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A Novel Synthetization Approach for Multi Coupled Line Section Impedance Transformers in Wideband Applications

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Abstract: In this paper, a novel synthetization approach is proposed for filter-integrated wideband impedance transformers (ITs). The original topology consists of N cascaded coupled line sections (CLSs) with $2N$ characteristic impedance parameters. By analyzing these characteristic impedances, a Chebyshev response can be derived to consume $N + 2$ design conditions. To optimize the left $N - 2$ variable parameters, CLSs were newly substituted by transmission lines (TLs) to consume the remaining variable parameters and simplify the circuit topology. Therefore, there are totally $2N - N - 2$ substituting possibilities. To verify the proposed approach, 25 cases are listed under the condition of $N = 5$, and 7 selected cases are compared and discussed in detail. Finally, a 75–50 Ω IT with 100% fractional bandwidth and 20 dB bandpass return loss (RL) is designed and fabricated. The measured results meet the circuit simulation and the EM simulation accurately.

Keywords: Chebyshev response; coupled line section (CLS); transmission line (TL); wideband impedance transformer

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1. Introduction

Impedance transformers (ITs) have been widely used in microwave circuits and systems as a basic circuit component [1]. A diagram of the typical end-fed antenna system is shown in Figure 1, where the IT is used to match the antenna impedance to the coax line impedance (usually 50 Ω). Many IT and filter structures have been developed in recent years [2–13], and multi-function ITs which are integrated by filters [14–20], phase shifters [21], power dividers [22], and antennas [23] have also been widely researched.

Figure 1. The diagram of typical end-fed antenna system.

In [2,3], a quarter-wave IT is presented using transmission lines (TLs) for single-band applications with a limited operating range. Several IT structures have been designed to

meet the requirements of multi-band communication systems [4–9], in which multi-section structures are an effective way to design multi-band systems and match impedance [4–7]. Inserting open/short stubs is also available to realize impedance matching [8,9].

In wideband IT applications, filter-integrated ITs have been proposed and developed in recent years to enhance the out-band suppression and reduce the circuit size [14,16–20]. In [14], the lumped elements are used to realize wideband response. However, lumped elements are limited in high-frequency applications, and the soldering will also involve undesirable parasitic effects. Instead of using lumped components, open/short stubs can be used to provide better high-frequency performance for the filtering ITs [16–20]. The equal-ripple response can be realized by lumped elements and J-inverters [24–27]. A Chebyshev polynomial is also used to constrain the equal-ripple response in filter and IT designs [28–30]. To realize DC block and reduce the circuit size, coupled line sections (CLSs) are widely used in wideband ITs and filters [31–45]. For example, two CLSs are added to each terminal of the stepped-impedance TLs symmetrically to integrate the filtering function [30].

To the best of the author’s knowledge, there still are some relationships among these transformers and filters that are not mentioned. For example, in [26,28–30,33], filters and ITs with a Chebyshev response are constituted by cascaded CLSs and TLs. Although the similar design synthesis was reported, the numbers and positions of CLSs and TLs were not clear.

In this paper, a novel synthetization approach is presented for filter-integrated multi-section wideband ITs. From the point of view of the number of variable parameters, the wideband IT, which consists of N cascaded CLSs, is provided with a Chebyshev performance, where $N + 2$ design conditions are consumed, and $N - 2$ variable parameters remain. To further optimization, CLSs are newly substituted by TLs to consume the remaining variable parameters and simplify the circuit topology. Therefore, there are $2^N - N - 2$ substituting possibilities in total for N -section ITs, and the design of the IT can be more flexible for both symmetric and asymmetric structures with the same wideband filtering performance.

2. The Principle of the Synthetization Approach

2.1. The Number of Variable Parameters of Conventional N -Section IT

Figure 2 shows the topology of N -section IT. It consists of N cascaded CLSs with the electrical length θ , where Z_{evi} and Z_{odi} are the even- and odd-mode characteristic impedances of the i th CLS, and $i = 1, 2, \dots, N$. Z_S and Z_L are two different terminal loads, and its impedance ratio k is defined as $k = Z_S/Z_L$, $Z_L = 1$ is defined in this paper; thus, $k > 1$.

Figure 2. Topology of conventional N -section IT [28].

Based on [28], the $ABCD$ matrix of the i th CLS can be summarized as

$$\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \frac{\sin \theta}{T_i} \begin{bmatrix} qS_i & \frac{j}{2} [T_i^2 + q^2(T_i^2 - S_i^2)] \\ 2j & qS_i \end{bmatrix} \quad (1)$$

where $q = \cot \theta$, and

$$S_i = Z_{evi} + Z_{odi} \quad (2)$$

$$T_i = Z_{evi} - Z_{odi} \quad (3)$$

Then, the $ABCD$ matrix of N -section filter-integrated IT can be written as

$$\begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdots \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \cdots \begin{bmatrix} A_{N-1} & B_{N-1} \\ C_{N-1} & D_{N-1} \end{bmatrix} \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix} \quad (4)$$

where

$$A_T = \sum a_p \cos^p \theta \quad (5)$$

$$B_T = \frac{j}{\sin \theta} \sum b_q \cos^q \theta \quad (6)$$

$$C_T = \frac{j}{\sin \theta} \sum c_q \cos^q \theta \quad (7)$$

$$D_T = \sum d_p \cos^p \theta \quad (8)$$

a_p and d_p are polynomial functions with degrees p . b_q and c_q are polynomial functions with degrees q . p and q defined in Equation (9). All polynomial functions can be determined by the even-and odd-mode characteristic impedances, respectively.

$$\begin{cases} p = 1, 3, 5, \dots, N & q = 0, 2, 4, \dots, N+1 \text{ when } N \text{ is odd number} \\ p = 0, 2, 4, \dots, N & q = 1, 3, 5, \dots, N+1 \text{ when } N \text{ is even number} \end{cases} \quad (9)$$

S -parameters of the N -section IT are

$$S_{11} = \frac{A_T Z_L + B - C Z_S Z_L - D Z_S}{A_T Z_L + B + C Z_S Z_L + D Z_S} \quad (10)$$

$$S_{21} = \frac{2\sqrt{Z_S Z_L}}{A_T Z_L + B + C Z_S Z_L + D Z_S} \quad (11)$$

The amplitude-squared transfer function can be expressed as

$$|S_{21}^{circuit}(\theta)|^2 = \frac{1}{1 + |F_{circuit}|^2} = \frac{1}{1 + \left| (A_T + B_T - kC_T - kD_T) / 2\sqrt{k} \right|^2} \quad (12)$$

where $F_{circuit}$ is the characteristic function of the circuit:

$$F_{circuit} = \sum (a_p - kd_p) \cos^p \theta / 2\sqrt{k} + \frac{j}{\sin \theta} \sum (b_q - kc_q) \cos^q \theta / 2\sqrt{k} \quad (13)$$

Conversely, based on the synthesis approach in [46], firstly, we define two variables: $x = \cos \phi = \alpha \cos \theta = \cos \theta / \cos \theta_c$, $y = \cos \xi = x \sin \theta_c / \sin \theta$, where θ_c is the electrical length at the lower cutoff frequency f_c of the passband, and α is a quantity used to determine the bandwidth of a bandpass filter. The Chebyshev transfer function can be written as

$$|S_{21}^{formula}(\theta)|^2 = \frac{1}{1 + |F_{formula}|^2} = \frac{1}{1 + \varepsilon^2 \cos^2(N\phi + \xi)} \quad (14)$$

where $\varepsilon = \sqrt{10^{0.1L_A} - 1}$, and L_A is the in-band ripple level factor.

The characteristic function of Chebyshev formula $F_{formula}$ can be rewritten as:

$$F_{formula} = \varepsilon \frac{\sum u_q \cos^q \theta}{\sin \theta} \quad (15)$$

From $F_{circuit} = F_{formula}$, the general design conditions of N -section IT are summarized as

$$\begin{cases} a_p - kd_p = 0 \\ (b_q - kc_q) / 2\sqrt{k} = \varepsilon u_q \end{cases} \quad (16)$$

Therefore, there are $2N$ parameters of characteristic impedances in the N -section IT, where $N + 2$ design conditions are used to derive the Chebyshev response in Equation (16), and $N - 2$ variable parameters can be designated arbitrarily.

2.2. The Proposed Synthesization Approach for Simplification

To simplify the conventional topology [28] and decrease the number of extra variable parameters, TLs were used to substitute CLSs in this paper.

In Figure 3, the i th CLS is substituted by a single TL for example, where Z_i and θ are the characteristic impedance and electrical length of the single TL. $Z_{evi} = 2Z_i$ and $Z_{odi} = 0$ must be maintained. Then, the $ABCD$ matrix of TL can be rewritten as

$$\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \begin{bmatrix} \cos \theta & j \sin \theta Z_{evi} / 2 \\ 2j \sin \theta / Z_{evi} & \cos \theta \end{bmatrix} \quad (17)$$

Figure 3. The i th CLS is substituted by a single TL.

The $ABCD$ matrix form of the total topology will not be changed. In other words, when any single CLS (i th CLS) can be substituted by a single TL, Equation (4) can also be derived, and the characteristic impedances in all polynomial functions (a_p , b_p , c_p and d_p) will be slightly different because of $Z_{evi} = 2Z_i$ and $Z_{odi} = 0$. Meanwhile, one variable parameter will be consumed, and the number of variable parameters will decrease to $N - 3$. Furthermore, the consumption of more variable parameters, different positions, and more CLSs can be substituted by TLs, where the maximum number of substituted TLs is no more than $N - 2$.

Based on the discussion above, the general procedure of the proposed synthesization approach in Figure 4 is summarized as follows: (1) according to the actual requirements, determine the section number N in the conventional IT; (2) determine the number of CLSs needing to be substituted; (3) select the substituting positions of the CLSs; (4) calculate the $ABCD$ matrix; (5) calculate all the characteristic impedances by the Chebyshev response; (6) if the results cannot meet the design requirements, go back to (2) and re-design the circuit.

Figure 4. The proposed synthesization approach for N -section IT.

In summary, for N -section IT ($N \geq 2$) in Figure 2, the basic principle of substituting TL for CLS is summarized as follows:

- (I) There are $2N$ parameters in the N -section IT [28];
- (II) $N + 2$ design conditions are used to derive the Chebyshev response; thus, $N - 2$ variable parameters can be designated arbitrarily;
- (III) Consuming one variable parameter, a single TL could substitute one CLS to maintain the Chebyshev response. In other words, maximum number of TLs will reach to $N - 2$;
- (IV) After running through all the positions of substituted TLs in every possible combination, the number of the substitutable cases are listed in Table 1, and the total number is:

$$C_N^1 + C_N^2 + \dots + C_N^{N-2} = 2^N - N - 2 \quad (18)$$

Table 1. The number of substitutable cases for N -section IT.

Number of Substituted CLSs by TLs	Number of Substitutable Combinations
1	C_N^1
2	C_N^2
...	...
$N - 3$	C_N^{N-3}
$N - 2$	C_N^{N-2}

3. Design Examples for the Proposed Synthetization Approach

Several substituting cases have been discussed in previous works ($N = 4$) [28,30]. To demonstrate the proposed synthetization approach, $N = 5$ was selected to design the N -section IT. By using the proposed synthetization approach for simplification, there were 25 cases in total. All the cases are listed and compared in this section. In order words, refs. [28] and [30] are two cases in the proposed synthetization approach; the former have not yet been reported.

3.1. Design Example for the Conventional 5-Section IT

The topology of 5-section IT is shown in Figure 5. From Equation (4), A_T , B_T , C_T and D_T can be calculated as follows:

$$A_T = a_5 \cos^5 \theta + a_3 \cos^3 \theta + a_1 \cos \theta \quad (19)$$

$$B_T = \frac{j}{\sin \theta} (b_6 \cos^6 \theta + b_4 \cos^4 \theta + b_2 \cos^2 \theta + b_0) \quad (20)$$

$$C_T = \frac{j}{\sin \theta} (c_6 \cos^6 \theta + c_4 \cos^4 \theta + c_2 \cos^2 \theta + c_0) \quad (21)$$

$$D_T = d_5 \cos^5 \theta + d_3 \cos^3 \theta + d_1 \cos \theta \quad (22)$$

Figure 5. Topology of N -section IT [28] under the condition of $N = 5$.

a_p , d_p ($p = 1, 3, 5$), b_q and c_q ($q = 0, 2, 4, 6$) are the coefficients of the polynomial functions which are determined by characteristic impedances of Z_{ev1} , Z_{od1} , Z_{ev2} , Z_{od2} , Z_{ev3} , Z_{od3} , Z_{ev4} , Z_{od4} , Z_{ev5} and Z_{od5} (the detailed equations are listed in Table A1 in Appendix A).

After the complicated arithmetical operation from Equations (10) to (13), $F_{circuit}$ can be written as

$$F_{circuit} = \frac{1}{T_1 T_2 T_3 T_4 T_5} (X_5 \cos^5 \theta + X_3 \cos^3 \theta + X_1 \cos \theta) + \frac{j}{T_1 T_2 T_3 T_4 T_5 \sin \theta} (Y_6 \cos^6 \theta + Y_4 \cos^4 \theta + Y_2 \cos^2 \theta + Y_0) \quad (23)$$

where

$$X_p = (a_p - k \cdot d_p) / 2\sqrt{k} \quad (24)$$

$$Y_q = (b_q - k \cdot c_q) / 2\sqrt{k} \quad (25)$$

Similarly, from Equations (10) to (13), $F_{formula}$ can be written as

$$F_{formula} = \varepsilon \cos(5\varphi + \xi) = \frac{\varepsilon}{\sin \theta} [u_6 \cos^6 \theta + u_4 \cos^4 \theta + u_2 \cos^2 \theta + u_0] \quad (26)$$

where

$$u_6 = \frac{16}{\cos^6 \theta_c} (1 + \sqrt{1 - \cos^2 \theta_c}) \quad (27)$$

$$u_4 = -\frac{4}{\cos^4 \theta_c} (5 + 7\sqrt{1 - \cos^2 \theta_c}) \quad (28)$$

$$u_2 = \frac{1}{\cos^2 \theta_c} (5 + 13\sqrt{1 - \cos^2 \theta_c}) \quad (29)$$

$$u_0 = -1 \quad (30)$$

From $F_{circuit} = F_{formula}$, the seven design conditions can be summarized as follows to maintain the Chebyshev response.

$$\begin{cases} a_5 - k \cdot d_5 = 0 \\ a_3 - k \cdot d_3 = 0 \\ a_1 - k \cdot d_1 = 0 \end{cases} \text{ and } \begin{cases} (b_6 - k \cdot c_6) / 2\sqrt{k} = \varepsilon u_6 \\ (b_4 - k \cdot c_4) / 2\sqrt{k} = \varepsilon u_4 \\ (b_2 - k \cdot c_2) / 2\sqrt{k} = \varepsilon u_2 \\ (b_0 - k \cdot c_0) / 2\sqrt{k} = \varepsilon u_0 \end{cases} \quad (31)$$

For the section number $N = 5$, there are 10 parameters (Z_{ev1} , Z_{od1} , Z_{ev2} , Z_{od2} , Z_{ev3} , Z_{od3} , Z_{ev4} , Z_{od4} , Z_{ev5} and Z_{od5}) corresponding to seven design conditions for the Chebyshev response; thus, three variable parameters can be utilized to simplify the circuit topology.

3.2. Design Example for Proposed Synthesization Approach

Since there were 3 variable parameters, different 25 cases could be divided into three categories, Categories I, II and III, which will be discussed in this section.

3.2.1. Category I: One CLS Is Substituted by TL

When one CLS is substituted by TL, there are two variable parameters, and the number of the substitutable cases is $C_5^1 = 5$. Any of the CLSs (first, second, third, fourth, fifth) can be substituted by TL. All the possible cases are listed in Table 2. This is the simplest category; therefore, the detailed discussion will be omitted.

Table 2. The substitutable cases in category I.

The i th CLS Substituted By TL	1st	2nd	3rd	4th	5th
Index of cases	Case 1	Case 2	Case 3	Case 4	Case 5

3.2.2. Category II: Two CLSs Are Substituted by TL

When two CLSs are substituted by TLs, there is only one variable parameter left. The number of substitutable cases is $C_5^2 = 10$, and all the cases are listed in Table 3. Case 11 is selected as an example and discussed in detail. The topology of case 11 is shown in

Figure 6. For fabrication convenience, the odd-mode characteristic impedance of the third CLS is fixed to 0.6Ω ($Z_{od3} = 0.6 \Omega$). Since the second and fourth CLSs are substituted by TLs, $Z_{ev2} = 2Z_2$, $Z_{od2} = 0$ and $Z_{ev4} = 2Z_4$, $Z_{od4} = 0$ can be derived.

Table 3. The substitutable cases in category II.

The i th CLS is substituted by TL	The i th CLS Is Substituted by TL				
	1st	2nd	3rd	4th	5th
	1st	Case 6	Case 7	Case 8	Case 9
	2nd		Case 10	Case 11	Case 12
	3rd			Case 13	Case 14
	4th				Case 15
	5th

Figure 6. The topology of case 11 in category II.

Then, the relationship among the characteristic impedances of CLSs (Z_{evi} and Z_{odi}) and TLs (Z_i), impedance ratio k (where $1.5 < k < 5.0$), and electrical length θ_c (where $30^\circ < \theta_c < 60^\circ$) are shown in Figure 7. As k increase and θ_c decrease, the coupling strengths C_i of all CLSs will become stronger, where $C_i = -20\log(Z_{evi} - Z_{odi})/(Z_{evi} + Z_{odi})$, $i = 1, 3$, and 5 . Therefore, the sum of all coupling strengths, C_Σ , will become smaller, where $C_\Sigma = C_1 + C_3 + C_5$, and the availability section is about $1.5 < k < 3.5$ and $30^\circ < \theta_c < 60^\circ$.

Figure 7. Relationships among characteristic impedance, k and θ_c of case 11 under the condition of $RL = 20$ dB. (a) Relationships about Z_{ev1} and Z_{ev5} . (b) Relationships about Z_{od1} and Z_{od5} . (c) Relationships about Z_{ev3} and k . (d) Relationships about Z_2 and Z_4 .

3.2.3. Category III: Three CLSs Are Substituted by TL

When three CLSs are substituted by TLs, there is no variable parameter. The number of substitutable cases is $C_5^3 = 10$, and all the cases are listed in Table 4. Case 21 is selected as an example and discussed in detail. The topology of case 21 is shown in Figure 8. Since the first, third, and fifth CLSs are substituted by TLs, $Z_{ev1} = 2Z_1$, $Z_{od1} = 0$; $Z_{ev3} = 2Z_3$, $Z_{od3} = 0$; and $Z_{ev5} = 2Z_5$, $Z_{od5} = 0$ can be derived.

Table 4. The substitutable cases of category III.

The i th CLS Is Not Substituted by TL	The i th CLS Is Not Substituted by TL				
	1st	2nd	3rd	4th	5th
	1st	Case 16	Case 17	Case 18	Case 19
	2nd		Case 20	Case 21	Case 22
	3rd			Case 23	Case 24
	4th				Case 25

Figure 8. The topology of case 21 in category III.

The relationships among characteristic impedances of CLSs (Z_{evi} and Z_{odi}) and TLs (Z_i), impedance ratio k (where $1.5 < k < 5.0$), and electrical length θ_c (where $30^\circ < \theta_c < 60^\circ$) are shown in Figure 9. The same results can be summarized accordingly: as k increases and θ_c decreases, the coupling strengths C_2 and C_4 will become stronger, and C_Σ will become smaller, where $C_\Sigma = C_2 + C_4$. The first, third, and fifth CLSs are substituted by TLs; therefore, the availability section is narrower than case 11, which is about $1.5 < k < 2.0$ and $30^\circ < \theta_c < 35^\circ$.

Figure 9. Cont.

Figure 9. Relationships among characteristic impedance, k and θ_c of case 21 under the condition of $RL = 20$ dB. (a) Relationships about Z_{ev2} and Z_{ev4} . (b) Relationships about Z_{od2} and Z_{od4} . (c) Relationships about Z_1 and Z_5 . (d) Relationships about Z_3 and k .

3.3. Comparison and Summary

To compare these three categories, seven selected examples are listed in Table 5 under the same design conditions ($\theta = 90^\circ$ at $f_0 = 2$ GHz, $k = 1.5$, $\theta_c = 45^\circ$ and $RL = 20$ dB). Their circuit simulation results are shown in Figure 10. The following conclusions are clearly summarized as:

Table 5. Typical simplification examples with detailed parameters, under the condition of $Z_S = 1.5$ Ω, $Z_L = 1.0$ Ω, $k = 1.5$, $\theta = 90$ at $f_0 = 2$ GHz, $\theta_c = 45^\circ$, $RL = 20$ dB.

Number of Variable Parameters	Substitutable Cases	Characteristic Impedances of the i th CLSs or TLs (Ω)					Symmetrical/Asymmetrical Topology
		1st	2nd	3rd	4th	5th	
3	Conventional topology [28] Former work	CLS $Z_{ev1} = 4.1030$ $Z_{od1} = 0.4371$	CLS $Z_{ev2} = 4.7546$ $Z_{od2} = 0.6000$	CLS $Z_{ev3} = 4.7474$ $Z_{od3} = 0.6000$	CLS $Z_{ev4} = 4.2000$ $Z_{od4} = 0.6000$	CLS $Z_{ev5} = 2.8441$ $Z_{od5} = 0.1826$	Symmetrical
2	Case 2 This work	CLS $Z_{ev1} = 3.8756$ $Z_{od1} = 0.6643$	TL $Z_2 = 1.5268$	CLS $Z_{ev3} = 3.8798$ $Z_{od3} = 0.6643$	CLS $Z_{ev4} = 3.9323$ $Z_{od4} = 0.6000$	CLS $Z_{ev5} = 2.8214$ $Z_{od5} = 0.2053$	Asymmetrical
1	Case 11 in Figure 5 This work	CLS $Z_{ev1} = 3.8447$ $Z_{od1} = 0.6953$	TL $Z_2 = 1.4813$	CLS $Z_{ev3} = 3.0487$ $Z_{od3} = 0.6000$	TL $Z_4 = 1.1271$	CLS $Z_{ev5} = 2.6067$ $Z_{od5} = 0.4200$	Symmetrical
1	Case 10 This work	CLS $Z_{ev1} = 3.7642$ $Z_{od1} = 0.7758$	TL $Z_2 = 1.1076$	TL $Z_3 = 0.9148$	CLS $Z_{ev4} = 3.0366$ $Z_{od4} = 0.6000$	CLS $Z_{ev5} = 2.7415$ $Z_{od5} = 0.2852$	Asymmetrical
1	Case 18 This work	CLS $Z_{ev1} = 3.8158$ $Z_{od1} = 0.7242$	TL $Z_2 = 1.2486$	TL $Z_3 = 1.4696$	CLS $Z_{ev4} = 5.5788$ $Z_{od4} = 1.5118$	TL $Z_5 = 1.5133$	Asymmetrical
0	Case 19 [30] Former work	CLS $Z_{ev1} = 3.7113$ $Z_{od1} = 0.8287$	TL $Z_2 = 0.7888$	TL $Z_3 = 0.4592$	TL $Z_4 = 0.6071$	CLS $Z_{ev5} = 2.4931$ $Z_{od5} = 0.5336$	Symmetrical
0	Case 21 in Figure 7 This work	TL $Z_1 = 2.2700$	CLS $Z_{ev2} = 8.5051$ $Z_{od2} = 2.1307$	TL $Z_3 = 2.8948$	CLS $Z_{ev4} = 5.8497$ $Z_{od4} = 1.2408$	TL $Z_5 = 1.5133$	Symmetrical

Figure 10. Circuit simulations of the 7 selected examples in Table 5. (a) Circuit simulations of S_{11} and S_{21} . (b) Enlarged circuit simulations of S_{21} and group delay.

(1) Although all the topologies (25 cases) are quite different, by using the proposed synthetization approach, the same Chebyshev frequency responses (S_{11} , S_{21} and group delay) can be derived.

(2) Symmetrical or asymmetrical topology will not affect the Chebyshev frequency responses, which are only determined by all the characteristic impedances of CLSs and TLs.

(3) When the category is determined, considering coupling strengths and impedance ratio, CLSs are preferred to connect the source and load terminals in symmetrical topology.

(4) More variable parameters will provide more flexible design procedures and wider realization ranges. When the number of variable parameters is zero, the most simplification topology appears with the unique solution of characteristic impedances. Although the realization range is quite limited, the circuit is highly convenient for fabrication.

4. Experimental Results

To validate the proposed synthetization approach, a $75\ \Omega$ – $50\ \Omega$ IT (case 11) was designed, fabricated, and measured, where $\theta_c = 45^\circ$, RL = 20 dB, and the center frequency $f_0 = 2$ GHz. The characteristic impedances and the physical parameters of the fabricated IT are listed in Tables 6 and A1.

Table 6. Parameters of the fabricated IT.

Characteristic Impedance (Ω)	Coupling Strength (dB)	Physical Parameters (mm)
$Z_{ev1} = 192.23$ $Z_{od1} = 34.77$	3.18	$W_1 = 4.0$ $L_1 = 27.5$ $S_1 = 10.8$
$Z_2 = 69.06$		$W_2 = 1.4$ $L_2 = 27.0$
$Z_{ev3} = 152.43$ $Z_{od3} = 30.00$	3.46	$W_3 = 4.9$ $L_3 = 26.7$ $S_3 = 8.4$
$Z_4 = 56.35$		$W_4 = 2.1$ $L_4 = 26.0$
$Z_{ev5} = 130.33$ $Z_{od5} = 20.10$	2.82	$W_5 = 7.6$ $L_5 = 26.4$ $S_5 = 10.3$

The slot-coupled directional (SCD) structure was used in the circuit fabrication. The 3D view of the proposed IT is shown in Figure 11. The structure was constructed on the Roger's RT/duroid 5880 substrate with a dielectric constant (ϵ_r) of 2.2, a loss tangent ($\tan\delta$) of 0.0009, and a thickness of 0.787 mm.

Figure 11. The 3D view of the proposed impedance transformer.

Figure 12a,b shows the front- and back-side photographs of the fabricated IT. Its circuit simulation, EM simulation, and measured results are shown in Figure 13. The S_{11} of EM simulation and measured results were lower than -17.1 dB and -14.5 dB in the passband, respectively. The errors between the EM simulation and measured results were mainly caused by fabrication error and parasitic effect around the terminal $75\ \Omega$ resistor. Four reflection zeros within the passband can clearly be observed in the measurement results.

Figure 12. Photographs of the fabricated transformer. (a) Front-side view; (b) back-side view.

Figure 13. Circuit simulation, EM simulation and measured results.

5. Conclusions

In this paper, a novel synthetization approach for the N -section IT has been proposed. Based on the principles of this proposed approach, TLs could substitute CLSs for flexible circuit design, simplified circuit topology, and easy fabrication; moreover, the same Chebyshev response will be maintained. Furthermore, through analyzing the relationship between the number of variable parameters and the number of design conditions, at most, $N - 2$ CLSs can be substituted by TLs, and the substituting positions can be selected flexibly. Therefore, both symmetric and asymmetric substituting structures can be realized. To validate the proposed theory, a $75\ \Omega$ – $50\ \Omega$ filtering IT was fabricated and measured. The measured results met the circuit simulation and the EM simulation effectively. This work has proposed a novel synthetization approach for IT design, and a detailed mathematical approach has been presented, which can be significant in IT and the design of other components. However, parts of the circuits were not easy to fabricate because of the strong coupled strength. In our future works, we will focus on changing the structure to decrease

the coupled strength and provide extra transmission zeros to improve the selectivity of the IT.

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Appendix A

For the 5-section IT [28], the detail parameters of a_p , b_q , c_q and d_p are summarized and listed in Table A1.

Table A1. Detailed parameter for the 5-section IT [28].

$a_5 = \frac{1}{T_1 T_2 T_3 T_4 T_5} [S_1 (S_1 + S_2) (S_2 + S_3) (S_3 + S_4) (S_4 + S_5)]$ $a_3 = -\frac{1}{T_1 T_2 T_3 T_4 T_5} [(S_2 + S_3) (S_3 + S_4) (S_4 + S_5) T_1^2 + S_1 (S_3 + S_4) (S_4 + S_5) T_2^2 + S_1 (S_1 + S_2) (S_4 + S_5) T_3^2 + S_1 (S_1 + S_2) (S_2 + S_3) T_4^2]$ $a_1 = \frac{1}{T_1 T_2 T_3 T_4 T_5} [S_1 T_2^2 T_4^2 + (S_2 + S_3) T_1^2 T_4^2 + (S_4 + S_5) T_1^2 T_3^2]$
$b_6 = -\frac{1}{2 T_1 T_2 T_3 T_4 T_5} [S_1 S_5 (S_1 + S_2) (S_2 + S_3) (S_3 + S_4) (S_4 + S_5)]$ $b_4 = \frac{1}{2 T_1 T_2 T_3 T_4 T_5} [S_5 (S_2 + S_3) (S_3 + S_4) (S_4 + S_5) T_1^2 + S_1 S_5 (S_3 + S_4) (S_4 + S_5) T_2^2 + S_1 S_5 (S_1 + S_2) (S_4 + S_5) T_3^2 + S_1 S_5 (S_1 + S_2) (S_2 + S_3) T_4^2$ $+ S_1 (S_1 + S_2) (S_2 + S_3) (S_3 + S_4) T_5^2]$ $b_2 = -\frac{1}{2 T_1 T_2 T_3 T_4 T_5} [(S_2 + S_3) (S_3 + S_4) T_1^2 T_5^2 + S_5 (S_4 + S_5) T_1^2 T_3^2 + S_5 (S_2 + S_3) T_1^2 T_4^2 + S_1 (S_1 + S_2) T_3^2 T_5^2 + S_1 (S_3 + S_4) T_2^2 T_5^2 + S_1 S_5 T_2^2 T_4^2]$ $b_0 = \frac{T_1 T_3 T_5}{2 T_2 T_4}$
$c_6 = -\frac{2}{T_1 T_2 T_3 T_4 T_5} (S_1 + S_2) (S_2 + S_3) (S_3 + S_4) (S_4 + S_5)$ $c_4 = \frac{2}{T_1 T_2 T_3 T_4 T_5} \{ (S_1 + S_2) (S_2 + S_3) (S_3 + S_4) (S_4 + S_5) + (S_3 + S_4) (S_4 + S_5) T_2^2 + (S_1 + S_2) (S_4 + S_5) T_3^2 + (S_1 + S_2) (S_2 + S_3) T_4^2 \}$ $c_2 = -\frac{2}{T_1 T_2 T_3 T_4 T_5} [(S_3 + S_4) (S_4 + S_5) T_2^2 + (S_1 + S_2) (S_4 + S_5) T_3^2 + (S_1 + S_2) (S_2 + S_3) T_4^2 + T_2^2 T_4^2]$ $c_0 = \frac{2 T_2 T_4}{T_1 T_3 T_5}$
$d_5 = \frac{1}{T_1 T_2 T_3 T_4 T_5} S_5 (S_1 + S_2) (S_2 + S_3) (S_3 + S_4) (S_4 + S_5)$ $d_3 = -\frac{1}{T_1 T_2 T_3 T_4 T_5} [S_5 (S_3 + S_4) (S_4 + S_5) T_2^2 + S_5 (S_1 + S_2) (S_4 + S_5) T_3^2 + S_5 (S_1 + S_2) (S_2 + S_3) T_4^2 + (S_1 + S_2) (S_2 + S_3) (S_3 + S_4) T_5^2]$ $d_1 = \frac{1}{T_1 T_2 T_3 T_4 T_5} [S_5 T_2^2 T_4^2 + (S_1 + S_2) T_3^2 T_5^2 + (S_3 + S_4) T_2^2 T_5^2]$

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