applied sciences

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Review

Complex Mathematical Modeling of the Well Drilling Process

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Abstract: Recently, the technologies for the global modeling of the process of oil well drilling have become widespread. Mathematical modeling is used in well design, virtual testing of various drilling equipment, simulations of various emergency situations, and personnel training. Complex modeling of the well drilling process includes the simulation of such phenomena as the dynamics of the drill string and its contact interaction with walls, the flow of the drilling fluid and its interaction with the soil (considering influxes and leakages), soil crushing by the drill, the transfer of cuttings particles by the drilling fluid, heat exchange with the soil, and others. This paper provides a detailed review of the existing modeling approaches to solving such problems. Most of the studies included in the review focus on building a detailed mathematical model of one or several of the above processes. Moreover, all these processes mutually influence each other, which also needs to be considered in the analysis. It appears that further development of such a multiphysics approach will be the main direction of research in this area in the near future.

Keywords: well drilling process modeling; drill string dynamics; drilling fluid flow

Citation: Churilova, M.; Lupuleac, S.; Shaposhnikov, N. Complex Mathematical Modeling of the Well Drilling Process. *Appl. Sci.* **2022**, *12*, 11369. https://doi.org/10.3390/app122211369

Academic Editor: Daniel Dias

Received: 30 September 2022
Accepted: 2 November 2022
Published: 9 November 2022

1. Introduction

Drilling the geological exploration wells and the production wells for oil and gas is a complex and expensive process. Modern research is largely focused on optimizing the drilling process, improving the existing and inventing new equipment, increasing its reliability and fault tolerance, and preventing emergency situations. Carrying out the full-size physical experiments is difficult and expensive; therefore, the mathematical modeling is used to replace some of them with virtual ones.

First, let us very briefly consider the main elements of oil drilling, as well as the phenomena that affect the process. Details can be found in the relevant technical literature (e.g., see [1,2]). The simplified general scheme of the oil well drilling process is shown in Figure 1. The drilling bit is connected to the surface by the drill string that can be up to several kilometers long. The drill string is constantly rotating to avoid getting stuck in the well. As the well deepens, the drill string is built up by adding the new drill pipes on top. The drill string is not homogeneous; its lower part is formed from collared (heavier) pipes. The drill string experiences vibrations in torsional, axial, and radial directions, which violate the stability of the drilling process, contribute to the collapse of the borehole walls due to contact with the drill string, and cause sticking and the failure of the drill string due to fatigue stress.

During drilling, a constant flow of drilling fluid is maintained. Inside the drill string, the fluid flows from the top to the bottom, and in the annular space between the drill string and the borehole walls, the fluid flows back to the surface. The presence of flowing drilling fluid has a strong effect on the drill string dynamics by damping the vibrations. On the other hand, due to the vibrations of the drill string, the flow area of the drilling fluid is constantly changing, significantly affecting the flow parameters. Such phenomena are called fluid–structure interactions. Strictly speaking, in mathematical modeling, the processes of the drill string vibrations and the flow of drilling fluid should be considered as coupled.

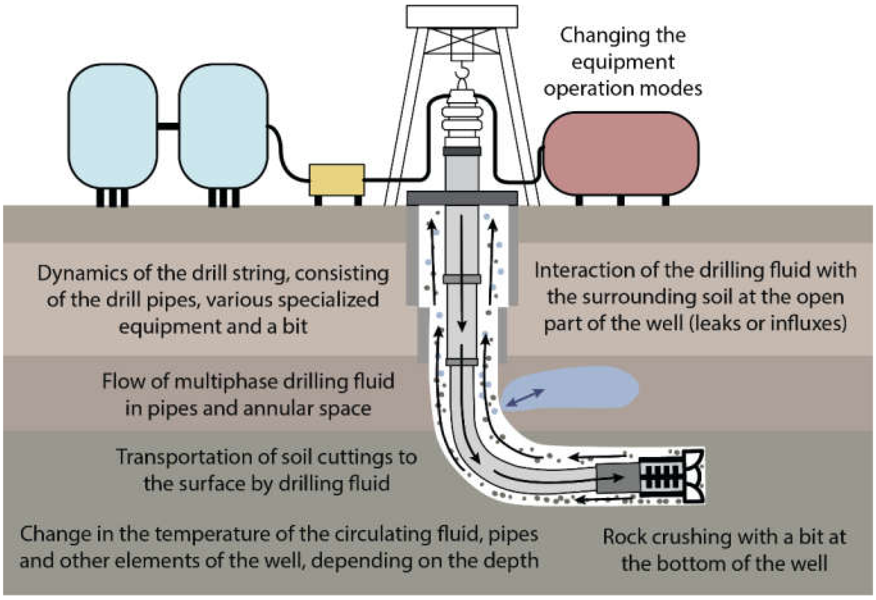

Figure 1. The drilling process scheme.

The drilling bit interacting with the rock is located at the bottom of the drill string. Thus, the torsion and axial pressure applied to the bit (weight on bit) are determined by the drill string. In its turn, the bit has a decisive influence on the dynamics of the drill string, coupling all types of vibrations (torsional, axial, and bending) and setting the conditions at the end of the string. Among other things, the flow of the drilling fluid transports cuttings to the surface. The total volume and distribution by size and mass of the drilled rock particles determine the composition of the drilling fluid flowing to the surface. In turn, the viscosity and flow rate of the drilling fluid determine the possibility of carrying particles to the surface and preventing the formation of plugs and sticking of the drill string. Thus, the process of crushing rock with a bit turns out to be coupled with both the dynamics of the drill string and the flow of the drilling mud.

In the course of the flow of the drilling fluid, there is a mass exchange with the surrounding soil (leaks of the drilling fluid and influxes of formation fluids—water, oil, or gas) in the open part of the well without casing. Inflows and leaks affect the composition of the drilling fluid, pressure distribution, and other characteristics. On the other hand, the leak and influx rates can be reduced to a certain extent by changing the composition and flow rate of the drilling fluid injected into the well. Thus, here again the mutually influencing processes take place.

The temperature of the formation surrounding the wellbore increases with depth. During the well drilling, a complex process of heat exchange takes place. Cooled at the surface, the drilling fluid heats up as it flows inside the drill string, thereby cooling it, and the fluid flows back in the annulus, as well as the surrounding formation. With increasing temperature, the physical properties of the drilling fluid, drill string, bit, and other underground equipment change significantly. In turn, the mechanical processes (rock crushing, friction, etc.) affect the temperature distribution. Thus, the heat exchange process is also strongly coupled with the other processes taking place during drilling.

The problem of modeling the process of well drilling is a classic multiphysics problem when it is required to simultaneously simulate several inter-related processes of different physical nature. Note that each of the processes listed above is quite complex, even if it is considered in isolation from the others. An advanced mathematical model is required to describe each of them.

There are dozens of commercial software packages that simulate one of these processes including Oliasoft WellDesign software for designing the well (trajectory, casing, etc.) [3]; TADPRO software from Pegasus Vertex Inc. for torque and drag calculation; CTEMP for wellbore circulating temperature prediction; CDEx for casing design; HYDPRO for drilling hydraulics modeling [4]; software from Halliburton Landmark [5]; DrillPlan for digital well construction planning and OLGA for prediction of key operational conditions from Schlumberger [6]; and many others.

Most scientific works in the field of the mathematical modeling of the process of well drilling are also devoted to modeling one of the processes listed here. At present, there are several reviews devoted to the mathematical modeling of certain processes. First of all, the review by Ghasemloonia et al. [7] should be noted, which is devoted to modeling the dynamics of the drill string. A review of Livescu [8] on modeling the flow of drilling fluid is also worth mentioning. At the same time, as far as we know, currently there is no review that combines the studies on modeling each of the processes listed here that occur during the drilling of oil wells. This review aims to fill this gap. The emphasis is on those few studies where an attempt is made to build the complex model with several processes.

Criteria for inclusion of an article in this review were as follows:

- The main topic of the article is the construction, application, or experimental verification of a mathematical model of at least one of the processes taking place during the well drilling.
- The article should be devoted to modeling the processes occurring directly in the well during drilling. The studies devoted to modeling the processes occurring in the reservoir (for example, fluid filtration) and those devoted to the operation of surface equipment were excluded from consideration.
- The model considered in the paper is not specific to any particular equipment or material used in drilling. For example, studies devoted to the fatigue analysis of some particular type of drill pipes were not included in the review.

The rest of the paper is organized in the following way. Section 2 gives a review of the papers on drill string dynamics. The papers devoted to drilling fluid circulation are presented in Section 3. Next, Section 4 is about the mathematical modeling of rock crushing with a drill bit. Sections 5 and 6 are devoted, respectively, to mass and heat transfer between the drilling fluid and surrounding soil. Section 7 describes the attempt to build the complex model that involves different processes occurring during drilling. The main conclusions are given in the final section.

2. Drill String Dynamics

The main part of the drill string consists of long, thin-walled pipes, with a total length of up to several kilometers. The lower part includes collars or reamers, stabilizers, and various equipment for monitoring the drilling process. The length of the lower (heavy) part is about several hundred meters. As a rule, while drilling, the upper drill string part, consisting of lighter pipes, is kept under tension, and the lower, weighted part is under compression [1]. The rotational speed of the drill string is in the range of 50–200 rad/min.

One of the important problems of the oil industry is finding out the causes of vibrations in the drill string and the development of ways to suppress them. Intensive uncontrolled vibrations can cause accidents. Such factors as damping by the drilling fluid, contact of the drill string with the borehole walls, and "accidental" interaction of the drill with the rock play a significant role in the dynamics of the drill string [9]. The most common application of drill string models is the prediction of the tool tuning effects (vibration sources) or the evaluation of key drilling parameters, such as rotation speed, torque, and axial load on the bit [10]. The main processes that need to be considered in the model of the drill string are shown in Figure 2, the drilling fluid flow is shown with blue arrows and the string possible movement with black ones.

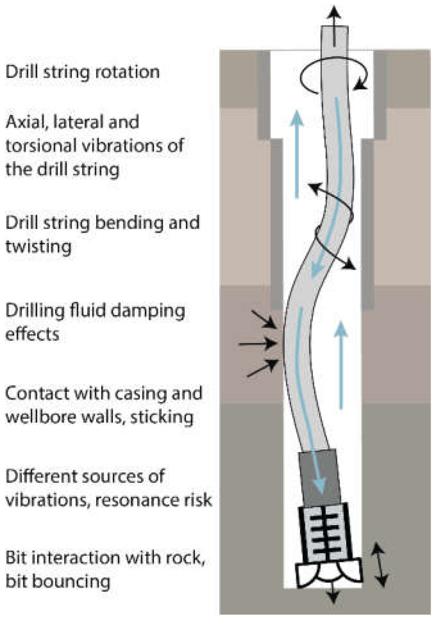

Figure 2. The drill string scheme.

Drill string vibrations are usually divided into three main components: axial, lateral, and torsional. Dangerous cases of each of these vibrations are, respectively, "bouncing", "vortex" movement, and jerky vibrations. Their main causes are the interaction of the bit and the rock, eccentricity, variable points of contact between the drill string and the borehole walls, and vibrations generated by the tools. All three types of vibrations are related. Axial vibrations can cause lateral ones, while severe lateral vibrations in the bottom of the drill string can cause axial and torsional vibrations that can be measured in the top of drill string [11]. In vertical wells, the axial vibrations are much more noticeable and dangerous than torsional ones.

In the past 70 years, the drill string vibration modeling has been actively developed, and the accuracy of predicting borehole vibrations and determining the effect of input parameters on drill string vibrations has been improved. However, despite the active development of modern computing systems, the modeling of a drill string (whose size ratios are comparable with the size ratio of a human hair) remains a complex physical and mathematical problem. Extensive theoretical material on modeling the dynamics of a drill string in a well is presented by Marquez et al. [12] and Azar and Samuel [13]. A detailed review of drill string dynamics was made by Ghasemloonia et al. [5]. Equations and effects regarding the axial vibration of the drill string can be found in the studies by Kreisle and Vance [14] and Li et al. [15].

Drill string stability is discussed by Dunayevsky et al. [16]. The positive effect of axial vibration on the drilling process is studied by Dareing [17]. In studies on axial vibrations, it is indicated that the length of the bottom part of the drill string plays a key role in the occurrence of vibrations, since this part is the heaviest, which means that by changing its length it is possible to adjust the position of the neutral point of the string (where tension gives way to compression). It is also important to understand that resonance must not be allowed to occur in the axial direction. If a harmonic force acts at the lower end of the drill string, and the drill beat frequency is close to the natural frequency of the string, this can lead to an uncontrolled increase in vibrations.

Studies on the bending vibrations of drill strings and their bending dynamics can be found in papers by Rao [18] and Ghasemloonia et al. [19]. Flexural vibrations and the propagation of flexural waves are considered in the studies by Gulyaev and others [20–23]. Due to the high flexibility of the drill string and its low rotation speed, it is generally accepted that the Bernoulli–Euler beam model accurately describes the dynamics of bending vibrations. An example of modeling the dynamics of drill string bending vibrations with the Timoshenko beam model can be found in the study by Beck et al. [24]. The paper by Zare et al. [25] is focused on the numerical aspects of modeling the drill string dynamics, and a description of modeling using the finite element approach is given.

The bending vibrations of drill strings are divided into two main types: in-plane vibrations and full-scale spatial vibrations. Typically, in-plane vibrations are described in polar coordinates. Since the lower part of the column is in compression, and the upper part is in tension, the lower part is subject to basic vibrations. The upper pipes begin to vibrate at higher frequencies than the standard column rotation speeds. Flexural vibrations are the most dangerous because the flexural waves propagate along the drill string only at certain frequencies, so most of the resulting flexural vibrations cannot be captured at the surface where the sensors are commonly placed.

Most of the literature is devoted to the modeling of torsional vibrations. Usually, the inverted pendulum model is used. The downhole mechanism is considered as a completely rigid body, and the drill string (drill pipes and extensions) is considered as a torsion spring with stiffness depending on the drill string parameters. A detailed description of such a model, as well as its modifications, can be found in the studies by Lin and Wang [26] and Tucker and Wang [27]. In the same articles, one can also find a model of viscous friction between the downhole mechanism and the walls of the well, as well as the hydrodynamic damping by the drilling fluid. A discussion of the torsional dynamics stability can be found in the articles by Palmov et al. [28], Balanov et al. [29], and Cuhna Lima et al. [30]. An interesting effect of the interaction of the bit with the rock is considered in the study by Gulaev et al. [31]. The phenomenon of "jerky vibrations" is considered in the studies by Besselink et al. [32] and Barton et al. [33].

As a rule, the drill string vibrates in one of three modes: bending–axial, torsion–axial, or bending–torsional. When it comes to combined movement, nonlinear models need to be considered. This applies both to the equations themselves and to the quantities included in these equations. Nonlinearity, if any, should remain in the boundary conditions. The reason for combining directly axial and torsional vibrations is the interaction of the drill with the rock at the lower end of the drill string, which is expressed in nonlinear contact interaction and nonlinear friction. It is assumed that these factors also give rise to the phenomenon of "jerky vibrations".

There are several papers describing the axial–torsional dynamics of the drill string. It is worth noting the studies by Sampaio et al. [34], Kamel [35], and Tucker [27]. In the study by Zamanian et al. [36], one can find a model with three degrees of freedom—one for the axial motion and two for the torsional one. Elsayed et al. [37] studies the dynamics of the torsion–axial vibrations of a polycrystalline diamond drill. The model of the bending–torsional vibration dynamics is discussed in the studies by Yigit and Christoforou [38,39]. A model of dynamics and control over unwanted vibrations is presented by Al-Hiddabi [40]. Leine et al. [41] study the problem of "vortex whirling"—the transition of the phenomenon of "jerky vibrations" into "vortex whirling" using the bifurcation theory. The study considers the influence of the flowing fluid as well as contact interaction.

The articles by Yigit and Christoforou [42] and Yigit et al. [43] play a key role in describing the bending–axial models. The first one investigates the bending–axial behavior of the downhole mechanism without considering the rotation of the string. The second shows the same dynamics of combined bending and axial movement but taking into account the string rotation. The bending–axial dynamics are also investigated by Hakimi and Moradi [44]. This article deals with the natural frequencies of a single-span beam in a plane. The article by Mahyari et al. [45] poses the problem of the optimal installation of stabilizers to maintain the stability of the column. Trindade [46] provides a finite element formulation for the nonlinear problem of bending–axial movements of the drill string.

There are relatively few models that contain fully combined vibrations (axial, torsional, and bending). Typically, the models consider the effect of axial load on bending–torsional vibrations or the effect of load in the presence of "jerky vibrations". Nevertheless, there are works that consider all three types of vibrations. One such model is described by Yigit and Christoforou [47]. Moreover, it is worth noting the study by Baumgart [48], which is one of the first with a model based on the nonlinear differential equations in three planes. The paper studies the influence of the drilling fluid and its pressure on the vibration of the drill string. The influence of hydrodynamic factors on such phenomena as "jerky vibrations", "bouncing", and the loss of stability of the equilibrium form is considered. The modeling of drill string vibrations by the special finite elements is presented by Khulief and Al-Naser [49]. In the study by Khulief et al. [50], a coupled model of bending–torsional vibrations was constructed, which includes bending–axial nonlinear geometric interaction. The results of an experimental validation of this model can be found in the study by Khulief and Al-Sulaiman [51].

The nonlinear dynamics of the drill string system model, considering the influence of fluid–structure coupling and the effect of support stiffness, are investigated by Wang et al. [52]. The model allows the simulation of the chaotic and periodic motion, providing a better explanation of the nonlinear dynamic characteristics of the deep-hole drill string system. The influence of the core on drill string vibration is considered by Sun et al. [53]. With a reduced-order model of drill string motion, the authors investigate the resonance of the drill pipe in coring drilling, and some measures to suppress resonance are given in this paper. The nonlinear dynamics of the drill string using the lumped parameter method was investigated by Khajiyeva et al. [54]. The authors model lateral vibrations of a vertical drill string under the action of a gas flow moving at supersonic speed.

3. Drilling Fluid Circulation in the Wellbore

The drilling fluid performs various functions: carrying rock cuttings to the surface, maintaining the stability of the wellbore, balancing formation pressure, cooling the bit, and supplying hydraulic energy to the bit to increase the rate of penetration (see Figure 3, the thin blue arrows represent the drilling fluid flow, big blue and orange arrows show the heat transfer—cooling the fluid in the top and heating in the bottom, respectively, gray arrows illustrate the formation pressure, dark blue arrow goes for fluid (mass) transfer through the wall). Drilling fluid can be based on water, oils (petroleum), air, steam, and foam. Water and oil have insufficient viscosity and density. Viscosity is needed to carry the cuttings, and density is needed to resist the pore pressure in the rock. Therefore, special solid particles are added to the fluid. Each type of drilling fluid contains three components: liquid, active, and inert solid particles. Active particles react chemically with the liquid, affecting its rheological properties.

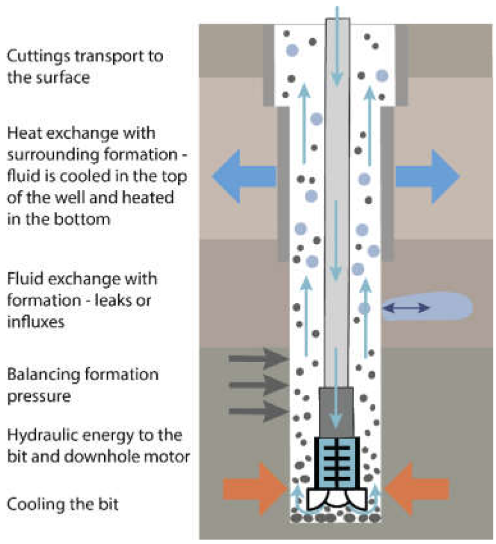

Figure 3. The drilling fluid circulation scheme.

A paper by Livescu [8] is devoted to a review of the literature on modeling the flow of drilling fluid in wells and the flow of crude oil in the oil pipelines. The work contains an extensive bibliography. A list of works devoted to the experimental determination of the liquid properties is presented. The experimental studies available in the literature are described, confirming the complex rheology of drilling fluids, including its dependence on temperature, pressure, time, and the magnitude of shear deformation. The author describes the main rheological fluid models: Bingham, Casson, Herschel–Bulkley, and others. As a general conclusion, the author notes that all the cited studies contain strict simplifications, and, therefore, their results can be used for a narrow range of real problems. For example, in most works it is assumed that the drilling fluid is incompressible and single-phase and that its properties do not depend on pressure and temperature. Moreover, the validation and verification of theoretical models is often neglected.

Modeling the Newtonian and non-Newtonian fluid flow in circular tubes and annular space is discussed by Nouri et al. [55]. Kim et al. [56] involve in the consideration the rotation of the drill string. Similar topics are discussed also by Scheid et al. [57] and Platonov [58]. The monograph by Wilkinson [59] is devoted to the features and the modeling of various flows of non-Newtonian fluids. The flow of Bingham fluids is discussed in the study by Gnoyevoy and Klimova [60], where the problems of flow in channels and cavities with wall shapes varying in time and space are considered. Beloglazov et al. [61] propose a new rheological model for a heavy-oil flow in oil-field pipelines.

The monograph by Bulatov, Makarenko, and Proselkov [62] is devoted to the drilling and cement fluids, including their properties and production methods. Studies by Gukasov et al. [63], Skalle [64], Makovei [65], and Mirzadzhanzade and Entov [66] are devoted to hydrodynamic calculations in drilling. The hydraulic codes are presented that allow determining the flow rate and rheological properties of a fluid that satisfy various conditions. Examples of hydraulic calculations can also be found in [67,68].

The most used fluid models are Newtonian (viscous stresses are linearly correlated to the local strain rate); Bingham viscoplastic (behaves as a rigid body at low stresses but flows as a viscous fluid at high stress); and power-law models (power-law relationship between stress and shear rate, which better reproduces the behavior of a fluid at low shear rates). There are also three main fluid models, which are some combination of the Bingham and power-law models: Casson, Robertson–Stiff, and Herschel–Bulkley models. The two-parameter Casson model is widely used in industry but rarely applied to the drilling fluids. The Robertson–Stiff model includes a gel strength parameter and is used to a limited extent in the petroleum industry. The Herschel–Bulkley model is a power law with ultimate shear stress. Unfortunately, in most applications this model leads to analytically unsolvable expressions; therefore, it is not used for calculations in the oil industry. Some companies use the pseudo-Herschel–Bulkley model, which does not directly relate the viscometric analysis to the hydraulic calculations.

The selection of a drilling fluid rheological model is discussed by Wiśniowski et al. [69]. Various models are considered: the Newtonian, Bingham Plastic, Casson, Ostwald de Waele, Herschel–Bulkley, Vom Berg, and Hahn–Eyring, with linear and nonlinear regression methods. The authors conclude that, for the used measurements of drilling fluid properties, the Vom Berg and Hahn–Eyring rheological models are best fitted to the description of drilling fluid rheological parameters. The Herschel–Bulkley viscosity model practical usage for drilling fluids is discussed in the study by Saasen and Ytrehus [70].

The main purpose of calculating the drilling fluid flow in the circulation system of the well is to determine the distribution of pressure along its depth. It is used to monitor the fulfillment of the necessary conditions: creating a pressure that prevents the flow of formation fluids and gases into the well and preventing hydraulic fracturing; control of pressure in pumps; control of cuttings removal; and control of the pressure difference in the pipes and in the annular space (pipe strength condition).

There are two approaches to obtain the pressure distribution along the well depth. The first approach is to determine the total pressure loss based on precise analytical or empirical formulas for the pressure loss in each element of the circulation system. Multiphase fluid (fluid with inclusions) in this approach is modeled as a single-phase fluid with variable viscosity. The advantages of this approach are the simplicity of implementation and high speed of calculations using exact formulas. An obvious disadvantage is limited application due to the possibility of obtaining exact solutions only for a narrow class of fluid types, flow regimes, and configurations of regions with a significant number of assumptions, as well as the lack of universality of empirical dependencies.

The second approach is to numerically solve the equations for the flow of a multiphase fluid in the well circulation system. Depending on the models used, this approach makes it possible to accurately consider various aspects of the multiphase flow of drilling fluid. The more detailed models more accurately describe the drilling fluid flow in the circulation system. At the same time, the use of such models leads to problems with large dimensions that require large computational resources.

Multiphase flow consists of two or more different phases (which do not mix with each other). The most studied two-phase flows can be classified according to the type of their phases: gas–solid, liquid–solid, and gas–liquid. Often one of the phases consists of many physical particles (solid particles, drops, bubbles)—such flow is called dispersed. Particles form a dispersed phase, and the medium which contains these particles forms a continuous or carrier phase. The nature of the interaction of the dispersed and continuous phases mainly depends on the size of the particles and their distribution density. If this density is low and the particles are small, then the liquid affects the movement of the particles, but the particles do not affect the flow of the liquid. In this case, there is one-way interaction between the phases. If both the liquid affects the particles and the particles affect the flow of the liquid, then there is a two-way interaction. It is assumed that the interaction of particles with each other can be neglected.

The main approaches to the modeling of multiphase flow are Euler–Lagrange and Euler–Euler. Since the motion of the continuous phase is usually described in the framework of the Euler approach, these approaches differ in the way of describing the motion of the dispersed phase, which can be considered in the framework of both the Euler and Lagrangian approaches.

Nowadays, the local instant formulation is used quite often as a method for describing the multiphase fluid flows: each particle of the dispersed phase is described according to the Lagrange approach; the particle positions are found from the equations of motion; in the area where there are no particles, the equations for the carrying phase are solved; at the phase boundary (surfaces of particles) these solutions are connected. Due to the use of significant computational resources, this approach can be used only for small geometries and a small number of physical particles of the dispersed phase. The Euler–Euler approach is considered, for example, in the monograph by Leonov and Isayev [68], as well as in the thesis by Isaev [71], in the articles by Kashevarov et al. [72], and with some modifications in the studies by Fan et al. [73] and Wang and Sun [74].

Due to the great practical importance of the multiphase flow theory, there are many approaches to the different aspects of the subject. This concerns both problem formulation and the derivation of the constitutive equations and methods for their solution. In addition to deriving the basic equations, it is required to specify additional closing relations. Their formulation often depends on the specific problem, which may include experimental results, may not be unique, and may contain the speculative hypotheses. Therefore, for flows with different combinations of phases their own branches of theory arise, and it is advisable to review them with the specific applications in mind. In this review, we consider only the most general approaches to modeling the multiphase flows.

The most complete monograph on multiphase flows is the book by Yeoh and Tu [75]. Another detailed work in which all aspects of multiphase flows are considered is the handbook on multiphase flows edited by Crowe [76]. The study by Brennen [77] also contains a lot of material on this subject. Gross and Reusken [78] discuss the application of the finite element method for multiphase flows. The proceedings of the symposium on computational approaches to multiphase flows are contained in [79], with articles on all the approaches discussed in the review of multiphase flows. In the monographs by Wallis [80] and Brill and Beggs [81], one-dimensional flows are considered. Chhabra [82] studies the motion of individual particles of different natures in a non-Newtonian fluid. In the study by Fang et al. [83], the gas–liquid two-phase flow model based on the drift flux model is developed to describe the characteristics of transient multiphase flow in the wellbore. It is used for calculation of the combination of wellhead backpressure and displacement for the managed pressure drilling technique with a narrow density window under complex geological environments.

4. Rock Crushing with a Drill Bit

Soil crushing with a drill bit can be described as a determining process in the drilling of oil wells. In this regard, great efforts have been made in the field of the mathematical modeling of this process in order to solve the following problems: determining the dynamics of the bit in order to ensure stable drilling and reduce the wear of the drilling tool, and evaluating the penetration rate and particle size distribution of the drilled rock.

The models of bit dynamics as a rule include torsional and axial degrees of freedom that are subject to corresponding linear dynamic equations involving inertial, stiffness, and damping terms and describing the process of vibrations. The model of bit–rock interaction allows the coupling of the torsional and axial vibrations and the building of a unified system of nonlinear equations describing the bit oscillations. Richard et al. proposed the model of bit–rock interaction [84] where the friction forces and torques in the system are connected by a time-delay term determined by the penetration of the bit blades into the rock. This approach was used in several works (e.g., see [85] and the references given there). Tengesdal et al. in [86] have supplemented this model by adding into consideration the drill string represented as a series of alternating springs and point masses.

The direct numerical simulation study on the rock-breaking process is presented by Wang et al. [87]. The dynamic rock breaking process under the compound impact is simulated by the finite element method using special cohesive interface elements. As a result of modeling, in particular, it is possible to determine both the rate of penetration and the distribution of the size and shape of the particles of the drilled rock. At the same time, such a direct method for modeling the interaction between a drill and soil is extremely complex and time-consuming. Therefore, the use of this approach in a complex global model of the drilling process seems hardly possible. The enormous amount of different uncontrollable factors in the process of drilling cannot be directly considered in the model. Thus, the probabilistic approaches are commonly used for modeling such processes.

It is quite natural to use the approach based on the concept of a branching process for modeling particle distribution over mass or size as it is done by Sevastyanov [88]. In general, the branching process describes the growth of a particle population from the probabilistic point of view [89,90]. Several studies have been devoted to the development of rock fragmentation models, including Filippov [90], Cheng and Render [91], and Fowler and Scheu [92].

For the particle mass probability density, one can obtain the integro-differential equation (see [90,91]). A power-law dependence of fragmentation intensity for the mass is commonly used (e.g., see [90]). Models for solid body crushing are presented in [93–96].

It is possible to consider three types of soil in relation to its interaction with a bit. The first type (“rocky soil”) is the soil with characteristic particle size which is comparable with the size of the bit. This type of soil will be crushed into smaller parts by the drilling bit. Soil of the second type (“sandy soil”) consists of particles that are much smaller than the bit. This soil is not crushed but mixed during drilling. The third type is “mixed soil” which is a mixture of the first and the second types, which are to be modeled separately. Thus, the problem is reduced to the description of soils of the first and second types.

There are models based on the concept of a branching process, both for the soil of the first and second type, but there is a fundamental difference between these soils. The first type of soil is crushed with a bit, while the particles of sandy soil are not transformed during drilling.

To build a branching process model, it is necessary to set the characteristic time of the process: the time needed for the initial set of particles to be fragmented into smaller ones. Two options are possible: either such fragmentation occurs after a certain fixed time step for all particles at the same time, or each particle is fragmented after some random time (different for each particle). Both approaches are used in the studies cited above. It is worth noting that the first approach is simpler, while the second better reflects the specifics of the crushing process. In addition, it is necessary to assume that all particles are crushed independently, and the crushing process does not depend on the prehistory of the particle. The latter means that the model of a branching Markov process is used.

Moreover, it is necessary to set the random function describing the fragmentation of each particle. This function returns the number and mass distribution of the new generation of particles obtained from a certain particle of a given mass. All the above characteristics are affected by the type of drill. In particular, the bit shape determines the intensity of the crushing of rock particles [97].

There are other physical aspects related to the bit that can be considered. Kessai et al. [98] analyze the drill bit deformations caused by the stick-slip vibrations. Their model is based on a 'mass-spring-damper' approach—a proxy system containing masses attached to springs and dampers, which includes the drive, drill pipes, and bit. The characteristics of the downhole bit load and longitudinal vibration of the drill string under different conditions were studied by Xu et al. [99]. The method of energy conversion efficiency from the drill string vibration to spring potential energy is proposed, along with the experimental analysis. Energy consumption and energy savings during the well drilling process are analyzed by Ivanova et al. [100]. The effect of changing the drilling tool and pipe design on the energy consumption required for well drilling is discussed. Moisyshyn et al. [101] investigate drilling large diameter wells with two rock cutting tools in the bottom hole assembly, which allow better management of the well trajectory. The mathematical model for wellbore trajectory calculation is presented, considering two rock cutting tools—a bit and a reamer. Bottomhole processes of rock breaking with the bit are investigated by Ihnatov [102]. The hydromechanical pellet impact drilling technology is discussed, in which hard pellets are used instead of the bit, moved by the drilling fluid flow in the bottom of the well and crushing the rock (a detailed description of the rock-breaking device is described in the work). A comparison with traditional bits is provided.

5. Mass Transfer between the Drilling Fluid and Soil in the Open Section of the Well

The "mass transfer" denotes here both the absorption of the drilling fluid filtrate by the formation, and the influx of formation fluids (water, oil, or gas) into the well. Solving the problem of mass transfer allows setting the sources or sinks in the model for calculating the circulation of the drilling fluid in the annular space. The change in rheological properties of the drilling fluid due to mass transfer are considered. If the fluid filtrate displaces the formation fluid in the near-wellbore zone, then its loss is estimated. The main result of solving the problem is determining the filtration rate at the well–reservoir interface. The calculated changes in the drilling fluid rheology affects all drilling process. Additionally, for the heat transfer problem the filtration rate inside the reservoir in the near-wellbore space is calculated.

When studying the issues of mass transfer in oil and gas production, the filtration equations in the porous medium (Darcy's law, continuity equation, equations of state) with equations of diffusion and mass transfer in a porous medium (generalized Fick's law, equation of mass balance, equations of kinetics, and isotherms of matter exchange between phases) are used. As a result, the system of differential equations is solved. The solution determines the velocity field associated with the pressure gradient at a given porosity and density, which may depend on the pressure distribution in the well and in the formation.

Models of axisymmetric and plane-radial filtration are usually used for simulation of mass transfer between an open well part and the surrounding soil. The stationary flat-radial filtration models allow the analytical solution under certain assumptions. Moreover, these models are given by the boundary value problems for ordinary differential equations. Usually, these 2D models are used to obtain a quick estimate of the required values. When using an axisymmetric filtration model, the formation fluid influx and drilling fluid loss are considered without internal mass transfer between the phases. The filtration velocity field can be found after calculating the pressure distribution in the area. When the drilling fluid filtrates into formation, the formation fluid is displaced without mixing.

The generally used approach is the separate consideration of the filtration problem for each layer as it was performed by Rabinovich [103], in which the top and bottom boundaries of the layers are assumed to be impenetrable. Some models involve anisotropic filtration parameters. The formation properties for fluid loss and influx in such models may significantly differ.

One of the main functions of drilling fluid is to hold the wellbore pressure greater than the formation pressure, thus preventing an influx (or so-called kick) of formation fluids into the well. Dynamic numerical simulation and analyzing the hydrodynamic properties of drilling fluid allows detecting a kick and developing effective well control strategies; see, for example, Manikonda and Kaushik [104].

The mathematical model for the formation–wellbore mass transfer can have different levels of complexity. According to the number and quality of available physical experiments and measurements, it can be made more complicated and therefore more difficult to implement for the complex multiphysical calculations.

An overview of the relevant studies is presented by Policarpo in [105]. In this study, the pressure decay measurements are used to estimate the mass transfer of gas (methane or carbon dioxide) into the liquid phase. The transient-state equilibrium diffusion model is used. The transient fully coupled model for the two-phase flow of CO_2 and water-based fluid in a wellbore is presented by Xinxin et al. in [106]. The authors consider the mass and heat transfer and dynamic coupling between the wellbore and reservoir. The integrated model is solved with a fully implicit scheme with constant space steps and varying time steps. The gas transport caused by diffusion with filtrate flow into the formation (against the gas transport) was considered by Bodwadkar et al. in [107]. The diffusion coefficients are determined with physical experiments in the laboratory at various temperatures and pressures. The authors show that the gas flux drops at some pressure differential threshold value, irrespective of the temperature.

The use of complex mathematical models of filtration requires time and computational costs, as well as the availability of an experimental base for finding the parameters of the model and its verification (see, for example, Chevalier et al. [108]). Simpler models, by contrast, are empirical and have lower accuracy.

6. Heat Exchange between the Well and the Surrounding Soil Massive

The depth of wells often reaches 5 km and sometimes even up to 10 km. Moreover, with a shortage of surface deposits of petroleum products in the future, it is possible that the average well depth will increase up to 15 km or more. For example, the Deepwater Horizon well sits at more than 10 km beneath the surface. The oil well known as Z-44 Chayvo goes over 12 km into the ground. The temperature of the rocks surrounding the wellbore at this depth reaches 300–400 °C and becomes one of the main causes of various complications and accidents during the construction of the well. Under the influence of high temperature, the rheological properties of drilling and cementing fluid, the composition and parameters of formation fluids and rocks, and the operating conditions of submersible motors and pipes, drill bits, and surface equipment change dramatically.

The well itself is a complex heat exchange system. The main heat exchanger in the well is the rock formation that is unbounded in the radial direction and has its natural temperature field. The drilling fluid is pumped into the well through the drill pipes at a certain initial speed; it moves down to the bit, washing the drill pipes from the inside (and, accordingly, exchanging heat with them). Coming out of the bit holes, the drilling fluid rises along the annular gap, contacting both the rock (or casing) and drill pipes. At the beginning of the ascent from the bit, the fluid is heated from the rock, the temperature of which at the well bottom can be quite significant. In addition, rising through the drill pipes the drilling fluid gives off some of the heat to the drilling fluid moving downward inside the drill pipes. At a certain depth, the temperature of the rising drilling fluid becomes equal to and then higher than the temperature of the rock. After that, the fluid gives off heat not only to the fluid moving downward but also to the rock formation.

There are two main approaches to modeling the heat transfer in a well: analytical and numerical. Examples of analytical models can be found in [109] by Holmes and Swift. Such models are usually simple to calculate. However, they are extremely approximate and do not consider many important factors that can significantly affect the temperature distribution in the wellbore and the rock formation. The approximate formulas obtained within the framework of such analytical models can be very useful in the initial analysis of thermal effects in the well. Although, their application in more accurate calculations is difficult due to many restrictions and assumptions made during their construction.

A fairly large number of studies have been devoted to the numerical models of heat transfer in the wellbores [110–113]. Many widely used models share a common approach to the choice of basic equations. As a rule, it is assumed that the diameter and thickness of the drill pipes and the thickness of annular space are negligible compared to the depth of the well. There is also a reason to assume that the temperature does not change in the circumferential direction. This makes it possible to consider a one-dimensional heat transfer model in the well, where the temperature of the drilling fluid in the pipes and annular space, as well as the temperature of the drill pipes, depend only on the depth of the well and on time.

The temperature in the rock formation changes not only in depth but also along the radius, especially near the well. That is why a two-dimensional model is often considered for a rock formation, and therefore a two-dimensional heat equation is solved.

In general, the mathematical models available in the literature have much in common. The differences lie in the details: whether energy release is considered or not; whether a one-dimensional or two-dimensional model is used for the rock formation; whether the fluid losses to the formation are taken into account; whether the presence of casing and cement is accounted for or not; and so on.

7. Multiphysical Model of the Drilling Process

The present section describes the authors' experience in the modeling of the well drilling process. The developed model drilling complex consists of several submodels corresponding to different phenomena: drill string dynamics; the circulation of drilling fluid; filtration in the annular space; and heat exchange and soil crushing into particles of different sizes. All submodels are combined and exchange data during simulation. Specially developed in-house software was used for simulation by applying methods from different branches of computational mechanics.

The drilling fluid was non-Newtonian multiphase liquid with complex rheology. It contained solid particles and gas inclusions. The process of liquid infiltration to/from formation layers was accounted for during the drilling fluid flow simulation because this phenomenon affects both the flow rate and physical properties of the fluid. In turn, the simulation of liquid infiltration needs to consider the formation of filter cakes, the anisotropy of formation layers, and so on. The temperature of formation layers dramatically increases during well deepening. Therefore, the process is highly non-isothermal, and heat exchange with formation also needs to be accounted for. The drilling fluid transports cutting particles of different size, density, and shape, and the drilling speed and size distribution of the particles also depend on the load and moment in the drill string as well as on its rotation speed.

The implemented drilling process computation procedure is iterative. Several computation modules are executed on each time step [114,115]. The global time step is defined by the computation control system and the internal time step is defined individually for every computational module as some fraction of a global step, because of different calculation approaches used in each module. Input/output data for all computational modules form a unified data structure; thus, the computation results of one module are transferred as an input data for another as it is shown in Figure 4. The computational modules, which form the global computational model, are drilling fluid flow, heat exchange, mass transfer, drill string dynamics, and drilling (rock crushing).

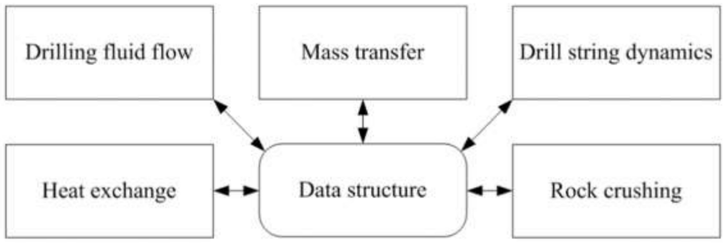

Figure 4. The combined model structure.

In the drilling fluid module, a two-phase flow model [116–118] based on the mixture approach [119] was implemented. The time-dependent equations were solved to calculate the volume fractions and velocities of phases (solid and liquid) as well as the mixture pressure. The one-dimensional flow model in the well was used, which is fully applicable due to high elongation of the flow channel [120–122].

Three types of formation were considered for modeling the interaction of the bit and the drilled formation layers: rock, sand, and mixed. The rock crushing with a bit was simulated with Markov's branching process [123], the sand particles remained as they were, and the drilling of the mixed soil type was simulated by a combination of these two approaches. The drill penetration rate was computed according the Rehbinder law [124]. The corresponding module calculates the penetration rate and distribution of the particles by the size to be used in the simulation of their transportation by the drilling fluid to the surface. A more detailed description of the combination of the first two modules can be found in [125].

The module of heat exchange was based on solving the energy transfer equation inside the drill string and the annulus. The heat equation was solved for pipe walls or casing and the surrounding formation [111,113]. The unknowns were the temperature fields of drilling fluid, drill string, and surrounding formation.

The mass transfer module was used to simulate the infiltration of drilling fluid into/from the open walls of the well. The continuity equations and equations of state closed by Darcy or nonlinear filtration laws [126,127] were solved numerically [128] to obtain the pressure gradients and the rate of drilling fluid filtrate invasion. Porous and fracturing rocks were considered.

The Bernoulli–Euler beam finite elements and a general alpha method were used for the simulation of drill string dynamics in the corresponding module [129–131]. Bending, longitudinal, and torsional oscillations were simulated regarding contact interaction with walls. The friction, drag, inertia, and damping of the drilling mud were also considered.

An important condition in the selection and development of the models within the framework of this project was the need to build such a scheme that could be easily used in engineering analysis and calculations on standard personal computers. One of the main criteria for the successful implementation of the calculation scheme in engineering practice is computational time. In the process of developing the engineering solutions, it is always necessary to test many options. Therefore, if the calculation within the framework of the constructed model takes significant computational time, such a calculation scheme will be poorly applied in practice. When developing the model, special attention was paid to finding a balance between the adequacy of the physical description of considered processes and computational efficiency.

8. Conclusions

Hundreds of works are devoted to modeling the processes that take place during well drilling. However, they usually consider local effects, for example, wear of the bit cutting parts. Or, vice versa, the process is simulated in the entire well, for example, the drilling fluid flow, but with significant simplifications—the movement of the drill string is not considered. This happens because even the modeling of each process itself is a very complex and time-consuming task with many nuances. However, all the described processes strongly influence each other. Table 1 shows some of the works that consider simultaneously the modeling of two or more physical processes in a well.

Table 1. Complex modeling of well drilling processes.

Article	Drill String Dynamics	Drilling Fluid Circulation	Drill Bit Interaction with Rock	Fluid Exchange with Soil (Mass Transfer)	Heat Exchange with Soil
Liao et al. [132], 2021	−	+	−	+	+
Ritto et al. [133], 2009	+	+	−	−	−
Zhang et al. [134], 2019	−	+	−	−	+
Guo et al. [135], 2017 Yang et al. [136], 2022	−	+	−	+	+
Tengesdal et al. [86], 2021	+	−	+	−	−
Lupuleac et al. [114], 2017 Churilova et al. [115], 2020	+	+	+	+	+

However, in industrial applications, it is required to consider all the processes that occur during drilling at once, wherein a simulation should not take much computational time, since it is usually used for multivariate calculations and optimization. To fulfill both these requirements is highly challenging.

The complex model of the well drilling process considered in Section 7 can be called multiphysical. This class includes models in which phenomena of different natures are simultaneously considered: hydrodynamics, dynamics of deformable structures, heat, and mass transfer, etc. This circumstance makes it very difficult to develop such models for the following two reasons:

- Expertise in each of the areas under consideration is needed for solving these problems. It is often difficult to find such expertise in one scientific group.
- The computation time of the numerical solution of each of the considered subproblems should be comparable. Otherwise, the simulation does not make practical sense, since in practice it is always required to carry out multiple calculations corresponding to the various scenarios under consideration.
- However, there is one important circumstance that facilitates the numerical solution of the problem under consideration. The mutual influence of the considered processes on each other is not so strong that special iterative procedures (e.g., Aitken's method [137]) are required to ensure the convergence of the numerical process at each time step. It is enough to provide the data exchange between separate solvers at each time step.
- The suggested multiphysical model is a first step on a way to model the complex physical processes in the well and their impact on each other. This topic needs further research to include more specific physical aspects in the model, for example, the wear of the bit cutting tools while leaving the model applicable for engineering calculations that are not very time-consuming to perform.

Note also that in the present study the main emphasis is on modeling the processes that take place in the well itself during drilling. At the same time, if necessary, it is also possible to use advanced coupled models of processes occurring in the reservoir, surface equipment, etc. Studies devoted to these approaches are beyond the scope of this review.

Author Contributions: Conceptualization, N.S. and S.L.; methodology, S.L.; software, M.C.; investigation, S.L. and M.C.; resources, N.S.; writing—original draft preparation, M.C.; writing—review and editing, N.S. and S.L.; supervision, N.S.; project administration, S.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was partially funded by the Ministry of Science and Higher Education of the Russian Federation under the strategic academic leadership program 'Priority 2030' (Agreement 075-15-2021-1333 dated 30 September 2021).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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