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Disturbance Observer-Based Robust Take-Off Control for a Semi-Submersible Permeable Slender Hybrid Unmanned Aerial Underwater Quadrotor

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Abstract: The development of hybrid unmanned aerial underwater vehicles (HAUVs) compatible with the advantages of the aerial vehicles and the underwater vehicles is of great significance. This paper presents the first study on a new HAUV layout using four rotors to realize the medium crossing motion of a transverse slender body similar to the fuselage of a missile or a submarine, that is, the hybrid aerial underwater quadrotor (HAUQ). Then, a robust control strategy is proposed for the take-off HAUQ on the water in the presence of unknown disturbances and complex model dynamic uncertainties. As a semi-submersible HAUQ rises straight from the water, the inside of the slender fuselage placed horizontally is filled with water. The center of the mass, the moment of inertia, and the arm of the force of the HAUQ will change rapidly in the take-off phase from the water because of the rapid nonuniform change in mass caused by the passive fast drainage. It is difficult to establish an accurate mathematical model of the complex dynamic changes caused by the multi-media dynamics, the fast changing buoyancy, and the added mass crossing the air–water surface. Therefore, an uncertain kinematic and dynamic model is established through the passive, fast, nonuniform change and the complex dynamics are considered as the unknown terms, and the external disturbances of gust and other factors are assumed as the bounded disturbance input. A robust design approach is introduced to deal with the fast time-varying mass disturbance based on the input-to-state stability (ISS) theorem. The complex dynamics are estimated using the basis function and the unknown weight parameters, and the adaptive laws are adopted for the on-line estimation of the unknown weight parameters. Considering the residual disturbance of the uncertain nonlinear system as a total disturbance term, a disturbance observer is introduced for disturbance observation. The numerical simulation shows the feasibility and robustness of the proposed algorithm.

Keywords: hybrid unmanned aerial underwater quadrotor; robust control; disturbance observer; adaptive laws

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1. Introduction

A great amount of significant research has been made in the last decades with the development of advanced robotic systems, especially about autonomous underwater vehicles (AUVs) and unmanned aerial vehicles (UAVs). As AUVs and UAVs are good at completing their tasks in their respective fields, they can efficiently achieve marine and aerial observations and attack missions, respectively. However, when faced with a multi-domain task, it seems impossible to accomplish the mission whether AUVs or UAVs. The heterogeneous multi-robot systems have been used to accomplish multi-domain environment monitoring [1–3]. And the multi-robot cooperation enriches the observation and improvement in work efficiency. Nevertheless, some new challenges are brought by using such a multi-robot system for users. The difficulties of establishing and maintaining may create multi-robot system operational difficulties. What is more, it is extremely difficult for the heterogeneous robots working in the different types of medium to communicate with each other because of the attenuation effect of water for the electromagnetic waves. Therefore, a single platform, that is, the hybrid aerial underwater vehicle (HAUV) which can be capable of moving in both air and water, is needed to be developed to complete multi-domain missions.

Because of the huge differences in the physical properties between air and water, it is a great challenge to design an HAUV capable of both air flight and underwater navigation, especially the efficient control technique. According to the existing literature, many vehicles of various layouts such as fixed-wing [4–6], variable-swept wing [7], flapping wing [8], and multi-rotor systems [9–12] have been adopted for aerial and underwater missions, which have shown the high practical value and fruitful results. And the controllability, payload, and practicality of HAUVs has been assessed [13].

However, the existing layouts of HAUVs are not compatible with the advantages of the aerial vehicles and the underwater vehicles. A transverse permeable slender body similar to the fuselage of a missile or a submarine is a suitable scheme for the fuselage of HAUVs. Then, a new HAUV layout is proposed which uses four rotors to realize the medium crossing motion of a transverse slender body similar to the fuselage of a missile or submarine, that is, the hybrid unmanned aerial underwater quadrotor (HAUQ) as shown in Figure 1. The HAUQ layout can help to use the morphing wing technology and the fixed-wing hybrid quadrotor technology to realize the underwater streamlined fuselage navigation and the fixed-wing flight. The key of the whole flight process is the trans-media motion control from water to air.

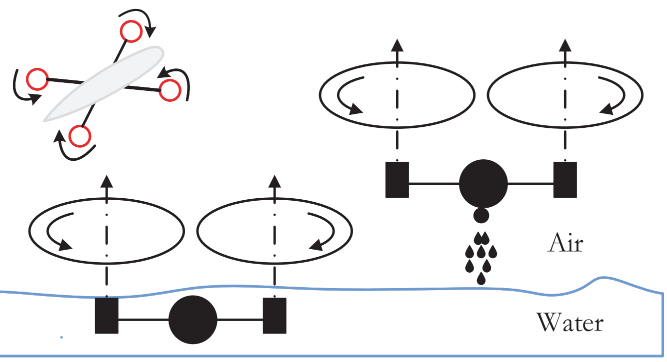


Figure 1. Take-off for a semi-submersible hybrid aerial underwater quadrotor on the water.

The controller design of the hybrid aerial underwater quadrotor (HAUQ) is a typical gas–liquid coupling problem. The water–air interface crossing motion has strong nonlinearities and uncertainties, such as the multivariable strong couplings, the nonlinear hydrodynamic characteristics of the water–air two-phase flow, the impact of water waves, the gust, and the passive drainage disturbance. For the HAUV, it includes the water–air transition section and the air–water transition section. To realize the stable and reliable conversion between the discontinuous media, the control scheme of the HAUV is one of the key issues of the cross-medium motion. The control models and the control arithmetics of the existing vehicles including the surface vessel and the underwater vehicle have some limitations and strong disturbances such as winds, waves, and currents which will challenge the control issues. For the marine environment interference, most of the research objects are underwater vehicles. The influence of the ocean current is studied on the motion control of the low-speed AUV [14]. A robust navigation algorithm is developed for the recovery of AUVs [15]. For the operability of an AUV, the least square method is introduced to estimate the ocean current [16]. However, the research on the modeling and control algorithm design of the trans-media motion in a complex environment is extremely rare.

For the control design problem of the water-to-air transition motion process of HAUQs, a water–air crossing motion control design scheme is proposed based on the lift provided by four rotors. The basic idea of the control algorithm of the HAUQ is to use four rotors in the air to pull up the streamlined body which is submerged in the water and drain the water inside the body in a short time. The traditional four-rotor UAVs is the most common and representative UAV in a multi-rotor system, and it is a multiple-input and multiple-output nonlinear system, including the nonlinearities, the multiple variables, the underactuated characteristics, the weak anti-jamming ability, and the complex couplings [17,18]. The HAUQ inherits these characteristics.

The HAUQ is facing a flight environment different from the traditional quadrotor UAV. When the permeable slender body of the HAUQ crosses from water to air, the medium changes, and there are many complex changes whose mechanism is not clear enough, such as the additional mass caused by the fast drainage, the multi-media dynamics, and the fast-changing buoyancy. And the body is filled with water and needs to be discharged in a very short time such that the mass of the whole HAUQ changes dramatically when it takes off from the water to the air. The nonuniform drainage will also cause the change in the center of mass and the arm of force. The main contributions of the manuscript can be summarized as follows:

1. A new water–air trans-medium pattern is proposed for the HAUQ with a permeable slender body. Compared with the existing layouts of the HAUVs, the HAUQs with a permeable slender body can help to keep the streamlined fuselage needed for underwater and air navigation.
2. A general mathematical model is established for HAUQs by employing the Newton–Euler formulation as the factors exist, including the strong uncertainties caused by the fluid dynamics in the complex water–air mixed environment, the fast time-varying added mass caused by the fluid dynamics and the residual water inside the slender body, the influence of the passive drainage, and the external disturbance.
3. A disturbance observer-based robust control scheme is proposed for HAUQs. The robust control is adopted to compensate for the fast time-varying mass uncertainty. For the uncertainties of the multi-media complex dynamics modeling on the position and attitude dynamic equations, it is estimated by considering it as a combination of the specific basis functions, and an adaptive method is used to estimate the unknown weight parameters. The rapid and uncontrollable drainage will cause the mass and the center of mass to change during take-off on the water surface. Meanwhile, the length of the arm of force and the moment of inertia matrix will change unpredictably, and they are considered as the bounded uncertainty of the moment of inertia matrix and the force arm variation. Then, a comprehensive dynamic disturbance term is formed together with the bounded additional disturbances, and a disturbance observer structure in [19] is introduced to estimate it under the assumption that the total disturbance is measurable. The idea of using a disturbance observer to estimate the system disturbances is to introduce feedforward compensation in the process of controller design to improve the control performance of the system, and it is widely used in aircraft control [20–29]. The input-to-state stability theorem is an effective method to study nonlinear systems with noises and disturbances [30]. This method can obtain the bounded states by suppressing the bounded disturbances; therefore, the stability of the position and attitude control of the HAUQ is analyzed by the input–state stability theorem. Finally, a nonlinear robust control algorithm is proposed consisting of three parts: a nonlinear robust take-off controller for HAUQs, an adaptive control law, and a disturbance observer. The simulation results show the effectiveness of the proposed algorithm.

The organization of this paper is as follows. Section 2 presents the kinematic equations with the uncertainties of the hybrid aerial underwater quadrotor in different coordinate systems. The design of the position controller and attitude controller as well as their stability analysis is discussed in Section 3. Finally, the controllers designed in this paper are tested via simulation in Section 4.

2. Mathematical Model

2.1. Dynamic Model of Water Surface Take-Off in Body Coordinate System

The take-off of an HAUQ on the water surface mainly refers to the process that as the HAUQ body is totally or partially submerged in the water medium, the pulling force generated by the rotation of the quadrotor pulls it out of the water and drains the residual water in the body. In order to establish the dynamic equation of an HAUQ, the inertial coordinates $\{\mathcal{F}^I\} = \{O, X, Y, Z\}$ of water–air integration are established. In this coordinate system, a point on the water surface where the flight path is located is the reference system origin. Any direction of the water surface is taken as the positive OX direction. The OZ-axis is perpendicular to the water surface and upward. The water depth is negative, and the height is positive. The OY is perpendicular to the OXZ plane and determined by the right-hand rule. Define the body coordinate system as $\{\mathcal{F}^b\} = \{O^b, X^b, Y^b, Z^b\}$. $R_v \in SO(3)$ is the velocity transform matrix from coordinate system $\mathcal{F}^b$ to coordinate system $\mathcal{F}^I$. Define the HAUQ position as $P^I = [x, y, z]^T$ which represents the position of the trans-media UAV in the inertial coordinates $\{\mathcal{F}^I\} = \{O, X, Y, Z\}$. The flight attitude is $\Theta = [\phi, \theta, \psi]^T$ and represents the roll angle, the pitch angle, and the yaw angle, respectively. $V^b \in \mathbb{R}^3$ is the velocity component in the body coordinate system. ω^b is the angular velocity in the body coordinate system. Let the mass of the HAUQ without water be M, and the mass of the HAUQ is $M + M_\Delta(t)$ during the take-off process on the water surface. $M_\Delta(t) \to 0$ caused by the fast drainage holds and changes rapidly. It is worth noting that the $M_\Delta(t)$ does not include the added mass caused by underwater navigation, which is considered as the uncertainty. The rapid mass change also brings about the change in the center of mass position and the moment of inertia. The moment of inertia matrix is defined as $J + J_\Delta(t)$.

We have

$$\dot{P}^I = R_v V^b, \ \dot{R}_v = R_v S(\omega^b) \tag{1}$$

where

$$S(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}, \forall x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

From Newton's laws of motion, we have Equation (3),

$$\begin{aligned}
(M + M_\Delta(t))\dot{V}^b &= -(M + M_\Delta(t))S(\omega^b)V^b + A_v(V^b) + \begin{bmatrix} 0 \\ 0 \\ -(F_1 + F_2 + F_3 + F_4) \end{bmatrix} + R_v^T \begin{bmatrix} 0 \\ 0 \\ (M + M_\Delta(t))g \end{bmatrix} + \Delta_1 \\
(J + J_\Delta(t))\dot{\omega}^b &= \begin{bmatrix} -(l + \Delta l_1(t))F_1 + (l + \Delta l_2(t))F_2 - (l + \Delta l_3(t))F_3 + (l + \Delta l_4(t))F_4 \\ (l + \Delta l_1(t))F_1 + (l + \Delta l_2(t))F_2 - (l + \Delta l_3(t))F_3 - (l + \Delta l_4(t))F_4 \\ -C(F_1 - F_2 + F_3 - F_4) \end{bmatrix} \\
&\quad - S(\omega^b)(J + J_\Delta(t))\omega^b + B_\omega(\omega^b) + \Delta_2
\end{aligned} \tag{2}$$

where $\Delta M(t)$ represents the change in the extra mass during the drainage process of cross-medium flight, $\Delta l_i(t), i = 1, ..., 4$ is the change in the arm of force caused by the change in the mass center, $\Delta J(t)$ is the change moment of inertia with time t, and $A_v(V^b)$ and $B_\omega(\omega^b)$ are the unknown terms including forces and moments caused by the complex multi-media dynamics and the fast-changing buoyancy. $\Delta M(t) \to 0$, $\Delta l_i(t) \to 0, i = 1, ..., 4$, and $\Delta J(t) \to 0$ during the fast drainage. $\Delta_i,\ i = 1,2$ is the external disturbance. Define $F^b = [F_1, F_2, F_3, F_4]^T$ and

$$N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 \end{bmatrix},\ G = \begin{bmatrix} 0 \\ 0 \\ M \end{bmatrix},\ G_\Delta(t) = \begin{bmatrix} 0 \\ 0 \\ M_\Delta(t) \end{bmatrix},\ L = \begin{bmatrix} -l & l & -l & l \\ l & l & -l & -l \\ -C & C & -C & C \end{bmatrix},$$
$$L_\Delta(t) = \begin{bmatrix} -\Delta l_1(t) & \Delta l_2(t) & -\Delta l_3(t) & \Delta l_4(t) \\ \Delta l_1(t) & \Delta l_2(t) & -\Delta l_3(t) & -\Delta l_4(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then, Equation (3) can be given by

$$\begin{aligned} (M + M_\Delta(t))\dot{V}^b =& -(M + M_\Delta(t))S(\omega^b)V^b + A_v(V^b) + NF^b + R_v^T(G + G_\Delta(t))g + \Delta_1 \\ (J + J_\Delta(t))\dot{\omega}^b =& -S(\omega^b)(J + J_\Delta(t))\omega^b + B_\omega(\omega^b) + (L + L_\Delta(t))F^b + \Delta_2 \end{aligned} \tag{3}$$

where l is the distance between the propeller axis and the mass center of the HAUQ when it is not in water, and $C \in \mathbb{R}$ is a constant number determined by the characteristics of the rotor motor.

2.2. Dynamic Model of Water Surface Take-Off in Inertial Coordinate System

In this section, the dynamic model established in the body coordinate system $\mathcal{F}^b$ is transformed into the water–air integrated inertial coordinate system $\mathcal{F}^I$ for the convergence of controller design. Let R_Θ be a transformation matrix from $\mathcal{F}^b$ to $\mathcal{F}^I$, and the dynamic equation of the HAUQ in the coordinate system can be written as

$$\begin{bmatrix} V^b \\ \omega^b \end{bmatrix} = \begin{bmatrix} R_v^T & 0_{3\times3} \\ 0_{3\times3} & R_\Theta^{-1} \end{bmatrix} \begin{bmatrix} \dot{P} \\ \dot{\Theta} \end{bmatrix} \tag{4}$$

So, there is

$$\begin{aligned} (M + M_\Delta(t))\dot{V}^b =& (M + M_\Delta(t))\left(\dot{R}_v^T\dot{P} + R_v^T\ddot{P}\right) \\ =& (M + M_\Delta(t))\left(R_v^T\ddot{P} - S(\omega^b)V^b\right) \end{aligned} \tag{5}$$

Then,

$$\begin{aligned} &(M + M_\Delta(t))R_v^T\ddot{P} = (M + M_\Delta(t))\left(\dot{V}^b + S(\omega^b)V^b\right) \\ &(M + M_\Delta(t))\left(R_v^T\right)^{-1}R_v^T\ddot{P} = (M + M_\Delta(t))\left(R_v^T\right)^{-1}\dot{V}^b + (M + M_\Delta(t))\left(R_v^T\right)^{-1}S(\omega^b)V^b \end{aligned} \tag{6}$$

According to the property $R_v^T = R_v^{-1}$ of the orthogonal matrix, we have

$$\begin{aligned} (M + M_\Delta(t))\ddot{P} &= (M + M_\Delta(t))\left(R_v\dot{V}^b + R_vS(\omega^b)V^b\right) \\ &= R_v\big(-(M + M_\Delta(t))S(\omega^b)V^b + NF^b + A_v(V^b) + R_v^T(G + G_\Delta(t))g + \Delta_1\big) + (M + M_\Delta(t))R_vS(\omega^b)V^b \\ &= R_vA_v(V^b) + (G + G_\Delta(t))g + R_vNF^b + R_v\Delta_1 \end{aligned} \tag{7}$$

In addition, $\dot{\omega}^b$ is given by

$$\dot{\omega}^b = \dot{R}_\Theta^{-1}\dot{\Theta} + R_\Theta^{-1}\ddot{\Theta} \tag{8}$$

Thus,

$$
\begin{aligned}
(J + J_{\Delta}(t))R_{\Theta}^{-1}\ddot{\Theta} =& (J + J_{\Delta}(t))\dot{\omega}^{b} - (J + J_{\Delta}(t))\dot{R}_{\Theta}^{-1}\dot{\Theta} \\
=& - S(\omega^{b})(J + J_{\Delta}(t))\omega^{b} + B_{\omega}(\omega^{b}) - (J + J_{\Delta}(t))\dot{R}_{\Theta}^{-1}\dot{\Theta} + (L + L_{\Delta}(t))F^{b} + \Delta_2
\end{aligned}
\tag{9}
$$

In summary, the dynamic equation of the HAUQ in the water–air integrated inertial coordinate system is

$$
\begin{cases}
(M + M_{\Delta}(t))\ddot{P} = R_v A_v(V^b) + R_v N F^b + (G + G_{\Delta}(t))g + R_v\Delta_1 \\
(J + J_{\Delta}(t))R_{\Theta}^{-1}\ddot{\Theta} = -S(\omega^b)(J + J_{\Delta}(t))\omega^b - (J + J_{\Delta}(t))\dot{R}_{\Theta}^{-1}\dot{\Theta} + (L + L_{\Delta}(t))F^b + B_{\omega}(\omega^b) + \Delta_2
\end{cases}
\tag{10}
$$

It can seen from Equation (10) that the HAUQ also has four independent inputs $F_i,\ i = 1, \ldots, 4$ compared with the traditional quadrotor, but it has six degrees of freedom, $x(t)$, $y(t)$, $z(t)$, $\phi(t)$, $\theta(t)$, and $\psi(t)$. There exists complex coupling relations between states. The sudden change in the medium causes the complex forces and the additional mass which change quickly in a short time.

3. Nonlinear Robust Control Laws

3.1. Control Model of Water Surface Take-Off

It is necessary to simplify the dynamic model before designing the control law. Firstly, let the moment of inertia matrix be

$$
J = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}, J_{\Delta}(t) = \begin{bmatrix} \Delta I_{xx} & \Delta I_{xy} & \Delta I_{xz} \\ \Delta I_{yx} & \Delta I_{yy} & \Delta I_{yz} \\ \Delta I_{zx} & \Delta I_{zy} & \Delta I_{zz} \end{bmatrix}
\tag{11}
$$

Consider the small angle of the HAUQ, and the dynamics can be rewritten as

$$
\begin{cases}
(M + M_{\Delta}(t))\ddot{P} = R_v A_v(\dot{P}) + R_v N F^b + (G + G_{\Delta}(t))g + R_v\Delta_1 \\
(J + J_{\Delta}(t))R_{\Theta}^{-1}\ddot{\Theta} = -S(\dot{\Theta})(J + J_{\Delta}(t))\dot{\Theta} + (L + L_{\Delta}(t))F^b + B_{\omega}(\omega^b) + \Delta_2
\end{cases}
\tag{12}
$$

where $R_v A_v(\dot{P})$ and $B_{\omega}(\omega^b)$ are unknown terms. $M_{\Delta}(t)$, $J_{\Delta}(t)$, and $L_{\Delta}(t)$ are time-varying variables. $R_v\Delta_1$ and Δ_2 are bounded disturbances. Note that the small-angle assumption is reasonable, because the HAUQ needs to move horizontally with a slight incline to ensure its stability such that the water inside the body can not shake violently and the high-speed rotating propellers are not damaged by water. The HAUQ control system is divided into the position control subsystem and the attitude control subsystem, that is,

$$
\Pi_1:\ \ddot{P} = \frac{1}{M + M_{\Delta}(t)} A_P(t, \dot{P}) + \frac{1}{M + M_{\Delta}(t)} R_v N F^b + \mathcal{G}g + \Delta_P
\tag{13}
$$

$$
\Pi_2:\ (J + J_{\Delta})R_{\Theta}^{-1}\ddot{\Theta} = - S(\dot{\Theta})(J + J_{\Delta}(t))\dot{\Theta} + B_{\omega}(\omega^b) + (L + L_{\Delta}(t))F^b + \Delta_2
\tag{14}
$$

where

$$
A_P(t, \dot{P}) = R_v A_v(\dot{P}),\ \mathcal{G} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, \Delta_P = \frac{1}{M + M_{\Delta}(t)} R_v \Delta_1
$$

As we use the small-angle assumption, the take-off process actually involves some actual physical constraints. Therefore, the following assumption is considered as well.

Assumption 1. *The water's surface keeps flat as the HAUQ flies from water to air.*

Assumption 1 actually means that the modeling and impact of waves is not considered, and some tiny waves can be considered as the disturbance under the condition of tolerant performance of the proposed controller, and it is also an acceptable assumption about the flight environment.

3.2. Robust Adaptive Position Controller Water–Air Crossing

The trans-media flight adopts the strategy of slowly climbing to a certain height to drain water. Make P_d the desired position point, and then $\dot{P}_d = 0$. Define $\eta_1 = p - p_d$, $\eta_2 = \dot{p}$. Subsystem Π_1 is written as

$$\begin{cases} \dot{\eta}_1 = \eta_2 + \dot{p}_d \\ \dot{\eta}_2 = \dfrac{1}{M + M_\Delta(t)} A_P(t, \eta) + \dfrac{1}{M + M_\Delta(t)} R_v F^P + \mathcal{G}g + \Delta_P \end{cases} \tag{15}$$

where $\eta = [\eta_1, \eta_2]^T$ and $F^P = NF^b$ holds. Assume that the independent element $A_P^i(t, \dot{P})$ in $A_P(t, \dot{P})$ can be written as a combination of N basis functions $\varphi_i(\eta)$ as follows,

$$A_P(t, \eta) = \Omega^T \Phi(\eta) + o(\eta) \tag{16}$$

where Ω is the unknown constant parameter vector and $o(\eta)$ is the high-order component, and

$$\Phi(\eta) = \big(\varphi_1(\eta), \varphi_2(\eta), ..., \varphi_N(\eta)\big)^T \in \mathbb{R}^N$$

is a known regression vector. Define $e_1 = \eta_1$ and $e_2 = \eta_2 - \eta_2^*$, and subsystem Π_1 is given by

$$\begin{cases} \dot{e}_1 = e_2 + \eta_2^* \\ \dot{e}_2 = \dfrac{1}{M + M_\Delta(t)} \Omega^T \Phi(\eta) + \dfrac{1}{M + M_\Delta(t)} R_v F^P + \mathcal{G}g + \Delta_P + \dfrac{1}{M + M_\Delta(t)} o(\eta) - \dot{\eta}_2^* \end{cases} \tag{17}$$

The additional mass $M_\Delta(t)$ is time-varying but satisfies $0 \le M_\Delta(t) \le M_\Delta^{max}$, where M_Δ^{max} is the maximum value of the additional mass. The robust adaptive control law is given by

$$\begin{aligned} F^p &= (M + M_\Delta^{max}) R_v^{-1} \Big(-K_2 e_2 - \left(\frac{1}{2\varepsilon_1^2} + \frac{1}{2\varepsilon_2^2} \right) e_2 - e_1 - \frac{1}{M + M_\Delta^{max}} \widehat{\Omega}^T \Phi(\eta) - \mathcal{G}g + \dot{\eta}_2^* \Big) \\ \eta_2^* &= -K_1 e_1 \\ \dot{\eta}_2^* &= -K_1 (e_2 + \eta_2^* + \dot{p}_{zd}) \\ \dot{\widehat{\Omega}} &= \Phi(\eta) e_2^T - K_3 \widehat{\Omega} \end{aligned} \tag{18}$$

where ε_1, ε_2, K_1, K_2, and K_3 are the positive constants.

Theorem 1. *For the closed-loop system with (17) and the robust adaptive control law (18), choose the appropriate parameters $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ $K_2 > 0$, $K_3 > 0$, and*

$$1 \le K_1 \le \frac{M_\Delta^{max}}{4M} + \sqrt{1 + \left(\frac{M_\Delta^{max}}{4M} \right)^2}$$

Consider the controller and the adaptive law (18), and the closed-loop system of system (17) and error $\Delta\Omega = \Omega - \widehat{\Omega}$ are input-to-state stable (ISS).

Proof of Theorem 1. Consider the following Lyapunov function,

$$V(e_1, e_2, \Delta\Omega) = \frac{1}{2}e_1^T e_1 + \frac{1}{2}e_2^T e_2 + \frac{1}{2(M + M_\Delta(t))} tr\left(\Delta\Omega^T \Delta\Omega\right) \tag{19}$$

where $\Delta\Omega = \Omega - \widehat{\Omega}$ is the parameter estimation error. Then, the derivative of V is

$$\begin{aligned}
\dot{V} =& e_1^T \dot{e}_1 + e_2^T \dot{e}_2 + \frac{1}{M + M_\Delta(t)} tr\left(\Delta\Omega^T \Delta\dot{\Omega}\right) \\
=& e_1^T (e_2 + \eta_2^*) + e_2^T \left(\frac{1}{M + M_\Delta(t)} \Omega^T \Phi(\eta) + \frac{1}{M + M_\Delta(t)} R_v F^P + \mathcal{G}g + \Delta_P + \frac{1}{M + M_\Delta(t)} o(\eta) - \dot{\eta}_2^* \right) \\
&+ \frac{1}{M + M_\Delta(t)} tr\left(\Delta\Omega^T \Delta\dot{\Omega}\right)
\end{aligned} \tag{20}$$

Substituting the virtual control law η_2^* into Equation (20), we have

$$\begin{aligned}
\dot{V} \leq& -K_1 e_1^T e_1 + e_2^T \left(e_1 + \frac{1}{M + M_\Delta(t)} \Omega^T \Phi(\eta) + \frac{1}{M + M_\Delta(t)} R_v F^P + \mathcal{G}g - \dot{\eta}_2^* \right. \\
&\left. + \Delta_P + \frac{1}{M + M_\Delta(t)} o(\eta) \right) + \frac{1}{M + M_\Delta(t)} tr\left(\Delta\Omega^T \Delta\dot{\Omega}\right) \\
\leq& -K_1 e_1^T e_1 + e_2^T \left(e_1 + \mathcal{G}g - \dot{\eta}_2^* + \frac{1}{M + M_\Delta(t)} \Omega^T \Phi(\eta) + \frac{1}{M + M_\Delta(t)} R_v F^P \right) \\
&+ e_2^T \left(\Delta_P + \frac{1}{M + M_\Delta(t)} o(\eta) \right) + \frac{1}{M + M_\Delta(t)} tr\left(\Delta\Omega^T \Delta\dot{\Omega}\right)
\end{aligned} \tag{21}$$

According inequalities are

$$e_2^T \left(\Delta_P + \frac{1}{M + M_\Delta} o(\eta) \right) \leq \frac{1}{2\varepsilon_1^2} e_2^T e_2 + \frac{\varepsilon_1^2}{2} \left(\Delta_P + \frac{1}{M + M_\Delta} o(\eta) \right)^T \left(\Delta_P + \frac{1}{M + M_\Delta} o(\eta) \right) \tag{22}$$

We have

$$\begin{aligned}
\dot{V} \leq& -K_1 e_1^T e_1 + e_2^T \left(e_1 + \frac{1}{2\varepsilon_1^2} e_2 + \frac{1}{M + M_\Delta(t)} \Omega^T \Phi(\eta) + \frac{1}{M + M_\Delta(t)} R_v F^P + \mathcal{G}g - \dot{\eta}_2^* \right) + \frac{1}{M + M_\Delta(t)} tr\left(\Delta\Omega^T \Delta\dot{\Omega}\right) \\
&+ \frac{\varepsilon_1^2}{2} \left(\Delta_P + \frac{1}{M + M_\Delta(t)} o(\eta) \right)^T \left(\Delta_P + \frac{1}{M + M_\Delta(t)} o(\eta) \right) \\
\leq& -K_1 e_1^T e_1 + e_2^T \left(e_1 + \frac{1}{2\varepsilon_1^2} e_2 + \frac{1}{M + M_\Delta(t)} \Omega^T \Phi(\eta) + \frac{1}{M + M_\Delta(t)} R_v F^P + \mathcal{G}g - \dot{\eta}_2^* \right) + \frac{1}{M + M_\Delta(t)} tr\left(\Delta\Omega^T \Delta\dot{\Omega}\right) \\
&+ \frac{\varepsilon_1^2}{2} \left(\Delta_P + \frac{1}{M + M_\Delta(t)} o(\eta) \right)^T \left(\Delta_P + \frac{1}{M + M_\Delta(t)} o(\eta) \right)
\end{aligned} \tag{23}$$

Substituting F^P into (23), Equation (23) is written as

$$
\begin{aligned}
\dot{V} \leq & -K_1 e_1^T e_1 + e_2^T \bigg(e_1 + \frac{1}{M+M_\Delta(t)} \Omega^T \Phi(\eta) + \frac{1}{2\varepsilon_1^2} e_2 + \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} R_v R_v^{-1} \bigg(-\Big(K_2 \\
& + \frac{1}{2\varepsilon_1^2} + \frac{1}{2\varepsilon_2^2} \Big) e_2 - e_1 - \mathcal{G}g - \frac{1}{M+M_\Delta^{max}} \widehat{\Omega}\Phi(\eta) - K_1(e_2+\eta_2^*) \bigg) + \mathcal{G}g + K_1(e_2+\eta_2^*) \bigg) \\
& + \frac{1}{M+M_\Delta(t)} tr\Big(\Delta\Omega^T \Delta\dot{\Omega}\Big) + \frac{\varepsilon_1^2}{2} \left(\Delta_P + \frac{1}{M+M_\Delta(t)} o(\eta) \right)^T \left(\Delta_P + \frac{1}{M+M_\Delta(t)} o(\eta) \right) \\
\leq & -K_1 e_1^T e_1 + \frac{1}{M+M_\Delta(t)} tr\Big(\Delta\Omega^T \Delta\dot{\Omega}\Big) + e_2^T \bigg(\Big(1-K_1^2\Big) \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) e_1 \\
& + \frac{1}{M+M_\Delta(t)} \Omega^T \Phi(\eta) - \frac{1}{M+M_\Delta(t)} \widehat{\Omega}^T \Phi(\eta) + \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) \mathcal{G}g \\
& + \left(K_1 - \frac{(K_1+K_2)(M+M_\Delta^{max})}{M+M_\Delta(t)} \right) e_2 + \frac{1}{2\varepsilon_1^2} \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) e_2 \\
& - \frac{M+M_\Delta^{max}}{2\varepsilon_2^2 (M+M_\Delta(t))} e_2 \bigg) + \frac{\varepsilon_1^2}{2} \left(\Delta_P + \frac{1}{M+M_\Delta(t)} o(\eta) \right)^T \times \left(\Delta_P + \frac{1}{M+M_\Delta(t)} o(\eta) \right) \\
\leq & -K_1 e_1^T e_1 + \Big(1-K_1^2\Big) \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) \left(-\frac{1}{2}(e_2-e_1)^t (e_2-e_1) + \frac{1}{2} e_2^T e_2 + \frac{1}{2} e_1^T e_1 \right) \\
& + e_2^T \bigg(\frac{1}{M+M_\Delta(t)} \Omega^T \Phi(\eta) - \frac{1}{M+M_\Delta(t)} \widehat{\Omega}^T \Phi(\eta) + \Delta_G + \frac{1}{2\varepsilon_1^2} \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) e_2 \\
& + \left(K_1 - \frac{(K_1+K_2)(M+M_\Delta^{max})}{M+M_\Delta(t)} \right) e_2 \bigg) - \frac{M+M_\Delta^{max}}{2\varepsilon_2^2 (M+M_\Delta(t))} e_2 \\
& + \frac{1}{M+M_\Delta(t)} tr\Big(\Delta\Omega^T \Delta\dot{\Omega}\Big) + \frac{\varepsilon_1^2}{2} \left(\Delta_P + \frac{1}{M+M_\Delta(t)} o(\eta) \right)^T \left(\Delta_P + \frac{1}{M+M_\Delta(t)} o(\eta) \right)
\end{aligned} \tag{24}
$$

where

$$
\Delta_G = \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) \mathcal{G}g
$$

Because of Young's inequality

$$
e_2^T \Delta_G \leq \frac{1}{2\varepsilon_2^2} e_2^T e_2 + \frac{\varepsilon_2^2}{2} \Delta_G^T \Delta_G \tag{25}
$$

and $M + M_\Delta(t) \leq M + M_\Delta^{max}$, define $K_1 \geq 1$, and then

$$
\begin{aligned}
\dot{V} \leq & -\left(K_1 - \frac{1}{2}\Big(1-K_1^2\Big) \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) \right) e_1^T e_1 - \left(\frac{(K_1+K_2)(M+M_\Delta^{max})}{M+M_\Delta(t)} - K_1 \right) e_2^T e_2 \\
& + \frac{1}{2\varepsilon_1^2} \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) e_2^T e_2 + \frac{1}{2\varepsilon_2^2} \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) e_2^T e_2 + \frac{1}{M+M_\Delta(t)} tr\Big(\Delta\Omega^T \Delta\dot{\Omega}\Big) + \frac{\varepsilon_2^2}{2} \Delta_G^T \Delta_G \\
& + e_2^T \left(\frac{1}{M+M_\Delta(t)} \Omega^T \Phi(\eta) - \frac{1}{M+M_\Delta(t)} \widehat{\Omega}^T \Phi(\eta) \right) + \frac{\varepsilon_1^2}{2} \left(\Delta_P + \frac{1}{M+M_\Delta(t)} o(\eta) \right)^T \left(\Delta_P + \frac{1}{M+M_\Delta(t)} o(\eta) \right) \\
\leq & -\left(K_1 - \frac{1}{2}\Big(1-K_1^2\Big) \left(1 - \frac{M+M_\Delta^{max}}{M+M_\Delta(t)} \right) \right) e_1^T e_1 - \left(\frac{(K_1+K_2)(M+M_\Delta^{max})}{M+M_\Delta(t)} - K_1 \right) e_2^T e_2
\end{aligned}
$$

$$
\begin{aligned}
&+e_2^T\left(\frac{1}{M+M_\Delta(t)}\Omega^T\Phi(\eta)-\frac{1}{M+M_\Delta(t)}\widehat{\Omega}^T\Phi(\eta)\right)+\frac{1}{M+M_\Delta(t)}tr\left(\Delta\Omega^T\Delta\dot{\Omega}\right)\\
&+\frac{\varepsilon_1^2}{2}\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)^T\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)+\frac{\varepsilon_2^2}{2}\Delta_G^T\Delta_G\\
&\le-\left(K_1-\frac{1}{2}\left(1-K_1^2\right)\left(1-\frac{M+M_\Delta^{max}}{M+M_\Delta(t)}\right)\right)e_1^Te_1-\left(\frac{(K_1+K_2)\left(M+M_\Delta^{max}\right)}{M+M_\Delta(t)}-K_1\right)e_2^Te_2+\frac{\varepsilon_2^2}{2}\Delta_G^T\Delta_G\\
&+\frac{1}{M+M_\Delta(t)}\left(e_2^T\Delta\Omega^T\Phi(\eta)+tr\left(\Delta\Omega^T\Delta\dot{\Omega}\right)\right)+\frac{\varepsilon_1^2}{2}\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)^T\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)
\end{aligned}
\tag{26}
$$

According to the property of the vector product,

$$
e_2^T\Delta\Omega^T\Phi(\eta)=tr\left(\Delta\Omega^T\Phi(\eta)e_2^T\right)
\tag{27}
$$

Thus,

$$
\begin{aligned}
\dot{V}&\le-\left(K_1-\frac{1}{2}\left(1-K_1^2\right)\left(1-\frac{M+M_\Delta^{max}}{M+M_\Delta(t)}\right)\right)e_1^Te_1-\left(\frac{(K_1+K_2)\left(M+M_\Delta^{max}\right)}{M+M_\Delta(t)}-K_1\right)e_2^Te_2+\frac{\varepsilon_2^2}{2}\Delta_G^T\Delta_G\\
&+\frac{1}{M+M_\Delta(t)}\left(tr\left(\Delta\Omega^T\Phi(\eta)e_2^T\right)+tr\left(\Delta\Omega^T\Delta\dot{\Omega}\right)\right)+\frac{\varepsilon_1^2}{2}\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)^T\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)\\
&\le-\left(K_1-\frac{1}{2}\left(1-K_1^2\right)\left(1-\frac{M+M_\Delta^{max}}{M+M_\Delta(t)}\right)\right)e_1^Te_1-\left(\frac{(K_1+K_2)\left(M+M_\Delta^{max}\right)}{M+M_\Delta(t)}-K_1\right)e_2^Te_2\\
&+\frac{1}{M+M_\Delta(t)}\left(tr\left(\Delta\Omega^T\Phi(\eta)e_2^T\right)-tr\left(\Delta\Omega^T\dot{\widehat{\Omega}}\right)\right)+\frac{\varepsilon_2^2}{2}\Delta_G^T\Delta_G\\
&+\frac{\varepsilon_1^2}{2}\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)^T\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)
\end{aligned}
\tag{28}
$$

Substituting the adaptive law $\dot{\widehat{\Omega}}$ into (29), we have

$$
\begin{aligned}
\dot{V}&\le-\left(K_1-\frac{1}{2}\left(1-K_1^2\right)\left(1-\frac{M+M_\Delta^{max}}{M+M_\Delta(t)}\right)\right)e_1^Te_1-\left(\frac{(K_1+K_2)\left(M+M_\Delta^{max}\right)}{M+M_\Delta(t)}-K_1\right)e_2^Te_2\\
&+K_3tr\left(\Delta\Omega^T\widehat{\Omega}\right)+\frac{\varepsilon_1^2}{2}\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)^T\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)+\frac{\varepsilon_2^2}{2}\Delta_G^T\Delta_G\\
&\le-\left(K_1-\frac{1}{2}\left(1-K_1^2\right)\left(1-\frac{M+M_\Delta^{max}}{M+M_\Delta(t)}\right)\right)e_1^Te_1-K_2e_2^Te_2+K_3tr\left(\Delta\Omega^T\widehat{\Omega}\right)+\frac{\varepsilon_2^2}{2}\Delta_G^T\Delta_G\\
&+\frac{\varepsilon_1^2}{2}\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)^T\left(\Delta_P+\frac{1}{M+M_\Delta(t)}o(\eta)\right)
\end{aligned}
\tag{29}
$$

Because $K_1\ge 1$, let

$$
K_1-\frac{1}{2}\left(1-K_1^2\right)\left(1-\frac{M+M_\Delta^{max}}{M+M_\Delta(t)}\right)>0
$$

Then,

$$
K_1-\frac{1}{2}\left(1-K_1^2\right)\left(1-\frac{M+M_\Delta^{max}}{M}\right)>0
$$

Thus,

$$\frac{M_{\Delta}^{max}}{2M}\left(K_1^2 - \frac{M_{\Delta}^{max}}{2M}K_1 - 1\right) < 0$$

Therefore, we have

$$K_1^2 - \frac{M_{\Delta}^{max}}{2M}K_1 - 1 < 0$$

We obtain

$$\left(K_1 - \frac{M_{\Delta}^{max}}{4M}\right)^2 < 1 + \left(\frac{M_{\Delta}^{max}}{4M}\right)^2$$

Therefore,

$$1 \leq K_1 \leq \frac{M_{\Delta}^{max}}{4M} + \sqrt{1 + \left(\frac{M_{\Delta}^{max}}{4M}\right)^2} \tag{30}$$

Define

$$K_e = K_1 - \frac{1}{2}\left(1 - K_1^2\right)\left(1 - \frac{M + M_{\Delta}^{max}}{M + M_{\Delta}(t)}\right)$$

Equation (29) is given by

$$\dot{V} \leq -K_e e_1^T e_1 - K_2 e_2^T e_2 + K_3 tr\left(\Delta\Omega^T \widehat{\Omega}\right) + \frac{\varepsilon_1^2}{2}\left(\Delta_P + \frac{1}{M + M_{\Delta}(t)}o(\eta)\right)^T\left(\Delta_P + \frac{1}{M + M_{\Delta}(t)}o(\eta)\right) + \frac{\varepsilon_2^2}{2}\Delta_G^T\Delta_G \tag{31}$$

Apply the equation

$$\begin{aligned} K_3 tr\left(\Delta\Omega^T\widehat{\Omega}\right) &= -\frac{K_3}{2}tr\left((\Omega - \widehat{\Omega})^T(\Omega - \widehat{\Omega})\right) + \frac{K_3}{2}tr\left(\Omega^T\Omega\right) - \frac{K_3}{2}tr\left(\widehat{\Omega}^T\widehat{\Omega}\right) \\ &\leq -\frac{K_3}{2}tr\left((\Omega - \widehat{\Omega})^T(\Omega - \widehat{\Omega})\right) + \frac{K_3}{2}tr\left(\Omega^T\Omega\right) \\ &= -\frac{K_3}{2}tr\left(\Delta\Omega^T\Delta\Omega\right) + \frac{K_3}{2}tr\left(\Omega^T\Omega\right) \end{aligned} \tag{32}$$

to obtain

$$\begin{aligned} \dot{V} \leq & -K_e e_1^T e_1 - K_2 e_2^T e_2 - \frac{K_3}{2}tr\left(\Delta\Omega^T\Delta\Omega\right) \\ & + \frac{K_3}{2}tr\left(\Omega^T\Omega\right) + \frac{\varepsilon_1^2}{2}\left(\Delta_P + \frac{1}{M + M_{\Delta}(t)}o(\eta)\right)^T\left(\Delta_P + \frac{1}{M + M_{\Delta}(t)}o(\eta)\right) + \frac{\varepsilon_2^2}{2}\Delta_G^T\Delta_G \\ \leq & -2\kappa V + \frac{\sigma}{2}\|\Delta\|^2 \end{aligned} \tag{33}$$

where $\sigma = \max\{\varepsilon_1^2, \varepsilon_2^2, K_3\}$, $\kappa = \min\{K_e, K_2, \frac{K_3}{2}\}$, and

$$\|\Delta\|^2 = \left(\Delta_P + \frac{1}{M + M_{\Delta}(t)}o(\eta)\right)^T\left(\Delta_P + \frac{1}{M + M_{\Delta}(t)}o(\eta)\right) + \Delta_G^T\Delta_G + tr\left(\Omega^T\Omega\right)$$

Solving the differential Equation (33), we obtain

$$V(e_1, e_2, \Delta\Omega) \leq e^{-2\kappa t} V(0) + \frac{\sigma}{4\kappa}\left(1 - e^{-2\kappa t}\right)\left(\sup_{0\leq\tau\leq t} \|\Delta\|^2\right) \tag{34}$$

Define $\xi = [e_1, e_2, \Delta\Theta]^T$, and then we have

$$\|\xi(t)\| \leq e^{-\kappa t}\|\xi(0)\| + \sqrt{\frac{\sigma}{2\kappa}(1 - e^{-2\kappa t})}\left(\sup_{0\leq\tau\leq t} \|\Delta\|\right) \tag{35}$$

where $\xi(0) = [e_1(0), e_2(0), \Delta\Omega(0)]^T$ and $\Delta\Theta(0) = \Omega(0) - \widehat{\Omega}(0)$. According to the definition of input–state stability, the whole position closed-loop system is ISS. Furthermore, if the uncertainty is small or does not exist, that is, $\Delta = 0$, we have $\|\xi(t)\| \leq e^{-\kappa t}\|\xi(0)\|$, and the closed-loop is exponentially stable. □

Denoting $u = [u_x, u_y, u_z]$ and $u = R_v F^P = R_v N F^b$, we have

$$u = (M + M_\Delta^{max})\left(-K_2 e_2 - \left(\frac{1}{2\varepsilon_1^2} + \frac{1}{2\varepsilon_2^2}\right)e_2 - e_1 - \frac{1}{M + M_\Delta^{max}}\widehat{\Omega}^T\Phi(\eta) - \mathcal{G}g + \dot{\eta}_2^*\right) \tag{36}$$

Calculate the desired attitude angle command $\Theta_d = [\phi_{cmd}, \theta_{cmd}, \psi_{cmd}]^T$ and the total force output NF^b through $u = R_V(\phi_{cmd}, \theta_{cmd}, \psi_{cmd})NF^b$.

3.3. Nonlinear Attitude Controller Based on Disturbance Observer

Consider the subsystem Π_2, and define $\xi_1 = \Theta - \Theta_d$ and $\xi_2 = \dot{\Theta} - \dot{\Theta}_d$, and we have

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = -R_\Theta J^{-1}S(\dot{\Theta})J\dot{\Theta} + R_\Theta J^{-1}LF^b + R_\Theta J^{-1}B_\omega(\omega^b) - \ddot{\Theta}_d - R_\Theta J^{-1}J_\Delta(t)R_\Theta^{-1}\ddot{\Theta} \\ \qquad + R_\Theta J^{-1}S(\dot{\Theta})J_\Delta(t)\dot{\Theta} + R_\Theta J^{-1}L_\Delta(t)F^b + R_\Theta J^{-1}\Delta_2 \end{cases} \tag{37}$$

Denote $\xi = [\xi_1, \xi_2]^T$. It is assumed that the unknown terms $R_\Theta J^{-1}B_\omega(\omega^b)$ caused by the multi-media dynamics can be rewritten as a combination of M basis functions $w_j(\xi)$, and we obtain

$$R_\Theta J^{-1}B_\omega(\omega^b) = \Xi^T W(\xi) + o_2(\xi) \tag{38}$$

where $\Xi \in \mathbb{R}^{m\times 3}$ is the unknown constant and $o_2(\xi)$ is the higher-order component, and

$$W(\xi) = (w_1(W(\xi), w_2(\xi), w_3(\xi), ..., w_M(\xi))) \in \mathbb{R}^M \tag{39}$$

is the known regression vector. Subsystem Π_2 can be rewritten as

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = -R_\Theta J^{-1}S(\dot{\Theta})J\dot{\Theta} + R_\Theta J^{-1}LF^b + \Xi^T W(\xi) - \ddot{\Theta}_d + \Delta_\xi \end{cases} \tag{40}$$

where

$$\Delta_\xi = -R_\Theta J^{-1}J_\Delta(t)R_\Theta^{-1}\ddot{\Theta} + R_\Theta J^{-1}S(\dot{\Theta})J_\Delta(t)\dot{\Theta} + R_\Theta J^{-1}L_\Delta(t)F^b + R_\Theta J^{-1}\Delta_2 + o_2(\xi)$$

Δ_ξ indicates the total disturbance. Define $e_\xi = \xi_2 - \xi_2^*$ where ξ_2^* is the virtual control law.

Equation (41) can be given by

$$\begin{cases} \dot{\xi}_1 = e_\xi + \xi_2^* \\ \dot{e}_\xi = -R_\Theta J^{-1} S(\dot{\Theta}) J \dot{\Theta} + R_\Theta J^{-1} L F^b + \Xi^T W(\xi) - \ddot{\Theta}_d - \dot{\xi}_2^* + \Delta_\xi \end{cases} \tag{41}$$

Then, we can design the attitude controller and obtain the main results.

Assumption 2. *Assume Δ_ξ is bounded, and there exists an unknown constant such that $\|\Delta_\xi\| \le \nu$.*

The observer design method in [19] is introduced to estimate the total uncertainty Δ_ξ and the adaptive estimate error $\Xi^T W(\xi) - \widehat{\Xi}^T W(\xi)$. Denote

$$\Delta_\Xi = \Xi^T W(\xi) - \widehat{\Xi}^T W(\xi) + \Delta_\xi \tag{42}$$

Assume $\widehat{\Delta}_\Xi$ is the estimate of Δ_Ξ, and the observer is given by

$$\begin{cases} \dot{\widehat{e}}_\xi = -R_\Theta J^{-1} S(\dot{\Theta}) J \dot{\Theta} + R_\Theta J^{-1} L F^b + \widehat{\Xi}^T W(\xi) - \ddot{\Theta}_d - \dot{\xi}_2^* + \widehat{\Delta}_\Xi + A\left(e_\xi - \widehat{e}_\xi\right) \\ \dot{\widehat{\Delta}}_\Xi = z_1 + B\left(e_\xi - \widehat{e}_\xi\right) \\ \dot{z}_1 = z_2 + C\left(e_\xi - \widehat{e}_\xi\right) \\ \dot{z}_2 = D\left(e_\xi - \widehat{e}_\xi\right) \end{cases} \tag{43}$$

Denoting the observer error $\widetilde{\Delta}_\Xi = \Delta_\Xi - \widehat{\Delta}_\Xi$ and $\widetilde{e}_\xi = e_\xi - \widehat{e}_\xi$, the error equation is given by

$$\begin{aligned} \dot{\widetilde{e}}_\xi &= \Xi^T W(\xi) - \widehat{\Xi}^T W(\xi) + \Delta_\xi - \widehat{\Delta}_\Xi - A\widetilde{e}_\xi \\ &= \Delta_\Xi - \widehat{\Delta}_\Xi - A\widetilde{e}_\xi \\ &= \widetilde{\Delta}_\Xi - A\widetilde{e}_\xi \\ \dot{\widetilde{\Delta}}_\Xi &= \dot{\Delta}_\Xi - \dot{\widehat{\Delta}}_\Xi \\ &= \dot{\Delta}_\Xi - z_1 - B\widetilde{e}_\xi \\ \ddot{\widetilde{\Delta}}_\Xi &= \ddot{\Delta}_\Xi - \dot{z}_1 - B\dot{\widetilde{e}}_\xi \\ &= \ddot{\Delta}_\Xi - z_2 - C\widetilde{e}_\xi - B\left(\widetilde{\Delta}_\Xi - A\widetilde{e}_\xi\right) \\ &= \ddot{\Delta}_\Xi - z_2 - B\widetilde{\Delta}_\Xi + (BA - C)\widetilde{e}_\xi \\ \dddot{\widetilde{\Delta}}_\Xi &= \dddot{\Delta}_\Xi - \dot{z}_2 - B\dot{\widetilde{\Delta}}_\Xi + (BA - C)\dot{\widetilde{e}}_\xi \\ &= \dddot{\Delta}_\Xi - D\widetilde{e}_\xi - B\dot{\widetilde{\Delta}}_\Xi + (BA - C)\left(\widetilde{\Delta}_\Xi - A\widetilde{e}_\xi\right) \\ &= \dddot{\Delta}_\Xi + \left((C - BA)A - D\right)\widetilde{e}_\xi + (BA - C)\widetilde{\Delta}_\Xi - B\dot{\widetilde{\Delta}}_\Xi \end{aligned} \tag{44}$$

Furthermore, an error state-space equation is constructed as follows:

$$\begin{bmatrix} \dot{\widetilde{e}}_\xi \\ \dot{\widetilde{\Delta}}_\Xi \\ \ddot{\widetilde{\Delta}}_\Xi \\ \dddot{\widetilde{\Delta}}_\Xi \end{bmatrix} = \begin{bmatrix} -A & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ (C - BA)A - D & BA - C & -B & 0 \end{bmatrix} \begin{bmatrix} \widetilde{e}_\xi \\ \widetilde{\Delta}_\Xi \\ \dot{\widetilde{\Delta}}_\Xi \\ \ddot{\widetilde{\Delta}}_\Xi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dddot{\Delta}_\Xi \tag{45}$$

Define $X = [\widetilde{e}_\xi, \widetilde{\Delta}_\Xi, \dot{\widetilde{\Delta}}_\Xi, \ddot{\widetilde{\Delta}}_\Xi]^T$ and

$$\mathcal{A} = \begin{bmatrix} -A & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ (C - BA)A - D & BA - C & -B & 0 \end{bmatrix}, \ \mathcal{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{46}$$

Then, we have

$$\dot{X} = \mathcal{A}X + \mathcal{B}\dddot{\Delta}_\Xi \tag{47}$$

Assume $|\dddot{\Delta}_\Xi| \leq \vartheta,\ \vartheta > 0$ and the control law can be given by

$$\begin{cases} LF^b = JR_\theta^{-1}\Big(-\xi_1 + R_\theta J^{-1}S(\dot{\Theta})J\dot{\Theta} - \widehat{\Xi}^T W(\xi) + \ddot{\Theta}_d + \dot{\xi}_2^* - \dfrac{1}{2\varepsilon_3^2}e_\xi - a_2 e_\xi - \widehat{\Delta}_\Xi \Big) \\ \xi_2^* = -a_1\xi_1 \\ \dot{\widehat{\Xi}} = W(\xi)e_\xi^T - a_3\widehat{\Xi} \end{cases} \tag{48}$$

where $e_\xi = \xi_2 - \xi_2^*$, $a_i, i = 1,2,3$, and ε_3 are the positive constants. Therefore, the following theorem can be obtained.

Theorem 2. *For the closed-loop system of the attitude error equation (41), the state observer error equation (44), the disturbance error equation (47), and the adaptive estimate error, if there exists constants $a_i > 0, i = 1,2,3$ and $\varsigma > 0$, and matrices $P > 0, Q > 0$ such that*

$$\mathcal{A}^T P + P\mathcal{A} = -Q,\ \varsigma > \frac{1}{\lambda_{min}(Q)} \tag{49}$$

The closed-loop system is input-to-state stable with the disturbance observer (43) and the robust adaptive controller (48). If all disturbances and uncertainties disappear, the closed-loop system is exponentially stable.

Proof of Theorem 2. For convenience of the stability analysis, denote $\widetilde{e}_\xi = e_\xi - \widehat{e}_\xi$ and $\widetilde{X} = \frac{X}{\varsigma}$ with $\varsigma > 0$ which is a small constant number. Equation (47) can be rewritten as

$$\varsigma\dot{\widetilde{X}} = \varsigma\mathcal{A}\widetilde{X} + \mathcal{B}\dddot{\Delta}_\Xi\Big(\Delta\Xi, J_\Delta, L_\Delta, \Delta_2, o_2, \ddot{\Theta}, \dot{\Theta}, \xi_1, e_\xi, X, t\Big) \tag{50}$$

Then, we redefine the error term as follows:

$$\dddot{\Delta}_\Xi\Big(\Delta\Xi, J_\Delta, L_\Delta, \Delta_2, o_2, \ddot{\Theta}, \dot{\Theta}, \xi_1, e_\xi, X, t\Big) \tag{51}$$

Equation (52) is given by

$$\varsigma\dot{\widetilde{X}} = \varsigma\mathcal{A}\widetilde{X} + \mathcal{B}\dddot{\Delta}_\Xi\Big(\Delta\Xi, J_\Delta, L_\Delta, \Delta_2, o_2, \ddot{\Theta}, \dot{\Theta}, \xi_1, e_\xi, \varsigma\widetilde{X}, t\Big) \tag{52}$$

Choose the Lyapunov function as

$$V(\xi_1, e_\xi, \Delta\Xi, \widetilde{X}) = \frac{1}{2}\xi_1^T\xi_1 + \frac{1}{2}e_\xi^T e_\xi + \frac{1}{2}tr(\Delta\Xi^T\Delta\Xi) + \varsigma\widetilde{X}^T P\widetilde{X} \tag{53}$$

where $P = P^T > 0$, and the time derivative of the Lyapunov function $V(\xi_1, e_\xi, \Delta\Xi, \widetilde{X})$ is

$$\begin{aligned} \dot{V} =& \xi_1^T\dot{\xi}_1 + e_\xi^T\dot{e}_\xi + tr(\Delta\Xi^T\Delta\dot{\Xi}) + \widetilde{X}^T P\dot{\widetilde{X}} + \dot{\widetilde{X}}^T P\widetilde{X} \\ =& \xi_1^T(e_\xi + \xi_2^*) + e_\xi^T\dot{e}_\xi + tr(\Delta\Xi^T\Delta\dot{\Xi}) + \varsigma\widetilde{X}^T P\dot{\widetilde{X}} + \varsigma\dot{\widetilde{X}}^T P\widetilde{X} \end{aligned} \tag{54}$$

Substituting ξ_2^* into (54), we have

$$\begin{aligned} \dot{V} =& -a_1\xi_1^T\xi_1 + \xi_1^T e_\xi + e_\xi^T\Big(-R_\Theta J^{-1}S(\dot{\Theta})J\dot{\Theta} + R_\Theta J^{-1}LF^b + \Xi^T W(\xi) - \ddot{\Theta}_d + \Delta_\xi\Big) + tr(\Delta\Xi^T\Delta\dot{\Xi}) \\ &+ \varsigma\widetilde{X}^T P\dot{\widetilde{X}} + \varsigma\dot{\widetilde{X}}^T P\widetilde{X} \end{aligned} \tag{55}$$

Using F^b in (55), then we obtain

$$\begin{aligned}
\dot{V} \leq & -a_1\xi_1^T\xi_1 - a_2 e_\xi^T e_\xi + e_\xi^T\left(\Xi^T W(\xi) - \widehat{\Xi}^T W(\xi) + \Delta_\xi - \widehat{\Delta}_\Xi - \frac{1}{2\varepsilon_3^2}e_\xi\right) + tr(\Delta\Xi^T\Delta\dot{\Xi}) + \varsigma\widetilde{X}^T P\dot{\widetilde{X}} + \varsigma\dot{\widetilde{X}}^T P\widetilde{X} \\
= & -a_1\xi_1^T\xi_1 - a_2 e_\xi^T e_\xi + e_\xi^T\left(\Delta\Xi^T W(\xi) + e_\Delta - \frac{1}{2\varepsilon_3^2}e_\xi\right) + \varsigma\widetilde{X}^T\left(\mathcal{A}^T P + P\mathcal{A}\right)\widetilde{X} \\
& + \ddot{\Delta}_\Xi^T\mathcal{B}^T P\widetilde{X} + \widetilde{X}^T P\mathcal{B}\ddot{\Delta}_\Xi + tr(\Delta\Xi^T\Delta\dot{\Xi}) \\
= & -a_1\xi_1^T\xi_1 - a_2 e_\xi^T e_\xi + e_\xi^T\left(\Delta\Xi^T W(\xi) + e_\Delta - \frac{1}{2\varepsilon_3^2}e_\xi\right) - \varsigma\widetilde{X}^T Q\widetilde{X} + 2\widetilde{X}^T P\mathcal{B}\ddot{\Delta}_\Xi + tr(\Delta\Xi^T\Delta\dot{\Xi}) \\
\leq & -a_1\xi_1^T\xi_1 - a_2 e_\xi^T e_\xi + e_\xi^T\left(\Delta\Xi^T W(\xi) + e_\Delta - \frac{1}{2\varepsilon_3^2}e_\xi\right) - \varsigma\widetilde{X}^T\lambda_{min}(Q)\widetilde{X} + 2\widetilde{X}^T P\mathcal{B}\ddot{\Delta}_\Xi + tr(\Delta\Xi^T\Delta\dot{\Xi})
\end{aligned} \tag{56}$$

where $e_\Delta = \Delta_\xi - \widehat{\Delta}_\Xi$ and $\mathcal{A}^T P + P\mathcal{A} = -Q$, $Q > 0$. Substituting inequalities

$$\begin{cases}
e_\xi^T e_\Delta \leq \dfrac{1}{2\varepsilon_3^2}e_\xi^T e_\xi + \dfrac{\varepsilon_3^2}{2}e_\Delta^T e_\Delta \\
\widetilde{X}^T P\mathcal{B}\ddot{\Delta}_\Xi \leq \dfrac{1}{2}\|\widetilde{X}\|^2 + \dfrac{1}{2}\|P\|^2\|\mathcal{B}\|^2\|\ddot{\Delta}_\Xi\|^2 \leq \dfrac{1}{2}\|\widetilde{X}\|^2 + \dfrac{1}{2}\|P\|^2\|\ddot{\Delta}_\Xi\|^2
\end{cases} \tag{57}$$

into (55), equation (58) is rewritten as

$$\dot{V} \leq -a_1\xi_1^T\xi_1 - a_2 e_\xi^T e_\xi + e_\xi^T\Delta\Xi^T W(\xi) - \varsigma\lambda_{min}(Q)\widetilde{X}^T\widetilde{X} + \widetilde{X}^T\widetilde{X} + \|P\|^2\|\ddot{\Delta}_\Xi\|^2 - tr(\Delta\Xi^T\Delta\dot{\widehat{\Xi}}) + \frac{\varepsilon_3^2}{2}e_\Delta^T e_\Delta \tag{58}$$

Because

$$e_\xi^T\Delta\Xi^T W(\xi) = tr\left(\Delta\Xi^T W(\xi)e_\xi^T\right) \tag{59}$$

Thus,

$$\dot{V} \leq -a_1\xi_1^T\xi_1 - a_2 e_\xi^T e_\xi - (\varsigma\lambda_{min}(Q) - 1)\widetilde{X}^T\widetilde{X} + tr\left(\Delta\Xi^T W(\xi)e_\xi^T\right) - tr(\Delta\Xi^T\Delta\dot{\widehat{\Xi}}) + \|P\|^2\|\ddot{\Delta}_\Xi\|^2 + \frac{\varepsilon_3^2}{2}e_\Delta^T e_\Delta \tag{60}$$

Use the adaptive law $\Delta\dot{\widehat{\Xi}}$, and we obtain

$$\begin{aligned}
\dot{V} \leq & -a_1\xi_1^T\xi_1 - a_2 e_\xi^T e_\xi - (\varsigma\lambda_{min}(Q) - 1)\widetilde{X}^T\widetilde{X} + tr\left(\Delta\Xi^T W(\xi)e_\xi^T\right) - tr\left(\Delta\Xi^T\left(W(\xi)e_\xi^T - a_3\widehat{\Xi}\right)\right) + \|P\|^2\|\ddot{\Delta}_\Xi\|^2 + \frac{\varepsilon_3^2}{2}e_\Delta^T e_\Delta \\
\leq & -a_1\xi_1^T\xi_1 - a_2 e_\xi^T e_\xi - (\varsigma\lambda_{min}(Q) - 1)\widetilde{X}^T\widetilde{X} + a_3 tr\left(\Delta\Xi^T\widehat{\Xi}\right) + \|P\|^2\|\ddot{\Delta}_\Xi\|^2 + \frac{\varepsilon_3^2}{2}e_\Delta^T e_\Delta
\end{aligned} \tag{61}$$

Term $a_3 tr\left(\Delta\Xi^T\widehat{\Xi}\right)$ can be rewritten as

$$\begin{aligned}
a_3 tr\left(\Delta\Xi^T\widehat{\Xi}\right) = & -\frac{a_3}{2}tr\left(\left(\Xi - \widehat{\Xi}\right)^T\left(\Xi - \widehat{\Xi}\right)\right) + \frac{a_3}{2}tr\left(\Xi^T\Xi\right) - \frac{a_3}{2}tr\left(\widehat{\Xi}^T\widehat{\Xi}\right) \\
\leq & -\frac{a_3}{2}tr\left(\left(\Xi - \widehat{\Xi}\right)^T\left(\Xi - \widehat{\Xi}\right)\right) + \frac{a_3}{2}tr\left(\Xi^T\Xi\right) \\
\leq & -\frac{a_3}{2}tr\left(\Delta\Xi^T\Delta\Xi\right) + \frac{a_3}{2}tr\left(\Xi^T\Xi\right)
\end{aligned} \tag{62}$$

Because

$$\lambda_{min}(P)\|\widetilde{X}\|^2 \le \widetilde{X}^T P \widetilde{X} \le \lambda_{max}(P)\|\widetilde{X}\|^2 \tag{63}$$

Then, we have

$$\begin{aligned} \dot{V} \le & -a_1\xi_1^T\xi_1 - a_2 e_\xi^T e_\xi - \frac{(\varsigma\lambda_{min}(Q)-1)}{\lambda_{max}(P)}\widetilde{X}^T P\widetilde{X} - \frac{a_3}{2}tr\left(\Delta\Xi^T\Delta\Xi\right) + \|P\|^2\|\ddot{\Delta}_\Xi\|^2 + \frac{a_3}{2}tr\left(\Xi^T\Xi\right) + \frac{\varepsilon_3^2}{2}e_\Delta^T e_\Delta \\ \le & -2\mu V + \frac{\iota}{2}\|\Delta_\Xi\|^2 \end{aligned} \tag{64}$$

where $\varsigma > \frac{1}{\lambda_{min}(Q)}$, $\iota = \max\{a_3, \varepsilon_3^2, 2\}$, and

$$\begin{aligned} \mu &= \min\left\{a_1, a_2, (\varsigma\lambda_{min}(Q)-1)/\lambda_{max}(P), \frac{a_3}{2}\right\} \\ \|\Delta_\omega\|^2 &= \|P\|^2\|\ddot{\Delta}_\Xi\|^2 + tr\left(\Xi^T\Xi\right) + e_\Delta^T e_\Delta \end{aligned}$$

With the integral on both sides of inequality (64), then we have

$$V \le e^{-2\mu t}V(0) + \frac{\iota}{4\mu}\left(1 - e^{-2\mu t}\right)\left(\sup_{0\le\tau\le t}\|\Delta_\omega\|\right) \tag{65}$$

Define $\chi = [\xi_1, e_\xi, \Delta\Xi, \widetilde{X}]^T$, and we have

$$\|\chi\| \le e^{-\mu t}\|\chi(0)\| + \sqrt{\frac{\iota}{2\mu}(1 - e^{-\mu t})}\left(\sup_{0\le\tau\le t}\|\Delta_\omega\|\right) \tag{66}$$

where $\chi(0) = [\xi_1(0), e_\xi(0), \Delta\Xi(0), \widetilde{X}(0)]^T$ and $\Delta\Xi(0) = \Xi(0) - \widehat{\Xi}(0)$, $\widehat{\Xi}(0) > 0$. Therefore, the closed-loop system is ISS. Moreover, if the uncertainty does not exist, that is, $\Delta_\omega = 0$, the closed-loop system is exponentially stable. □

Through the term Δ_Ξ which includes the unmodeled dynamics, the external disturbance, and the higher-order characteristics, the disturbance observer (43) can improve the robustness of the system without the accurate model for estimating objects.

4. Simulation Results

In this section, a numerical simulation of an HAUQ is presented to verify the effectiveness of the proposed position and attitude control algorithm and the observer strategy using the MATLAB platform. Design a water–air crossing flight scene in which the center of gravity of the HAUQ comes out of the water from point $(0, 0, -0.5\ m)$ and climbs to point $(0, 0, 0.5\ m)$ to drain the water inside the body. Then, the control effect of the proposed position and attitude control algorithm and the disturbance observer are verified. The design parameters of the HAUQ are given by

$$M = 7\ Kg,\ g = 9.8\ m/s^2,\ l = 0.5\ m,\ C = 1 \tag{67}$$

and the fundamental moment of inertia matrix is

$$J = \begin{bmatrix} 0.325 & 0 & 0 \\ 0 & 0.285 & 0 \\ 0 & 0 & 0.181 \end{bmatrix} \tag{68}$$

Assume that the disturbance change caused by a large amount of water in the slender body is

$$M_\Delta = 2 \times 0.005^t,\ J_\Delta = \begin{bmatrix} 0.01 \times 0.005^t & 0.01 \times 0.005^t & 0.01 \times 0.005^t \\ 0.01 \times 0.005^t & 0.01 \times 0.005^t & 0.01 \times 0.005^t \\ 0.01 \times 0.005^t & 0.01 \times 0.005^t & 0.01 \times 0.005^t \end{bmatrix}$$
$$L_\Delta = \begin{bmatrix} F_e^t(t) & F_e^t(t) & -F_e^t(t) & -F_e^t(t) \\ F_e^t(t) & F_e^t(t) & -F_e^t(t) & -F_e^t(t) \\ 0 & 0 & 0 & 0 \end{bmatrix},\ F_e^t(t) = 0.02 \times 0.005^t \tag{69}$$

The unmodeled items of the complex dynamics of the HAUQ are composed of state variable feedback, the wave disturbance signal, and the random noise which are given by

$$A_P(t,\eta) = \begin{bmatrix} 0.25 & 1.00 & 3.00 \\ 1.00 & 5.00 & 1.00 \\ 1.00 & 1.00 & 0.10 \end{bmatrix} \dot{P} + \begin{bmatrix} 0.25sin\left(\frac{3}{4}\pi t\right) \\ 0.25cos\left(\frac{3}{4}\pi t\right) \\ 0.25cos\left(\frac{3}{4}\pi t\right) \end{bmatrix} + \begin{bmatrix} \sqrt{2}rand(1) \\ \sqrt{2}rand(1) \\ \sqrt{2}rand(1) \end{bmatrix}$$
$$R_\Theta J^{-1} B_\omega(\omega^b) = \begin{bmatrix} 0.35 & 0.00 & 0.00 \\ 0.00 & 0.10 & 0.00 \\ 0.00 & 0.00 & 0.50 \end{bmatrix} \dot{\Theta} + \begin{bmatrix} \sqrt{0.0000001}rand(1) \\ \sqrt{0.0000001}rand(1) \\ \sqrt{0.0000001}rand(1) \end{bmatrix} \tag{70}$$

where $\sqrt{2}rand(1)$ is a Gaussian random signal with the standard deviation $\sqrt{2}$, the mean value 0, and the variance 1. $\sqrt{0.0000001}rand(1)$ is also a Gaussian random signal with the standard deviation $\sqrt{0.0000001}$, the mean value 0, and the variance 1. For the unknown uncertainty caused by complex dynamics, the polynomial regression method is adopted and the adaptive law is used to estimate the unknown weight. Define $\eta_1/1000 = [\eta_1^x, \eta_1^y, \eta_1^z]^T$, $\eta_2 = [\eta_2^x, \eta_2^y, \eta_2^z]^T$, $\xi_1 = [\xi_1^\phi, \xi_1^\theta, \xi_1^\psi]^T$, and $\xi_2 = [\xi_2^\phi, \xi_2^\theta, \xi_2^\psi]^T$, where the position feedback estimation η_1 after dividing by 1000 is used to estimate the uncertainty to avoid the large initial values. The basis functions are given by

$$\begin{aligned} \Phi(\eta) =& [(\eta_1^x)^2, (\eta_1^y)^2, (\eta_1^z)^2, (\eta_2^x)^2, (\eta_2^y)^2, (\eta_2^z)^2, \eta_1^x\eta_1^y, \eta_1^x\eta_1^z, \eta_1^x\eta_2^x, \eta_1^x\eta_2^y, \eta_1^x\eta_2^z, \\ & \eta_1^y\eta_1^z, \eta_1^y\eta_2^x, \eta_1^y\eta_2^y, \eta_1^y\eta_2^z, \eta_1^z\eta_2^x, \eta_1^z\eta_2^y, \eta_1^z\eta_2^z, \eta_2^x\eta_2^y, \eta_2^x\eta_2^z, \eta_2^y\eta_2^z, \eta_1^x, \eta_1^y, \eta_1^z, \eta_2^x, \eta_2^y, \eta_2^z, 1] \\ W(\xi) =& [(\xi_1^x)^2, (\xi_1^y)^2, (\xi_1^z)^2, (\xi_2^x)^2, (\xi_2^y)^2, (\xi_2^z)^2, \xi_1^x\xi_1^y, \xi_1^x\xi_1^z, \xi_1^x\xi_2^x, \xi_1^x\xi_2^y, \xi_1^x\xi_2^z, \xi_1^y\xi_1^z, \xi_1^y\xi_2^x, \\ & \xi_1^y\xi_2^y, \xi_1^y\xi_2^z, \xi_1^z\xi_2^x, \xi_1^z\xi_2^y, \xi_1^z\xi_2^z, \xi_2^x\xi_2^y, \xi_2^x\xi_2^z, \xi_2^y\xi_2^z, \xi_1^x, \xi_1^y, \xi_1^z, \xi_2^x, \xi_2^y, \xi_2^z, 1] \end{aligned} \tag{71}$$

The unknown matrix $\Omega \in \mathbb{R}^{21\times 3}$ and $\Xi \in \mathbb{R}^{21\times 3}$ are estimated though the adaptive laws (18) and (48). The estimate of $A_P(t,\eta)$ and $B^\omega(t,\omega^b)R_\Theta J^{-1}B_\omega(\omega^b)$ is

$$\widehat{A}_P(t,\eta) = \widehat{\Omega}^T\Phi(\eta), \widehat{B}^\omega(t,\omega^b) = \widehat{\Xi}^T W(\xi) \tag{72}$$

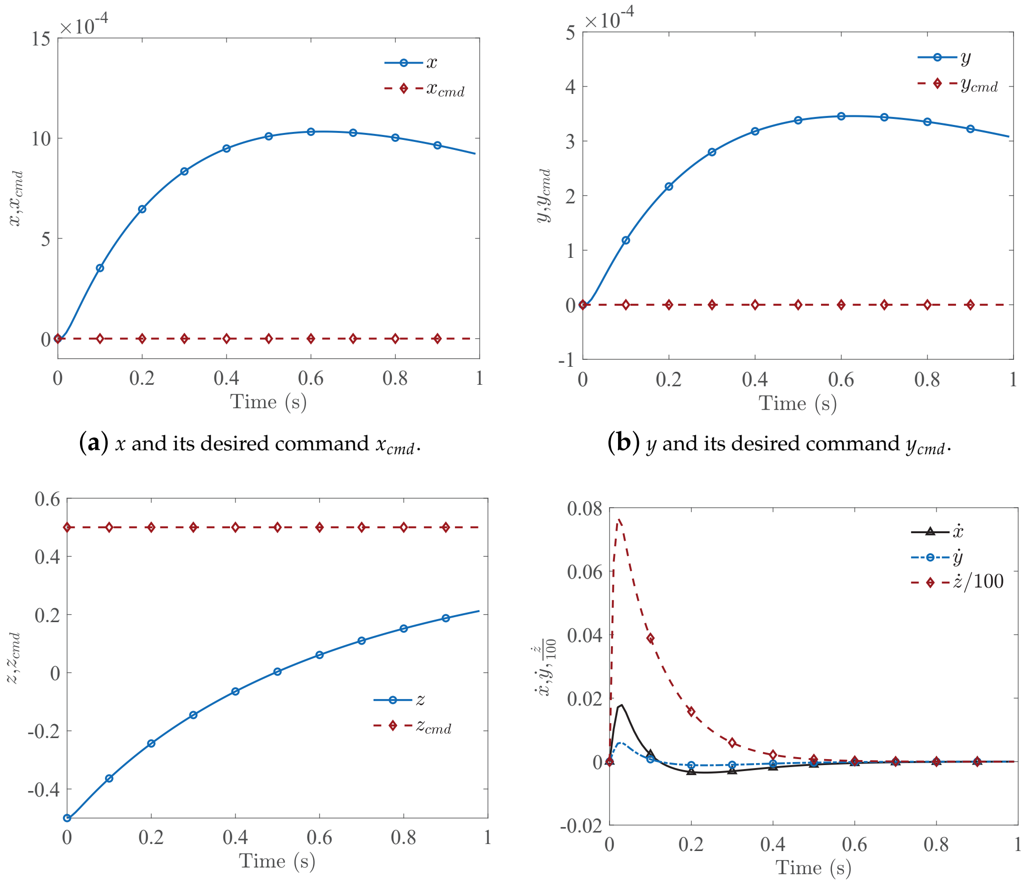
The disturbance caused by extra factors such as gusts is assumed as

$$\Delta_1 = 0.0005\sin\left(\frac{1}{4}\pi t\right), \Delta_2 = 0.0005\cos\left(\frac{1}{4}\pi t\right) \tag{73}$$

The gain parameter matrix of the control law and the adaptive control law are given by

$$A = \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}, B = \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}, C = \begin{bmatrix} 100.00 & 0.00 & 0.00 \\ 0.00 & 100.00 & 0.00 \\ 0.00 & 0.00 & 100.00 \end{bmatrix}, D = \begin{bmatrix} 0.001 & 0.00 & 0.00 \\ 0.00 & 0.001 & 0.00 \\ 0.00 & 0.00 & 0.001 \end{bmatrix} \tag{74}$$

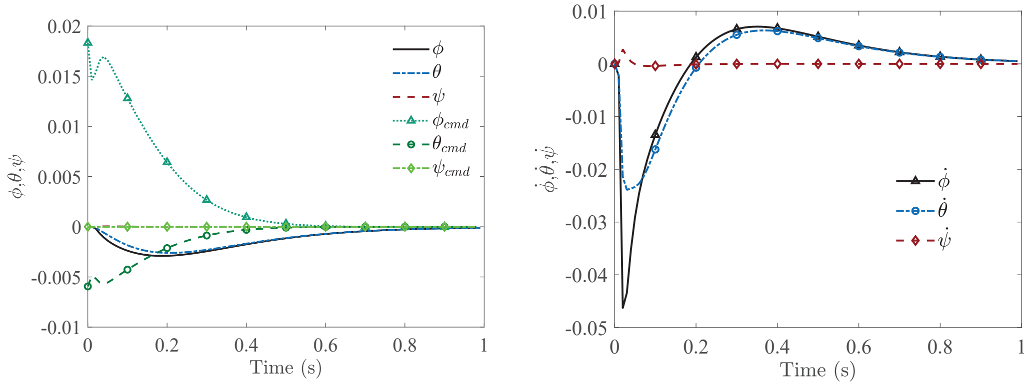
Denote $Q_{12\times12} = 10I_{12\times12}$, solve the linear matrix inequalities (LMI) (49), and we can obtain a proper $P_{12\times12} > 0$ where $I_{12\times12}$ is the identity matrix. In the simulation, the initial value of P is $P(0) = [0, 0, -0.5]^T$ and the others are 0. The simulation results are shown in Figures 2–7.



(**a**) x and its desired command x_{cmd}. (**b**) y and its desired command y_{cmd}.

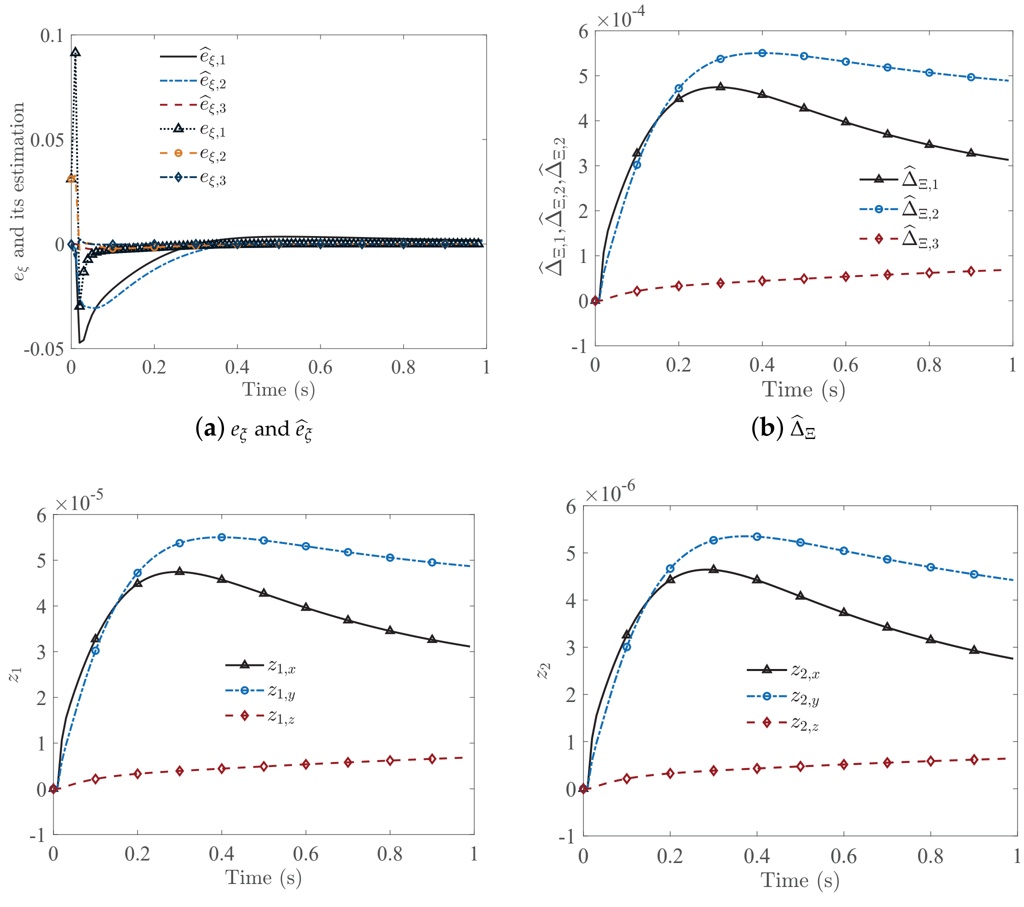
(**c**) z and its desired command z_{cmd}. (**d**) Derivative of position variable.

Figure 2. Position state variables of the hybrid aerial underwater quadrotor.



(**a**) Attitude and its desired attitude. (**b**) Derivative of attitude angle.

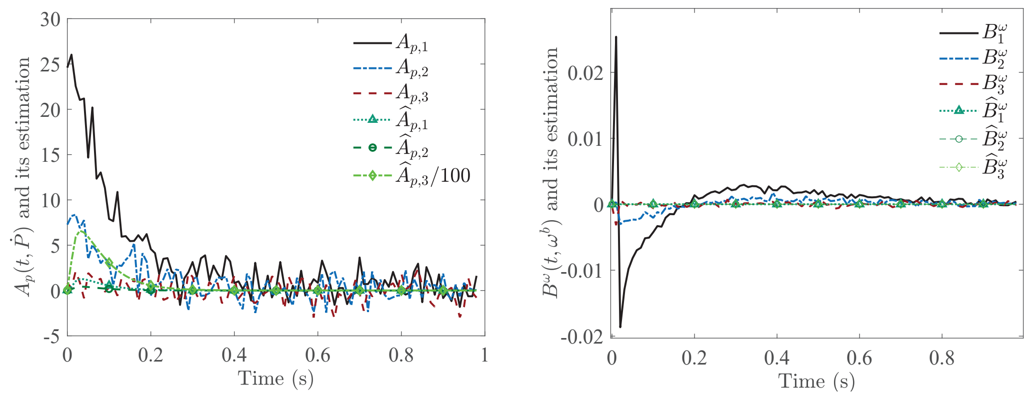
Figure 3. Attitude state variable of the hybrid aerial underwater quadrotor.



(a) e_ξ and $\widehat{e}_\xi$ **(b)** $\widehat{\Delta}_\Xi$

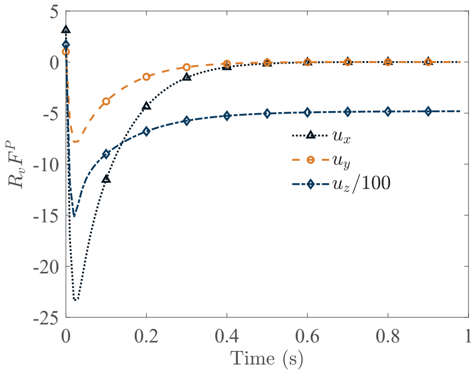
(c) Z_1 **(d)** Z_2

Figure 4. Disturbance observer state variable of the hybrid aerial underwater quadrotor.

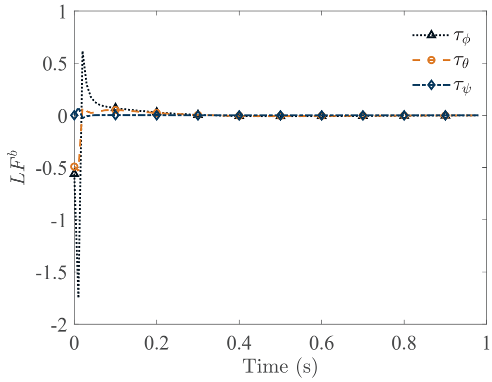


(a) $A_p(t,\dot{P})$ and its estimate $\widehat{A}_p(t,\dot{P})$ **(b)** $B^\omega(t,\omega^b)$ and its estimate $\widehat{B}^\omega(t,\omega^b)$

Figure 5. The unknown uncertainty and its estimate of the hybrid aerial underwater quadrotor.



(**a**) Force input R_vF^p of the hybrid aerial underwater quadrotor.



(**b**) Torque input LF^b of the hybrid aerial underwater quadrotor.

Figure 6. Control input variable of the hybrid aerial underwater quadrotor.

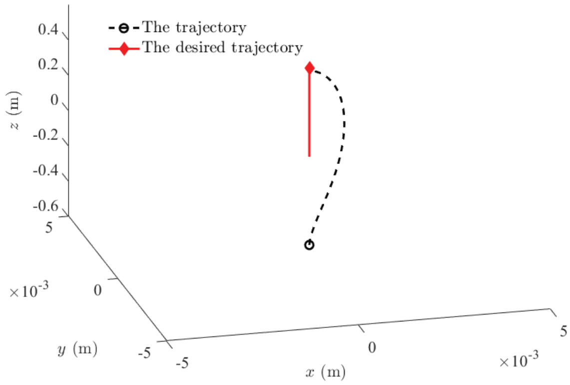


Figure 7. Three-dimensional trajectory of water surface take−off.

Figure 2 shows the position and the velocity curves in the (x, y, z) directions. Figure 2a,b illustrate that the position changes of the HAUQ in the two directions of the $x-$axis and $y-$axis are less than 10×10^{-4} m and 4×10^{-4} m, which shows that the HAUQ climbs. After $t > 0.6$ seconds, Figure 2c illustrates that the HAUQ reaches the fixed−point hover drainage position. Figure 2d gives the velocity curves of (x, y, z). The velocity in the $x-$axis and $y-$axis is less than 0.02 m/s, and that in the $z-$axis is less than 8 m/s. Figure 3a gives the change curves of the attitude angle (ϕ, θ, ψ) and its desired angle command $(\phi_{cmd}, \theta_{cmd}, \psi_{cmd})$. Figure 3b gives the change curves of the attitude angular rate. The roll rate, the pitch rate, and the yaw rate are all less than 0.05 rad/s. Figures 2 and 3 actually show that the trajectory tracking errors and the angle tracking errors rapidly converge to a small neighborhood of zero. Figure 4 shows the curves of the disturbance observer states $(e_\xi, \widehat{e}_\xi), \widehat{\Delta}_\xi, Z_1, Z_2$. Figure 4a shows that $\widehat{e}_\xi$ can realize the dynamic observation of e_ξ. Figure 4b gives the estimate $\widehat{\Delta}_\Xi$ of Δ_Ξ. The state variables Z_1 and Z_2 are shown in Figure 4c,d. The approximation effect of the uncertainty term caused by complex kinematics is shown in Figure 5. Figure 5a illustrates that our proposed method combined with the adaptive approach and the polynomial method has good effect, and the initial value of the disturbance estimation in the z direction of $A_p(t, \dot{P})$ reaches about 700 due to the position feedback in approximation. The disturbance term $B^\omega(t, \omega^b)$ and its estimation are shown in Figure 5b. The control input curves of position (x, y, z) and the attitude (ϕ, θ, ψ) are given in Figure 6. Finally, the three-dimensional flight trajectory of the mass center of the HAUQ in the water surface take-off is shown in Figure 7 which means that the HAUQ can achieve climbing and hover drainage as the fixed point (0, 0, 0.5 m) with the proposed control algorithm, the uncertainty estimator, and the disturbance observer. However, the unmodeled items, the time−varying drainage mass M_Δ, the time−varying disturbance moment of inertia matrix $J_{\Delta}(t)$, the external disturbance, and the time-varying change matrix of the arm of the force L_{Δ}, exist and are given by (69), (70), and (73), and the position and the attitude can achieve a successful nonlinear robust tracking of their desired values under the proposed algorithm with the robust adaptive law $\dot{\hat{\Omega}}$ and $\dot{\hat{\Xi}}$ and the disturbance observer (43) in this simulation scenario.

5. Conclusions

In order to solve the problem of climbing and draining water from a slender HAUQ, a robust position and attitude control law with the adaptive law of the unknown approximation weights and a four-order disturbance observer are proposed by using the robust control method, the uncertainty approximation approach, and the disturbance observer. The proposed control law can effectively compensate and suppress the model uncertainty and the additional disturbance caused by the drainage, the multi-media complex dynamics, the gust, and other factors. A numerical simulation shows its effectiveness.

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