

Article

Investigation of the Motion Characteristics of Parts on a Platform Subjected to Planar Oscillations

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Abstract: Positioning applications are very important in a variety of industrial processes, including automatic assembly. This paper proposes a technique for positioning applications that involves employing a platform subjected to planar oscillations along circular, elliptical, and complex trajectories. Dynamic and mathematical models of the motion of a part on the platform were developed to investigate the motion characteristics of the part. The research showed that when the platform was excited in two perpendicular directions by sinusoidal waves, different trajectories of the part's motion could be obtained by controlling excitation parameters such as the frequencies and amplitudes of the waves and the phase shift between the waves. Furthermore, by adjusting these parameters, the average displacement velocity of the part could be controlled. The results demonstrate that the part can be moved in any direction at a given velocity and can be subjected to complex dense positioning trajectories. Therefore, such a platform can be applied in feeding, positioning, and manipulation tasks.

Keywords: positioning; motion control; trajectory planning; oscillating platform; dynamics; modeling; frequency; sinusoidal waves; phase shift



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1. Introduction

The importance of automation in various industrial processes is constantly growing. Among them, assembly automation is one of the most challenging tasks to implement. Depending on the method of automatic assembly, the assembly process can be divided into several stages, including the feeding of parts into the assembly position, positioning, manipulation, joining, etc. The directional movement of parts is required to perform stages such as feeding, positioning, or manipulation. Typically, various manipulators [1–5], grippers [6,7], sophisticated robots [8–10], and other assembly and positioning devices are employed to implement assembly operations through passive [11–13], active [14–17], and hybrid [18] approaches.

Methods based on the active approach usually employ different types of sensors [19–21], feedback systems [22], computer vision systems [23,24], and sophisticated control algorithms [25]. However, all of these appliances are expensive and complex. This approach can also be associated with increased energy consumption [26]. Furthermore, these methods may not always deliver the desired efficiency. The economic interest of industries is to increase the efficiency of production and to reduce costs. With this in mind, the passive approach can give an edge in terms of efficiency and costs.

Currently, oscillating surfaces are mostly applied for transportation tasks. Directional motion is achieved when a net frictional force is created, i.e., when friction forces are not cancelled out over one cycle of oscillations. For example, this can be observed when the path of oscillations is not symmetric or the law of motion along this path is not symmetric

as well. This kind of asymmetry is widely applied to asymmetric vibratory feeders, vibratory conveying systems and machines [27–32]. An optimum wave form that ensures the maximum conveying velocity for asymmetric vibratory feeders was analyzed by Okabe et al. [33]. Frei et al. [34] proposed a method for transporting parts simultaneously with different trajectories on a platform composed of many surface elements controlled by individual actuators. The motion of parts was achieved through net friction forces caused by horizontal and vertical oscillations of the individual surface elements. Nath et al. [35] applied a platform subjected to spatially asymmetric lateral biharmonic oscillations to achieve a directed motion of parts due to the ratchet effect, which occurs when the driving force overcomes the threshold of static friction asymmetrically.

Directional motion can also be achieved when the forward motion of an oscillating surface takes longer than the backward motion in every period of oscillations [36]. Reznik et al. [36] carried out research on the dynamics of a part being transported over a straight-line path on a surface that oscillated in this manner. Mayyas [37,38] studied the dynamics of stick-slip motion achieved by different forward and backward accelerations caused by a leaf spring with direction-dependent elasticity that was applied as a suspension for the manipulation platform. Viswarupachari et al. [39] investigated directed transport of parts on a platform, as the forward motion of the platform took a different time than the backward motion in conjunction with a geometrically asymmetric motion profile.

Oscillating surfaces that exhibit direction-dependent frictional properties can also be used to achieve directional motion. Umbanhowar et al. [40] applied textured surfaces such as micromachined silicon and fabrics to create friction-induced velocity fields on the surface of an oscillating platform for vibratory manipulation of parts. Mitani et al. [41–43] demonstrated that the directional motion of microparts can be achieved on a symmetrically oscillating platform with a textured surface for feeding applications. Chen et al. [44] showed that the surface with fin-like asperities of a longitudinal oscillating trough can be applied for the directional motion of particles achieved due to the net force created by the fin-like asperities.

Recently, a technique for manipulation and transportation of various parts was demonstrated where the motion was achieved by the means of a periodic asymmetry of the frictional forces between the part and the harmonically oscillating surface that are dynamically controlled in a certain manner with respect to the period of the harmonic excitation [45,46].

The presented paper proposes a technique for positioning applications, which is accomplished by employing a platform subjected to planar oscillations along circular, elliptical, and complex trajectories. The aim of this research is to investigate the motion characteristics of a part on this platform. By adjusting the parameters that control the oscillations of the platform, the part can be moved in any direction at a given velocity and can be subjected to complex dense positioning trajectories. Therefore, such a platform can be applied for feeding, positioning, and manipulation tasks.

2. Dynamic and Mathematical Models of the Motion of a Part on a Horizontally Oscillating Platform

In order to carry out an investigation on the motion characteristics of a part on a platform subjected to planar oscillations along circular and elliptical trajectories for positioning applications, dynamic and mathematical models were developed. Figure 1 shows the dynamic model of the positioning of a part on a platform subjected to planar oscillations along circular and elliptical trajectories. The system consists of a stationary base (Figure 1, (1)), on which a horizontal movable platform (Figure 1, (2)) is attached. The origin of the base's coordinate system Σ is at the intersection of two horizontal stationary ξ - and η -axes. The coordinate system of the moving platform is named Σ_{pl} . This coordinate system moves with the platform.

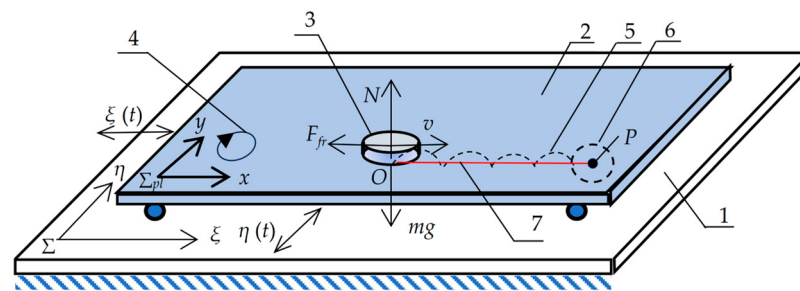


Figure 1. Dynamic model of the positioning on a platform subjected to planar oscillations along circular and elliptical trajectories: (1) stationary base; (2) mobile platform; (3) part to be positioned; (4) trajectory of the platform's motion; (5) trajectory of the part's transient sliding motion; (6) trajectory of the part's steady-state motion; (7) traveled path Δd .

The mobile platform is excited in two perpendicular horizontal directions by harmonic sinusoidal excitation. When the amplitude of the harmonic excitation is the same for both directions and the frequency is also the same, any point on the platform traces a circular trajectory (Figure 1, (4)). The trajectory of the platform movement is elliptical when the excitation amplitudes are not equal in different directions.

As the platform is excited with the same amplitude in both directions, the equations of motion of the platform are written as follows:

$$\begin{cases} \xi = \xi_0 + A_x \sin \omega t, \\ \eta = \eta_0 + A_y \sin(\omega t + \varepsilon), \end{cases} \quad (1)$$

where t is time, $A_x = A_y$ are the excitation amplitudes (A_x is the excitation amplitude in the ξ -axis, and A_y is the excitation amplitude in the η -axis), ω is the frequency of excitation, and ε is the phase shift between the harmonic waves along the ξ - and η -axes (it is equal $\varepsilon = \pi/2$ for circular motion).

The horizontal platform can be subjected to reversed planar oscillations. Then, the equations are written as follows:

$$\begin{cases} \xi = \xi_0 - A_x \sin \omega t, \\ \eta = \eta_0 - A_y \sin(\omega t + \varepsilon). \end{cases} \quad (2)$$

The excitation can also be shifted with respect to time. If it is shifted by $\pi/2$, the equations can be expressed as follows:

$$\begin{cases} \xi = \xi_0 + A_x \cos(\omega t + \varepsilon), \\ \eta = \eta_0 + A_y \cos \omega t. \end{cases} \quad (3)$$

In this case, the equations for the reversed planar oscillations are written as follows:

$$\begin{cases} \xi = \xi_0 - A_x \cos(\omega t + \varepsilon), \\ \eta = \eta_0 - A_y \cos \omega t. \end{cases} \quad (4)$$

By changing the phase shift between the perpendicular excitation directions, circular, elliptical, and linear shapes of the oscillations of the platform can be obtained, and the principle of changing orientation is identical to the principle of the Lissajous curve. When there is no phase shift, the platform oscillates in a linear trajectory.

To formulate the equations of motion of the part to be positioned (Figure 1, (3)) that is placed on the platform, it is assumed that the part is flat, and the mass of the part is concentrated in the center of mass, and the coefficient of dry friction between the part and

the platform is constant. When the part slides relative to the platform, the projections of the acceleration of the center of mass are written as follows:

$$\begin{cases} a_{\xi} = \ddot{x} + \ddot{\xi} = \ddot{x} - A_x \omega^2 \sin \omega t, \\ a_{\eta} = \ddot{y} + \ddot{\eta} = \ddot{y} - A_y \omega^2 \sin(\omega t + \varepsilon). \end{cases} \quad (5)$$

The motion of the part on the platform is influenced by the dry friction force F_{fr} , which acts in opposition to the relative velocity. The projections of the dry friction force are expressed as follows:

$$\begin{cases} F_{fr\xi} = -\mu mg \sin\left(\sqrt{\dot{x}^2 + \dot{y}^2}, x\right) = -\mu mg \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}}, \\ F_{fr\eta} = -\mu mg \sin\left(\sqrt{\dot{x}^2 + \dot{y}^2}, y\right) = -\mu mg \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}}, \end{cases} \quad (6)$$

where μ is the kinetic coefficient of dry friction, $\sqrt{\dot{x}^2 + \dot{y}^2}$ is the relative velocity, and $\dot{x}^2 + \dot{y}^2 \neq 0$.

The following equilibrium holds:

$$\begin{cases} ma_{\xi} = F_{fr\xi}, \\ ma_{\eta} = F_{fr\eta}. \end{cases} \quad (7)$$

Inserting Equations (1) and (6) into Equation (7) leads to the equations that determine the motion of the part to be positioned:

$$\begin{cases} \ddot{x} + \mu g \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = A_x \omega^2 \sin \omega t, \\ \ddot{y} + \mu g \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = A_y \omega^2 \sin(\omega t + \varepsilon). \end{cases} \quad (8)$$

The transient sliding motion of the part (Figure 1, (5)) varies depending on the excitation frequency and the frictional conditions, while the steady-state motion trajectory (Figure 1, (6)) varies depending on the excitation amplitude. Therefore, the velocity of the part can be controlled using these parameters. In this study, the average velocity is calculated by finding the path traveled (Figure 1, (7)) and dividing it by the time of traveling:

$$v = \frac{\Delta d}{\Delta t}, \quad (9)$$

where the Δd is the path traveled, and Δt is the time of traveling. The motion equations express the position and velocity vectors of the part with respect to the coordinate system Σ_{pl} .

3. Modeling Results

To investigate the influence of the input parameters μ , ω and A_e on the motion characteristics of the part, modeling was carried out using software developed for this purpose. The software was written using the MATLAB (Mathsoft, Cambridge, MA, USA) programming language. The motion equations were solved using the ode45 solver based on the Runge–Kutta (4, 5) formula and the Dormand–Prince pair. The value of $g = 9.81 \text{ m/s}^2$ was set for all of the modeled cases.

When a horizontal platform is subjected to planar oscillations, the part may initially move from the start point P_0 at a certain distance over a spiral or wavy trajectory (Figure 2, (1)). The characteristics of motion and the nature of the trajectory depend on the parameters of the system, such as the amplitude and frequency of the excitation, the dry friction conditions, or the phase shift between the harmonic waves along the ξ and η axes.

However, the part to be positioned eventually starts to oscillate around the equilibrium point P_c (Figure 2, (2)), and the nature of the steady trajectory depends mainly on the amplitude and the phase shift. It was observed that the part on the platform moves at a certain angle α . The displacement angle is measured from the starting point O to the center of the steady trajectory P_c .

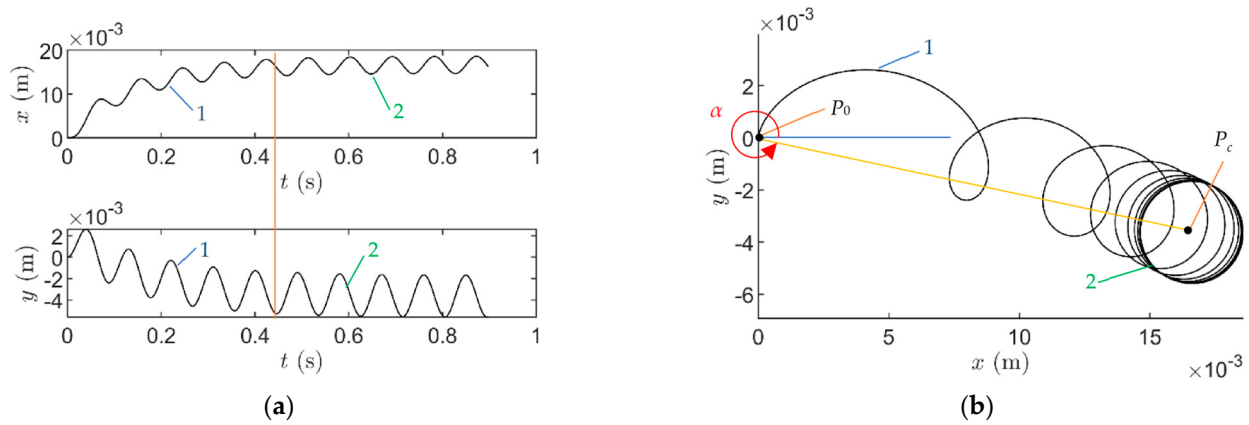


Figure 2. Trajectories of transient (1) and steady motion (2) of the part on the platform subjected to planar oscillations, when $\mu = 0.2$, $A_x = A_y = 0.002$ m, $\omega = 70$ rad/s, $\varepsilon = \pi/2$: (a) displacement of the part with respect to time in the x and y directions; (b) trajectory of the part to be positioned, where P_0 is the start point, and P_c is the center of the steady trajectory.

As can be seen in Figure 2, the part still slides around a certain point after a certain number of oscillation cycles of the platform. This oscillation around the equilibrium position is no longer necessary for positioning, since the part is positioned; therefore, the excitation of the platform can be terminated at this time. To determine the average velocity of the part, it is important to determine how long it takes for the part to settle in the equilibrium position. To do that, the developed software was registering the centers of the arcs and circles of each oscillation cycle and was checking the differences between the coordinates of two adjacent points. When the difference between the center points of adjacent circles was $\Delta p < 2 \cdot 10^{-4}$ m, the code was terminated, and the final positioning time was determined.

In this paper, the motion characteristics of parts on the platform are analyzed in the following four cases:

1. When the platform is excited in two perpendicular directions by sinusoidal waves of the same amplitude ($A_x = A_y$), and the phase shift between the perpendicular components is $\pi/2$. In this case, the platform moves in a circular trajectory.
2. When the platform is excited in two perpendicular directions by sinusoidal waves of different amplitudes ($A_x \neq A_y$), and the phase shift between the perpendicular components is $\pi/2$. In this case, the platform moves in an elliptical trajectory.
3. When the platform is excited in two perpendicular directions by sinusoidal waves of the same amplitude ($A_x = A_y$), and the phase shift between the perpendicular components varies in the range of $\pi/10 \leq \varepsilon < 9\pi/10$. These phase shift values are chosen since the part moves in a straight path at phase shift values of 0 and π , and that is not suitable for positioning. When the phase shift varies in the range of $\pi/10 \leq \varepsilon < \pi/2$, the motion of the platform follows inclined elliptical trajectories; then, at $\varepsilon = \pi/2$, the trajectory of the platform becomes circular, and when the phase shift increases even more ($\pi/2 < \varepsilon \leq 9\pi/10$), the trajectory becomes elliptical again, but the angle of inclination of the ellipse changes.
4. When the platform is excited in two perpendicular directions by sinusoidal waves of different frequencies ($\omega_x \neq \omega_y$).

The first two cases might be best suited for applications related to feeding systems or similar tasks, where there is no need to rapidly change the direction of motion. The third case is very well suited for applications related to non-prehensile manipulation tasks, as the phase shift employed as an additional control parameter makes it easier and faster to change the direction of motion of the part. The fourth case is best suited for applications related to positioning tasks or for the implementation of the search phase of automatic assembly, as, in this case, the platform moves along a complex trajectory. Nevertheless, all four techniques can be applied for the positioning of parts and non-prehensile manipulation tasks, as parts can be moved in any direction.

The trajectories of the part after 10 circular oscillation cycles are presented in Figure 3. As can be seen in Figure 3a, to position the part over a greater distance, it is necessary to increase the amplitude of the excitation of the platform in the direction of travel (in the x -direction, in this case). Different trajectories of the part's motion can be obtained by subjecting the platform to harmonic sinusoidal oscillations in two perpendicular directions with different values of phase shift. Linear, elliptical, or circular trajectories of the platform can be obtained by changing the phase shift, which results in a similar trajectory of the part placed on the platform. When the phase shift is equal to 0 or π , the part moves from the starting point along a line inclined at an angle of 45° or -45° with respect to the x -axis, respectively. Positioning along a straight line is only possible at these angles; therefore, these values of the phase shift were rejected in the investigation on the influence of the system parameters on the average velocity of the part. Figure 3b demonstrates the influence of the phase shift between the perpendicular components of the excitation on the nature and direction of the motion of the part. If the phase shift is in the ranges of $\pi/10 \leq \epsilon < \pi/2$ and $\pi/2 < \epsilon \leq 9\pi/10$, then the part moves along an inclined and stretched trajectory during the transient motion stage, and the trajectory of the steady motion stage takes the shape of an inclined ellipse. The trajectory of the steady motion stage is circular when the phase shift is $\pi/2$. It was also observed that the part moves a greater distance from the starting point when $\epsilon \neq \pi/2$.

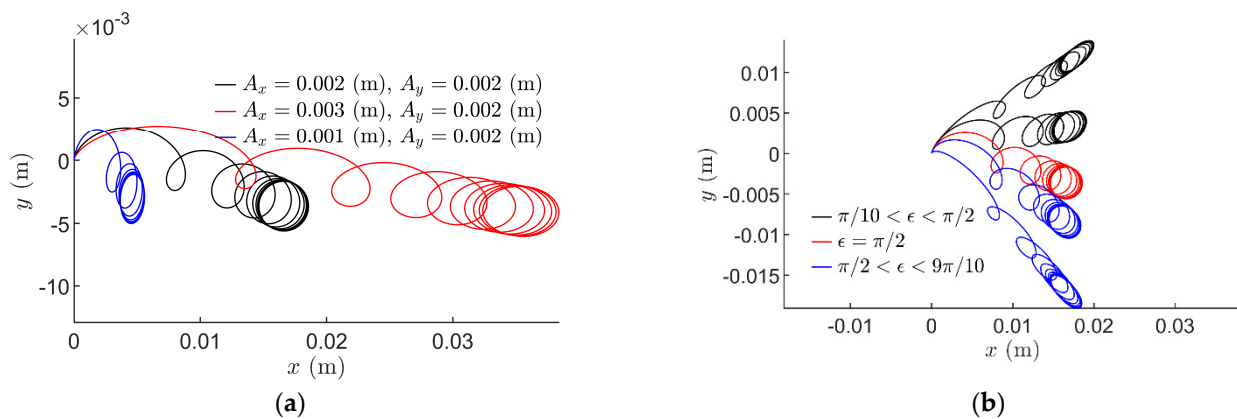


Figure 3. Trajectories of the part: (a) when the platform is excited in two perpendicular directions with a phase shift of $\epsilon = \pi/2$, where $\mu = 0.2$, $\omega = 70$ rad/s; (b) when the platform is excited in two perpendicular directions under various values of the phase shift, where $\mu = 0.2$, $\omega = 70$ rad/s, $A_x = A_y = 0.002$ m.

Figure 4 shows the positioning area achieved in the case when the platform is excited in two perpendicular directions by sinusoidal waves of the same amplitude ($A_x = A_y$) according to Equations (1)–(4), and the phase shift between the perpendicular components varies in the range of $\pi/10 \leq \epsilon < 9\pi/10$.

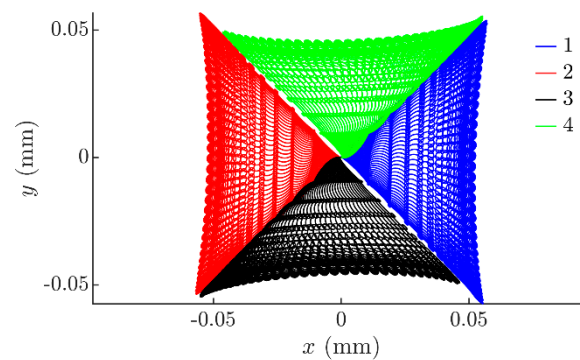


Figure 4. Positioning area, where $\mu = 0.2$, $\omega = 100$ rad/s, $A_x = A_y = 0.002$ m: (1) trajectories when the platform is excited according to Equation (1), as the phase shift changes in the range of $\pi/10 < \varepsilon \leq 9\pi/10$; (2) trajectories when the platform is excited according to Equation (2), as the phase shift changes in the range of $\pi/10 < \varepsilon \leq 9\pi/10$; (3) trajectories when the platform is excited according to Equation (3), as the phase shift changes $\pi/10 < \varepsilon \leq 9\pi/10$; (4) trajectories when the platform is excited according to Equation (4), as the phase shift changes in the range of $\pi/10 < \varepsilon \leq 9\pi/10$.

The analysis of the motion trajectories showed that the trajectories of the part cover a triangular area when the phase shift changes in the range of $\pi/10 < \varepsilon \leq 9\pi/10$. This means that the part can be positioned at any location in this triangle. Furthermore, the part can be positioned at any location in a certain square by changing the direction of the initial excitation (Figure 4).

The trajectory of the motion of the platform becomes complex when the platform is excited in two perpendicular directions by sinusoidal waves of different frequencies. Therefore, the part that is sliding on the platform also moves along a complex trajectory. Various complex trajectories of the part can be achieved by controlling the ratio between the frequencies of the waves k ($\omega_y = k\omega_x$). The trajectories that are the most suitable for positioning are presented in Figure 5.

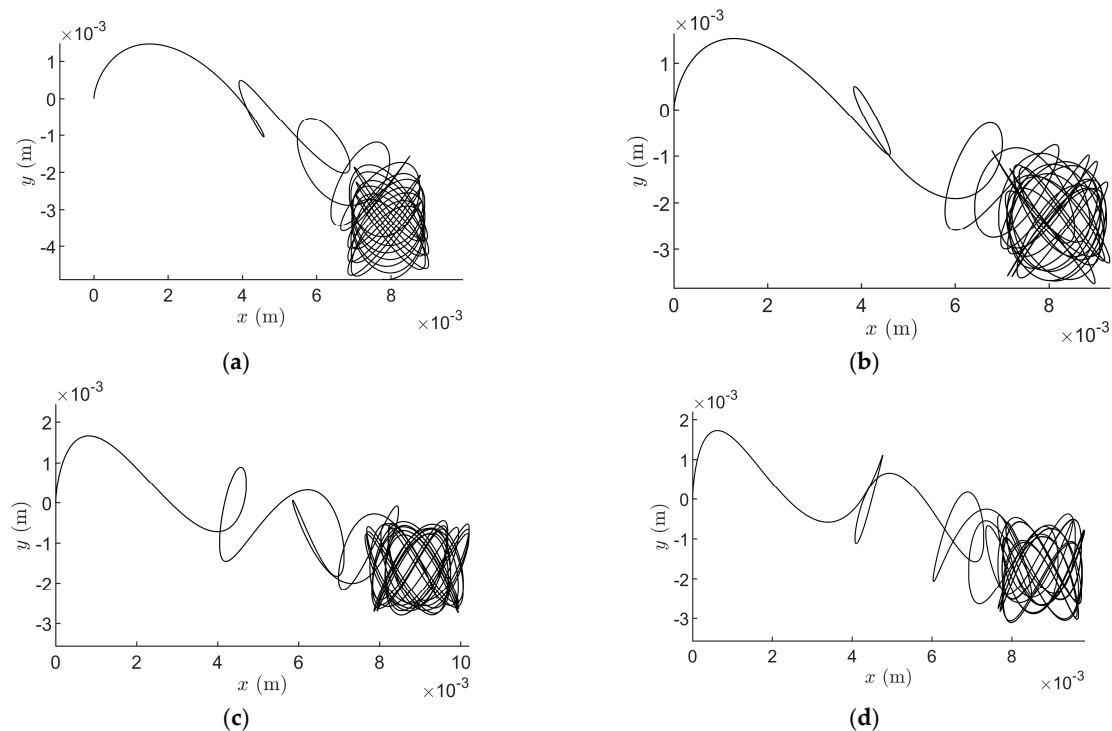


Figure 5. Cont.

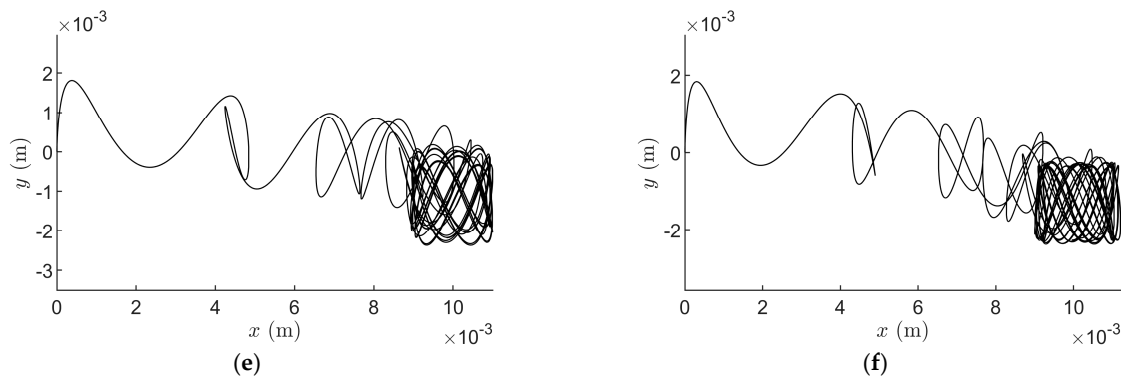


Figure 5. Trajectories of the sliding part when the platform is excited in two perpendicular directions by sinusoidal waves of different frequencies, where $g = 9.81 \text{ m/s}^2$, $\mu = 0.2$, $\omega_x = 100 \text{ rad/s}$, $A_x = A_y = 0.001 \text{ m}$, $\varepsilon = \pi/2$: (a) $k = 1.2$; (b) $k = 1.3$; (c) $k = 1.6$; (d) $k = 1.8$; (e) $k = 2.2$; (f) $k = 2.4$.

Such complex trajectories, in which the center of the part to be positioned makes loops, i.e., it moves very densely over a certain area, are perfectly suited to stages of automatic assembly such as the positioning and the search.

The analysis showed that when the frequency difference is equal to one ($|\omega_x - \omega_y| = 1$), the positioning trajectory of the part is arranged in a regular square (Figure 6). The size of the positioning square depends on the excitation amplitude of the platform. It was observed that choosing a higher frequency of the platform excitation yields a denser positioning trajectory.

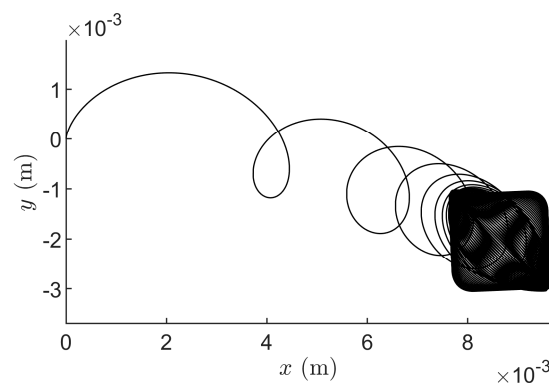


Figure 6. Positioning trajectory, where $g = 9.81 \text{ m/s}^2$, $\mu = 0.2$, $\omega_x = 100 \text{ rad/s}$, $\omega_y = 101 \text{ rad/s}$, $A_x = A_y = 0.001 \text{ m}$, $\varepsilon = \pi/2$.

3.1. Motion Characteristics of the Part When the Platform Is Excited in Two Perpendicular Directions by Sinusoidal Waves of the Same Amplitude, and the Phase Shift between the Perpendicular Components Is $\pi/2$

As mentioned earlier, in order to determine the influence of the system parameters on the average displacement velocity of the part, the positioning time was recorded up until the instant at which the part starts to repeatedly rotate around one point. It was found that when the excitation frequency increases, the average displacement velocity increases linearly (Figure 7a). A similar linear relation was observed between the excitation amplitude and the average displacement velocity (Figure 7b). However, an increase in the friction coefficient between the platform and the part results in a decrease in the average velocity of the part (Figure 7c). Thus, when the horizontal platform is excited in two perpendicular directions by sinusoidal waves of the same amplitude, the frequency and amplitude of the excitation have the greatest influence on the part's average displacement velocity.

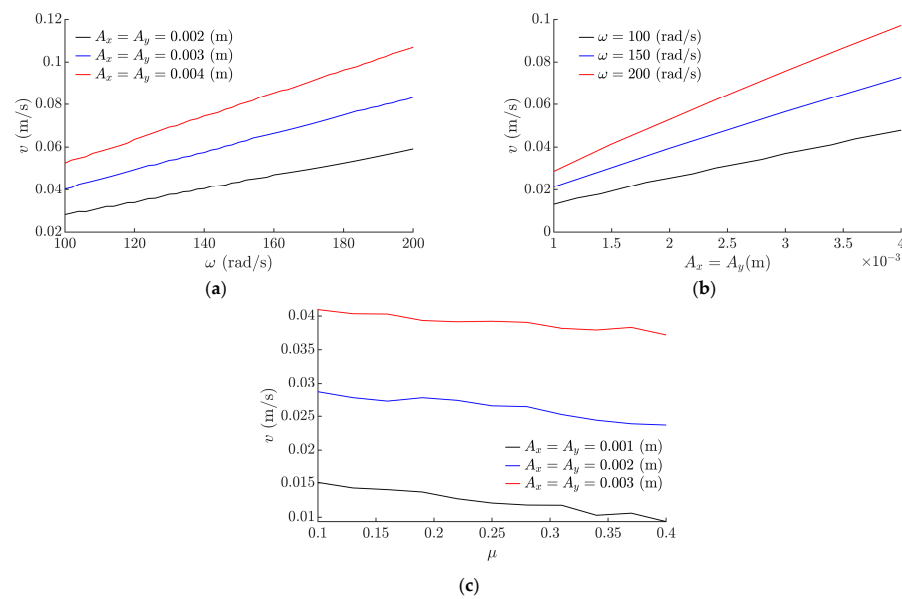


Figure 7. Average displacement velocity depending on: (a) excitation frequency ω , where $\mu = 0.15$, $\varepsilon = \pi/2$; (b) excitation amplitude, where $\mu = 0.15$, $\varepsilon = \pi/2$; (c) friction coefficient, where $\omega = 100$ rad/s, $\varepsilon = \pi/2$.

Figure 8 shows the dependencies of the displacement angle on the system parameters. It was found that the displacement angle of the sliding part also depends on all these parameters. The excitation frequency has the least influence on the displacement angle (Figure 8a). As it increases, the angle increases slightly (the angle varies within a range of about $\pi/36$). The excitation amplitude of the platform has a greater influence on the displacement angle of the part. When the excitation amplitudes A_x and A_y are increased to 0.003 m, the displacement angle increases more rapidly (Figure 8b). The friction coefficient has the greatest influence on the displacement angle: when it increases, the sliding angle decreases (Figure 8c). The effect of friction is greater on the angle at low amplitudes.

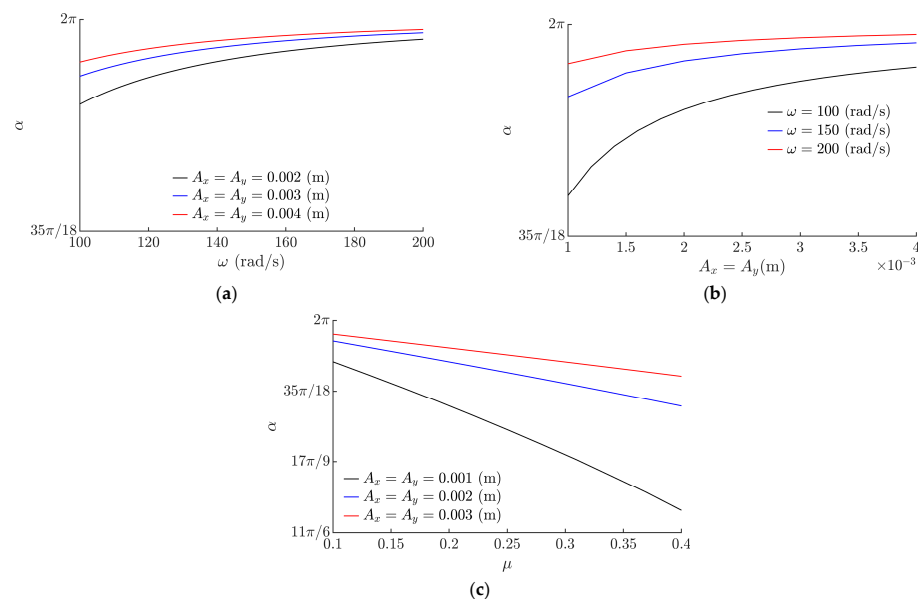


Figure 8. Displacement angle α depending on: (a) excitation frequency ω , where $\mu = 0.15$, $\varepsilon = \pi/2$; (b) excitation amplitude, where $\mu = 0.15$, $\varepsilon = \pi/2$; (c) friction coefficient μ , where $\omega = 100$ rad/s, $\varepsilon = \pi/2$.

3.2. Motion Characteristics of the Part When the Platform Is Excited in Two Perpendicular Directions by Sinusoidal Waves of Different Amplitudes, and the Phase Shift between the Perpendicular Components Is $\pi/2$

In this case, the following two options are possible:

1. The sinusoidal wave in the x -direction has a higher amplitude than the sinusoidal wave in the y -direction;
2. The sinusoidal wave in the y -direction has a higher amplitude than the sinusoidal wave in the x -direction.

In order to show the differences between these two options, the dependences of the average displacement velocity and the displacement angle on the system parameters are plotted. As in the case of circular excitation, an increase in the excitation frequency leads to an increase in the average displacement velocity (Figure 9a). When $A_x > A_y$, the part tends to move much faster than when $A_x < A_y$. This happens because the amplitude of A_x is oriented in the direction of the motion of the part. For the same reason, when the excitation amplitude of the platform in the x -direction increases, the average velocity of the part increases rapidly (Figure 9b). Meanwhile, the amplitude in the y -direction has no effect on the average sliding velocity (Figure 9c). The friction coefficient between the platform and the part has a low influence on the average displacement velocity. When the friction coefficient increases, the average displacement velocity decreases slightly (Figure 9c).

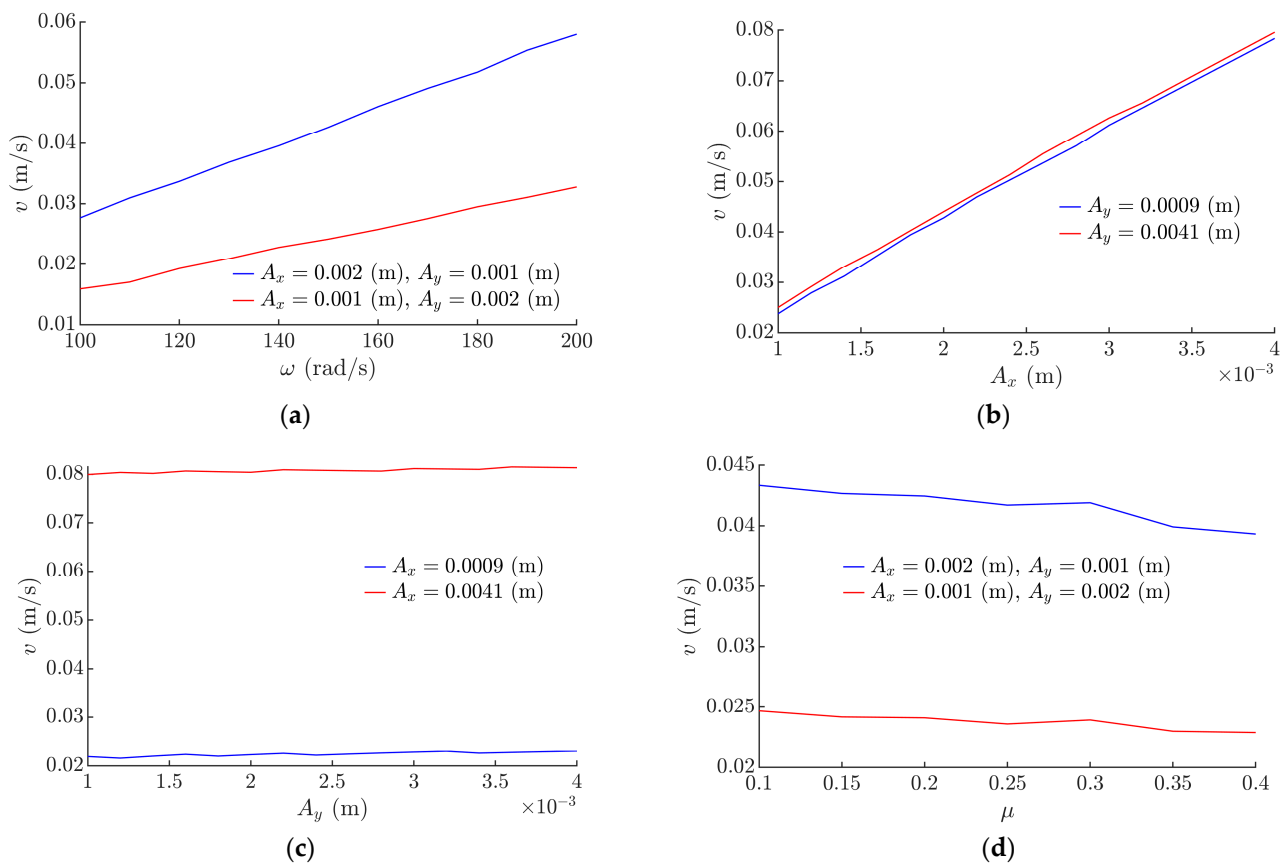


Figure 9. Average displacement velocity depending on: (a) excitation frequency ω , where $\mu = 0.15$, $\varepsilon = \pi/2$; (b) excitation amplitude A_x , where $\mu = 0.15$, $\omega = 150$ rad/s, $\varepsilon = \pi/2$; (c) excitation amplitude A_y , where $\mu = 0.15$, $\omega = 150$ rad/s, $\varepsilon = \pi/2$; (d) friction coefficient μ , when $\omega = 150$ rad/s, $\varepsilon = \pi/2$.

The displacement angle increases slightly as the excitation frequency increases (Figure 10a). When the excitation amplitude in the x -direction A_x increases, the average velocity increases (Figure 10b). The response of the displacement angle to the excitation amplitude in the y -axis A_y when $A_x > A_y$ is different from the response when $A_x < A_y$

(Figure 10c). When $A_x < A_y$, A_y does not significantly affect the displacement angle; meanwhile, when $A_x > A_y$, the displacement angle decreases slightly as A_y increases (Figure 10c). The friction coefficient has a greater influence on the displacement angle when $A_x < A_y$. As the friction coefficient increases, the displacement angle decreases linearly (Figure 10d).

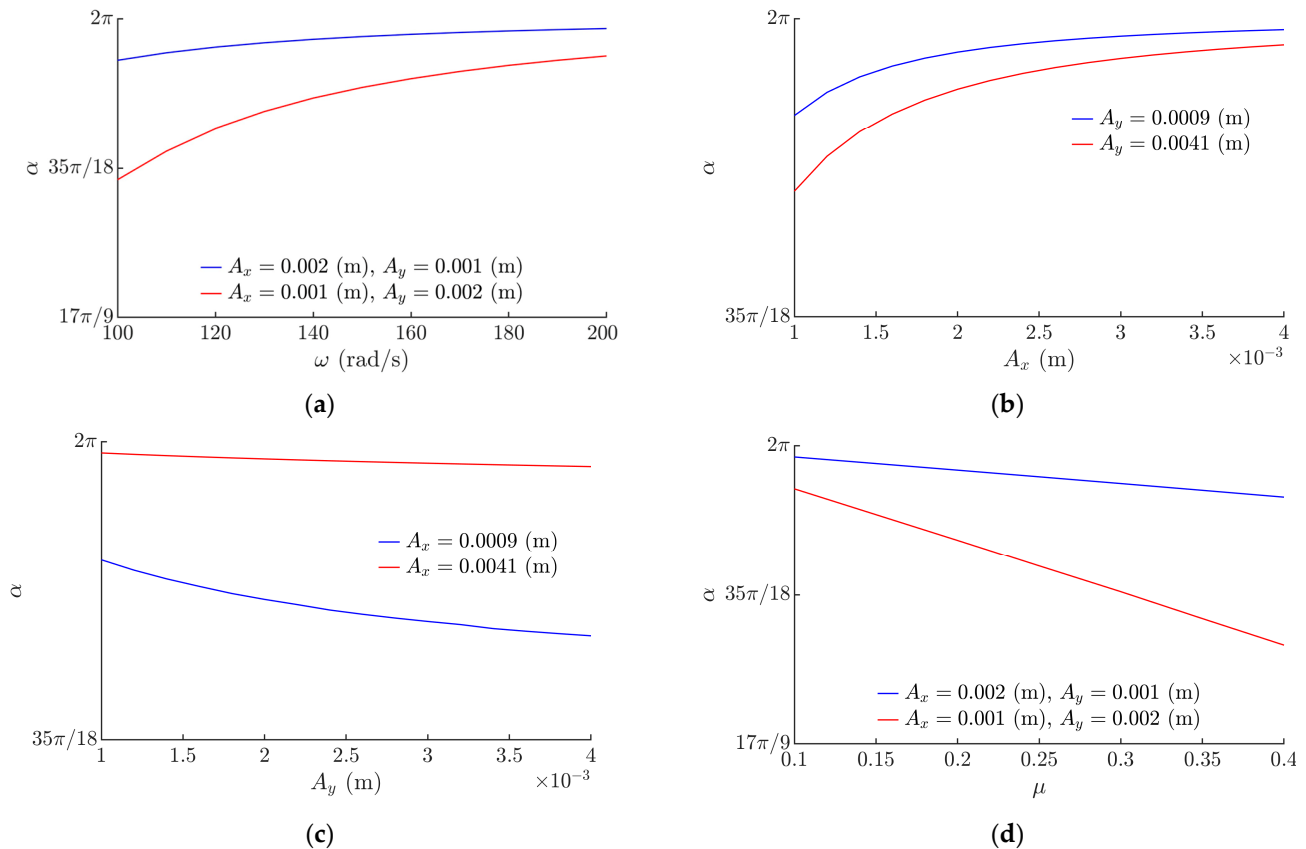


Figure 10. Displacement angle α depending on: (a) excitation frequency ω , where $\mu = 0.15$, $\varepsilon = \pi/2$; (b) excitation amplitude A_x , where $\mu = 0.15$, $\omega = 150$ rad/s, $\varepsilon = \pi/2$; (c) excitation amplitude A_y , where $\mu = 0.15$, $\omega = 150$ rad/s, $\varepsilon = \pi/2$; (d) dry friction coefficient μ , where $\omega = 150$ rad/s, $\varepsilon = \pi/2$.

3.3. Motion Characteristics of the Part When the Platform Is Excited in Two Perpendicular Directions by Sinusoidal Waves of the Same Amplitude, and the Phase Shift between the Perpendicular Components Varies in the Range of $\pi/10 \leq \varepsilon < 9\pi/10$

It is important to determine how the phase shift affects the average velocity and angle of the part's displacement on the platform when the platform is excited in two perpendicular directions by harmonic waves of the same amplitude with a phase shift. The average displacement velocity and the angle-related results are shown in Figures 11 and 12, respectively. It was found that the average displacement velocity decreases rapidly when the phase shift increases from $\pi/10$ to $\pi/2$, and the minimum displacement velocity is observed at $\pi/2$. A further increase in the phase shift results in an increase in the average displacement velocity (Figure 11). Higher values of the average displacement velocity of the part are obtained with higher values of both excitation frequency (Figure 11a) and amplitude (Figure 11b). The friction coefficient has little influence on the average displacement velocity (Figure 11c).

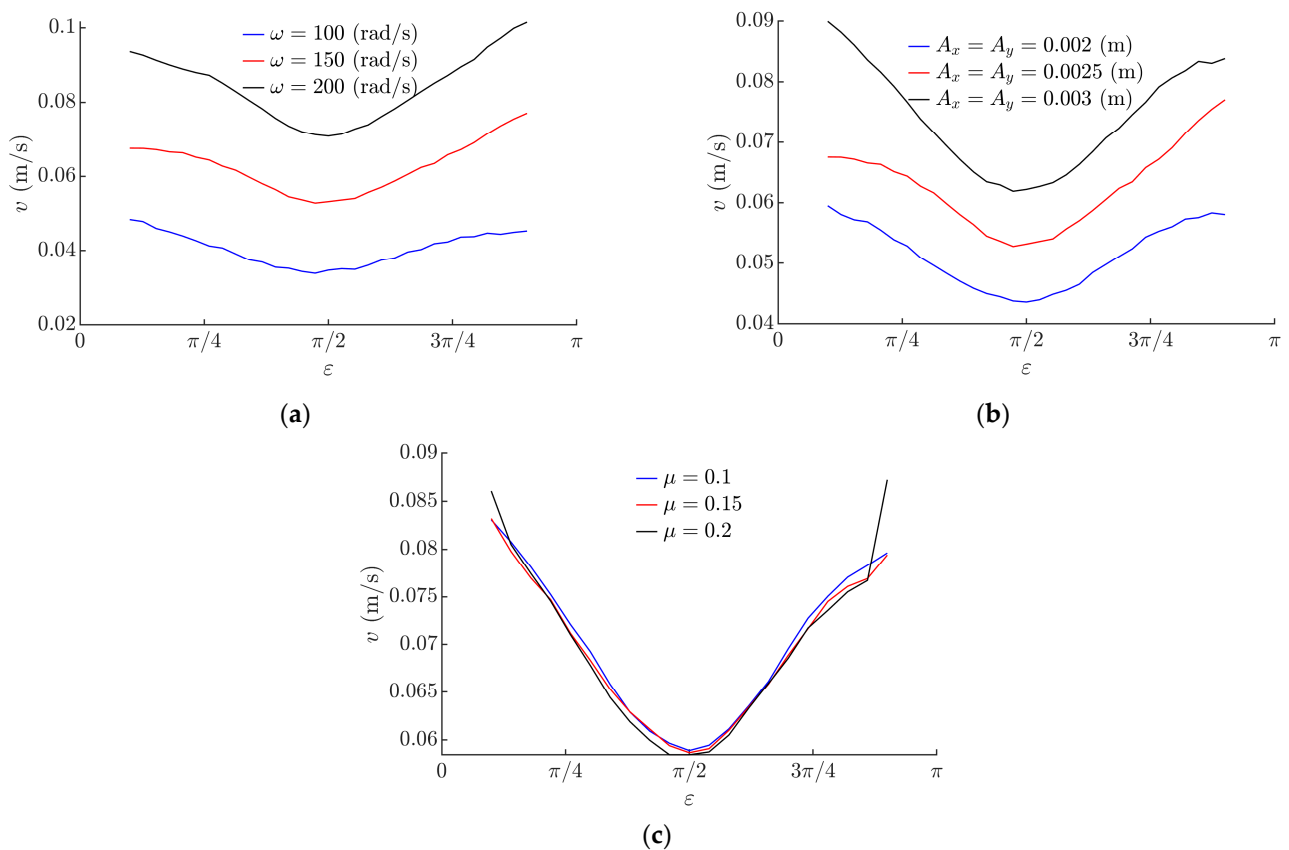


Figure 11. Average displacement velocity depending on phase shift: (a) at different excitation frequencies, where $\mu = 0.15$, $A_x = 0.0025$ m; (b) at different excitation amplitudes, where $\mu = 0.15$, $\omega = \text{rad/s}$; (c) at different values of the friction coefficient μ , where $\omega = 200$ rad/s, $A_x = 0.002$ m.

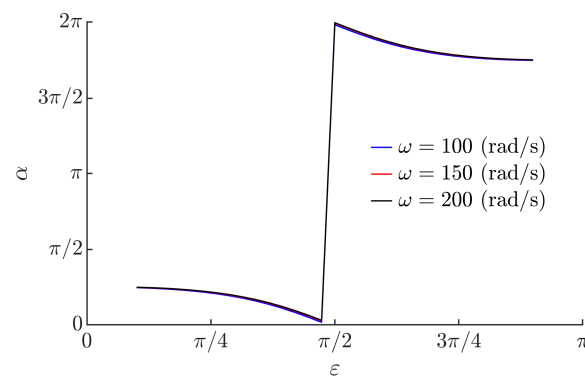


Figure 12. Displacement angle α depending on the phase shift, where $\mu = 0.15$, $A_x = 0.0025$ m.

The displacement angle of the part decreases from $\pi/4$ to 0 as the phase shift increases, and then there is an angle jump due to the crossing of the x -axis. Then, the displacement angle decreases to $3\pi/4$ as the phase shift increases further (Figure 12). All other system parameters have minimal influence on the displacement angle.

4. Conclusions

Dynamic and mathematical models of the motion of a part on a platform subjected to planar oscillations were developed to investigate the motion characteristics. The research showed that when the platform was excited in two perpendicular directions by sinusoidal waves, different trajectories of the part's motion can be obtained by controlling the excitation parameters, such as the frequencies and amplitudes of the waves and

the phase shift between the waves. When the platform is excited in two perpendicular directions by sinusoidal waves of the same amplitude, and the phase shift between the perpendicular components is $\pi/2$, the platform follows a circular trajectory, while at other values of phase shift, the platform follows an elliptical trajectory. The case in which the platform is excited in two perpendicular directions by sinusoidal waves with the same amplitude, and the phase shift between the perpendicular components varies in the range of $\pi/10 \leq \varepsilon < 9\pi/10$, is very well suited to applications related to non-prehensile manipulation tasks, as the phase shift employed as an additional control parameter makes it easier and faster to change the direction of motion of the part. The trajectory of the motion of the platform and the part on the platform becomes complex when the platform is excited in two perpendicular directions by sinusoidal waves of different frequencies. Various complex trajectories of the part can be achieved by controlling the ratio between the frequencies of the waves. Such complex trajectories, when the center of the part to be positioned performs loops, i.e., it moves very densely over a certain area, are perfectly suited for stages of automatic assembly such as the positioning and the search. It was found that choosing a higher frequency of the platform excitation yields a denser positioning trajectory.

In all of the analyzed cases, the frequency and amplitude of excitation have the greatest influence on the average displacement velocity of the part, with higher values of these parameters leading to higher velocities of the part. When the platform is excited in two perpendicular directions by sinusoidal waves of the same amplitude, the minimum displacement velocity is observed at a phase shift value of $\pi/2$. In the case where the platform is excited in two perpendicular directions by sinusoidal waves of different amplitudes, the values of the average displacement velocity are the highest when the amplitude is higher in the direction of the motion of the part.

The results demonstrated that parts on the platform can be moved in any direction at a given velocity and can be subjected to dense complex positioning trajectories by controlling excitation parameters such as the frequencies and amplitudes of the excitation waves and the phase shift between the waves. Therefore, such a platform can be applied in feeding, positioning, and manipulation tasks.

In future work, we plan to develop an experimental setup and carry out experimental tests of the functionality of the proposed methods.

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