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A Cost-Effective Triplet Lens Design with Chromatic Aberration Correction Based on Optimization Algorithm and Illustration Method

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Abstract: This paper proposes an optimization algorithm combined with an illustration method to select the three best glass materials for the design of a thickened triplet lens and a correction of the paraxial chromatic aberrations. In the thin lens thickening process, chromatic aberration arises from the deviation between the real and paraxial chromatic aberrations. To solve this problem, we propose an optimization algorithm and illustration method, which are integrated into a thickened triplet lens design. We optimize the eight thickened triplet lens groups to obtain the longitudinal chromatic aberration curve for the best group in the visible light range from 0.4861 μm to 0.6563 μm. The chromatic aberration curve area is 3.33×10^{-7} (μm × mm) for the longitudinal chromatic aberration and 2.11×10^{-5} (μm × mm) for the lateral chromatic aberration. Finally, the chromatic aberration of the thicken triplet lens is close to that of the thin triplet lens design, and the proposed method can obtain a good optical performance.

Keywords: triplet lens; chromatic aberration; optimization algorithm; illustration method

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1. Introduction

Chromatic aberration elimination is a critical part of the optical lens design. The correction of chromatic aberration of the long and short wavelengths at either end of the visible light range is called achromatic correction. However, after correction of the two wavelengths, residual chromatic aberrations at other wavelengths remain, which is called the secondary spectrum. Chromatic aberration correction of three wavelengths including the central wavelength produces a lens which is called apochromatic while four wavelength chromatic aberration corrections are needed for a super achromat [1]. Stephens [2,3] proposed a method for correction of chromatic aberration by selecting the four wavelengths of the F-line (wavelength 0.4861 μm), d-line (wavelength 0.5876 μm), C-line (wavelength 0.6563 μm), and the other wavelength between 0.3650 μm to 1.014 μm combining three different glasses. In 1970, Lessing [4] used a system of four lenses to eliminate the chromatic aberration of the four wavelengths of 0.3650 μm, 0.486 μm, 0.6563 μm, and 1.014 μm. In 1977, Wynne [5] proposed a way to correct secondary chromatic aberration using ordinary glass. Sharma [6] used a graph method to design a doublet lens. Robb [7] developed a method for correction of the longitudinal chromatic aberration for a doublet lens, which could correct the chromatic aberration for at least three wavelengths, and with some combinations for four wavelengths. Rayces and Rosete-Aguilar [8,9] designed an equal optical path system, which can correct spherical aberration, coma, and longitudinal chromatic aberration for a doublet lens. Duplov [10] proposed a system for an apochromatic telescope. Seong [11] used the Mach–Zehnder interferometer principle for the measurement of chromatic aberration. Benny [12] developed a wide-angle eye model and Ravikumar [13] calculated chromatic aberration correction on the retina. Ferrato [14] presented a microscope design for multi-band chromatic aberration correction. Sun et al. [15] proposed a new method for the optimization and correction of chromatic aberration with double prisms, and applied it to a stereo photographic system with achromatic double prism arrays [16] and a doublet lens [17]. In 2010, Sun [18] proposed an illustrating method for a triplet prism combination for minimizing the chromatic aberration.

In our previous publication [15], the triple prisms can be incorporated to correct the chromatic aberration for a thin triplet lens design by increasing the thickness of the thick lens and the spacing between the lenses while maintaining the focal length of the triplet lens. To correct for the increased chromatic aberration caused by the lens thickening, we used an optimization algorithm combined with an illustration method to ensure that the chromatic aberration of the thickened triplet lens system is close to that of a thin lens system. Finally, the triplet lens design is tested with satisfactory results.

2. Methods

2.1. Refractive Index versus Wavelength

The refractive index used in this study and coefficients for the SCHOTT glass are calculated by the Sellmeier equation [19], as shown in the following formula:

$$n^2(\lambda) - 1 = \frac{B_1\lambda^2}{\lambda^2 - D_1} + \frac{B_2\lambda^2}{\lambda^2 - D_2} + \frac{B_3\lambda^2}{\lambda^2 - D_3} \tag{1}$$

where B_1, B_2, B_3, D_1, D_2, and D_3 are the glass coefficients.

2.2. Chromatic Aberration of a Thin Lens

The formula for the refractive power of a thin lens is expressed as

$$K_\lambda = \frac{1}{f_\lambda} = (n_\lambda - 1)(C_1 - C_2). \tag{2}$$

where K_λ, f_λ, and n_λ are the refractive power, focal length, and refractive index of a thin lens at a certain wavelength, respectively. We define C_1 and C_2 as the curvature of the first and second surface of the thin lens, respectively.

Since the lateral chromatic aberration of the thin lens is zero, the corrected chromatic aberration is dominated by the longitudinal chromatic aberration. Assuming ε_{FC} to be the primary color, which is defined as the refractive power difference between the F-line and C-line of the thin lens,

$$\varepsilon_{FC} = K_F - K_C = \frac{K_d}{V_d}. \tag{3}$$

where K_d is the refractive power of the thin lens of d-line; V_d is the Abbe number; $V_d = (n_d - 1)/(n_F - n_C)$. Here, n_F, n_d, and n_C are the refractive indexes of the F-line, d-line, and C-line, respectively, and K_F and K_C are the refractive powers of the F-line and the C-line.

If ε_{dC} is the second spectrum and defined as the difference in the refractive power between the d-line and the C-line of the thin lens, then

$$\varepsilon_{dC} = K_d - K_C = \frac{P_{dC}}{V_d}K_d. \tag{4}$$

where $P_{dC} = (n_d - n_C)/(n_F - n_C)$ is the relative partial dispersion.

2.3. Correction of Longitudinal Chromatic Aberration of a Thin Triplet Lens

Assuming that the total refractive power K_d of the fixed three lenses is 0.1 mm^{-1}, and to calculate the primary color ε_{FC} and the second spectrum ε_{dC} for eliminating the longitudinal chromatic aberration, we apply the theoretical formulas shown in Equations (5)–(7):

$$K_d = \frac{1}{f_d} = K_{d_1} + K_{d_2} + K_{d_3} = 0.1\ mm^{-1}, \tag{5}$$

$$\varepsilon_{FC} = \frac{K_{d_1}}{V_{d_1}} + \frac{K_{d_2}}{V_{d_2}} + \frac{K_{d_3}}{V_{d_3}} = 0, \tag{6}$$

$$\varepsilon_{dC} = \frac{P_{dC_1}}{V_{d_1}} K_{d_1} + \frac{P_{dC_2}}{V_{d_2}} K_{d_2} + \frac{P_{dC_3}}{V_{d_3}} K_{d_3} = 0, \tag{7}$$

where f_d is the focal length of the thin triplet lens and f_d = 10 mm; K_{d_1}, K_{d_2}, and K_{d_3} are the refractive powers of the first, second, and third lenses of the thin triplet lens group, respectively; V_{d_1}, V_{d_2}, and V_{d_3} are the Abbe numbers of the first, second, and third lenses of the thin triplets, respectively; and P_{dC_1}, P_{dC_2}, and P_{dC_3} are the relative partial dispersion power of the first, second, and third lenses of the thin triplet lens group, respectively.

By solving Equations (5)–(7) simultaneously, the refractive power formulas for K_{d_1}, K_{d_2}, and K_{d_3} can be obtained.

$$K_{d_1} = \frac{V_{d_1}(P_{dC_2} - P_{dC_3})K_d}{V_{d_1}(P_{dC_2} - P_{dC_3}) + V_{d_2}(P_{dC_3} - P_{dC_1}) + V_{d_3}(P_{dC_1} - P_{dC_2})}, \tag{8}$$

$$K_{d_2} = \frac{V_{d_2}(P_{dC_3} - P_{dC_1})K_d}{V_{d_1}(P_{dC_2} - P_{dC_3}) + V_{d_2}(P_{dC_3} - P_{dC_1}) + V_{d_3}(P_{dC_1} - P_{dC_2})}, \tag{9}$$

$$K_{d_3} = \frac{V_{d_3}(P_{dC_1} - P_{dC_2})K_d}{V_{d_1}(P_{dC_2} - P_{dC_3}) + V_{d_2}(P_{dC_3} - P_{dC_1}) + V_{d_3}(P_{dC_1} - P_{dC_2})} \tag{10}$$

Equations (2) and (8)–(10) are solved to obtain the curvature difference of each thin lens as follows.

$$(C_1 - C_2) = \frac{K_{d_1}}{(n_{d_1} - 1)}, \tag{11}$$

$$(C_3 - C_4) = \frac{K_{d_2}}{(n_{d_2} - 1)}, \tag{12}$$

$$(C_5 - C_6) = \frac{K_{d_3}}{(n_{d_3} - 1)} \tag{13}$$

Here, C_1 and C_2 are the curvatures of the first and second surfaces of the first thin lens group; C_3 and C_4 are the curvatures of the first and second surfaces of the second thin lens group; and C_5 and C_6 are the curvatures of the first and second surfaces of the third thin lens group, respectively.

After obtaining the curvature of each thin lens, the refractive power of the thin triplet lens at a certain wavelength can be calculated by the following equation.

$$K_\lambda = (n_{\lambda_1} - 1)(C_1 - C_2) + (n_{\lambda_2} - 1)(C_3 - C_4) + (n_{\lambda_3} - 1)(C_5 - C_6). \tag{14}$$

The chromatic aberration correction of the three-piece glass lens is related to the glass's n_d, V_d, and P_{dC}. The 106 kinds of glass produced by the Schott Glass Company are numbered as shown in Table 1.

Table 1. Schott glass type and numbers.

No	Glass Type	No	Glass Type	No	Glass Type	No	Glass Type	No	Glass Type
1	F2	23	N-F2	45	N-LAK33B	67	N-SF2	89	P-LASF50
2	F5	24	N-FK5	46	N-LAK34	68	N-SF4	90	P-LASF51
3	FK5HTi	25	N-FK51A	47	'N-LAK7	69	N-SF5	91	P-SF68
4	K10	26	N-FK58	48	N-LAK8	70	N-SF57	92	P-SF69
5	K7	27	N-K5	49	N-LAK9	71	N-SF6	93	P-SF8
6	LAFN7	28	N-KF9	50	N-LASF31A	72	N-SF66	94	P-SK57
7	LASF35	29	N-KZFS11	51	N-LASF40	73	N-SF8	95	P-SK57Q1
8	LF5	30	N-KZFS2	52	N-LASF41	74	N-SK11	96	P-SK58A
9	LLF1	31	N-KZFS4	53	N-LASF43	75	N-SK14	97	P-SK60
10	N-BAF10	32	N-KZFS5	54	N-LASF44	76	N-SK16	98	SF1
11	N-BAF4	33	N-KZFS8	55	N-LASF45	77	N-SK2	99	SF10
12	N-BAF51	34	N-LAF2	56	N-LASF46A	78	N-SK4	100	SF11
13	N-BAF52	35	N-LAF21	57	N-LASF9	79	N-SK5	101	SF2
14	N-BAK1	36	N-LAF33	58	N-PK51	80	N-SSK2	102	SF4
15	N-BAK2	37	N-LAF34	59	N-PK52A	81	N-SSK5	103	SF5
16	N-BAK4	38	N-LAF35	60	N-PSK3	82	N-SSK8	104	SF56A
17	N-BALF4	39	N-LAF7	61	N-PSK53A	83	'N-ZK	105	SF57
18	N-BALF5	40	N-LAK10	62	N-SF1	84	N-ZK7A	106	SF6
19	N-BASF2	41	N-LAK12	63	N-SF10	85	P-BK7		
20	N-BASF64	42	N-LAK14	64	N-SF11	86	P-LAF37		
21	N-BK10	43	N-LAK21	65	N-SF14	87	P-LAK35		
22	N-BK7	44	N-LAK22	66	N-SF15	88	P-LASF47		

2.4. Merit Function

The real chromatic aberration is corrected for optimization. The damped least-squares method [20,21] is applied for optimization of the design for correction of the chromatic aberration. A merit function is defined as the summation of the squared values of the weighting differences between the aberrations and their target values. The formula can be written as

$$\phi = \sum_{i=1}^{m} w_i^2 (e_i - t_i)^2, \tag{15}$$

where m is the total summation number; w_i is the weighting factor; e_i is the aberration; and t_i is the target value. We define the function $f(x_1, x_2, \ldots ., x_n)$ as

$$f_i = w_i(e_i - t_i). \tag{16}$$

Before optimization, the n variables are denoted as $x_{10}, x_{20}, \ldots ., x_{n0}$; the m aberrations before the optimization are $f_{10}, f_{20}, \ldots ., f_{m0}$. After the optimization process, the variables are denoted as $x_1, x_2, \ldots, x_n$, and the aberrations as $f_1, f_2, \ldots ., f_m$. Here, we define a matrix A, in which the elements are

$$A_{ij} = \frac{\partial f_i}{\partial x_j}, \tag{17}$$

We get the equation

$$X = -\left(A^T A + pI\right)^{-1} A^T f_0, \tag{18}$$

where A^T is the transposed matrix of A; I is a unit matrix, p is a damping factor; and f_0 is the matrix containing the elements $f_{10}, f_{20}, \ldots, f_{m0}$. If x and x_0 are the matrices containing the elements $x_1, x_2, \ldots ., x_n$, and $x_{10}, x_{20}, \ldots ., x_{n0}$, respectively, we can obtain

$$x = x_0 + X. \tag{19}$$

3. The Combination of a Thin Triplet Lens Using the Illustration Method

With Equations (8)–(10), in addition to fixing the total refractive power of the triplet lens, the longitudinal chromatic aberrations for the three wavelengths of the F-line, d-line, and C-line can also be corrected, but longitudinal chromatic aberration between the F-line and d-line and between the d-line and C-line still remain. In order to reduce the residual longitudinal chromatic aberration, two new wavelengths, A-line and B-line, are added. If we let A-line (0.53685 μm) be the middle wavelength of the F-line and d-line, and B-line (0.62192 μm) be the middle wavelength of the d-line and C-line, then ε_{AB} is the longitudinal chromatic aberration between the A-line and the B-line. Equations (8)–(10) are substituted into Equation (20) to obtain Equations (21) and (22) to correct the chromatic aberrations of the three wavelengths for the F-line, d-line, and C-line:

$$\varepsilon_{AB} = \frac{P_{AB_1}}{V_{d_1}} K_{d_1} + \frac{P_{AB_2}}{V_{d_2}} K_{d_2} + \frac{P_{AB_3}}{V_{d_3}} K_{d_3}, \tag{20}$$

$$\varepsilon_{AB} = \frac{[P_{AB_1}(P_{dC_2} - P_{dC_3}) + P_{AB_2}(P_{dC_3} - P_{dC_1}) + P_{AB_3}(P_{dC_1} - P_{dC_2})]}{[V_{d_1}(P_{dC_2} - P_{dC_3}) + V_{d_2}(P_{dC_3} - P_{dC_1}) + V_{d_3}(P_{dC_1} - P_{dC_2})]} K_d, \tag{21}$$

$$\varepsilon_{AB} = \frac{E}{G} K_d, \tag{22}$$

where $P_{AB} = (n_A - n_B)/(n_F - n_C)$; $E = P_{AB_1}(P_{dC_2} - P_{dC_3}) + P_{AB_2}(P_{dC_3} - P_{dC_1}) + P_{AB_3}(P_{dC_1} - P_{dC_2})$; $G = V_{d_1}(P_{dC_2} - P_{dC_3}) + V_{d_2}(P_{dC_3} - P_{dC_1}) + V_{d_3}(P_{dC_1} - P_{dC_2})$.

If the ratio of E to G is smaller, a smaller ε_{AB} can be obtained, and so a smaller E value and a larger G value can be obtained by selecting the appropriate combination of three types of glass material (P_{AB}, P_{dC}, V_d).

3.1. $V_d - P_{dC}$ Diagram (G Diagram)

If the V_d of the glass material is taken as the horizontal axis and the P_{dC} is taken as the vertical axis, we can draw a figure called a G diagram. Taking three different pieces of glass (N-KZFS11, N-SF66, and N-PK52A) as an example, setting point A as N-KZFS11 (V_{d_1} = 42.41, P_{dC_1} = 0.3000) and point B as N-SF66 (V_{d_2} = 20.88, P_{dC_2} = 0.2822), then point C is N-PK52A (V_{d_3} = 81.60, P_{dC_3} = 0.3055), as shown in Figure 1.

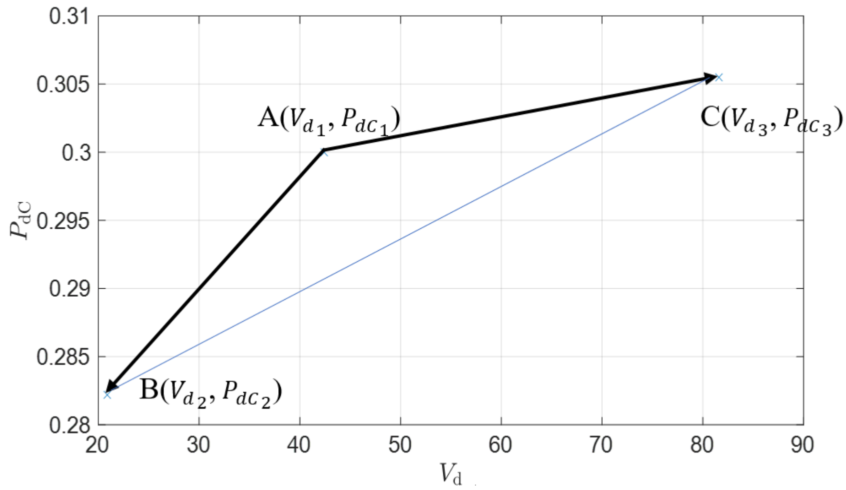

Figure 1. Plot of P_{dC} versus V_d for N-KZFS11, N-SF66, and N-PK52A glasses.

In Figure 1, the $\overrightarrow{AB}$ vector coordinate is $(V_{d_2} - V_{d_1}, P_{dC_2} - P_{dC_1})$ and the $\overrightarrow{AC}$ vector coordinate is $(V_{d_3} - V_{d_1}, P_{dC_3} - P_{dC_1})$. The absolute value formula of the cross product with the $\overrightarrow{AB}$ and $\overrightarrow{AC}$ vectors is shown in Equation (23).

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = V_{d_1}(P_{dC_2} - P_{dC_3}) + V_{d_2}(P_{dC_3} - P_{dC_1}) + V_{d_3}(P_{dC_1} - P_{dC_2}), \tag{23}$$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = G = 2 \times \Delta ABC. \tag{24}$$

Therefore, the G value is twice the area (ΔABC) enclosed by the three points ABC as shown in Figure 2 (namely, the $V_d - P_{dC}$ diagram).

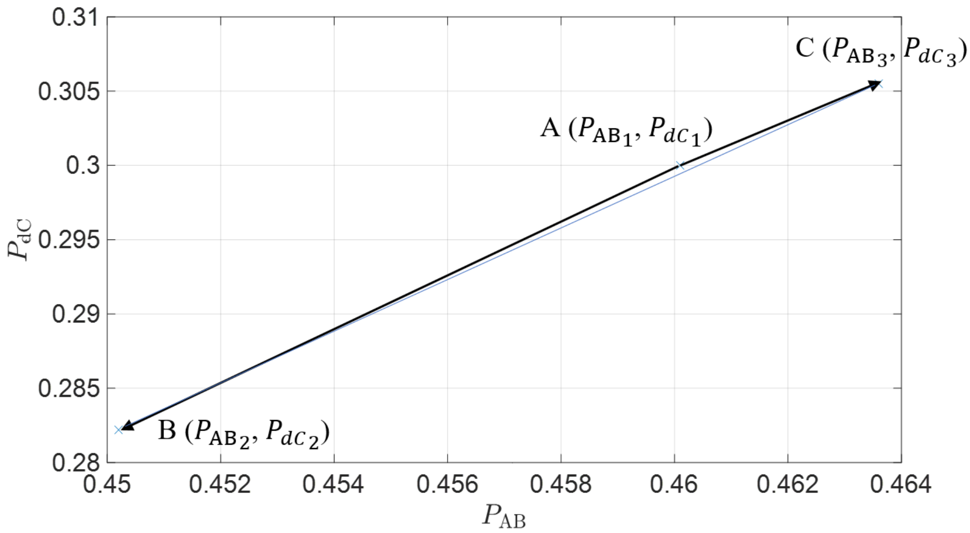

Figure 2. $P_{AB} - P_{dC}$ diagram of N-KZFS11, N-SF66, and N-PK52A.

3.2. $P_{AB} - P_{dC}$ Diagram (E Diagram)

Plotting the glass material P_{AB} on the horizontal axis and P_{dC} on the vertical axis, we obtain the E diagram. Taking the same three pieces of glass (N-KZFS11, N-SF66, and N-PK52A) as an example, with N-KZFS11 at point A (P_{AB_1} = 0.4604, P_{dC_1} = 0.3000) and N-SF66 at point B (P_{AB_2} = 0.4502, P_{dC_2} = 0.2822), then N-PK52A is at point C (P_{AB_3} = 0.4636, P_{dC_3} = 0.3055), as shown in Figure 2.

For Figure 3, the absolute value formula of the cross product with the $\overrightarrow{AB}$ and $\overrightarrow{AC}$ vectors is expressed as

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = P_{AB_1}(P_{dC_2} - P_{dC_3}) + P_{AB_2}(P_{dC_3} - P_{dC_1}) + P_{AB_3}(P_{dC_1} - P_{dC_2}), \tag{25}$$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = E = 2 \times \Delta ABC. \tag{26}$$

Therefore, the E value is twice the area (ΔABC) enclosed by the three points ABC as shown in Figure 3 (namely, in the $P_{AB} - P_{dC}$ diagram).

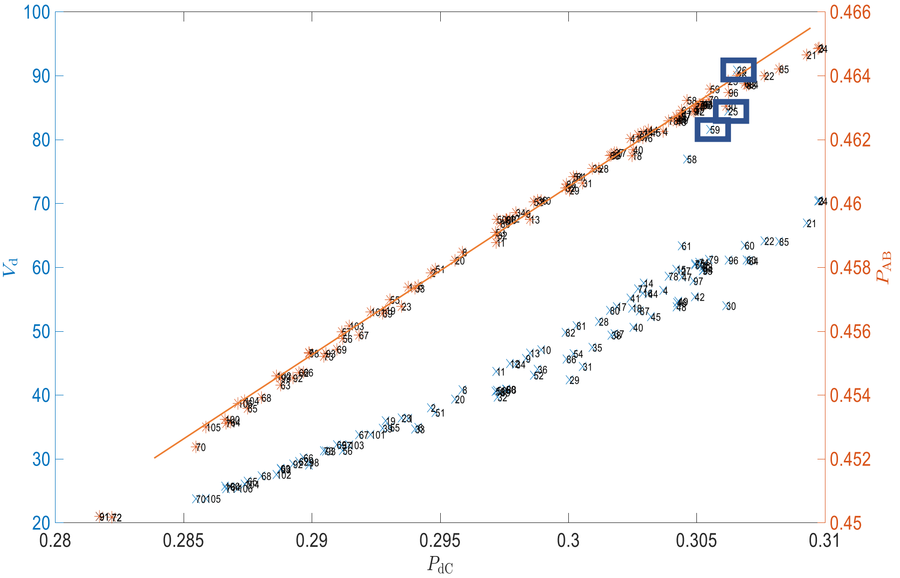

Figure 3. $V_d - P_{dC}$ and $P_{AB} - P_{dC}$ diagrams for the 106 SCHOTT glass materials.

3.3. Illustration Method

According to Equation (22), the achromatic formulation of a thin triplet lens is $\varepsilon_{AB} = (E/G)K_d$. If we want to obtain a smaller chromatic aberration ε_{AB} between the A-line and the B-line, then the larger the value of the denominator G and the smaller the value of the numerator E. Coordinate diagrams for the 106 kinds of glass material (V_d, P_{dC}, P_{AB}) being selected from the Schott Optical Glass Catalog and numbered as listed in Table 1 can be drawn. As shown in Figure 4, P_{AB} is on right vertical axis, V_d is on the left vertical axis, and P_{dC} is on the horizontal axis. Blue indicates data for the $V_d - P_{dC}$ coordinate group and orange indicates data for the $P_{AB} - P_{dC}$ group.

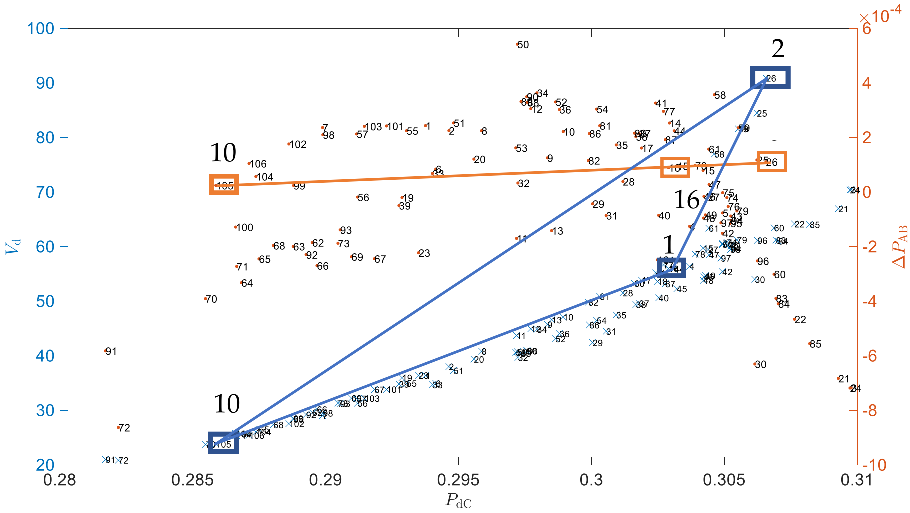

Figure 4. Diagram of $V_d - P_{dC}$ and $\Delta P_{AB} - P_{dC}$.

For the blue data group in the $V_d - P_{dC}$ diagram, the larger the area enclosed by the three glass options the better. Among them, blue No. 26 (N-FK58), No. 25 (N-FK51A), and No. 59 (N-PK52A) are farthest away from the blue data group, so these three types of glass material can be chosen as the benchmark, with the other two kinds farthest away from these three selected to obtain a larger area. However, the distribution of the orange data points in the $P_{AB} - P_{dC}$ diagram is denser, and the area surrounding the three types of glass should be the smallest, which is difficult to observe by viewing the diagram. Thus, we must expand the relative coordinates of the orange data points. A line shows the trend of the orange data points for linear adjustment. The function for calculating the trend is P_{AB} = 0.5278P_{dC} + 0.3021. Let P_{AB} be the adjusted coordinate value and $\Delta P_{AB} = P_{AB} - (0.5278P_{dC} + 0.3021)$. The triangular area E′ of the adjusted $\Delta P_{AB} - P_{dC}$ is expressed as follows:

$$E' = \Delta P_{AB_1}(P_{dC_2} - P_{dC_3}) + \Delta P_{AB_2}(P_{dC_3} - P_{dC_1}) + \Delta P_{AB_3}(P_{dC_1} - P_{dC_2}) \tag{27}$$

$$E' = P_{AB_1}(P_{dC_2} - P_{dC_3}) + P_{AB_2}(P_{dC_3} - P_{dC_1}) + P_{AB_3}(P_{dC_1} - P_{dC_2}) = E, \tag{28}$$

Figure 4 can be obtained by re-plotting the coordinate values of $\Delta P_{AB} - P_{dC}$. We use the illustration method to design an example of a triplet lens combination. Looking at the section with the blue data point G in the figure, we first select glass No. 26 (N-FK58), and then choose the other two glasses in the data group which are the farthest apart P_{dC}. The greater the base of the triangle, the larger the G value which can be obtained. At the same time, the three glass numbers selected also need to be close to the orange data point line, to obtain a smaller E value. Thus, we selected number 16 (N-KZFS11) and number 105 (P-SF6).

The focal length shift Δf_S is defined as the difference between the focal length of any wavelength (f_λ) and the focal length of the center wavelength (f_d), as shown in the following equation:

$$\Delta f_S = f_\lambda - f_d \tag{29}$$

The illustration method was used to select eight triplet lens combinations. Figure 5 shows the chromatic aberration curve, in which the vertical axis shows the focal length shift Δf_S and the horizontal axis shows the wavelength.

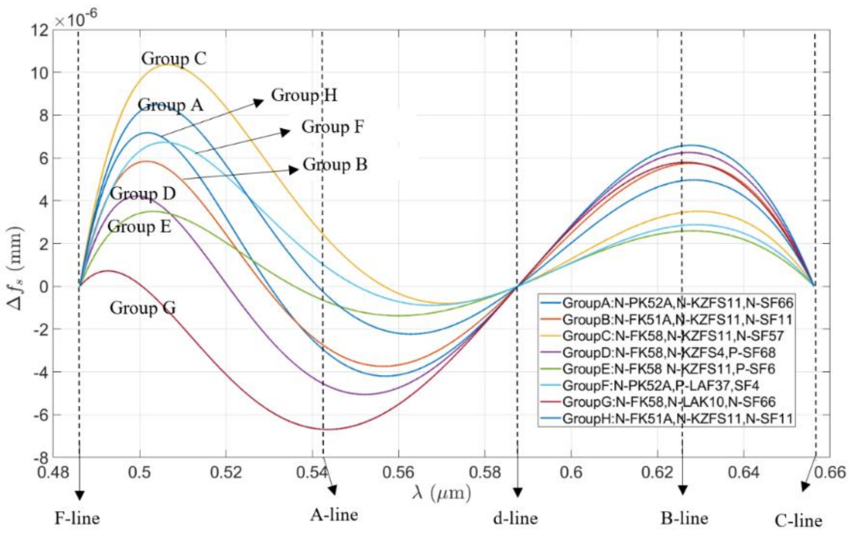

Figure 5. Chromatic aberration curves of the eight triplet lens combinations selected by the illustration method.

Figure 6 shows the chromatic aberration curves for the eight groups comprised of three thin lenses selected by the illustration method. The wavelength of the light source starts from 0.48613 μm (F-line) and ends at 0.65627 μm (C-line) for the calculation of Δf_s. Zero indicates the position of the Δf_s of the center wavelength of 0.58613 μm (d-line). A total of eight chromatic aberration curves, labelled A to H, are obtained. The design parameters of the eight groups of triplet lenses are shown in Table 2. The value of the total chromatic aberration is defined as the area of the curve enclosed by the straight line with zero Δf_s used to calculate the area of the curve. The longitudinal chromatic aberration area (LCA) can be obtained from Equation (30). The parameters and data used in the calculation are listed in Table 2.

$$LCA = \sum_{0.48613\ \mu\text{m}}^{0.65627\ \mu\text{m}} |\Delta f_S| \Delta\lambda. \tag{30}$$

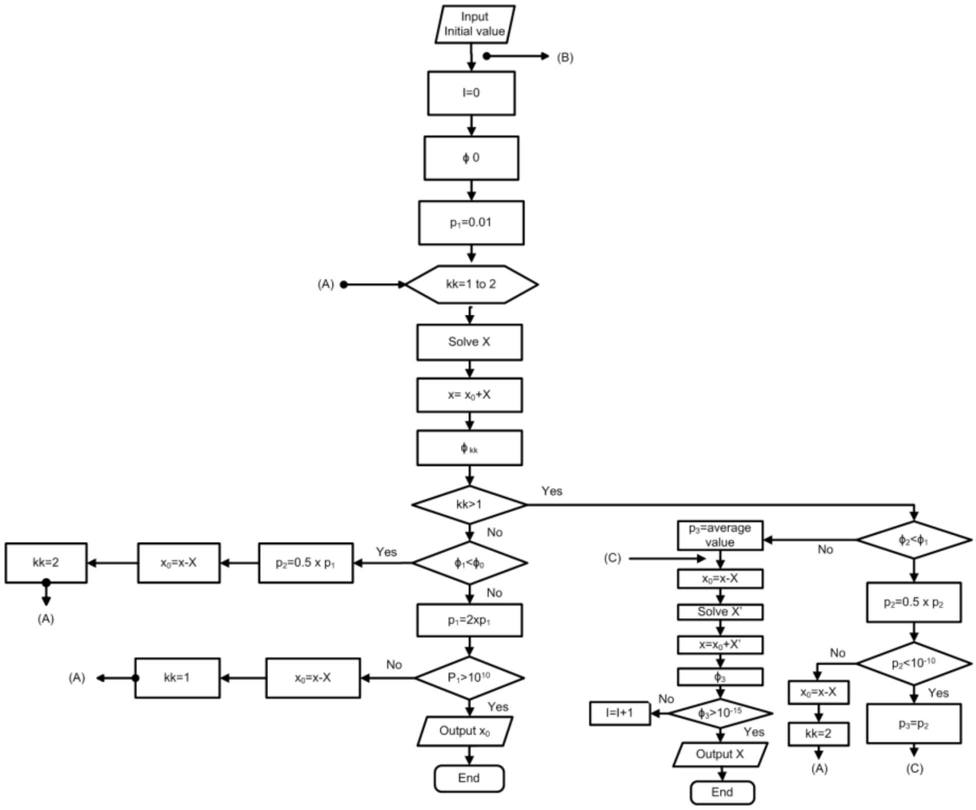

Figure 6. Flow chart of the optimization program.

Table 2. Design parameters for the eight groups of thin triplet lenses selected by the illustration method.

Number	K_{d_1} (mm^{-1})	K_{d_2} (mm^{-1})	K_{d_3} (mm^{-1})	$C_1 - C_2$ (mm^{-1})	$C_3 - C_4$ (mm^{-1})	$C_5 - C_6$ (mm^{-1})	f_d (mm^{-1})	LCA (μm × mm)
A	0.2504	−0.1701	0.0197	0.5039	−0.2667	0.0213	10	6.4531×10^{-7}
B	0.2468	−0.1819	0.0351	0.5072	−0.2852	0.0477	10	5.5363×10^{-7}
C	0.2264	−0.1530	0.0265	0.4966	−0.2399	0.0313	10	5.8096×10^{-7}
D	0.2336	−0.1509	0.0172	0.5124	−0.2460	0.0171	10	5.8880×10^{-7}
E	0.2224	−0.1407	0.0183	0.4878	−0.2207	0.0182	10	2.6743×10^{-7}
F	0.2351	−0.1528	0.0177	0.5155	−0.2491	0.0192	10	3.9891×10^{-7}
G	0.2645	−0.1766	0.0121	0.5800	−0.2452	0.0131	10	6.2705×10^{-7}
H	0.2474	−0.1769	0.0296	0.5084	−0.2774	0.0349	10	5.6752×10^{-7}

4. The Refractive Power of the Thick Triplet Lens Design

The illustration method described in the previous section is applied to design a thin triplet lens. The refractive power of the thin triplet lens and the characteristics of the glass materials are used to find the best chromatic aberration correction. The thickness of the thin lens is generally ignored (i.e., the thickness is assumed to be zero), but in fact, all lenses have thickness. If the thickness of the thin lens is considered, it will affect both the refractive power of the single lens and the refractive power of the triplet lens which will increase the longitudinal and lateral chromatic aberrations.

4.1. The Refractive Power of a Thick Lens

The formulation of the refractive power of a single thick lens is expressed as

$$K'_{\lambda} = K_{\lambda_1} + K_{\lambda_2} - \frac{d}{n_{\lambda}} K_{\lambda_1} K_{\lambda_2}, \tag{31}$$

$$K'_{\lambda} = (n_{\lambda} - 1)(C_1 - \frac{h_2}{h_1} C_2), \tag{32}$$

where K'_{λ} is the refractive power of the thick lens; K_{λ_1} and K_{λ_2} are the refractive powers of the first and second surfaces of the thick lens; d is the thickness of the thick lens; and h_1 and h_2 are the heights of the marginal rays on the first and second surfaces, respectively.

Equations (2) and (32) are respectively multiplied by h_1. If h_1 = 0.5 D_{en}, where D_{en} is the entrance pupil diameter, then Equations (33) and (34) are obtained as follows:

$$h_1 K_{\lambda} = (n_{\lambda} - 1)(h_1 C_1 - h_1 C_2) = (n_{\lambda} - 1)(\alpha_1 - \alpha_2), \tag{33}$$

$$h_1 K'_{\lambda} = (n_{\lambda} - 1)(h_1 C_1 - h_2 C_2) = (n_{\lambda} - 1)(\alpha_1 - \alpha_2), \tag{34}$$

Set the curvature factor α_i as follows:

$$\alpha_i = h_i C_i, i = 1, 2 \tag{35}$$

where α_1 and α_2 are the curvature factors of the first and second surfaces of the lens, respectively. The marginal ray height is $h_1 = h_2$ for the thin lens.

In order to ensure that the refractive powers of the thin lens remains unchanged, if the curvature factors α_1 and α_2 remain unchanged after the lens is thickened, then the refractive power of the thick lens must be consistent with that of the thin lens.

4.2. The Refractive Power of a Thick Triplet Lens

The refractive power K' of the thick triplet lens is

$$K' = K_{\lambda_1} + \frac{h_3}{h_1} K_{\lambda_2} + \frac{h_5}{h_1} K_{\lambda_3}, \tag{36}$$

$$K' = (n_{\lambda_1} - 1)\left(C_1 - \tfrac{h_2}{h_1} C_2\right) + \tfrac{h_3}{h_1}(n_{\lambda_2} - 1)\left(C_3 - \tfrac{h_4}{h_3} C_4\right) + \tfrac{h_5}{h_1}(n_{\lambda_2} - 1)\left(C_5 - \tfrac{h_6}{h_5} C_6\right), \tag{37}$$

where K_{λ_1}, K_{λ_2}, and K_{λ_3} are the refractive powers of the first, second, and third lenses, respectively; C_1 and C_2 are the curvatures of the first and second surfaces of the first lens, respectively; C_3 and C_4 are the curvatures of the first and second surfaces of the second lens; C_5 and C_6 are the curvatures of the first and second surfaces of the third lens, respectively; h_1 and h_2 are the heights of the marginal rays on the first and second surfaces of the first lens, respectively; h_3 and h_4 are the heights of the marginal rays on the first and the second surfaces of the second lens, respectively; and h_5 and h_6 are the heights of the marginal rays on the first and the second surfaces of the third lens, respectively.

In the three thin lens designs it is assumed that the thickness of the lens is zero, the distance between the lenses is also zero, and the refractive power of the lens group is K_{λ} = 0.1 mm^{-1}. Equations (14) and (37) are respectively multiplied by h_1, which is the height of the marginal ray on the first surface of the triplet lens group. If h_1 = 0.5 D_{en}, where D_{en} is the entrance pupil aperture, the following equations are obtained:

$$h_1 K_{\lambda} = (n_{\lambda_1} - 1)(\alpha_1 - \alpha_2) + (n_{\lambda_2} - 1)(\alpha_3 - \alpha_4) + (n_{\lambda_3} - 1)(\alpha_5 - \alpha_6) \tag{38}$$

$$h_1 K'_{\lambda} = (n_{\lambda_1} - 1)(\alpha_1 - \alpha_2) + (n_{\lambda_2} - 1)(\alpha_3 - \alpha_4) + (n_{\lambda_3} - 1)(\alpha_5 - \alpha_6) \tag{39}$$

Comparing Equation (38) with Equation (39), it can be seen that the surface curvature factor α_i is kept fixed. After the lens is thickened, the refractive power of each lens element and the refractive power of the triplet lens remain unchanged, that is, $K_\lambda = K'_\lambda = 0.1\ \text{mm}^{-1}$.

5. Optimization Program and Design Results

When the thickness of the thin lens is added and the distance between the lenses is adjusted, the height of the marginal ray on the surface of the lens will change. The curvature of the surface will also change, the longitudinal chromatic aberration will increase, and the lateral chromatic aberration in the thin lens system will be zero. When the thickness of the lens increases, the lateral chromatic aberration also increases. In order to maintain the desired chromatic aberration result for the thin lens design, we use an optimization program to eliminate both the increase in the longitudinal and lateral chromatic aberrations due to the influence of the lens thickness. A flow chart of the optimization program is shown in Figure 6.

The eight triplet lens combinations selected by the illustration method (as shown in Table 2) are used as the starting values for optimization. During the optimization process, the thickness of the lens and the distance between the triplet lens are appropriately increased. The selected variables x_0 are the respectively refractive powers of the triplet lenses k_{d_1}, k_{d_2}, and k_{d_3}. The optimization functions include the focal length f_d of the triplet lens, the longitudinal chromatic aberration curve area (LCA), and the lateral chromatic aberration curve area (TCA) of the triplet lens group. The designed target value sets are $t_{fd} = 10$ mm, $t_{LCA} = 0$, and $t_{TCA} = 0$. The optimization function is

$$\phi = \sum_{i=1}^{3} f_i^2 = w_1^2(f_d - 10)^2 + w_2^2(LCA - t_{LCA})^2 + w_3^2(TCA - t_{TCA})^2, \quad (40)$$

where ϕ stands for the merit function; f_i is the design target; and w_i is the corresponding weight for the design target.

The optimization results for the correction of the real longitudinal aberration and lateral chromatic aberration for the eight triplet lens groups selected by the illustration method are shown in Table 3, Figures 7 and 8. Figures 7 and 8 show graphs of the real longitudinal and lateral chromatic aberrations, respectively. The optimized focal length is 10 mm, and the real longitudinal chromatic aberration area value is up to 10^{-7}, which is similar to the chromatic aberration area obtained with the thin-lens illustration method, and the real lateral chromatic aberration area ranges from 10^{-5} to 10^{-6}.

Table 3. Optimization results for the eight triplet lens groups selected by the illustration method.

	f_d (mm)	LCA (mm×µm)	TCA (mm×µm)
A	10	5.6328×10^{-7}	8.2488×10^{-6}
B	10	4.8447×10^{-7}	9.2612×10^{-6}
C	10	5.8346×10^{-7}	8.1079×10^{-5}
D	10	4.5206×10^{-7}	3.7359×10^{-5}
E	10	9.7842×10^{-7}	4.4330×10^{-6}
F	10	4.3850×10^{-7}	1.8181×10^{-5}
G	10	3.3362×10^{-7}	2.1194×10^{-5}
H	10	5.4256×10^{-7}	1.4754×10^{-5}

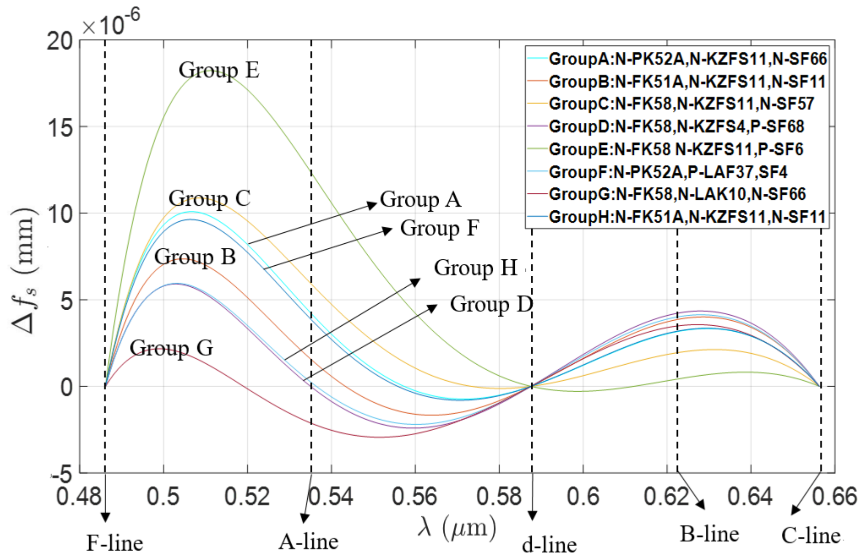

Figure 7. Real longitudinal chromatic aberration curve optimized for the eight triplet lens groups selected by the illustration method.

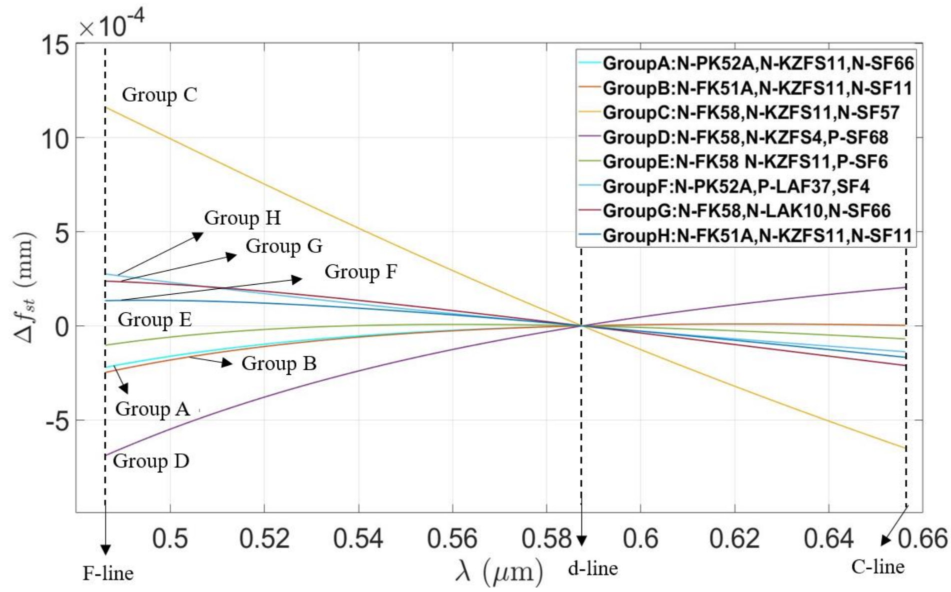

Figure 8. Optimized real lateral chromatic aberration curve for the eight triplet lens groups selected by the illustration method.

6. Conclusions

To design a thickened triplet lens with chromatic aberration correction, it is more efficient and probably faster to start with the proposed illustration method and optimization program. For the design of a thin triplet lens group, the optimization algorithm associated with illustration method can be used to compare the $\mathrm{P_{AB}} - \mathrm{P_{dC}}$ and $\mathrm{V_d} - \mathrm{P_{dC}}$ diagrams of the glass materials, and the selection of the three different types of glass materials with the smallest chromatic aberration can be performed effectively and quickly. Finally, eight lens groups of three thin elements were selected for examination. In the visible light range from 0.4861 μm to 0.6563 μm, the longitudinal chromatic aberration curve area of the optimal thin triplet lens is 2.67×10^{-7} (μm × mm). After the thickening of the thin lenses in the thin triplet lens combination selected by the proposed method and fixing the total refractive power, an optimization program combined with the illustration method is used to optimize the eight thickened triplet lens groups to obtain the longitudinal chromatic aberration curve for the best group in the visible light range. The curve area is 3.33×10^{-7} (μm × mm) for the longitudinal chromatic aberration and 2.11×10^{-5} (μm × mm) for the lateral chromatic aberration.

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References

1. Welford, W.T. *Aberrations of the Symmetrical Optical System*; Harcourt Brace Jovanovich: New York, NY, USA, 1974.
2. Stephens, R.E. Selection of glasses for three-color achromats. *J. Opt. Soc. Am.* **1959**, *49*, 398–401. [CrossRef]
3. Stephens, R.E. Four-color achromats and superchromats. *J. Opt. Soc. Am.* **1960**, *50*, 1016–1019. [CrossRef]
4. Lessing, N.V.D. Selection of optical glasses in superchromats. *Appl. Opt.* **1970**, *9*, 1665–1668. [CrossRef] [PubMed]
5. Wynne, C.G. Secondary spectrum correction with normal glasses. *Opt. Commun.* **1977**, *21*, 419–424. [CrossRef]
6. Sharma, K.D.; Rama Gopal, S.V. Design of achromatic doublets: Evaluation of the double-graph technique. *Appl. Opt.* **1983**, *22*, 497–500. [CrossRef] [PubMed]
7. Robb, P.N. Selection of optical glasses. 1: Two materials. *Appl. Opt.* **1985**, *24*, 1864–1877. [CrossRef] [PubMed]
8. Rayces, J.L.; Rosete-Aguilar, M. Selection of glasses for achromatic doublets with reduced secondary spectrum. I. Tolerance conditions for secondary spectrum, spherochromatism, and fifth-order spherical aberration. *Appl. Opt.* **2001**, *40*, 5663–5676. [CrossRef] [PubMed]
9. Rosete-Aguilar, M.; Raycesd, J.L. Selection of glasses for achromatic doublets with reduced secondary spectrum. II. Application of the method for selecting pairs of glasses with reduced secondary spectrum. *Appl. Opt.* **2001**, *40*, 5677–5686. [CrossRef] [PubMed]
10. Duplov, R. Apochromatic telescope without anomalous dispersion glasses. *Appl. Opt.* **2006**, *45*, 5164–5167. [CrossRef] [PubMed]
11. Seong, K.; Greivernkamp, J.E. Chromatic aberration measurement for transmission interferometric testing. *Appl. Opt.* **2008**, *47*, 6508–6511. [CrossRef] [PubMed]
12. Benny, Y. Wide-angle chromatic aberration corrector for the human eye. *J. Opt. Soc. Am. A* **2007**, *24*, 1538–1544. [CrossRef]
13. Ravikumar, S.; Thibos, L.N.; Bradley, A. Calculation of retinal image quality for polychromatic light. *J. Opt. Soc. Am. A* **2008**, *25*, 2395–2407. [CrossRef] [PubMed]
14. Ferraro, P.; Miccio, L.; Grilli, S.; Paturzo, M.; De Nicola, S.; Finizio, A.; Osellame, R.; Laporta, P. Quantitative phase microscopy of microstructures with extended measurement range and correction of chromatic aberrations by multiwavelength digital holography. *Opt. Express* **2007**, *15*, 14591–14600. [CrossRef] [PubMed]
15. Sun, W.S.; Tien, C.L.; Sun, C.C.; Lee, C.C. A Low-cost Optimization Design for Minimizing Chromatic Aberration by Doublet Prisms. *J. Opt. Soc. Korea* **2012**, *16*, 336–342. [CrossRef]
16. Sun, W.S.; Tien, C.L.; Chu, P.Y.; Hung, Z.Y. Optimization design of a stereo-photographic system based on achromatic double-prism arrays. *Appl. Opt.* **2018**, *57*, 8034–8043. [CrossRef]
17. Sun, W.S.; Chu, C.H.; Tien, C.L. Well-Chosen Method for an Optimal Design of Doublet Lens Design. *Opt. Express* **2009**, *17*, 1414–1428. [CrossRef] [PubMed]
18. Sun, W.S. Illustrating Method of Triplet Prisms for Minimizing the Chromatic Aberration. In *International Optical Design Conference*; Optica Publishing Group: Jackson Hole, WY, USA, 2010; p. JMB33.
19. Schott, Inc. TIE-29: Refractive index and dispersion. In *Proceedings of Schott Technical Information*; Schott Inc.: Jena, Germany, 2015.
20. Wynne, C.G.; Worme, P.M.J.H. Lens design by computer. *Appl. Opt.* **1963**, *2*, 1233–1238. [CrossRef]
21. Jamieson, T.H. *Optimization Techniques in Lens Design*; American Elsevier: New York, NY, USA, 1971.