

Article

Electromagnetic Fields from Cloud-to-Cloud Horizontal Lightning Channel on Perfect Conducting Soil: Induced Potentials in Flying Aircraft

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Featured Application: The proposed model can be used to evaluate the effect of a cloud-to-cloud lightning strike on a power transmission line or any metal structure on the ground, such as a lightning rod or airplane in flight.

Abstract: Calculation expressions for the electric and magnetic fields produced by a horizontal cloud-to-cloud lightning channel, assuming a perfectly conducted ground, are proposed in this paper. These expressions depend on the current model traveling through the channel and serve as the starting point to calculate the induced fields and potentials at any point in space. The derived expressions for the fields are used to calculate the induced potentials by the channel on metallic structures such as vertically driven rods in the ground and aircraft in flight. The influence of soil with finite conductivity is discussed, and an estimation of the induced potentials in this situation is proposed.

Keywords: cloud-to-cloud lightning channel; perfect conducting soil; induced potentials in aircraft



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1. Introduction

Atmospheric discharges in which a vast amount of electric charge is exchanged between clouds and the ground or even between two clouds give rise to the formation of channels through which a high current intensity surges in waves. The process by which these channels are generated, especially between clouds and the ground, is quite well understood [1], and the effects caused by them are of great practical interest. The problem of calculating the electric and magnetic fields generated by a vertical cloud-to-ground lightning channel has been widely investigated by various authors under the assumption that the ground is a perfect conductor [2]. In these works, two types of calculation methods can be found, the time-domain-based method [3,4] and the one that uses Fourier phasors as calculation elements in the frequency domain [5]. For the problem being studied, air is the medium in which the lightning channel develops. This is a poor conductive medium for which a virtually null electrical conductivity must be assumed except when it is ionized, as is the case with the channel that carries the electrical charge between the cloud and the ground. Due to the null conductivity of air, the frequency-based method leads to equations for the induced electric field whose solutions have a singularity for zero frequency, that is, for the contribution to the induced electric field of the constant component of the source [5]. For this reason, calculations in the time domain are much more frequently found in the literature. However, it must be mentioned that considering that the ground has a finite conductivity, that is, it is not a perfect conductor, the contribution to the electromagnetic fields due to the ground is much more direct and clearer if worked in the frequency domain. Due to the difficulties described before with zero frequency, very few contributions tackle this problem [6]. On the other hand, the study of horizontal cloud-to-cloud lightning

channels is difficult to find in the literature, perhaps because their effects at ground level are too small to represent a problem, although this may not be entirely true [7–9]. However, some installations, such as communication antennas or power lines, are located at a certain height above mountain peaks or elevated buildings, which shortens the distance to the horizontal channel, potentially generating significant disturbances in the installations with respect to their normal behavior. The evaluation of the effects of intense fields generated by a horizontal cloud-to-cloud channel on closer and isolated metallic elements, such as flying aircraft, is also of great interest. This work tackles the calculation of the electric and magnetic fields produced by a horizontal cloud-to-cloud lightning channel in the time domain, in the simplest case of considering the ground to be a perfect conductor. As an application example, the double-exponential discharge intensity will be used, and the electromagnetic fields will be evaluated at various points in space. To this end, the paper is structured as follows. After this introduction, the proposed mathematical model is developed in Section 2. Then, a numerical application example is considered, followed by a section discussing the influence of finite-conductive soil. Finally, the conclusions and comments on what has been revealed throughout the work are summarized at the end of the paper.

2. The Model Backgrounds

A small current element is considered in a horizontal lightning channel located in the air above a conductive ground at height z_p as indicated in Figure 1.

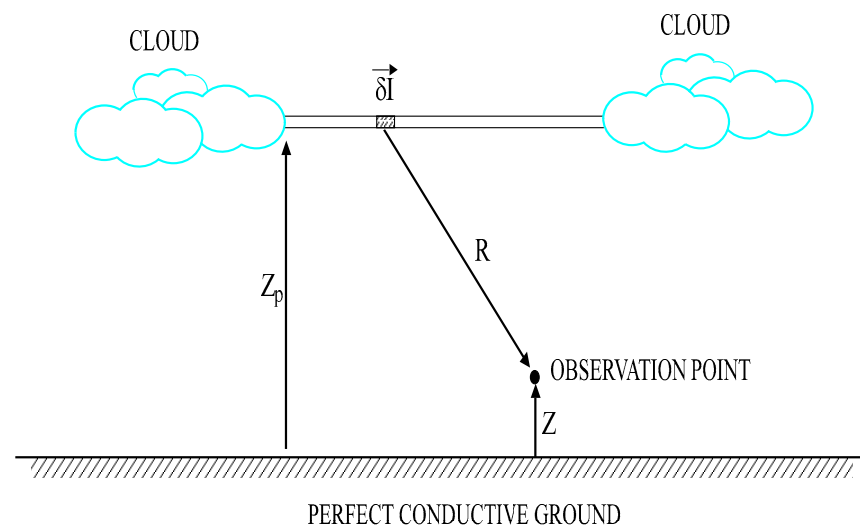


Figure 1. Geometry of a horizontal lightning channel over a perfect conductive ground.

In the frequency domain, the phasor at frequency ω of the vector potential must satisfy the equations,

$$\begin{aligned} \nabla^2 \vec{A} + \frac{\omega^2}{c_0^2} \vec{A} &= -\mu_0 \delta \vec{I} \quad (z > 0) \\ \nabla^2 \vec{A} + \frac{\omega^2}{c_1^2} \vec{A} &= 0 \quad (z < 0) \end{aligned} \quad (1)$$

where $c_0^2 = (\epsilon_0 \mu_0)^{-1}$ is the speed of light in a vacuum and $c_1^2 = (\mu_1 (\epsilon_1 + \frac{\sigma_1}{j\omega}))^{-1}$. The current element can be expressed as $\delta \vec{I} = \delta I \cdot \vec{u}_x$ located at the position (x_p, y_p, z_p) . The electrical parameters in the air carry the subscript 0 with $\sigma_0 = 0$, while for the conductive medium, the subscript is 1. The vector potential must also satisfy the boundary conditions at the air–ground interface, which are expressed in terms of the generated electric and

magnetic fields. For a horizontal waveguide along the x -axis, at a point with coordinates (x, y, z) located in the air, the vector potential has the following components.

$$\begin{aligned} A_x(x, y, z, \omega) &= \frac{\mu_0 \delta I}{4\pi} \int_0^\infty \left(\frac{\lambda}{\lambda_0} e^{-\lambda_0 |z-z_p|} + f(\lambda) e^{-\lambda_0 (z-z_p)} \right) J_0(\lambda r) d\lambda \\ A_y(x, y, z, \omega) &= 0 \\ A_z(x, y, z, \omega) &= \frac{\mu_0 \delta I}{4\pi} \frac{x-x_p}{r} \int_0^\infty g(\lambda) e^{-\lambda_0 (z-z_p)} J_1(\lambda r) d\lambda \end{aligned} \quad (2)$$

where $r = \sqrt{(x-x_p)^2 + (y-y_p)^2}$ and the functions $f(\lambda)$ and $g(\lambda)$ must be determined from the aforementioned boundary conditions. Once the boundary conditions are imposed, the expressions for these functions are the following,

$$\begin{aligned} f(\lambda) &= \frac{\lambda}{\lambda_0} \cdot \frac{\lambda_0 - \lambda_1}{\lambda_0 + \lambda_1} \cdot e^{-2\lambda_0 z_p} \\ g(\lambda) &= \frac{\gamma_0^2 - \gamma_1^2}{\gamma_0^2 \lambda_1 + \gamma_1^2 \lambda_0} \cdot \frac{2\lambda^2}{\lambda_0 + \lambda_1} \cdot e^{-2\lambda_0 z_p} \end{aligned} \quad (3)$$

In the Equation (3), $\gamma_i^2 = -\frac{\omega^2}{c_i^2}$ ($i = 0, 1$) and $\lambda_i = \sqrt{\lambda^2 + \gamma_i^2}$. By substituting Equation (3) into (2) and using the Lipschitz identity [10]

$$\frac{e^{-\beta |\vec{r} - \vec{r}_p|}}{|\vec{r} - \vec{r}_p|} = \int_0^\infty \frac{\lambda}{\sqrt{\lambda^2 + \beta^2}} e^{-\sqrt{\lambda^2 + \beta^2} |z-z_p|} J_0(\lambda r) d\lambda \quad (4)$$

results

$$\begin{aligned} A_x(x, y, z, \omega) &= \frac{\mu_0 \delta I}{4\pi} \cdot \frac{e^{-jk_0 R}}{R} + \frac{\mu_0 \delta I}{4\pi} \cdot \frac{e^{-jk_0 R'}}{R'} - \frac{\mu_0 \delta I}{4\pi} \int_0^\infty \frac{2\lambda}{\lambda_0} \cdot \frac{\lambda_1}{\lambda_0 + \lambda_1} e^{-\lambda_0 (z+z_p)} J_0(\lambda r) d\lambda \\ A_y(x, y, z, \omega) &= 0 \\ A_z(x, y, z, \omega) &= \frac{\mu_0 \delta I}{4\pi} \frac{x-x_p}{r} \int_0^\infty \frac{\gamma_0^2 - \gamma_1^2}{\gamma_0^2 \lambda_1 + \gamma_1^2 \lambda_0} \cdot \frac{2\lambda^2}{\lambda_0 + \lambda_1} e^{-\lambda_0 (z+z_p)} J_1(\lambda r) d\lambda \end{aligned} \quad (5)$$

where $k_i = \frac{\omega}{c_i}$ ($i = 0, 1$), $R = \sqrt{(x-x_p)^2 + (y-y_p)^2 + (z-z_p)^2}$ and, on the other hand, $R' = \sqrt{(x-x_p)^2 + (y-y_p)^2 + (z+z_p)^2}$. If the soil is a perfect electric conductor, $\sigma_1 \rightarrow \infty$, it is straightforward to prove that $\gamma_1^2 \rightarrow \infty$, $\lambda_1 \rightarrow \infty$ and $\frac{\gamma_1^2}{\lambda_1} \rightarrow \infty$, which gives rise to

$$\begin{aligned} A_x(x, y, z, \omega) &= \frac{\mu_0 \delta I}{4\pi} \cdot \frac{e^{-jk_0 R}}{R} - \frac{\mu_0 \delta I}{4\pi} \cdot \frac{e^{-jk_0 R'}}{R'} \\ A_z(x, y, z, \omega) &= 0 \end{aligned} \quad (6)$$

which in practice entails evaluating the contribution of the actual horizontal channel and subtracting the contribution of the image horizontal channel with respect to the ground interface. Given that the current element can be expressed as $\delta I = I(x_p, \omega) dx_p$, the potential vector in the frequency domain is.

$$A_x(x, y, z, \omega) = \frac{\mu_0 I(x_p, \omega)}{4\pi} \cdot \frac{e^{-jk_0 R}}{R} dx_p - \frac{\mu_0 I(x_p, \omega)}{4\pi} \cdot \frac{e^{-jk_0 R'}}{R'} dx_p \quad (7)$$

and its expression in the time domain is,

$$A_x(x, y, z, t) = \frac{\mu_0 i(x_p, t - R/c_0)}{4\pi R} dx_p - \frac{\mu_0 i(x_p, t - R'/c_0)}{4\pi R'} dx_p \quad (8)$$

For the calculation of the fields, the well-known expressions $\vec{B} = \vec{\nabla} \times \vec{A}$ for the magnetic field and $\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$ for the electric field will be used, in which the scalar potential will be evaluated, according to the Lorenz gauge, as

$$\phi(R, t) = -c_0^2 \int_0^t \vec{\nabla} \vec{A} \cdot d\tau \quad (9)$$

The electric field at point (x, y, z) created by the current element of the actual horizontal channel at position (x_p, y_p, z_p) , has the following Cartesian components.

$$\begin{aligned} dE_x &= \frac{\mu_0}{4\pi\epsilon_0} dx_p \left[\frac{3(x-x_p)^2 - R^2}{R^5} \int_0^t i(x_p, \tau - \frac{R}{c_0}) d\tau + \frac{3(x-x_p)^2 - R^2}{c_0 R^4} i(x_p, t - \frac{R}{c_0}) + \frac{(x-x_p)^2 - R^2}{c_0^2 R^3} \frac{\partial}{\partial t} i(x_p, t - \frac{R}{c_0}) \right] \\ dE_y &= \frac{\mu_0}{4\pi\epsilon_0} dx_p \left[\frac{3(x-x_p)(y-y_p)}{R^5} \int_0^t i(x_p, \tau - \frac{R}{c_0}) d\tau + \frac{3(x-x_p)(y-y_p)}{c_0 R^4} i(x_p, t - \frac{R}{c_0}) + \frac{(x-x_p)(y-y_p)}{c_0^2 R^3} \frac{\partial}{\partial t} i(x_p, t - \frac{R}{c_0}) \right] \\ dE_z &= \frac{\mu_0}{4\pi\epsilon_0} dx_p \left[\frac{3(x-x_p)(z-z_p)}{R^5} \int_0^t i(x_p, \tau - \frac{R}{c_0}) d\tau + \frac{3(x-x_p)(z-z_p)}{c_0 R^4} i(x_p, t - \frac{R}{c_0}) + \frac{(x-x_p)(z-z_p)}{c_0^2 R^3} \frac{\partial}{\partial t} i(x_p, t - \frac{R}{c_0}) \right] \end{aligned} \quad (10)$$

Regarding the magnetic field,

$$\begin{aligned} dB_x &= 0 \\ dB_y &= -\frac{\mu_0}{4\pi} \cdot \frac{z-z_p}{R} \cdot \left(\frac{1}{c_0 R} \frac{\partial}{\partial t} i(x_p, t - \frac{R}{c_0}) + \frac{1}{R^2} i(x_p, t - \frac{R}{c_0}) \right) dx_p \\ dB_z &= \frac{\mu_0}{4\pi} \cdot \frac{y-y_p}{R} \cdot \left(\frac{1}{c_0 R} \frac{\partial}{\partial t} i(x_p, t - \frac{R}{c_0}) + \frac{1}{R^2} i(x_p, t - \frac{R}{c_0}) \right) dx_p \end{aligned} \quad (11)$$

which must include the contributions from the image channel subtracted, which are obtained by substituting $z_p \rightarrow -z_p$ in Equations (10) and (11). The different terms in Equation (10) represent the static, induction, and radiation components of the electric field.

The contribution of the entire horizontal channel to the fields will be obtained by integrating the Equations (10) and (11) in the variable x_p along the channel length L

$$\begin{aligned} E_i(x, y, z) &= \int_L dE_i(x_p) \\ B_i(x, y, z) &= \int_L dB_i(x_p) \end{aligned} \quad (12)$$

as it is pointed out in Equation (12).

3. Application Examples

In the contributions where the fields generated by a vertical lightning channel are studied, several models for the discharge are used [10]. In the so-called MTLE engineering model, an intensity wave that moves along the channel at velocity v is used. The model meets the equation $i(z_p, t) = i(0, t - \frac{z_p}{v}) \cdot P(z_p) \cdot u(t - \frac{z_p}{v})$, where $u(t)$ is the step function, $P(z_p)$ is an attenuation function of the discharge in its transit between the cloud and the earth at height z_p , and $i(0, t)$ is the intensity signal at the origin of the channel, which can take different forms [11].

In this work, a horizontal channel along the X axis with length $L = 4000$ m and height $z_p = H = 4000$ m will be considered for the evaluation of the electric and magnetic fields at different heights. For the current intensity that travels from one side of the channel to the other, the expression $i(x_p, t) = i(0, t - \frac{x_p}{v}) \cdot u(t - \frac{x_p}{v})$ will be assumed, and the attenuation $P(z_p)$ will be suppressed. Among the models of the current $i(0, t)$, the most commonly used are the ones that will be employed in this paper. These models include the double-exponential model and the Heidler model. In the first model, a double-exponential shape of the signal $i(0, t) = I_{\max} \cdot (a \cdot e^{b \cdot t} + c \cdot e^{d \cdot t})$ will be adopted with $I_{\max} = 15.6$ kA,

$a = 1$, $b = -4 \cdot 10^5 \text{ s}^{-1}$, $c = -1$, $d = -6 \cdot 10^6 \text{ s}^{-1}$. The Heidler model [12] is defined by the following equation,

$$i(0, t) = \frac{I_{01}}{\eta} \cdot \frac{(t/\tau_1)^2}{(t/\tau_1)^2 + 1} \cdot \exp(-t/\tau_2) + I_{02} \cdot (\exp(-t/\tau_3) - \exp(-t/\tau_4)) \quad (13)$$

where the values of the parameters chosen are: $I_{01} = 9.9 \text{ kA}$, $I_{02} = 7.5 \text{ kA}$, $\eta = 0.845$, $\tau_1 = 0.072 \text{ } \mu\text{s}$, $\tau_2 = 0.072 \text{ } \mu\text{s}$, $\tau_3 = 0.072 \text{ } \mu\text{s}$, $\tau_4 = 0.072 \text{ } \mu\text{s}$. The speed of the intensity wave along the channel will be taken as $v = \frac{2}{3}c_0$ [13]. Thus, the time interval to consider for induction phenomena will be determined by the ratio of the channel length to the speed at which the current signal travels through it. In our case, $t_{\max} = 2 \cdot 10^{-5} \text{ s}$, although the time elapsed due to the distance between the source element and the field point that traveled at the speed of light must also be taken into account. The shape of the current models and their temporal derivative at the beginning of the channel are shown in Figure 2.

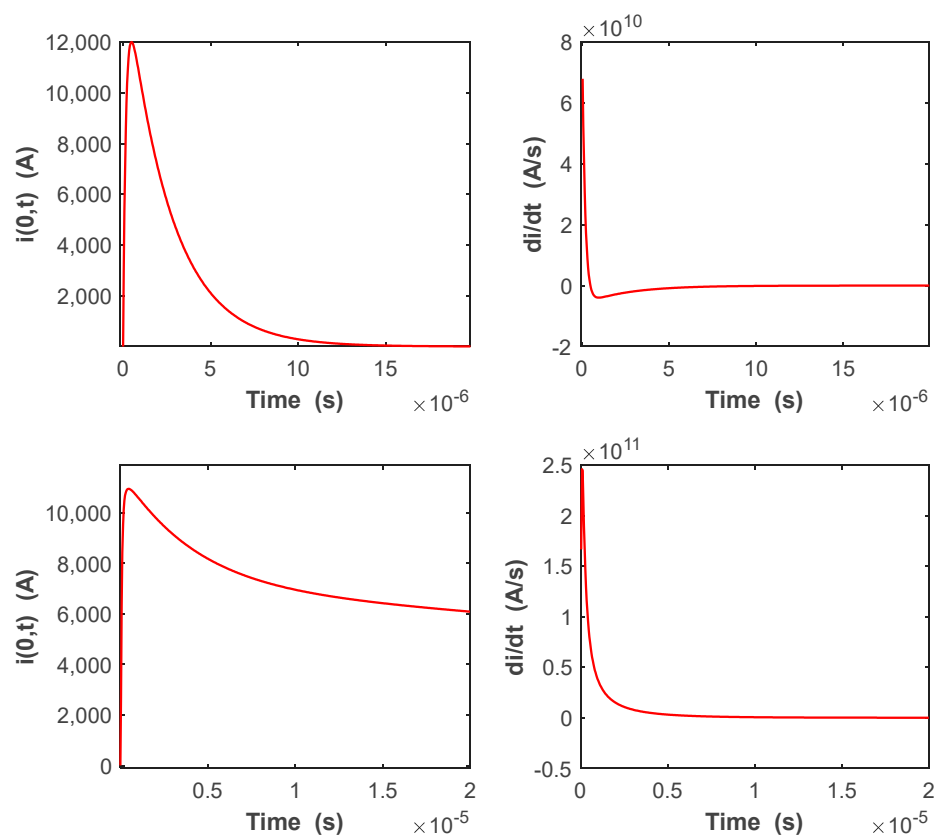


Figure 2. The double-exponential current model and the current first derivative at the beginning of the channel (upper panels). The Heidler current model and its first derivative in the lower panels.

In practice, field calculations are performed by segmenting the lightning channel into portions ranging from 10 cm to 1 m in length, depending on the proximity of the field points to the channel. For each segment, the fields are evaluated using the theoretical Equations (10) and (11), and through superposition, the final result is obtained. A critical aspect of the calculation is the accurate definition of the chosen intensity model and its first derivative. In our case, the double-exponential and Heidler models have been tested, as illustrated in Figure 2, demonstrating suitable behavior for simulating inter-cloud discharges. The calculation algorithm has been implemented using MatLab, where some of the built-in routines of the software are used, although it can be adapted to any other programming language as well.

Two types of application examples will be considered. On the one hand, the fields will be calculated at points close to the ground, where power transmission lines and lightning

rods are located. On the other hand, the induced potentials in isolated metallic structures closer to the lightning channel, such as an aircraft, will be calculated.

3.1. Induced Potentials at Points near Ground Level: Induced Potential Difference in a Vertical Rod with One End on the Ground

Figure 3 represents a lightning horizontal channel under which a vertical metallic rod rests on the ground with the length Z_c . The task is to calculate the induced potential difference that appears between the ends of the rod due to the lightning channel. The rod is 10 m tall and is located approximately in the middle of the channel.

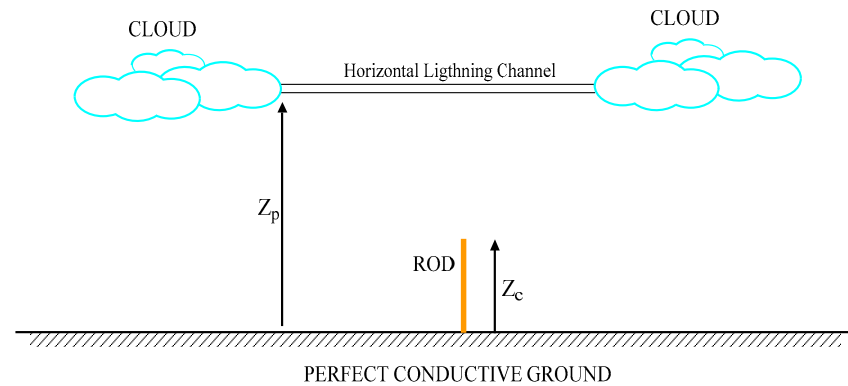


Figure 3. A vertical conductive rod under the horizontal lightning channel.

The induced potential difference that appears between the ends of the rod can be obtained from

$$\Delta V_\epsilon = \int_0^{Z_c} \vec{E} \cdot d\vec{r} \quad (14)$$

although other authors prefer to estimate this induced potential based on the scalar potential difference $\Delta V = \phi(0) - \phi(Z_c)$, both ways of defining ΔV and ΔV_ϵ only coincide for a DC current. Actually, ΔV_ϵ can be interpreted as an electromotive force between the ends of the conductor, which will cause a movement of free carriers, that is, an induced current. The magnitude of this current will depend on the impedance of the conductor and can be significant if the impedance is small. The line integral (14) is numerically computed by dividing the integration path into segments, with the segment size being unimportant due to the minimal variation of vector fields along this path.

Considering that the scalar potential difference can be directly measured using a voltmeter and based on the relationship $\vec{E} = -\vec{\nabla}\phi - \frac{\partial}{\partial t}\vec{A}$, it is obtained that

$$\Delta V = \phi(0) - \phi(Z_c) = \int_0^{Z_c} \vec{E} \cdot d\vec{r} + \int_0^{Z_c} \frac{\partial}{\partial t} \vec{A} \cdot d\vec{r} \quad (15)$$

This magnitude will be considered representative of the effect caused by the horizontal lightning channel. Figure 4 shows the potential difference ΔV as a function of the channel response time. In this example, the contribution of the temporal derivative of the vector potential is irrelevant due to the proximity of the rod to the perfectly conducting ground. Thus, the induced potentials ΔV and ΔV_ϵ are similar in practice.

As can be seen in Figure 4, a peak-to-peak potential difference of around 250 volts is induced in the rod if the double-exponential current model is considered. On the other hand, a slightly higher induced potential difference is obtained when using the Heidler current model. Since the frequencies involved are very high, the current that can flow through the rod is mainly determined by the inductive impedance of the rod. The electrical power transferred to the rod is, therefore, of an inductive nature.

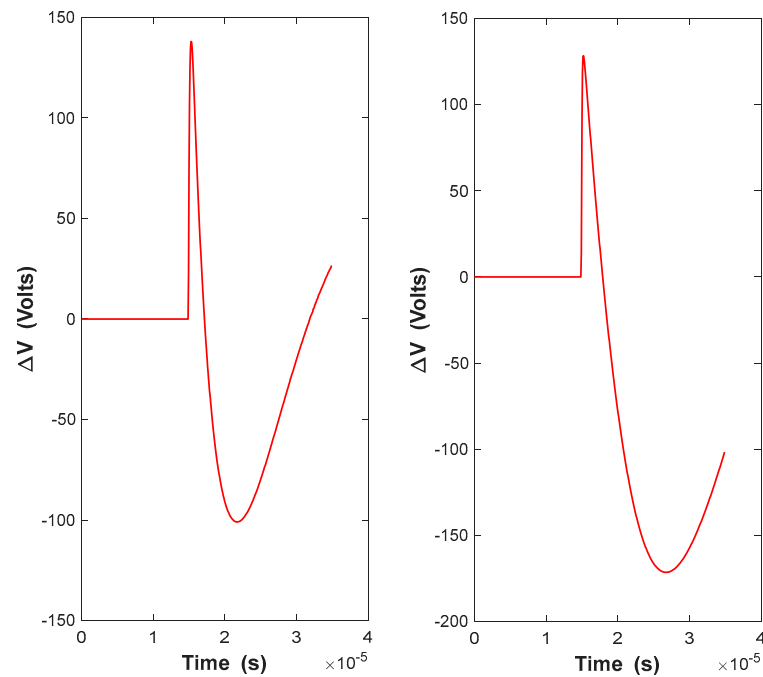


Figure 4. Induced potential between the rod ends as a function of the channel response time for the two current channel models, double-exponential (left panel) and Heidler (right panel).

3.2. Induced Potentials on an Aircraft in Flight

Figure 5 represents a horizontal cloud-to-cloud lightning channel and an aircraft flying $h = 1000$ m below the channel with its fuselage oriented along the channel direction and its wings perpendicular to it, positioned approximately in the middle of the channel and laterally displaced from the channel by a distance of 1000 m.

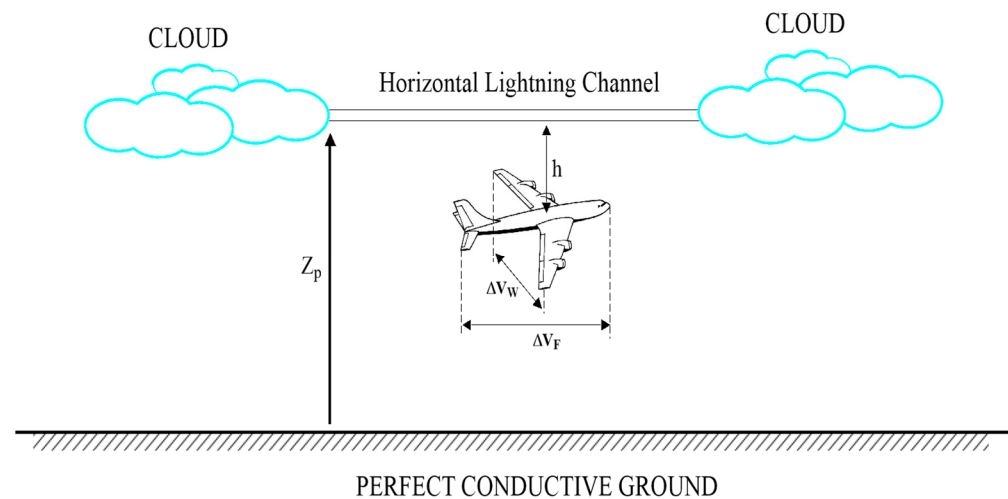


Figure 5. A flying aircraft under a horizontal lightning channel.

The goal is to evaluate the induced potential difference that appears between the ends of the fuselage $\Delta V_F = \int_{Fus} \vec{E} \cdot d\vec{r}$ and between the ends of the aircraft wings $\Delta V_W = \int_{Wings} \vec{E} \cdot d\vec{r}$. Figure 6 shows the potential differences defined as a function of the channel response time when a double-exponential current model is used.

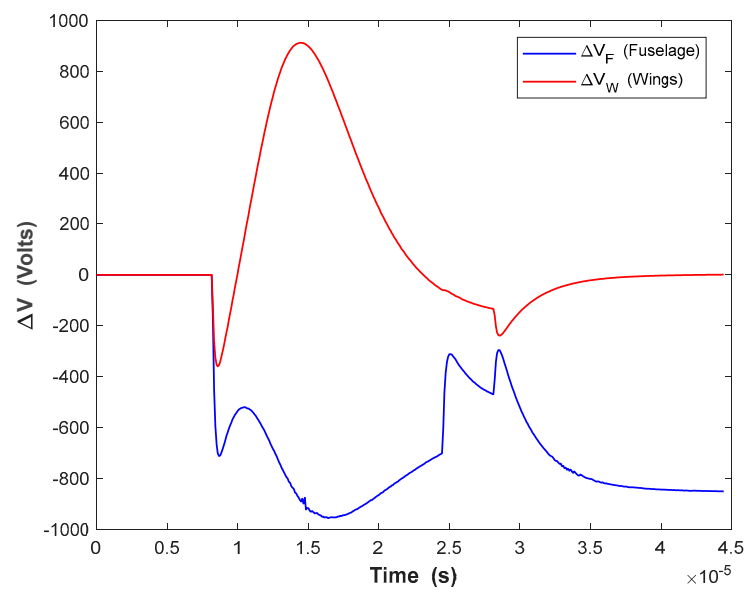


Figure 6. Induced potentials between the nose and tail (blue line) and between the wingtips (red line) of the aircraft when a double-exponential current model is used.

Significant differences are found when comparing the above-shown induced potentials with the induced scalar potential difference. In Figure 7, the comparison between ΔV and ΔV_ϵ for the previous example is shown. The left panel displays the two ways of expressing the induced potential difference in the fuselage, while the right panel shows the same for the aircraft wings. It is interesting to note that for the wings, there is no distinction between both ways of expressing the induced potential difference since the vector potential has no component along the wings, while noticeable differences are observed for the length of the fuselage.

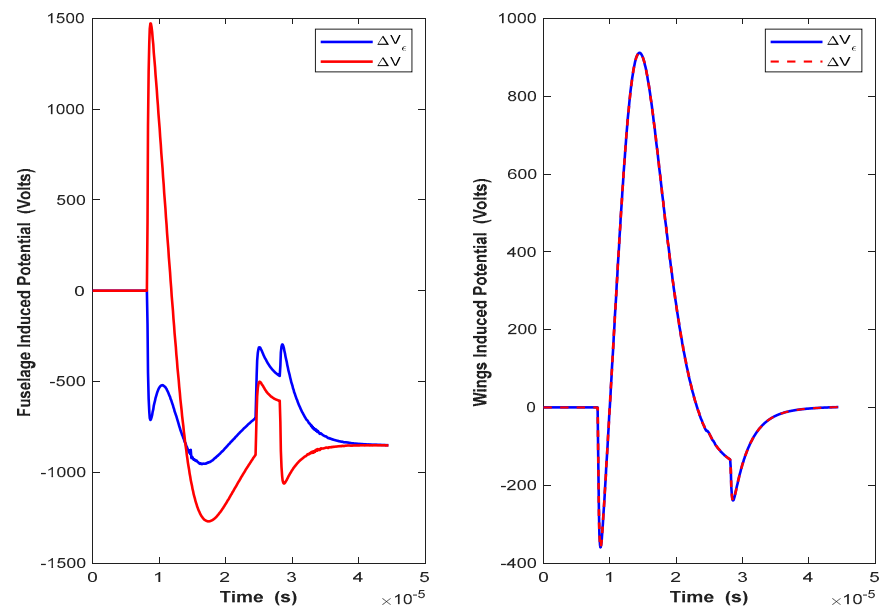


Figure 7. Differences between ΔV and ΔV_ϵ for the airplane example.

To conclude this section, the induced potentials on the aircraft are addressed when it is closer to the ground, at 1000 m above it, and again maintaining the lateral displacement of 1000 m. The double-exponential model for the current channel is used. Figure 8 shows the induced potentials along the fuselage and between the wingtips when using the induced potential from Equation (14).

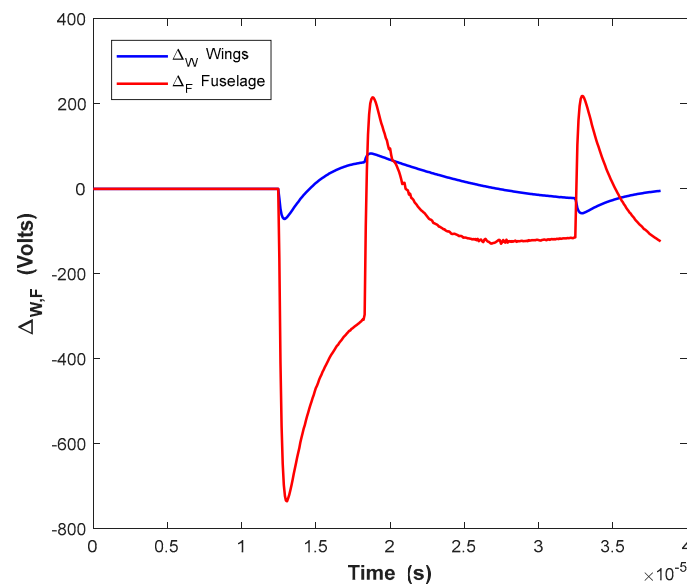


Figure 8. Induced potentials between the nose and tail (red line) and between the wingtips (blue line) of the aircraft when it is 1000 m above the ground in the middle of the lightning channel.

If, on the other hand, Equation (15) is used to calculate the induced potential, Figure 9 shows the appearance of this potential along the fuselage compared to the one obtained using Equation (14). The comparison for the induced potential on the wings is not shown for the same reasons mentioned when discussing Figure 7.

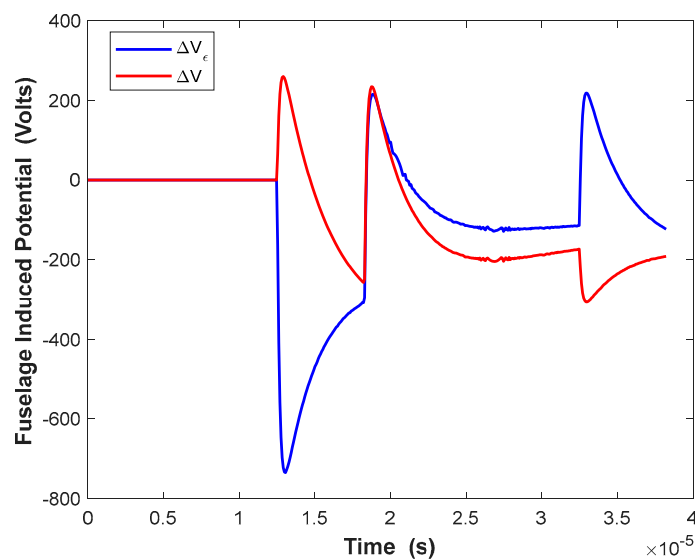


Figure 9. Induced potential ΔV_ϵ (blue line) and ΔV (red line) along the aircraft fuselage.

As can be seen in the presented figures, the energy transferred by the electromagnetic field generated by the lightning channel to the aircraft structure can generate undesired effects that can impact safety, even at large distances from the channel.

4. Effect of a Non-Perfect Conducting Ground

The finite conductivity of the soil has repercussions on the terms of Equation (5) involving improper integrals of the Sommerfeld type. As previously seen, these terms become simplest when considering the soil as a perfect conductor.

$$\begin{aligned} A_x^\sigma(x, y, z, \omega) &= -\frac{\mu_0 \delta I}{4\pi} \int_0^\infty \frac{2\lambda}{\lambda_0} \cdot \frac{\lambda_1}{\lambda_0 + \lambda_1} e^{-\lambda_0(z+z_p)} J_0(\lambda r) d\lambda \xrightarrow{\sigma \rightarrow \infty} -\frac{2\mu_0 \delta I}{4\pi} \cdot \frac{e^{-jk_0 R'}}{R'} \\ A_z^\sigma(x, y, z, \omega) &= \frac{\mu_0 \delta I}{4\pi} \frac{x-x_p}{r} \int_0^\infty \frac{\gamma_0^2 - \gamma_1^2}{\gamma_0^2 \lambda_1 + \gamma_1^2 \lambda_0} \cdot \frac{2\lambda^2}{\lambda_0 + \lambda_1} e^{-\lambda_0(z+z_p)} J_1(\lambda r) d\lambda \xrightarrow{\sigma \rightarrow \infty} 0 \end{aligned} \quad (16)$$

The main problem with the terms associated with finite soil conductivity (16) is that they do not have a closed expression in the time domain and must be evaluated in the Fourier domain. Additionally, Sommerfeld-type integrals are difficult to evaluate and require special numerical techniques [14]. In this paper, the objective is not to numerically calculate the contribution of these terms but to estimate bounds on the values of the induced potentials in the extreme cases associated with infinite soil conductivity and zero soil conductivity. A soil with finite conductivity lies between these extreme situations, and therefore, the induced potential will fall within the bounds associated with these extreme cases.

When the soil is considered a medium with zero conductivity, such as air, and with electric parameters similar to air, the Equation (5) along with property (4) yield the following results,

$$\begin{aligned} A_x^\sigma(x, y, z, \omega) &= -\frac{\mu_0 \delta I}{4\pi} \int_0^\infty \frac{2\lambda}{\lambda_0} \cdot \frac{\lambda_1}{\lambda_0 + \lambda_1} e^{-\lambda_0(z+z_p)} J_0(\lambda r) d\lambda \xrightarrow[\substack{\sigma \rightarrow 0 \\ \varepsilon_1 = \varepsilon_0 \\ \mu_1 = \mu_0}]{\sigma \rightarrow \infty} -\frac{\mu_0 \delta I}{4\pi} \cdot \frac{e^{-jk_0 R'}}{R'} \\ A_z^\sigma(x, y, z, \omega) &= \frac{\mu_0 \delta I}{4\pi} \frac{x-x_p}{r} \int_0^\infty \frac{\gamma_0^2 - \gamma_1^2}{\gamma_0^2 \lambda_1 + \gamma_1^2 \lambda_0} \cdot \frac{2\lambda^2}{\lambda_0 + \lambda_1} e^{-\lambda_0(z+z_p)} J_1(\lambda r) d\lambda \xrightarrow[\substack{\sigma \rightarrow 0 \\ \varepsilon_1 = \varepsilon_0 \\ \mu_1 = \mu_0}]{\sigma \rightarrow \infty} 0 \end{aligned} \quad (17)$$

Thus, one bound on the induced potential value will be obtained by replacing the soil with air, where the only contribution to the fields comes from the vector potential of the actual horizontal lightning channel. On the other hand, the other bound will be obtained by considering the soil as a perfect conductor, and the contribution to the fields will be the difference between the contribution from the vector potential of the actual channel and the contribution from the image channel with respect to the soil surface. For a soil with finite conductivity, the potential will lie between the two mentioned bounds.

It can be expected that the influence of finite soil conductivity will be more pronounced in points close to the surface since both the real lightning channel and the image channel are at comparable distances. However, in points near the real lightning channel, the influence of the soil is heavily limited by the distance.

Figure 10 depicts the upper and lower bounds of the induced potential on the vertical rod shown in Figure 3. The blue line corresponds to soil with zero conductivity, equivalent to replacing the conductive medium of the soil with air. The red line, representing the minimum bound of the induced potential, corresponds to a perfectly conducting soil. The graph associated with a soil of finite and non-zero conductivity will lie between the two curves shown in the figure.

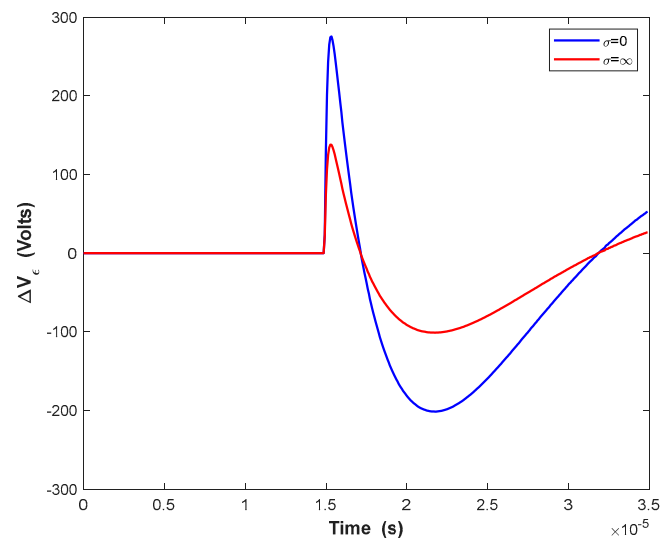


Figure 10. Induced potential bounds for the vertical rod of Figure 2.

5. Conclusions

The expressions for calculating the electric and magnetic fields generated by a horizontal lightning channel at points located on the surface of a perfectly conducting soil have been proposed in this work. Using these expressions, the induced potentials on a vertical rod supported by the ground and the induced potentials on the metallic structure of an aircraft in various scenarios have been calculated. Due to the non-stationary nature of the problem studied, the concept of induced potential possesses some ambiguity. As a result, two methods of evaluating the potential have been employed and compared. It can be concluded from this work that for structures close to the soil surface, the induced potential can be calculated using the scalar potential. However, in points near the lightning channel, significant differences are found between the two descriptions. The induced potentials are highly dependent on the current model used to define the lightning channel, so two popular models, namely the double-exponential and Heidler models, have been employed in this work. The calculations demonstrate that the induced potentials are not negligible for the proposed configuration, although they may be somewhat less intense in real situations involving greater distances from the channel.

The contribution of soil with finite conductivity has also been addressed in this paper. It is concluded that the induced potential should lie between the upper and lower bounds corresponding to a perfectly conducting soil and a perfectly insulating soil.

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