



# Article Modeling of Traffic Flows Sustainability on Highway Network Stretches

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Abstract: Assessing the transport flow robustness is a significant aspect of a qualitative solution to traffic management problems. Therefore, management should be based on appropriate criteria, accounting for different factors characterizing traffic flow sustainability. That's why it is crucial to establish the impact rate for each group of factors on the robustness criterion. Therefore, the current study aims to obtain the dependence of the criterion changes for traffic flow sustainability on the traffic jam occurrence when changing the gradients' product of traffic flow density and its speed. The value of the robustness criterion allows for performing an impact rating for input factors on traffic flow sustainability. All factors affecting transport flow robustness are divided into three groups. Based on simulation results, factors rating that impact the robustness margin value of the traffic flow is presented. Length and weight of automobiles are at first place according to impact terms on the sustainability loss of the traffic flow. In second place of impact on sustainability loss are the temporary factors group and factors group that considers the roadway environment's infrastructure. Hence, the results can be used to analyze sustainability traffic flows in controlled highway network stretches and develop measures to increase sustainability reserve.

**Keywords:** sustainability traffic flow; mathematical modelling; dynamic model; density gradient; velocity gradient; stability criterion; robustness traffic flow; sustainable supply chain

# 1. Introduction

Traffic flow and traffic jams are major societal, economic, and environmental issues caused by road transportation in developed nations' urban areas. Therefore, traffic monitoring on congested highways requires a clear understanding of what causes road congestion, which determines traffic jams' time and place, how congestion spreads along the road network, etc. [1]. Over the past fifty years, numerous traffic flow theories and models have been developed to address these research questions. Using traffic flow modeling is a cost-effective way of studying transportation, which allows for reliable predictions of delays and traffic congestion. The variety of traffic controlling options leads to using numerous criteria, making it possible to model and predict the sustainability of traffic flows and traffic jam conditions [2,3].

Transport flow density is the main factor in traffic modeling [4–6]. For example, in research [4], the transport flow density is considered an equilibrium, constant value. In this case, equilibrium flux densities were obtained analytically by the authors of this study. The



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**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). investigation [5] provides obtaining method for transport flow density corresponding to driving near intersections. The research [6] shows the application of variable flow density in models. The authors obtained solutions for different density values. This approach improves prediction accuracy.

An executed review of current research on solving traffic management problems has provided the following framework for the main approaches and methods applied in this field (Figure 1).



Figure 1. Classification of methods and models for solving traffic management problems.

Our study corresponds to the first group—mathematical models classified into three types (Figure 1). Therefore, a detailed literature review of the current studies for each mathematical model type is provided in the following sections.

# 1.1. General Models for Solving the Real-Time Traffic Management Problem and Predicting Traffic Flows

In the topic's context of our research, several prominent works can be distinguished on creating models [7–10] and modeling traffic flows in urban highway network sections [11–16].

Approach [7] proposes to unify two commonly used evaluations of traffic flow sustainability: probabilistic analysis of driving instability, accounting random nature of traffic speed, and analysis of non-fatal accidents. The method is used due to online monitoring by highway network stretches. It is acceptable on a road network that is equipped with appropriate controls. However, the approach falls short of a comprehensive sustainability assessment, considering the road environment and driver's behavior.

Study [8] presents traffic-changing possibilities, considering visibility on highway sections near intersections. The authors proposed a 3D model evaluating changes in the traffic flow parameters based on road infrastructure at intersections. The study offers only one sustainability criterion.

Papers [9,10] describe traffic sustainability dependencies on truck delays during route passes on urban highways. Regrettably, these studies only relate to sustainability indirectly because criteria for assessing traffic flow robustness weren't established. However, these studies identified a group of factors that slow down other vehicles in traffic.

Today, using intelligent transport systems is carried out not only for assessing traffic flow parameters but also for forecasting. Studies [11,12] describe how during predicting traffic flow, one can consider the geometric speed of the driver's driving in different weather and climatic conditions (fog, clear weather, fog with an on-board detection system, etc.) [11]. The second research in the same direction was carried out on the indirect road section, considering the driver's behavior simulation due to poor visibility, particularly fog. The results of these two studies are shown in examples of Brazilian highways. In our opinion, despite approach innovation, it evaluates traffic flow sustainability only from one perspective.

Traffic forecasting is carried out by comparing the expected active traffic management schemes for the four busiest highways in England [13]. This approach will improve sustainability and energy independence. The study shows only a general assessment and directions for finding solutions to increase robustness. Specific criteria are not applied, which is a lack.

Applying neural networks for driving speed prediction is another direction to improve traffic flow sustainability. The study is promising due to the smart cities' concept [14]. However, significant statistics are needed for such an assessment. The challenges of implementing neural networks are often linked to issues that arise during their training, such as self-learning duration and tuning quality. The solution is a hybrid algorithm using, considering data lack and factors restrictions [15].

Prediction of traffic flow parameters is provided according to signal retrospective [16]. In this case, the model accuracy could be doubted, especially when there are no signals or have hopping values that exceed the extreme traffic flow behavior.

The above-described models are general and have several restrictions on application but further highlight the significance of chosen research direction. Therefore, the criterion design for transport flow robustness should be based on different groups of factors, their modeling, assessment, and ranking due to each value.

Therefore, it is essential to examine each mathematical model type depicted in Figure 2 in greater detail. Because our research is completely correlated with such an approach. According to the above, it is possible to clearly identify knowledge gaps in current scientific papers devoted to traffic flow sustainability assessment based on mathematical modeling.



Figure 2. Research flowchart (main steps).

## 1.2. Macroscopic Models of the Traffic Flows

Modeling of heterogeneous traffic flows is presented in the works [17,18]. The authors of [17] have developed a new macroscopic model of a heterogeneous transport flow with partial connection to an automated environment based on the theory of a potential security field. In the study [18], the macroscopic model of heterogeneous transport flow considers the velocity gradient and the driver's delay time in making decisions.

The research study [19] suggested a macroscopic model that utilizes the relaxation time parameter. This parameter is used as a characteristic of traffic flow density change. According to research results, vehicle interaction in traffic affects relaxation time. Relaxation

times include driver behavior that affects the perception and handling of road situations and related activities. Research [20] considers data set on the flows and vehicle speeds driving along the highway, accumulated by fixed sensors and classified by lanes and vehicle classes. The authors extrapolate two important pieces of data with a neural network-based learning model: the traffic jams prediction and total vehicle quantity that presumably passes through a controlled section of the route. This information is used for improving simulation accuracy.

The studies [21,22] conclude that stochastic factors significantly contribute to the destabilization of the traffic flow. The research study [21] investigated the stochastic impact on transport flow sustainability using Lyapunov's method. Based on the obtained results, the authors concluded that the stochastic presence destabilized macroscopic models and modeling results. In the research study [22], a critical condition of transport flow sustainability was obtained. The motion of the density wave is simulated, which affects the occurrence of traffic jams.

The authors of the research [23] note that successfully using macroscopic models requires calibration. According to the scientists who developed these models, it is necessary to calibrate and check real data of traffic flows. Model calibration determines the optimal set of parameters that minimizes the discrepancy between simulation results and actual data. In addition, model validation is performed to confirm calibrated model accuracy.

A sustainable control model [24] considers uncertainties that could arise while driving (vehicle entrance/exit from the highway, traffic jams occurrence, and other factors.). The model defines the management effect that wholly improves transport network functioning and considers only traffic features. In our opinion, it is not enough to fully assess the traffic flow robustness.

The authors propose to use the ensemble method for short-term forecasting of traffic parameters [25]. Therefore, the success of this method relies heavily on the quantity of data gathered regarding the traffic flow. The model considers speed, density, occupancy, and a sufficient degree of traffic. The state of the road infrastructure and driver behavior are not considered.

The study proposed combines multitask learning (MTL) with a graph convolution network [26]. The current model is combined since it includes several graphs characterizing the state of the transport network at different intervals. In our opinion, the approach is interesting but requires significant resources to accumulate traffic information. Also, the model becomes bulkier every time. It complicates the efficiency of calculating the traffic flow robustness.

The paper [27] proposes a deep-learning architecture using models of folded sparse auto-encoders (SAE). It gives possibilities to accurately assess the traffic flow by the whole network with an already deployed set of sensors. Despite the promise, the model requires accurate and fine-tuning. Unfortunately, this is not always possible.

Summarizing the analysis of traffic flow publications in macroscopic models, it should be noted that such models are used for simulating traffic situations on large sections of highway networks, for example, predicting delays and traffic jams. The main factors in such models are transport flow density and speed. These factors are proportional to the transport flow "energy". However, in our opinion, such a choice of factors is a necessary but insufficient term for modeling.

## 1.3. Microscopic Models of the Traffic Flows

Another class of models is traffic flow's microscopic models. Microscopic models are devoted to studies [28–38]. The models presented in the articles [28–30] are devoted to studies of automobile driving behind the car in front. Models include the standard deviation of speed and allow reproducing fluctuations in traffic flow. According to the authors of the research, the modeling results can be used in advanced motion control systems in real-time to ensure accurate prediction of delays.

In the papers [31-34], the average heterogeneity effects of automobiles driving in several lanes' presence for travel and the emergence of conflict situations between vehicles when changing lanes were investigated. For example, in the study [32], a factor such as weather conditions is included in the model. In the article [33], such parameters as vehicle acceleration and braking when driving behind an automobile in front are added to the listed factors. The manuscript [34] considers different driving styles and differences in vehicle parameters (speed, acceleration, deceleration, etc.). Such factors were included in the model using principal component analysis and the *k*-means clustering method. According to the authors, considering such factors allows for increasing the microscopic mathematical models' adequacy to a real road situation and predicting emergencies and traffic jams occurrence.

Microscopic models include models for traffic situation simulation at intersections [35–37]. Investigation [35] considers an integrated microscopic model of transport flow for modeling vehicle maneuvers in conditions of their interaction. The model is designed on a cluster approach and considers the difference in driver response. The study [36] proposes a new method for vehicle trajectories simulating in two-dimensional space, considering their speed. The study [37] explains delays at unsignaled intersections and their entrance. A simulation was carried out based on technical parameters' data of the road infrastructure (road signs, markings, the presence of barriers reducing the driver visibility, etc.) at the intersections themselves and when approaching unsignaled intersections.

Studies [38,39] show the effect of vehicle stability on traffic flow sustainability while driving. According to the authors, an important aspect is knowing how the vehicle behaves depending on its driving trajectory. The authors show the automobile speed stability in the traffic flow using the example of a two-axle tractor and a semi-trailer. In this case, linear and non-linear wheel slip hypotheses are considered. It was concluded that the studied parameter significantly affects road train speed stability and the entire traffic flow sustainability, mainly if it contains a large percentage of trucks.

In papers [40,41], comparable studies are carried out reproducing a decrease in throughput at exits from highways and entrances to highways. The authors identified one of the main weaknesses of microscopic modeling of traffic at the exit and entrance to the highway—this is a decrease in throughput. The forced lane change model is proposed for vehicles entering the highway.

The study [42] proposes a features adaptive-R extended Kalman filter (AREKF) using a specific technique for traffic density estimation. The flow density estimate is selected as the basis criterion for traffic flow sustainability. The simulation was carried out for a highway stretch. This study does not consider the impact of other factors on traffic flow sustainability, which is a limitation.

The authors propose a new algorithm for mixed traffic control at unsignaled intersections in the paper [43]. This approach is carried out based on delays accounting during passing intersections. Although the experiment was conducted on a microscopic model, the approach is common and corresponds mainly to macroscopic modeling. The criterion for traffic flow robustness is poorly represented.

Summing up publications' analysis of traffic flows in microscopic models, it should be noted that such models are used for simulation road situations on small stretches of highways. The main factors in such models are driving speed, fluctuations in travel speed (acceleration, deceleration), and the driver's reaction to a change in the traffic situation.

## 1.4. Hybrid Models of the Traffic Flows

The third class of models is hybrid models of traffic flows. Hybrid models are devoted to papers [44–50]. These models combine macro and micro models' capabilities. The studies [44,45] provide hybrid models analysis of traffic flows over the past decade. The component modules of models and architectural solutions were analyzed. Such models are designed based on neural networks [45] with self-learning and self-control functions. A hybrid model is proposed to investigate the stopping and driving phenomenon caused by

passing vehicles in front, combining capabilities at macro and micro levels [46]. First, under the stochastic nature of driving behavior, Brownian noise is added to the speed difference to modify the model of vehicle passing. The traffic flow macroscopic characteristics are then added, and the two models are combined in the context of intensive transport flow. The basis for designing such models is traffic density estimation [47]. According to the authors, vehicle recognition and counting are two main steps for evaluating traffic density. Vehicle identification systems can be designed based on neural networks.

In [48–52], hybrid models are presented as intelligent traffic control systems. These system types effectively solve traffic jam issues in urban areas, increasing road capacity and ensuring driver safety. Intelligent traffic control systems' short-term forecasting of traffic flows is the main task.

After reviewing publications on hybrid models of traffic flows, it is important to mention that these models are primarily developed using neural networks. They are utilized for simulating road situations on both small and large stretches of highway networks. The main factors in such models are vehicle density and speed in traffic, fluctuations in speed (acceleration, deceleration), the driver's response to a change in the road situation, and the transport flow sustainability to traffic jams. We believe traffic flow density gradient and velocity gradient should be significant factors when using hybrid models. In the study [18], the velocity gradient of the transport flow was utilized, and in the paper [6], the alteration in flow density was analyzed. These parameters will make it possible to consider transport flow stochastic, which is noted in the investigation [21], and increase the prediction rate of the transport flow sustainability to traffic jams.

The authors used the same approach in the investigation [53]. This paper proposes a set of decision-making systems for optimizing transport flow to solve traffic jam issues in a specific area. To analyze actual driving conditions and calculate passing density and speed, a driving prediction model is designed and updated iteratively. Based on this model, the congestion rate on the current highway stretch is estimated, and therefore, methods of intelligent decision-making and consistent optimization are proposed. In addition, this article implements some application experiments on a controlled highway section with three intersections and provides perfect results for predicting density and speed. Besides, the presented model in this paper has higher accuracy, shorter prediction time, and stronger interference protection than other existing prediction methods. However, this study does not answer determining the sustainability limit on congestion occurrence.

In the paper [54], graph theory is used, according to relationships accounts between the spatial-local state of neighboring roads and the controlled one. In this case, traffic flow speed is predicted by delays. The model does not consider the driver's behavior and, therefore, their impact on traffic flow sustainability.

The study [55] proposed a criterion for estimating traffic based on robust vector equilibrium. To estimate the current rating of traffic flow sustainability, a subspecies of the minimax task is solved. Unfortunately, the application of this method to the real road network has not been shown. Therefore, the study is purely theoretical.

# 1.5. Knowledge Gap Identification and Research Novelty

The provided literature review allows a conclusion about criterion absence that could comprehensively assess traffic flow sustainability. A comprehensive evaluation should be based on multifactorial accounting. This approach provides new principles for solving traffic management problems. Therefore, there was a need to use a multivariate robustness criterion to establish the impact rate of each factor.

A distinctive feature of our study is the justification of the criterion for determining the moment or boundary of traffic jam occurrence. The proposed criterion, which has a "sustainability margin" physical sense to traffic jams, will allow a factors' rating impacting the traffic flow robustness margin. Applying this rating allows for choosing rational routes for goods transportation considering urban highway network, surveying an expert assessment of the existing transport system, and developing practical recommendations for improving traffic organization.

Therefore, the novelty of this study is to define dependencies of the robustness criterion value on the changes in gradients of density and speed traffic flow on the controlled highway stretch. These dependencies allow ranking the incoming factors by their impact rates on traffic flow sustainability due to traffic jam occurrence.

#### 1.6. Conceptualization of Current Research Framework and Study Objective

To increase the predicting accuracy of traffic jam occurrence in urban highways, it is necessary to have a criterion for assessing congestion occurrence moment (limit). It allows considering changing traffic situations over time. According to Lyapunov, the functioning of the transport flow should be considered a dynamic process, using the concepts of sustainability of technical systems [21].

This paper is a continuation results of the previous research [56], where a mathematical model structure is justified for ergonomic assessment of transport flow sustainability on different highway stretches under the external disturbances impact. The mathematical model takes into account the dynamic nature of the process development. In addition to traffic flow speed and density gradients, the dynamic vehicle properties, a multi-lane road network, and delays time at pedestrian crossings and traffic lights are considered. It is shown that a third-order differential equation describes the dynamic characteristics of the transport flows.

A continuation of the results provided in [56] is investigations of traffic flow sustainability (robustness) when vehicles driving on urban highways with a variation in input factors that affect the traffic flow sustainability. It will make it possible to obtain factors' rating that affects the value of the transport flow sustainability margin.

The study objective is to test and analyze the traffic flow sustainability mathematical model in various highway stretches under the external disturbances action and rank the factors affecting stability.

# 1.7. Purpose and Objectives of the Study

The main research aim is to study factor impacts characterizing traffic flow resistance to traffic jam occurrence, substantiate the criterion for assessing robustness, and rank the factors by their impact rates on sustainability.

The study includes the following research tasks (Figure 2).

#### 2. Materials and Methods

The paper [56] presents a third-order differential equation written in operator form, where p = d/dt, representing the differentiation operator:

$$(T_1T_2T_3)p^3 + (T_1T_2 + T_1T_3 + T_2T_3)p^2 + (T_1 + T_2 + T_3 + K_2K_3T_1)p + K_2K_3 + 1 = (K_1K_2T_3)p + K_1K_2$$
(1)

After analyzing the experimental data, the paper [56] clarified the time constants and gain factors of the mathematical model's adequacy.

The value of the time constant  $T_1$  is represented by an equation. It characterizes the driver's inertia, depending on traffic flow density and intensity values:

$$T_1 = \frac{t_1^2 \cdot N \cdot l_a \cdot \sigma_a}{70 \cdot L \cdot v}, s$$
<sup>(2)</sup>

where  $t_1$ —the driver's reaction time to changes in traffic situations can range from 0.6 to 1.4 s; *N*—vehicle quantity on the controlled highway stretch;  $l_a$ —vehicle length, m;  $\sigma_a$ —standard deviation of vehicle acceleration in traffic flow, m/s<sup>2</sup>, a formula for calculation is submitted in the paper [56]; *L*—the typical length of a controlled highway stretch is about 1000 m; *v*—vehicle speed in traffic, m/s.

The value of the time constant  $T_2$ , which characterizes the vehicle inertia and is expressed in maneuverability, is represented by the formula,

$$T_2 = \frac{M \cdot l_a^2}{1000 \cdot N_e \cdot t_2^2}, \, s$$
(3)

where *M*—vehicle weight, kg;  $N_e$ —rated engine power of the vehicle, W;  $t_2$ —time for a maneuver that the vehicle can use when changing the traffic situation in the controlled area, s.

The value of the time constant  $T_3$ , which characterizes the inertia in the traffic situation changing, is represented by the formula:

$$T_3 = \frac{n \cdot t_2^2}{5 \cdot t_3}, \, s \tag{4}$$

where *n*—lanes quantity on the roadway;  $t_3$ —total delay time in passing route, s.

The formula representing the gain coefficient,  $K_1$ , which indicates how traffic density affects a driver's reaction time rate, is as follows:

$$K_1 = \frac{N \cdot l_a}{L}.$$
(5)

Traffic density gradient:

$$gradp = \frac{\sigma_N}{L^2}, \, 1/m^2 \tag{6}$$

where  $\sigma_N$ —the standard deviation of vehicle quantity in the traffic flow, the formula for calculation is presented in the paper [56].

By comparing Equations (5) and (6), it can be concluded that the coefficient is  $K_1$  proportional to the density gradient of the transport flow.

The gain coefficient  $K_2$ , which characterizes the rate of dynamic traffic flow impact on the traffic delay time and sustainability loss, is represented by the formula:

$$K_2 = \frac{\sigma_a \cdot N^2 \cdot l_a^3}{v^2 \cdot L^2} \tag{7}$$

Traffic velocity gradient:

$$gradv = \frac{\sigma_a}{v \cdot L}, \ 1/ms \tag{8}$$

By comparing Equations (7) and (8), the  $K_2$  factor is proportional to the traffic velocity gradient.

The formula that represents the gain coefficient,  $K_3$ , which determines how much the delay time and sustainability are affected by changes in traffic situations during driving in the flow, is as follows:

$$K_3 = \frac{N \cdot l_a \cdot k^2 \cdot s^2}{10 \cdot L} \tag{9}$$

where *k*—pedestrian crossings' quantity in the controlled stretch, units; *s*—traffic lights quantity in the controlled stretch, units.

In differential Equation (1), the right-hand side describes the input signal as a disturbance that impacts the transport flow as a dynamic system. These are coefficient values  $K_1$  and  $K_2$  and the time constant  $T_3$ . In addition, on the right side of the Equation is the first derivative of the product of the listed parameters. The sustainability of traffic flow is affected by more than just the levels of  $K_1$ ,  $K_2$ , and  $T_3$ —it also depends on the rate at which they change over time. Therefore, it is possible to formulate a factor on which to perform modeling and studies of transport flow for sustainability. The product of the density gradient Equation (6) and velocity gradient Equation (8) is directly proportional to the gain coefficients  $K_1$  and  $K_2$ .

To increase the mathematical model adequacy, it is necessary to introduce a dynamic factor into the robustness criterion, which is justified in the study [56], which will be considered the right side of the differential Equation (1):

$$k_d = \frac{23,99 \cdot 10^6 \cdot K_1 \cdot K_2 \cdot T_3}{t_4},\tag{10}$$

where  $t_4$ —the time of the traffic situation on the controlled highway section, s. For instance, in an urban network, the duration of the green traffic light signal is essential for its proper functioning.

Considering the dynamic factor, the criterion of traffic flow robustness, which is presented in the paper [56], will take the form:

$$RR = \frac{\left[ (T_1T_2 + T_1T_3 + T_2T_3) \cdot (T_1 + T_2 + T_3 + K_2K_3T_1) \right]}{\left[ (T_1T_2T_3 \cdot K_2K_3) + T_1T_2T_3 \right] \cdot k_d}.$$
 (11)

The traffic flow sustainability criterion *RR* is named the robustness criterion. It evaluates the possibility of traffic jams. The criterion was obtained based on a theory of technical systems robustness found by Lyapunov [21]. The mathematical transformation of the criterion follows from the differential Equation (1).

# 3. Results

The study of the transport flow sustainability (robustness), RR, Equation (11), when driving along urban streets and highways, will be carried out by changing the differential Equation (1) right side. This parameter is shown as the density gradient product, Equation (6) and velocity gradient, Equation (8):

$$gradp \cdot gradv = \frac{\sigma_N \cdot \sigma_a}{L^3 \cdot v}, \left[\frac{1}{m^3 \cdot s}\right]$$
(12)

In addition, the input parameters on which the traffic flow robustness depends will be: vehicle length  $l_a$ , m; vehicle weight M, kg; vehicle engine power  $N_e$ , kW; driver's reaction time to the traffic situation  $t_1$ , s; vehicle maneuver time according to traffic situation changes  $t_2$ , s; total delays time driving route  $t_3$ , s; lanes quantity on the roadway n, unit; pedestrian crossings k and traffic lights quantities s in the controlled stretch, unit.

The next input parameter values (Table 1) were chosen during the simulation of traffic flow robustness. The sustainability of traffic flow depends on three indicator groups, which are displayed as parameters. During modeling, three levels of values were utilized for each parameter: the minimum, average, and maximum.

Table 1. Parameters Value for Transport Flow Robustness Simulation.

Parameter for Modeling	T 1 1	Value					
ratameter for wodening	Label	Minimal	Average	Maximal			
Vehicle Length, m	la	4	12	20			
Vehicle weight, kg	M	2000	10,000	18,000			
Vehicle engine power, kW	$N_e$	100	150	200			
Driver's reaction time to the traffic situation, s	$t_1$	0.6	1.0	1.4			
Vehicle maneuver time according to traffic situation changes, s	$t_2$	4	7	10			
Total delays time driving route, s	$t_3$	50	150	250			
Lanes quantity on the roadway, unit	п	1	2	4			
Pedestrian crossings and traffic lights quantities in the controlled stretch, unit	k/s	2/2	5/5	8/8			

For clarity displaying the modeling process conducted in the study, Figure 3 depicts the incoming and decision variables and obtained results as target functions. Figure 3 fully presents factors that influence the sustainability in the traffic flows model.



Figure 3. Modeling Structure.

Figure 4 presents the dependencies of traffic flow robustness across various density and speed gradients, with changes in vehicle lengths considered. If the robustness criterion is RR = 1, the traffic flow loses sustainability, i.e., traffic stops or a traffic jam is formed. According to a study [56], the presence of vehicle traffic in the flow can be identified by multiple values of the robustness criterion. When the value of the *RR* criterion is higher, it indicates that the margin for traffic robustness has increased. If the *RR* value is less than one, it indicates a traffic stop and the appearance of congestion.



**Figure 4.** Dependencies of traffic flow robustness range for different values of density and speed gradients when changing vehicle lengths:  $1-l_a = 4 \text{ m}$ ;  $2-l_a = 12 \text{ m}$ ;  $3-l_a = 20 \text{ m}$ .

According to the analysis of the dependencies displayed in Figure 4, vehicle length is a significant factor that affects traffic flow robustness. In curve 3, when the vehicle length measures 20 m, sustainability loss occurs at low gradient values as indicated by the crossing with RR = 1 line. Conversely, with a vehicle length of  $l_a = 4$  m, the sustainability margin is achieved at 40 units, as demonstrated in curve 1 for cars.

Vehicle weight is the second most crucial factor that impacts traffic flow robustness. Figure 5 illustrates the variations in traffic flow sustainability under different density and velocity gradient scenarios while accounting for changes in vehicle weight. According to displayed dependencies, increasing vehicle mass reduces the traffic flow robustness. Please note that the robustness range is reduced by half compared to changes in vehicle length and does not exceed a value of RR = 20. Loss of traffic flow sustainability occurs at higher values of gradients.





The vehicle engine power is the third most important factor that affects traffic flow robustness. Figure 6 showcases the dependencies of traffic flow robustness for varying density and velocity gradients, considering alterations in vehicle engine power.





According to the presented dependencies, increasing the vehicle engine power increases the traffic flow robustness. However, the level of impact is minimal and does not surpass RR = 7. Loss of traffic flow sustainability occurs at even high values of gradients.

Vehicle length, Figure 4, vehicle weight, Figure 5, and Vehicle engine power, Figure 6, constitute a group of design factors that characterize the vehicle and its effect on traffic flow robustness.

The next set of factors can be named temporary factors that consider: the driver's reaction time to the traffic situation, Figure 7; vehicle maneuver time according to traffic situation changes, Figure 8; the total delays time driving the route, Figure 9.



**Figure 7.** Dependencies of traffic flow robustness range for different values of density and speed gradients when changing the driver's reaction time to the traffic situation:  $1-t_1 = 0.6$  s;  $2-t_1 = 1.0$  s;  $3-t_1 = 1.4$  s.



**Figure 8.** Dependencies of traffic flow robustness range for different values of density and speed gradients when changing vehicle maneuver time according to traffic situation changes:  $1-t_2 = 4$  s;  $2-t_2 = 7$  s;  $3-t_2 = 10$  s.



**Figure 9.** Dependencies of traffic flow robustness range for different values of density and speed gradients when changing total delays time driving route:  $1-t_3 = 50$  s;  $2-t_3 = 150$  s;  $3-t_3 = 250$  s.

The average impact rate on the traffic flow's robustness range can be determined by analyzing the dependencies presented. Factors such as the driver's reaction time to the road situation ( $t_1$ ) and the total delay time when driving the route ( $t_3$ ) play a significant role in the current study. Note, that the robustness range of  $t_1$  and  $t_3$  does not exceed RR = 20. Loss of sustainability in traffic occurs at average gradients' values.

The extent of the Impact on the duration for maneuver,  $t_2$ , which can use vehicles when traffic situation changes, is presented in Figure 8. If a vehicle driver has more time to maneuver (according to curve 3), then traffic flow robustness will increase. When there is limited time for maneuvering, traffic will inevitably come to a stop (as shown in Curve 1). The robustness test value at the change of density and velocity gradient values does not exceed 6 units.

The third group of factors considers features of road environment design. Such factors include lane quantity on the roadway, Figure 10; pedestrian crossings, and traffic light quantities in the controlled stretch, Figure 11.



**Figure 10.** Dependencies of traffic flow robustness range for different values of density and speed gradients when changing lanes quantity on the roadway: 1-n = 4; 2-n = 2; 3-n = 1.



**Figure 11.** Dependencies of traffic flow robustness range for different values of density and speed gradients when changing pedestrian crossings and traffic lights quantities in the controlled stretch: 1-k = 2; s = 2; 2-k = 5; s = 5; 3-k = 8; s = 8.

Based on the displayed data, it appears that adding more lanes in the roadway results in an increased robustness criterion to RR = 22, as shown by curve 1. It helps to enhance traffic flow sustainability across a wide range of density and velocity gradients. Therefore, the probability of traffic jams is minimal. An increase in crosswalks, k, and traffic lights, s, on the controlled highway stretch leads to decreasing sustainability, promoting the emergence of traffic jams. It should be noted that the robustness range at k and s changing does not exceed RR = 6 values.

The results of the RR robustness test calculation for all values of the evaluation factors are displayed in the resulting Table 2.

Parameter for Modeling		Grad $p \cdot Grad v \cdot 10^{-9}$ , $1/m^3 \cdot s$										
		2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5
Vehicle length, m	4	10	9.3	8.6	8	7.3	6.6	6	5.3	4.6	3.3	2.6
	12 20	24 50	22.3 49	20.6 48	19 47	17.3 46	15.6 45	14 44	12.3 43	10.6 42	7.3 40	5.6 39
Vehicle weight, kg	2000	3.84	3.2	2.64	2.15	1.74	1.4	1.12	0.91	0.78	0.72	0.7
	10,000 18,000	6 19.7	4.15 14.07	3.17 11.55	2.7 10	2.36 9	2.12 8.25	1.94 7.65	1.8 7.17	1.68 6.77	1.58 6.43	1.5 6.1
Vehicle engine power, kW	100	5.13	4.65	4.18	3.73	3.29	2.88	2.48	2.1	1.73	1.38	1.05
	150 200	5.65 6.35	5.13 5.8	4.63 5.25	4.15 4.72	3.7 4.21	3.24 3.72	2.8 3.25	2.38 2.8	1.98 2.35	1.6 1.92	1.22 1.51
Driver's reaction time to the traffic situation, s	0.6	5	4	3.17	2.52	2	1.6	1.27	1	0.8	0.64	0.51
	$1.0 \\ 1.4$	7.18 11.44	5.7 9.1	4.52 7.24	3.6 5.75	2.85 4.58	2.26 3.64	1.8 2.9	1.42 2.3	1.13 1.83	$0.9 \\ 1.45$	0.71 1.15
Vehicle maneuver time according to traffic situation changes, s	4	0.84	0.71	0.6	0.5	0.4	0.34	0.28	0.25	0.22	0.21	0.2
	7 10	1.61 5.7	1.4 5.0	1.21 4.37	1.04 3.8	0.9 3.25	0.8 2.76	0.7 2.32	0.62 1.93	0.58 1.58	0.55 1.29	0.5 1.0
Total delays time driving route, s	50	8.75	8.23	7.7	7.18	6.6	6.14	5.62	5.1	4.58	4.06	3.54
	150 250	12.5 22.5	11.7 21.0	10.91 19.54	10.12 18.06	9.3 16.58	8.54 15.1	7.75 13.62	6.95 12.14	6.16 10.66	5.37 9.18	4.58 7.7
Lanes quantity on the roadway, unit	1	9.6	8.69	7.86	7.11	6.44	5.82	5.27	4.77	4.31	3.9	3.53
	2 3	14.04 23.07	12.54 20.42	11.2 18.07	10.0 16.0	8.93 14.16	7.98 12.53	7.12 11.09	6.36 9.82	5.68 8.69	5.07 7.69	4.53 6.81
Pedestrian crossings and traffic lights quantities in the controlled stretch, unit	2/2	5.25 5.41	3.15 4 7	2.34 4.0	1.9 3.52	1.6 3.05	1.4 2.65	1.25	1.13	1.04	0.96 1 49	0.9 1 29
	8/8	5.53	5.26	5.0	4.76	4.53	4.31	4.1	3.9	3.71	3.53	3.35

Table 2. Results of the RR Test Calculation due to Evaluation Factors.

# 4. Discussion

By analyzing Equation (11), which is caused as a criterion for transport flow robustness, one can identify the parameters that impact sustainability. Regarding the above mathematical formulations by which the  $K_2$  gain coefficients are calculated, the transport flow sustainability is  $K_3$  affected by traffic density and intensity. These parameters should be calculated according to Equations (7) and (9) for each stretch of the road network or highways, which should be investigated and substituted into Equation (11). The time constants  $T_1$ ,  $T_2$ , and  $T_3$  depend on driver qualification and psychophysiological properties, the fatigue rate, the dynamic properties of vehicles, and traffic conditions. The values for these parameters can be calculated using Equations (2)–(4) and then plugged into Equation (11).

If the criterion value equals one, the traffic flow functions on the verge of sustainability loss. If the traffic flow sustainability is lost, the criterion value would be less than one, resulting in traffic coming to a halt due to traffic jams occurring. When the criterion value exceeds one, it indicates that the traffic flow is sustainable and operates smoothly without any disruptions or congestion. The bigger criterion value means a greater sustainability margin.

The *RR* robustness criteria should be applied when analyzing the road network for delays during traffic and traffic jams and when designing a new urban highways network.

The obtained result differs from previous studies in this field because it allows for determining sustainability loss limits by modeling—traffic jams occurring. Limit values determining transport flow density and intensity and their gradients will allow designing measures to prevent traffic jams considering multi-lane driving.

These results are based on previous research discussed in papers [56,57]. The accounting of process development dynamics is the main difference in the current study from others in the same scientific sphere. In addition to velocity and density gradients of traffic flows that are used in these researchers, the dynamic features of vehicles and the multi-lane highway nature, as well as delay time at pedestrian crossings and traffic lights, are considered.

Based on theoretical studies, it has been found that the mathematical model for traffic flow sustainability has restrictions to its application. Limitations are related to source data finding for the simulation. On the controlled highway stretch, it is necessary to determine flow density and intensity, vehicles' speed, and their acceleration in traffic. Modern electronic controllers are capable of providing such measurements. In addition, statistically determined values are drivers' reaction time to changes in traffic and the appearance of pedestrian crossings and traffic lights on monitored highway stretches.

It is important to acknowledge that the proposed approach has several limitations in application:

- The study of factors that impact traffic flow sustainability was only conducted in the values range presented in Table 1;
- robustness criterion assessment was only applied for a highway stretch with a fixed length equal to one kilometer;
- the group of factors characterizing weather conditions was not considered when determining the dependencies of factors that impact the target function *RR*;
- the robustness criterion is only applicable to evaluating traffic flow sustainability on a
  particular highway section and cannot be used to assess the entire urban transport network;
- *RR* did not directly consider the vehicle intensity, although the traffic flow density and speed gradients were used among the estimated factors.
- We hope these limitations do not reduce the scientific and practical significance of the proposed approach.

The next step in the research should focus on creating reliable models to measure transportation flow density and intensity. These models should also be able to predict changes in vehicle density and speed within the traffic. It allows the development of a unified methodology for modeling traffic flow sustainability to provide forecasts of traffic jams and transport loads on highways.

# 5. Conclusions

The study of the traffic flow sustainability (robustness) when driving vehicles on urban streets and highways was carried out. The transport flow sustainability is shown to be a product function of the traffic flow density gradient and its velocity gradient.

All input factors affecting transport flow sustainability are proposed to be divided into three groups. The first factors characterize the vehicle design, particularly automobile length, weight, and engine power.

The second group of factors, named temporary factors, considered: the time of the driver's reaction to traffic situation changing; the time for a maneuver that a vehicle can use when changing a traffic situation; the total delay time in route passing.

The third group of factors considers the peculiarities of infrastructure building of the traffic. Such factors include the lane quantity on the roadway, pedestrian crossings, and traffic light quantity in the analyzed stretch.

Based on the simulation results, the factors rating that impact the value of the transport flow sustainability margin is shown. Vehicle length and weight are in the first place according to impact terms on the traffic flow sustainability loss. The groups of temporary factors and factors considering road environment infrastructure are in second place according to the impact criterion on the traffic flow sustainability loss. The impact rating of these factors can be considered the same.

It has been shown that the mathematical model and simulating results can be used to analyze the traffic flow sustainability in monitored highway stretches and develop measures to increase the stability margin.

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