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A Theoretical Model to Identify the Fundamental Frequency of Simply Supported Girders from a Passing Heavy Vehicle

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Abstract: This paper discusses the influence of vehicle parameters on the identification accuracy of bridge fundamental frequency from a passing vehicle and validates such influence through laboratory tests. In the process of theoretical derivation, the interaction between a vehicle and bridge is no longer ignored to uncouple the equation group and the stepwise integration method is used to solve the equation group. Key parameters such as the vehicle speed α_v , vehicle-to-bridge mass ratio α_{mv} , and frequency ratio μ_1 , are dimensionless. Based on the simplified vehicle-to-bridge system model, the numerical analysis method is used to analyze their influence on the identified bridge frequency. Through parameter analysis, it is found that the vehicle-to-bridge mass ratio and vehicle speed have a great impact on the patterns of identified bridge frequency; and when the vehicle frequency is greater than or less than the bridge frequency, the identified bridge frequency exhibits different patterns. The bridge frequency deviation rate can be greater than 20% when the vehicle speed and vehicle-to-bridge mass ratio increases. In order to ensure the accuracy of bridge frequency identification, it is recommended that the vehicle-bridge mass ratio should not exceed 10%. Besides, appropriately increasing the vehicle-to-bridge mass ratio will increase the probability of accurately identifying the bridge frequency under certain conditions. In addition, a novel frequency extraction method was proposed to more accurately filter out the frequency from the vehicle acceleration spectrum.

Keywords: bridge; fundamental frequency; vehicle mass; frequency deviation; vehicle-bridge interaction; laboratory test

Citation: Wu, Q.; Han, B.; Yu, J.; Yan, W.; Zhang, J.; Xie, H. A Theoretical Model to Identify the Fundamental Frequency of Simply Supported Girders from a Passing Heavy Vehicle. *Appl. Sci.* **2022**, *12*, 2422. <https://doi.org/10.3390/app12052422>

Academic Editor: César M. A. Vasques

Received: 10 January 2022

Accepted: 19 February 2022

Published: 25 February 2022

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1. Introduction

Structural performance degradation is an extensively concerned issue for in-service bridges, and the inspection and evaluation is an essential and necessary work to keep bridges in good operating condition. However, a traditional bridge inspection method usually requires considerable preparatory work, such as field installation of sensors and undesirably traffic closure, which require massive time and economic costs. The disadvantages of a traditional bridge inspection method limit its wide application to highways and urban bridges. To meet the increasing maintenance requirements of these bridges, rapid detection methods and techniques are urgently needed and have received more attention in recent years [1–4].

The indirect bridge detection method was first proposed by Yang [5] as early as 2004 and has gained great popularity because of its low cost and low maintenance effort. Unlike the traditional bridge inspection method [6–8], this approach employed the dynamic response of a passing vehicle instead of a bridge itself. A vehicle acts as an exciter and a signal collection device simultaneously in this method. In the initial stage, researchers use

the acceleration signal of a passing vehicle to extract the bridge natural frequency. The accuracy of the result was verified through a theoretical derivation [9,10], laboratory investigation [11–13], and field tests [14–16]. With further development of this method, more bridge modal parameters were identified from vehicles' acceleration signal, such as modal shape and damping parameter [17–19]. In order to reduce vehicle response interference in the signal, the response of the contact point of the vehicle with the bridge, rather than the vehicle itself, is proposed [20]. On this basis, the effect of identifying bridge parameters using the acceleration signals of passing vehicles is further improved, especially the extraction of bridge frequency [21–24]. The vehicle model is also developed from a single-degree-of-freedom (DOF) model to a single-axle vehicle model. The single-axle vehicle is modeled as a two DOF system, including both the rocking and vertical motions [25]. In bridge modal analysis, Malekjafarian and Obrien present a novel algorithm for the estimation of bridge mode shapes using the response measured on a passing vehicle [17]. A rescaling process is proposed to construct the bridge mode shapes using the amplitude of the responses measured on two following axles. And the concept of subtraction is employed to remove the effect of the road profile in the measured responses. They also present a novel algorithm for bridge damage detection based on the mode shapes estimated from a passing vehicle [26]. Different from previous studies, a measurement system based on laser vibrometers and accelerometers is employed to measure the bridge response at moving coordinates on a passing vehicle. The bridge mode shape is estimated with a high resolution by using an improved short time frequency domain decomposition method. Moreover, a number of signal processing methods were introduced to achieve better parameter identification by reducing false-positive feedbacks extracted from the roughness of road surfaces [27–29]. A novel indirect method of extracting bridge fundamental frequency was proposed by Wang [30], which aimed to eliminate the roughness influence in vehicle signals. A particle filter in combination with a random walk model is used to estimate the displacement inputs at the front and rear tires. Then the bridge's fundamental frequency can be clearly shown through the subtraction between the displacement inputs at the front and rear tires. Bridge damage identification methods can be divided into frequency-based and mode shape-based methods, depending on whether or not they rely on modal parameters [31]. They are both capable of damage detection [14,32,33], while only the mode shape-based methods are able to realize damage localization [19,34,35]. In addition, signal processing and machine learning techniques, which do not rely on modal parameters, perform well in identifying structural damage location and degree [36–38].

A bridge dynamic response can be amplified by regulating the vehicle frequency, increasing vehicle weight or speed. Increasing vehicle weight becomes a feasible solution, when considering the negative effects of increasing vehicle speed and the inconvenience of altering vehicle stiffness. When analyzing the vehicle-bridge interaction (VBI) system, we usually assume that the effect of vehicle vibration on the bridge can be ignored because the vehicle weight is so small compared to that of the bridge. In Shi's paper [39], a new assumption is made to uncouple the equation group, which is that the vehicle acceleration magnitude is much lower than the gravitational acceleration constant. Although the vehicle mass is as high as 100% of the total bridge mass in Shi's research, the vehicle was maintained at a low speed in order to meet Shi's assumption. These two assumptions both ignore the interaction between the vehicle and bridge. However, this assumption tends to be invalid when the vehicle becomes heavier and the bridge becomes lighter or the vehicle speed becomes faster. In such cases, the frequency variation of the bridge coupled with the moving vehicle should be taken into account to avoid the identification deviations.

Natural frequencies are mainly sensitive to global stiffness modifications and not so much to local ones [2]. However, the global stiffness is often influenced by variations in environmental factors or heavy vehicles crossing the bridge. That influence can be high enough to completely mask the presence of even severe damage [40]. In Dimitrios's paper [41], a steel single-span tied-arch bridge was monitored for a year. An increase of 10 °C in temperature resulted in a relative decrease of about 0.2~0.25% for the natural

frequencies of the arch modes. In the field experiment in Chang's paper [42], a decrease of 2.6% in bridge frequency was observed when a truck with mass ratio of 0.082 to the experiment bridge was parked at the midspan of the bridge. In the laboratory experiments, a decrease of 10.4% was observed when a model vehicle with mass ratio of 0.154 to the scaled bridge was parked at the midspan of the bridge. The bridge frequency also changes under daily traffic conditions. From the 24-h ambient vibration monitoring on a cable-stayed bridge, it was found that the natural frequencies of the global modes can exhibit as much as 1% variation [43]. Piotr et al. [44] showed that the natural frequencies of the bridge modes can exhibit variation between 1.7% and 5.5%. When heavy vehicles pass, the frequency of simply supported Nongro bridge was approximately 5.7% larger than that when light vehicles pass [45]. Kim et al. [46] found that frequency shifts up to 5.4% for a short span and relatively high (vehicle to bridge) mass ratio. In addition, previous studies mainly discussed the variation rule of bridge frequency with different vehicle stiffness through theoretical analysis and laboratory test [47,48]. The analytical solution of a bridge and vehicle frequency when the vehicle is in a different position of the bridge was given, however fail to account for the applicability of bridge frequency extraction in indirect bridge detection methods.

In this paper, the equation group for the general VBI system cannot be uncoupled under the strong vehicle-bridge interaction. A strong vehicle-bridge interaction is achieved through adjusting vehicle parameters, such as a large vehicle-to-bridge mass ratio and high vehicle speed. The influence of vehicle parameters on frequency extraction from a passing vehicle is discussed in detail and the accuracy of frequency identification is demonstrated. The main parameters are made dimensionless, including vehicle speed, frequency ratio, and vehicle-to-bridge mass ratio. Euler–Bernoulli beams and a spring-mass system are used to simulate bridge and vehicle, respectively. It must be mentioned that the road surface roughness and damping coefficient are not considered in this study. In addition, a frequency extraction method was proposed to more accurately filter out the frequency from the vehicle acceleration spectrum. The proposed method is verified and discussed by the experimental tests.

2. Vehicle-Bridge Interaction System

The mathematical model for the VBI system is shown in Figure 1. The vehicle is modeled as a one degree-of-freedom oscillator moving over a simply-supported beam. The beam is assumed as a Euler–Bernoulli beam with constant cross-sections. Damping of the entire VBI system, and road surface roughness are also neglected. In the mathematical model, the vehicle moves along the x -axis of the beam to point x at time t . In Figure 1, v , k_v , and m_v is the speed, stiffness, and mass of the vehicle, respectively; m_b , EI , and L is the unit mass, section stiffness, and span of the bridge, respectively; y_v , y_b , and y_w is the vertical displacement of vehicle, bridge, and wheel, respectively; and g is the for acceleration due to gravity. The equations of motion at time t for the vehicle and bridge can be written each as follows,

$$m_v \ddot{y}_v + k_v (y_v - y_{b,x=vt}) = 0 \quad (1)$$

$$m_b \frac{\partial^2 y_b(x, t)}{\partial t^2} + EI \frac{\partial^4 y_b(x, t)}{\partial x^4} = f_N \delta(x - vt) \quad (2)$$

where $f_N = m_v(g - \ddot{y}_v) + m_w(g - \ddot{y}_w)$.

Figure 1. The mathematical model for the vehicle-bridge interaction (VBI) system.

The assumptions made in this study are: 1. The bridge follows Hooke's law, Navier's hypothesis, Saint-Venant's principle, and small-deflection theory; 2. the wheels of the vehicle stay on the deck constantly while it passes through the bridge; 3. the bridge is initially at rest before the vehicle starts; and 4. the vehicle speed and its path are constant numbers and the suspension spring is considered to be linearly elastic.

By making the above equation dimensionless, the dimensionless frequencies of vehicles and bridges can be expressed as:

$$\Omega_v = \sqrt{\frac{m_b}{EI}} L^2 \omega_v \quad (3)$$

$$\Omega_{bn} = \sqrt{\frac{m_b}{EI}} L^2 \omega_{bn} = \sqrt{\frac{m_b}{EI}} L^2 \left(\frac{n\pi}{L} \right)^2 \sqrt{\frac{EI}{m_b}} = (n\pi)^2. \quad (4)$$

Using modal superposition, $\alpha_{yb}(s, \tau) = \sum_{n=1}^{\infty} H_n(\tau) \sin(n\pi s)$, where $\sin(n\pi s)$ and $H_n(\tau)$ is the n -th mode shape and mode displacement, respectively. Substituting the aforementioned expressions into Equations (1) and (2), multiplying both sides of Equation (2) by $\sin(n\pi s)$ and integrating s from 0 to 1, we obtain:

$$\begin{aligned} \ddot{H}_m(t) + 2\alpha_{mw} \sum_{n=1}^{\infty} \ddot{H}_n(\tau) \sin(n\pi s) \sin(m\pi s) + 4\alpha_{mw}\alpha_v \sum_{n=1}^{\infty} (n\pi) \dot{H}_n(\tau) \cos(n\pi s) \sin(m\pi s) \\ + \Omega_{bm}^2 H_m(\tau) - 2\alpha_{mw}\alpha_v^2 \sum_{n=1}^{\infty} (n\pi)^2 H_n(\tau) \sin(n\pi s) \sin(m\pi s) \\ + 2\alpha_{mv}\Omega_v^2 \sum_{n=1}^{\infty} H_n(\tau) \sin(n\pi s) \sin(m\pi s) - 2\alpha_{mv}\Omega_v^2 \alpha_{yv} \sin(m\pi s) = 2(\alpha_{mv} + \alpha_{mw})g^* \sin(m\pi s) \end{aligned} \quad (5)$$

$$\frac{\partial^2 \alpha_{yv}}{\partial \tau^2} + \Omega_v^2 \alpha_{yv} - \Omega_v^2 \sum_{n=1}^{\infty} H_n(\tau) \sin(n\pi s) = 0 \quad (6)$$

where, α_{yb} , α_{yv} , α_{mv} , α_{mw} , α_v , s , τ , and g^* are the dimensionless parameters of y_b , y_v , m_v , m_w , v , x , t , and g , respectively. Their expressions are detailed in Appendix A. The above equations can be solved by using the stepwise integration method.

It is generally believed that the mass of the vehicle is far less than the mass of the bridge, which means the impact of the vehicle vibration on bridge vibration is ignored. Thus, we have $m_v \ll m_b L$, $m_w \ll m_b L$, namely $\alpha_{mv} \rightarrow 0$, $\alpha_{mw} \rightarrow 0$. Equation (5) can be simplified as:

$$\ddot{H}_m(\tau) + \Omega_{bm}^2 H_m(\tau) = 2(\alpha_{mv} + \alpha_{mw})g^* \sin(m\pi s). \quad (7)$$

Under zero initial conditions, we obtain:

$$H_m(\tau) = \frac{2(\alpha_{mv} + \alpha_{mw})g^*}{\Omega_{bm}^2 (1 - S_m^2)} [\sin(m\pi s) - S_m \sin(\Omega_{bm} \tau)] \quad (8)$$

where $S_m = \frac{n\pi\alpha_v}{\Omega_{bn}}$.

Substituting Equation (8) into Equation (6), vehicle response α_{yv} can be solved from Equation (6) by Duhamel's integral. Then, α_{yv} can be differentiated twice to yield the acceleration of the vehicle as:

$$\ddot{\alpha}_{yv} = A_1 \cos \Omega_v \tau + A_2 \cos 2n\pi s + A_3 \cos(\Omega_{bn} - n\pi\alpha_v)\tau + A_4 \cos(\Omega_{bn} + n\pi\alpha_v)\tau \quad (9)$$

where, $\mu_n = \Omega_v/\Omega_{bn}$ is the ratio of vehicle frequency to bridge frequency, in which n represents the n -th frequency of the bridge. The expressions of A_1 , A_2 , A_3 , and A_4 are detailed in Appendix B.

It can be seen from Equation (9), the acceleration of the vehicle $\ddot{\alpha}_{yv}$ is mainly composed of four parts, including vehicle frequency term Ω_v , bridge frequency term $\Omega_{bn} \pm \pi\alpha_v$, and speed term $2n\pi\alpha_v$. Apparently, the bridge frequency derived from Equation (9) is $\Omega_{bn} \pm \pi\alpha_v$, which is only related to the speed term. The frequency deviation rate can be defined as:

$$\Delta_b = \frac{\Omega_{bex} - \Omega_{bn}}{\Omega_{bn}} \quad (10)$$

where Ω_{bex} represents the bridge frequency identified through vehicle acceleration. When the identified bridge frequency is less than the corresponding bridge frequency, it is called the left-biased frequency, corresponding to $\Delta_b < 0$. On the contrary, it is called the right-biased frequency, corresponding to $\Delta_b > 0$. Therefore, disregarding vehicle mass, the bridge frequency deviation rate is:

$$|\Delta_b| = \left| \frac{\Omega_{bex} \pm \Omega_{b1}}{\Omega_{b1}} \right| = \frac{\pi\alpha_v}{\Omega_{b1}}. \quad (11)$$

As mentioned in previous research results [5], the value of Δ_b is only related to vehicle speed. In summary, the vehicle speed is positively related to the frequency deviation rate Δ_b .

3. Peak Frequency Identification Method

Judgment principles need to be defined primarily on how to identify the fundamental frequency of the bridge from the acceleration spectrum of the vehicle. The central idea is to select the peaks worth paying attention to according to specific indicators from the many peaks in the spectrogram within a specific frequency range. Then, known peaks are removed from the selected peaks and result in the isolation of the target peak. Combined with the issue at hand, the fundamental problem is to select the vehicle frequency item, bridge frequency item, and speed item from the spectrogram. When the vehicle frequency item and speed item are eliminated, the bridge frequency can be obtained.

Firstly, the following parameters are introduced: Peak amplitude F_{pv} , peak aspect ratio F_{hw} , peak area F_{ae} , and peak significance F_{sr} . Their expressions are explained in the following section and in Figure 2.

$$F_{pv}(i) = Y_i^{max} \quad (12)$$

$$F_{ae}(i) = \frac{1}{2} [Y_{i-1}^{min} + Y_{i+1}^{min}] Y_i^{max} \quad (13)$$

$$F_{hw}(i) = \frac{Y_i^{max} - \frac{Y_{i-1}^{min} + Y_{i+1}^{min}}{2}}{X_{i+1}^{min} - X_{i-1}^{min}} \quad (14)$$

$$F_{sr}(i) = \min \left(\frac{Y_i^{max}}{Y_{i-1}^{min}}, \frac{Y_i^{max}}{Y_{i+1}^{min}} \right) \quad (15)$$

where, X_i^{max} or X_i^{min} represents the frequency value corresponding to the i -th extreme point and Y_i^{max} or Y_i^{min} represents the amplitude of the i -th extreme point. The superscript max

and min indicate the maximum value and minimum value, respectively. The physical meaning of F_{hw} is a quantity describing the peak shape. The larger the value of F_{hw} , the more concentrated the frequency corresponding to the peak. F_{ae} and F_{sr} describe the ease in which the peak can be identified. The larger their value, the easier the peak is to be observed. F_{pv} indicates the degree of concern for the peak. Generally, it can be considered that the corresponding frequency with a large value of F_{pv} is the main component of the signal.

Figure 2. Schematic diagram of peak parameters.

Suppose there are n maximum points in the frequency range of interest, so the basic score of each maximum point can be defined as:

$$V_{FB}(i) = F_{hw}(i) + F_{st}(i) + F_{ae}(i) / \max(F_{ae}(k)) \quad (16)$$

where $k = 1, 2, \dots, n$.

In order to improve identification accuracy of the peaks, it is necessary to make the following supplements:

1. To ensure the significance of the maximum point between two adjacent minimum points, it is recommended that the amplitude of the maximum point should be at least 10% greater than that of the adjacent minimum point, which means $F_{sr}(i) > 1.1$.
2. In order to ensure the visibility of peaks, the area corresponding to each peak should not be too low. The ratio of the area corresponding to the peak in the same spectrogram to the maximum area should not be too low. To be conservative, take $F_{ae}(i) / \max(F_{ae}(k)) > 0.01$ in this paper. Thus, the peak discrimination function is defined as:

$$W_{pi}(i) = \begin{cases} 1 & F_{sr}(i) > 1.1 \quad \text{or} \quad F_{ae}(i) / \max(F_{ae}(k)) > 0.01 \\ 0 & \text{else} \end{cases} \quad (17)$$

A special case, as shown in Figure 3, has multiple successively decreasing small peaks (boxed in red) on the side of a major peak. This situation becomes more prominent as the vehicle crossing the bridge slows down, the number of sampling points increases, and the vehicle-to-bridge mass ratio increases. Preliminary analysis indicates that this phenomenon is caused by real-time changes in the frequency of vehicles and bridges in the VBI system as vehicles pass through the bridge. Applying Fast Fourier transformation alone to the signal cannot satisfactorily reflect the real-time frequency change in the spectrogram.

Figure 3. Vertical acceleration spectrum of vehicle with different (a) α_{mv} and (b) α_v .

For illustrative purposes, two cases are demonstrated briefly. The value of each parameter is shown in Table 1. The Hilbert transform was used to analyze the vehicle and bridge terms in the vehicle acceleration signal, as well as to obtain their time-frequency curve during the passage of the vehicle on the bridge, shown in Figure 4.

Table 1. Value of parameters.

Case 1				Case 2		
μ_1		1.5			1.5	
α_{mv}	0.01	0.15	0.35		0.15	
α_v		0.3		0.15	0.3	0.45

Figure 4. Frequency variation of vehicle and bridge when vehicle passing the bridge. (a) With different vehicle mass; (b) with different speed.

In order to minimize the interference of these small peaks and highlight the frequency of the vehicle and bridge, gain factors are defined for each major peak. The approach of assigning gain coefficient is as follows: (1) The top three with the largest value of F_{pv} are given 5, 2, and 1 in turn:

$$W_{G1}(i) = \begin{cases} 5 & \text{Rank}(F_{pv}(i)) = 1 \\ 2 & \text{Rank}(F_{pv}(i)) = 2 \\ 1 & \text{else} \end{cases} \quad (18)$$

For the maximum point in the maximum point, assign 5:

$$W_{G2}(i) = \begin{cases} 5 & F_{pv}(i) \text{ is the maximum point among the maximum points} \\ 1 & \text{else} \end{cases} \quad (19)$$

In summary, the score of the maximum point is:

$$V_F(i) = V_{FB}(i) \times W_{pi}(i) \times \max(W_{G1}(i), W_{G2}(i)). \quad (20)$$

Peaks in the spectrogram were sorted according to the above scoring principle. The acceleration of the vehicle contains the vehicle frequency term, bridge frequency term, and speed term, which correspond to the vehicle frequency, the bridge frequency, and the speed frequency in the vehicle vertical acceleration spectrum, respectively. In this paper, most of the top three scoring peaks correspond to the bridge frequency, vehicle frequency, and speed frequency. The intuitive performance on the spectrogram is three significant peaks. After excluding the vehicle frequency Ω_{vex} and speed frequency Ω_{sex} from these three frequencies, the resulting bridge frequency Ω_{bex} can be obtained successfully. However, changes in the vehicle-to-bridge mass ratio α_{mv} , vehicle-to-bridge frequency ratio μ_1 , and vehicle speed α_v , pose difficulties in bridge frequency recognition.

1. When α_{mv} is small and α_v is large, the vehicle-bridge interaction is weak. At the same time, the number of collection points is small. Although Ω_{vex} can be identified, the bridge frequency term cannot be fully reflected in the vehicle acceleration signal, which leads to the failure of extracting Ω_{bex} .
2. When the vehicle speed gradually increases to meet $\alpha_v > 2\pi\Omega_v$, which is $L/v < 1/2\pi\omega_v$, the time for the vehicle to cross the bridge is less than its vibration period. The vehicle has left the bridge before finishing a complete vibration cycle. This will result in that the vehicle acceleration signal does not include the vehicle frequency term, and Ω_{vex} cannot be obtained.
3. When μ_1 is small, the vehicle frequency is too low to distinguish between vehicle frequency and speed terms in the spectrogram.
4. The vehicle frequency coincides with the high-order frequency of the bridge. While the overlap between the vehicle frequency and bridge fundamental frequency is avoided in this paper, the vehicle frequency term coincides with the n -th order bridge frequency term in the spectrogram when μ_1 is close to n^2 ($n > 2$). As a result, it is impossible to solve for Ω_{vex} , however it generally does not affect the identification of Ω_{bex} .

In summary, Ω_{bex} is calculated corresponding to the highest scoring peak excluding Ω_{vex} and Ω_{sex} . Alternatively, the frequency corresponding to the highest peak can be temporarily interpreted as the bridge frequency Ω_{bex} until further verification is conducted provided with the values of each parameter.

4. The Vehicle-to-Bridge Mass Ratio on Frequency Deviation Rate

4.1. Numerical Examples

The frequency of the vehicle is generally between 0~15 Hz in reality [2,49]. Based on the numerical examples in Yang's study [5], the range of each dimensionless parameter is as follows: $\mu_1 = 0.2 \sim 0.9$, $1.2 \sim 15$, $\alpha_{mv} = 0.01 \sim 0.5$, $\alpha_v = 0.03 \sim 0.75$, and $\alpha_w = 0.001$.

Substituting the above parameters into Equations (5) and (6), the acceleration time history curve of vehicles passing the bridge can be obtained. Then Fast Fourier transform can be used to obtain the vehicle acceleration spectrum. Using the method described Section 3, the dimensionless bridge fundamental frequency can be identified, and then the corresponding frequency deviation rate can be obtained using Equation (10). The main results of the numerical calculation are shown in Figures 5 and 6, and the shaded part in the figure indicates that the recognition failed. The colors in Figures 5 and 6 gradually change from blue to red, representing the gradual change of the bridge fundamental frequency deviation rate from the minimum value to the maximum value. Blue and red represent the minimum and maximum values in their respective graphs. The different colors indicate the relative magnitude of the bridge fundamental frequency deviation rate in each graph. Therefore, the absolute values of the bridge fundamental frequency deviation rate represented by the colors in different pictures are not the same.

Figure 5. Bridge fundamental frequency deviation rate ($\mu_1 < 1$). (a) $\alpha_v = 0.03$, (b) $\alpha_v = 0.15$, (c) $\alpha_v = 0.30$, (d) $\alpha_v = 0.45$, (e) $\alpha_v = 0.60$, (f) $\alpha_v = 0.75$.

Figure 6. Bridge fundamental frequency deviation rate ($\mu_1 > 1$). (a) $\alpha_v = 0.03$, (b) $\alpha_v = 0.15$, (c) $\alpha_v = 0.30$, (d) $\alpha_v = 0.45$, (e) $\alpha_v = 0.60$, (f) $\alpha_v = 0.75$.

4.2. Results Description

Without considering the mass of the vehicle, the bridge fundamental frequency deviation rate $|\Delta_b|$ is 0.72%, 3.6%, 7.2%, 10.8%, 14.4%, and 18.0%, their corresponding value of α_v is 0.03, 0.15, 0.30, 0.45, 0.60, and 0.75, respectively. However, as shown in Figures 5 and 6 even when α_{mv} is 0.01, the Ω_{bex} will change with the change of μ_1 , and both the left-biased and right-biased frequencies will appear. Therefore, it should be noted that introducing vehicle mass also brings the influence of the vehicle-to-bridge frequency ratio.

As shown in Figures 5 and 6, at different speeds (take α_v as 0.03, 0.15, 0.30, 0.45, 0.60, and 0.75, respectively) Δ_b varies with μ_1 and α_{mv} in inconsistent patterns.

In the case of $\mu_1 > 1$, the recognition accuracy becomes increasingly worse with the increase of α_v . When α_v is small $\Delta_b < 0$, and Δ_b increases with the increase of μ_1 and gradually stabilizes. As α_v increases, Δ_b will become greater than zero when α_{mv} and μ_1 are small, and the range of this situation becomes larger. Δ_b gradually decreases as μ_1 increases

under the condition of $\Delta_b < 0$. Generally speaking, when $\mu_1 > 1$, Ω_{bex} is mainly left-biased frequency and $|\Delta_b|$ increases as α_v and μ_1 decrease. Specifically, Δ_b decreases as α_v and α_{mv} increase when μ_1 is constant, and Δ_b decreases as α_{mv} increase when α_v is constant.

Compared to the case where μ_1 is greater than 1, the changing pattern of Δ_b is relatively complicated when μ_1 is less than 1. Similar to the previous case, as α_v increases, the recognition accuracy becomes increasingly worse. As shown in Figure 4, the shaded part indicates that the fundamental frequency of the bridge cannot be identified. When α_v is small, $\Delta_b > 0$. There will be a situation where Δ_b is less than zero when α_{mv} or μ_1 is small as α_v increases. Especially, within specific range of μ_1 , the area where $\Delta_b < 0$ will extend to where the value of α_{mv} is comparatively large (as shown in the red frame of Figure 5c,d). Generally speaking, Ω_{bex} is mainly a right-biased frequency when $\mu_1 < 1$. Specifically, as α_{mv} and α_v increases, Δ_b increases and decreases, respectively when μ_1 is constant, however when α_v continues to increase, the above pattern shows the opposite trend. In most cases, Δ_b increases as μ_1 increases while α_{mv} remains constant, however the pattern is different in the area where $\Delta_b < 0$; Δ_b increases as μ_1 and α_{mv} increases while α_v remains constant, however decreases when α_v becomes larger.

4.3. Probability of Accurately Identifying Bridge Frequencies

Here, we define that when $|\Delta_b| < 0.05$, the bridge fundamental frequency is considered to be accurately identified. Since vehicle speed and mass mentioned in the above analysis process are accurately known, they are considered as controllable parameters. Within the specified range of parameters in the previous section, α_{mv} and α_v are selected as variables. The probability of accurately identifying the bridge fundamental frequency was obtained, as shown in Figure 7. In Figure 7a, when $\alpha_{mv} < 0.05$ and $\alpha_v < 0.3$, the probability of accurately identifying the bridge fundamental frequency is greater than 80%. However, the probability of the bridge fundamental frequency being accurately identified is less than 50% when $\alpha_{mv} > 0.25$. If the standard for accurately identifying fundamental bridge frequencies is $|\Delta_b| < 0.1$ and the results are shown in Figure 7b. It can be seen from Figure 7b that the resulting probability has increased significantly. The probability of the bridge fundamental frequency being accurately identified is greater than 80% when $\alpha_{mv} < 0.1$ and $\alpha_v < 0.6$. From the conclusion drawn from Figure 7, when $0.3 < \alpha_v < 0.6$ and $\alpha_{mv} < 0.1$, selecting the value of α_{mv} carefully can improve the probability of accurately identifying the bridge fundamental frequency to a certain extent. To ensure the accuracy of identifying the bridge fundamental frequency, the vehicle-to-bridge mass ratio should not be selected as too high and the vehicle speed should not be too fast. If either of them is too large, it will cause a large deviation in the identified frequency.

Figure 7. Probability of identifying bridge fundamental frequencies. (a) $|\Delta_b| < 0.05$, (b) $|\Delta_b| < 0.1$.

In the parameter analysis, in order to avoid resonance between the vehicle and bridge, μ_1 is not selected as a value between 0.9 and 1.2. However, for short and rigid bridges, when vehicles move at a relatively low speed on the bridge, vehicle frequencies may overlap

with the bridge frequencies. Faced with this problem, the most direct way is to change the frequency of vehicles or bridges so that the two are no longer close. Obviously, for bridges that must be used continuously, as only the frequency of the vehicles can be adjusted. Adjusting the frequency of the vehicle can be accomplished by increasing or decreasing the vehicle's mass, or by changing the vehicle's suspension stiffness. In comparison, the method of changing vehicle mass is relatively simple. In order to make the frequency change obvious, it is recommended that the adjusted vehicle mass should not be less than 2 times the original mass or more than half of the original mass. After adjusting the vehicle frequency, try to identify the bridge frequency again.

It should be noted that due to the limitation of the simplified VBI model, the results of the numerical simulation in this paper should not be directly applied to the actual bridge. In addition, for the actual bridge, the value of α_v may not reach the upper limit of the parameter analysis. Different bridges have different α_v value ranges. Therefore, for vehicle speed, we can only give advice not to be too fast. For large-span bridges, due to the large self-weight, the problem of the obvious shift in identified frequency is generally not considered. Instead, the focus should be on how to obtain a stronger bridge response. For small-span bridges or bridges with small self-weight, attention should be paid to the frequency deviation when the vehicle mass is large or there are heavy vehicles, such as buses or trucks, passing through the bridge. Therefore, it is recommended that the vehicle-to-bridge mass ratio α_{mv} be less than 0.1, or even 0.05, to ensure the success of accurately identifying bridge frequencies.

4.4. Parameters Sensitivity Analysis

The Morris method is used to qualitatively analyze the effect of the vehicle-bridge mass ratio α_{mv} , vehicle-bridge frequency ratio μ_1 , and vehicle speed α_v on the bridge frequency deviation rate Δ_b . The Morris method was first proposed by Max D. Morris in 1991 [50]. It is a discrete search method based on parameter space, which studies model parameters on a global scale. Its main idea is to assume elementary effects (EE) that measure the sensitivity of the parameter x_i obeys probability distribution f_i , and the global sensitivity of the parameter x_i can be determined by measuring the mean and standard deviation of the distribution. If the mean value of the parameter x_i is larger, it has a greater impact on the model output. The standard deviation σ is used to measure the interaction between parameters. The larger the value, the greater the interaction and vice versa.

For a model with k parameters, assume that the range of parameters is $[0, 1]$. For a given parameter vector:

$$X = (x_1, x_2, \dots, x_k). \quad (21)$$

where each x_i may take on value $\left\{0, \frac{1}{p-1}, \frac{2}{p-1}, \dots, 1\right\}$ and p is the number of levels. The elementary effect of the i -th input is calculated by the formula below:

$$EE_{ij} = \frac{Y(x_1, \dots, x_j + \Delta_j, \dots, x_k) - Y(X)}{\Delta_j} \quad (22)$$

where $Y(X)$ is the model output corresponding to the initial parameter vector and $Y(x_1, \dots, x_j + \Delta_j, \dots, x_k)$ is the model output after the j -th parameter of the initial parameter vector is changed; Δ is the magnitude of the step between 0 and $1 - 1/(p - 1)$, which is a multiple of $1 - 1/(p - 1)$. To calculate EE of the remaining $k-1$ parameters, the above procedure is repeated. Then, randomly generate n initial vectors and repeat the above process to calculate the n elementary effects of the k parameters, which is EE_{ij} ($j = 1, 2, \dots, k \quad i = 1, 2, \dots, n$). The mean and standard deviation of EE_{ij} can be expressed as:

$$\mu_i = \sum_{j=1}^n EE_{ij} / n \quad (23)$$

$$\sigma_i = \sqrt{\sum_{j=1}^n (EE_{ij} - \mu_i)^2 / n}. \quad (24)$$

Combined with the actual situation, the frequency of the vehicle is generally between 0~15 Hz [2,49]. Therefore, during parameter analysis, the value range of each dimensionless parameter are $\mu_1 = 0.2 \sim 0.9$, $1.2 \sim 15$, $\alpha_{mv} = 0.01 \sim 0.5$, and $\alpha_v = 0.03 \sim 0.75$. Take the identified bridge fundamental frequency increment Δ_b as the model output. There are three variables, and the parameter space is divided into five equal parts. The sampling calculation results are shown in Table 2 to discuss the global sensitivity of each parameter by using the Morris method.

Table 2. Parameters' sensitivity analysis results.

	μ_1	$\mu_1 < 1$ α_{mv}	α_v	μ_1	$\mu_1 > 1$ α_{mv}	α_v
μ	5.650×10^{-1}	4.258×10^{-1}	5.417×10^{-1}	5.333×10^{-2}	6.200×10^{-1}	3.625×10^{-1}
σ	3.180×10^{-1}	5.106×10^{-1}	4.010×10^{-1}	3.409×10^{-3}	2.378×10^{-1}	2.251×10^{-2}

It can be seen from Table 2 that when $\mu_1 > 1$, α_{mv} and α_v have great impact to Δ_b . Moreover, the interaction between μ_1 and the other two parameters is not obvious. When $\mu_1 < 1$, μ_1 , α_{mv} , and α_v all have a great impact on Δ_b . There is also a similar degree of interaction between them. The above conclusion is mutually verified with the pattern described in the previous section. For instance, when $\mu_1 > 1$, the value of Δ_b tends to be stable as μ_1 increases; when $\mu_1 < 1$, the rule of change of Δ_b caused by changing the values of μ_1 and α_{mv} will also change accordingly as α_v takes on a different value.

5. Laboratory Test Verification

The test uses steel beams to simulate the bridge, and the design frequency of steel beams is about 7 Hz. The design drawing of the steel beam is shown in Figure 8. The steel beam is made of 3-mm thick steel plate, which is 3 m long and 30 cm wide. The steel plate is equipped with 2 longitudinal ribs and 5 transverse ribs. The ribs are made of iron slats with a width of 3 cm and a thickness of 3 mm. Two longitudinal ribs are arranged symmetrically with an interval of 15 cm, and 5 transverse ribs are respectively arranged at the end, 1/4 span, and mid-span of the beam. Dimensions of the support is shown in Figure 8. The finite element model of the steel beam is established by using shell elements. The first three vertical frequencies of the steel beam are 7.00 Hz, 25.24 Hz, and 55.05 Hz, and the corresponding vibration modes are shown in Figure 9. During the test, the beam body adopts single-piece steel beam and double-piece steel beam to simulate the increase of bridge mass. The first three tested vertical frequencies of the single-piece steel beam are 6.39 Hz, 25.45 Hz, and 54.33 Hz. The first three vertical frequencies of the double-piece steel beam are 6.29 Hz, 25.17 Hz, and 55.32 Hz. The time history curve and frequency spectrum of the test are shown in Figures 10 and 11 and the structural parameters of the single-piece steel beam are shown in Table 3.

Figure 8. Design drawing of the steel beam: (a) main view and (b) side view.

Figure 9. The first three vertical modes of the steel beam: (a) 7.00 Hz, (b) 25.24 Hz, and (c) 55.05 Hz.

Figure 10. Vertical acceleration response of free vibration at 1/4 span: (a) single-piece and (b) double-piece steel beam.

Figure 11. Vertical acceleration spectrum of free vibration at 1/4 span: (a) single-piece and (b) double-piece steel beam.

Table 3. Structural parameters of single-piece steel beam.

Parameters	L	m_b	EI
Value	3 m	8.6 kg/m	$1.22 \times 10^4 \text{ N}\cdot\text{m}^2$

The main structure of the test vehicle is also made of 3-mm thick steel plates, the length and width are 35 cm and 25 cm, respectively. As shown in Figure 12, the test vehicle is a double-layer structure, with 4 springs symmetrically arranged between the upper and lower layers. Steel plates with dimensions 35 cm $\times$ 25 cm $\times$ 3 mm are used as the counterweight above the springs in the upper layer of the vehicle. The frequency of the vehicle itself can be altered by adjusting the mass of the upper structure, which is the number of steel plates, and the stiffness of the spring. The vehicle mass is differentiated into the light and heavy categories. Light vehicles have no upper structure, and their frequency is much higher than the bridge frequency. Heavy vehicles have an upper layer mass of 6.18 kg. The vehicle frequency is tested to be 9.33 Hz. The time history curve and frequency spectrum of the test are shown in Figure 13. In the case of light and heavy vehicles, the vehicle passes through single- and double-steel beams at different speeds, respectively. Specific conditions of the four testing scenarios are listed in Table 4. As shown in Figure 12, the test vehicle is symmetrical. Due to the limitation of the vertical rod, the upper layer of the vehicle is mainly vertical movement, and there is almost no front and rear rocking movement. When the test vehicle is simplified to a one degree-of-freedom oscillator, the front and rear springs of the test vehicle are simplified to a single spring element in parallel. Therefore, when applying Equations (5) and (6), the position of the center of the test vehicle, which is the same distance as the front and rear wheels, is used as the contact point between the vehicle and bridge.

Figure 12. Design drawing of the test vehicle: (a) main view and (b) top view.

Figure 13. Free vibration of heavy vehicle: vertical acceleration (a) response and (b) spectrum.

Table 4. Test parameters and results.

Steel Beam	Vehicle	μ_1	α_{mv}	Δ_{st}	α_v	Identified Steel Beam Frequency	Experimental Δ_b	Theoretical Δ_b
Single-piece	Light	—	0.16	−14.40%	0.027 0.045 0.066	5.67 Hz 5.60 Hz 5.58 Hz	−11.27% −12.36% −12.67%	−11.22% −10.26% −9.78%
	Heavy	1.46	0.39	−29.26%	0.031	4.62 Hz	−27.54%	−27.06%
Double-piece	Light	—	0.08	−7.79%	0.031	5.82 Hz	−7.47%	−5.47%
					0.050	5.84 Hz	−7.15%	−5.47%
					0.091	5.90 Hz	−6.20%	−4.99%
	Heavy	1.48	0.20	−11.76%	0.030 0.051 0.094	5.22 Hz 5.17 Hz 5.16 Hz	−17.01% −17.81% −17.97%	−17.94% −16.98% −15.54%

The layout of laboratory experiment is shown in Figure 14. The acceleration sensors are respectively arranged in the center of the vehicle, 1/4 (3/4) span and mid span of the bridge, respectively. The speed of the vehicle is controlled by an adjustable speed motor. A one-car length of space, which is 35 cm away from the two ends, is set as the entry and exit region. Result data excluding the entry and exit region is used for further analysis. At a distance of 35 cm from the support, a protrusion is set. When the vehicle passes through the protrusion, the acceleration signal will produce a significant fluctuation, which is used as a sign of the vehicle entering and leaving the bridge. A set of vehicle acceleration signals contains four obvious fluctuations, and the data between 0.5 s after the second fluctuation and 0.5 s before the third fluctuation are selected for analysis. This ensures that the front and rear wheels of the vehicle are on the bridge at the same time. Moreover, because the speed of the test vehicle passing through the bridge is different, the red box in Figure 15 is only for illustration. The test parameters and results are dimensionless as shown in Table 4. Partial experimental results and their spectrum are shown in Figure 15. To compare the frequency change pattern and provide references for frequency identification, let Δ_{st} denote the frequency deviation rate when the vehicle is stationary in the middle of the span.

Figure 14. Layout of experiment: (a) laboratory model; (b) vehicle; and (c) steel beam.

From Table 4, most of the test results show that the measured frequency deviation is larger than the theoretical simulated value. The theoretical Δ_b differs from the experimental Δ_b by about 12% on average. It is obvious that with the change of α_{mv} , the identified steel beam frequency changes significantly. As α_{mv} increases, the degree of the identified bridges frequency deviation increases. If the bridge frequency deviation rate $|\Delta_b| < 10\%$ is used as the condition for successful identification, the steel beam frequency is successfully identified only when the light vehicle and double-piece beam are used. This corresponds to the conditions for identifying success in Section 4.3, that is $\alpha_{mv} < 0.1$. In this case, the changing pattern of Δ_b is in good agreement with the theory, that is, $|\Delta_b|$ decreases with the increase of α_v . After the test vehicle speed is converted into a dimensionless parameter by using the expression in Table A1, the value of α_v is only 0.027~0.094 due to the limitation of the experimental conditions. However, in the parameter analysis in Section 4, the value of α_v ranges from 0.03 to 0.75. As a result, in the test results, the effect of α_v on $|\Delta_b|$ cannot be well reflected. In this study, the magnitudes of these frequency deviation rates are also greater than those frequency deviation rates calculated by Equation (11). Although, experimental results are limited because of the chosen scaling of the experimental models, the bridge frequency deviation obtained by the bridge dynamic response should be taken seriously. In particular, the bridge frequency deviation caused by passing vehicles must be excluded.

The overall test results are in good agreement with the theoretical calculations and mostly conform to the pattern described in the previous section. However, there are inevitable deviations between the test results and theoretical calculations. For the abnormal phenomena in the above test results, possible reasons for preliminary analysis are as follows: 1. One degree-of-freedom oscillator is used as a vehicle model in the theoretical derivation, which is not the case for an actual vehicle. 2. The vehicle damping and beam damping are ignored in the theoretical calculation. 3. Improper installation of vehicle components and unsmoothed wheels will affect the interaction between the vehicle and bridge to a certain extent, which, in turn, interferes with the signal. 4. The vehicle will experience a short period of acceleration until reaching the desired speed. Although a previous study [51] shows that little difference exists between the vehicle passing the bridge at a constant speed or constant acceleration. Due to the large vehicle-to-bridge mass ratio in this test, the influence of the non-uniform speed may be amplified, resulting in errors.

Figure 15. Vertical acceleration response and spectrum of vehicle: (a,b) light vehicle passing through single-piece steel beam with $v = 0.027$; (c,d) heavy vehicle passing through single-piece steel beam with $v = 0.031$; (e,f) light vehicle passing through double-piece steel beam with $v = 0.065$; and (g,h) heavy vehicle passing through single-piece steel beam with $v = 0.066$.

6. Conclusions

In this paper, the one degree-of-freedom oscillator and a simply-supported Euler-Bernoulli beam model were used to establish a vehicle-bridge interaction system. With a large vehicle-bridge mass ratio, dimensionless parameters, mainly from the three aspects, vehicle speed, vehicle-bridge frequency ratio, and mass ratio, were used to discuss the patterns of an extracted bridge fundamental frequency from the acceleration signal of a

passing vehicle. In general, under the influence of vehicle mass, the bridge fundamental frequency identified from the vehicle acceleration signal shows a different pattern than that of when the vehicle mass is ignored. An intuitive display of the accuracy of the extracted bridge fundamental frequency is given. Through the model test in the laboratory, the theoretically calculated patterns are verified. In addition, the corresponding screening principles and methods are proposed on how to select the appropriate peak frequency from the spectrogram to achieve successful bridge frequency identification. The main conclusions are summarized as follows:

1. The right-biased frequency does not always occupy the main advantage in the identified bridge fundamental frequencies, especially when $\mu_1 > 1$, the identified bridge fundamental frequency is dominated by the left-biased frequency.
2. When $\mu_1 > 1$, Δ_b is greatly affected by α_{mv} and α_v , and the interaction between μ_1 and the other two parameters is small; when $\mu_1 < 1$, μ_1 , α_{mv} , and α_v have a greater impact on Δ_b , and there is a similar degree of interaction between them.
3. To ensure the accuracy of identifying the bridge fundamental frequency, the vehicle-bridge mass ratio and vehicle speed should not be too large. It is recommended that the vehicle-bridge mass ratio should not exceed 10%, or even 5%.
4. This paper proposed a few basic principles and reference methods for effective bridge frequency identification using spectrograms of vehicle acceleration, which provide greater accuracy and rigorous for bridge frequency extraction.

The conclusions of this paper can provide useful suggestions for the application of bridge indirect monitoring techniques. Due to the limitations of the simplified model and the scale of the experimental model, further investigations are required in order to understand the pattern of bridge frequency deviation. First, the models of vehicles and bridges should be refined. In future parametric analyses, the effects of vehicle damping and bridge damping on identifying bridge frequencies from a passing vehicle's acceleration will be considered. Second, field tests will be carried out. For a real bridge, the conditions under which the frequency deviation phenomenon occurs should be studied.

Author Contributions: Conceptualization, B.H.; methodology, Q.W. and H.X.; software, Q.W. and W.Y.; validation, Q.W., B.H. and J.Y.; formal analysis, Q.W.; investigation, Q.W. and W.Y.; writing—original draft preparation, Q.W.; writing—review and editing, B.H., J.Z. and H.X.; visualization, Q.W.; supervision, B.H. and J.Z.; project administration, B.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Fundamental Research Funds for the Central Universities under grant no. 2018YJS102; the Science and Technology Project of Beijing Transportation Commission under grant no. KJ-2017-5; and the Fundamental Research Funds for the Central Universities under grant no. 2019RC012.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

The expressions for α_{yb} , α_{yv} , α_{mv} , α_{mw} , α_v , S , τ , and g^* are shown in Table A1.

Table A1. Structural parameters of single-piece steel beam.

Parameters	Expression	Implication
α_{yb}	y_b/L	bridge vertical displacement
α_{yv}	y_v/L	vehicle vertical displacement
α_{mv}	$m_v/m_b L$	vehicle mass
α_{mw}	$m_w/m_b L$	wheel mass
α_v	$\sqrt{\frac{m_b}{EI}} v L$	vehicle speed
s	x/L	vehicle position
τ	$\sqrt{\frac{EI}{m_b}} t/L^2$	time
g^*	$\frac{m_b L^3}{EI} g$	acceleration due to gravity

Appendix B

Substituting Equation (8) into Equation (6), vehicle response α_{yv} can be solved from Equation (6) by Duhamel's integral. Then, α_{yv} can be differentiated twice to yield the acceleration of the vehicle as:

$$\ddot{\alpha}_{yv} = \sum_{n=1}^{\infty} \frac{2(\alpha_{mv} + \alpha_{mw})g^*\Omega_v^2}{\Omega_{bn}^2(1-S_n^2)} \left[\cos \Omega_v \tau + \frac{(2S_n/\mu_n)^2 \cos 2n\pi s - \cos \Omega_v \tau}{1-(2S_n/\mu_n)^2} + S_n \frac{(1-S_n)^2/\mu_n^2 \cos(\Omega_{bn} - n\pi\alpha_v)\tau - \cos \Omega_v \tau}{1-(1-S_n)^2/\mu_n^2} - S_n \frac{(1+S_n)^2/\mu_n^2 \cos(\Omega_{bn} + n\pi\alpha_v)\tau - \cos \Omega_v \tau}{1-(1+S_n)^2/\mu_n^2} \right]. \quad (A1)$$

After simplification, the expressions of A1, A2, A3, and A4 are as follows:

$$A_1 = \sum_{n=1}^{\infty} \frac{2(\alpha_{mv} + \alpha_{mw})g^*\Omega_v^2}{\Omega_{bn}^2(1-S_n^2)} \left[-\frac{(2S_n/\mu_n)^2}{1-(2S_n/\mu_n)^2} - \frac{S_n}{1-(1-S_n)^2/\mu_n^2} + \frac{S_n}{1-(1+S_n)^2/\mu_n^2} \right] \quad (A2)$$

$$A_2 = \sum_{n=1}^{\infty} \frac{2(\alpha_{mv} + \alpha_{mw})g^*\Omega_v^2}{\Omega_{bn}^2(1-S_n^2)} \left[\frac{(2S_n/\mu_n)^2}{1-(2S_n/\mu_n)^2} \right] \quad (A3)$$

$$A_3 = \sum_{n=1}^{\infty} \frac{2(\alpha_{mv} + \alpha_{mw})g^*\Omega_v^2}{\Omega_{bn}^2(1-S_n^2)} \left[\frac{S_n(1-S_n)^2}{\mu_n^2 - (1-S_n)^2} \right] \quad (A4)$$

$$A_4 = \sum_{n=1}^{\infty} \frac{2(\alpha_{mv} + \alpha_{mw})g^*\Omega_v^2}{\Omega_{bn}^2(1-S_n^2)} \left[-\frac{S_n(1+S_n)^2}{\mu_n^2 - (1+S_n)^2} \right]. \quad (A5)$$

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