

Article

Kinematic Modeling of a Trepanation Surgical Robot System

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Abstract: This paper presents the concept of a parallel medical robotic service system to assist in a surgical procedure involving precise exploratory trepanation holes in a patient's skull. The target position and orientation of the trepanation tool in the cranial region is determined using a prior intracranial image analysis using an external medical imaging system. A trepanning actuation system is attached to the end-effector of the parallel robot. The end-effector will act as an accurate positioner for the trepanning drill in the medical intervention area. The conceptual design of the mechanical actuation subsystem of a trepanning robot was developed in the SolidWorks 2022 software environment. The virtual model of the kinematic chain of the robot and the assumed design parameters were used to analytically derive the equations describing the inverse kinematics task. An analysis of the forward kinematics task of the parallel manipulator was also carried out using analytical and numerical methods. A workspace analysis was performed using Matlab based on the kinematic model of the parallel robot. This paper significantly advances the field by presenting the conceptual design of the actuation subsystem, deriving the kinematics equations, conducting a thorough workspace analysis, and establishing a foundation for subsequent control-algorithm development.

Keywords: trepanning; medical robotics; parallel robots; surgical robots; kinematics modeling; positioning



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1. Introduction

The neurosurgical procedures of cranial trepanation belong to the group of highly complex, medically invasive procedures. Historically, these procedures have been carried out since prehistoric times, as evidenced by documented archaeological finds of human and animal skulls dating back to 5000 BC [1–3]. Contemporary archaeological studies of skulls found from that time indicate a high mortality rate of patients, mainly due to infection and brain damage acquired during the procedure. The skulls found in many cases also suggest that a large proportion of patients survived such procedures and were likely to continue to function well for years or decades after the procedure [4,5]. Archaeological research material also suggests that such procedures were performed on deceased patients, in many cases most likely to refine techniques in trepanning practice [6]. Considering the dated age of these archaeological finds and identifying the possible tools that were probably used to perform the procedures (predominantly specially formed flint, which allows for scraping, circular grooving, drilling and cutting, and linear cutting with angular intersections [7]), they can be classified in the category of very controversial, from an ethical point of view, early medical procedures.

The main indication for cranial trepanation surgery (or burr hole drainage surgery) is to restore the pressure balance between the nerve tissue that makes up the brain, the cerebrospinal fluid, and the vessels through which the blood flows. This situation caused,

for example, by post-accident trauma resulting in an intracranial haematoma or other factors causing an increase in brain swelling, e.g., an oncological tumour, hydrocephalus, a physical fracture, or a fracture of a skull bone which, by compression, leads to an increase in the volume of organs and tissues (an increase in intracranial pressure). Such procedures are also carried out for exploratory purposes or for the purpose of a biopsy.

The modern surgical instruments used for cranial trepanation are, in the vast majority of cases, technically only a further development of the instruments used in the 18th to 20th centuries AD. Their overall form technically resembles hand tools for wood or metalworking rather than modern surgical instruments. Skull-trepanation procedures in the reality of the modern hospital are, of course, also supported by modern medical imaging equipment such as MRI (magnetic resonance imaging) and CT (computer tomography) scanners, which can provide a full skull data set in just a few seconds [8], and coupled robotic systems. This already represents 21st-century technology.

One of the many technical and technological challenges of modern medical robotics is to build systems that can assist or replace some of the activities performed by a surgeon. The use of robots and manipulators to assist the surgeon in the operating theatre is being investigated by many research centres nationally and internationally [9–11]. For example, according to the Intuitive Surgical Company [12], there are currently more than 7500 Da Vinci surgical robot systems in use in hospital operating theatres in 70 countries around the world, including, according to the manufacturer, neurosurgery. Other well-known commercial service robotic solutions dedicated to neurosurgical procedures are the Zeus robotic systems still in use (this system is currently out of production) from Computer Motion Company [13,14], or the Neuromate stereotactic robot—a platform solution for a wide range of functional neurosurgical procedures [15]. The SurgiScope system is the next example of a robotic tool-holding and tool-positioning system for neurosurgery [16]. The same functionality had robot neuroArm. This is an image-guided MRI-compatible robot for brain surgery [17]. All the robotic systems identified above fall into the category of serial robots. The literature on the subject also defines robot designs based on closed parallel structures, including mainly tripod, quattro, and hexapod robots [18–21]. For each of the above types of parallel robots, researchers around the world are trying to develop new methods for the structural analysis and synthesis of such mechanisms, using both analytical and numerical methods [19,21]. Important engineering issues to be considered in the practical implementation and use of parallel robots, in the various applications of parallel mechanisms, are above all the problems of determining the singular positions of the mechanism, as well as the selection of optimal design parameters of the links forming the kinematic chain, so as to obtain the largest possible working space [22–24]. However, most of these technical solutions are currently at the stage of laboratory prototypes or test or development platforms when it comes to medical applications. An example of such a solution is the hexapod functional prototype robot developed by The Fraunhofer-Institut für Produktionstechnik und Automatisierung (IPA), Stuttgart, Germany, dedicated to microsurgery [25]. Another interesting solution is a novel skull-mounted robot with a compact and lightweight parallel mechanism for positioning surgical tools in minimally invasive neurosurgery [26]. An analysis of technical solutions based on a parallel mechanism for the RCM (remote centre of motion) of surgical robots, with possible application to neurological procedures, is also presented in reference [27]. An overview of the different solutions of neurosurgical robots that have been used and developed in the last 30 years has been described in references [28,29].

A conceptual solution for a parallel robot to assist in cranial trepanation surgery is presented in this paper. The main motivation for the topic presented in this paper was the need to create a controllable, accurate robot to support the surgeon's activities in the operating theatre, with a kinematic structure as light and rigid as possible, with the possibility of attaching it to the operating table, in order to increase the accuracy of trepanation openings performed during neurological surgical procedures. The robotic structure developed should allow the future integration of the developed robotic mechanism into

medical imaging systems based on MRI and CT so that the surgical tool can be precisely positioned and thus have the smallest possible size after the procedure to close the holes through the bone wax. The steps taken to create a new parallel robotic solution to support selected neurosurgical procedures, in particular cranial trepanation, are mainly due to the fact that the source analysis carried out on existing technical solutions on the market and prototype service robotic solutions in the respective area of interest showed that this area of application is practically “undeveloped” scientifically, except for the solution presented in reference [25].

The organization of the contents of this paper is as follows: (1) the first section (Section 1) is an introductory chapter on the application of the trepanning robot technology discussed; (2) the next section (Section 2.1) describes the design of a parallel manipulator for positioning a trepanation tool in the cranial region of the patient; this section also describes in detail the components of the manipulator and the advantages of the use of closed-loop manipulators for accurate tool positioning; (3) the next section (Section 2.2) presents the mathematical analytical equations describing the inverse kinematics task of a parallel manipulator; (4) a forward kinematics task for a manipulator is solved analytically in the fourth section (Section 2.4); (5) in the next section (Section 3.1), the workspace analysis of the robot system is performed, and (6) the last section (Section 4) is a summary of the results obtained, defining, among other things, the areas for further development and future research of a parallel robot dedicated to assisting in cranial trepanation procedures.

2. Methods

2.1. Design Criteria

Taking into account the functional characteristics of parallel robots, in particular the high rigidity of the structure, which translates into a sufficiently high positioning accuracy and speed of the robot end-effector, due to the lower mass of the structure in relation to the volume and thus the lower inertial forces generated, as well as the high natural frequency of the robot structure, the development of this type of parallel robot has many advantages. Unfortunately, compared to serial robots, the end-effector of a parallel robot can be positioned in a much smaller working area, which translates into limited use in practical applications. Another major engineering difficulty is the need to position all the end-effector actuators simultaneously, the existence of singular points in the robot's workspace, and the lack of an analytical solution to a simple kinematics problem.

The genesis of the development of a parallel robotic system to support the surgeon's activities during the medical procedure of skull trepanation was born from an in-depth literature analysis of parallel robotic solutions considered worldwide and the performance characteristics that distinguish them from serial robotic structures. Several works carried out at the Department of Automation and Robotics, BUT [30], including two diploma theses of engineering students, K. Kalinowski [31] and K. Zajkowski [32], created a solid basis for the further development of the construction and testing of the solution with a view to increasing its functionality and eliminating the inconveniences of the design developed at that time. Changing the control strategy of the end-effector armed with a trepanning tool is the main focus of today's development research.

Figure 1 shows the design of the mechanical actuation subsystem of a parallel robot, in the form of a mobile platform with a parallel robot in series connected by linear electric actuators with an absolute encoder to the support frame of the structure. This connection makes it possible to increase the size of the working area of the end-effector equipped with a trepanning drill. As shown in Figure 1, we can distinguish the main components that make up the kinematic structure of the trepanning robot: (1) the main supporting frame; (2) the mobile frame carrying a parallel manipulator; (3) a parallel robot with an end-effector (4) armed with a surgical trepanning tool (5); (6) electric linear actuators for positioning mobile frame; (7) electric linear actuators with absolute encoders for positioning the platform with the end-effector; (8) a box containing the electronics for controlling the robot (in the prototype is an Arduino Mega controller, and in the future solution the authors plan to use

an FPGA controller board for robot control) and controllers with a power supply system for the electric linear actuators; (9) a system for stabilizing the position of the patient's skull, which can be adjusted to different sizes and shapes; (10) a baseplate for mounting the robot components and the system for comfort and stabilization of the patient's head position; it is also used to attach the robot to the headrest of the operating table.

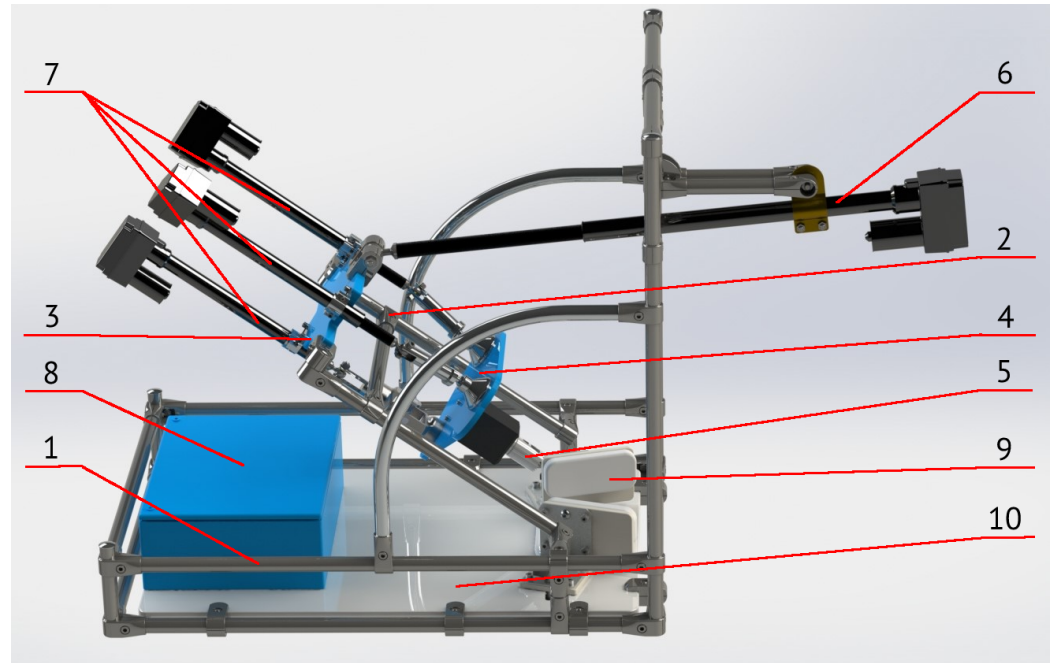


Figure 1. A view of the structure of the trepanation robot system: 1—the main supporting frame; 2—the mobile frame; 3—the base of a parallel robot; 4—end-effector; 5—the trepanning drill; 6—the actuator for positioning the mobile frame; 7—the actuators of the parallel robot; 8—the control and the power supply system of the robot; 9—a system for stabilizing the position of the patient's skull; 10—a baseplate for mounting all components.

The main considerations in the design of the robotic structure for surgeon assistance in cranial trepanation procedures were primarily: (1) the possibility of integrating the robot into existing operating theatres by attaching it to the auxiliary table mounts produced by the main manufacturers on the market; the robot should be able to be quickly attached and detached from the operating table; (2) the possibility of carrying out exploratory operations, in particular those caused by a herniation of the brain (*herniatio cerebri*); (3) the main dimensions of the device components were taken from an analysis based on an atlas of human measurements and data for ergonomic design and evaluation and adopted for the study, which correspond to the max dimensions of male specimens according to the 95C centile model [33]; (4) the ability to integrate the controller with medical imaging systems through proprietary software; (5) designed using medical-grade bacteriostatic materials, with the option of additional sterile sleeves for components in close and direct contact with the patient's body, resistant to substances and physical agents used to sterilise robot components; (6) electric linear actuators and absolute sensor systems should be placed as far away as possible from the working area of the trepanning drill so as not to interfere with other instruments used during surgery; (7) the overall dimensions of the robot should allow it to be positioned between the elements of a traditional operating theatre, without the need to reorganise it in order to provide a functional working environment for the medical staff (that is: width max—650 mm, length max—910 mm, and height max—860 mm).

The tripod structure used to design the trepanation surgical robot system is lighter than the robot base on Stewart platform structures and is easier to control. From a cost point of view, a series of actuators with encoders is cheaper to manufacture. These were the main factors in choosing the new robot design over alternative options.

The robot supporting the skull trepanation procedure in combination with the operating table is shown in Figure 2. The human skeleton shown in Figure 2 symbolizes the patient's body, and its placement here is only intended to show its functionality in combination with other components of the operating theatre equipment. In this case, the robot is attached to the headrest of the operating table from the side of the end-effector, while on the other side, it is supported on structural legs, which ensure support of the structure and its stabilization on the floor. Figures 1 and 2 do not show the connections of the electrical cables and their arrangement, in order to improve the readability of the technical solution of the parallel robot movement device.

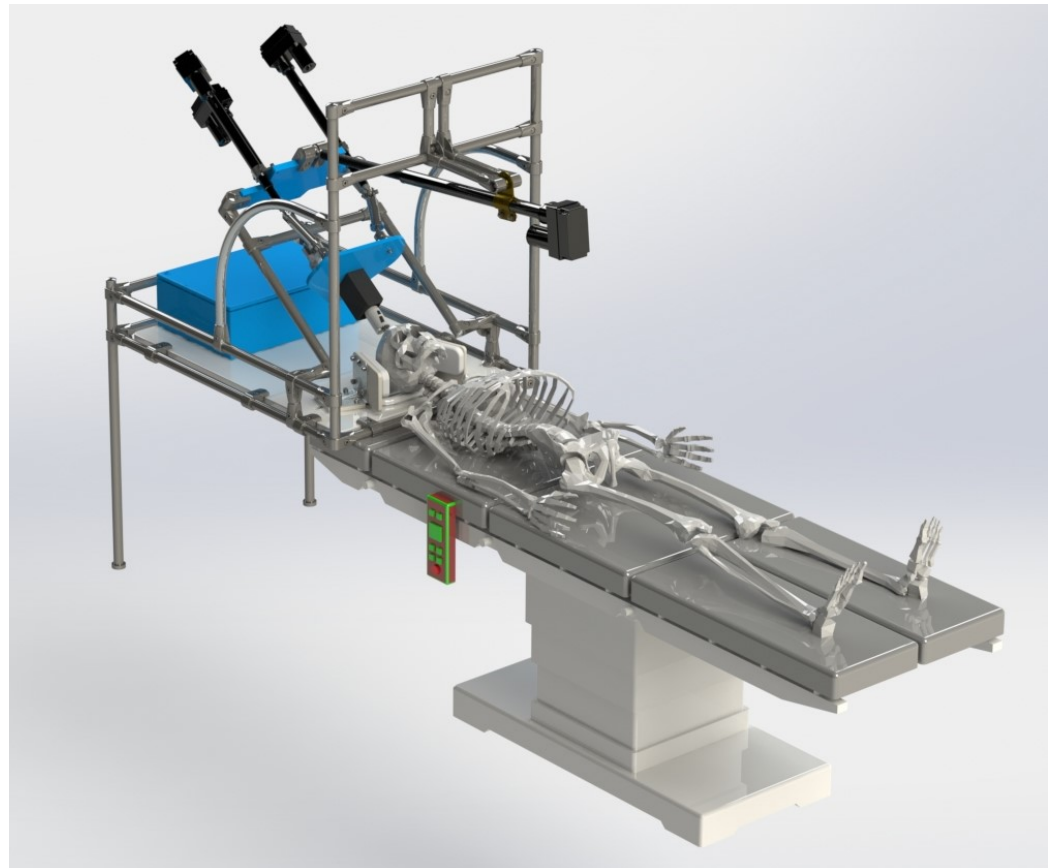


Figure 2. A view of the concept of the use of the trepanation robot with surgical table and patient.

This article does not consider the robot's electronic information subsystem, nor the human goal-setting subsystem, but only the mechanical actuation subsystem. It is for this that the rest of the paper will provide mathematical formulas describing the inverse and forward kinematics task. The task of determining the configuration space based on the joint space and the set of joint coordinates based on the current configuration of the end-effector (its position and orientation) is a fundamental task in robotics. This knowledge is used to construct the position and orientation control of the robot tool, i.e., the positioning of the trepanning drill.

2.2. Inverse Kinematics

In this section, the procedure for solving the inverse kinematics problem of the trepanation robot system is explained. In solving the inverse kinematics problem, we must find the values for joint displacements q_{1-4} when given the desired pose of the end-effector A_T .

Since the kinematic chain construction limits the available degrees of freedom, we need to find the IK function of the following form:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = IK({}^A x, {}^A z, {}^A r_x, {}^A r_y) \quad (1)$$

where x and z are the end-effector coordinates and r_x and r_y are the orientation, both expressed in the base $\{A\}$ frame of reference.

Since the proposed trepanation robot system consists of a parallel tripod mechanism attached in a series to a revolving frame, it is convenient to solve separately the inverse kinematics problem of the revolving frame and of the tripod effector. Let us first investigate the former.

The diagram of the frame and effector chain is shown in Figure 3. In the diagram, the base frame of reference is denoted as $\{A\}$, the tripod base frame is $\{B\}$, the tripod platform frame is $\{C\}$, and the end-effector frame is $\{E\}$. We are therefore aiming to calculate the joint displacements q_{1-4} according to the desired end-effector transform ${}^A_E T$.

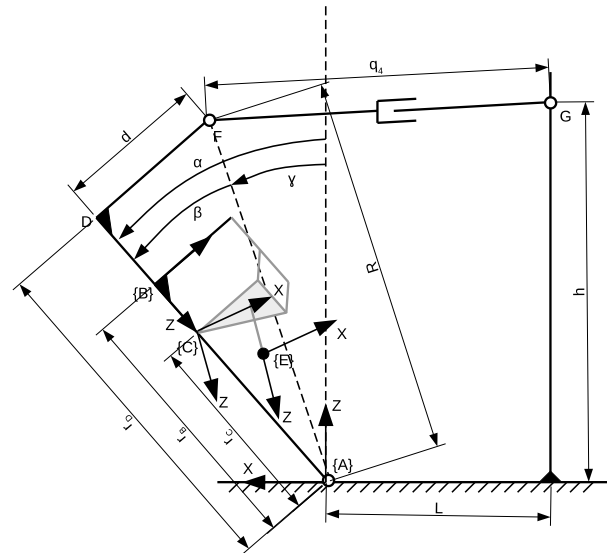


Figure 3. Diagram of the frame kinematics. The base frame is $\{A\}$ and the end-effector frame is $\{E\}$.

In order to find the joint displacements, let us first find the position of point C_1 in the $\{A\}$ frame of reference:

$${}^A C_1 = {}^A_C T \cdot {}^C C_1 = {}^A_E T \cdot {}^E_C T^{-1} \cdot {}^C C_1 \quad (2)$$

where ${}^E_C T$ denotes a transform between the tripod platform and the end-effector that is known and constant. From the ${}^A C_1$ coordinates, we can calculate the angle α :

$$\alpha = \tan^{-1} \frac{{}^A x_{C1}}{{}^A z_{C1}} \quad (3)$$

We can also now find the angle γ :

$$\gamma = \alpha - \beta = \alpha - \tan^{-1} \frac{d}{r_d} \quad (4)$$

The coordinates of the point F are therefore:

$${}^A F = [R \sin \gamma, 0, R \cos \gamma, 1]^T \quad (5)$$

and the joint displacement q_4 can be calculated as the distance between points F and G :

$$q_4 = \sqrt{(R \sin \gamma + L)^2 + (R \cos \gamma - h)^2} \quad (6)$$

In order to find the joint displacements q_{1-3} we first need to establish the transform ${}^B_C T$ between the tripod base and the tripod platform. This can be found as:

$${}^B_C T = {}^A_B T^{-1} \cdot {}^A_E T \cdot {}^C_E T^{-1} \quad (7)$$

In the equation above, ${}^A_E T$ is given and ${}^C_E T$ is known. ${}^A_B T$ can be found in the following way.

The transform between any two frames $\{1\}$ and $\{2\}$ can be described using a translation vector and the orientation expressed in EAA (equivalent axis-angle) notation as:

$${}^1_2 T = \text{Trans}(x, y, z) \cdot \text{Rot}([k_x, k_y, k_z], \theta) \quad (8)$$

where the $\text{Rot}([k_x, k_y, k_z], \theta)$ is:

$$\text{Rot}(k, \theta) = \begin{bmatrix} c\theta + k_x^2 t & k_x k_y t - k_z s\theta & k_x k_z t + k_y s\theta & 0 \\ k_x k_y t + k_z s\theta & c\theta + k_y^2 t & k_y k_z t - k_x s\theta & 0 \\ k_x k_z t - k_y s\theta & k_y k_z t + k_x s\theta & c\theta + k_z^2 t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

with $c\theta = \cos(\theta)$, $s\theta = \sin(\theta)$ and $t = 1 - \cos(\theta)$ and $\text{Trans}(x, y, z)$ is:

$$\text{Trans}(x, y, z) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

The coordinates of the origin of $\{B\}$ can be calculated as:

$${}^A_B = [r_B \sin \gamma, 0, r_B \cos \gamma, 1]^T \quad (11)$$

and the orientation of $\{B\}$ with respect to $\{A\}$ is around the y_A axis with an angle of ${}^A_B r_y$:

$${}^A_B r_y = \pi + \alpha \quad (12)$$

Therefore, ${}^A_B T$ can be expressed as:

$${}^A_B T = \text{Trans}(x, 0, z) \cdot \text{Rot}([0, 1, 0], {}^A_B r_y) \quad (13)$$

Therefore, we can find the transform ${}^B_C T$ according to Equation (7).

2.3. Inverse Kinematics of the Tripod Effector

In this section, we describe the inverse kinematics of the tripod effector, which is the latter part of the kinematic chain of the proposed robot system. The tripod effector diagram is shown in Figure 4. The base frame of the effector is marked as $\{B\}$, the frame associated with the platform is $\{C\}$, and the end-effector frame is $\{E\}$. The platform is actuated through three prismatic joints with displacements denoted as q_{1-3} . The circles represent spherical joints.

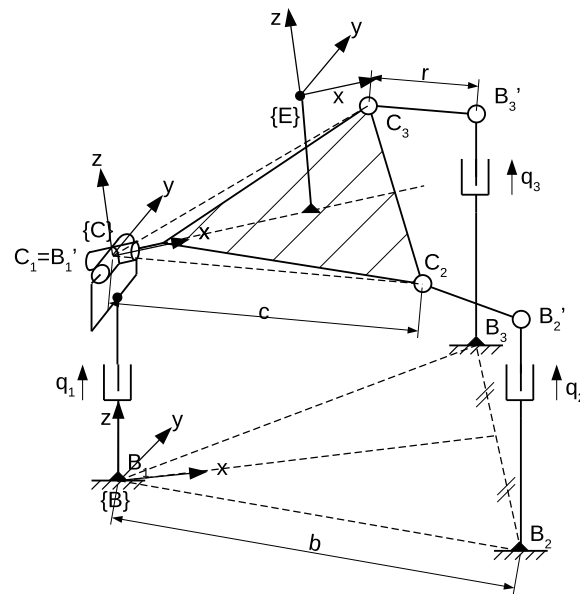


Figure 4. Diagram of the tripod effector.

Let us consider the positions of points C_1 , C_2 , and C_3 when the platform is positioned in pose ${}^B T_C$:

$${}^B C_i = {}^B T \cdot {}^C C_i, \quad i = 1, \dots, 3 \quad (14)$$

Since the kinematic chain structure of the tripod effector only allows for the movement in the ${}^B \hat{z}$ direction and rotation about the ${}^B \hat{x}$ and ${}^B \hat{y}$ axes, the pose of the $\{C\}$ frame can be constrained to:

$${}^B T_C = \text{Trans}(0, 0, z) \cdot \text{Rot}([k_x, k_y, 0], \theta) \quad (15)$$

Once the positions of points C_2 and C_3 are known, it is possible to find the positions of points B'_2 and B'_3 , representing the ends of the prismatic effectors q_2 and q_3 . Since points $B_{2,3}$ are connected with points $C_{2,3}$ with spherical joints, the positions of B'_2 and B'_3 can be found considering the intersection of spheres with radius r placed at $C_{2,3}$ and vertical lines placed at $B_{2,3}$.

The intersection of a sphere with radius r with the center at position (x_0, y_0, z_0) and a vertical line going through a point (x_1, y_1) is given as:

$$\begin{cases} x = x_1 \\ y = y_1 \\ (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2 \end{cases} \quad (16)$$

The solution of this system of equations is:

$$\begin{cases} x = x_1 \\ y = y_1 \\ z = \sqrt{r^2 - (x_1 - x_0)^2 - (y_1 - y_0)^2} - z_0 \end{cases} \quad (17)$$

Therefore, the equations describing the positions of points B'_i in the $\{B\}$ frame of reference can be written as:

$${}^B z_{B1'} = {}^B z_{C1} \quad (18)$$

$${}^B z_{B2'} = \sqrt{r^2 - ({}^B x_{B2} - {}^B x_{C2})^2 - ({}^B y_{B2} - {}^B y_{C2})^2} - {}^B z_{C2} \quad (19)$$

$${}^B z_{B3'} = \sqrt{r^2 - ({}^B x_{B3} - {}^B x_{C3})^2 - ({}^B y_{B3} - {}^B y_{C3})^2} - {}^B z_{C3} \quad (20)$$

Once the positions of points B_i' are known, it is straightforward to find the joint displacements q_{1-3} :

$$q_i = {}^B z_{B_i'} - {}^B z_{B_i}, \quad i = 1, \dots, 3 \quad (21)$$

2.4. Forward Kinematics

In this section, the forward kinematics (FK) solution of the tripod effector is discussed. In order to produce the loop-closure constraints, the closed-chain mechanism is modeled as three open chains starting from the fixed frame $\{B\}$ to the body frame $\{E\}$ via frames $\{C_1\}$, $\{B_2\}$, and $\{B_3\}$ (see Figure 5). In order to find a closed-form solution, a simplification is made, assuming that all the joints are rotational according to the frames of Figure 5. The modified D-H (Denavit–Hartenberg) parameters [34] are shown in Tables 1–3.

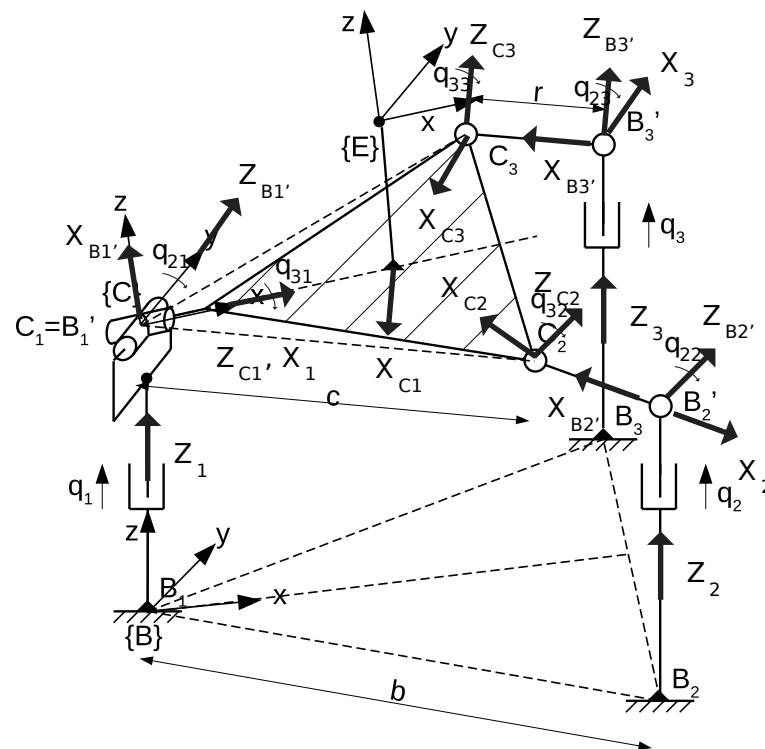


Figure 5. Diagram of the mechanism with the labeled reference frames.

Table 1. The modified D-H parameters from the the fixed frame $\{B\}$ to the body frame $\{E\}$ via $\{C_1\}$.

i	α_{i-1}	L_{i-1}	d_i	q_i
1 ($\{B\}, \{1\}$)	0	0	q_1	0
2 ($\{1\}, \{B_1\}$)	$-\pi/2$	0	0	$q_{21} - \pi/2$
3 ($\{B_1\}, \{C_1\}$)	$-\pi/2$	0	CE	$q_{31} + \pi/2$
4 ($\{C_1\}, \{E\}$)	$\pi/2$	0	0	$-\pi/2$

Table 2. The modified D-H parameters from the the fixed frame $\{B\}$ to the body frame $\{E\}$ via $\{B_2\}$.

i	α_{i-1}	L_{i-1}	d_i	q_i
1 ($\{B\}, \{2\}$)	0	b	q_2	$-\pi/4$
2 ($\{2\}, \{B_2\}$)	$-\pi/2$	0	0	$q_{22} - \pi$
3 ($\{B_2\}, \{C_2\}$)	0	$B_2'C_2$	0	q_{32}
4 ($\{C_2\}, \{E\}$)	$-\pi/2$	C2E	0	$-5\pi/8$

Table 3. The modified D-H parameters from the the fixed frame $\{B\}$ to the body frame $\{E\}$ via $\{B_3\}$.

i	α_{i-1}	L_{i-1}	d_i	q_i
1 ($\{B\}, \{3\}$)	0	b	q_3	$\pi/4$
2 ($\{3\}, \{B_3\}$)	$-\pi/2$	0	0	$q_{23} - \pi$
3 ($\{B_3\}, \{C_3\}$)	0	$B_3'C_3$	0	q_{33}
4 ($\{C_3\}, \{E\}$)	$\pi/2$	C3E	0	$5\pi/8$

The angles q_{21} and q_{31} that appear in Table 1 are used for the rotations around the $Z_{B_1'}$ and Z_{C_1} axes, respectively (see Figure 5). Similarly, the angles q_{22} and q_{32} that appear in Table 2 are used for the rotations around the $Z_{B_2'}$ and Z_{C_2} axes, respectively. Finally, the angles q_{23} and q_{33} that appear in Table 3 are used for the rotations around the $Z_{B_3'}$ and Z_{C_3} axes respectively.

The loop-closure equations are the following:

$$\begin{aligned} {}^B T_1 \cdot {}^1 T_{B_1'} \cdot {}^{B_1'} T_{C_1} \cdot {}^{C_1} T_E &= {}^B T_2 \cdot {}^2 T_{B_2'} \cdot {}^{B_2'} T_{C_2} \cdot {}^{C_2} T_E \\ {}^B T_1 \cdot {}^1 T_{B_1'} \cdot {}^{B_1'} T_{C_1} \cdot {}^{C_1} T_E &= {}^B T_3 \cdot {}^3 T_{B_3'} \cdot {}^{B_3'} T_{C_3} \cdot {}^{C_3} T_E \end{aligned} \quad (22)$$

Using the position part of the loop-closure equations, we are able to determine five out of six variables (q_{21} , q_{32} , q_{33} , q_{22} , and q_{23}). The variable q_{31} can be found from the orientation part. The forward kinematics set of solutions is the following:

$$\begin{aligned} q_{21} &= \cos^{-1} \frac{b}{\|CE\|} \\ q_{32} &= \cos^{-1} \frac{(t_1 - t_2 - \|CE\| \cdot \sin q_{21})^2 - \|C_2E\|^2 - \|B_2C_2\|^2}{2\|C_2E\| \cdot \|B_2C_2\|} \\ q_{33} &= \cos^{-1} \frac{(t_1 - t_3 - \|CE\| \cdot \sin q_{21})^2 - \|C_3E\|^2 - \|B_3C_3\|^2}{2\|C_3E\| \cdot \|B_3C_3\|} \\ q_{22} &= \tan^{-1} \frac{\|C_2E\| \cdot \cos q_{32} + \|B_2C_2\|}{\|C_2E\| \cdot \sin q_{32}} \\ q_{23} &= \tan^{-1} \frac{\|C_3E\| \cdot \cos q_{33} + \|B_3C_3\|}{\|C_3E\| \cdot \sin q_{33}} \\ q_{31} &= \sin^{-1} \left(\frac{\sqrt{2}}{2} \sin(q_{22} + q_{32}) \right) \end{aligned} \quad (23)$$

2.5. Numerical Solution for the Forward Kinematics of the Tripod End-Effector

When concerned with parallel robots, since very often the solution to the forward kinematics problem is unavailable in closed form, it is very common to use numerical methods instead. In this section one such solution based on the formulation of the optimization problem is presented.

The forward kinematics problem for the tripod end-effector can be formulated as a task of finding the pose of the end-effector when the configuration variables are known:

$${}^B_E T = FK(q_1, q_2, q_3) \quad (24)$$

Since the end-effector E is fixed to the tripod platform and the ${}^E_{C_1}T$ transform is known, the problem can be reduced to the issue of finding the transform ${}^B_{C_1}T$. It is immediately evident that the vertical displacement between the points B_1 and C_1 is equal to the configuration variable q_1 . The transform ${}^B_{C_1}T$, taking into account its degrees of freedom, can thus be parametrized in the following way:

$${}^B_{C_1}T(q_1, {}^B_{C_1}r_x, {}^B_{C_1}r_y) = \text{Trans}(0, 0, q_1) \cdot \text{Rot}([0, 1, 0], {}^B_{C_1}r_y) \cdot \text{Rot}([1, 0, 0], {}^B_{C_1}r_x) \quad (25)$$

Next, the positions of the points C_2 and C_3 can be found as:

$$\begin{aligned} {}^B_{C_2} &= {}^B_{C_1}T \cdot {}^C_{C_2} \\ {}^B_{C_3} &= {}^B_{C_1}T \cdot {}^C_{C_3} \end{aligned} \quad (26)$$

The positions of the ends of the actuators B'_2 and B'_3 associated with the configuration variables q_1 and q_2 , respectively, can be found as:

$$\begin{aligned} {}^B_{B'_2} &= \text{Trans}(0, 0, q_2) \cdot {}^B_{B_2} \\ {}^B_{B'_3} &= \text{Trans}(0, 0, q_3) \cdot {}^B_{B_3} \end{aligned} \quad (27)$$

We can now observe that the solution to the forward kinematics problem is only correct when the distances between the calculated pairs of points $({}^B_{B'_2}, {}^B_{C_2})$ and $({}^B_{B'_3}, {}^B_{C_3})$ satisfy the distance constraints imposed by the lengths r of the links connecting the ends of the actuators q_2 and q_3 to the tripod platform. Let us construct the objective function:

$$\begin{aligned} f({}^B_{C_1}r_x, {}^B_{C_1}r_y) &= (||({}^B_{B'_2}(q_2), {}^B_{C_2}(q_1, {}^B_{C_1}r_x, {}^B_{C_1}r_y))|| - r)^2 \\ &+ (||({}^B_{B'_3}(q_3), {}^B_{C_3}(q_1, {}^B_{C_1}r_x, {}^B_{C_1}r_y))|| - r)^2 \end{aligned} \quad (28)$$

We can now find the orientation coordinates of the tripod platform by solving the optimization problem formulated as:

$$[{}^B_{C_1}r_x, {}^B_{C_1}r_y] = \text{argmin} f({}^B_{C_1}r_x, {}^B_{C_1}r_y) \quad (29)$$

Once these orientation coordinates are found, the forward kinematics solution can be expressed as:

$${}^B_E T = FK(q_1, q_2, q_3) = \quad (30)$$

$$\begin{aligned} &= {}^B_{C_1}T(q_1, {}^B_{C_1}r_x, {}^B_{C_1}r_y) \cdot {}^C_E T \\ &= \text{Trans}(0, 0, q_1) \cdot \text{Rot}([0, 1, 0], {}^B_{C_1}r_y) \cdot \text{Rot}([1, 0, 0], {}^B_{C_1}r_x) \cdot {}^C_E T \end{aligned} \quad (31)$$

We have implemented the solution presented above in Matlab, where the typical solution time is around 0.02 s. This performance matches our criterion for the desired controller frequency for the robot, and we believe that the implementation designed in a better-performing language (such as C++) will provide an even better margin.

3. Results

In this section we present the results achieved through the numerical simulation of the kinematics of the proposed robot platform. In Section 3.1 we present the analysis of the workspace of the robot, and in the Section 3.2 we present a numerical analysis of the manipulability of the robot.

3.1. Workspace

In order to prove the solutions presented for the inverse (see Section 2.2) and forward kinematics (see Section 2.4) of the trepanning robot, we have performed an experiment in simulation to establish the shape and the volume of the workspace of the system.

The workspace analysis was performed in Matlab R2022, where a Monte Carlo approach was used to generate the point cloud of the feasible end-effector positions [35]. We have performed this analysis separately for the tripod end-effector as well as the complete system. In both cases, $N = 25,000$ random joint space configurations were sampled and the forward kinematics problem was solved numerically.

The forward kinematics solutions were limited to a set where the B and C points of the tripod platform are placed at larger displacements away from the platform base than the end of the effectors q_2 and q_3 .

Next, a boundary of the obtained point cloud was calculated, and a mesh was generated to represent the approximated workspaces. These workspaces are shown in Figure 6 for the end-effector and Figure 7 for the complete system.

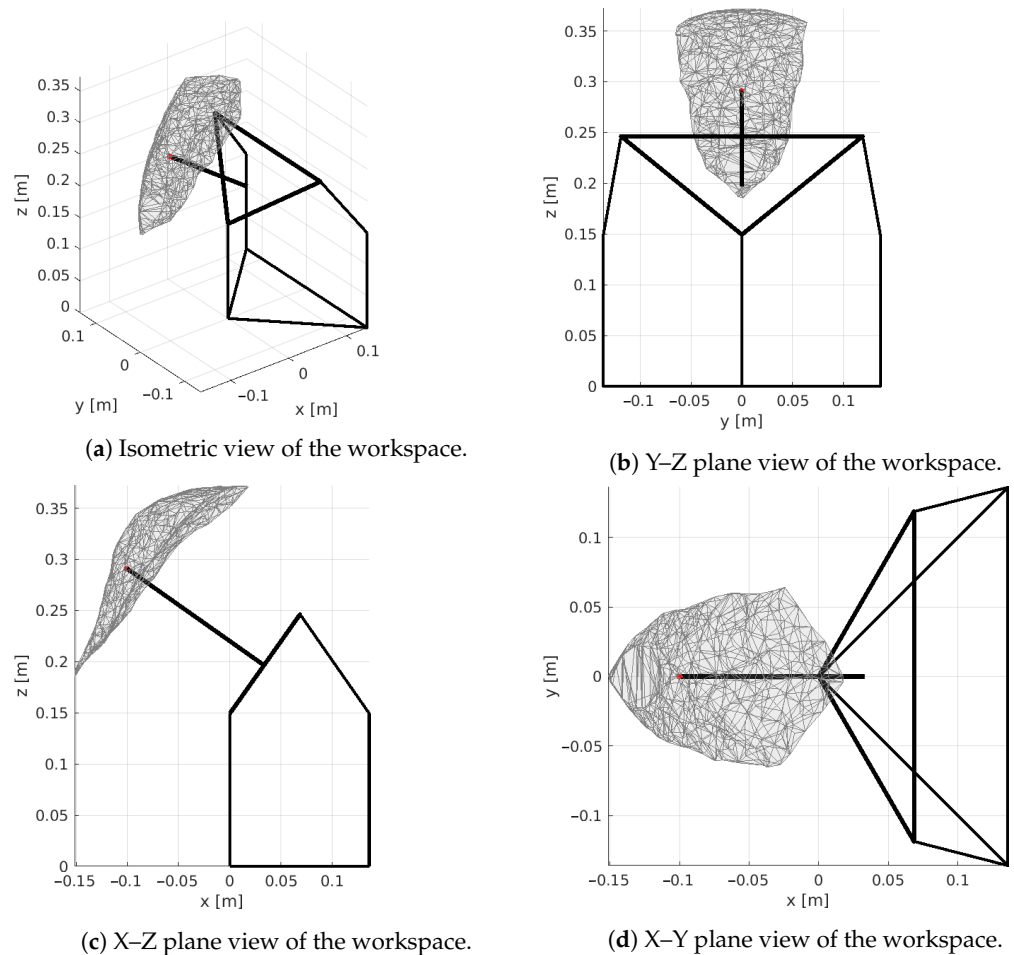


Figure 6. Diagram of the tripod effector workspace. The red dot indicates the position of the end effector in the default configuration of the system.

The dimensions of the robot were obtained from the SolidWorks virtual model file. The dimensions of the system are as follows:

$b = 0.1924 \text{ [m]}$	$c = 0.1678 \text{ [m]}$	$r = 0.1193 \text{ [m]}$
$e = 0.1637 \text{ [m]}$	$L = 0.4020 \text{ [m]}$	$h = 0.5350 \text{ [m]}$
$d = 0.1867 \text{ [m]}$	$r_b = 0.4904 \text{ [m]}$	$r_d = 0.4904 \text{ [m]}$

We have measured the minimum and maximum ranges for the joint variables as follows:

$$q_1 \in [0.0994 \text{ m}; 0.1994 \text{ m}]$$

$$q_2 \in [0.0994 \text{ m}; 0.1994 \text{ m}]$$

$$q_3 \in [0.0994 \text{ m}; 0.1994 \text{ m}]$$

$$q_4 \in [0.4216 \text{ m}; 0.7215 \text{ m}]$$

The shape of the workspace of the tripod end-effector is shown in Figure 6. The volume of the workspace is 0.435 dm^3 .

The shape of the workspace of the complete trepanation robot setup is shown in Figure 7. The volume of the workspace is 2.3 dm^3 .

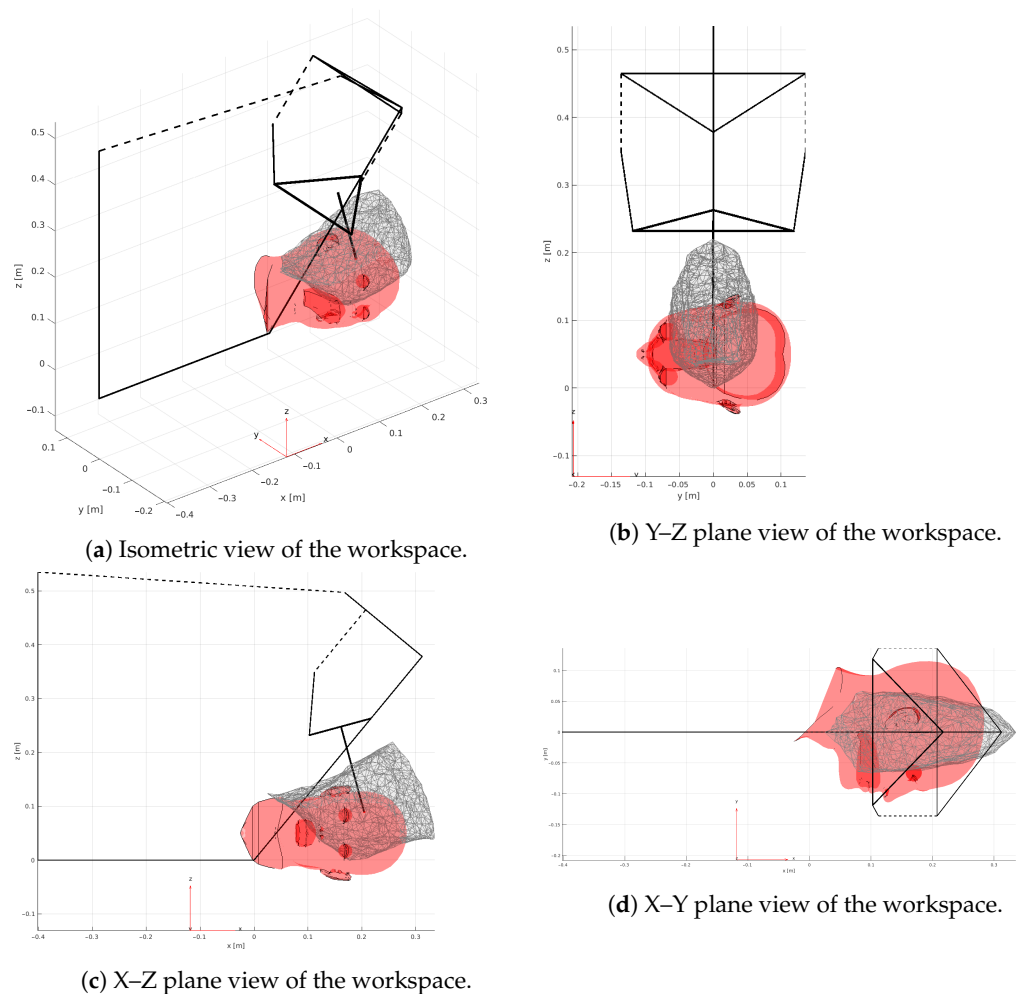


Figure 7. Diagram of the trepanation robot's workspace with a model of the human skull highlighted in red in lateral position during a selected surgical procedure [36].

The robot is designed to perform exploratory surgery, particularly for brain hernias (herniatio cerebri) [37]. Brain hernias are classified according to the structure through which the tissue has herniated. The points of medical intervention required for these trepanning procedures can also be achieved by changing the patient's position during the procedure: supine, lateral, abdominal. Research into the determination of the surgical trepanning robot's workspace confirms that the workspace achieved is sufficient to perform the indicated procedures.

3.2. Manipulability

In this section, we consider the kinematic performance of the robot system in terms of the manipulability index. Due to the hybrid (series–parallel) nature of the system and the fact that in the closed kinematic chains the issues associated with the singularities are the most insidious, in this section we focus on the analysis of the tripod end-effector.

We have performed the manipulability index analysis within the designed workspace of the robot. In this configuration space, we have calculated the Jacobian matrix for the pose of the end-effector E in the base $\{B\}$ frame of reference. The Jacobian is estimated numerically using the following procedure.

Let us consider the pose offsets ${}^B_E\Delta T_i$ due to small changes in the configuration variables q_i for $i = 1, 2, 3$:

$${}^B_E\Delta T_i = {}^B_E T(q_i)^{-1} \cdot {}^B_E T(q_i + \epsilon) \quad (32)$$

The form of the obtained matrices ${}^B_E\Delta T_i$ can be written as:

$${}^B_E\Delta T_i = \epsilon \frac{1}{\Delta q_i} \begin{bmatrix} 0 & -\Delta r_z & \Delta r_y & \Delta x \\ \Delta r_z & 0 & -\Delta r_x & \Delta y \\ -\Delta r_y & \Delta r_x & 0 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (33)$$

The form presented above allows us to find the derivatives of the end-effector coordinates (x, y, z, r_x, r_y, r_z) due to changes in the configuration variables q_i for $i = 1, 2, 3$. Thus, the Jacobian matrix ${}^B_E J$ can be constructed as:

$${}^B_E J = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial y}{\partial q_1} & \frac{\partial z}{\partial q_1} & \frac{\partial r_x}{\partial q_1} & \frac{\partial r_y}{\partial q_1} & \frac{\partial r_z}{\partial q_1} \\ \frac{\partial x}{\partial q_2} & \frac{\partial y}{\partial q_2} & \frac{\partial z}{\partial q_2} & \frac{\partial r_x}{\partial q_2} & \frac{\partial r_y}{\partial q_2} & \frac{\partial r_z}{\partial q_2} \\ \frac{\partial x}{\partial q_3} & \frac{\partial y}{\partial q_3} & \frac{\partial z}{\partial q_3} & \frac{\partial r_x}{\partial q_3} & \frac{\partial r_y}{\partial q_3} & \frac{\partial r_z}{\partial q_3} \end{bmatrix} \quad (34)$$

$$= \frac{1}{\epsilon} \begin{bmatrix} {}^B_E\Delta T_{1/1,4} & {}^B_E\Delta T_{1/2,4} & {}^B_E\Delta T_{1/3,4} & {}^B_E\Delta T_{1/3,2} & {}^B_E\Delta T_{1/1,3} & {}^B_E\Delta T_{1/2,1} \\ {}^B_E\Delta T_{2/1,4} & {}^B_E\Delta T_{2/2,4} & {}^B_E\Delta T_{2/3,4} & {}^B_E\Delta T_{2/3,2} & {}^B_E\Delta T_{2/1,3} & {}^B_E\Delta T_{2/2,1} \\ {}^B_E\Delta T_{3/1,4} & {}^B_E\Delta T_{3/2,4} & {}^B_E\Delta T_{3/3,4} & {}^B_E\Delta T_{3/3,2} & {}^B_E\Delta T_{3/1,3} & {}^B_E\Delta T_{3/2,1} \end{bmatrix} \quad (35)$$

where symbol ${}^B_E\Delta T_{k/i,j}$ denotes the element of matrix ${}^B_E\Delta T_k$ located in the i -th row and the j -th column.

The manipulability index $w(q_1, q_2, q_3)$ is calculated as a function of the configuration variables q_i for $i = 1, 2, 3$:

$$w(q_1, q_2, q_3) = \sqrt{\det({}^B_E J \cdot {}^B_E J^T)} \quad (36)$$

Figure 8 presents the value of the manipulability index w in the designed workspace of the robot. Since the full three-dimensional surface of the w function is impossible to visualize, we have chosen to present a few slices of the function in the (q_2, q_3) , (q_1, q_3) , and (q_2, q_1) planes intersecting the midpoint of the workspace. In the figure, we have limited the range of the z -axis to 40 to focus on the relevant range of the manipulability index. The noise observed in the function values is an artifact of the numerical method used to compute the manipulability index. As observed, the minimum value of the manipulability index in the designed workspace is $w_{min} = 0.65$. The lack of zeros of the manipulability function suggests that there are no singular robot configurations located within the desired workspace of the robot system.

Indeed, the limits of the actuators in the designed system were chosen such that the robot operates within the well-conditioned workspace and far away from singularities where undesired effects such as posture change may occur.

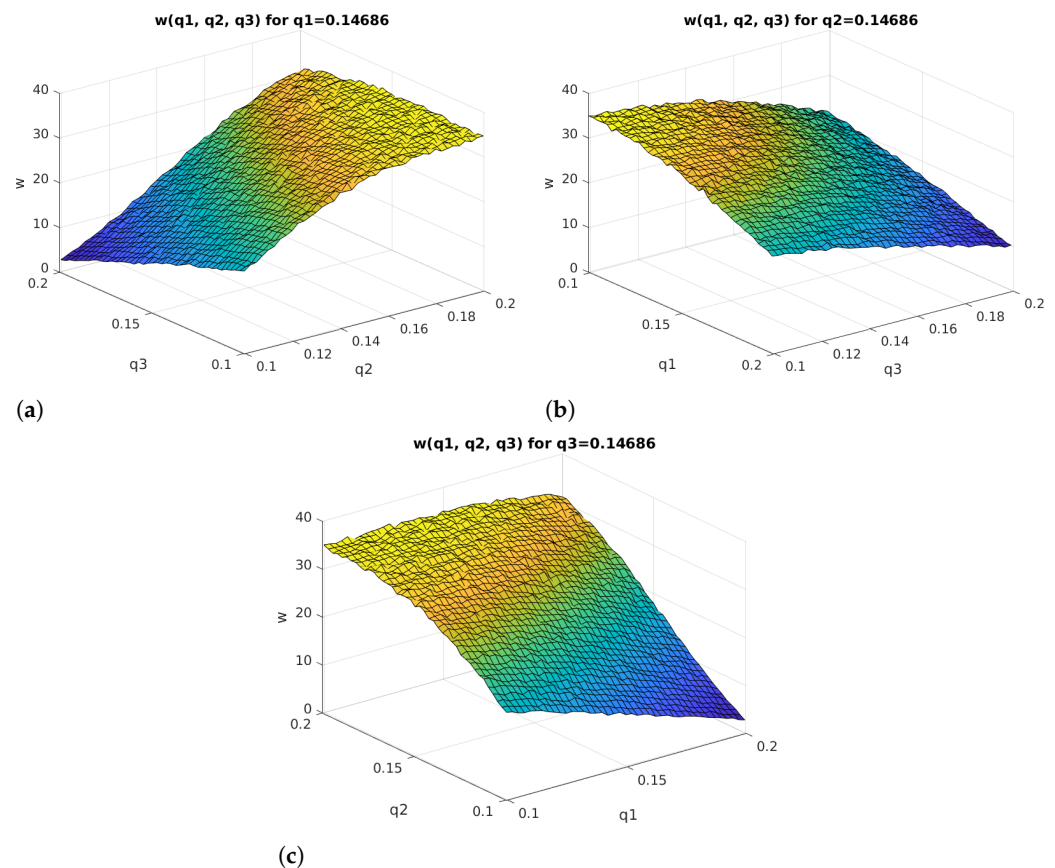


Figure 8. Analysis of the manipulability index of the tripod end-effector. (a) Slice of the manipulability index $w(q_2, q_3)$ for $q_1 = 0.14686$. (b) Slice of the manipulability index $w(q_1, q_3)$ for $q_2 = 0.14686$. (c) Slice of the manipulability index $w(q_1, q_2)$ for $q_3 = 0.14686$.

4. Conclusions

The technical solution adopted in this paper is a prototype version of a robot with a series-parallel structure. The main task, in this case, was to verify the hypotheses and test the validity of the technical solution adopted. Further development of the project will focus on changing the construction materials and the way in which they are joined to those that will ensure adequate rigidity of the structure and compatibility with medical applications. Another area of research will be the selection of accurate, safety-compliant medical equipment in direct contact with the patient, linear electric drives with higher accuracy, and absolute encoders made for use in operating-room conditions. The method of secure and stable attachment of the robot to the operating table and floor in the prototype robot solution has not been finally resolved and will be the subject of further development of the proposed design. The current stage of development of the prototype of a trepanning robot makes it possible to test the engineering assumptions adopted and to lay a solid foundation for the development of control and safety systems based on the solution of a forward and inverse kinematics task. The study of the dynamics of this type of robot is usually neglected due to the very low dynamics of the displacements (low speed of the drives) of the parts of the robot that support the surgical procedures. Basically, the robot remains in a static position once it has reached a certain position. This is to provide support and guidance for the trephine drill. An important direction for future research into the design of the robot and its use in the operating theatre will be the development of a human-machine interface (HMI) that allows medical staff to easily use the robotic tool for specific medical procedures while integrating medical imaging from MRI and CT sources.

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Symbols and Abbreviations

The following symbols and abbreviations are used in this manuscript:

MRI	magnetic resonance imaging
CT	computer tomography
IPA	The Fraunhofer-Institut für Produktionstechnik und Automatisierung
RCM	remote centre of motion
FK	forward kinematics
IK	inverse kinematics
EAA	equivalent axis-angle
D-H	Denavit–Hartenberg (parameters)
MC	Monte Carlo
q_1, q_2, q_3, q_4	configuration variables for joints 1 to 4
$q_{21}, q_{22}, q_{23}, q_{31}, q_{32}, q_{33}$	additional configuration variables for the redundant joints of the parallel kinematic structure
$\{F\}$	F frame of reference
${}^A x_B, {}^A y_B, {}^A z_B$	x, y , and z coordinates of point B expressed in A frame of reference
${}^A T_B$	a 4×4 homogeneous transformation matrix from A to B frame of reference
$\text{Trans}(x, y, z)$	a 4×4 homogeneous transformation matrix denoting translation
$\text{Rot}([k_x, k_y, k_z], \theta)$	a 4×4 homogeneous transformation matrix denoting rotation about the axis k with the angle of magnitude θ
$\{A\}$	the base frame of reference
$\{B_{1-3}\}$	frames of reference attached to the tripod effector actuator mounting points
$\{B'_{1-3}\}$	frames of reference attached to the tripod effector actuator end points
$\{C_{1-3}\}$	frames of reference attached to the tripod platform bearing mounting points
$\{E\}$	end-effector frame of reference
b	length of the side of the tripod base plate
c	length of the side of the tripod effector platform
r	length of the arm connecting the tripod actuators to the tripod platform
e	distance from the tripod platform to the end-effector
L	distance between the axis of the tilting frame to the fourth actuator mounting plane
h	height of the fourth actuator mounting point from the device base plane
d	length of the shorter side of the tilting frame
r_b	distance from the tilting frame axle to the tripod base mounting point
r_d	length of the longer side of the tilting frame
α, β, γ	angles used in the solution of the IK problem for the system

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