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Efficient Realization for Third-Order Volterra Filter Based on Singular Value Decomposition

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Abstract: Nonlinear distortion in loudspeaker systems degrades sound quality and must be properly compensated for by linearization techniques. One technique to reduce nonlinear distortion is to use a Volterra Filter, which approximates the nonlinearity of the target loudspeaker using the Volterra series expansion. In general, the Volterra Filter is computationally very expensive, and the amount of computation needs to be reduced for real-time processing. In this paper, we propose an efficient implementation of the third-order Volterra filter based on singular value decomposition. The proposed method determines the necessary coefficients based on the symmetry of the third-order Volterra filter and applies singular value decomposition to them. In the filter structure consisting of singular values and their corresponding singular vector, the computational complexity of the third-order Volterra filter can be reduced by eliminating the part of the filter with small singular values. By focusing on the magnitude of the singular values, the proposed method can improve the computational efficiency of the third-order Volterra filter without decreasing its approximation accuracy. Simulation results show that the proposed method can improve the computational efficiency by 60% while maintaining the nonlinear distortion compensation performance of the micro-speaker for smartphones by about 8 dB.

Keywords: nonlinear signal processing; volterra filter; compensation of nonlinear distortions; singular value decomposition; micro-speaker

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1. Introduction

The performance of an acoustic or audio system is highly dependent on the performance of the loudspeaker located at the final stage of the system. Generally, the performance of a loudspeaker is evaluated by its frequency response, especially its amplitude response, but it is not sufficient to evaluate the loudspeaker assuming it is a linear system. This is because nonlinearities in loudspeakers become more pronounced as the diaphragm is driven at higher amplitudes to increase the sound volume. The details of loudspeaker nonlinearity are described in the literature [1]. The two types of nonlinearity are: the nonlinearity caused by the magnetic flux density changing with amplitude as the diaphragm is driven at large amplitudes and the nonlinearity caused by the stiffness of the edges and dampers that support the diaphragm changing with amplitude as the diaphragm is driven at large amplitudes.

Loudspeaker nonlinearities generate nonlinear distortions that degrade sound quality. Here, nonlinear distortion is classified into harmonic distortion, in which frequency components of integer multiples of the input frequency are generated, and intermodulation distortion, in which frequency components of the sum and difference of multiple input frequencies are generated. In particular, second- and third-order nonlinear distortions are dominant in loudspeakers, and reducing these nonlinear distortions is essential for improving sound quality. To compensate for nonlinear distortions, it is important to accurately model the nonlinearities of the loudspeaker.

A mirror filter [2,3] that relies on the mechanism that generates nonlinearity based on the nonlinear motion equation of the loudspeaker has been proposed as a method of modeling the nonlinearity of the loudspeaker. The mirror filter also reduces the nonlinear distortion generated in the loudspeaker, and can be implemented using a nonlinear IIR filter [4–7], which requires a low amount of computation. However, nonlinear compensation using a mirror filter or nonlinear IIR filter is based on the nonlinear motion equation of the loudspeaker and can only compensate for nonlinear distortion caused by the nonlinearity expressed in the motion equation. As a result, the compensation performance is known to be quite limited and the intermodulation distortion cannot be compensated [8].

On the other hand, a method has been proposed by assuming the loudspeaker as a general nonlinear system and approximating its nonlinearity by the Volterra series expansion [9], which is an extension of the Theler expansion to systems with memory and is composed of Volterra kernels that represent the impulse response of a linear system [10]. It is composed of Volterra kernels that represent second-, third-, and higher-order nonlinearities, corresponding to the impulse response of a linear system. The Volterra filters [11,12], which realize these Voltera kernels as digital filters, can be used to model the nonlinearities of loudspeakers. Here, the coefficients of the Volterra filter are identified using the adaptive Volterra filter [13–15] or frequency response method [16]. The Volterra filter coefficients obtained by modeling can then be used to construct a linearization system [17,18] to compensate for nonlinear distortion and placed in the front stage of the loudspeaker to improve sound quality. The Volterra filter can be used to compensate for nonlinear distortion not only in loudspeakers, but also in parametric array loudspeakers [19–21] using ultrasonic waves because the nonlinear distortion compensation is independent of the cause of the nonlinear distortion [22].

However, Volterra filters are very computationally expensive because of their multi-dimensional filter structure. In particular, the third-order Volterra filter used for compensation of third-order nonlinear distortions has a three-dimensional filter structure, which requires a very large amount of computation. Therefore, parallel-cascade truncated Volterra filters [23,24], tensor decomposition realization [25], diagonalization [26], and subband realization [27–31] have been proposed as efficient implementations of third-order Volterra filters, respectively. However, in the parallel cascaded truncated Volterra filter, some of the parameters used to design the approximate model of the target third-order Volterra filter are determined by adaptive signal processing, and the accuracy of the approximate model is reduced due to the adaptation error. In addition, the tensor decomposition realization and diagonalization do not take into account the symmetry of the Volterra kernel, which limits the improvement in computational efficiency. Furthermore, in improving computational efficiency in subband realization, there is a problem that it is not guaranteed to reduce computational complexity because it strongly depends on the nonlinearity of the target loudspeaker.

In this paper, we propose an efficient realization method for third-order Volterra filters based on singular value decomposition. Using the symmetry of the target third-order Volterra filter, the proposed method decomposes the filter into several 2D filters of different shapes and performs singular value decomposition on each 2D filter. The resulting singular values and their corresponding singular vectors are then used to construct each 2D filter. By removing the filter sections with small singular values, the approximation accuracy of the original third-order Volterra filter is not degraded and the computational efficiency is improved.

This paper is organized as follows. In Section 2, a Volterra filter and the linearization system to compensate for nonlinear distortions are explained. Then, the proposed efficient realization method for the third-order Volterra filter is described in Section 3. Simulation results are given in Section 4 to demonstrate the effectiveness of the proposed efficient realization and a conclusion is given in Section 5.

2. Linearization System for Loudspeaker

Loudspeakers increase volume as well as nonlinear distortion. This is due to the fact that as the diaphragm amplitude increases, the voice coil is displaced from the magnetic flux formed by the permanent magnet, causing the magnetic flux density of the voice coil to change depending on the diaphragm amplitude, and the stiffness of the edges and dampers that support the diaphragm to change depending on the diaphragm amplitude. Therefore, the loudspeaker must be modeled as a nonlinear system. Since the nonlinearity in a loudspeaker has a memory, like the impulse response in a linear system, it can be approximated by a Volterra series expansion [9] that can accurately model a nonlinear system with a memory. Then, the Volterra kernels representing each term in the Volterra series expansion are represented as multidimensional digital filters, which are called Volterra filters [13], and are one of the representative examples of nonlinear digital filters. Now, if the nonlinearity of the nonlinear system under consideration can be terminated up to the third order, in other words, if the nonlinearity above the fourth order is relatively small and negligible, the nonlinear input–output relation of the Volterra filter can be described as:

$$\begin{aligned} y(n) &= \sum_{k_1=0}^{N-1} h_1(k_1)x(n-k_1) \\ &+ \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} h_2(k_1,k_2)x(n-k_1)x(n-k_2) \\ &+ \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\sum_{k_3=0}^{N-1} h_3(k_1,k_2,k_3)x(n-k_1)x(n-k_2)x(n-k_3), \end{aligned} \tag{1}$$

where $x(n)$ and $y(n)$ are input and output signals with discrete time n, N is the memory length of the Volterra filter, and $h_1(k_1)$, $h_2(k_1,k_2)$, and $h_3(k_1,k_2,k_3)$ are the filter coefficients of the first-, second-, and third-order Volterra filters, respectively. From (1), the second term is represented by a 2D digital filter and the third term is represented by a 3D digital filter. Hence, as the computation of the third term requires huge computational cost, the efficiency realization is required. In general, Volterra filters (kernels) have symmetry, for example, the symmetric property for the third-order Volterra filter is given by:

$$\begin{aligned} h_3(k_1,k_2,k_3) &= h_3(k_1,k_3,k_2) \\ &= h_3(k_2,k_1,k_3) \\ &= h_3(k_2,k_3,k_1) \\ &= h_3(k_3,k_1,k_2) \\ &= h_3(k_3,k_2,k_1). \end{aligned} \tag{2}$$

A block diagram of the linearization system that compensates for nonlinear distortion in a loudspeaker is shown in Figure 1. $H_1(z)$, $H_2(z_1,z_2)$, and $H_3(z_1,z_2,z_3)$ shown in the loudspeaker section in this figure are the first, second, and third order responses of the loudspeaker, respectively. Note that these are expressed in the form of multidimensional z-transformed Volterra filters for each order in (1). Furthermore, $\hat{H}_2(z_1,z_2)$ and $\hat{H}_3(z_1,z_2,z_3)$ in the linearization system section represent the multidimensional z transform of the pre-identified second- and third-order Volterra kernels of the loudspeaker, respectively. Furthermore, $\hat{H}_1^{-1}(z)$ is the transfer function of the approximate linear inverse filter of the identified first-order Volterra kernel (linear response) of the loudspeaker, which is designed to satisfy the following condition:

$$H_1(z) \cdot H_1^{-1}(z) = z^{-\Delta}, \tag{3}$$

where Δ is called the inverse delay and generally set to half the filter length of $\hat{H}_1^{-1}(z)$. Second- and third-order nonlinear distortions in loudspeakers can be fully compensated for by the linearization system if the second- and third-order Volterra kernels are identified with high accuracy and the approximate linear inverse filter is properly designed.

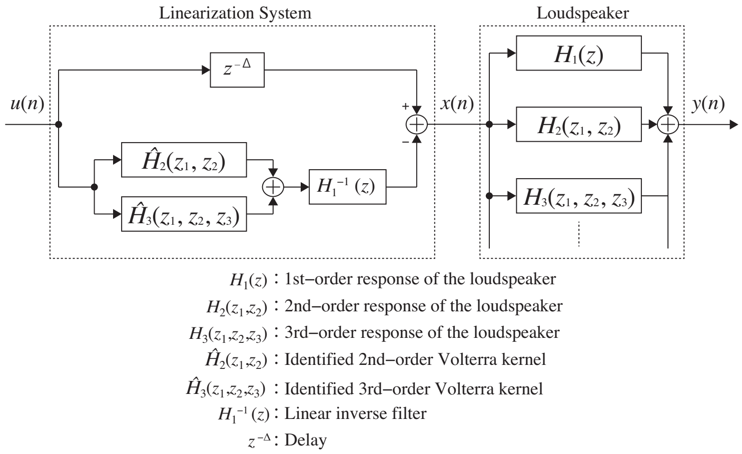

Figure 1. Block diagram of the 3rd-order linearization system for loudspeakers with higher-order nonlinearity.

3. Efficient Realization for Third-Order Volterra Filter Using Singular Value Decomposition

In this section, we describe the proposed efficient realization of the third-order Volterra filter based on singular value decomposition. Figure 2 shows a schematic diagram to understand the proposed realization method. Figure 2a shows all elements of the third-order Volterra kernel when the memory length is $N = 4$. It can be seen that the third-order Volterra kernel is a three-dimensional digital filter. Next, since the third-order Volterra kernel has symmetry given by (2), the elements required to compute the output signal of the third-order Volterra filter, denoted by the third term in (1), are shown in Figure 2b. Then, Figure 2c shows the four 2D elements obtained by slicing the structure shown in (b) in the l direction. This operation results in the decomposition of a three-dimensional filter structure into multiple two-dimensional filter structures. Thus, the l-layer 2-D filter can be represented by the following matrix:

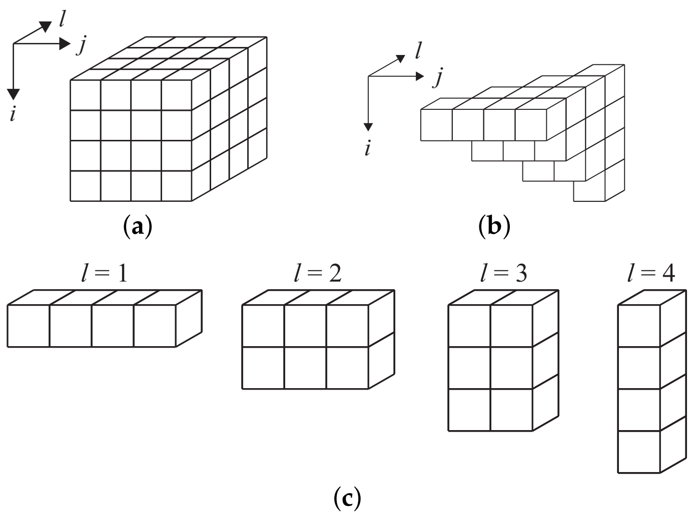

Figure 2. Third-order Volterra kernel considering its symmetry characteristic and its separated layers. (**a**) All elements ($N = 4$). (**b**) Elements required for convolution. (**c**) Elements of each layer.

$$\mathbf{H}_{3,l} = \begin{pmatrix} h_3(0,l-1,l-1) & h_3(0,l,l-1) & \cdots & h_3(0,N-1,l-1) \\ h_3(1,l-1,l-1) & h_3(1,l,l-1) & \cdots & h_3(1,N-1,l-1) \\ \vdots & \vdots & \ddots & \vdots \\ h_3(l-1,l-1,l-1) & h_3(l-1,l,l-1) & \cdots & h_3(l-1,N-1,l-1) \end{pmatrix}. \quad (4)$$

Then, the input–output relation equation of the l-layer 2D filter is:

$$y_{3,l}(n) = \sum_{k_1=0}^{l-1} \sum_{k_2=l-1}^{N-1} h_3(k_1,k_2,l-1)x(n-k_1)x(n-k_2), \quad (5)$$

or,

$$y_{3,l}(n) = \mathbf{x}^T(n)\mathbf{H}_{3,l}\mathbf{x}'(n), \quad (6)$$

where $\mathbf{x}(n)$ and $\mathbf{x}'(n)$ are the input signal vectors represented by:

$$\mathbf{x}(n) = [x(n)\ x(n-1)\ \cdots\ x(n-l+1)]^T, \quad (7)$$
$$\mathbf{x}'(n) = [x(n-l+1)\ x(n-l)\ \cdots\ x(n-N+1)]^T. \quad (8)$$

Thus, the output of the third-order Volterra filter is:

$$y_3(n) = \sum_{l=1}^{N} y_{3,l}(n)x(n-l+1). \quad (9)$$

In the proposed realization, these matrices $\mathbf{H}_{3,l}$ are decomposed by singular value decomposition as follows:

$$\mathbf{H}_{3,l} = \sum_{m=1}^{M} \sigma_{m,l}\mathbf{u}_{m,l}\mathbf{v}_{m,l}^T, \quad (10)$$

where M is the rank of $\mathbf{H}_{3,l}$, $\sigma_{m,l}$ is the l-layer mth singular value, $\mathbf{u}_{m,l}$ is a left singular vector, and $\mathbf{v}_{m,l}$ is a right singular vector, respectively. Hence, the l-layer output is given by:

$$y_{3,l}(n) = \sum_{m=1}^{M} \sigma_{m,l}\left[\mathbf{x}^T(n)\mathbf{u}_{m,l}\mathbf{v}_{m,l}^T\mathbf{x}'(n)\right]. \quad (11)$$

This input–output relationship is represented by a block diagram shown in Figure 3. Figure 3 shows that this filter structure has independent paths for each singular value. Thus, even if we remove paths with small singular values, we expect to be able to model the original l-layer 2D filter with high accuracy. If only the paths involving the singular values up to the a_lth are kept and the other paths are removed, the input–output relation is:

$$y_3(n) = \sum_{l=1}^{N} y_{3,l}(n)x(n-l+1), \quad (12)$$
$$y_{3,l}(n) = \sum_{m=1}^{a_l} \sigma_{m,l}\left[\mathbf{x}^T(n)\mathbf{u}_{m,l}\mathbf{v}_{m,l}^T\mathbf{x}'(n)\right]. \quad (13)$$

The concrete procedure of the proposed implementation method is as follows.

1. Obtain the third-order Volterra kernel of the target loudspeaker using an identification method such as FRM [16].
2. Reduce the obtained third-order Volterra kernel to only the necessary components as shown in Figure 2b based on the symmetry of (2).

3. Slice the third-order Volterra kernel with only the necessary components as shown in Figure 2b into multiple two-dimensional components $H_{3,l}$, as shown in Figure 2c.
4. Apply SVD to each of the sliced 2D components (matrices) $H_{3,l}$ to obtain singular value $\sigma_{m,l}$, left singular vector $\mathbf{u}_{m,l}$, and right singular vector $\mathbf{v}_{m,l}$.
5. Based on the pre-defined reduction ratio r, leave the singular value $\sigma_{m,l}$, corresponding left singular vectors $\mathbf{u}_{m,l}$ and right singular vectors $\mathbf{v}_{m,l}$ up to the upper a_lth, and implement the configuration in Figure 3.
6. Apply the configuration in Figure 3 to the $\hat{H}_3$ block in Figure 1 to compensate for the third-order nonlinear distortion of the target loudspeaker.

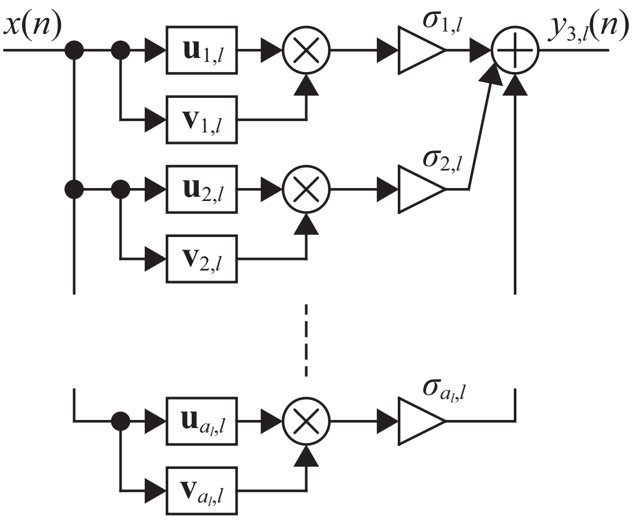

Figure 3. Block diagram of lth layer of the proposed structure based on singular value decomposition.

4. Reduction of Computational Cost and Compensation Performance for the Third-Order Harmonic Distortion

In this section, some simulation results are shown to verify the effectiveness of the proposed method. In the simulations, we first compare the path reduction in the proposed method with the compensation performance of the third-order harmonic distortion. Next, we compare the compensation performance of the third-order harmonic distortion in the case of using the parallel cascaded truncated Volterra filter [23,24] with that of the proposed method. The subject of distortion compensation is a micro-speaker for smart phones, which generally has large nonlinearities. The specifications of the target micro-speaker are shown in Table 1. Here, the third-order Volterra kernel of the micro-speaker was identified by the frequency response method [16]. Table 2 and Figure 4 show the identification conditions of the Volterra kernel and the distribution of singular values of the target third-order Volterra kernel, respectively. Figure 4 shows that there are many singular values with small values, which is expected to reduce the amount of computation while maintaining compensation performance.

Table 1. Specifications of the target micro-speaker.

Micro-speaker	HDR9267 (Hosiden)
Dimensions of target micro-speaker	$9 \times 16 \times 2.4$ mm
Rated power	0.4 W
Input impedance	7.3 Ω
Resonance frequency	1008 Hz
Enclosure type	closed-box
Inside dimensions	$26 \times 26 \times 20$ mm

Table 2. Identification conditions of the 3rd-order nonlinearity of the target micro-speaker.

Input voltage	1.2 Vrms
Sampling frequency	48,000 Hz
Frequency range	240–12,000 Hz
Tap length of 1st-order Volterra kernel	200
Tap length of 2nd-order Volterra kernel	200 × 200
Tap length of 3rd-order Volterra kernel	200 × 200 × 200

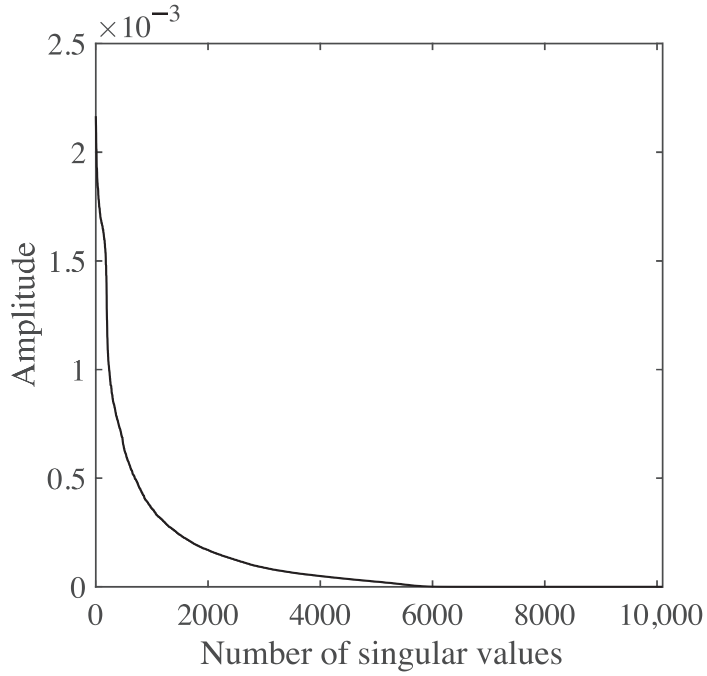

Figure 4. Distribution of singular values for the third-order Volterra kernel of the target micro-speaker.

4.1. Computational Complexity of the Proposed Method

In this section, we compare the numbers of multiplications for each realization. First, the number of multiplications for (1) considering the symmetry property given by (2) is given by:

$$\frac{N(N^2+3N+2)}{2}. \quad (14)$$

Next, the total number of multiplications for all paths obtained by singular value decomposition is given below for even and odd numbers of N, respectively. If N is even,

$$2\sum_{l=1}^{N/2}\left\{l^2+l(N-l+1)+2l\right\}+N=2\sum_{l=1}^{N/2}\{l(N+3)\}+N, \quad (15)$$

where the first term of the left-hand side indicates the total multiplications of the left singular vectors $\mathbf{u}_{m,l}$, the second term indicates the total multiplications of the right singular vectors $\mathbf{v}_{m,l}$, the third term indicates the multiplications of each singular vector output and the multiplications of singular values $\sigma_{m,l}$ in Figure 3, and the fourth term indicates the multiplications in (9), respectively. In the same way, if N is odd, we have:

$$2\sum_{l=1}^{(N-1)/2}\{l(N+3)\}+(N^2+4N+3)/2+N. \quad (16)$$

Applying the formula for the sum of arithmetic sequence to each of these expressions yields:

$$\frac{N(N^2+5N+10)}{4} \quad (N \text{ is even}), \tag{17}$$

$$\frac{N^3+7N^2+23N+9}{8} \quad (N \text{ is odd}). \tag{18}$$

Finally, we find the number of multiplications when we remove paths with small singular values, which is the proposed method. In this case, the length of the singular vector depends on the layer number (e.g., $l = 1$ and $l = 2$ in Figure 2), so the number of multiplications cannot be determined explicitly. Therefore, by assuming that r [%] of the multiplications required for all layers are removed, the total number of multiplications can be approximated as:

$$\frac{N(N^2+5N+6)}{4}(1-r/100)+N. \tag{19}$$

For the cases of $N = 200$ and r set to 0, 60, 75, and 90%, the numbers of multiplications of the proposed method for the third-order Volterra kernel are shown in Table 3. Table 3 shows that the amount of computation can be reduced to about half by applying singular value decomposition compared to computing the output of the third-order Volterra filter in (1) considering the symmetry property given by (2). The computational complexity can be further reduced by applying the proposed method, which removes paths with small singular values.

Table 3. Comparison of number of multiplications for the ratio of removed singular values.

Removed Path Ratio r [%]	# of Multiplications	Average Compensation [dB]
original (no SVD)	4,060,200	13.5 dB
0 %	2,050,500	13.5 dB
60 %	820,320	7.9 dB
75 %	512,775	3.5 dB
90 %	205,230	−1.2 dB

4.2. Compensation of Third-Order Harmonic Distortion

Next, the relationship between the reduction ratio of the amount of computation and the compensation performance of the third-order harmonic distortion is examined. By applying singular value decomposition to the identified third-order Volterra filter in the linearization system shown in Figure 1 and applying the proposed method to remove paths with small singular values, the compensation performance of the third-order harmonic distortion is evaluated through simulations. For the evaluation of the third-order harmonic distortion, a sweep sine is used as the input signal, and the sampling frequency is 48,000 Hz and the sweep frequency is 240–5760 Hz. The third-order nonlinear distortion compensation was simulated using MATLAB.

Figure 5 shows compensation results for third-order harmonic distortions. Here, the ratio of removed singular values r was set to 0, 60, 75, and 90%. Furthermore, the average compensation performance is shown in Table 3. Figure 5 shows that when the ratio of paths removed was 60%, the compensation performance for harmonic distortion up to about 6000 Hz was almost as good as when all paths were used. The harmonic distortion was also reduced for frequencies above 6000 Hz, although the compensation performance was slightly lower. Next, when the ratio of paths removed was 75% and 90%, the compensation performance for harmonic distortion up to about 4000 Hz was almost as good as when all paths were used. However, the 75% case shows little compensation for harmonic distortion above 4000 Hz, while the 90% case shows an increase in distortion above 4000 Hz. Table 3 also shows that the average amount of compensation decreased as the ratio of paths removed increased. Therefore, for the micro-speaker used in this experiment, it is reasonable to limit the reduction of paths to about 60%. However, even with a 60% reduction, the amount of calculation was sufficiently reduced, thus demonstrating that the proposed method can reduce the amount of computation while maintaining the performance of harmonic distortion compensation.

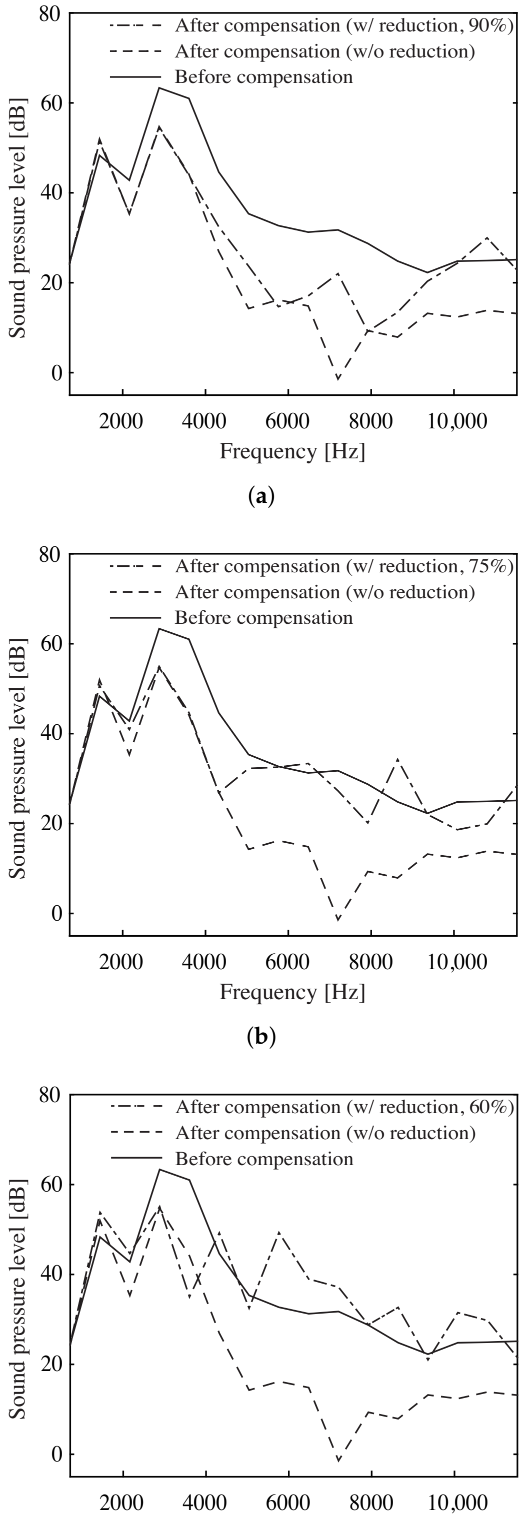

Figure 5. Simulation results for compensation of third-order harmonic distortion. (**a**) Reduction rate $r = 60\%$. (**b**) Reduction rate $r = 75\%$. (**c**) Reduction rate $r = 90\%$.

4.3. Comparison between Proposed Method and Parallel-Cascade Truncated Volterra Filter

Finally, the compensation performance was compared for the proposed method and the parallel-cascade truncated Volterra filter [23,24] via simulation. This paper focuses on nonlinear distortion compensation of micro-speakers. As shown in [10], the Volterra series expansion is an effective nonlinear model for loudspeakers from both theoretical and experimental perspectives. Therefore, this paper focuses on improving the computational efficiency of the Volterra filter and compares it with the Parallel-Cascade realization, which is a method for improving the computational efficiency of the Volterra filter, in this section. Therefore, comparisons with other nonlinear models (e.g., functional link networks with Legendre and Chebychev kernels) are out of the scope of this paper and will be discussed in the future.

The simulation conditions are the same as the previous section and the ratio of removed singular values (paths) r is set to 75%. Moreover, the number of branches of the parallel-cascade truncated Volterra filter is set to 26 so that the number of multiplications is almost the same as that for the proposed method with $r = 75\%$. Figure 6 shows that the parallel-cascade truncated Volterra filter is almost incapable of compensating for harmonic distortion with the same amount of calculations as the proposed method. On the other hand, the proposed method can compensate harmonic distortion up to approximately 4000 Hz, while reducing the amount of computation by 75%. Therefore, it has been demonstrated that the proposed method, which slices the third-order Volterra kernel into multiple layers, applies singular value decomposition to the two-dimensional filter in each layer, and removes paths with small singular values, can reduce the amount of computation while maintaining the harmonic distortion compensation performance.

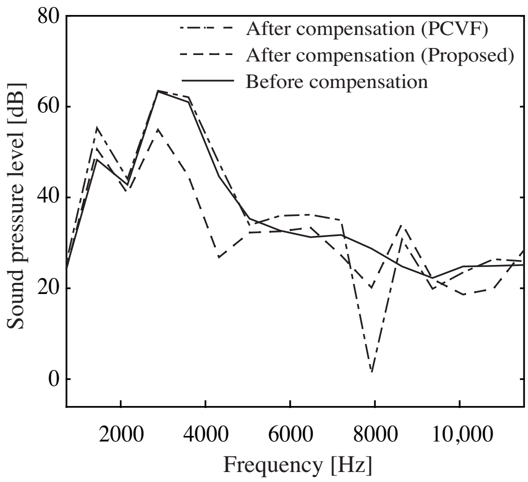

Figure 6. Comparison of compensation of third-order harmonic distortion between proposed method and parallel-cascade truncated Volterra filter.

5. Conclusions

In this paper, we proposed a method for reducing the computational complexity of third-order Volterra filters based on singular value decomposition, and evaluated its performance in reducing computational complexity and compensating third-order harmonic distortion. In the proposed method, the target third-order Volterra kernel is sliced into multiple two-dimensional filters by considering its symmetry, singular value decomposition is applied to the sliced two-dimensional filters, and the paths with small singular values are removed to achieve the reduction of computational cost while maintaining distortion compensation performance. The proposed method also achieves a reduction in computational complexity while maintaining the compensation performance of the nonlinear distortion. In addition, unlike conventional computational complexity reduction methods, the proposed method does not use adaptive signal processing, so the accuracy of the approximate model with reduced computational complexity is not degraded due to its adaptive error. Simulation results demonstrate that the proposed method has better harmonic distortion compensation performance than the parallel cascade truncated Volterra filter for the same amount of computation. Future works will include the application of the proposed method to nonlinear acoustic echo cancelers and nonlinear active noise control. In addition, it is necessary to compare the amount of computation and the performance of nonlinear distortion compensation with other nonlinear models.

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