

Article

Design and Ground Performance Evaluation of a Multi-Joint Wheel-Track Composite Mobile Robot for Enhanced Terrain Adaptability

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Abstract: The tracked-wheeled mobile robot has gained significant attention in military, agricultural, construction, and other fields due to its exceptional mobility and off-road capabilities. Therefore, it is an ideal choice for reconnaissance and exploration tasks. In this study, we proposed a multi-jointed tracked-wheeled compound mobile robot that can overcome various terrains and obstacles. Based on the characteristics of multi-jointed robots, we designed two locomotion modes for the robot to climb stairs and established the kinematics/dynamics equations for its land movement. We evaluated the robot's stability during slope climbing, its static stability during stair climbing, and its ability to cross trenches. Based on our evaluation results, we determined the key conditions for the robot to overcome obstacles, the maximum height it can climb stairs, and the maximum width it can cross trenches. Additionally, we developed a simulation model to verify the robot's performance in different terrains and the reliability of its stair-climbing gait. The simulation results demonstrate that our multi-jointed tracked-wheeled compound mobile robot exhibits excellent reliability and adaptability in complex terrain, indicating broad application prospects in various fields and space missions.

Keywords: kinematics; stability; terrestrial performance; wheel-tracked composite mobile robot

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1. Introduction

Mobile robots are intelligent systems that integrate various functions, including environmental perception, dynamic decision-making and planning, behavior control, and execution. These robots find wide applications in various fields, such as rescue, reconnaissance, and military operations. Based on their mode of mobility, mobile robots can be classified into four broad categories [1], namely, wheeled robots, legged robots, tracked robots, and composite structural robots. The walking mechanism of wheeled mobile robots is relatively simple, with wheels as the main body. They possess advantages, such as high efficiency, being lightweight, and ease of control, but their environmental adaptability is weak, making it difficult for them to navigate structured environments, such as staircases. Legged mobile robots are highly adaptable to their surroundings and can adjust their leg posture to traverse rugged terrains. However, these robots tend to have complex postural control and mechanisms, owing to their complex structure. Crawler robots boast a relatively stable structure and can move steadily over uneven surfaces. Unlike wheeled robots, they consume much power, owing to higher friction resistance, which limits their maximum speed. Composite mobile robots bring together two or more basic forms to achieve higher overall performance and can scale special terrain. As such, composite robots are currently the focus of much scholarly research. Some of the more common composite robots include: (1) wheel-leg composite mobile robots [2,3], which combine the stability of wheeled robots with the strong obstacle-crossing ability of leg robots; (2) wheel-track composite mobile

robots [4], which employ wheels for high-speed long-distance movement and tracks for adaptation to various complex terrains, thereby enhancing overall environmental adaptability; and (3) wheel-track-leg composite mobile robots [5], which combine even more factors and have a more complex structure. By combining the efficiency of movement on flat ground with the ability to negotiate complex terrain, wheel-tracked composite mobile robots have gained widespread attention and use. The performance of mobile robots on land is reflected in their posture, ability to cross obstacles, and movement stability [6].

Several scholars have designed and analyzed the terrestrial performance of various types of robots. Bruzzone, L. created a versatile platform that combines tracked motion on irregular and yielding terrain, energy-efficient wheeled motion on flat and compact surfaces, and stair climbing/descending capabilities. Unfortunately, the platform's gait becomes overly complicated and inefficient while ascending stairs due to its limited swing arms of only two [7]. Niimi, H. studied the relationship between the position of the center of gravity of the crawler-type mobile robot and the height of the step that can be climbed [8]. Zhang, S., et al., designed a wheel/track morphological reconfigurable mobile robot and established the relationship between the robot and the height of the step pendulum angle and the step height h for climbing obstacles, as well as the maximum slope that can be climbed. However, the robot's structure is overly complex and requires a specialized wheel/track conversion device for implementation [9]. Yonggan Wang et al., analyzed the crucial movements for their self-designed amphibious delivery robot on water and land. However, the robot's large body mass (resulting from the swing arm wheel being the same size as the main wheel) and the overly complicated and inefficient gait when climbing stairs are due to structural limitations [10]. Zhang, S., et al., proposed an adaptive wheel-legged shape reconfigurable mobile robot and conducted a kinematics analysis of the robot's ability to climb obstacles, stairs, and gullies. However, the robot's structure remains somewhat complex, and the analysis of only two wheels failed to determine its load capacity [11]. Tanyildizi designed a hybrid wheeled fire extinguisher robot with a five-piece transformable wheel system enabling swift movement on flat ground. The wheel mechanism opens and transforms into a star-shape, allowing the robot to climb and grasp onto obstructions in challenging terrain [12]. Barthuber, L. introduced a novel tracked mobile robot design capable of teleporting across various unstructured terrains within the physical environment. Despite being tested on different terrains, the robot features a single tracked structure and moves at a relatively slow speed [13].

Overall, while there have been various designs and analyses of robots' terrestrial performance, each design has its unique set of advantages and limitations. Further research and development are necessary to improve the robots' functionality and address structural constraints to better navigate challenging terrains.

In this paper, the focus is on analyzing the ground performance of an urban mobile robot operating in a complex environment. To enhance movement efficiency, obstacle traversal capabilities, and overall stability in complex environments, a multi-joint wheel-rail composite mobile robot was designed by drawing from the literature [10]. The rest of the paper is structured as follows. The structure of the wheel-track composite mobile robot is described in Section 2. The relationship between the center of mass of the mobile robot and the angle between the front and rear swing arms are analyzed. According to the change in the center of mass, the gait of the mobile robot climbing steps on the land is planned, and the relationship between the height of the steps and the posture of the front and rear swing arms is obtained in Section 3. The mobile robot's ability to climb slopes on land and cross trenches is analyzed in Section 4. Simulations are used to verify the rationality of the mobile robot design and the performance of the barrier crossing in Section 5, providing a theoretical foundation for the development of this kind of robot.

The main contributions of this paper are as follows. (1) A multi-joint wheel-track composite mobile robot is designed, equipped with four-swing arms that enhances stability and flexibility when traversing different terrains. (2) To increase climbing efficiency, two step-climbing gaits are planned for both high and low steps. (3) A systematic analysis

of the robot's performance in climbing steps, crossing obstacles, ascending slopes, and crossing trenches was conducted, aimed at providing a theoretical foundation for further development of such robotic systems.

2. Mobile Robot Structure Design

As the degree of urbanization continues to increase, the urban environment becomes increasingly complex and diverse, posing a huge challenge to the mobility of large equipment. Therefore, mobile robots have gradually become an indispensable choice. Mobile robots can adapt well to various urban environments and better meet the various needs and challenges of modern cities. However, in the continuous changes of the urban environment, ground mobile robots also face challenges. Mobile robots must have the ability to explore natural terrain and artificial environments, such as stairs, steps, slopes, etc., while having flexible mobility and portability to adapt to the diversity of urban environments.

There are currently many wheeled and tracked hybrid mobile robots available, such as the VIPER robot developed by Israeli Elbit Systems, the Lizard intelligent robot, developed by the Shenyang Institute of Automation, and research robots cited in the literature [9–13]. These robots take advantage of the benefits of combining wheels and tracks, providing good obstacle traversal capabilities and adaptability to various complex terrains. These robots have widespread application fields, including military reconnaissance, exploration and surveying, medical care, logistics delivery, and other areas.

The structural design of robots is adjusted and optimized according to different application requirements. To ensure that our multi-jointed tracked-wheeled compound mobile robot performs reliably in complex terrain, we considered potential real-world imperfections that could impact its functionality. We recognize that the terrain can be varied and unpredictable, and imperfections, such as sensor noise [14] or dynamic effects [15] can destabilize the robot's control performance [16]. Therefore, during our design process, we incorporated contingencies to account for these challenges, such as developing two distinct locomotion modes for the robot to climb stairs and establishing kinematics/dynamics equations for its land movement.

In this paper, we focus on urban reconnaissance robots. The mobile system of the wheel and shoe composite mobile robot designed in this paper has certain climbing and obstacle surmounting ability, flexible movement ability, and good passive adaptability to terrain. The design of the mobile system should mainly meet the following design objectives based on the characteristics of the urban environment and the task requirements of the mobile robot (good portability, mobility, and ability to cross obstacles). (1) The climbing slope is greater than 30° , and it can overcome obstacles higher than 300 mm and deep ditches more than 250 mm wide. (2) The mass of the mobile subsystem shall not exceed 15 kg, and the speed shall not exceed 15 km/h. (3) The robot has strong passive terrain adaptation ability and is easy to control.

Integrating the characteristics of wheeled and tracked walking mechanisms, a multi-joint wheel-track composite mobile robot has been designed. It consists of four wheels, two intermediate tracks, and four swing arm tracks, which can ensure both the obstacle-crossing capability and rapid driving on flat ground. The mobile robot's ability to cross barriers is related to the mechanism's size, and the wheeled robot's ability is limited by its wheel diameter. To overcome this limitation, the design idea of a folding and deforming mechanism is adopted to allow the robot to fold in a non-working state for storage and transportation. When it is in a working state, the robot unfolds, expanding in size, and thus achieving its obstacle-crossing function. The wheel-tracked composite robot is shown in Figure 1.

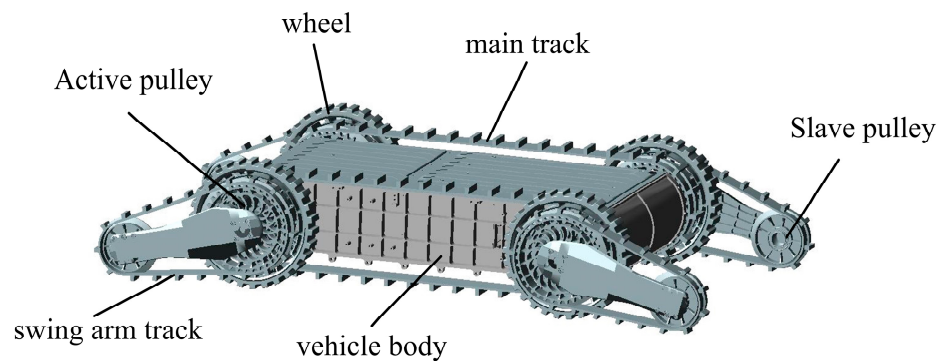


Figure 1. Wheel-tracked mobile robot.

Compared to the design in the literature [10], our robotic platform has undergone significant improvements. One of the major changes we made was to adopt a pendulum wheel design with different sizes between the front and rear. This design enhances the flexibility and operability of our robot, enabling it to traverse over obstacles and uneven terrain more effectively. Moreover, we configured the front and rear pendulum wheels on the same axis, which further enhances the stability and control of our platform. These changes were carefully designed to optimize the performance of our robot while meeting specific application requirements.

When the robot is running normally, the track swing arm is in a retracted state, and the wheels are in contact with the ground for land travel. This feature makes it ideal for driving on flat roads, allowing for fast and efficient movement. However, when the robot encounters obstacles, its gait can be changed by any angle combination between the front and rear swing arms. This allows the robot to cross obstacles easily and with flexibility.

The swingarm is a crucial component of the robot's capability to overrun and should be designed in a way that maximizes its overrunning ability without compromising the robot's flexibility by making the arm too bulky. The robot tracks should feature a high coefficient of friction, which will enable the robot to traverse slippery dirt surfaces. Moreover, pendulum arms are designed to provide the robot with stability when moving across uneven and slippery terrain. The arms counterbalance the robot's movement, preventing it from tipping over and ensuring a smooth gait. These design features serve to improve the robot's efficiency and reduce the likelihood of damage to its components. With these considerations in mind, the robot's pendulum arms are designed, as shown in Figure 2, each of which can be rotated 360° to adjust the robot's walking posture. With these design features, the robot has a higher chance of successfully traversing rough terrains and overcoming obstacles, making it a highly useful tool for search and rescue operations, exploration, and military applications.

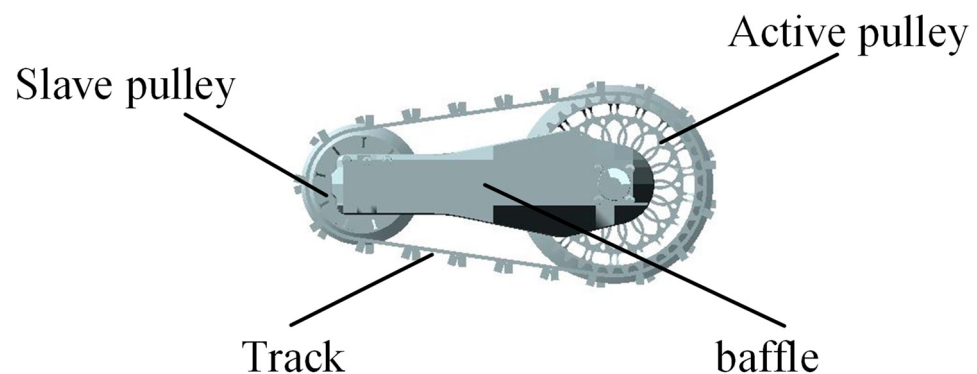


Figure 2. Swinging Leg.

3. Step-Climbing Ability

Steps are a common obstacle found in both structured and unstructured environments. Studying how a tracked robot crosses steps clarifies its ability to navigate through other obstacles and is a crucial indicator of its overrunning performance [17]. The robot's journey over the steps can be divided into two main parts. First, the robot step climbing stage is considered. A process whereby the robot is driven with a sufficiently large force to enable the robot to hitch onto the outer corner of the step is investigated. Second, the robot steps across the stage. At this stage, as the robot continues to advance, the angle of elevation of the robot increases, and the center of gravity of the robot gradually approaches and overtakes the outer corner line of the step, both horizontally and vertically [18]. The following assumptions were made to analyze the performance of the robot across the steps. (1) No slipping of the swing arm tracks and main track with the steps occurs; (2) The contact with the step is rigid; (3) The robot travels without deflection and in the same direction throughout its journey over obstacles. (4) The robot's power section has ample driving power.

3.1. Relationship between the Center of Mass and Track Angle

The center-of-mass variation law has an essential influence on a robot's ability to overturn obstacles during the overturning process [19]. It is beneficial to understand the change in the robot center of mass with the pitch angle of the robot body and the swing angle of the joint track to understand the change in the robot center of mass in the process of obstacle crossing and to plan the robot's obstacle crossing posture better.

As shown in Figure 3, with xoy as the implicated coordinate system, the mass of the main body is M , the masses of the two swing arms are m_1 and m_2 , the length of the active track is L , the lengths of the two swing arm tracks are L_1 and L_2 , the center of mass of the main body is G , the centers of mass of the two swing arms are G_1 and G_2 , the radius of the main pulley is R , the radius of the middle pulley is R_1 , the radii of the front and rear swing arm pulleys are R_2 , and the joint angles between the tracks are θ_1 , θ_2 , and θ_3 . The angle of the track L_2 axis around the track L axis in the counterclockwise direction is positive, and the angle of the track L_1 axis around the track L axis in the counterclockwise direction is positive. The x -coordinate C_x and y -coordinate C_y of the center of mass C in the xoy coordinate system are calculated by the following equation.

$$c = \frac{\sum_{i=1}^n m_i C_i}{\sum_{i=1}^n m_i} \quad (1)$$

$$C_x = \frac{m_2 a_2 \cos \theta_2 + M l \cos \alpha + m_1 (L + a_1 \cos \theta_1)}{M + m_1 + m_2} \quad (2)$$

$$C_y = \frac{m_2 a_2 \sin \theta_2 + M l \sin \alpha + m_1 a_1 \sin \theta_1}{M + m_1 + m_2} \quad (3)$$

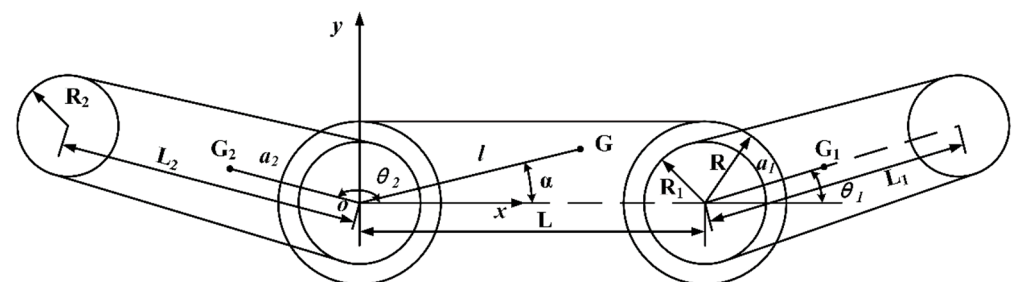


Figure 3. Schematic diagram of the land walking center of mass.

The relationship between the position of the center of mass and the angle (θ_1 and θ_2) are shown in Figure 4a,b).

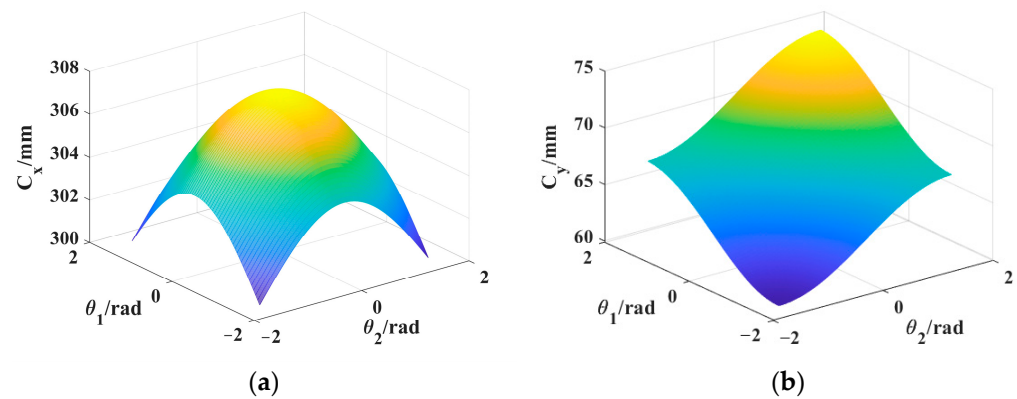


Figure 4. Center of mass position versus angle. (a) C_x varies with the swing arm angle. (b) C_y varies with the swing arm angle.

3.2. Gait Planning

The capability of a mobile robot to traverse a step obstacle is primarily determined by the maximum height it can successfully climb over. By coordinating the control of the main body's posture and the front and rear swing arms, the robot's capacity to negotiate obstacles can be optimized [20]. In the case of steps with varying height ranges, the robotic platform can utilize different gaits to climb them effectively. We redesigned the gait of the robot climbing stairs, improving its efficiency and performance.

- (1) At low-height steps, to make the climbing process faster for the robot, the steps are climbed in Figure 5a.
 - (a) Assuming the robot is positioned at the front end of the step, the front swing arm rotates clockwise by a certain angle to contact the low step and the rear swing arm rotates counterclockwise by 180° , as shown in the 1→2 process in Figure 5a.
 - (b) The robot advances and the rear swing arm track rotates counterclockwise, causing the robot body to rotate counterclockwise by a certain angle, as shown in the 2→3 process in Figure 5a.
 - (c) The rear swing arm rotates counterclockwise as the robot body advances, and when the robot body falls on the step, the rear swing arm rotates clockwise, as shown in the 4→5→6 process in Figure 5a.
- (2) At high steps, the posture of the robot differs from that when climbing low steps. The gait of the robot when climbing high steps is shown in Figure 5b.
 - (a) Assuming that the robot platform is just in front of the step, the rear pendulum arm rotates clockwise, causing the body to rotate counterclockwise and raising the center of mass of the body. The front pendulum arm rotates clockwise and contacts the step, as shown in process 1→2 in Figure 5b.
 - (b) The robot advances and the rear swing arm rotates clockwise to raise the center of mass of the vehicle, as in the process 3→4 in Figure 5b.
 - (c) The rear swing arm continues to rotate clockwise as the robot body advances, while simultaneously adjusting the angle of the front and rear swing arms to successfully cross the step, as shown in process 5→6 in Figure 5b.

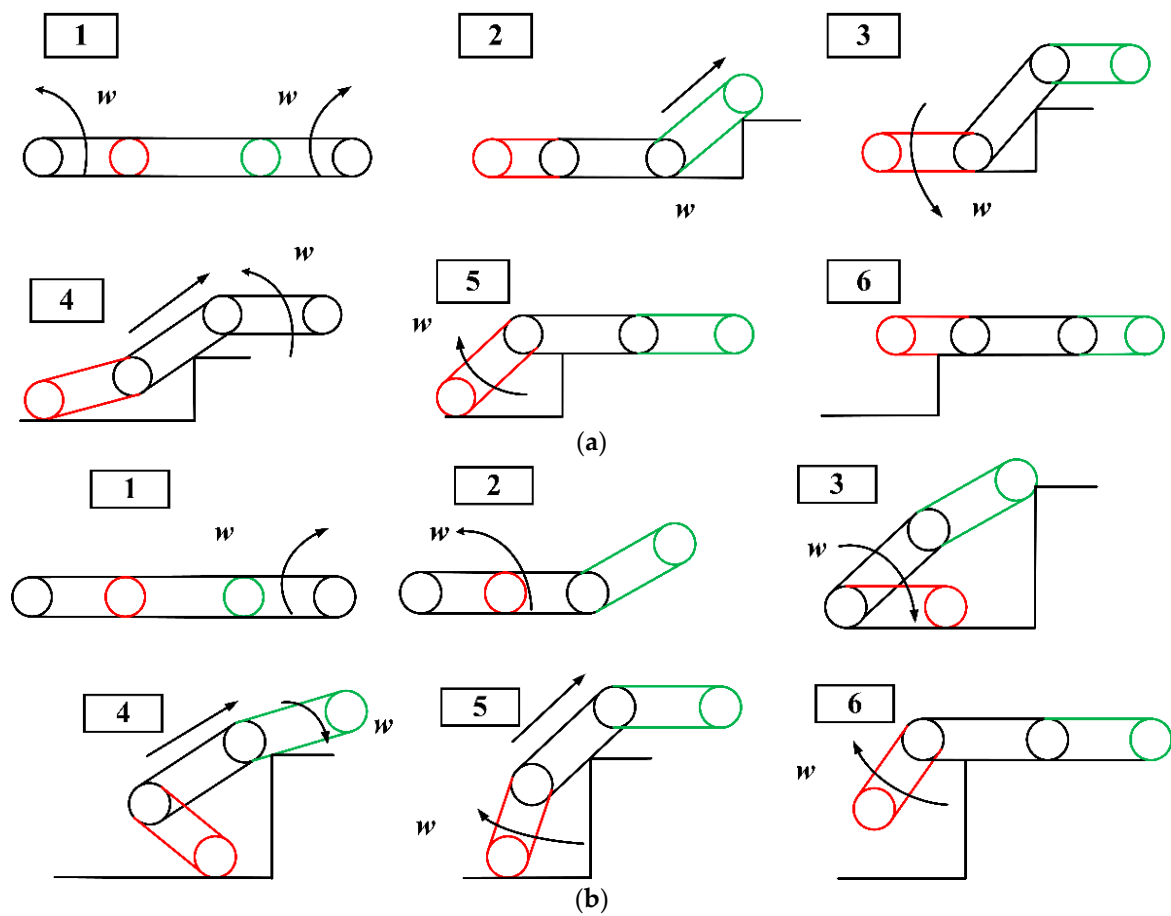


Figure 5. Step climbing gait planning for mobile robots. (a) Sketch of gait planning for a mobile robot climbing a low step. (b) Sketch of gait planning for a mobile robot climbing a high step.

3.3. Step Climbing Height Analysis

The height that the robot can climb steps depends on the effective radius R_e of the front wheels and the value of the height that the front swing arm can lift [21]. The robot climbs the low step height, as shown in Figure 6.

$$h(\theta_1) = R + L_1 \sin \theta_1 - R_2 \cos \theta_1 \quad (4)$$

$$R_e = \frac{x}{2 \sin \beta \cos \beta} \quad (5)$$

$$\sin \beta = \frac{x}{\sqrt{x^2 + y^2}} \quad \cos \beta = \frac{h}{\sqrt{x^2 + y^2}} \quad (6)$$

$$x = L_1 \cos \theta_1 + R_2 \sin \theta_1 \quad (7)$$

$$R_e(\theta_1) = \frac{L_1^2 + R^2 + R_2^2 - 2RR_2 \cos \theta_1 + 2L_1 R \cos \theta_1}{2L_1 \sin \theta_1 + 2R - 2R_2 \cos \theta_1} \quad (8)$$

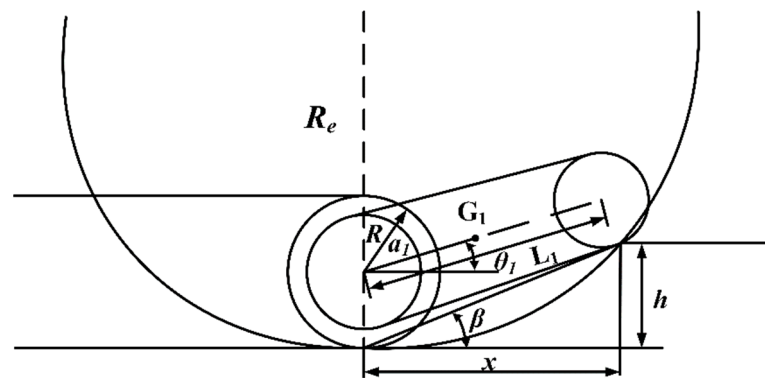


Figure 6. Diagram of robot climbing low step height.

With the height of the climbing low step $h(\theta_1)$ as the objective function and the angle θ_1 between the centerline of the front swing arm and the horizontal plane as the variable, the height model of the climbing step is as follows.

$$\begin{cases} \text{MAX } Z = h(\theta_1) \\ \text{ST } h(\theta_1) - R_e(\theta_1) \leq 0 \\ 0 \leq \theta_1 \leq \pi/2 \end{cases} \quad (9)$$

The height of the steps that the robot can climb when climbing high steps is determined by the effective radius of the main body and the front swing arm, as shown in Figure 7.

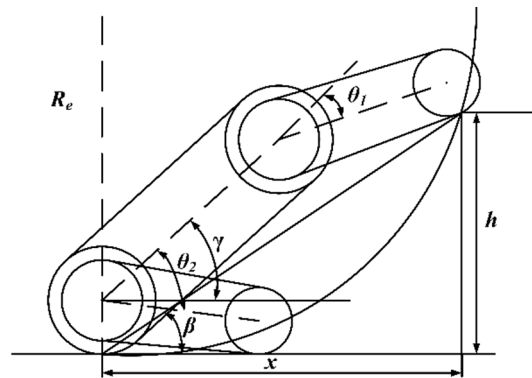


Figure 7. Diagram of the robot climbing a high step height.

As for the analysis of climbing high steps, the height and effective radius of climbing high steps are as follows.

$$\delta = \theta_2 - \gamma = \arcsin(R - R_2/L_2) \quad (10)$$

$$h = R + L\sin(\theta_2 - \delta) + L_1\sin(\theta_2 - \delta - \theta_1) - R_2\cos(\theta_2 - \delta - \theta_1) \quad (11)$$

$$x = L\cos(\theta_2 - \delta) + L_1\cos(\theta_2 - \delta - \theta_1) + R_2\sin(\theta_2 - \delta - \theta_1) \quad (12)$$

$$R_e(\theta_1, \theta_2) = \frac{[R + L\sin(\theta_2 - \delta) + L_1\sin(\theta_2 - \delta - \theta_1) - R_2\cos(\theta_2 - \delta - \theta_1)]^2 + [L\cos(\theta_2 - \delta) + L_1\cos(\theta_2 - \delta - \theta_1) + R_2\sin(\theta_2 - \delta - \theta_1)]^2}{2[L_1\cos(\theta_2 - \delta - \theta_1) + L\cos(\theta_2 - \delta) + R_2\sin(\theta_2 - \delta - \theta_1)]} \quad (13)$$

Taking the height of the climbing step $h(\theta_1, \theta_2)$ as the objective function, we use the angle θ_1 (between the centerline of the front swing arm and the centerline of the middle

track) and the angle θ_2 (between the centerline of the middle track and the centerline of the rear swing arm as variables). The height model of the climbing step is as follows.

$$\begin{cases} \text{MAX } Z = h(\theta_1, \theta_2) \\ \text{S.T. } h(\theta_1, \theta_2) - R_c(\theta_1, \theta_2) \leq 0 \\ L_2 - x \leq 0 \\ -\frac{\pi}{4} \leq \theta_1 \leq 0 \\ 0 \leq \theta_2 \leq \frac{\pi}{3} \end{cases} \quad (14)$$

3.4. Crossing the Height of the Steps

The robot reaches a critical state of overturning the step obstacle when the line of gravity (the vertical line over the center of mass) of the robot can pass exactly through the outer corner of the step. The center of the rear balance wheel circle is the origin of the coordinate system xoy , and the horizontal direction is the x -direction and the vertical direction is the y -direction, as shown in Figure 8.

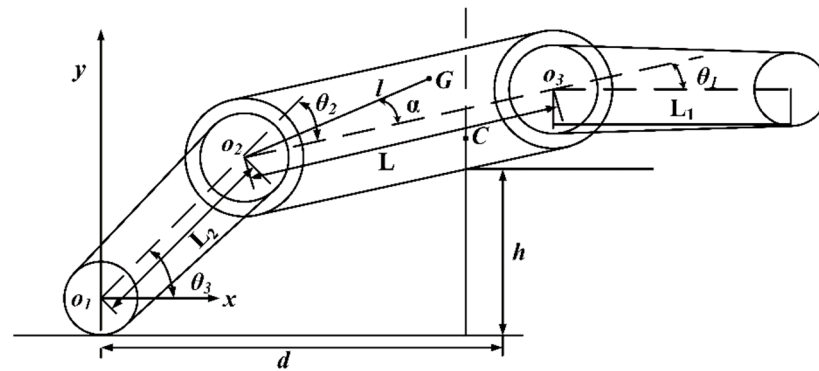


Figure 8. Schematic diagram of the robot platform going over the steps.

Then, the location of the center of mass of the robot platform is as follows.

$$X_c = \frac{m_2 a_2 \cos \theta_3 + M[L_2 \cos \theta_3 + L \cos(\theta_3 - \theta_2 + \alpha)] + m_1[L_2 \cos \theta_3 + L \cos(\theta_3 - \theta_2) + a_1 \cos(\theta_1 + \theta_2 - \theta_3)]}{M + m_1 + m_2} \quad (15)$$

$$Y_c = \frac{m_2 a_2 \sin \theta_3 + M[L_2 \sin \theta_3 + L \sin(\theta_3 - \theta_2 + \alpha)] + m_1[L_2 \sin \theta_3 + L \sin(\theta_3 - \theta_2) + a_1 \sin(\theta_1 + \theta_2 - \theta_3)]}{M + m_1 + m_2} \quad (16)$$

The distance from the center of the rear swing arm wheel to the step is d . When the critical condition of the overturning step is reached, $X_c = d$, thus, the height of the overturned step is H .

$$H = L_2 \sin \theta_3 + R_2 + (d - L \cos \theta_3 - R \sin(\theta_3 - \theta_2)) \tan(\theta_3 - \theta_2) \quad (17)$$

When H reaches the maximum, there must be $\frac{\partial H}{\partial \theta_1} = 0$, and then we can find $\theta_1 + \theta_2 - \theta_3 = 0$, that is, $\theta_1 = \theta_3 - \theta_2$, and, thus, we can obtain:

$$H(\theta_1, \theta_3) = \left(\frac{m_2 a_2 \cos \theta_3 + M[L_2 \cos \theta_3 + L \cos(\theta_1 + \alpha)] + m_1[L_2 \cos \theta_3 + L \cos(\theta_1) + a_1]}{M + m_1 + m_2} - L \cos \theta_3 - R \sin \theta_1 \right) \tan \theta_1 + R_2 + L_2 \sin \theta_3 \alpha \quad (18)$$

The height H of the overturning step serves as the objective function, with the angle θ_1 between the centerline of the front swing arm and the centerline of the middle track and

the angle θ_3 between the centerline of the rear swing arm and the x-direction as variables. The overturning height model is as follows.

$$\begin{cases} \text{MAX } Z = H(\theta_1, \theta_3) \\ -\frac{\pi}{2} < \theta_1 < \frac{\pi}{2} \\ 0 \leq \theta_3 \leq \pi \end{cases} \quad (19)$$

4. Slope Climbing and Trenching Capability

4.1. Trench Crossing Capability

Trenching is a relatively common obstacle in complex environments and is one of the typical obstacles in structured and unstructured environments. Analyzing the motion mechanism of a mobile robot across a trench is essential in the validation of its design and assessing the overall motion performance of the mobile robot. This analysis is also an integral part of the overall motion performance assessment of the mobile robot [22]. There are two main situations in which a robot can cross a trench: the width of the trench is less than the maximum width the robot can drive through, or the maximum width the robot can drive through is less than or equal to the width of the trench. In the first case, when the robot crosses this type of obstacle, it does not tip over because its center of gravity is in a vertical line the proximal edge line of the trench, or it has not yet arrived when the front swing arm just touches the distal edge line of the trench. When the robot's body crosses the trench, its center of gravity aligns vertically with the distal edge line of the trench or has already crossed it, while the rear swing arm is located at the proximal edge line of the trench. Therefore, it will not overturn either, as shown in Figure 9.

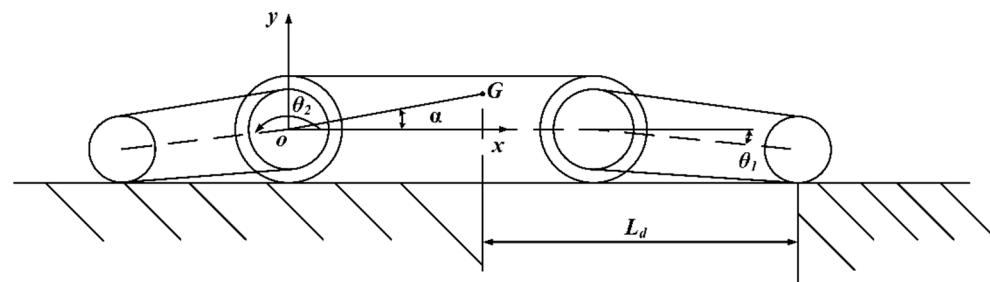


Figure 9. Schematic diagram of the robot crossing trench.

The maximum width that the robot platform can have over the trench is determined by the minimum distance between the center of mass and the top of both the front and rear pulleys.

$$L_d = \min\{L_2 \cos \theta_2 + C_x, L - C_x + L \cos \theta_1\} \quad (20)$$

4.2. Slope Climbing Ability Study

In mobile robot design, the stability of the robot's large slope motion is a key factor in measuring the success or failure of mobile robot design. It is an essential indicator of the robot's ability to move longitudinally or laterally on a slope without tipping over, and it is closely related to the robot's ability to smoothly traverse a rough terrain. Every concave and convex cell on the ground can be treated as a slope, either significant or minor. Therefore, the anti-tip performance of a mobile robot on a slope can be simplified as stability of its travel on a rough road [23]. Slopes can be classified into two types based on the orientation of the mobile robot's movement on the sloping terrain—positive slope and side slope terrain. Positive slope terrain refers to the situation where the robot moves along the slope direction of the slope, while side slope terrain refers to the condition where the robot moves along the slope direction of the vertical slope [24]. The size of the robot's climbing ability is related to the road adhesion coefficient and the size of its structure.

The height of the center of mass should be adjusted by altering the angle between the front and rear swing arms. This adjustment ensures that any variations in the slope

angle of the slope will not cause the robot to tip over. The slope angle γ is a function of θ_1 , θ_2 , and θ_3 .

As shown in Figure 10, the robot climbs the slope longitudinally. To avoid tipping, the robot must meet the following condition, where G_c represents the robot's gravity.

$$G_c \cos \gamma_1 (C_x + L_2 \cos \theta_2) > G_c \sin \gamma_1 (C_y + R) \quad (21)$$

$$\gamma_1 < \arctan \left(\frac{C_x + L_2 \cos \theta_2}{C_y + R} \right) \quad (22)$$

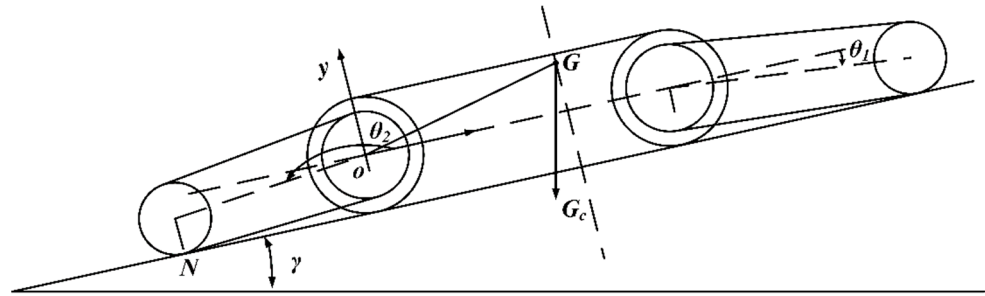


Figure 10. Robot climbing slopes longitudinally.

During the climbing process, the robot has the interaction of the air resistance F_1 , and the traction force F of the swing arm tracked robot, in addition to the gravitational force, the support force, to complete the climbing process of the swing arm tracked robot [25]. Let the coefficient of friction between the robot track and the slope be μ_1 , and the conditions for climbing the slope without slipping are as follows.

$$F - F_1 - \mu_1 G_c \cos \gamma_2 - G_c \sin \gamma_2 \geq 0 \quad (23)$$

The speed of the robot is relatively slow, and the air resistance of the swing arm tracked robot is also very low when moving against the wind, which is generally negligible. Therefore, the above equation can be simplified at this point:

$$F - \mu_1 G_c \cos \gamma_2 - G_c \sin \gamma_2 \geq 0 \quad (24)$$

$$\gamma_2 \leq \arcsin \frac{F}{G_c \sqrt{\mu_1^2 + 1}} - \arctan \frac{1}{\mu_1} \quad (25)$$

In summary, the maximum slope angle that a robot can climb longitudinally without tipping or slipping is:

$$\gamma_{max} = \min (\gamma_1, \gamma_2) \quad (26)$$

Realistic terrain slopes are not all single slopes, and sometimes the terrain may be shaped as an inverted figure—the “Chinese eight”. In such cases, the width of the flat part at the bottom may be smaller than the width of the main body of the robot. If the robot is allowed to move directly in its original state, the outermost sides of the track and wheels may encounter the inverted figure, a “Chinese eight” slope, making it difficult for the swing-arm tracked robot to continue moving. For the robot to move effectively, it needs to move completely to one side of the slope, where the track and wheel sections are in full contact with the slope, as shown in Figure 11.

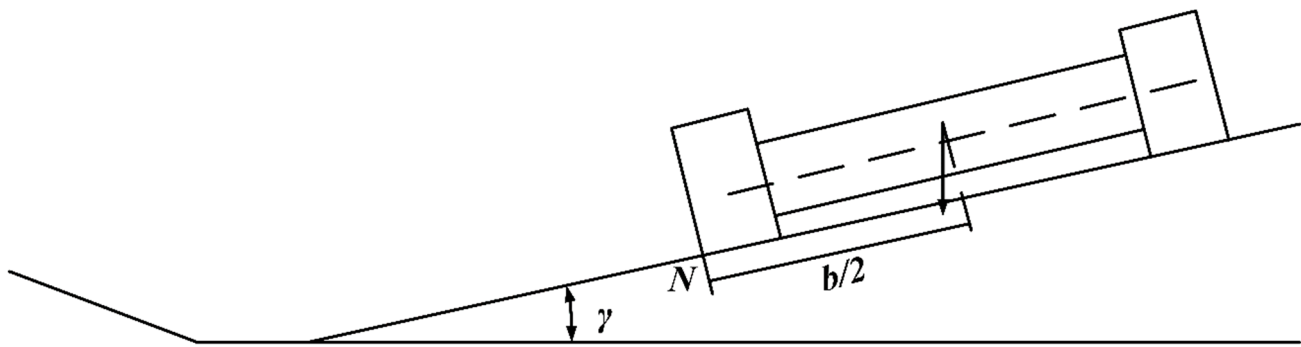


Figure 11. Schematic diagram of lateral slope climbing.

The robot moves laterally on the slope, and the maximum slope angle at which the robot does not tip over is obtained. The condition for lateral movement on a slope without tipping is as follows.

$$G_c \cos \frac{b}{2} > G_c \sin \gamma_3 (C_y + R) \quad (27)$$

$$\gamma_3 < \arctan \left[\frac{b}{2(C_y + R)} \right] \quad (28)$$

The coefficient of friction between the robot's lateral motion and the slope is μ_2 . The conditions under which the lateral motion does not slip are as follows.

$$\mu_2 G_c \cos \gamma_4 > G_c \sin \gamma_4 \quad (29)$$

$$\gamma_4 < \arctan \mu_2 \quad (30)$$

The maximum slope angle that the robot can climb without tipping or slipping is:

$$\gamma_{max} = \min (\gamma_3, \gamma_4) \quad (31)$$

5. Simulation and Numerical Calculation

Based on the analysis and our task requirements, we have carefully designed and developed a robot for urban environments. To gain a better understanding of the robot's performance, we have listed its relevant parameters in Table 1. The robot we have manufactured is shown in the following, Figure 12.

Table 1. Robot construction parameters.

Name	Parameters	Value	Unit
Front swing arm mass	m_1	1.02	kg
Rear swing arm mass	m_2	1.02	kg
Intermediate body mass	M	16.05	kg
Length of front swing arm	L_1	200	mm
Rear swing arm length	L_2	200	mm
Length of main body	L	520	mm
Distance from sidetrack to centerline	b	568	mm
Distance of main body center of mass from point o	l	314.42	mm
Angle between the center of mass of main body and x -axis	α	13.96	°
Distance from the center of mass of the swing leg to o	a_2	65.62	mm
Wheel radius	R	91	mm
Active pulley radius	R_1	78	mm
Slave pulley radius	R_2	48	mm

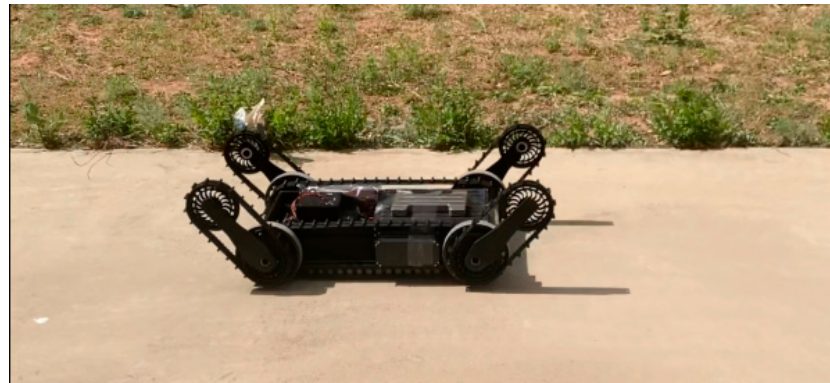


Figure 12. The robot for urban environments.

Substitute the parameters of the robot into Equations (4)–(19). After calculation, for the robot climbing low steps, it reaches the maximum height of climbing $h_{max} = 206.65$ mm when $\theta_1 = 13.49^\circ$. As for climbing high steps, the maximum height of the robot climbing high step is $h_{max} = 542.47$ mm. For the height of the robot platform overturning, the maximum overturning height is $H_{max} = 505.3$ mm. Thus, the robot's maximum step climbing height is 505.3 mm. When the height is below 206.65 mm, the robot adopts the first gait to improve climbing efficiency. For heights exceeding 206.65 mm, the second gait is used to maximize the climbing ability.

Figure 13a shows the curve of the swing arm track's effective radius R_e and step height h , when the robot climbs a low step. The intersection of the two curves is the maximum height the robot can climb under a low step. Figure 13b shows the surface plot of the swing arm's effective radius R_e and step height h , when the robot climbs a high step. The highest point on the intersection line of the two surfaces is the maximum height that the robot can climb under the high step. Figure 13c shows the surface plot of the relationship between the overturning height of the robot and the angle of the swing arm track. The highest point on the surface corresponds to the maximum height that the robot can overturn.

The virtual prototype of the wheel-tracked robot and the obstacle model were created in the three-dimensional modeling software Unigraphics NX 8.0 and imported into ADAMS. The four main wheels of the robot and the ground were set as the contact collision type, and the dynamic friction and static friction coefficients were redefined so that the robot wheels can contact the road and generate sufficient friction for movement. This allowed for the robot model movement on the road. The initial simulation settings and obstacle parameter settings are listed in Table 2.

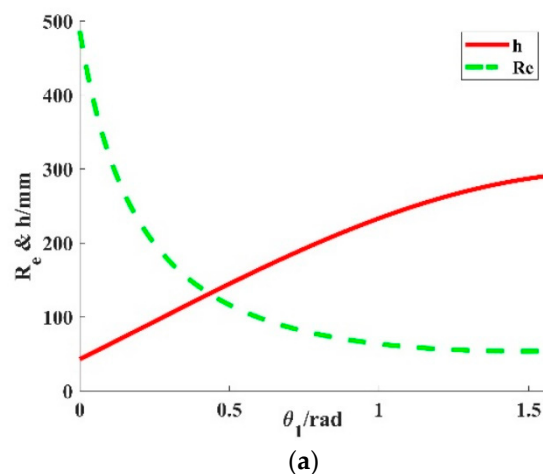


Figure 13. Cont.

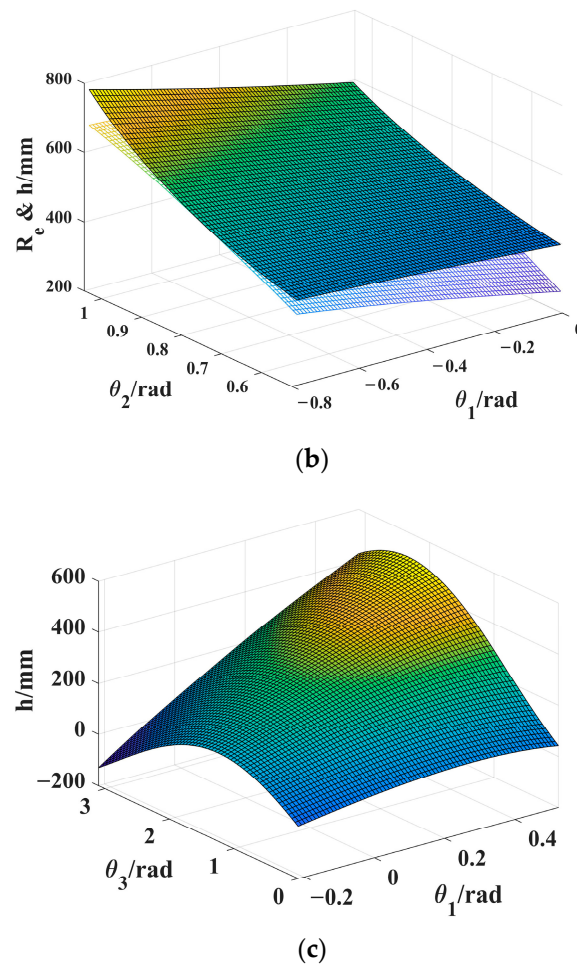


Figure 13. Numerical solution for robot climbing stairs. (a) Low step-down climbing height and effective radius curve. (b) Climbing height and effective radius curve under high steps. (c) Robot over-height surface.

Table 2. Crossing step simulation parameters.

Parameter	Numerical Value
Surface roughness	1.2
Robot speed/(m/s)	1
Obstacle height/(mm)	300
Static Coefficient	0.3
Dynamic Coefficient	0.25
Gully width/mm	300
Ramp angle/(°)	30

The linear motion speed curve of the robot is shown in Figure 14. The model of the robot undergoes a linear acceleration process within 0.4 s after startup. The model can fully drive into the simulated road surface after 1.2 s of trial calculation because of the non-excitation road surface at the beginning of driving. After approximately 0.41 s, the robot mobile reaches its maximum motion speed and enters a uniform linear motion state. The robot ultimately meets the preset requirement of 1 m/s.

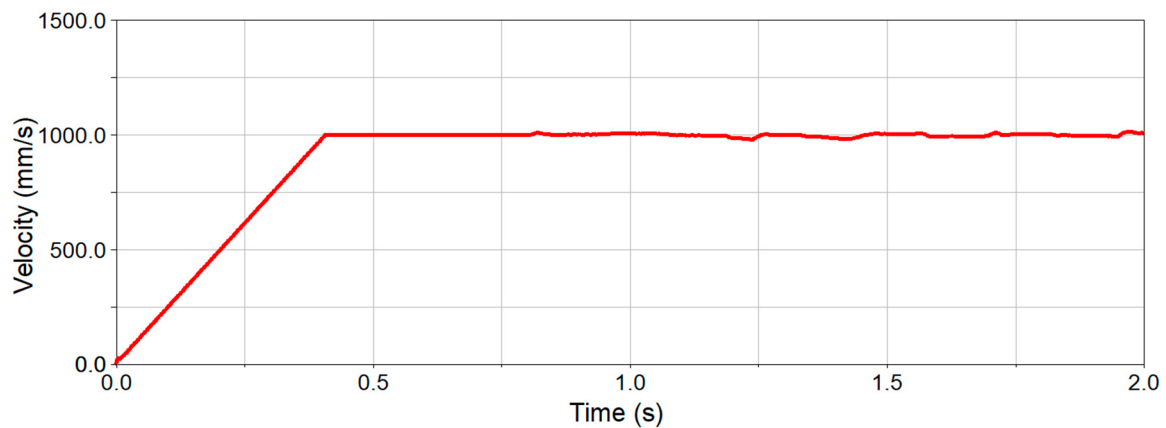


Figure 14. Robot speed curve.

The simulation of the robot climbing a step with a height of 300 mm is shown in Figure 15. The robot first approaches the step at a steady speed. Then, the rear swing arm rotates clockwise to raise the robot's body, and the front swing arm rotates clockwise to contact the step. As the robot advances, the rear swing arm rotates clockwise, and the robot lifts until it successfully crosses the high step. Overall, the simulation demonstrates the robot's ability to climb steps with ease and accuracy. The use of swing arms and the coordinated motion of the robot's body allowed it to elevate itself and cross the step effectively. This proves the effectiveness of our gait planning. Future work can focus on enhancing the robot's ability to climb higher steps and improving its stability during the climbing process.

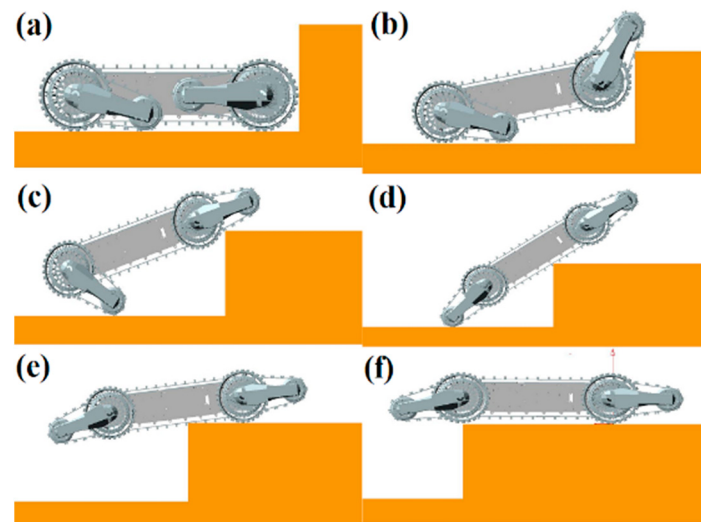


Figure 15. Step climbing (300 mm). (a) Climbing high steps stage 1. (b) Climbing high steps stage 2. (c) Climbing high steps stage 3. (d) Climbing high steps stage 4. (e) Climbing high steps stage 5. (f) Climbing high steps stage 6.

Substitute the parameters of the robot into Equations (21)–(26). When the rubber timing belt drives on a smoother road surface (without considering the influence of track spurs on the size of adhesion force), the adhesion coefficient f can be taken as 0.7, or, when the rubber timing belt drives on the road surface with higher friction, the adhesion coefficient can be taken as 1.2. According to the calculation, the maximum slope angle of the robot in longitudinal climbing is $\gamma = 35^\circ$ for an attachment coefficient of 0.7 and $\gamma = 50.2^\circ$ for an attachment coefficient of 1.2.

Substitute the parameters of the robot into Equations (27)–(31). According to the calculation, the maximum slope angle of the robot in longitudinal climbing is $\gamma = 35^\circ$ for an attachment coefficient of 0.7 and $\gamma = 50.2^\circ$ for an attachment coefficient of 1.2. Establish the slope climbing simulation model of the mobile robot.

The process of the robot's ascent up a slope of 30° has been simulated, and the simulation findings are elaborated in Figure 16. The initial scenario in Figure 16a shows the robot moving along steadily at a consistent speed prior to reaching the slope. During this stage, the robot's swing arm is held in a contracted state. Moving on to the subsequent stage in Figure 16b, the robot is depicted in an open swing arm state, negotiating the 30° slope. Additionally, the trajectory of the robot while it climbs up the slope is illustrated in Figure 17. When observing Figure 17, we can see that, from 0 to 0.66 s on the time axis, the robot was driving on a flat surface until it reached the slope position. At 0.66 s, the robot started to climb the stairs, and at 1.09 s, the robot had fully entered the slope and continued to travel on it.

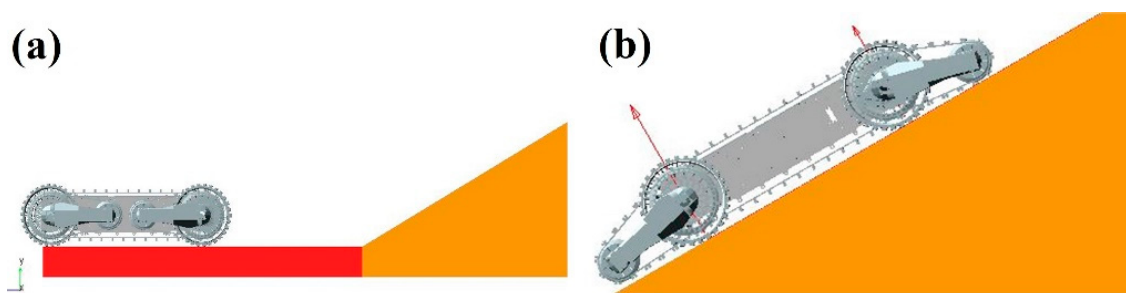


Figure 16. Slope climbing simulation (30°). (a) Moving on flat ground. (b) Moving on a slope.

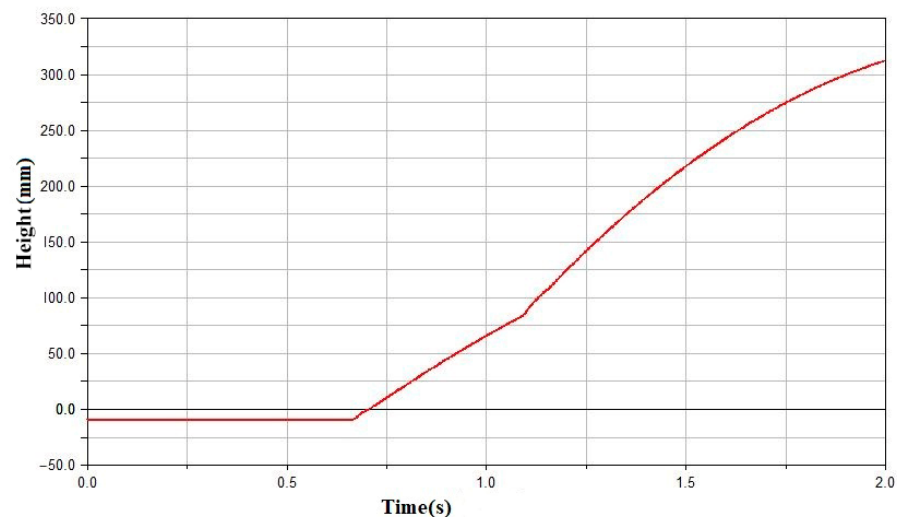


Figure 17. The centroid trajectory during uphill climbing.

Substitute the robot's parameters into Equation (20). According to the calculation, when $\theta_1 = -12.42^\circ$ and $\theta_2 = 192.42^\circ$, the maximum width of the transverse trench $L_d = 388.05$ m. Now, establish a simulation model of the robot that can cross the trench.

The simulation of the robot crossing the trench is shown in Figure 18. Initially, the robot moves at a constant speed to the near edge of the trench and subsequently enters the crossing phase with its front wheels in suspension and the front swing arm touching the far edge of the trench. Then, the front swing arm crosses the distal edge line of the trench, straddles the trench, and the front drive wheels move from the original overhang to the distal edge line of the trench. The robot proceeds to move forward, while its rear drive wheels remain suspended near the side edge line of the trench. The rear pair of drive

wheels of the robot moves from the overhang to the far side edge line, and the rear pair of swing arms begin to enter the overhang. Ultimately, the robot is positioned fully on the side of the distal edge line of the trench, successfully crossing it. The trajectory of the robot crossing the trench centroid is shown in Figure 19. It can be observed from the figure that the robot is capable of smoothly crossing the trench.

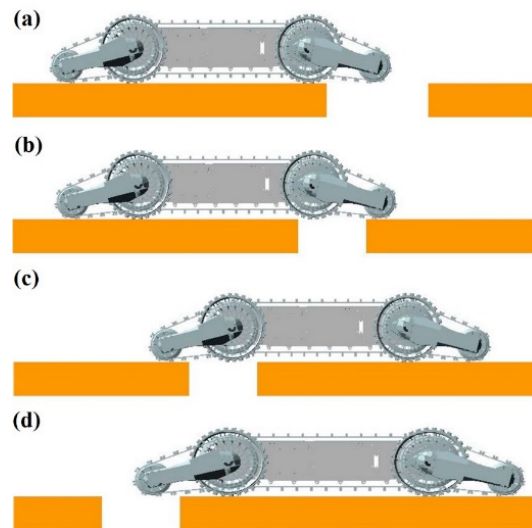


Figure 18. Cross Trench Simulation. (a) Crossing trench stage 1. (b) Crossing trench stage 2. (c) Crossing trench stage 3. (d) Crossing trench stage 4.

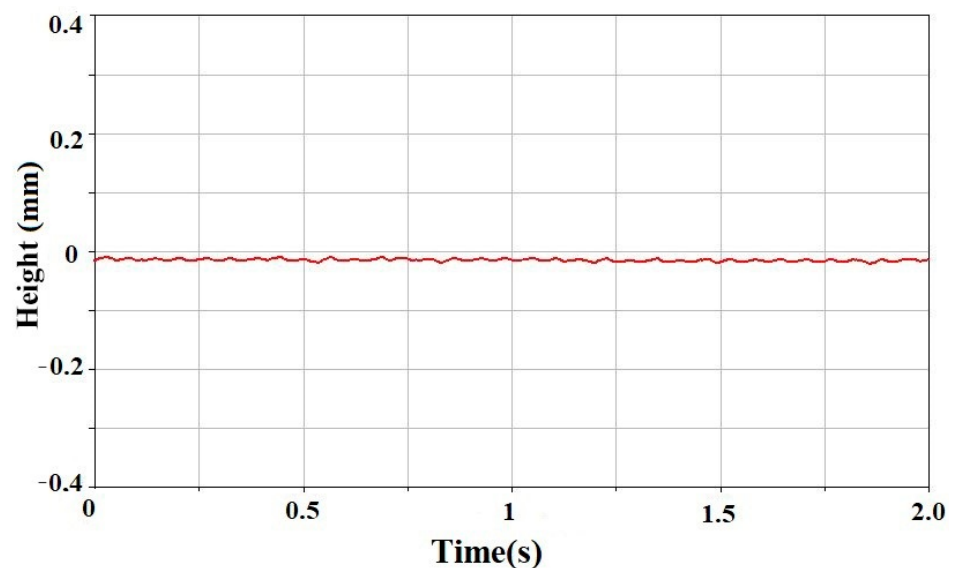


Figure 19. Centroid trajectory of trench crossing.

6. Conclusions

In this paper, we specifically designed a composite mobile robot with both wheeled and tracked structures. We planned corresponding gaits for it and analyzed its performance in climbing stairs, slopes, and crossing ditches. The research results are as follows.

- (1) Based on the change rule of the robot's center of mass, we planned stair-climbing gaits for different terrains. The maximum climbable height is 505.3 mm; if it is less than 206.65 mm, the first gait is used; otherwise, the second gait is adopted.

- (2) For slope climbing, we analyzed the platform torque changes and anti-skid conditions of the robot. When the adhesion coefficient is 0.7 and 1.2, the maximum longitudinal and lateral climbing angles are 35° and 50.2° , respectively.
- (3) We studied the geometric relationship between the robot's center of mass and the ditch during the ditch-crossing process, and we concluded that the maximum passable ditch width is 388.05 mm.
- (4) The simulation validation shows that the robot can successfully complete tasks, such as climbing stairs, going uphill, and crossing ditches and ravines, proving the effectiveness and rationality of the planned gait.

In future work, we will conduct in-depth research on robot performance, including three-dimensional analysis, to fully understand its performance in complex environments. We will focus on the coordinated operation at different heights in obstacle scenarios and explore other influencing factors to optimize the gait planning, improve the stability and adaptability of practical applications. In summary, we will analyze and validate from multiple perspectives, aiming to achieve effective robot applications in various environments.

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