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Methods and Tools Supporting the Learning and Teaching of Mathematics Dedicated to Students with Blindness

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Abstract: Teaching mathematics to blind people is a challenge of modern educational methods. This article presents a method of preparing the adapted material and its usage in the learning process of mathematics by blind people, as well as the results of evaluating the proposed approach. The presented results were obtained based on a mathematical analysis course conducted in two classes—with and without using the developed method. The developed method uses the conceptualization of knowledge as a graph. The learning process is supported by feedback processes that consider the mechanisms of knowledge and error vectors, on which a personalized adaptation of the learning path is made for each particular student. The evaluation process has shown a statistically significant improvement in learning results achieved by blind students. The average final test score in the group working with the platform during learning increased by 14%. In addition, there was an increase in cooperation between blind students who had the opportunity to take on the role of a teacher, which was observed in 27% of the participants. Our results indicate the effectiveness of the developed approach and motivate us to evaluate the method in a broader group of students. The engagement of students indirectly indicates overcoming the barriers known from the state of the art: uncertainty, poor motivation, and difficulties in consolidating the acquired skills.

Keywords: assistive technology; education; tutoring system; alternative presentation; math to students with blindness

Citation: Maćkowski, M.; Kawulok, M.; Brzoza, P.; Spinczyk, D. Methods and Tools Supporting the Learning and Teaching of Mathematics Dedicated to Students with Blindness. *Appl. Sci.* **2023**, *13*, 7240. https://doi.org/10.3390/app13127240

Academic Editors: Ante Bilic Prcic, Marko Periša, Dragan Peraković and Matjaz Gams

Received: 19 May 2023
Revised: 13 June 2023
Accepted: 15 June 2023
Published: 17 June 2023

1. Introduction

Nowadays, with technology advancing at a rapid pace, limitations in the learning of mathematics by individuals with blindness often lead to an insufficient acquisition of skills crucial in various aspects of life. This urges the development of algorithms, methods, and tools for effectively teaching science. An additional aspect worth considering in this context is adapting educational content to the needs of blind people. The main barriers that affect the learning of mathematics by blind people include the availability of adapted educational materials, communication between students and teachers, and challenges in evaluating the students' learning progress [1–3].

Despite the significant technological advancements in IT tools, we still face difficulties in adapting educational materials to suit the needs of blind individuals, especially when it comes to mathematical formulas and images. Adapting such materials requires extensive knowledge and experience from the person (transcriber) responsible for that [4–6]. Furthermore, such a process is very time-consuming and expensive—it requires specialized equipment such as Braille printers or monitors. Another challenge is the use of various mathematical Braille notations, which may differ from the individual's native language [7,8]. These notations include the Nemeth Code, Unified English Braille, the Marburg Mathematics Code, French Mathematical Braille, and Polish Braille Notation. Most students have only basic knowledge of a specific mathematical notation, which hinders their ability to correctly read complex mathematical formulas. Consequently, this limits their access to adapted materials. Moreover, these notations rely on a linear representation of mathematical expressions, whereas the original structure is presented visually and in a two-dimensional form. As a result, when dealing with more complex formulas, the Braille notation can become quite lengthy, leading to cognitive overload and potential misunderstandings or ambiguous interpretations of the math equation's structure [9,10].

Another challenge lies in the communication between teachers and blind students in the classroom. Students use Braille notebooks and mathematical notations to solve exercises during lessons to write expressions and formulas. Supervising a student's work in this scenario necessitates the teacher's knowledge of Braille and the appropriate mathematical notation to read the student's written material. Unfortunately, when the teacher does not know such a writing convention, they must converse with the student, dictate to them the content of the exercise to be solved, and then the student must read their noted solutions aloud. This individualized approach to working with students is time-consuming and restricts the teacher's ability to work with multiple students simultaneously [11]. Consequently, assessing students' learning progress in the classroom becomes challenging, laborious, and often requires individualized attention. An alternative to Braille-based mathematical presentation is voice communication, where the teacher writes the exercise on the board and simultaneously reads it aloud. It allows blind students to understand and take notes. However, this method of alternative communication requires precise descriptions of the equations and their structure to ensure unambiguous understanding.

Currently, e-learning applications and platforms are commonly and increasingly used in education, offering promising solutions that can potentially overcome the information and communication barriers faced by blind students, as mentioned earlier. However, the effectiveness of these tools relies on the availability of digital educational materials and applications. The next chapter presents an overview of the available methods and tools for blind students to learn mathematics.

This study presents methods and tools aimed at supporting the teaching and self-study of mathematics for blind students. Based on the literature review and our previous research involving blind participants and their teachers, as well as the use of assistive technologies, we have identified numerous challenges related to the accessibility of digital educational materials and the usability of educational software. Our approach focuses on facilitating the creation and supporting the process of preparing interactive mathematical exercises. Thanks to the proposed solution, we can define a graph of concepts for a specific mathematics course and assign subsequent stages of the exercise to specific elements in the graph. Each exercise presents the content, text, and mathematical formulas in an alternative adapted format, enabling their description and structure to be read aloud via a speech synthesizer.

Another method we employ involves supervised learning and the automatic selection of subsequent exercises for students based on their mastered material and identified mistakes. To assess the effectiveness of these methods, we developed a platform utilizing the concept of knowledge and error vectors. This platform automatically assesses students' learning progress and selects materials for further mastery. The evaluation of our methods was conducted in a group of high school students and involved comparing the effectiveness of traditional classroom learning with Braille textbooks to using our developed platform. The research results indicate an improvement in the teaching process when using the platform. Teachers could easily oversee the students' progress and address any difficulties, facilitating communication and enhancing the overall educational process. Students who utilized the platform provided positive feedback related to their experience and positively commended the user-friendly platform interface.

2. Related Works

Many research projects have been designed to overcome barriers and issues related to accessibility for blind and visually impaired students in the process of learning mathematics.

Based on the results of studies [12–14], it can be concluded that the traditionally used (static) form of presenting educational materials adapted for the blind poses several challenges in the teaching process. Teachers are required to have knowledge of Braille mathematical notation, and a teacher who lacks familiarity with such a notation must engage in dialog with the student, which hinders their interaction with other students in the classroom.

The second group of solutions allows for the dynamic/interactive presentation of mathematical content in an adapted format using assistive technologies such as screen readers, Braille monitors, and speech synthesizers. Numerous tools have been developed to offer alternative ways of presenting mathematical formulas and expressions to visually impaired students. For instance, one such approach involves the automated conversion of mathematical expressions into audio descriptions or mathematical Braille notation which describe their structure [15]. Additionally, methods for automatically converting expressions using formats such as MathML are well known [16]. Additionally, other researchers have proposed various methods of implementing mathematical notation in electronic form using specialized tools such as Lambda Editor, InftyEditor, and Duxbury Braille Translator [17,18]. The design of new mathematical notations often relies on extending established products such as BlindMoose and MathType for Microsoft Word, odt2braille for Open/Libre Office Writer, and the MathML extension of the DAISY format.

There are many tools and screen readers available on the market that support the automatic reading of mathematical formulas displayed on websites. An example of such a tool is Microsoft Immersive Reader. The tool offers a range of features that support reading, including font customization, background and spacing adjustments, text-to-speech capabilities, word highlighting during reading, text translation into different languages, and more in Microsoft products. This tool is particularly useful for individuals with reading difficulties and students with dyslexia, as well as visually impaired or low-vision individuals [19]. Unfortunately, the tool does not perfectly read mathematical formulas in Polish. Additionally, JAWS and NVDA screen readers do not correctly read mathematical formulas written in MathML in Polish. In many cases, current screen readers provide limited support with semantic navigation based on content type [20,21]. In addition, we tried to create a platform that was supported in various web browsers, e.g., Chrome and Firefox, and supported by various screen readers, e.g., JAWS, Windows-Eyes, SuperNova, etc.

Based on previous research involving blind students and their teachers, it can be concluded that many educational platforms developed for learning mathematics and STEM (science, technology, engineering, and mathematics) subjects, such as ALEKS and Khan Academy [22,23], strive to ensure the accessibility of their offered content by complying to the WCAG 2.1 (Web Content Accessibility Guidelines) [24]. However, not all educational materials provided are fully accessible to individuals who are visually impaired. Moreover, the usability of many online educational platforms and their user interfaces poses significant challenges for blind students [25]. These students often rely on screen readers or assistive technologies to navigate the content. In some cases, the educational platforms may not be optimized and prepared to work seamlessly with these assistive technologies, resulting in difficulties for blind students to access and interact with the materials effectively [26,27]. Another aspect that hampers accessibility is the use of complex formats, such as PDF files, that are often not properly tagged. Such formats can pose barriers to blind students who rely on screen readers or other assistive technologies to comprehend the content [28]. To address these issues, educational platforms need to prioritize accessibility in their design and development processes. They should provide comprehensive alternative text descriptions for visual content, ensure compatibility with assistive technologies, and adopt correct formats for content delivery.

Moreover, authors of many studies [29,30] emphasize the importance of applying the user-centered design (UCD) approach in designing platforms for individuals who are visually impaired. The UCD is a design methodology that focuses on the needs, expectations, and preferences of users to provide them with an optimal experience. Applying the UCD in the design of platforms for individuals with visual impairments helps to create educational environments that are accessible and effective for these users. This approach ensures that blind people receive full support in the learning process and have access to educational content adapted to their needs, enabling their development in areas such as mathematics and other STEM subjects [31]. Taking into account our previous experience in designing solutions for individuals who are visually impaired [32–34] and many consultations with blind students and their teachers, we also attempt to merge a user-centered design approach with our solution.

In conclusion, while lots of educational platforms make efforts to ensure accessibility by complying with WCAG 2.1 guidelines, they are yet to meet the needs of visually impaired individuals. The usability of these platforms and their user interfaces remains an important area to address, along with the assurance of adding alternative text descriptions and accessible formats. By overcoming these challenges, educational platforms can create a more inclusive learning environment for blind students in mathematics and other STEM subjects.

In addition to ensuring accessibility, another crucial aspect of our work focuses on automating the assessment of the student's knowledge and assigning exercises that are appropriate to their skills. Intelligent learning environments have recently emerged as practical tools in guiding students through course materials [35,36]. The literature review reveals different approaches to implementing these environments, with two prominent ones being data-driven and knowledge-engineering-based methods [35]. The data-driven approach is extensively described in the book [37], which provides an overview of available recommender systems designed to support learning. This approach relies on analyzing data, such as student performance and behavior, to make recommendations and personalize the learning experience. On the other hand, knowledge engineering methods, such as the knowledge vector [38,39], involve a comprehensive analysis of the exercises solved by the student. This analysis considers the common misconceptions and specific difficulties faced by the student, and other factors necessary for designing hints and feedback messages. Our platform incorporates this knowledge engineering approach to gain detailed insights into the potential learning problems encountered by students.

The knowledge vector method is widely recognized in education as a way to organize the learning process and reduce the student's effort by targeting areas that require further mastery [39]. In our platform, we have implemented this approach by highlighting a graph that represents a typical knowledge vector in a specific subject area. This graph contains all of the relevant terms and concepts related to the subject. Building upon this approach, we have extended its functionality by analyzing the types of mistakes made by students while solving math exercises with the use of error vectors. By analyzing the categories of mistakes, our platform gains more profound insights into the specific areas where students face difficulties. It allows us to provide targeted support, adapted hints, and feedback messages that address the particular challenges that students encounter. This approach goes beyond a generic assessment of correct or incorrect answers and provides a more detailed understanding of the specific misconceptions or gaps in knowledge that students may have.

As a contribution to the field, the presented platform utilizes algorithms that monitor the student's mastery of the material and select appropriate exercises adapted to each student's abilities. This approach offers a solution for overcoming barriers in the education of visually impaired individuals. By utilizing concepts such as knowledge and error vectors, the platform allows for a personalized and adaptive learning experience. Overall, our platform combines the knowledge vector method with an analysis of the student's mistakes, enabling us to create a more personalized and adaptive learning experience. By highlighting these techniques, we aim to optimize the learning process, focus students on the areas in which they need to improve, and provide adapted support to enhance their understanding and mastery of mathematical concepts.

The system's ability to adapt the learning process based on a student's current knowledge state contributes to advancing inclusive education. The importance of the developed method was also proven in our previous works [40]. Furthermore, the platform's application in the classroom facilitates collaboration between visually impaired and sighted students, creating an inclusive learning environment. The development of this platform brings new insights and possibilities to the field of educational technology, particularly in supporting the needs of visually impaired learners and promoting equal access to education.

3. Materials and Methods

The designed platform is a web application developed in accordance with the WCAG 2.1 standard [24]. Sighted students use a graphical user interface (GUI) with mouse and keyboard controls. On the other hand, visually impaired students have access to the platform using a keyboard and assistive technologies such as a screen reader and a speech synthesizer.

All exercises on the platform are prepared by mathematics teachers in TeX format using LaTeX tags. Based on the developed exercises, HTML content is generated (using a previously prepared conversion tool), which includes images of mathematical formulas. The decision to present mathematical content in a graphical form was made due to the accurate display and scaling capabilities in various web browsers. For this reason, an alternative description is prepared for each mathematical formula in a semi-automatic way, which can then be read by a blind person using a screen reader. This gives the teacher the opportunity to change the description of the mathematical formula by including additional information about the structure of the formula, according to the level of experience of the student. Alternative descriptions for inexperienced students who have trouble understanding mathematical formulas can be much more detailed.

In addition, such alternative descriptions can be read piece by piece by students using different screen renderers, which gives an easy way to navigate through the structure of the mathematical formula. The way of interacting with the platform is through an interface containing the following:

- Multiple choice lists;
- Drop-down lists;
- Text fields/boxes to enter values.

Moreover, all of the mentioned ways of interaction are available using a keyboard, and the blind student is notified (through the use of a screen reader) as to whether a particular list item is selected or not.

As part of our study, we developed a method whose particular stages are illustrated in Figure 1. Each of the presented stages is described in detail below.

3.1. Structuring Knowledge into a Graph of Concepts

The most important elements of the presented method involved developing a group of concepts related to selected branches of mathematics and organizing them into a directional graph of concepts. We first identified the issues included in the prepared course material to achieve this. Then, we placed the identified concepts into a table and created a directional graph. Each issue had a successor—a student needed to first master the material contained in a given concept before moving onto the next issue. The aim was to navigate the entire directional graph and reach the node representing the final element. The prepared directional graph is shown in Figure 2. Each node of the graph corresponds to a given concept from a given area of mathematics. It is worth mentioning that some nodes in the graph had already been highlighted (green color). We did this to better illustrate and provide an example of the solution from the later part of this article. The nodes marked in green helped to map the direction of the learning process and the issues necessary to solve the exercise presented in Section 3.4.

Table A1 (see Appendix A) presents the previously mentioned list of identified concepts in the developed material of the math analysis course at a high school level.

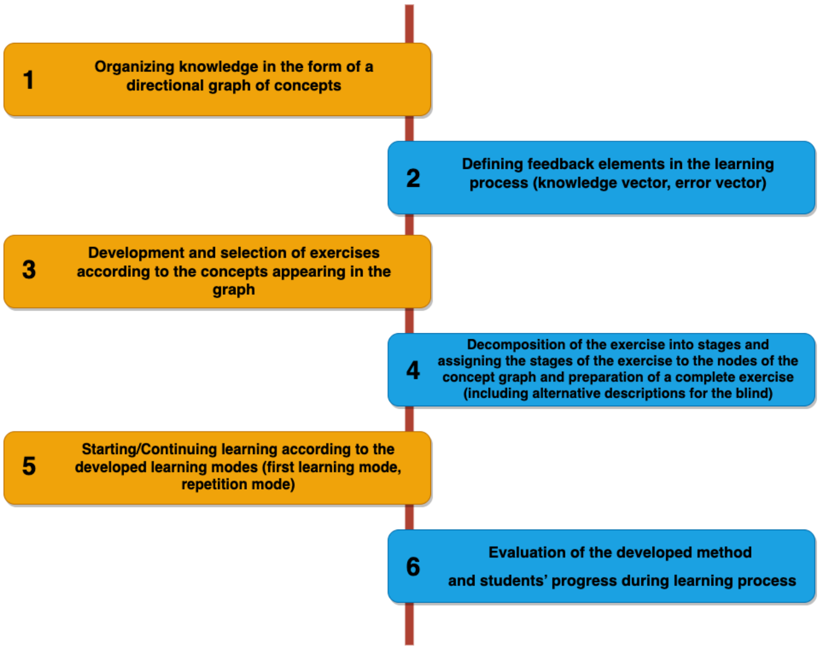

Figure 1. Stages of the proposed method supporting learning/teaching mathematics regarding students with blindness.

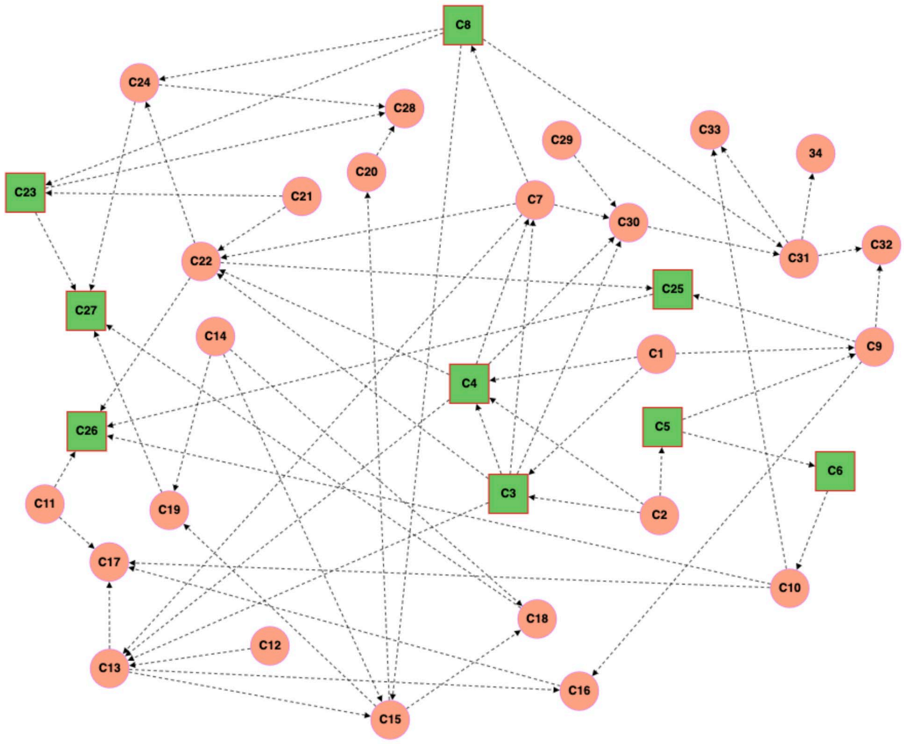

Figure 2. Concept graph developed for the mathematics course on mathematical analysis (green elements come from the exercise described in Section 3.4).

3.2. Elements of Feedback Used during Learning in the Developed Platform

A very important phase while designing the method was the development of a mechanism for handling feedback during the learning process, which consists of 2 elements:

- Error vector: Creates a learning history for a given student and accumulates mistakes that are grouped into classes. With a given node in the concept graph, multiple types of defined errors can be associated.

- Knowledge vector: stores information about the concepts mastered by the student based on correctly solved exercises.

The number of components of the error/knowledge vector results unequivocally from the number of nodes in the concept graph. For a given node, the type of error directly relates to the concept, e.g., the error in determining the domain of a function relates to the concept of defining the domain of a function. In the error vector component, we stored a list with timestamps corresponding to the moment of making an error while solving a step of the exercise assigned to the related concept. The length of the list determines how often a given error was made by a given user.

The knowledge vector is symmetrical to the structure of the concept graph—it is a list containing timestamps of phases of correctly solved exercises assigned to a related concept. The current state of the knowledge vector reflects the student's mastery of the material and determines the path that the user follows in the graph.

Figure 3 illustrates a scenario where the student is at a specific stage of the exercise corresponding to monotonicity intervals of a polynomial. This stage corresponds to concept C25 in the knowledge vector, which has a value of 0. The mistake made by the student while solving this stage of the exercise results in entering a value of 1 into the error vector. According to the proposed learning mode, the system requests the student to solve this stage again. Another mistake leads to entering the value 2 into the error vector and presenting a similar example in a step-by-step mode. After studying the example exercise, the student attempts to solve this specific step for the third time. In the case of a correct solution, the value 1 is set in the C25 field of the knowledge vector. However, if the solution is incorrect, the system redirects the student to the node selected from the set of predecessors (according to the proposed algorithm), the one with the largest value in the error vector. In the example exercise, it is the C9 node. The pseudocode of selecting the predecessor in the concept graph and the corresponding next exercise in case of student errors during the learning process is illustrated in the Algorithm 1.

Algorithm 1 Pseudocode of exercise selection during learning mode

require: concept graph C, knowledge_vector, error_vector
require: list of all exercises E corresponding for each element in graph C

input: current node N in concept graph C

M = C.get_predecessors(N)
W[] = 0
M_predecessor = None
for M_i in M:
    W_i = error_vector[M_i]
end for
if all W_i in W == 0:
    M_predecessor = select M_i from M where min(knowledge_vector[M_i].time_stamp)

else:
    maxW = max(W[])
    M_temp = select all M_i from M where W_i == maxW
    M_predecessor = select M_i from M_temp where max(error_vector[M_i].time_stamp)

Z = select random exercise from E corresponding to M_predecessor

output: Z

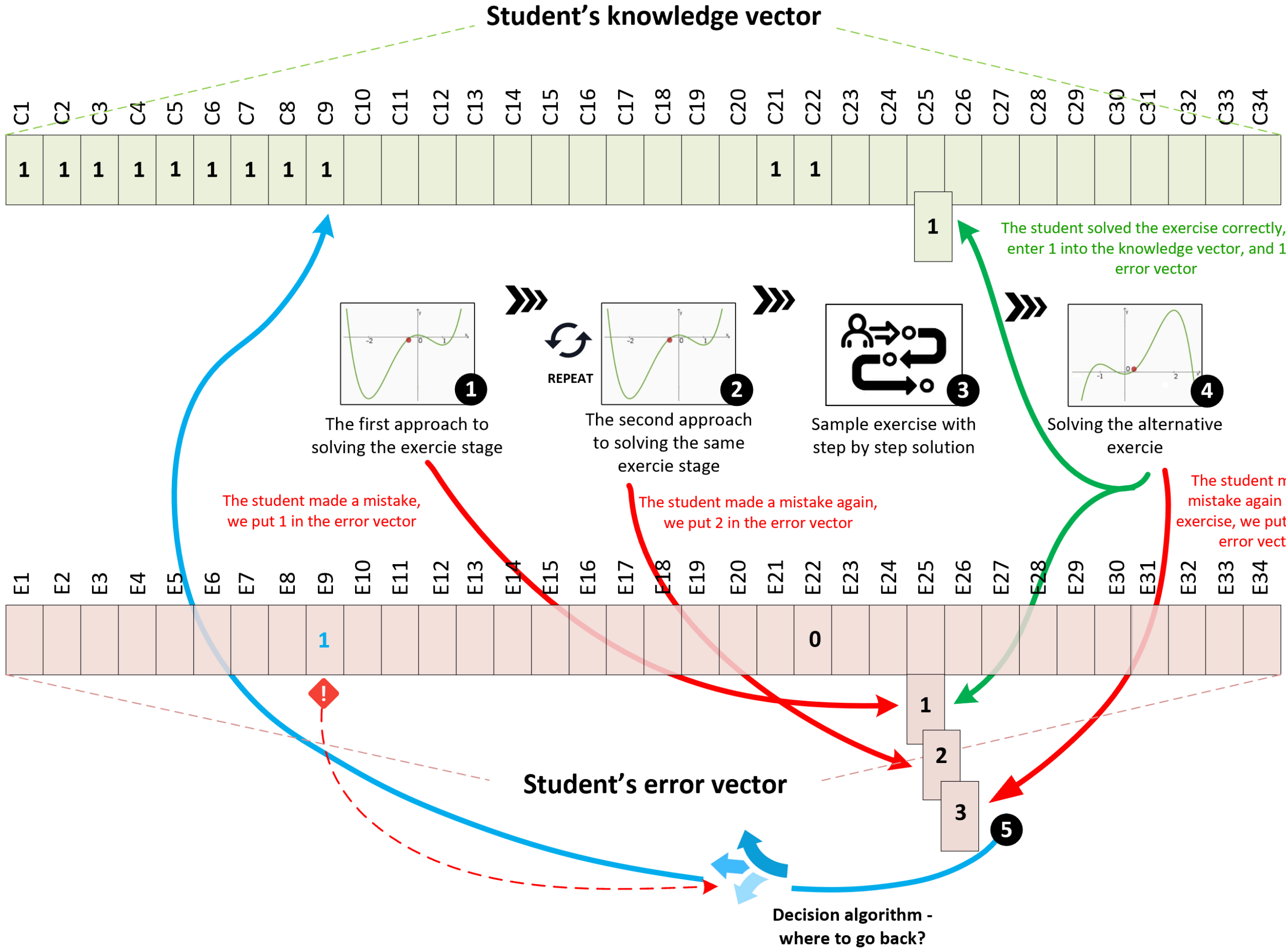

Figure 3. Elements of feedback used during learning in the developed platform (green—knowledge vector, red—mistakes vector).

3.3. Selection/Development of a Set of Exercises

We created the directional graph of concepts to facilitate the teacher's work. It allows the teacher to select or arrange a set of exercises based on it, aiming to cover as many essential issues as possible for the students to master. In other words, the developed graph in Section 3.1 is the range of material for which the teacher creates exercises to cover the entire graph.

3.4. Decomposition of the Exercise into Phases

The next stage of our method was to split exercises into phases. For this purpose, the teacher divides the exercises so that each phase is associated with at least one node from the directional concept graph. An example exercise is shown below:

"Find the local extrema of the function: $f(x) = 4x^3 - x^2 - 4x + 1$".

The decomposition of the above exercise is shown in Figure 4. It consists of the following stages: function domain, function subdomain, graph, root of a function, derivative of a function, monotonicity intervals of the function, and extreme of the function.

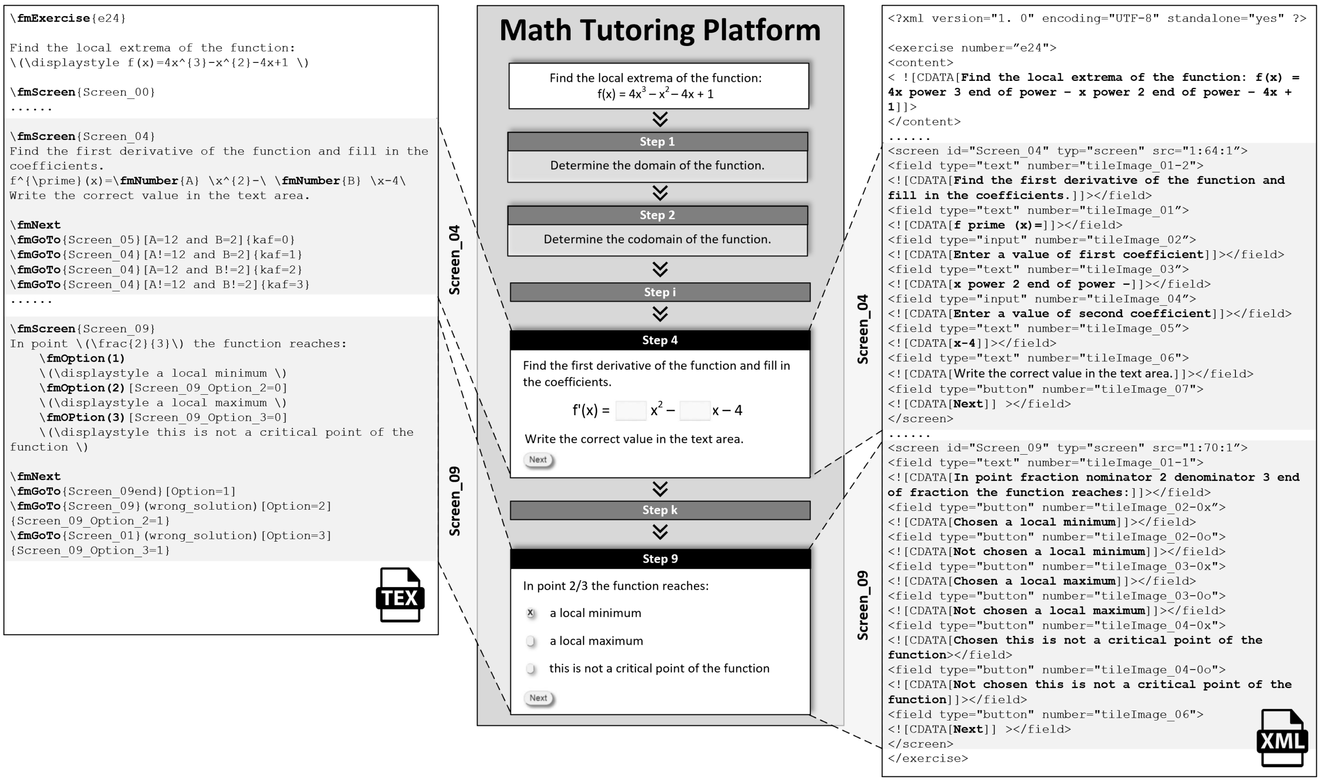

Figure 4. The decomposition of a sample exercise into stages.

Technically, in this stage, the method uses the extended syntax of the TEX compiler developed by the authors. The use of the mentioned syntax allowed us to go to the phase of generating the exercise into a form possible to be rendered in a web browser equipped with a screen reader that the blind person uses. One of the elements that the exercise contains is the generated conditions of the graph, which allow moving to the next stages [41]. In addition, the platform generates the structures for a given stage in an XML file containing alternative descriptions for individual elements.

To make the alternative description of a mathematical equation understandable for a blind person, it should contain additional elements describing the formula's structure, which are unnecessary for sighted people. This process can be simplified by defining a namespace in the XML standard. An example of the proposed notation for a selected stage of the exercise is shown in Figure 4. On the left side of Figure 4, there is a fragment of the TEX file that contains the exercise content. The bold font indicates additional tags that extend the TEX syntax. On the right side of Figure 4, there is a fragment of an XML file that contains alternative descriptions of mathematical expressions. The bold font indicates alternative descriptions that describe the structure of the expression, for example, "f(x) = 4x power 3 end of power − x power 2 end of power − 4x + 1". Our method used the extended syntax of the TEX compiler. The reason for such a choice was the fact that it is a standard for the description of mathematical exercises. Details of the extended syntax were presented in our earlier publication related to the topic of mathematics [18,42].

3.5. Starting/Continuing Learning

In the method, we utilized the defined material for the student to master, the directional graph of concepts, and the error and knowledge vectors described above to establish two learning modes. By learning mode, we mean the support provided in making decisions regarding the selection of the next exercise.

3.5.1. Learning Mode 1—Mastering New Material

This mode can be presented in several steps. At the beginning, in the knowledge vector, the subset of mastered concepts is zero. During the learning process, the student solves exercises consisting of stages, where each stage is associated with a specific node in the concept graph. Each step has two solving options: correct or incorrect. If the solution in an exercise current step is correct, the method indicates the next step in the exercise to be solved and assigns a value of 1 to the correctly solved step in the knowledge vector. In case of a mistake, the following algorithm has been proposed.

If an error occurs for the first time (possibly a random error), the system signals an incorrect solution and asks the student to solve the current step again. If the student makes a mistake again, the system provides a hint in the form of an analogous example containing a solution for the given stage. Optionally, the student has the opportunity to familiarize themselves with the theory related to the given stage in context (e.g., definitions of occurring terms, and formulas). If the student continues to make mistakes at a given stage of solving the exercise, the algorithm looks for the associated list of predecessors of concepts from the graph. In this case, the predecessor with the highest priority is selected. The priority is determined individually based on historical values contained in the error vector. The student must solve the selected exercise from the beginning and go through all of the previous stages.

Each concept in the graph has a list of exercise definitions (highly granular), in which the last step is the solution to the given concept. It allows the student's attention to be focused on a concept and issue that they have not yet mastered.

3.5.2. Learning Mode 2—Material Review

In the repetition mode, the proposed pseudocode (Algorithm 2) for selecting exercises in repetition mode considers the smallest subset from existing exercises that covers all desired concepts for repetition from the concept graph. As an assumption, we chose the longest exercises, as they were the most comprehensive ones.

The level of mastery of the material by individual students was verified based on their current values in the knowledge and error vectors.

3.6. Experiments and Evaluation Method

Nine mathematics teachers, experts in working with blind students, tested and evaluated the elements of the developed method. The teachers had a range of experience, with the least advanced teacher having 2 years of experience and the most advanced teacher having 10 years of experience teaching mathematics to blind students.

Next, we implemented and tested the system among students who attended a school for blind students. We evaluated the system with two groups of students. The pilot research focused on the course on algebra and mathematical analysis at a secondary school level. A parallel class learned the same material without using the proposed platform. The research group, consisting of 14 blind students aged 17–18, utilized the developed platform for learning. The second group, comprising 12 individuals of the same age, used traditional teaching methods for mathematics using Braille books. It is important to note that students in both groups had no other diagnosed disabilities that could hinder the learning process.

The stages of the proposed method to be evaluated were as follows:

1. Selection of the scope of the material—the evaluation covered the correctness of the preparation of the concept graph.
2. Selection or design of new exercises—the evaluation included the preparation of exercises in the extended syntax of the TEX compiler.
3. Division of the exercises into stages—the evaluation included the assignment stages of exercise to concepts in the graph.
4. Creating alternative descriptions—the evaluation included applying the developed rules for generating alternative descriptions.

Algorithm 2 Pseudocode of exercise selection during revision mode.

```
require: student's error_vector instance—E
require: list of exercises Z = [Z1, Z2, Z3 ... ] with information about covered concept nodes
where:
Zi = [z1, z2, ... , zn]
is a binary vector of length n equals to the length of error vector.
Every exercise Zi is represented as a vector, where each non zero component
represents the corresponding covered concept node.

E = [e1, e2, ... , en]
is a vector of length n equals to the length of error vector, where each non
zero component represents the number of errors made by the student in a
particular concept.

input: E

n = E.length()
E_repetition[] = 0
Z_selected[] = None
Z_copy = Z
for j in n:
      if E[j] > 0:
            E_repetition[j] = 1
end for
while (E_repetition.contains(1)):
   for Zi in Z_copy:
         covered_concepts[] = 0
         for j in n:
               if zj * ej == 1: covered_concepts[i]++
         end for
   end for

   Z_temp = Z_copy[index of max(covered_concepts[])]
   Z_selcted.append(Z_temp)
   Z_copy.remove(Z_temp)

   E_covered[] = 0
   for j in n:
         E_covered[j] = E_covered[j] * Z_temp[j]
         E_repetition[j] = E_repetition[j] − E_covered[j]
   end for
end while

outuput: Z_selected
```

Evaluating the student's learning progress:

5. Using the system for learning—the evaluation included verifying the feedback mechanisms: the error vector and the knowledge vector in the process of mastering the developed material by students.

The evaluation process encompassed the following quantitative measures:

- Additional teacher time needed to prepare the course:
  a. Preparation of the concept graph.
  b. Selection of the exercise.
  c. Division of exercises into stages, assignment of every exercise stage to the node in the concept graph, and preparation of alternative descriptions.

- Evaluation of the impact of the developed method on students' progress through the average grades of students in mastering the developed material:
    a. Comparison of grades between two classes in the final test (one using the developed method and one not using it).
- Evaluation of the possibility of knowledge consolidation by blind students through assisting less advanced peers in learning a new topic—the percentage of students who attempted to support their peers.

4. Results

As part of the evaluation of points 1–4, we assessed the teacher's additional time needed to prepare the entire course. The process of preparing the materials was distinguished into the following stages:

- Preparation of the concept graph (average time of preparing a graph of concepts for the teacher's course)—between 10 and 18 h, with an average value of 15.2 h.
- Selection of a set of exercises that cover all of the concepts from the concept graph within the chosen branch of mathematics. The exercises should represent different difficulty levels, and each element of the graph should be associated with five different exercises (average time for a teacher to choose or prepare an exercise)—between 8 and 16 h, with an average value of 9.7 h.
- Dividing the exercises into stages, assigning stages to the nodes in the concept graph, and preparing alternative descriptions (average time per exercise)—from 2 to 8 h, with an average of value 5.3 h.

The study assessed the impact of the developed method on student progress by comparing the average grades obtained from mastering the developed material in two classes. We compared the results in both classes—one learning with the developed platform and one learning without it—using traditional methods. Verification took place via a test/exam taken by the students of both classes (the student took the exam in a traditional Braille form). The obtained results are shown in Figure 5.

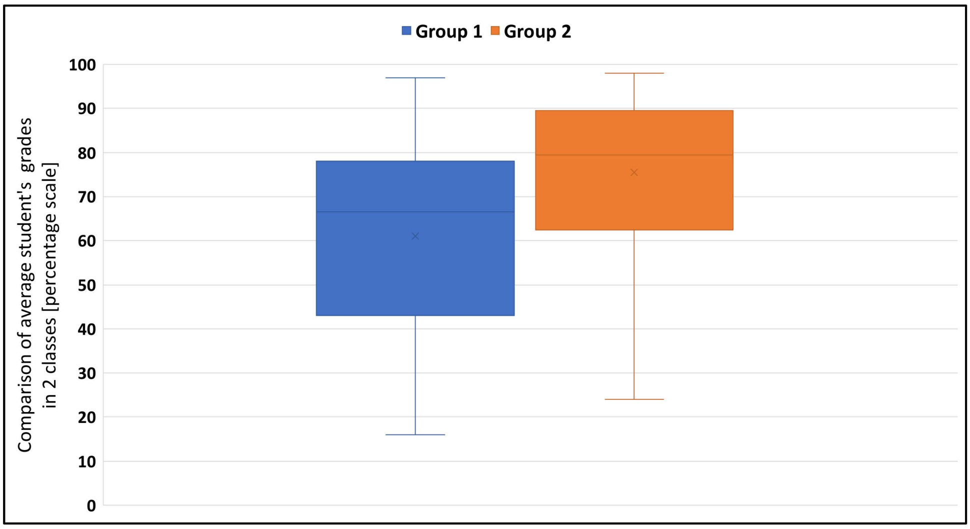

Figure 5. Comparison of average grades in 2 classes in the final exam (with and without the developed method).

To determine whether the obtained results are statistically significant, an appropriate statistical test was conducted at a significance level of $p = 0.05$. After verifying the normality distribution in both groups (using the Shapiro–Wilk test), we used the Student's *t*-test to compare the average values in both groups. A statistically significant difference between the average values amounted to approximately 14%. In the class that used the tool, the average was 75%, while in the class without using the proposed methods, it was 61%.

In the context of evaluating the possibility of consolidating the knowledge of blind students by helping less advanced peers in learning a new topic, we determined the percentage of students who provided support through interactions. The obtained value of 27% is important in the context of overcoming the barrier of low self-esteem and the inability to consolidate knowledge. It creates the possibility of using knowledge in other issues and subjects.

5. Discussion and Conclusions

As the literature shows, a significant difficulty in learning mathematics for blind individuals is low self-esteem, motivation, poor learning progress, and difficulties in consolidating acquired skills. The results of the current study indicate an improvement in learning outcomes and, at the same time, an increased number of interactions between students. The students also supported their peers by explaining the material that they had already mastered. Thus, we can assume that the increased number of interactions allows students to be more actively involved (overcoming low motivation), overcome the barrier of poor results, and better consolidate the material. Additionally, we noticed that an additional advantage of the developed method was supporting students' self-study.

In the traditional teaching method using Braille textbooks, teachers have to work individually with each student, engaging in dialog and encouraging interaction. Additionally, they need to verify the student's answers written in Braille, which requires the teacher's knowledge of Braille notation and can be pretty laborious. In comparison, the developed method also requires the teacher to spend time preparing the course, but this effort is a one-time occurrence, and the materials can be reused multiple times. The teacher also gains the ability to assess the student's learning progress with detailed identification of challenging issues for individual students. Furthermore, by utilizing feedback mechanisms such as the knowledge vector and the error vector, each student has a personalized automatic selection of subsequent exercises. It relieves the teacher of the need to individually assess the knowledge of each student, allowing them to focus on the most challenging aspects of the material.

Referring to the limitations of the proposed method, the evaluation included a limited number of students divided into two groups. We decided to limit the scope of the educational material to a selected course in mathematical analysis at a high school level. The evaluation of the developed method focused on alternative descriptions in the Polish language. It would be worthwhile to compare students' results in classes using different languages.

To sum up, the most valuable features of the developed method are the algorithmic selection of material (represented in the form of a directional graph of concepts), the preparation process (exercise decomposition into stages), and enhancement in the learning process (feedback elements: knowledge vector and error vector used during graph navigation, reflecting the learning process). Our approach, including the above stages, provides new opportunities, such as the following:

- Standardization of the educational material and the phases of its acquisition;
- The possibility of a quantitative assessment of the student's knowledge;
- The possibility of a detailed assessment of challenging elements;
- The ability to easily compare the knowledge of students;
- The ability to choose lesson materials corresponding to the level of knowledge of students in the class;
- The possibility of cooperation between students via individual consultations;
- The development of various ways of navigation and presentation of the exercise content, as well as the formula itself, in visual forms and alternative audio presentations, depending on the stage of the exercise and its context;
- Semi-automatic generation of alternative presentations of mathematical formulas, including additional structural information based on the TEX notation, which is a standard used for writing mathematical expressions; the choice of widely used TEX

notation serves as a bridge between the commonly used presentation and the proposed alternative presentation. The developed method of alternative presentation can be used in different languages. It requires adapting the alternative descriptions to the rules of a specific language, while the TEX syntax modifications remain unchanged. The paper presents the method and demonstrates its application with alternative descriptions in the Polish language.

The article presents a complete method explicitly developed for supporting learning and teaching mathematics for students with blindness, addressing the following issues: preparation of exercises involving the alternative audio presentation of mathematical formulas, communication between a blind student and a teacher during the process of learning and teaching mathematics, and the automation of knowledge assessment. Based on the developed method, an educational platform has been created that can be utilized by people with low vision. It allows them to work in a visual mode with a high magnification factor and the ability to adjust the contrast individually.

Further research directions will focus on evaluating the developed method and platform and the accompanying educational materials in the context of inclusive education. This will help us to verify the feasibility of simultaneous platform use by both sighted and blind students. We also plan to research the effectiveness of learning with the developed platform when using a larger group of students and materials from other areas of mathematics and other STEM subjects such as physics, geography, chemistry, etc.

Author Contributions: Conceptualization, M.M. and D.S.; methodology, M.M. and M.K.; software, M.M. and M.K.; validation, P.B. and D.S.; formal analysis, M.M.; investigation, M.K.; resources, M.M. and M.K.; data curation, M.M.; writing—original draft preparation, M.M. and D.S.; writing—review and editing, P.B., M.K. and M.M.; visualization, M.M.; supervision, M.M.; project administration, M.M. and P.B. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. The list of concepts from the developed material.

Node Number	Concept (Math Term)	Successor Node	Mistake Node [1]	Mistake Type and Description
C1	Number intervals	C3, C4, C9	E1	Number set type definition error; range boundary definition error; range boundary type error
C2	Function definition	C3, C4, C5	E2	Function definition understanding error; function definition error
C3	Definition of the domain of a function	C4, C7, C13, C22, C30	E3	Determination of value constraints in the domain of the function error; determination of the boundary of the range of the domain of the function error

Table A1. *Cont.*

Node Number	Concept (Math Term)	Successor Node	Mistake Node [1]	Mistake Type and Description
C4	Definition of the codomain of a function	C7, C13, C22, C30	E4	Determination of value constraints in the counter domain of the function error; determination of the boundary of the range of the counter domain of the function error
C5	Definition of the derivative of a function	C6, C9	E5	Function derivative definition error; function derivative definition understanding error
C6	Definition of the second derivative of the function	C10	E6	Second function derivative definition error; second function derivative understanding error
C7	Definition of the function graph	C8, C13, C22, C30	E7	Determination of the value of the function for the specified value of the argument error; inability to determine the shape of the function graph error
C8	Definition of a root of a function	C15, C23, C24, C31	E8	Inability to understand the definition of zero error; inability to calculate zero error
C9	Definition of monotonicity interval	C16, C25, C32	E9	Inability to understand the idea of monotonicity error; inability to calculate the interval error; calculation of monotonicity interval error
C10	Definition of the inflection point of a function	C11, C17, C26, C33	E10	Inability to understand the idea of an inflection point error; inability to calculate inflection point error; inflection point calculation error
C11	Definition of the extreme of a function	C17, C26	E11	Inability to understand the function extreme error; distinguishing the type of extreme error; inability to calculate the extreme of a function error; function extreme calculation error
C12	Quadratic function	C13	E12	Inability to understand the formula of a quadratic function error; formula of a quadratic function writing error
C13	Quadratic function graph	C15, C16, C17	E13	Inability to determine the shape of the graph of a quadratic function error; inability to determine the properties of the graph of a quadratic function error (vertex, direction of parabola arms)
C14	Quadratic equation	C15, C18, C19	E14	Inability to understand the ways used to solve quadratic equation error; quadratic equation calculation error
C15	Roots of a quadratic function	C18, C19, C20	E15	Inability to understand the formulas for the roots of a quadratic function error; quadratic equation root calculation error
C16	Monotonicity intervals of a quadratic function	C17	E16	Inability to understand how to calculate monotonicity intervals of a quadratic function error; monotonicity intervals of a quadratic function calculation error
C17	The extreme of a quadratic function	-	E17	Extreme of a quadratic function calculation error

Table A1. *Cont.*

Node Number	Concept (Math Term)	Successor Node	Mistake Node [1]	Mistake Type and Description
C18	Square root equations	C27	E18	Inability to understand how to solve an equation with a square root error; root equation calculation error
C19	Equations to be solved by square substitution	C27	E19	Inability to understand how to solve an equation with substitution error; solving an equation with substitution calculation error
C20	Quadratic inequalities	C28	E20	Inability to understand how to solve a quadratic inequality error; square inequality calculation error
C21	Polynomial	C22, C23	E21	Inability to understand the formula of polynomial function error; polynomial function formula writing error
C22	Polynomial graph	C24, C25, C26	E22	Inability to determine the shape of the graph of a polynomial function error; inability to determine the properties of a polynomial graph error
C23	Division of polynomials	C27, C28	E23	Inability to understand the method of dividing polynomial error; polynomial dividing error
C24	Roots of a polynomial	C27, C28	E24	Inability to understand the formulas for polynomial root error; polynomial root calculation error
C25	Monotonicity intervals of a polynomial	C26	E25	Inability to understand how to calculate polynomial monotonicity interval error; polynomial monotonicity interval calculation error
C26	The extreme of a polynomial	-	E26	Polynomial extreme calculation error
C27	Polynomial equation	-	E27	Inability to understand how to solve a polynomial equation error; polynomial equation calculation error
C28	Polynomial inequalities	-	E28	Inability to understand how to solve a polynomial inequality error; polynomial inequality calculation error
C29	Trigonometric function—types	C30	E29	Inability to understand the types of trigonometric function error; inability to determine the period of trigonometric function error
C30	Graphs of trigonometric functions	C31	E30	Inability to distinguish the type of trigonometric function based on the graph error; period of trigonometric function determination error
C31	The zero of the trigonometric function	C32, C33, C34	E31	Inability to understand inverse trigonometric function error; zero calculation error
C32	Monotonicity intervals of trigonometric functions	-	E32	Inability to understand the properties of graphs of trigonometric function error; trigonometric function monotonicity interval determination error

Table A1. *Cont.*

Node Number	Concept (Math Term)	Successor Node	Mistake Node [1]	Mistake Type and Description
C33	Trigonometric equations	-	E33	Inability to understand how to solve trigonometric equation error; trigonometric equation calculation error
C34	Trigonometric inequalities	-	E34	Inability to understand how to solve trigonometric inequality error; trigonometric inequality calculation error

[1] Exemplary values of the error vector corresponding to the content of the exercise from Section 3.4 are shown in Figure 3.

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