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Correction

Correction: Martinović et al. Mathematical Considerations for Unmanned Aerial Vehicle Navigation in the Magnetic Field of Two Parallel Transmission Lines. *Appl. Sci.* 2021, *11*, 3323

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The authors wish to make the following changes to their paper [1].

1. Changes in Section 2.2: Orientation Determination

The results regarding the roll angle calculation in Section 2.2: Orientation Determination are wrong and should be corrected. The following original text is affected:

"The roll angle γ can also be computed by exploiting the zero field property (2). However, the rotation matrix $R_G^{L_u}$ does not in general contain γ in its x-row. Thus, a different sensor k must have a different local frame L_k in order for the zero to appear in the y- or z-row containing all three angles. This can be achieved by, for example, placing the sensor so that it is rotated relative to the UAV body frame L_u by an angle δ about the y-axis, i.e., it is:

$$R_{L_u}^{L_k} = R_y(\delta) \tag{11}$$

Then, from Equation (5), it follows that after applying the coordinate transformation $A = R_G^{L_u} R_y^T(\delta) R_{L_u}^G$, the vector $\vec{B}\left(\vec{p}_k\right)$ is expressed in L_u:

$$\begin{aligned} \hat{\vec{B}}(\vec{p}_k) = A\vec{B}(\vec{p}_k) = R_G^{L_u} R_y^T(\delta) R_{L_u}^G R_G^{L_u} R_y(\delta)\vec{v}_k, \\ \text{for } \delta = \tfrac{\pi}{2} \Rightarrow \begin{pmatrix} \hat{B}_x(\vec{p}_k) \\ \hat{B}_y(\vec{p}_k) \\ 0 \end{pmatrix} = R_G^{L_u}\vec{v}_k. \end{aligned} \tag{12}$$

From Equation (12), it follows that the rotation of $\vec{B}\left(\vec{p}_k\right)$ for a given angle $\delta = \frac{\pi}{2}$ around the y-axis of L_u has forced the z-component of the resulting vector $\hat{\vec{B}}\left(\vec{p}_k\right)$ to zero:

$$\hat{B}_z\left(\vec{p}_k\right) = 0 = \left(R_G^{L_u}\vec{v}_k\right) \cdot \vec{e}_z, \tag{13}$$

where $\vec{e}_z$ is the Cartesian unit vector. Then, in a first step, Equation (13) can be written as:

$$0 = \left(s_\gamma s_\alpha + c_\gamma c_\alpha s_\beta\right) v_{kx} + \left(c_\gamma s_\beta s_\alpha - c_\alpha s_\gamma\right) v_{ky} + c_\gamma c_\beta v_{kz}.$$

Next, the division by c_γ forms the tangent function t_γ, which can be isolated and finally solved for γ:

$$\begin{aligned} 0 = (t_\gamma s_\alpha + c_\alpha s_\beta) v_{kx} + (s_\beta s_\alpha - c_\alpha t_\gamma) v_{ky} + c_\beta v_{kz} \\ \Rightarrow \gamma = atan\left(\tfrac{v_{ky} s_\beta s_\alpha + v_{kx} s_\beta c_\alpha + v_{kz} c_\beta}{v_{ky} c_\alpha - v_{kx} s_\alpha}\right) \end{aligned} \tag{14}$$

The sensor can be placed anywhere on the UAV. However, the result only applies to the local frame definition in (11). Theoretically, arbitrary local frame definitions can be chosen,

check for updates

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but in this case, there is no zero in any coordinate, and the whole system of Equation (12) must be solved since the sensor position $\vec{p}_k$ is unknown. However, the equations are complicated, and it is questionable whether an analytical solution exists. The presented formulas are valid if the UAV angles are within the definition range D, respectively, α, β, $\gamma \in D = [-\frac{\pi}{2}, \frac{\pi}{2}]$. The limits of the equations presented are discussed in Section 3." should be replaced by "Following the same approach to calculate the roll angle γ by using the zero field property does not lead to any result, since the analytic equation obtained is linearly dependent. The state of the art now is that the roll angle cannot be calculated analytically. On the one hand, it can be calculated numerically using Equation (5), which leads to a very complicated system of equations since it now additionally contains the sensor location, which is unknown. On the other hand, the roll angle can be measured by additional sensors, e.g., by an inertial measurement unit (IMU), which is present in almost all UAVs. In this case, the yaw and pitch angles calculated by Equations (9) and (10) can be used as correction information, since the IMU provides all three angles. For the following considerations in connection with the UAV position calculation, all three angles are assumed to be known, either through calculation or measurement."

Error explanation: The assumptions and results in this original text that relate to the roll angle of the unmanned aerial vehicle (UAV) are incorrect. The reason is explained as follows, and it is shown that the roll angle cannot be calculated analytically but must be measured with additional sensors.

To determine the roll angle, the paper pursued the idea of using a third magnetometer k rotated 90 degrees around its y-axis relative to the UAV body frame $L_{\boldsymbol{u}}$, which should lead to a third linearly independent equation. In this configuration, when the UAV body frame is aligned with the global frame $\{G\}$, the magnetometer measures a zero value on its z-axis due to the zero-field property (2). Linking this to the sensor measurement via a coordinate transformation then leads to Equation (13), which appears to yield the roll angle as the only unknown quantity. However, this result is incorrect, because in derivation (12), the coordinate transformation A was set incorrectly, namely $A = R_G^{L_u} R_y^T R_{L_u}^G$, where R_y represents the rotation of the magnetometer by 90 degrees around its y-axis. The correct transformation is $A = R_y^T R_G^{L_u} R_y$, which can be derived as follows: the rotated magnetometer k measures $\vec{B} = R_G^{L_u} R_y \vec{v}_k$ referred to the global frame according to Equation (5), where $\vec{v}_k$ is the measurement in the local sensor frame. If the UAV has no orientation, i.e., $R_G^{L_u} = I$, it is $\vec{B}_k = R_y \hat{\vec{B}}_k$, where $\hat{\vec{B}}_k$ has a zero z-component (due to its 90-degree rotation). From this follows the correct A and further $\hat{\vec{B}}_k = R_y^T R_G^{L_u} R_y \vec{v}_k$. Now, following the same approach as in the paper, the calculation of $\hat{\vec{B}}_k \cdot \vec{e}_z = \left(R_y^T R_G^{L_u} R_y \vec{v}_k \right) \cdot \vec{e}_z = 0$, fails, because the corrected A now has no roll angle variable in the z-row.

2. Changes in Section 2.3: Position Determination

The numbering of the equations beginning from Equation (15) should be changed, i.e., Equation (15) should start with value (11). Additionally, the corresponding references in the text should be updated. The following original text is affected:

"Thus, to obtain $\vec{p}_D$, it is sufficient to solve one of these rows in (15)." should be changed to "Thus, to obtain $\vec{p}_D$, it is sufficient to solve one of these rows in (11)."

"This means, on the one hand, that the two equations in (16) are linearly independent and theoretically allow the determination of the unknowns y_i and z_i." should be changed to "This means, on the one hand, that the two equations in (12) are linearly independent and theoretically allow the determination of the unknowns y_i and z_i."

"On the other hand, the equations are nonlinear, and the question arises whether it is possible to solve (16) analytically and, if so, how." should be changed to "On the other hand, the equations are nonlinear, and the question arises whether it is possible to solve (12) analytically and, if so, how."

"The first row in (16) can be solved for one coordinate y or z, which is then inserted into the second row to compute the other coordinate;" should be changed to "The first row in (12) can be solved for one coordinate y or z, which is then inserted into the second row to compute the other coordinate;"

"The result $y\left(z, V_y\right)$ has the same structure, but is not used, as the easiest final equation can be found by inserting (17) into $B_y^2(y,z) = V_y^2$. With D from (17), it becomes:" should be changed to "The result $y\left(z, V_y\right)$ has the same structure, but is not used, as the easiest final equation can be found by inserting (13) into $B_y^2(y,z) = V_y^2$. With D from (13), it becomes:"

"In summary, it is difficult to impossible to derive an analytical solution only from the equation system (16). In this context, the idea is to consider the signal power as an additional equation. As will be shown in the next steps, its combination with (16) leads to a compact analytical solution, which is derived below:" should be changed to "In summary, it is difficult to impossible to derive an analytical solution only from the equation system (12). In this context, the idea is to consider the signal power as an additional equation. As will be shown in the next steps, its combination with (12) leads to a compact analytical solution, which is derived below:"

"Next, with the positive solution of (20), the equation $B_z\left(y, z^2(y,P)\right) = V_z$ can be written as:" should be changed to "Next, with the positive solution of (16), the equation $B_z\left(y, z^2(y,P)\right) = V_z$ can be written as:"

"Then, with the coefficients a_n given in (21), the four large expressions for y are greatly reduced and become:" should be changed to "Then, with the coefficients a_n given in (17), the four large expressions for y are greatly reduced and become:"

"The same result is obtained for the negative solution of Equation (20)." should be changed to "The same result is obtained for the negative solution of Equation (16)."

"**Theorem 1.** *Two of the y-solutions in (22) can immediately be excluded, because they are complex*." should be changed to "**Theorem 1.** *Two of the y-solutions in (18) can immediately be excluded, because they are complex*."

"The square root addends in Equation (22) represented by S now can be replaced by this result, leading to:" should be changed to "The square root addends in Equation (18) represented by S can now be replaced by this result, leading to:"

"with r and φ according to (24)." should be changed to "with r and φ according to (20)."

"The hypothesis is true if the absolute value of the square root addends in Equation (22) is always greater than $C|V_z|P^{-2}$." should be changed to "The hypothesis is true if the absolute value of the square root addends in Equation (18) is always greater than $C|V_z|P^{-2}$."

"Further, with a, b from (23), it follows for the left and right side:" should be changed to "Further, with a, b from (19), it follows for the left and right side:"

"The two real solutions $y_{1,2}$ can again be obtained from (25) or from the general Equation (22)." should be changed to "The two real solutions $y_{1,2}$ can again be obtained from (21) or from the general Equation (18)."

"Once the corresponding y is determined, in a final step, z can be calculated from Equation (20):" should be changed to "Once the corresponding y is determined, in a final step, z can be calculated from Equation (16):"

"There are also situations where, for a given y, a subset of the z-solutions in (28) can become complex." should be changed to "There are also situations where, for a given y, a subset of the z-solutions in (24) can become complex."

"For all solutions $y_{1,2,3}$ obtained, $g \geq 0$ holds, since they were obtained using the z^2-expression (20)." should be changed to "For all solutions $y_{1,2,3}$ obtained, $g \geq 0$ holds, since they were obtained using the z^2-expression (16)."

3. Changes in Section 2.4: Selecting the Right Solution

"Equation (28) yields four z-solutions for each y-solution of Cases (I)–(IV)." should be changed to "Equation (24) yields four z-solutions for each y-solution of Cases (I)–(IV)."

"where $y_{1,2}$ comes from Equation (27) and z^2 comes from (20)." should be changed to "where $y_{1,2}$ comes from Equation (23) and z^2 comes from (16)."

"Now, the idea is to investigate which of the eight remaining positions lead to a match with the sensor measurement $\vec{V}$, i.e., which positions satisfy Equation (16)." should be changed to "Now, the idea is to investigate which of the eight remaining positions lead to a match with the sensor measurement $\vec{V}$, i.e., which positions satisfy Equation (12)."

"**Theorem 3.** *Given a sensor k that measures* $\vec{V}_k = \left(V_{ky}, V_{kz}\right)^T$ *and has two y-solutions according to (25) in Case (I), for each of the two values, there are four associated z-solutions according to Equation (28).*" should be changed to "**Theorem 3.** *Given a sensor k that measures* $\vec{V}_k = \left(V_{ky}, V_{kz}\right)^T$ *and has two y-solutions according to (21) in Case (I), for each of the two values, there are four associated z-solutions according to Equation (24).*"

"In summary, there are only two possible locations for a single sensor, so that Equation (16) is satisfied." should be changed to "In summary, there are only two possible locations for a single sensor so that Equation (12) is satisfied."

"Since both points must be in areas with the same direction of the magnetic field lines to satisfy Equation (16), one is outside the circle and one is inside." should be changed to "Since both points must be in areas with the same direction of the magnetic field lines to satisfy Equation (12), one is outside the circle and one is inside."

4. Changes in Section 3.2: Ambiguities Due to Signal Symmetry

"The fact that it is impossible to uniquely determine the sensor position can even be seen directly from Equation (25). It contains the prefactor C, which, as shown in (17), depends on the current;" should be changed to "The fact that it is impossible to uniquely determine the sensor position can even be seen directly from Equation (21). It contains the prefactor C, which, as shown in (13), depends on the current;"

5. Changes in Section 3.3: Resolving the Ambiguities

"If $\vec{v}_i$ does not satisfy (31), $-\vec{v}_i$ does, and vice versa." should be changed to "If $\vec{v}_i$ does not satisfy (27), $-\vec{v}_i$ does, and vice versa."

"Since the rotation matrix in (31) is used, it must be ensured that the angles can be computed independently of the direction of I." should be changed to "Since the rotation matrix in (27) is used, it must be ensured that the angles can be computed independently of the direction of I."

"Equation (14) for the roll angle γ uses the measurement from only one sensor as the input. Thus, it is also independent of the sign of I." should be discarded.

6. Changes in Section 4.1: SIL Design and Test Procedure

The notation of the Gazebo rotation matrix in Section 4.1: SIL Design and Test Procedure is wrongly stated and should be changed. The following original text is affected:

"For this purpose, Equations (4) and (5) were used, where the rotation matrix was used according to the convention in Gazebo, i.e., $R_G^{\hat{L}_u} = R_x^T(\hat{\gamma})R_y^T(\hat{\beta})R_z^T(\hat{\alpha})$." should be changed to "For this purpose, Equations (4) and (5) were used, where the rotation matrix was used according to the convention in Gazebo, i.e., $R_G^{\hat{L}_u} = R_z^T(\hat{\alpha})R_y^T(\hat{\beta})R_x^T(\hat{\gamma})$."

"Select the correct sign for the measurement values $\vec{v}_i$ according to the selection criterion (31) (the SIL skips this step because it always calculates with the equivalent static current)." should be changed to "Select the correct sign for the measurement values $\vec{v}_i$ according to the selection criterion (27) (the SIL skips this step because it always calculates with the equivalent static current)."

"Calculate the y-coordinates y_{i1} and y_{i2} using Equation (25)." should be changed to "Calculate the y-coordinates y_{i1} and y_{i2} using Equation (21)."

"Select the correct signs in Equation (28) using the Theorem 3 in order to reduce the amount of possible z-solutions and calculate the z-coordinates z_{i1} and z_{i2}." should be changed to "Select the correct signs in Equation (24) using Theorem 3 in order to reduce the amount of possible z-solutions and calculate the z-coordinates z_{i1} and z_{i2}."

As the roll angle cannot be calculated, it is assumed to be zero. Thus, it does not make sense anymore to mention the roll angle in the figures and in the corresponding text passages in Section 4 Simulation Results. They should be changed to refer to the pitch angle instead:

Text referring to step 2 of SIL description:

"Calculate the UAV angles using Equations (9), (10) and (14) for all possible sensor combinations and calculate the average. The results are $\overline{\alpha}$, $\overline{\beta}$, and $\overline{\gamma}$." should be changed to "Calculate the UAV angles using Equations (9) and (10) for all possible sensor combinations and calculate the average. The results are $\overline{\alpha}$ and $\overline{\beta}$. The roll angle $\overline{\gamma}$ cannot be calculated and is assumed to be zero."

7. Changes in Section 4.3: Test Results

The results referring to Figure 10 should be updated:

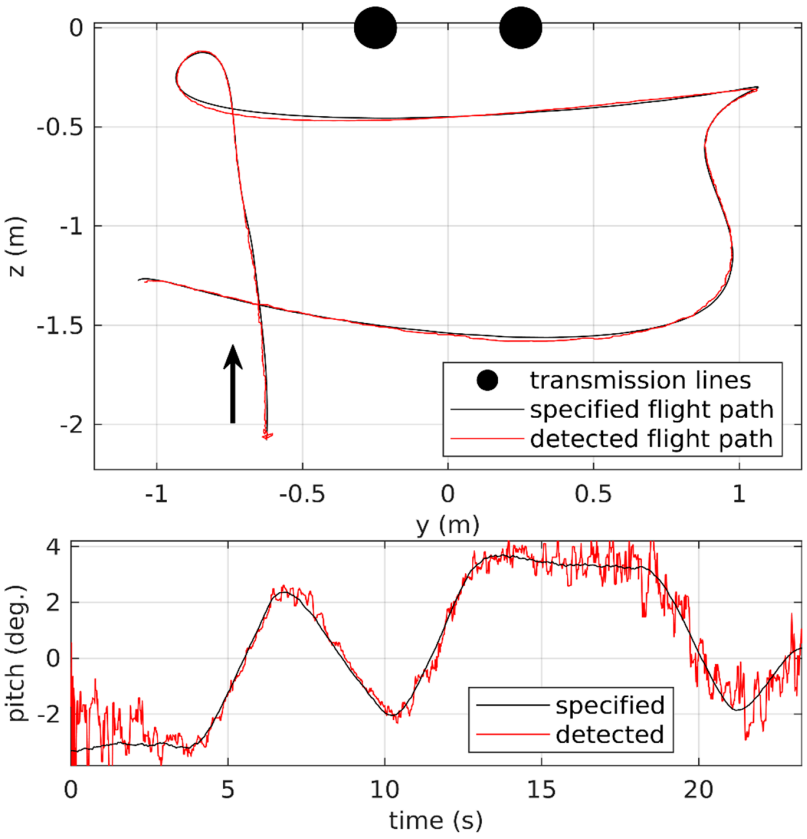


Figure 10. Localization quality, good shielding: σ = 1 μT.

"The mean deviation from the specified flight path was 21 mm, with a maximum error of 54 mm. The standard deviation of the roll angle error was 0.4°, and its mean was very close to zero, around 0.025°. Only the roll angle is shown here, as it had the largest values since the UAV moved almost exclusively in the yz-plane in the simulation." should be changed to "The mean deviation from the specified flight path was 19 mm, with a maximum error of 61 mm. The standard deviation of the pitch angle error was 0.6°, and its mean was very close to zero, around −0.076°. Only the pitch angle is shown here as the yaw angle is of similar quality."

Figure 11 should be replaced by:

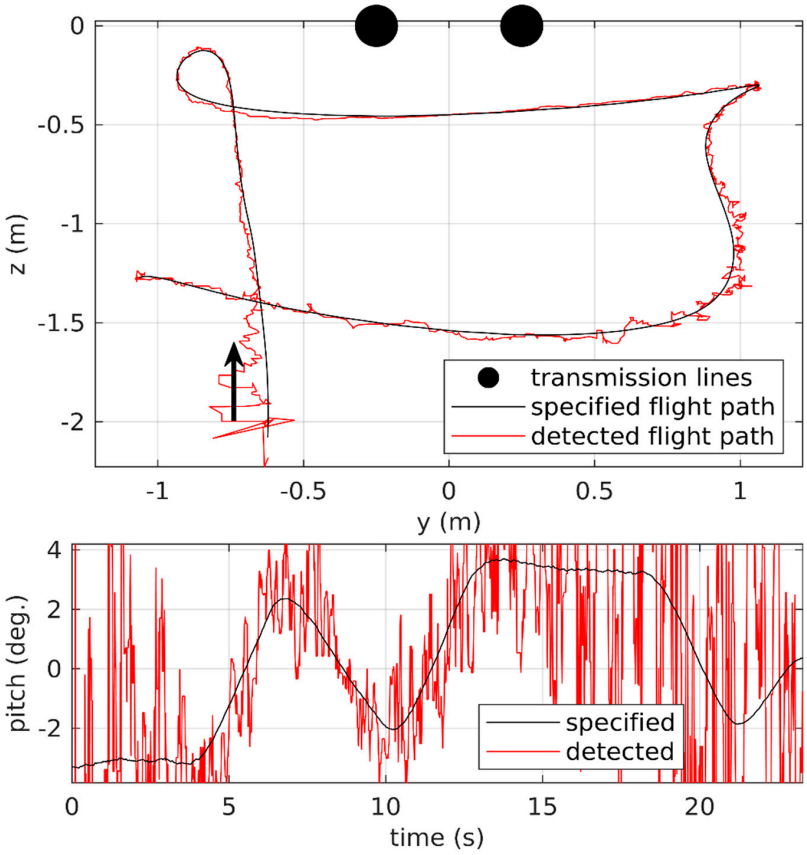


Figure 11. Localization quality, poor shielding: σ = 8 μT.

In the original publication, there was a mistake in "Figure 12. Path and roll angle errors, poor shielding: σ = 8 μT." as published. The corrected "Figure 12. Path and pitch angle errors, poor shielding: σ = 8 μT." appears below.

Figure 12 should be replaced by:

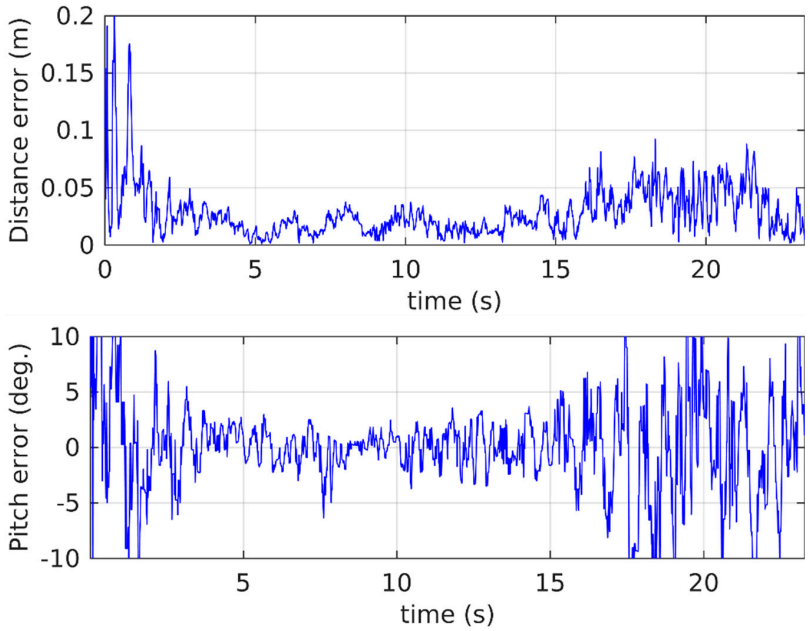


Figure 12. Path and pitch angle errors, poor shielding: σ = 8 μT.

The results referring to Figure 12 should be updated:

"Figure 12 shows the deviation from the specified flight path and the specified roll angle for each waypoint. The mean deviation from the specified flight path was 41 mm, where the maximum error was 221 mm. The mean deviation from the specified roll angle was −0.22°, where the maximum error was 12.3°. The standard deviation of the roll angle error was 2.96°." should be changed to "Figure 12 shows the deviation from the specified flight path and the specified pitch angle for each waypoint. The mean deviation from the specified flight path was 29 mm, where the maximum error was 204 mm. The mean deviation from the specified pitch angle was 0.15°, where the maximum error was 28.4°. The standard deviation of the pitch angle error was 4.4°."

Figure 13 should be replaced by:

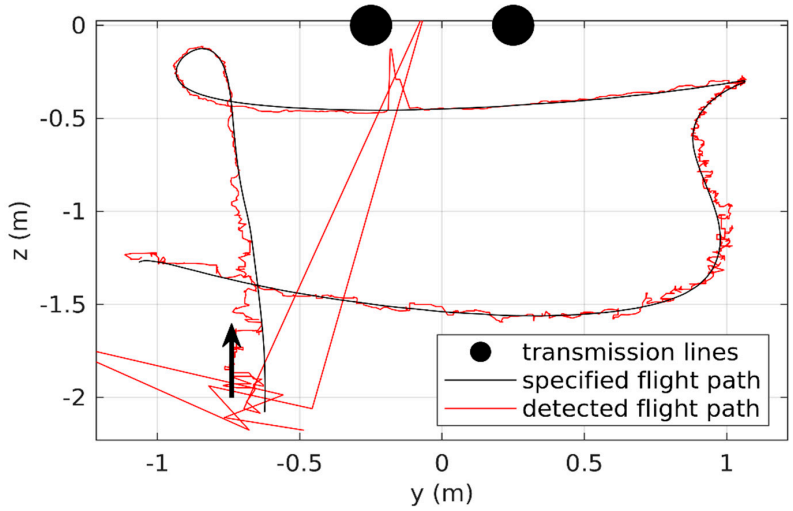


Figure 13. Localization quality, three sensors, σ = 8 µT.

The results referring to Figure 13 should be updated:

"The mean deviation from the specified flight path was 91 mm. In areas far from the transmission lines, as in this case, very strong outliers could occur occasionally, so that the maximum error was now about 2260 mm." should be changed to "The mean deviation from the specified flight path was 34 mm. In areas far from the transmission lines, as in this case, very strong outliers could occur occasionally, so that the maximum error was now about 2202 mm."

Figure 14 should be replaced by:

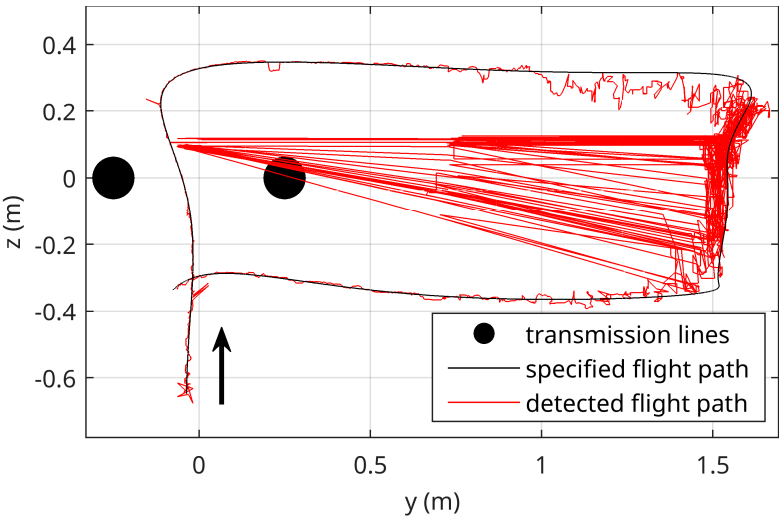


Figure 14. Localization quality close to the transmission lines and far away, three sensors, σ = 8 µT.

Figure 15 should be replaced by:

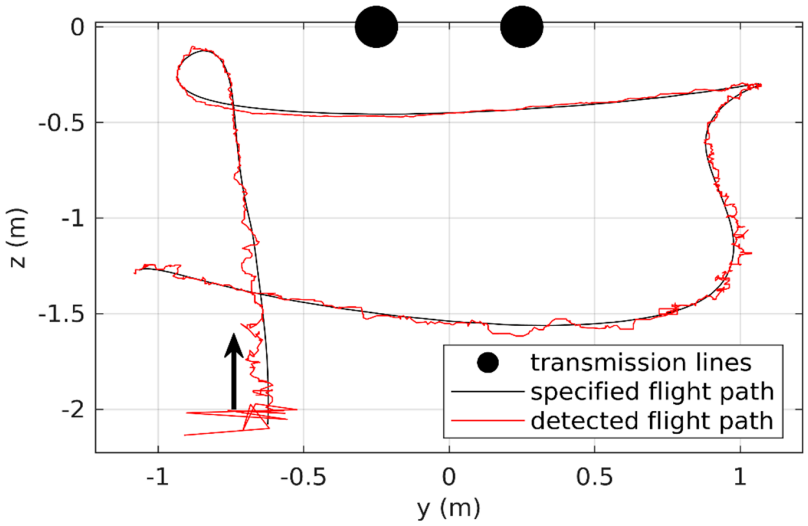


Figure 15. Localization quality in the case of low transmission line current of 100 A, four sensors, $\sigma = 1$ μT.

8. Changes in Section 6: Conclusions

Two sentences should be updated in Section 6: Conclusions:

"In this context, equations were derived to be able to calculate the UAV orientation analytically. It was shown that only three magnetometers are needed for this purpose and that neither the current, nor the magnetic field equations need to be known." should be changed to "In this context, equations were derived to be able to calculate the UAV's yaw and pitch angle analytically. It was shown that only two magnetometers are needed for this purpose and that neither the current nor the magnetic field equations need to be known."

The authors apologize for any inconvenience caused and state that the scientific conclusions are unaffected. This correction was approved by the Academic Editor. These changes do not affect the conclusions presented in the paper. The original article has been updated.

Reference

1. Martinović, D.; Bogdan, S.; Kovačić, Z. Mathematical considerations for unmanned aerial vehicle navigation in the magnetic field of two parallel transmission lines. *Appl. Sci.* **2021**, *11*, 3323. [CrossRef]

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