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An Improved Vicsek Model of Swarms Based on a New Neighbor Strategy Considering View and Distance

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Abstract: Collective behaviors in nature and human societies have been intensively studied in recent decades. The Vicsek model is one of the typical models that explain self-ordered particle systems well. In the original Vicsek model, the neighbor strategy takes all its neighbors' mean directions into account when updating particles' directions, which leads to a longer convergence time and higher computation cost due to the excess number of neighbors. In this paper, we introduce a new neighbor strategy to the Vicsek model. It defines that each particle will only select a certain number of particles with the farthest distance that fall into its vision sector as its neighbors. In addition, we classify the Vicsek model as the static model and the dynamic model according to whether the features of particles in the model are constant or not. Moreover, we design a new rule to apply the new neighbor strategy to dynamic Vicsek models. The simulation results indicate that our new neighbor strategy can significantly decrease the average number of particles' neighbors but still be able to further enhance the Vicsek model's convergence performance. The comparative results found that the static and dynamic model applied with the new neighbor strategy outperforms the models that only apply view restriction or remote neighbor strategy in noiseless and noisy conditions.

Keywords: Vicsek model; self-ordered particle systems; collective behavior; neighbor strategy



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1. Introduction

Collective behaviors of biological swarms have gathered interest from researchers in various fields in the past decades. The collective motion demonstrated in biological systems, including swarms of insects [1,2], flocks of birds [3,4], and schools of fish [5,6], has been widely studied. Creatures in such swarms coordinate their movements in the absence of an external control center or an internal leader [7].

Modeling is developed to understand collective behaviors in nature better, as well as to effectively control artificial swarms, such as multi-robot systems [8] and unmanned vehicles [9–11]. Reynolds proposed the Boid model and ran the first computer simulation of flocking based on three heuristic rules, known as cohesion, separation, and alignment [12]. Later, Vicsek et al. proposed a more straightforward model that concentrated on the direction alignment in self-propelled particle systems [13], which can be regarded as the earliest work in the field of active matter physics. More recently, as matter physics develops, new research branches occur, including dry aligning dilute active matter [14,15]. Our work focuses on the fundamental Vicsek model (VM), where particles move synchronously in a two-dimensional square plane with periodic boundary conditions. At each discrete time step, the velocity direction of each particle is updated following the average velocity direction of its neighbors. Neighbors of a particle are defined as other particles that fall into the disc centered at this particle. Simulations of VM demonstrate that all particles will try to align their movements and achieve highly coordinated movements under certain circumstances. However, the original VM is too simple to simulate the complex behaviors

of real creatures. Moreover, due to limited convergence performance, VM needs to be improved to be applied in practical applications. Many dynamic features of VM also need to be studied. Thus, research on VM has attracted numerous researchers' attention.

Adjustments of the original VM realize the simulation of real creatures, such as bacterial colonies [16] and marching locusts [2]. More recent studies introduce response delay [17], obstacles [18], evolutionary game theory [19], and proportional-integral control strategy [20] to VM. In addition, phenomena including scale-free chaos [21] and multi-species flocking [22] were also studied. Another study proposes a neighbor selection strategy based on FNN [23]. Methods to accomplish improvements in the convergence performance of the original VM are another popular research branch, and this paper divides them into two categories. The first category realizes improvement by providing particles in VM with more information when updating their directions [24–28], and the second category is ultimately the opposite, providing less information [19,29–31]. Examples of the first category include the adaptive velocity model proposed by Li Wei et al., where particles adjust their speed constantly according to additional consensus information provided [28]. Xue et al. introduced hierarchical society into VM [24], where each particle has additional social status information. Both these two improved VM above improve convergence performance with more environmental information provided for particles in the VM. On the contrary, research on the second category found that too much environmental information obtained by particles in the system may obstruct synchronization [29,30], which is counter-intuitive. The decrease in environmental information provided for particles is mainly realized by reducing the average number of neighbors. In particular, Tian et al. proposed an improved VM with a restricted vision field (RVFVM) [29]. Similar to the limited vision field of real creatures [32,33], RVFVM limits the vision field of particles in the model, leading to fewer neighbors. Similarly, the improved VM with remote neighbor strategy (RNSVM) also reduces the average number of neighbors by selecting the farthest proportion of particles that fall into the visual field as neighbors [31]. In RVFVM and RNSVM, less environmental information is provided for particles by reducing their neighbors in different ways, and the convergence performance is improved. Table 1 sums up the research progress related to the improvement of VM.

Table 1. Research progress related to the improvement of VM.

Category	Method
Providing more information	Hierarchical society [24], Neighbors' degree [25] Weight difference [26], Direction preference [27] Adaptive velocity [28]
Providing less information	Evolutionary game theory [19], Line of sight [30] RVFVM [29], RNSVM [31] The proposed new neighbor strategy

Reducing the number of neighbors has practical value, as it means a smaller amount of computation in artificial swarms, leading to a decrease in cost and an increase in performance. Therefore, this paper focuses on the second VM improvement method, reducing the environmental information for particles in VM. Present research aiming at this method mainly considers only one factor, such as RVFVM and RNSVM, as mentioned. However, the effect of multi factors on the performance of VM and the coupling relationship between these factors is unclear. Additionally, the present VM improvement studies are limited by the velocity updating strategy of particles in the model. It is worth exploring a new strategy to improve VM's convergence performance further. Thus, the goal of this paper is to improve the velocity updating strategy of VM by combining multi factors that can reduce the number of neighbors and study the effect of these factors on the performance of VM and the coupling relationship between these factors. To achieve this goal, we improve the velocity updating strategy by proposing a new neighbor strategy for particles in VM to

choose their neighbors when updating the velocity. The new neighbor strategy applies view restriction and remote neighbor strategy simultaneously. More specifically, each particle will only consider the farthest part of particles that fall into its vision sector, which is a new way of reducing the number of neighbors. Two new factors are also introduced to determine how neighbors are selected: the angle of the vision sector and the proportion by which the farthest neighbors are selected. Then, we compare the performance of the improved VM with the new neighbor strategy, RVFVM, RNSVM, and the original VM.

The remainder of this paper is organized as follows. In Section 2, the details of the original VM are introduced. In Section 3, we propose the improved VM with the new neighbor strategy based on view and distance. The innovative strategy design is elaborated in this section. In Section 4, indicators are applied to evaluate the performance of the improved VM and settings of simulation experiments are introduced. Then, simulation and discussion are given in Section 5. In the end, conclusions of the findings are given in Section 6.

2. The Vicsek Model

In the original VM, N particles move synchronously with the same velocity v_0 in an $L \times L$ square region with periodic boundary conditions. Initially, all particles are randomly distributed in the square region, and their initial directions are also randomly distributed within $[0, 2\pi]$. At each discrete time step k , particle i updates its direction as an average direction of its neighbors $\langle \theta_i(k) \rangle$ with random noise $\Delta\theta$ added. The range of the random noise added is $[-\eta, \eta]$, $0 \leq \eta \leq \pi$. The direction of particle i is updated according to Equation (1):

$$\theta_i(k+1) = \langle \theta_i(k) \rangle + \Delta\theta, \quad i = 1, 2, \dots, N. \quad (1)$$

Therefore, the position of particle i is updated according to Equation (2):

$$\vec{X}_i(k+1) = \vec{X}_i(k) + v_0 e^{i\theta_i(k)} \quad (2)$$

In Equation (2), $\vec{X}_i(k)$ denotes the position vector of particle i at time step k . The neighbors of particle i are defined as particles that fall into the circle with radius R centered at the position of particle i , as is shown in Figure 1. Therefore, particle j is the neighbor of particle i if and only if $\|\vec{X}_i(k) - \vec{X}_j(k)\| \leq R$.

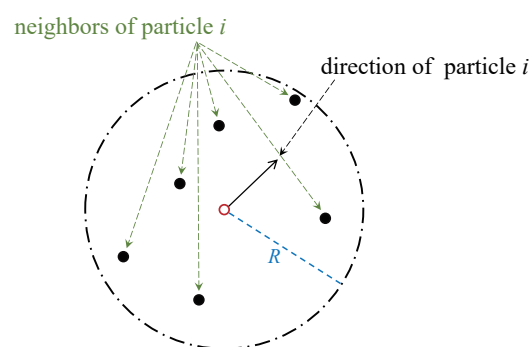


Figure 1. Illustration of a particle's neighbor set of the original VM.

3. The Framework of the Proposed VM

This research proposes an innovative neighbor strategy that combines view restriction and remote neighbor strategy. The improved VM based on this new neighbor strategy provides particles in the model with a new way to select their neighbors when updating their velocity directions. The proposed new neighbor strategy is distinguished from strategies proposed in previous studies, and it aims at further reducing the number of neighbors as well as improving the convergence performance of the VM simultaneously.

The VM with the new neighbor strategy is initially built as the static type, defined as the improved VM with static parameters (SPVM) in this study. In SPVM, the angle of the

vision field and the proportion by which particles select their neighbors are static. When noise is introduced to VM, particles will be affected by random noise when determining the next time step's velocity direction, making the overall environment unstable and the model difficult to converge. The static feature of SPVM makes particles unable to adapt to the constantly changing environment. To perform better in conditions with noise, the static type of model is evolved into the dynamic type, defined as the improved VM with dynamic parameters (DPVM) in this work. In DPVM, the view angle and the selection proportion are assumed to be dynamic, which helps particles choose the strategy most suitable to the current environment nearby, thus improving the performance of VM in noisy conditions.

3.1. The Improved SPVM

Different from the original VM, in SPVM, the way a particle selects its neighbors and updates its direction is changed, as illustrated in Figure 2. Firstly, the vision field of each particle is a sector symmetric about its current velocity direction. The view angle ω determines the size of the vision field, which is consistent for all particles in SPVM. Mathematically, particle j is inside the vision field of particle i at step k if and only if $\|\vec{X}_i(k) - \vec{X}_j(k)\| \leq R$ and $\text{angle}(\vec{v}_i(k), \vec{ij}(k)) \leq \omega/2$ are satisfied. $\vec{v}_i(k)$ denotes the velocity vector of particle i ; $\vec{ij}(k)$ denotes the vector which starts at the position of particle i and ends at the position of particle j ; $\text{angle}(\vec{a}, \vec{b})$ denotes the angle between vector $\vec{a}$ and vector $\vec{b}$. Then, $N_{i,\omega}$ particles that fall into the vision field of particle i are sorted according to the Euclidean distance between particles $N_{i,\omega}$ and particle i . To further reduce the number of neighbors, we assume that only the farthest $n_i(k)$ particles could be chosen as neighbors of particle i at time step k . Moreover, we design that $n_i(k) = p \times N_{i,\omega}(k)$, where p is the selection proportion and $p \in [0, 1]$. The neighbor set of particle i at time step k is $\Gamma_i(k)$. Thus, the average direction of the neighbors of particle i is calculated by Equation (3):

$$\langle \theta_i(k) \rangle = \arctan\left(\frac{\sum_{j=1}^{n_i(k)} \sin\theta_j(k)}{\sum_{j=1}^{n_i(k)} \cos\theta_j(k)}\right), \quad j \in \Gamma_i(k) \quad (3)$$

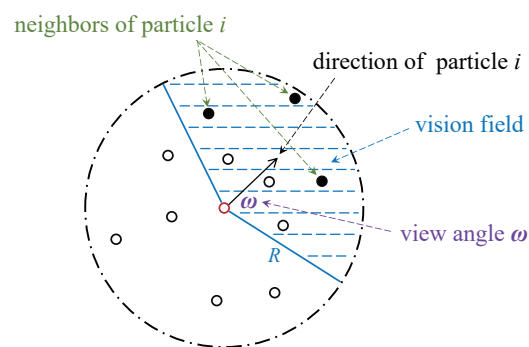


Figure 2. Illustration of particles' neighbor strategy of the improved VM proposed.

3.2. The Improved DPVM

Previous research has proved that adding a dynamic feature or non-homogeneous feature to the VM can improve the performance of VM [24,28]. Additionally, the model of dynamic type can adapt to conditions with noise better, as mentioned before. Therefore, this paper proposes DPVM to improve SPVM further, overcoming SPVM's weakness in noisy conditions. The DPVM is assumed to share the same rules with SPVM, except that each particle has a different view angle ω and a selection proportion p , which change along with the simulation time and the local level of consensus. The dynamic features of parameters ω and p , as well as the features that each particle has different ω and p , could help the model perform better in noisy conditions.

The following order parameter $V_{local}(k)$ measures the degree of movement consensus of the neighbors of particle i . The information $V_{local}(k)$ provides will determine the view angle and selection proportion of certain particle i at the next time step $k + 1$:

$$V_{local}(k) = \frac{1}{n_i(k)} \left| \sum_{j=1}^{n_i(k)} e^{i\theta_j(k)} \right|, j \in \Gamma_i(k). \quad 0 \leq V_{local}(k) \leq 1 \quad (4)$$

In Equation (4), only particles belonging to the neighbor set of particle i are measured when calculating $V_{local}(k)$ of particle i , so $V_{local}(k)$ represents the degree of consensus of the neighbors of particle i at time step k . The definition of neighbor degenerates to that of the original VM when calculating $V_{local}(k)$, which means all particles falling into the disc with radius R centered at particle i are regarded as neighbors. In other words, view angle ω is not taken into consideration when calculating $V_{local}(k)$. When particles move in a highly synchronized manner, all particles move in almost the same direction, and thus, the value of index $V_{local}(k)$ will be close to 1.

In DPVM, the view angle ω and selection proportion p of particle i are not constants. Instead, they are functions of $V_{local}(k)$. Therefore, in DPVM, each particle will constantly adjust its way of choosing neighbors according to its nearby consensus level. There is no limitation to the design of the functions, but by applying different functions, the model will demonstrate different characteristics and performance. DPVM will degenerate to SPVM if the functions applied are constant functions.

4. Simulation and Experiments

Firstly, the setting of the parameters of the model applied in the simulation will be introduced. Based on simulation results, the improvement of SPVM and DPVM will be evaluated. Therefore, order parameter $V_a(k)$, convergence time T , and indicator V_{stable} are introduced, in order to quantitatively indicate the convergence performance of SPVM and DPVM from different aspects, and the average number of neighbors $\bar{n}(k)$ evaluates the amount of information a particle obtains from the environment.

4.1. Parameter Settings of Experiments

To investigate the performance of the newly proposed neighbor strategy, this study considers N particles moving in a $L \times L$ square region with periodic boundary conditions, where $N = 400$ and $L = 10$. Furthermore, the interaction radius R is fixed as 1; the velocity of particle v_0 is fixed as 0.1. By controlling the value of these parameters, the coupling effect of parameters ω and p will be better understood. In cases without noise, $\eta = 0$; in cases with noise, $\eta = 1$.

4.2. Evaluation Indicators of Model Performance

Order parameter $V_a(k)$, shown in Equation (5), is widely adopted to quantitatively evaluate the degree of direction consensus of all the N particles in VM [26,28–30,34]:

$$V_a(k) = \frac{1}{N} \left| \sum_{j=1}^N e^{i\theta_j(k)} \right|, \quad 0 \leq V_a(k) \leq 1 \quad (5)$$

where when all particles move in the same direction, $V_a = 1$. When the model is at a disordered state where there is barely a consensus of moving direction, V_a is close to 0. Snapshots of moving particles in the model with different values of V_a are demonstrated in Figure 3.

In a noiseless model ($\eta = 0$), order parameter V_a will be very much close to its maximum value (which equals to 1) after certain steps, except for extreme values of parameters (e.g., N , p or ω is too small). The convergence time T is an indicator of the model's convergence performance in noiseless cases, which is defined as the time steps

required by the model for V_a to surpass 0.99 from the random initial state when V_a is close to 0.

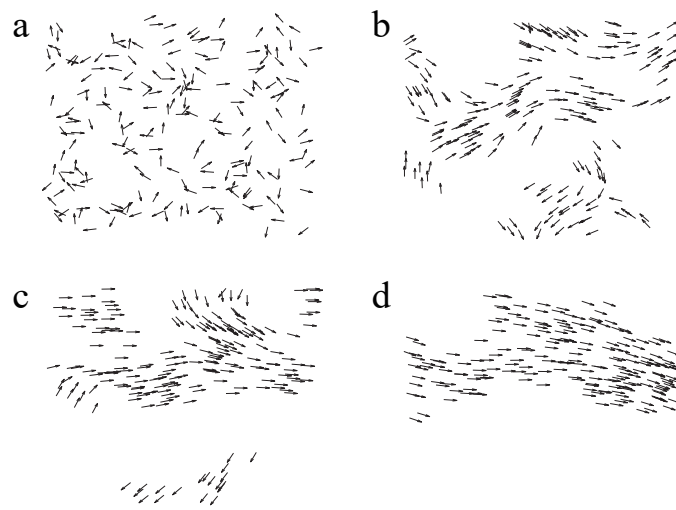


Figure 3. Snapshots of moving particles in the VM at different time steps with the same set of parameters: $N = 200$, $L = 10$, $R = 1$, $v_0 = 0.1$, $\omega = 2\pi$, $p = 1$, $\eta = 0$. Each arrow represents a particle and the direction of the arrow represents its moving direction. (a) step = 0, $V_a = 0.0369$; (b) step = 19, $V_a = 0.5346$; (c) step = 42, $V_a = 0.7507$; (d) step = 78, $V_a = 0.9908$.

For cases when noise is considered ($\eta > 0$), the maximum value that V_a can actually reach will change as model parameters change. The system will be in a state between highly ordered (V_a close to 1) and disordered (V_a close to 0). In fact, after 8000 time steps, order parameter V_a will fluctuate around a certain value, which is defined as V_{stable} statistically in Equation (6):

$$V_{stable} = \frac{1}{2000} \sum_{k=8000}^{10000} V_a(k) \quad (6)$$

$$V_a(k) = \frac{1}{20} \sum_{time=1}^{20} V_{time}(k) \quad (7)$$

The value of V_{stable} is defined as the average of a consecutive series of V_a over 2000 time steps after 8000 time steps of iteration. V_a at time step k is obtained by averaging over 20 random initializations to ensure the reliability of the results, which is defined in Equation (7). The value of V_{stable} reflects the maximum degree of synchronization that the model can achieve in the presence of noise.

In addition, the average number of neighbors of all N particles at time step k is defined as $\bar{n}(k)$, where $n_i(k)$ represents the number of neighbors of particle i at time step k (Equation (8)); $\bar{n}(k)$ represents the average amount of information particles obtain from the environment to decide their movements at the next time step.

$$\bar{n}(k) = \frac{1}{N} \sum_{i=1}^N n_i(k) \quad (8)$$

5. Results and Analysis

We simulate SPVM in both noiseless and noisy conditions and compare it with the original VM, RVFVM, and RNSVM. Since the dynamic feature of DPVM will not bring further improvement in performance in noiseless conditions, only simulations of DPVM under noisy conditions will be included in this section. Furthermore, a comparison among DPVM and several typical SPVM will also be involved.

5.1. Simulation and Analysis of SPVM

In SPVM, view angle ω and selection proportion p are the key parameters that determine the performance of the model. Therefore, this section will analyze the coupling effect of ω and p on the performance of the model and the coupling relationship between ω and p .

- Simulation of SPVM under noiseless conditions

The time step k -order parameter V_a relationship of various Vicsek models, including VM, RVFVM, RNSVM, and SPVM, are compared in noiseless conditions. It is noticed that the original VM is a special case of SPVM with $\omega = 2\pi$ and $p = 1$; RVFVM is a special case of SPVM with $p = 1$; RNSVM is a special case of SPVM with $\omega = 2\pi$. In RVFVM, there is an optimal view angle ω_{opt} which leads to the shortest convergence time T and $\omega = \omega_{opt} = 1.4\pi$ is adopted in the simulation. Similarly, regarding RNSVM, $p = p_{opt} = 0.4$ is adopted to perform the simulation. Regarding SPVM, the view angle and the selection proportion are set as $\omega = 1.2\pi$ and $p = 0.4$.

The simulation results are shown in Figure 4a. It can be seen that the proposed SPVM reaches the ordered state ($V_a = 1$) with the least time steps. It is worth mentioning that the value of view angle ω set for RVFVM and the value of selection proportion p set for RNSVM are optimum. Additionally, SPVM can accelerate the convergence speed further by combining view restriction and remote neighbor strategy. Moreover, as shown in Figure 4b, SPVM also has the least number of neighbors. To sum up, the proposed SPVM can further improve the model's convergence performance while reducing neighbor information obtained from the environment simultaneously.

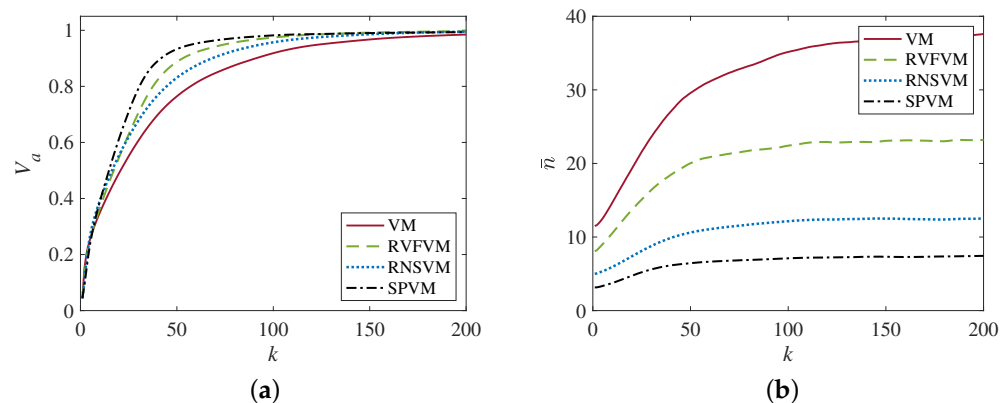


Figure 4. Comparisons of various Vicsek models. (a) V_a and (b) $\bar{n}$ as a function of time step k . For other parameters: $N = 400$, $L = 10$, $v_0 = 0.1$, $R = 1$, $\eta = 0$. Each data point is obtained by averaging over 300 different realizations.

The combinatory effect of view angle ω and selection proportion p on the convergence time T of SPVM is shown in Figures 5 and 6. In the following simulations based on SPVM, ω or p changes, while other parameters are set as: $N = 400$, $L = 10$, $v_0 = 0.1$, $R = 1$, $\eta = 0$. Firstly, the convergence time T as a function of ω for different values of p is shown in Figure 5a. A smaller value of T means that the system reaches the ordered state faster, indicating a better convergence performance. From Figure 5a, a trend shows that for a fixed p , when ω increases, T decreases first, then increases. It means that for different cases of $p = p_0$, there exists an optimal value of $\omega = \omega_{opt, p=p_0}$ that leads to the shortest convergence time. When p is too small, convergence time T will get longer, but the relationship between ω and T observed before will be lost. Then, ω is set as a certain value and the relationship between p and T is shown in Figure 5b. For a fixed ω , when p increases, T decreases first and then increases, which means there exists an optimal value of $p_{opt, \omega=\omega_0}$ that leads to the shortest convergence time when $\omega = \omega_0$. Figure 6 demonstrates the relationship between view angle ω , selection proportion p , and convergence time T . Obviously, when ω or p is too small, the convergence time T will be longer because particles in the system

do not have enough information to determine what direction to choose. However, when ω or p is too large, the convergence time will also be longer because of the excessive information obtained. Therefore, it is necessary to obtain the right amount of information and the right part of the information that really helps the model synchronization. By setting $\omega = 1.2\pi$ and $p = 0.4$, SPVM has the shortest convergence time T , which means combining view restriction and remote neighbor strategy can bring further improvement to convergence speed. If we narrow view angle ω or reduce selection proportion p further, the average number of neighbors $\bar{n}$ will decrease, whereas convergence time T will increase. It means SPVM with $\omega = 1.2\pi$ and $p = 0.4$ can reduce the environment information each particle obtained to its minimum and bring the convergence performance of the model to its maximum simultaneously.

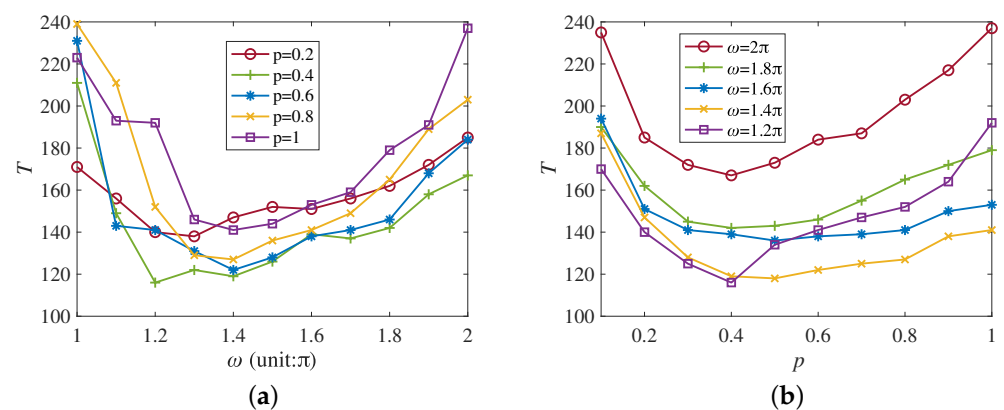


Figure 5. Convergence time T as a function of ω or p . (a) T as a function of ω with fixed p . (b) T as a function of p with fixed ω . For other parameters: $N = 400$, $L = 10$, $v_0 = 0.1$, $R = 1$, $\eta = 0$.

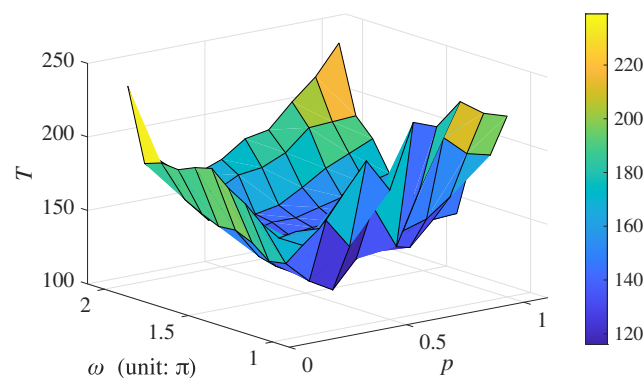


Figure 6. Three-dimensional plot of the relationship between ω , p and T . For other parameters: $N = 400$, $L = 10$, $v_0 = 0.1$, $R = 1$, $\eta = 0$.

- Simulation of SPVM under noisy conditions

When noise is considered ($\eta = 1$), V_{stable} is used to evaluate the convergence performance of the model. Simulations based on SPVM with noise is shown in Figures 7 and 8, where view angle ω or selection proportion p changes, but other parameters are set as: $N = 400$, $L = 10$, $v_0 = 0.1$, $R = 1$, $\eta = 1$. Firstly, selection proportion p is constant, and V_{stable} as a function of view angle ω is shown in Figure 7a. When ω decreases, V_{stable} decreases first, then increases slightly to its maximum and decreases significantly when ω decreases further. In addition, there exists an optimal value of $\omega = \omega_{opt, p=p_0}$ for different cases of $p = p_0$ that leads to the highest level of V_{stable} , and the value of $\omega_{opt, p=p_0}$ approximately equals to 0.9π even p_0 varies from 0.2 to 1. The value of $\omega_{opt, p=p_0}$ shows an independence from p_0 . Then, ω is set as a certain value and the relationship between p and V_{stable} is shown in Figure 7b. When p decreases with ω fixed, V_{stable} increases first then decreases, which is contrary to the trend when ω decreases with p fixed, since reducing

p or ω both lead to fewer neighbors. Figure 8 demonstrates the relationship between ω , p and V_{stable} . V_{stable} reaches its maximum when $\omega = 0.9\pi$ and $p = 0.6$, so by limiting the environment information particles obtain, the model can reach an ordered state of a higher level. Furthermore, when $\omega > 0.9\pi$, V_{stable} remains at a high level even p varies significantly, which indicates ω has relatively more impact on V_{stable} .

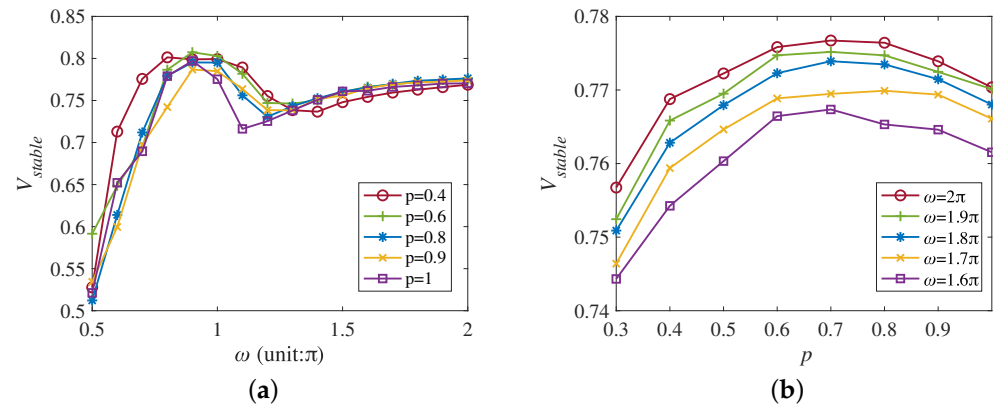


Figure 7. V_{stable} as a function of ω or p . (a) V_{stable} as a function of ω with fixed p . (b) V_{stable} as a function of p with fixed ω . For other parameters: $N = 400$, $L = 10$, $v_0 = 0.1$, $R = 1$, $\eta = 1$.

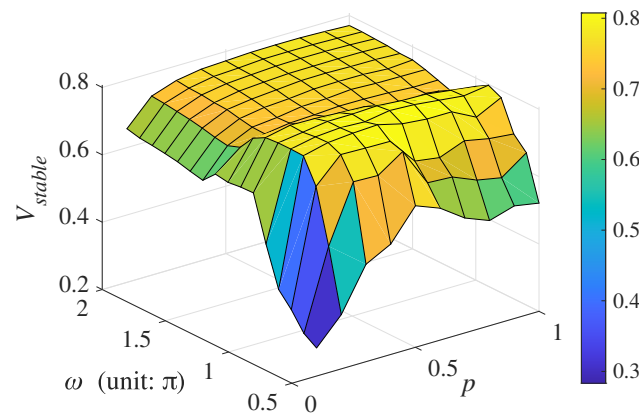


Figure 8. Three-dimensional plot of the relationship between ω , p and V_{stable} . For other parameters: $N = 400$, $L = 10$, $v_0 = 0.1$, $R = 1$, $\eta = 1$.

However, SPVM with the same V_{stable} performs differently. Comparisons between several SPVM with approximately the same value of V_{stable} but four different sets of parameters (SPVM(1–4)) are shown in Figure 9.

- SPVM(1): $\omega = 2\pi, p = 0.7$
- SPVM(2): $\omega = \pi, p = 1$
- SPVM(3): $\omega = 0.8\pi, p = 0.2$
- SPVM(4): $\omega = 0.9\pi, p = 0.6$

In Figure 9a, after 8000 time steps, V_a of SPVM(1), SPVM(2), and SPVM(3) oscillates around the same value which is the value of V_{stable} of the three models. However, before V_a saturates at the same V_{stable} , these three models perform quite differently from time step 0 to time step 8000, which means indicator V_{stable} does not comprehensively evaluate the convergence performance of the model under conditions with noise. Despite a higher level of synchronization of SPVM(4) after 4000 time steps, SPVM(1) apparently outperforms SPVM(4) when V_a of SPVM(4) has yet to pass V_a of SPVM(1). Therefore, it would be better if a balance between the faster convergence speed of SPVM(1) and the higher stabilized consensus level of SPVM(4) can be reached.

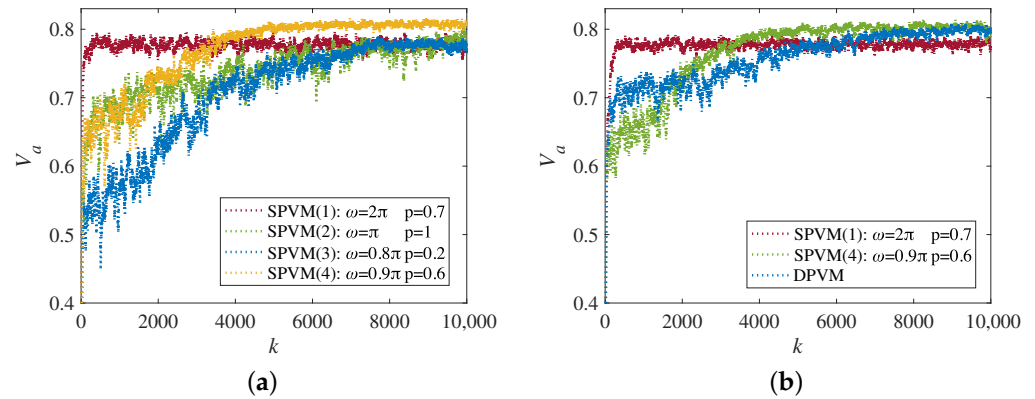


Figure 9. Order parameter V_a as a function of time step k . Each data point is obtained by averaging over 20 (a) and 30 (b) random initializations. $N = 400, L = 10, v_0 = 0.1, R = 1, \eta = 1$.

5.2. Simulation and Analysis of DPVM

As is shown in Figure 9a, SPVM cannot reach the balance between the faster convergence speed and the higher stabilized consensus level in noisy conditions. In DPVM, the updating functions of view angle ω and selection proportion p are the key factors determining the model's performance. Therefore, simulations are conducted to verify that if the updating functions of ω and p are properly designed, DPVM will be able to reach the balance.

Since SPVM(1) can reach and oscillate around its maximum consensus level in less than 500 steps, SPVM(1) represents the model of a fast convergence speed. SPVM(4) has the highest value of V_{stable} , and thus, SPVM(4) is the model that has the highest stabilized consensus level among all. Therefore, SPVM(1) and SPVM(4) are used to compare with DPVM from aspects of convergence speed and the stabilized consensus level. Simulation based on SPVM(1), SPVM(4), and DPVM is shown in Figure 9b. In order to achieve a better performance, we have compared several function designs of the updating functions of view angle ω and selection proportion p . Eventually, we set the functions as Equations (9) and (10). In the simulation, DPVM demonstrates a balance between faster convergence speed and higher stabilized consensus level.

$$\omega_i(k+1) = \begin{cases} 2\pi & 0 \leq V_{local}(k) < 0.72 \\ 0.9\pi & 0.72 \leq V_{local}(k) \leq 1 \end{cases} \quad (9)$$

$$p_i(k+1) = \begin{cases} 0.7 & 0 \leq V_{local}(k) < 0.72 \\ 0.6 & 0.72 \leq V_{local}(k) \leq 1 \end{cases} \quad (10)$$

The simulation result has been demonstrated in Table 2. Quantitatively, V_{stable} of SPVM(1) equals to 0.7783; V_{stable} of SPVM(4) equals to 0.8032; V_{stable} of DPVM equals to 0.7973. In order to evaluate the convergence speed of the models, the threshold value of the definition of convergence time T need to be adjusted. Because of the presence of noise, the maximum value V_a can reach is 0.8 based on current parameters, so the convergence speed cannot be measured if the threshold of T remains at 0.99. Therefore, the threshold value of the definition of convergence time T is adjusted from 0.99 to 80% of the maximum value V_a can reach under the present parameters. Convergence time T here is, thus, redefined as the time steps required by the model for V_a to surpass 0.64 from the random initial state when V_a is close to 0. The value of 0.64 can effectively serve as a threshold value because separate groups of particles moving in an orderly manner have been formed when $V_a = 0.64$ according to our observation. $T_{threshold=0.64}$ of SPVM(1) equals to 55; $T_{threshold=0.64}$ of SPVM(4) equals to 1579; $T_{threshold=0.64}$ of DPVM equals to 96. By comparing these two performance indicators of the three models, it can be concluded that DPVM is the only

model that is capable of reaching a balance between faster convergence speed and higher stabilized consensus level.

Table 2. Performance comparison between SPVM and DPVM.

	SPVM(1)	SPVM(4)	DPVM
V_{stable}	0.7783	0.8032	0.7973
$T_{threshold} = 0.64$	55	1579	96

6. Conclusions

In this paper, an improved neighbor strategy is introduced to VM. The improved strategy enables particles in the model to select their neighbors in a new way, which helps them converge faster with less local environmental information compared with previous models. The improved neighbor strategy states that each particle in VM will only consider the farthest part of particles that fall into its vision sector with a specific angle. Two improved VM (SPVM and DPVM) are introduced based on this rule. In SPVM, the selection proportion and view angle of particles are constants. In DPVM, each particle's selection proportion and view angle are different and change with time according to the degree of movement consensus nearby. Compared to the original VM and other improved VM such as RVFVM and RNSVM, SPVM can further reduce the average number of neighbors while improving convergence speed in simulations without noise. In simulations with noise, SPVM can enhance the level of direction consensus further, whereas it cannot reach the balance between a higher level of direction consensus and a faster convergence speed. Thus, DPVM is proposed, and simulation results prove that such a balance can be reached with an adequately designed updating function of view angle and selection proportion. Since the updating functions in DPVM and the optimum parameters of the model are designed empirically in this work, future work will focus on applying the machine learning method to help particles automatically learn the optimum functions or values of parameters. Additionally, experiments under more parameter combinations will be conducted to further consolidate our findings.

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