the calculation process of each control gate is shown in Equations (9) to (10):

$$r = \sigma(x_{t+1}^k U^r + s_t^k W^r) \quad (9)$$

$$z = \sigma(x_{t+1}^k U^z + s_t^k W^z) \quad (10)$$

$$h = \tan h(x_{t+1}^k U^h + s_t^k \otimes r) W^h \quad (11)$$

$$s_{t+1}^k = z \otimes s_t^k + (1 - z) \otimes h \quad (12)$$

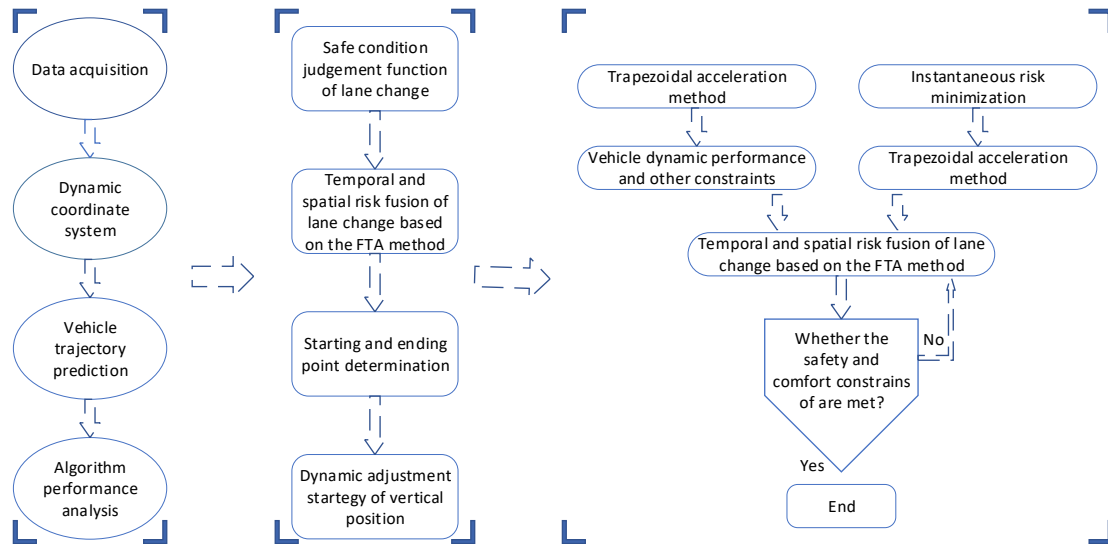


Figure 8. Schematic diagram of CLTP model [135].

In Equations (9) to (12), r represents the reset gate, z represents the update gate, and h represents the contents of the current memory. s_t^k represents the memory of the adjacent vehicles k with the current time step, σ represents the sigmoid activation function, x_t^k represents the coordinates (x_{ij}^k, y_{ij}^k) of the adjacent vehicles k at time t , U^i represents the update transfer matrix, W^i represents the weight matrix, $i = r, z, h$, and the symbol $\otimes$ represents the multiplication of vector elements.

The experiment adopts the natural driving vehicle trajectory data contained in the HighD dataset [136], and the GRU algorithm collects the vehicle movement information of adjacent lanes of lane-changing vehicles in the HighD dataset. Using fault tree analysis [137, 138], as shown in Figure 9, a spatial-temporal risk recognition model of lane change events was established. Based on the overtaking rule of acceleration and deceleration and the ladder acceleration method, the longitudinal and lateral motion schemes during a lane change were planned, and the motion parameters were obtained by using the genetic algorithm. The experimental results effectively improved the safety of a lane change.

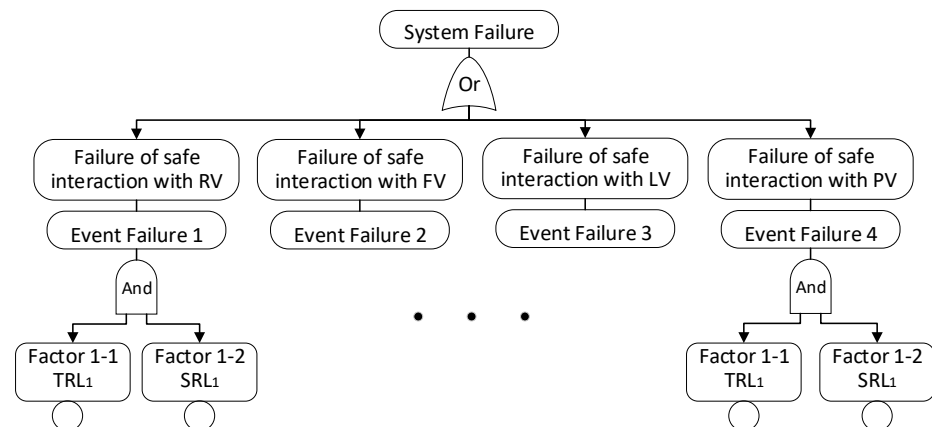


Figure 9. FTA analyzes safe cross-lane change behavior [135].

Vehicles can cause turbulence in the traffic flow when performing dangerous lane changes, and even cause traffic accidents during dangerous lane changes [1,2]. Therefore, the effective prediction of lane change risk can improve the safety of drivers when making lane changes. Shangguan et al. [139] found that most of the research on L/C risk prediction did not include driver intention recognition, and the real-time performance of L/C in the process of risk prediction was difficult to guarantee. Therefore, considering the driving risk difference between the left lane (LCL) and right lane (LCR), a letter-of-credit risk prediction framework is proposed, which integrates the letter-of-credit driving intention identification module and the letter-of-credit risk prediction module, as shown in Figure 10.

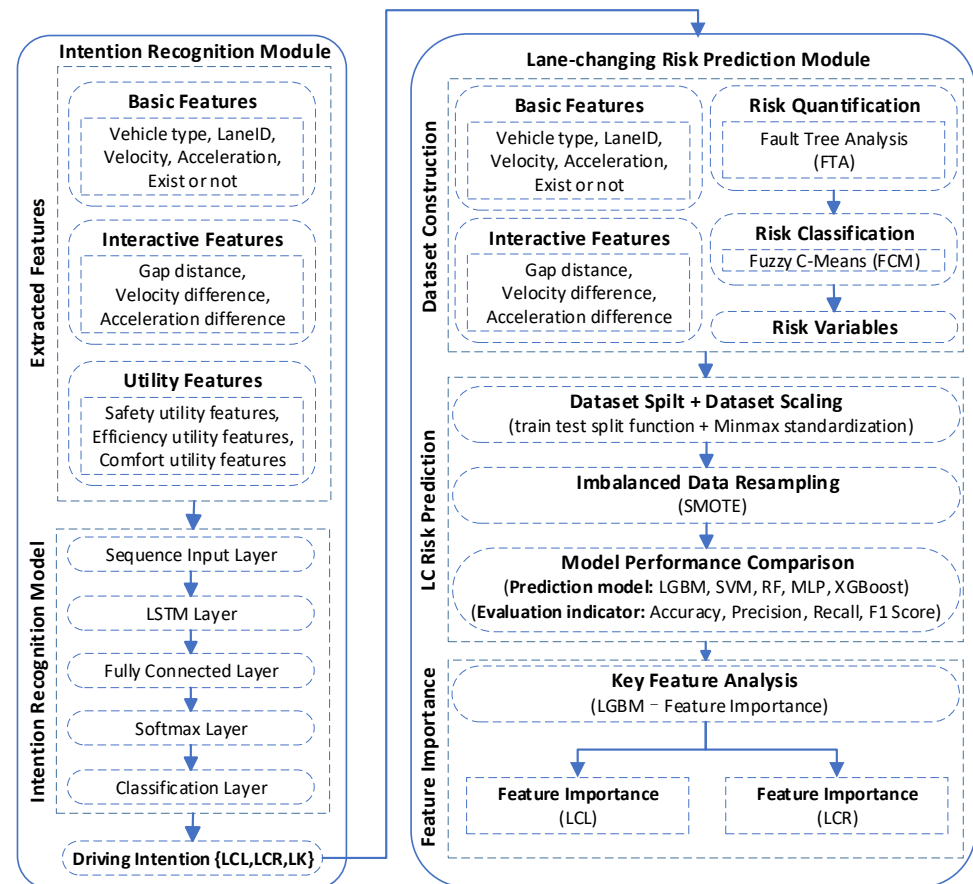


Figure 10. Credit risk prediction framework [139].

Drivers' lane change intention is influenced by several complex factors [106]. In the cited study, vehicle type, vehicle motion characteristics, speed difference, acceleration difference, distance between adjacent vehicles, safety, efficiency, and comfort characteristics were extracted as part of the drivers' lane change intention. A long short-term memory (LSTM) neural network was used to identify a driver's lane change intention. In order to ensure the effectiveness of that intention, vehicle track data on vehicle motion features were extracted for risk prediction. The risk of lane change was quantified using fault tree analysis, and the risk of collision with adjacent vehicles was also calculated [140]. The optical gradient intensifier (LGBM) algorithm [141] was then used to predict the risk of lane changes. Finally, the key characteristics of lane change risk were verified by using the HighD trajectory dataset. Changing lanes is a complex driving behavior. Assisting vehicles to understand and accurately predict the lane change process can help road users to better navigate the environment around the driving vehicle and thus avoid potential safety hazards during the lane change process. Yuan et al. [142] developed a lane change intention recognition (LC-IR) model and a lane change state prediction (LC-SP) model. An

integrated temporal convolutional network (TCN-LSTM) of long- and short-term memory units was proposed, as shown in Figure 11 below.

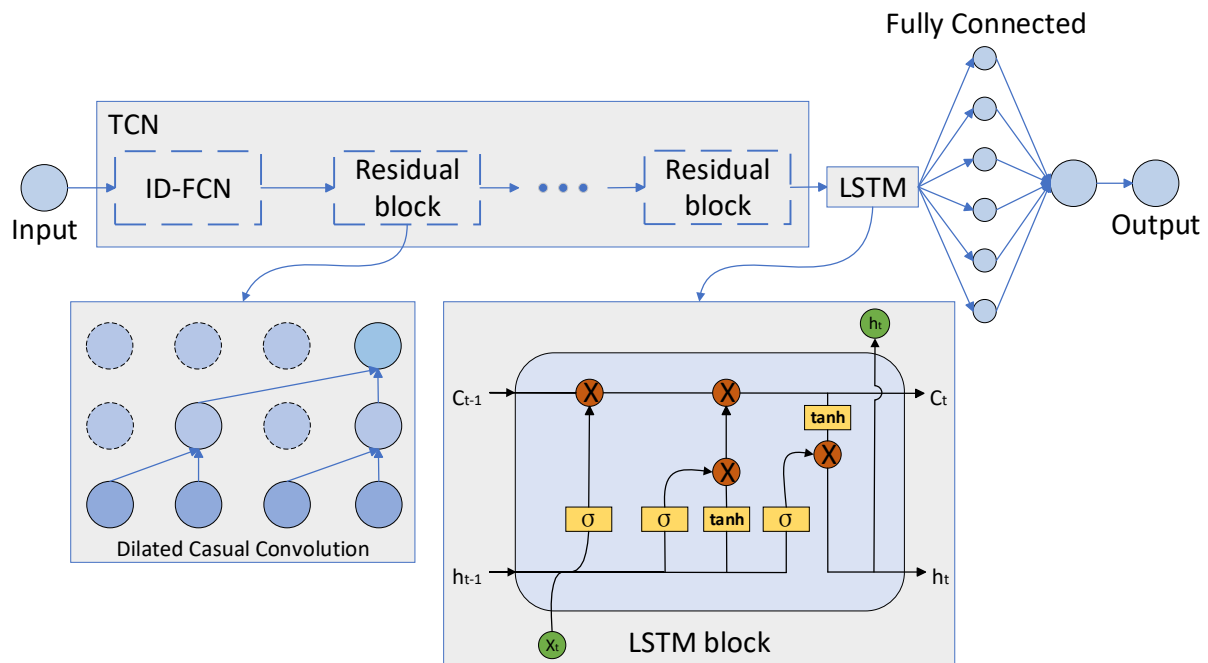


Figure 11. Integrated time convolutional network of long short-term memory units. From left to right are the input layer, the TCN network structure, the detailed structure of the TCN residual block, the detailed structure of the LSTM module, the fully connected layer and the network output layer [142].

The LSTM network has the problem of gradient disappearance and an inability to perform parallel computations [143]. The temporal convolutional network TCN [144] can capture the time-scale dependencies in the input sequence through the causal convolutional layer. Therefore, a temporal convolutional network (TCN-LSTM) is established to collect long-term dependencies in sequence data. In addition, three multi-task models, MLT-LSTM, MLT-TCN, and MLT-LSTM-TCN [145,146], were also established to compare the relationships between driving state variables, as shown in Figures 12–14.

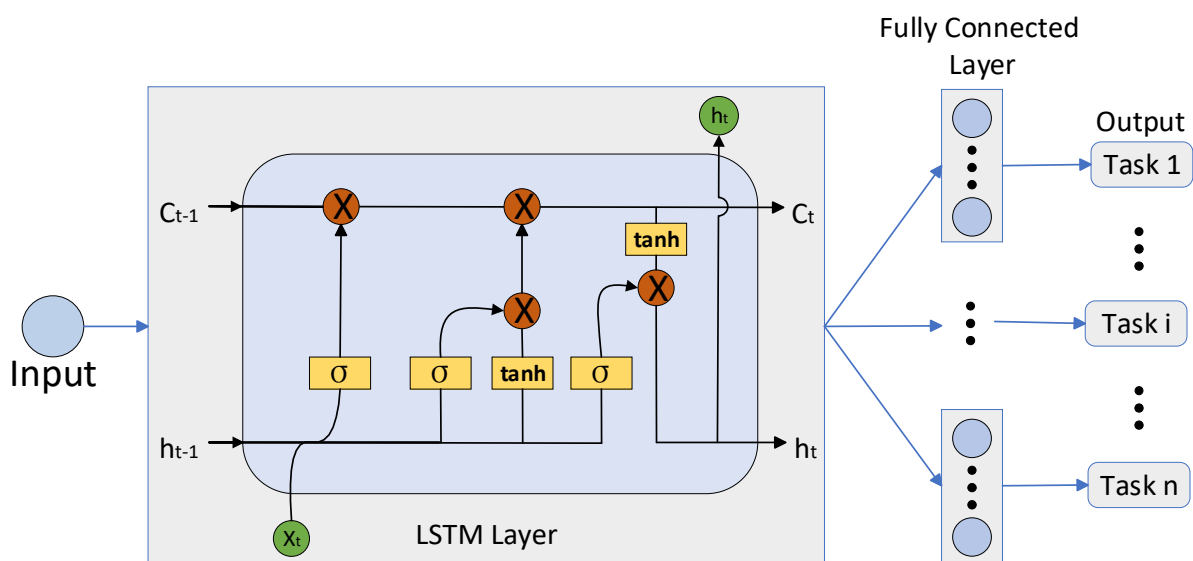


Figure 12. MLT-LSTM network structure [142].

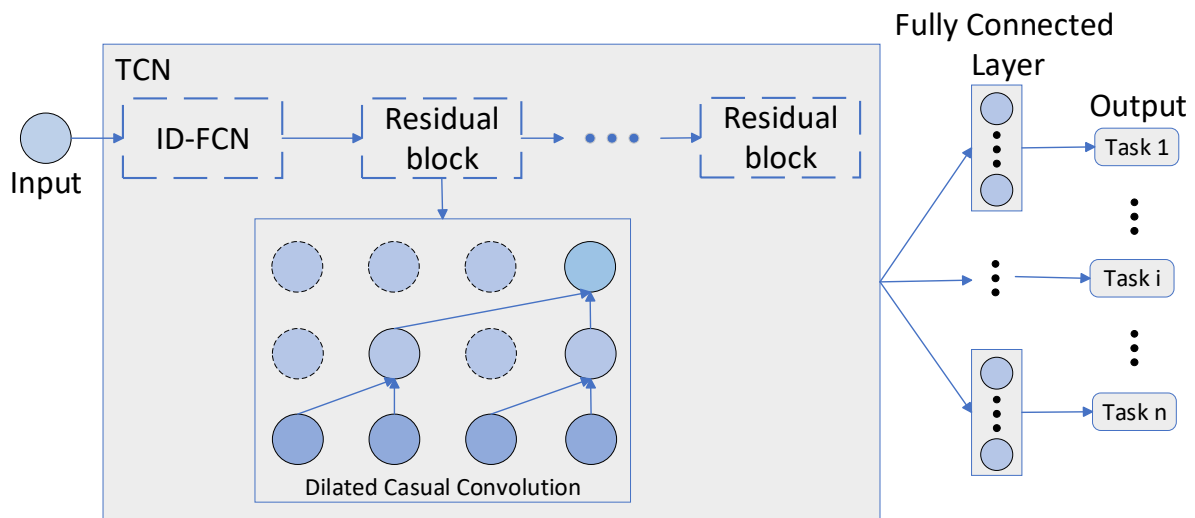


Figure 13. MLT-TCN structure [142].

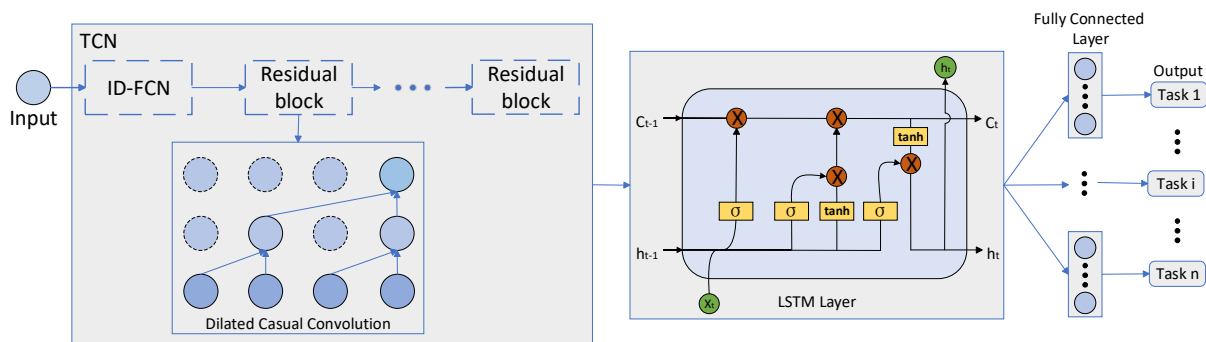


Figure 14. MLT-LSTM-TCN structure [142].

Based on vehicle trajectory data, a framework for lane change intention recognition and driving state prediction (LC-IR-SP) was proposed. For experimental verification, 1023 vehicle trajectories were selected from the CitySim [147] dataset. Pearson's coefficient was used to determine the relevant indicators. The experimental results showed that the TCN-LSTM model is better than the TCN and LSTM models in the classification of a driver's lane change intention. The performance of the three multi-task learning models is significantly better than that of the single-task learning model. Table 9 gives a summary of the mixed models. At present, although the mixed models improve the performance of the lane change model to a certain extent, they still need to be further optimized to meet the requirements of vehicle lane change.

Table 9. Input parameters of different lane change mixed models and key problems to be solved.

Paper	Input Variable	Algorithm	Key Issues to Be Resolved
[132]	HighD	CLTP	Improve lane change safety for self-driving cars
[138]	HighD	LSTM + LGBM	It can help drivers make safe driving decisions based on the movement characteristics of surrounding vehicles
[142]	CitySim	TCN-LSTM + MTL-LSTM + MTL-TCN + MTL-TCN-LSTM + LC-IR-SP	It can help traffic participants to better identify potential lane change safety hazards and improve traffic safety level

3. The Development Trend of Future Lane Change Models

Lane changing is a very complex traffic behavior, involving the driving dynamics of the lane, adjacent lanes, and target lanes. Lane change behavior is an important part of the lane change decision model and has an important impact on traffic flow characteristics and traffic safety. Therefore, researchers have developed a variety of lane change behavior models, including the single-type and mixed lane change models. The single-type lane change model considers incomplete information, easily loses details, and has a mediocre detection performance when dealing with complex scenes. The mixed lane change model combines several different algorithm models, comprehensively considers the factors and characteristics of lane change behavior, and combines the advantages and characteristics of different lane change models to describe the movement trajectory of lane change behavior more accurately, such as in the following examples. (1) Decision trees have a more intuitive decision flow, but do not capture nonlinear relationships. The neural network can capture more complex features and relationships by learning a large number of sample data, to improve the accuracy and generalizability when predicting the driver's lane change decision. (2) The information-processing model mainly records the driver's perception and judgement of information in the traffic environment, and it can process and understand important traffic information, but the model is unable to obtain nonlinear relations. In this case, the combination of machine learning models can improve the predictive power of the model. Machine learning models can mine patterns and associations by learning instance data, to improve the ability to make lane change decisions. The results obtained with the information-processing model can be used as inputs to the machine learning model to extract higher-level behavioral features. (3) A cognitive mapping model (such as the driver's mental model) is mainly used to model the driver's cognitive processes during a lane change in combination with the behavior planning model. The cognitive mapping model contains the driver's intention, goal, and decision basis in the process of a lane change, but the cognitive model cannot be accurately translated into specific behavioral strategies. The behavior planning model can generate specific behavior strategies according to the state and goal to support the execution of the lane change behavior. Therefore, different types of lane change models can be combined to form a mixed model that overcomes the limitations of the single lane change model. The mixed model can comprehensively consider the factors and characteristics of the lane change behavior, to improve the lane change. In the future, the mixed lane change model will be one of the trends in our research and development, aiming to reduce traffic accidents between a car and neighboring vehicles during lane changes.

4. Conclusions

In this paper, the empirical, physical, cognitive, and mixed models of lane changes were summarized. Since a lane change of vehicles is a complex traffic behavior, a single lane change model has certain limitations and cannot comprehensively take into account various parameters involved, so the advantages of different algorithms should be combined. In this review, we have explored the logit, reinforcement learning, kinematic, dynamic, control, decision tree, and artificial neural network, and information-processing models. When considering the different model types and key problems to solve, we have found that to further improve the effectiveness of the lane change model, it is necessary to obtain a large sample dataset, which may capture complex features and relationships. Complex linear and nonlinear features and relationships require multiple algorithm models to work together. This paper has introduced the input parameters of different vehicle lane change models, the main problems solved, and the advantages and disadvantages of different models. Its summary can help researchers in the field to quickly understand the status quo of different vehicle lane change models.

This research review is a summary produced through the authors' original research, which has certain limitations. First, the specific research methods and contents need to be considered, which requires referring to the original texts. For instance, the specific research

methods and data results need to be framed within the hardware equipment used, the algorithm models, the parameters of the algorithm models, and the datasets of experimental tests. Second, the type of mixed model proposed in this paper can improve the accuracy of the algorithm model to some extent, but it will also increase the consumption of hardware computing power, which should be taken into account when training the mixed model. Accordingly, we hope that this paper has been read in full consideration of the various factors that affect the algorithms, along with the shortcomings of this paper.

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