

Article

Improvement of Position Repeatability of a Linear Stage with Yaw Minimization

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Abstract: Recently, due to the miniaturization of electronic products, printed circuit boards (PCBs) have also become smaller. This trend has led to the need for high-precision electrical test equipment to check PCBs for disconnections and short circuits. The purpose of this study is to improve the position repeatability of the platform unit up to $\pm 2.5 \mu\text{m}$ in linear stage type test equipment. For this purpose, the causes of the position errors of the platform unit are analyzed. The platform unit holding the PCB is driven by a single-axis linear ball screw drive system offset from its geometric center due to design constraints. The yaw rotation of the platform is found to have a dominant effect on position repeatability. To address this problem, adding balancing weights to the platform unit and adjusting the stiffness of the LM Guides are proposed. These methods reduce the yaw rotation by moving the centers of mass and stiffness closer to the linear ball screw actuator. In the verification tests, the position repeatability was decreased to less than $\pm 1.0 \mu\text{m}$.

Keywords: precision stage; balancing weight; drive force offset; yaw motion; error prediction; low-cost stage; linear motion guide (LM Guide); ANSYS bushing joint



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1. Introduction

Electrical test equipment measures the resistance of a circuit pattern on a printed circuit board (PCB) to inspect any abnormalities. An electrical inspection is performed with thousands or more probe pins concurrently positioned on the upper and lower surfaces of the PCB along the circuit checkpoints. Since the recent miniaturization and thinning of electronic products has resulted in fine-pitch patterned PCBs, there is a need for high-precision alignment of the probe pins. Lowering the manufacturing cost of the equipment is also required.

The electrical test equipment has an XY stage that moves the PCB to the position coordinates and precisely aligns the PCB with the probe pins. The stage is a module of significant importance that must meet productivity and precision standards. Figure 1 shows the schematic diagram of the XY stage.

The stage has a platform with a central opening to allow the probe pins onto the upper/lower surfaces of the PCB. A coordinate system is chosen such that the x-axis is in the length direction of the platform while the y-axis is in the width direction. The platform is driven by linear ball screw actuator systems offset from the platform's center. Due to the offsets, yaw moments are generated during the operation of the stage. The major contributor to the yaw motion is driving force offset, resulting in deteriorated precision due to the yaw angle [1]. In the stacked XY stage structure, the dynamic behavior of the supported stage (y-stage in Figure 1) is supposed to be affected by that of its supporting stage (x-stage), which is driven by its actuator system. Judging from the studies on single-stage dynamics, the interaction between the stacked stages is expected to be very complicated. In most cases,

the supported stage is much lighter than its supporting stage, as depicted in Figure 1. In this work, the subtle dynamics of inter-stage interaction are ignored and left for future study.

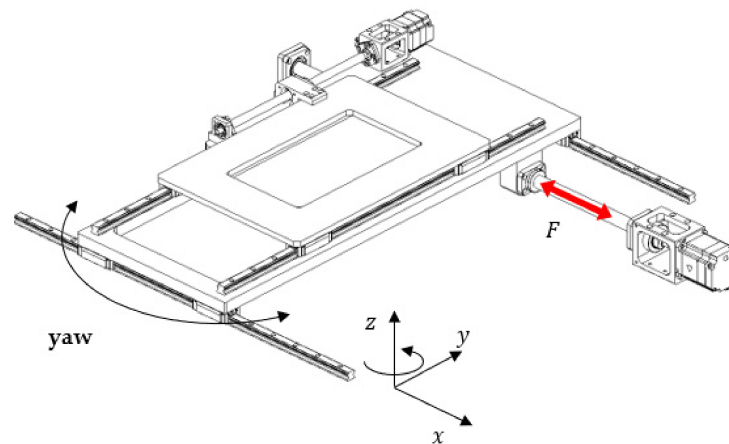


Figure 1. Scheme of offset-driven XY stage.

To reduce the yaw moment, the linear stage may be driven by synchronous control with driving units installed on both sides of the guide. Since the parallelism between the two linear motion guides (LM Guide) on both sides is imperfect, the actuator loads may vary during operation. Repeated load fluctuations may cause structural distortions over time, which ultimately may lead to jamming. Thus, the synchronous control method has many shortcomings, such as increased manufacturing and maintenance costs and complexity of control programs compared to the single motor control method.

Typically, a single-axis linear motion stage consists of a pair of LM Guides. Each rail of the configured LM Guides is exposed to shape errors, machining errors on the installation surface, and assembly errors. As a result, unintended 5-degree of freedom (DOF) error motions occur. Among these, errors in the translational motion direction can be corrected to some extent by the averaging effect, whereas errors arising from the angular motions, such as yaw, are difficult to correct [1,2].

Various studies have been conducted to analyze the error motions caused by the geometrical error of the LM Guide's rail. Major studies have simplified the balls of the LM Guide, which are the most elastic parts deformed by force, to spring models. Such simplification has been used to predict the translational error motions of stages with the LM Guide rail shape error and ball stiffness [3–16]. Linear stages for electrical test machines are designed with high acceleration and deceleration capabilities for productivity. As a result, significant yaw motion and vibration of the platform could occur. Most of the previous studies, however, employed static equilibrium equations for the platform without inertial effects.

The position accuracy of the stage may be supplemented by measuring the motion error of the platform and compensating with corrected position coordinates. For this purpose, the position repeatability of the platform is crucial [15,17,18]. However, as in the x-axis shown in Figure 1, if yaw motion occurs, the position repeatability of the platform varies depending on the magnitude and direction of the driving force (F) and the bearing friction between the platform and the rails. Since the friction is a result of complex tribological interaction between the balls and the rails subjected to statistical uncertainties [19–21], platform yaw is most unpredictable. In this case, it is extremely difficult to enhance the position accuracy even with compensation.

In this work, a rigid body dynamics model is proposed to analyze the contributions of various factors to platform yaw motion subjected to a single offset driving force. Further, a method for minimizing the yaw motion is presented along with experimental verifications of its effectiveness. This effort led to the development of a low-cost precision stage, satisfy-

ing the platform acceleration of 1 g and the position repeatability of $\pm 1.0 \mu\text{m}$ required for advanced electrical test equipment.

In Section 2, a rigid body dynamics model of the moving platform subjected to an offset driving force is described. In Section 3, contributions from the various factors on platform yaw motion are analyzed. The position error due to platform yaw is predicted and improved by a new XY stage design in Section 4. The improved platform position repeatability with optimized centers of mass and stiffness is verified with a test bed in Section 5.

2. Dynamic Model for Moving Platform

The single-axis stage has a pair of LM Guides jointed to a platform unit driven by a linear motor or an assembly consisting of a ball screw and motor. In the following discussion, the motion of the platform is represented by a rigid body dynamics model.

2.1. Yaw Error Motion

A single-axis linear stage consists of a moving platform mounted on a pair of LM Guides. The platform is driven by a linear actuator, as illustrated in Figure 2. The system may be characterized by the center of mass of the platform, the center of stiffness of the LM Guides, and the driving force offset with respect to the geometric center of the platform. In this system, the ratio of platform length (l) in the motion direction to the width (w) is very important for drive stability. Typically, $l/w > 1$ is required, and 1.6 or higher is recommended [1].

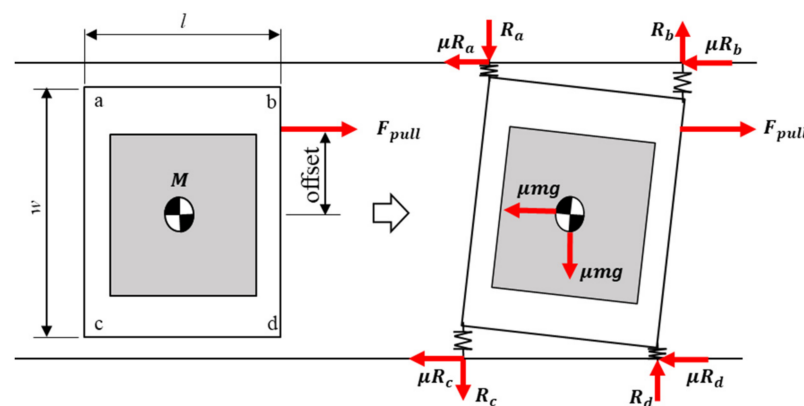


Figure 2. Scheme of an offset-driven platform mounted on two linear motion guides.

When the driving force (F) offset is significant, the yaw motion of the platform is induced by its inertia, reaction forces (R_a to R_d), and friction forces (μR_a to μR_d), leading to possible jamming.

2.2. Stiffness of LM Guide

An LM Guide is composed of a rail, one or two blocks, and balls, as shown in Figure 3a. The balls recirculate, enabling the block to perform a linear motion along the rail. The stiffness between the block and rail is analogous to a set of springs that resist external forces applied to the block in horizontal and vertical directions, as shown in Figure 3b [3–15].

Table 1 shows the stiffness data of the LM Guides as shown in THK's HSR models. The stiffness values were extracted from the catalog's experimental load vs. displacement graphs. These values are used for modeling but should generally be considered as having an uncertainty of 1–2%.

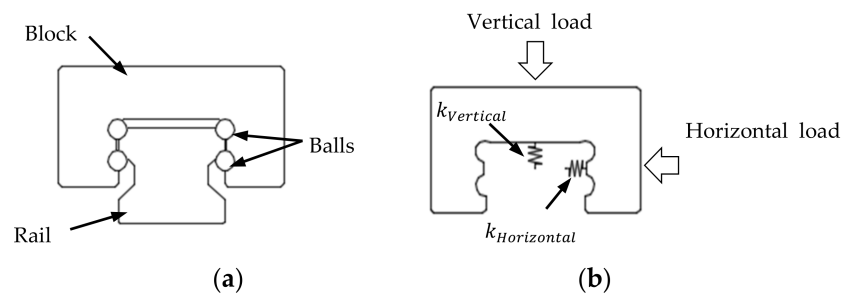


Figure 3. LM Guide (a) components and (b) stiffness elements.

Table 1. Stiffness of HSR LM Guides.

	Horizontal Stiffness (kN/mm)	Vertical Stiffness (kN/mm)	Effective Load (kN)
HSR20	250	349	1.4
HSR20L	335	472	2.3
HSR25L	335	488	2.8
HSR30L	453	645	3.8
HSR35L	495	681	5.5

2.3. Equations of Motion

A free body diagram is shown in Figure 4, presenting the reactions between the moving platform and two LM Guides. The platform guided by rail #1 and rail #2 is driven by force (F) offset from the platform's center of geometry by the distance (d).

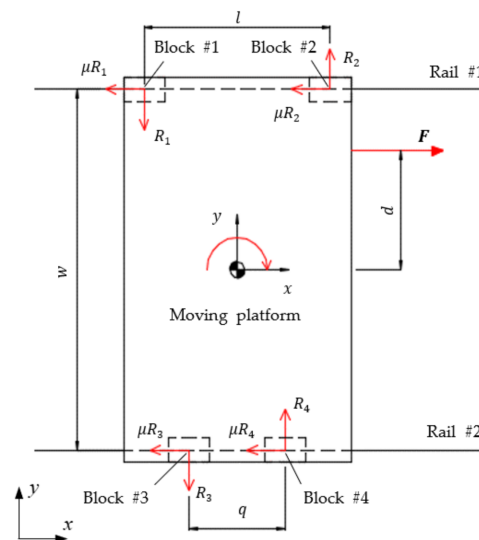


Figure 4. Free body diagram of the moving platform.

The blocks of rail #1 are numbered 1 and 2, respectively, while those of rail #2 are numbered 3 and 4, which are symmetric about the y -axis by distances of l and q , respectively. The blocks have a horizontal stiffness of k_n and a damping coefficient of c_n . ($n = 1, 2, 3, 4$).

The Cartesian coordinate system represents the movement direction and the width direction as x and y and the direction perpendicular to the x - y plane as z .

Figure 5 shows a schematic diagram of the platform rotated by an angle of θ by the driving force (F). The coordinates (x_c, y_c) and (x_p, y_p) refer to the center of mass and the center of rotation, respectively.

$$\begin{aligned}\sum F_x &= m\ddot{x}_p \\ \sum F_y &= m\ddot{y}_p \\ \sum M &= I\ddot{\theta}\end{aligned}\quad (1)$$

The force equilibrium equation for the platform is given as Equation (1), and the rotational inertia is given as Equation (2).

$$I = \int_{Platform} r^2 dm + m((x_p - x_c)^2 + (y_p - y_c)^2) \quad (2)$$

The rotational inertia (I) is calculated with respect to the rotation center. The r is the distance from the center of rotation to the point mass (dm) of the moving platform.

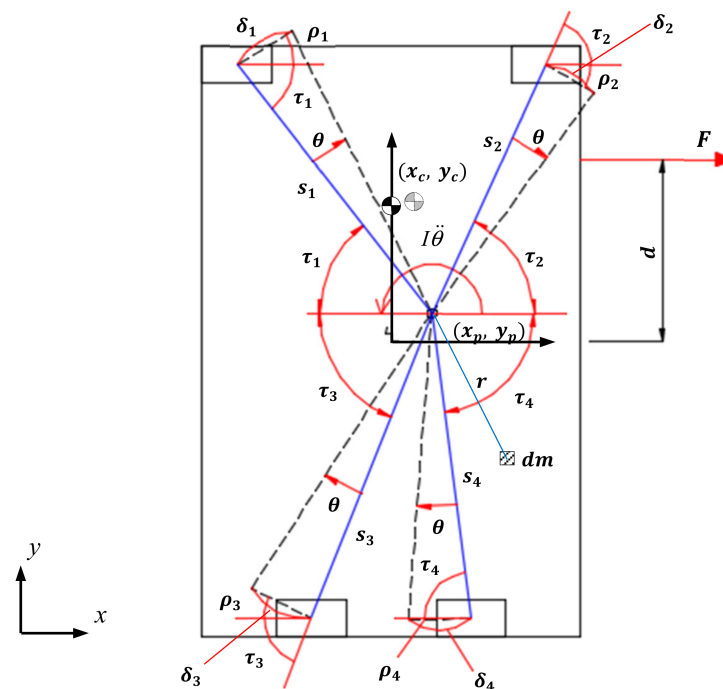


Figure 5. Schematic diagram of yaw motion.

The position of the rotational center (Pole) is a function of time, and the lengths of the lines (s_n) from the pole to the block centers are given as,

$$\begin{aligned}s_1(t) &= \sqrt{(l/2 + x_p(t))^2 + (w/2 - y_p(t))^2} \\ s_2(t) &= \sqrt{(l/2 - x_p(t))^2 + (w/2 - y_p(t))^2} \\ s_3(t) &= \sqrt{(q/2 + x_p(t))^2 + (w/2 + y_p(t))^2} \\ s_4(t) &= \sqrt{(q/2 - x_p(t))^2 + (w/2 + y_p(t))^2}\end{aligned}\quad (3)$$

while the angles (τ_n) between the lines s_n and the x -axis are given as,

$$\begin{aligned}\tau_1(t) &= \tan^{-1}\left(\frac{w/2 - y_p(t)}{x_p(t) + l/2}\right) \\ \tau_2(t) &= \tan^{-1}\left(\frac{w/2 - y_p(t)}{-x_p(t) + l/2}\right) \\ \tau_3(t) &= \tan^{-1}\left(\frac{w/2 + y_p(t)}{x_p(t) + q/2}\right) \\ \tau_4(t) &= \tan^{-1}\left(\frac{w/2 + y_p(t)}{-x_p(t) + q/2}\right)\end{aligned}\quad (4)$$

The angle (ρ_n) between the block center displacement vectors (δ_n) and the x-axis can be derived as shown in Equation (5).

$$\begin{aligned}\rho_1(t) &= -\frac{\theta(t)+2\tau_1(t)}{2} + \frac{\pi}{2} \\ \rho_2(t) &= \frac{\theta(t)-2\tau_2(t)}{2} + \frac{\pi}{2} \\ \rho_3(t) &= \frac{\theta(t)-2\tau_3(t)}{2} + \frac{\pi}{2} \\ \rho_4(t) &= -\frac{\theta(t)+2\tau_4(t)}{2} + \frac{\pi}{2}\end{aligned}\quad (5)$$

The displacement vectors (δ_n) are decomposed into δ_{nx} and δ_{ny} , as shown in Figure 6. The displacements and velocities in the y-direction are given as,

$$\delta_{ny} = s_n(t)\theta(t) \sin \rho_n(t) \quad (6)$$

$$\dot{\delta}_{ny} = \dot{s}_n \theta \sin \rho_n + s_n(t) \dot{\theta} \sin \rho_n(t) + s_n(t) \dot{\rho}_n(t) \cos \rho_n(t) \quad (7)$$

The horizontal reaction forces (R_n) on the blocks are functions of the displacements (δ_{ny}) given as

$$R_n = k_n \delta_{ny} + c_n \dot{\delta}_{ny} \quad \text{where } n = 1, 2, 3, 4 \quad (8)$$

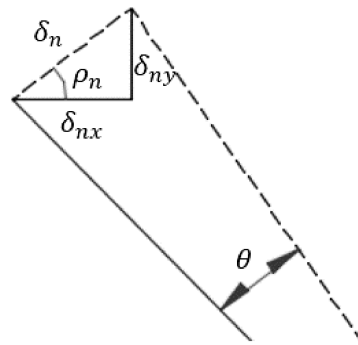


Figure 6. Vector decomposition of rotation block displacement.

The coordinate of the LM block n is (x_n, y_n) , and (x_g, y_g) is the geometric center, as shown in Figure 7. (x_{mid1}, y_{mid1}) and (x_{mid2}, y_{mid2}) are the midpoints of blocks 1 and 2 and blocks 3 and 4, respectively. The relation between the center of mass (x_c, y_c) and the center of rotation (x_p, y_p) is given by the theorem of similar triangles as follows: where $y_c(0)$ refers to the initial y-coordinate value of the platform's center of mass.

$$\begin{aligned}x_1 &= x_p - s_1 \cos(\tau_1 + \theta), \quad y_1 = y_p + s_1 \sin(\tau_1 + \theta) \\ x_2 &= x_p + s_2 \cos(\tau_2 - \theta), \quad y_2 = y_p + s_2 \sin(\tau_2 - \theta) \\ x_3 &= x_p - s_3 \cos(\tau_3 - \theta), \quad y_3 = y_p - s_3 \sin(\tau_3 - \theta) \\ x_4 &= x_p + s_4 \cos(\tau_4 + \theta), \quad y_4 = y_p - s_4 \sin(\tau_4 + \theta) \\ x_g &= \frac{4x_p - s_1 \cos(\tau_1 + \theta) + s_2 \cos(\tau_2 - \theta) - s_3 \cos(\tau_3 - \theta) + s_4 \cos(\tau_4 + \theta)}{4} \\ y_g &= \frac{4y_p + s_1 \sin(\tau_1 + \theta) + s_2 \sin(\tau_2 - \theta) - s_3 \sin(\tau_3 - \theta) - s_4 \sin(\tau_4 + \theta)}{4} \\ x_c &= \frac{y_c(0)}{w/2} \left(\frac{-s_1 \cos(\tau_1 + \theta) + s_2 \cos(\tau_2 - \theta) + s_3 \cos(\tau_3 - \theta) - s_4 \cos(\tau_4 + \theta)}{4} \right) + \frac{4x_p - s_1 \cos(\tau_1 + \theta) + s_2 \cos(\tau_2 - \theta) - s_3 \cos(\tau_3 - \theta) + s_4 \cos(\tau_4 + \theta)}{4} \\ y_c &= \frac{y_c(0)}{w/2} \left(\frac{s_1 \sin(\tau_1 + \theta) + s_2 \sin(\tau_2 - \theta) + s_3 \sin(\tau_3 - \theta) + s_4 \sin(\tau_4 + \theta)}{4} \right) + \frac{4y_p + s_1 \sin(\tau_1 + \theta) + s_2 \sin(\tau_2 - \theta) - s_3 \sin(\tau_3 - \theta) - s_4 \sin(\tau_4 + \theta)}{4}\end{aligned}\quad (9)$$

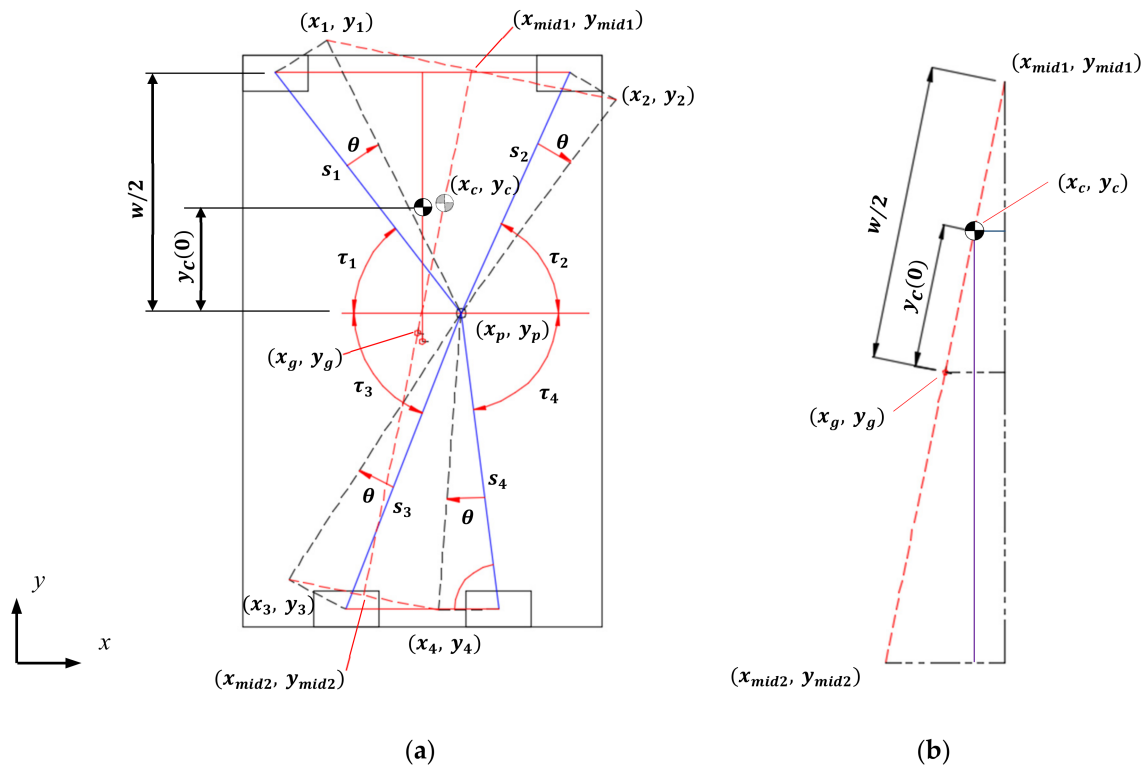


Figure 7. Schematic diagrams of (a) the relationship between the center of mass and the geometric center and (b) the zoomed view of the rotated center of mass.

Substituting Equations (2)–(8) into the force equilibrium Equation (1) of the free body diagram shown in Figure 4 leads to the following equations of yaw motion:

$$\begin{aligned} \sum F_x : F - \mu mg - \mu(R_1 + R_2 + R_3 + R_4) &= m\ddot{x}_p \\ \sum F_y : -R_1 + R_2 - R_3 + R_4 - \mu mg &= m\ddot{y}_p \\ \sum M : F(d - y_p) - \left(x_p - \mu y_p + \frac{l + \mu w}{2}\right)R_1 + \left(x_p + \mu y_p - \frac{l + \mu w}{2}\right)R_2 - \left(x_p - \mu y_p + \frac{q - \mu w}{2}\right)R_3 + \left(x_p + \mu y_p - \frac{q - \mu w}{2}\right)R_4 &= I\ddot{\theta} \end{aligned} \quad (10)$$

3. Factors Affecting the Yaw Motion

The yaw motion of the moving platform depends on the platform aspect ratio, position of the platform's center of mass, driving force offset ratio, parallelism between the two LM Guides, and position of the LM Guides' center of stiffness [1,17,18].

These factors can be classified into controllable and uncontrollable parameters, depending on the situation. In this work, the platform's aspect ratio and driving force offset were uncontrollable under the given conditions, while the parallelism between the LM Guides, the center of mass, and the center of stiffness was defined as a controllable parameter for design and implementation.

The yaw motion of the moving platform may be described by the yaw angle (θ) and settling time (λ), respectively, as functions of parallelism (PA), the center of mass (CM), the center of stiffness (CS), and force offset (FO).

$$\text{Yaw motion} = f(\theta, \lambda) \quad (11)$$

$$\begin{aligned} \theta &= \theta(\text{PA}, \text{CS}, \text{CM})|_{\text{FO}} \\ \lambda &= \lambda(\text{PA}, \text{CS}, \text{CM})|_{\text{FO}} \end{aligned} \quad (12)$$

The yaw angle can be predicted for the given size of the platform, the installation positions of the LM Guides, and the driving force from Equation (10).

Numerical solutions to Equation (10) were obtained using the GEKKO package of python as a solver. GEKKO is a package for machine learning and the optimization of mixed-integer and differential-algebraic equations [22].

3.1. Uncontrollable Parameters: Given Conditions

3.1.1. Aspect Ratio of Platforms

The effects of platform aspect ratio and force offset have been investigated, as shown in Table 2 and Figure 8. Four platforms with different sizes were selected. The length (l) in the moving direction was fixed at 0.5 m, and the width (w) was varied from 0.3–1.0 m, resulting in the l/w ratio of 1.7 to 0.5. The LM Guide model HSR20 was selected with blocks on the four corners of the platform. The driving motion profile is trapezoidal with an acceleration of 1 g (9.8 m/s^2) and an acceleration time of 0.1 s.

Table 2. Dimensions of moving platform.

No.	Length, l (m)	Width, w (m)	Weight (kg)	l/w
1	0.5	0.3	29.3	1.7
2	0.5	0.5	48.8	1.0
3	0.5	0.8	78.0	0.6
4	0.5	1.0	97.5	0.5

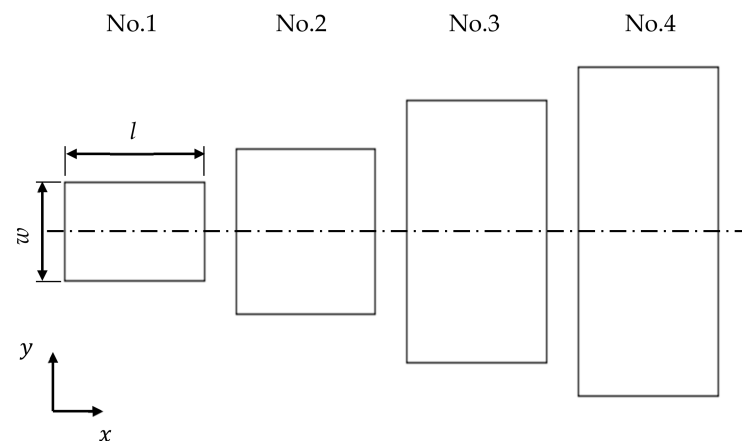


Figure 8. Comparison of moving platforms.

The friction force and damping coefficient of LM Guides vary depending on the operating environment, maintenance, and driving conditions [20,21]. Even for LM blocks of the same model number, the friction coefficients and damping coefficients may vary. To proceed, the friction coefficient (μ) and damping coefficient (c) were fixed to 0.02 and 10 Nsec/mm, respectively. The effects of their variations will be discussed later.

3.1.2. Force Offset

The force offset (FO) may be defined in terms of the distance between the ball screw linear actuator, illustrated in Figure 1, and the geometric center of the moving platform. In its normalized form, the force offset is defined as

$$FO(\text{ForceOffset}) = \frac{d}{w/2} \times 100 (\%) \quad (13)$$

where d is the offset distance and w is the width of the platform. Figure 9 illustrates the parameters for FO and examples of different FO configurations.

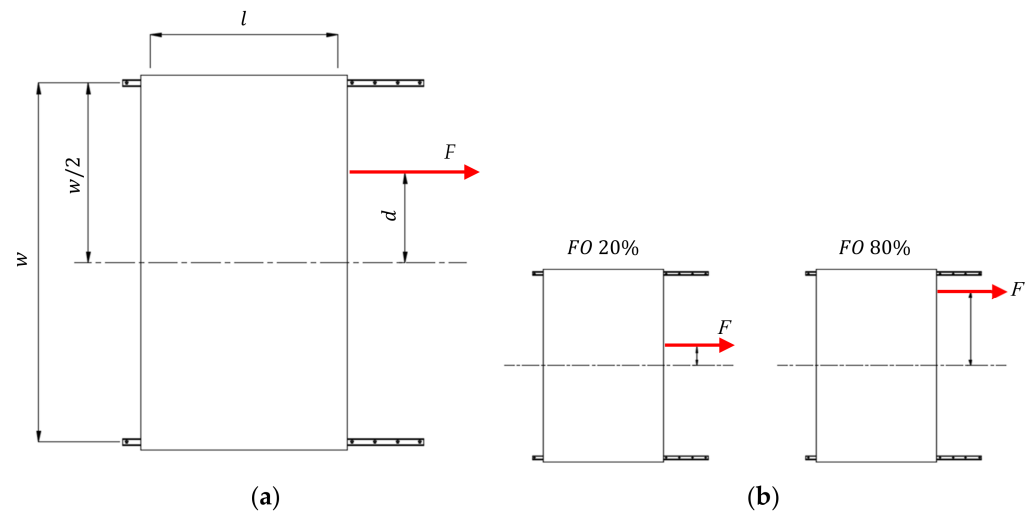


Figure 9. (a) Schematic diagram of FO definition and (b) comparison of FO 20% and FO 80%.

Figure 10 shows the yaw vibration of the No. 4 platform (Figure 8) driven with four different values of FO . The vibration response of the moving platform in Figure 10 is typical for the trapezoidal motion profile [23]. The yaw angle is observed to converge after a certain period of time, indicating that the higher the FO , the higher the convergence value becomes.

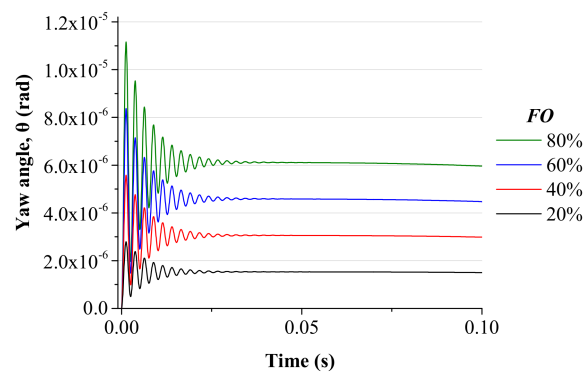


Figure 10. Yaw angle vs. time for various force offsets (No. 4).

Figure 11 presents the convergence value of the yaw angle vs. FO value for the four platform types, which demonstrates that the larger the l/w value, the smaller the FO effect on the yaw angle.

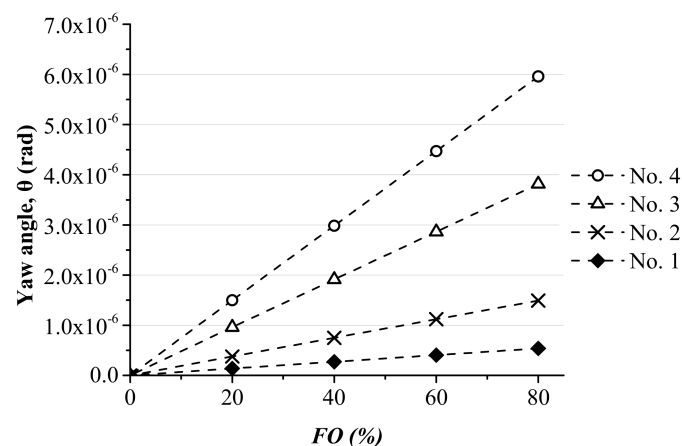


Figure 11. Yaw angle vs. force offset for various platforms.

Figure 12 is an enlarged graph of a FO of 80% in Figure 10. The settling time (λ) is when the vibration value becomes less than the error band, 0.1×10^{-6} rad, about 33 ms.

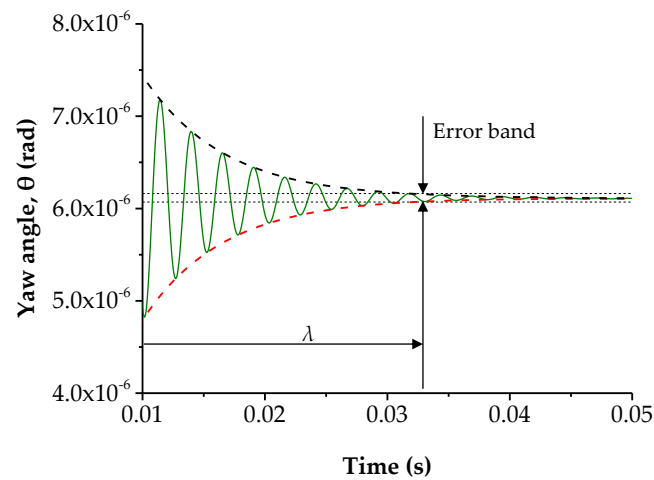


Figure 12. Zoomed yaw angle graph: settling time (λ) and error band.

3.1.3. Verification of Dynamics Model

The predictions with Equation (10) were compared with the finite element analysis results obtained with ANSYS, a commercial finite element analysis (FEA) tool. In many previous studies, the deformation of the LM Guide with respect to the load showed a high correlation between the experiment and the simulation [5–9]. In the ANSYS FEA model, the balls between the block and rail of the LM Guide were represented by a bushing joint, which requires two opposing contact surfaces, described as the reference and mobile in Figure 13a. The constraint of a bushing joint was applied with vertical and horizontal stiffness values (Figure 3) between the mobile and reference surfaces. Figure 13b shows the finite element mesh for FEA.

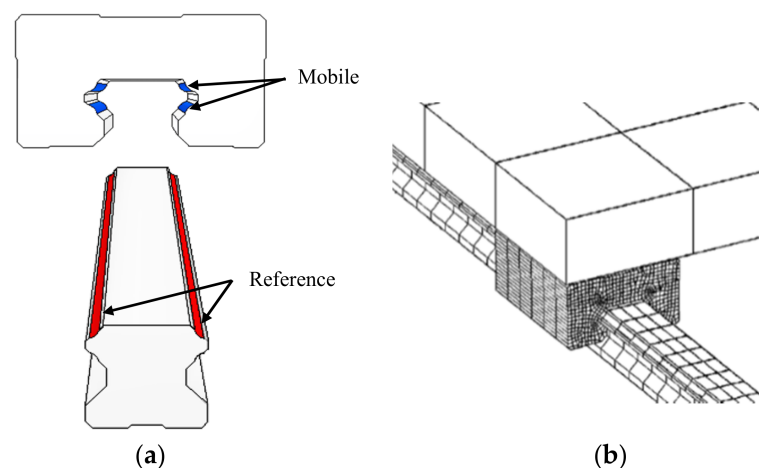


Figure 13. Finite element model for LM Guide: (a) contact surfaces of a bushing joint, and (b) finite element mesh.

Platform 4 in Table 2 was selected for the simulation model, and the analysis was performed in the 20–80% range of FO values.

The yaw-induced positioning error of the stage is represented by the difference between the x-axis coordinate differences of block two and block four, as illustrated in Figure 14. The yaw angle values from the FEM simulations and Equation (10) are listed in Table 3. With FO , the yaw angle tended to increase in both the results of the simulations and the dynamics Equation (10).

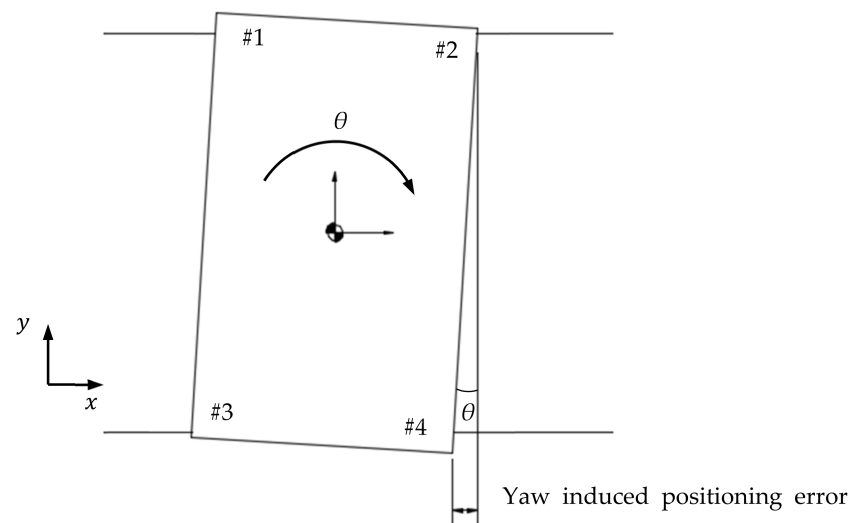


Figure 14. Schematic diagram of yaw error.

Table 3. Yaw angle predictions for the No. 4 platform (μrad).

FO	FEM Simulation ANSYS	Rigid Dynamics Model Equation (10)
0%	0.0	0.0
20%	0.0	1.5
40%	2.9	3.0
60%	5.7	4.4
80%	6.7	6.0

The difference between the predictions from the FEM model and the rigid body dynamics model may be due to the structural vibration ignored in the rigid body dynamics model. It may also be due to the difference between the bushing joint and simple spring model for the LM Guide stiffness. Even with these differences, a simpler and faster rigid body dynamics model may be a useful tool for predicting yaw error at the early stage of precision linear stage design.

The friction coefficient and damping coefficient affect the yaw angle and vibration damping, respectively. As the friction coefficient increases, the value of the yaw angle increases, and as the damping coefficient increases, the settling time decreases. For example, in the case of FO:80%, for the friction coefficient ranging between 0.002 and 0.2, No. 4 platform's yaw angle shows about 4% variation, as shown in Table 4.

Table 4. Convergence value of yaw angle according to friction coefficient.

Friction Coefficient (μ)	0.002	0.01	0.02	0.03	0.2
Yaw angle (μrad)	5.93	5.94	5.96	5.98	6.17

For the damping coefficient ranging between 0.01 and 100, the settling time shows a 0.6% variation, as shown in Table 5. These two factors do not demonstrate dominant effects on the predicted yaw angle for their respective practical ranges.

Table 5. Settling time according to damping coefficient.

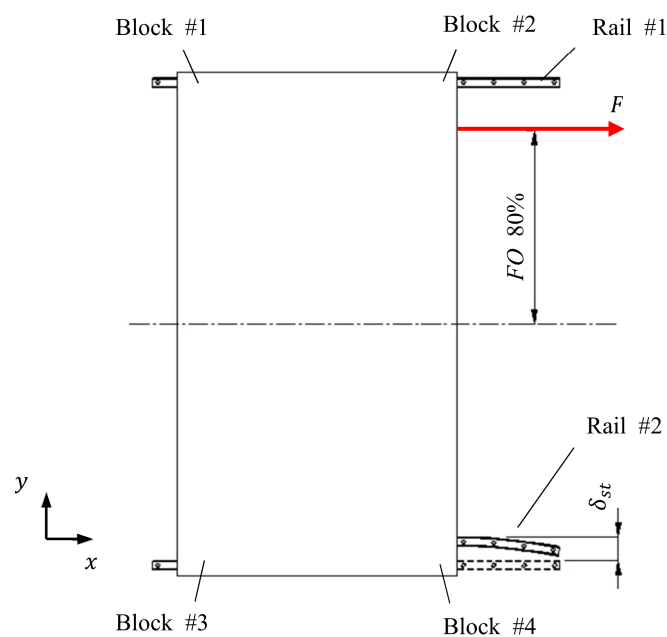
Damping Coefficient (Nsec/mm)	0.01	0.1	10	100
Settling time (ms)	32.02	32.02	31.92	31.87

3.2. Controllable Parameters: Design, Manufacturing Conditions

3.2.1. Parallelism between LM Guides

Ideally, a pair of LM Guide rails should be perfectly parallel. In reality, this cannot be the case due to an assembly error found in LM Guides and the machining error on the installation surface itself.

As shown in Figure 15, LM rail number one is an ideal straight line, and rail number two represents the straightness error in the y -direction by δ_{st} from the block number four position, which indicates the application of an external force of $\delta_{st} \times k_4$ to the number four location in the y -direction. The yaw angle was calculated by reflecting the additional external force in the yaw motion in Equation (10).

**Figure 15.** Parallelism error between LM Guides.

The driving force was applied to the four types of platforms at an FO of 80% with the trapezoidal motion profile mentioned earlier. The convergence values of the yaw angle according to the parallelism error (PA) are shown in Figure 16. The yaw angle at zero parallelism is equivalent to the value of an FO of 80% in Figure 11. With parallelism, the yaw angle is increased by several times or more regardless of l/w . Parallelism is the most dominant factor affecting the yaw motion, regardless of l/w ratio or FO.

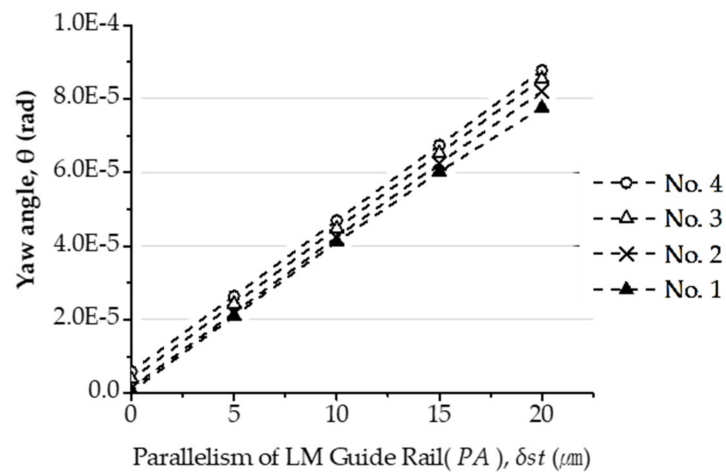


Figure 16. Results of the yaw angle of platforms.

3.2.2. Center of Mass & Center of Stiffness

The moving platform has several midpoints: centers of geometry, mass, and stiffness. These midpoints are defined as

$$\text{Center of geometry} = O(0,0) \quad (14)$$

$$\text{Center of mass} = O' \left(\frac{\int x dm}{m}, \frac{\int y dm}{m} \right) \quad (15)$$

$$\text{Center of stiffness} = O'' \left(\frac{\sum x_i k_i}{\sum k_i}, \frac{\sum y_i k_i}{\sum k_i} \right) \quad (16)$$

The midpoints are illustrated in Figure 17.

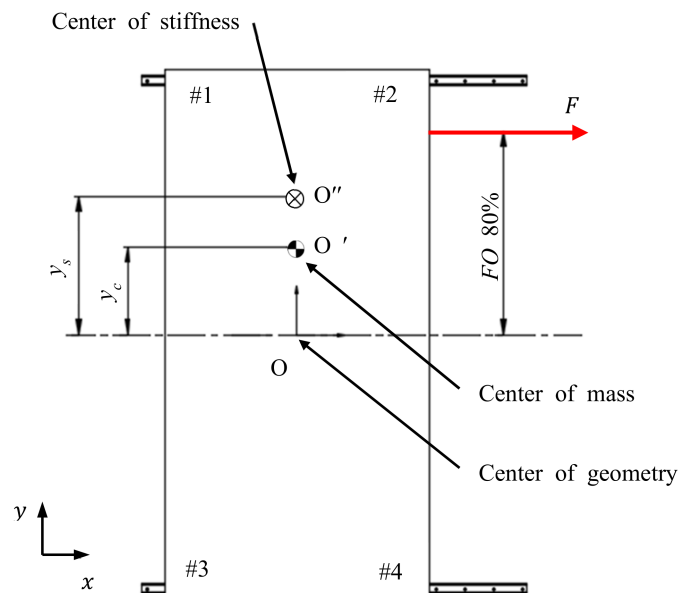


Figure 17. Midpoints of the platform.

Normalized y-coordinates of the platform mass center and LM guide stiffness center are defined as the ratios of the distances from the center of geometry (origin O) to each center point in the width direction to the half-width of the platform as

$$CM(\text{Normalized coordinate of platform mass center}) = \frac{y_c}{w/2} \times 100 (\%) \quad (17)$$

$$CS(\text{Normalized coordinate of LM guide stiffness center}) = \frac{y_s}{w/2} \times 100 (\%) \quad (18)$$

The center of mass can be modified by adding or removing weights for the given platform size. The center of stiffness can be modified by changing the positions of the LM Guides, installing additional guides, or replacing the LM Guide types.

In general, two blocks on an LM Guide rail are identical, and thus $k_1 = k_2$, $k_3 = k_4$. Table 6 lists some of the CS values for the k_3/k_1 ratios.

Table 6. Ratio of center of stiffness (CS) for k_3/k_1 values.

CS	0%	20%	40%	60%	80%	100%
k_3/k_1	1	0.7	0.4	0.30	0.1	0

The yaw motion dependence on the CS and CM for number four platform with zero parallelism at 80% of FO such as

$$\begin{aligned} \theta &= \theta(PA : 0, CS, CM) \big|_{FO:80\%, No4Platform} \\ \lambda &= \lambda(PA : 0, CS, CM) \big|_{FO:80\%, No4Platform} \end{aligned} \quad (19)$$

is described in Figures 18 and 19.

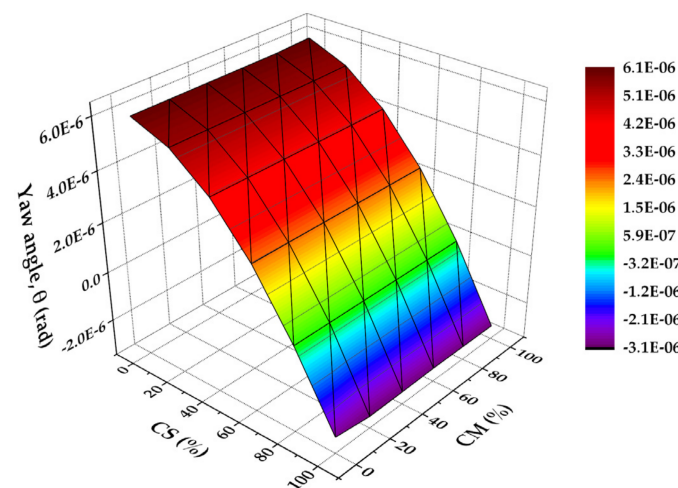


Figure 18. Convergence yaw angle (θ) vs. CM and CS.

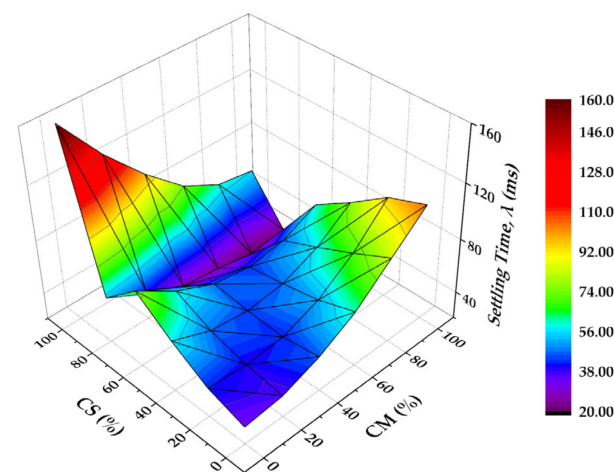


Figure 19. Settling time (λ) vs. CM and CS.

Figure 18 shows the value of the yaw angle for a range of CS and CM, and Figure 19 shows the settling time. The yaw angle is greatly affected by the CS, while the effect of the CM is minimal. The absolute value of the yaw angle is the smallest at a CS of 80% for an FO of 80%. Figure 19 shows that the settling time is also minimal when the CS is the same as the CM. The lowest settling time is also achieved at a CS of 80%.

As for the yaw angle, Figure 18 indicates that it is governed primarily by the CS, indicating that the LM Guides are the most dominant elements constraining the yaw rotation of the platform. For the lower range of CS values, the centers of stiffness and rotation are relatively close to each other. As the CS increases closer to the value of FO, the constraints from guide rail number two are partially reduced, and thus, the center of rotation also moves closer to the FO position. This will reduce the yaw moment by the driving force (F) about the center of rotation. Moreover, the rotational inertia of the platform is increased since the rotation center moves away from the center of mass.

Settling time is a result of the complex interplay among the CS, CM, and FO, as indicated in Figure 19. It turns out that the settling time reaches its minimum at the values of CS and CM similar to a FO of 80%. In this case, the centers of rotation, mass, and stiffness are closer to the FO position, reducing the yaw moment and rotational inertia.

The data for Figures 18 and 19 are listed in Tables 7 and 8.

Table 7. Convergence yaw angle for combinations of CM and CS, (μrad).

CS \ CM		0%	20%	40%	60%	80%	100%
CM	0%	5.96	5.38	4.19	2.39	0.0012	−2.83
	20%	5.96	5.38	4.19	2.40	0.0018	−3.01
	40%	5.96	5.38	4.19	2.40	0.0021	−2.98
	60%	5.96	5.38	4.19	2.40	0.0020	−2.94
	80%	5.96	5.38	4.19	2.40	0.0020	−2.94
	100%	−6.02	5.40	4.20	2.40	0.0020	−2.94

Table 8. Settling time for combinations of CM and CS, (ms).

CS \ CM		0%	20%	40%	60%	80%	100%
CM	0%	33	42	58	77	54	162
	20%	36	40	47	59	42	127
	40%	46	42	44	47	28	98
	60%	61	52	47	43	25	74
	80%	83	70	58	47	22	60
	100%	105	95	76	59	23	56

For the conditions where the CS and CM have an identical value, the effects of parallelism (PA) on the yaw angle and settling time, such as

$$\begin{aligned}\theta &= \theta(PA, CS = CM)|_{FO80\%, No4Platform} \\ \lambda &= \lambda(PA, CS = CM)|_{FO80\%, No4Platform}\end{aligned}\quad (20)$$

may be investigated.

Perfect parallelism between two LM Guide rails is impossible. The practical lower limit for PA is 10 μm for the moving platform. The maximum admissible settling time is 100 ms for the current motion profile of 1 g and 0.1 s for acceleration and deceleration.

Figure 20 suggests that when the value of the CS and CM is maintained close to an FO of 80%, the allowable yaw angle is achieved for a PA of 10 μm . Figure 21 shows that

settling time is maintained below 70 ms for the entire range of the CS and CM and for a PA of 20 μm and beyond. The data for Figures 20 and 21 are listed in Tables 9 and 10.

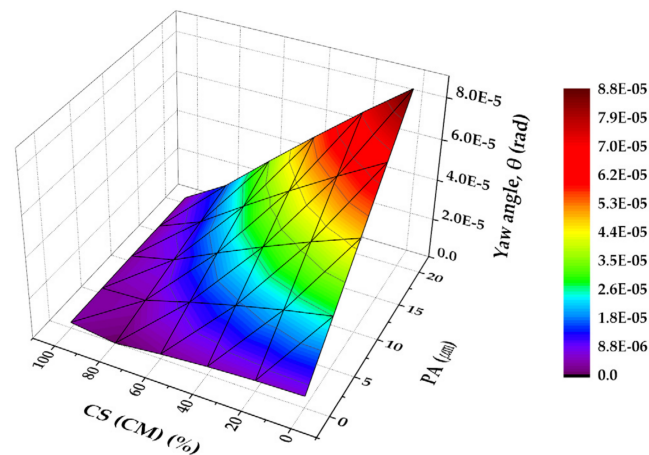


Figure 20. Yaw angle vs. CS and PA.

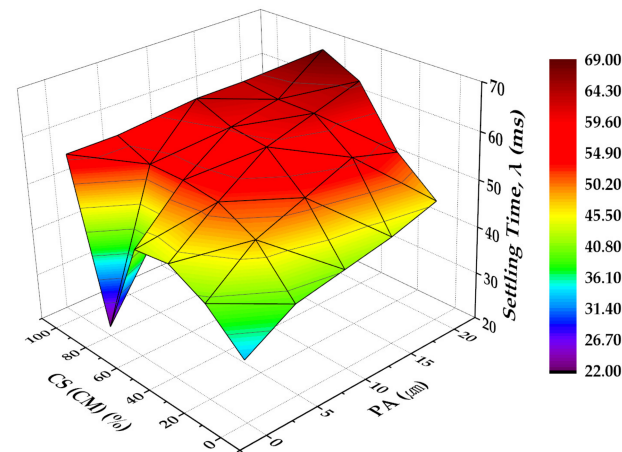


Figure 21. Settling time (λ) vs. CS and PA.

Table 9. Convergence yaw angle for combinations of PA and CS.

CM \ PA	PA					
	0 μm	5 μm	10 μm	15 μm	20 μm	
0%	5.96	26.4	46.9	67.4	87.8	
20%	5.38	21.8	38.2	54.6	71.1	
40%	4.19	16.5	28.8	41.1	53.4	
60%	2.40	10.6	18.7	26.9	35.1	
80%	0.002	4.04	8.08	12.1	16.2	
100%	2.92	2.96	3.00	3.04	3.08	

Table 10. Settling time for combinations of PA and CS.

$PA \backslash CM$		0 μm	5 μm	10 μm	15 μm	20 μm
0%		33	39	41	43	46
20%		40	48	52	56	53
40%		44	52	59	62	65
60%		43	53	60	62	69
80%		22	53	63	63	63
100%		56	56	56	56	56

At this point, it may be suggested that the values of the CS and CM need to be close to an FO of 80%, while PA may be maintained in the vicinity of 10 μm . This will require the ratio k_3/k_1 to be less than 0.1 and balancing weight to the platform for 80% of CM. In practice, it is difficult to accurately match the ratio k_3/k_1 with commercially available LM Guides. One alternative is to choose $k_3 = 0$ with a CS of 100%. Keeping the blocks of rail number two simply supported in the z-direction without constraints in the x- and y-directions can be a practical solution, as illustrated in Figure 22.

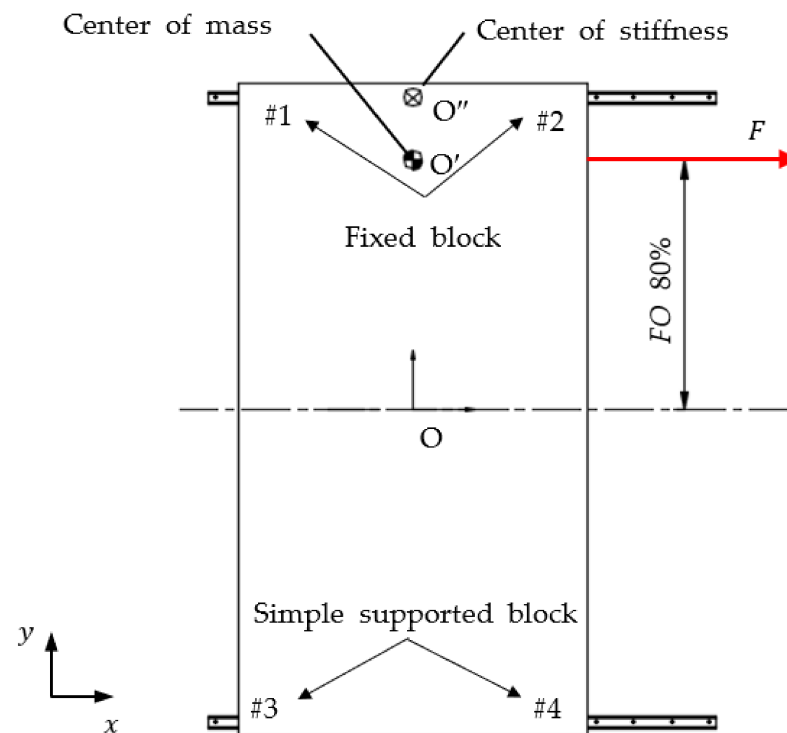
**Figure 22.** Modified platform for minimization of yaw angle.

Figure 23 shows three yaw angle waveforms for the deceleration stage of the trapezoidal motion profile for the number four platform at 80% FO. The original model with zero values for the CS and CM has a clockwise yaw angle. An improved model with a CS of 100% and CM of 80% has a counterclockwise yaw angle half the magnitude of the original model. After the stiffness of k_1 and k_2 are doubled, the magnitude of the yaw angle is further reduced to half.

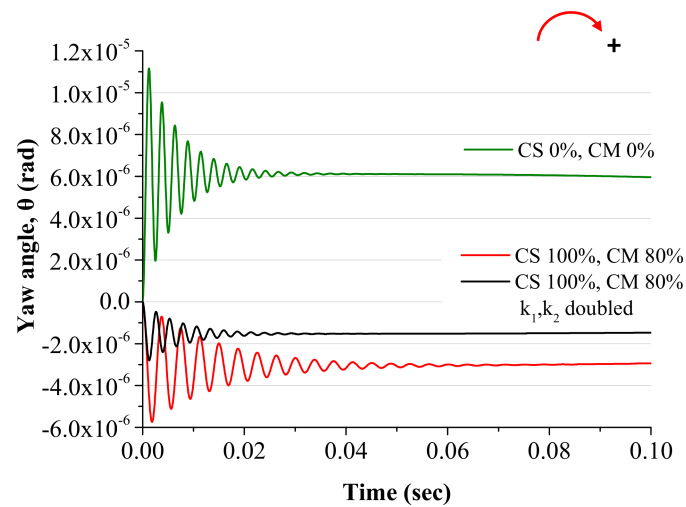


Figure 23. Yaw angles according to change of CM and CS.

4. Design Improvement of XY Stage

The results in the previous section have shown that the yaw angle of the stage decreases as the l/w ratio increases and as the parallelism of the LM Guides improves and as the platform's centers of stiffness and mass come closer to the driveline of the linear actuator. The position repeatability of the stage with respect to the reciprocating motion becomes better with a reduced yaw angle.

The PCB electrical test machine has an XY stage with offset-driven moving platforms. In particular, it has disadvantageous spatial constraints against precision driving with the aspect ratio l/w of 0.41 and an FO of 80%.

To overcome this challenge, the following design improvements are proposed. First, maximize k_1 and k_2 while keeping $k_3 = k_4 = 0$ with simple support of the blocks on rail number two. For this, THK's HSR35L and HSR20L were selected for rail number one and rail number two, respectively.

Furthermore, to best match the center of mass with the driveline, a balancing weight was added to the base of the moving platform assembly, as shown in Figure 24. Since the platform moves along the y-axis with a stroke of 300 mm, the CM fluctuates between 60–71%. The linear drive actuator system was composed of a 20 mm lead ball screw, a 750 W servo motor, and a linear encoder.

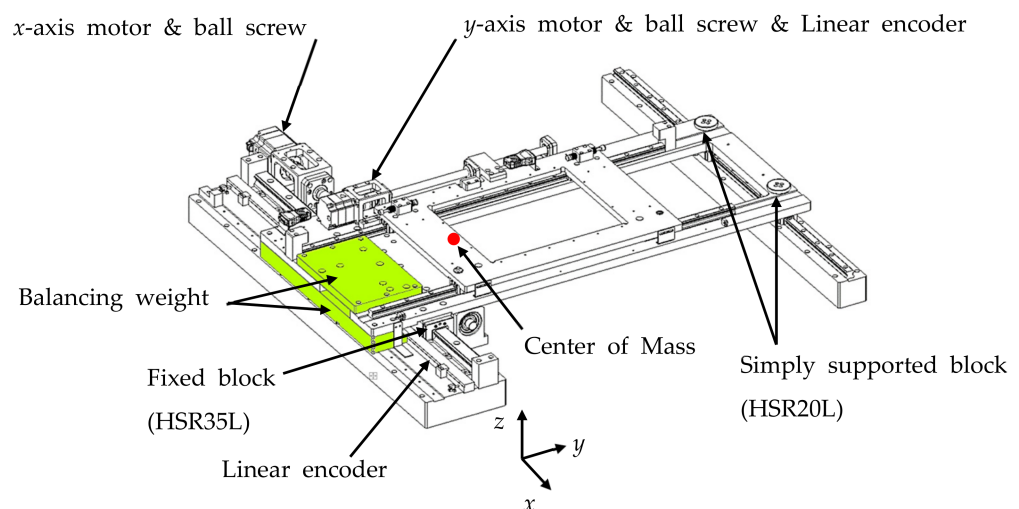


Figure 24. Design of 2-axis linear stage with balancing weight.

Table 11 shows the results of the yaw error calculated using Equation (10) for the improvements.

Table 11. Platform dimensions and predicted yaw error for the improved x-axis stage.

Axis	Length (mm)	Width (mm)	l/w	Balancing Weight (kg)	FO (%)	Yaw Error (μm)
x	370	895	0.41	17.8	80	± 0.67

Since the predicted yaw error is below the target repeatability of $\pm 1.0 \mu\text{m}$, a detailed design of the x-axis was performed. The improved design of the XY stage is shown in Figure 24. Figure 25 shows the details of the simply supported blocks in Figure 24. In Figure 25, the platform was mounted onto the support plate, and a pin was assembled in the platform hole. A urethane ring was inserted between the pin and the platform hole to absorb the parallelism error of the LM Guide to prevent any constraint on the platform. A pin top block was utilized to constrain the z-direction motion of the platform. The pin length was processed and assembled approximately $10 \mu\text{m}$ longer than the platform thickness.

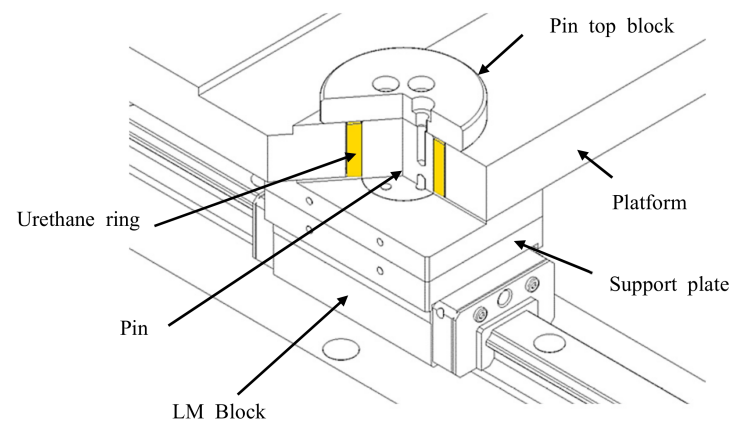


Figure 25. Sectional view of the simply supported blocks in Figure 24.

Experimental Verification of Effectivity of the Design Improvement of XY Stage

The manufactured test bed in Figure 26 includes an improved XY linear stage and a vision camera. Artifact glass, in Figure 27, was utilized for position calibration and the repeatability test.

Calibration artifacts were precisely printed to be 0.5 mm in diameter at 5 mm intervals by light exposure to glass.

The position compensation of the XY stage was performed recognizing this circular dot using the vision camera.

The XY stage started from the origin down a path passing through 15 points on the artifact glass, as shown in Figure 27, according to the actual electrical test machine operations. The cycles were repeated 25 times. The x and y data of each point were gathered through a camera. Ideally, the coordinates of the 15 points should be aligned with the center of the camera. The actual points deviated from the center due to errors from the stage.

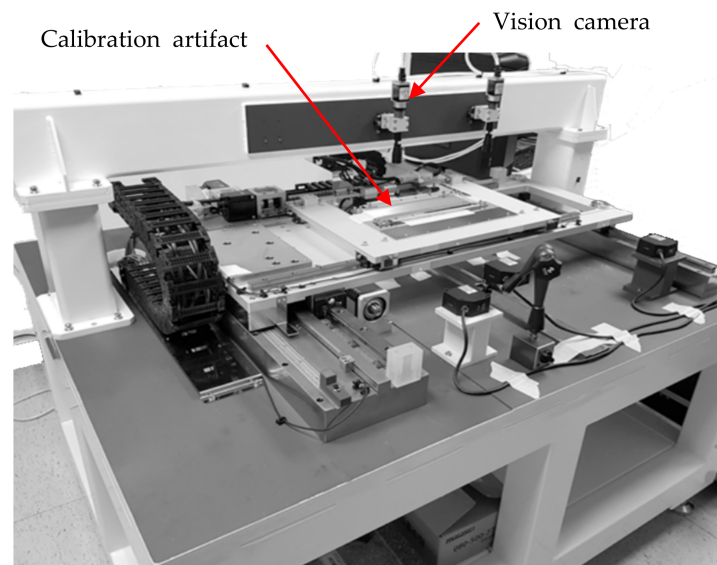


Figure 26. Test bed for improved XY linear stage.

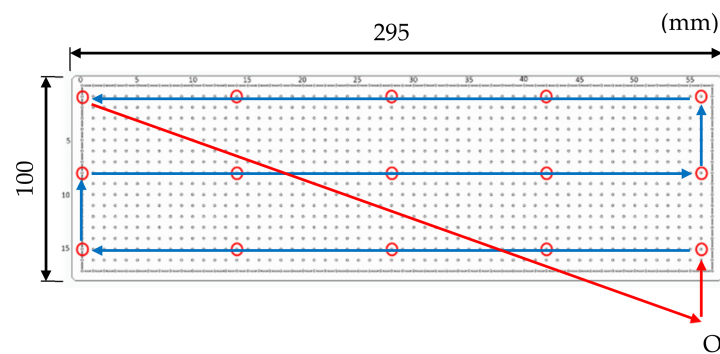


Figure 27. The calibration artifact with a pitch circle of 5 mm mounted on the platform of XY stage. The origin and 15 point track viewed from a stationary camera.

A set of the actual positions of the 15 points with respect to the center of the camera for the repeated 25 cycles is shown in Figure 28a points without the balancing weight and Figure 28b with the balancing weight. In Figure 28a, the scatter in the x -axis was large and out of the $\pm 1.0 \mu\text{m}$ target, and in Figure 28b, the scatter range was within the target. The six sigma analyses of the point coordinates show changes from 1.62σ to 5.51σ in the x -axis and from 3.68σ to 3.21σ in the y -axis. The process's capability has been greatly improved by more than 5σ in the x -axis with a slight setback in the y -axis, but still allowable beyond the practical limit of 3σ . Thus, the beneficial effects of CS and CM modifications were clearly verified.

The scatter patterns in Figure 28 seem to be elongated, with their major axes covering the second and the fourth quadrants. This is due to one-way travel in the x -axis direction during the 15 point track illustrated in Figure 27. The yaw and tilt of the platform cause the elongation through the camera.

It is evident that the application of a balancing weight to the y -axis motion would produce similar results.

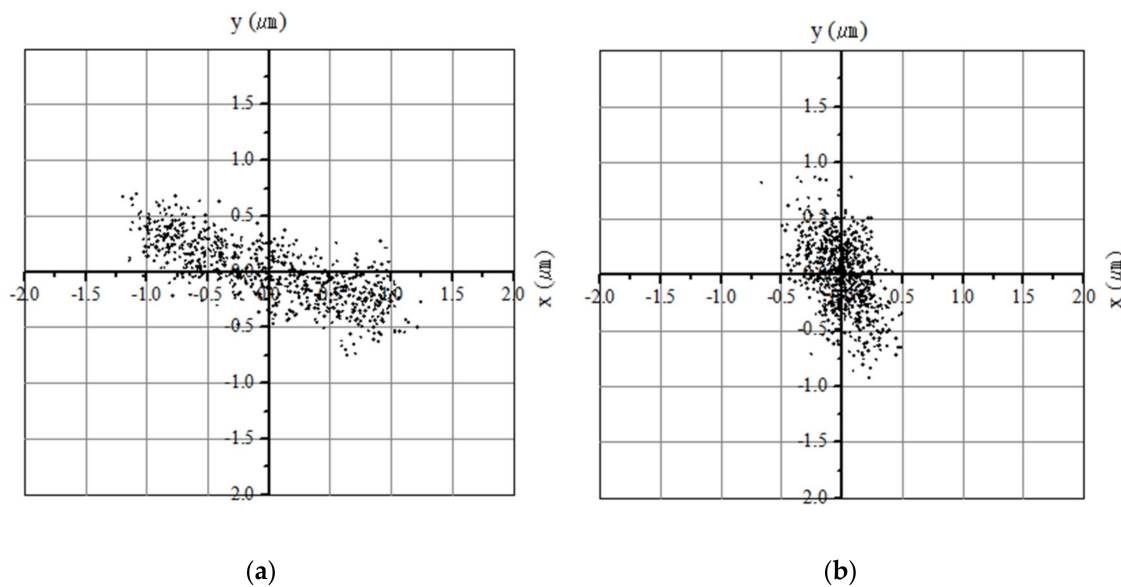


Figure 28. Markers of 15 points on calibration artifact glass for 25 cycles of repeatability test captured by camera: (a) without balancing weight and (b) with balancing weight.

5. Conclusions

The PCB electrical test machine has an XY stage with an open-frame moving platform. It has disadvantageous conditions for precision driving with a ratio l/w of 0.41 and an FO of 80%. Practical constraints on the moving platform make the choice unavoidable, even though such a configuration is considered most unfavorable [1,2]. Practically attainable position repeatability of such a configuration has been known to be in the order of tens of micrometers or more.

Since the platform yaw motion affects the stage's position repeatability significantly, its control is a crucial factor in stage design. Yawing of a moving platform is governed by various factors, including the driving force offset from the platform's geometric center, the positions of the stiffness, mass, and rotation centers. Other contributions are also expected from the parallelism between the two linear motion guides supporting the platform. The stiffness between the block and rail of the linear motion guides also contributes significantly.

In this work, it has been confirmed that yawing may be minimized by reducing the distances between the linear ball screw actuator and the platform's centers of mass and stiffness. This may be implemented by adding proper balance weights to the platform and partially relieving one of the linear motion guides from its constraints. The adverse effects of a parallelism error between the two linear motion guides may also be alleviated by this method. An example of design improvement along these lines has been presented with experimental validation.

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Nomenclature

F	Driving force
m	Mass of a moving platform
$k_{Vertical}$	Vertical stiffness of LM Guide
$k_{Horizontal}$	Horizontal stiffness of LM Guide
k_n	Horizontal stiffness of n^{th} block
PA	Parallelism of LM Guide
FO	Normalized coordinate of driving force position
CM	Normalized coordinate of platform mass center
CS	Normalized coordinate of LM Guide stiffness center
θ	Yaw angle of a moving platform
c	Damping coefficient
c_n	Damping coefficient of n^{th} block
μ	Friction coefficient
σ	Standard deviation

References

1. Slocum, A.H. *Precision Machine Design*; Society of Manufacturing Engineers Dearborn: Dearborn, MI, USA, 1992.
2. THK, Co., Ltd. THK Linear Motion Systems Catalog 513-1K. Available online: <https://www.thk.com/catalog/?lang=en> (accessed on 20 June 2021).
3. Shimizu, S. Stiffness Analysis of Linear Motion Rolling Guide. *J. Jpn. Soc. Precis. Eng.* **1998**, *64*, 1573–1576. [CrossRef]
4. Rahmanian, M.; Bleicher, F. Experimental and analytical investigations on normal and angular stiffness of linear guides in manufacturing systems. *Procedia CIRP* **2016**, *41*, 795–800. [CrossRef]
5. Shaikharaova, A.; Liang, Y.; Feng, H.; Xu, B. Study of Stiffness of Linear Guide Pairs by Experiment and FEA. *World J. Eng. Technol.* **2016**, *4*, 115–128. [CrossRef]
6. Ohta, H.; Hayashi, E. Vibration of Linear Guideway type recirculating Linear Ball Bearings. *J. Sound Vib.* **2000**, *235*, 847–861. [CrossRef]
7. Kim, H.Y.; Jeong, J.K.; Won, J.J.; Jeong, J.I. A Computational Modeling Reflecting Static and Dynamic Characteristics of LM Bearing for Machine Tools. *J. Korean Soc. Precis. Eng.* **2012**, *29*, 1062–1069. [CrossRef]
8. Jeong, J.; Kang, E.; Jeong, J. Equivalent stiffness Modeling of Linear Motion Guideways for Stage Systems. *Int. J. Precis. Eng. Manuf.* **2014**, *15*, 1987–1993. [CrossRef]
9. Chang, J.C.; Hung, J.P. Analytical and Finite Element Modeling of the Dynamic Characteristics of a Linear Feeding Stage with Different Arrangements of Rolling Guides. *Math. Probl. Eng.* **2014**, *11*, 1–11. [CrossRef]
10. Kim, K.H.; Park, C.H.; Lee, H.; Kim, S.W. Analysis of the Motion Errors in Linear Motion Guide. *J. Korean Soc. Precis. Eng.* **2002**, *19*, 139–148.
11. Khim, G.; Park, C.H.; Oh, J.S. A Method of Calculating Motion Error in a Linear Motion Bearing Stage. *Sci. World J.* **2015**, *2015*, 696417. [CrossRef]
12. Rahmanian, M.; Bleicher, F. Experimental and numerical studies of the influence of geometric deviations in the performance of machine tools linear guides. *Procedia CIRP* **2016**, *41*, 818–823. [CrossRef]
13. Khim, G.; Park, C.H.; Shamoto, E.; Kim, S.W. Prediction and compensation of motion accuracy in a linear motion bearing table. *Precis. Eng.* **2011**, *35*, 393–399. [CrossRef]
14. Li, X.; Liu, Y.; Liu, J.; Guo, H. Theory and experimental research on static stiffness of linear rolling guide. In Proceedings of the 2012 IEEE International Conference on Computer Science and Automation Engineering (CSAE), Zhangjiajie, China, 25–27 May 2012; pp. 85–87.
15. Hu, C.; Zhu, Y. Self-Calibration of Two-Dimensional Precision Metrology Systems. *New Trends Dev. Metrol.* **2016**, *Ch.8*, 185–210.
16. Yi, Y.S.; Kim, Y.Y.; Choi, J.S.; Yoo, J.H.; Lee, D.J.; Lee, S.W.; Lee, S.J. Dynamic analysis of a linear motion guide having rolling elements for precision positioning devices. *J. Mech. Sci. Technol.* **2008**, *22*, 50–60.
17. Ekinci, T.; Mayer, J. Relationships between straightness and angular kinematic errors in machines. *Int. J. Mach. Tools Manuf.* **2007**, *47*, 1997–2004. [CrossRef]
18. Yanga, P.; Takamuraa, T.; Takahashi, S.; Takamasua, K.; Sato, B.O.; Osawa, B.S.; Takatsuji, T. Development of high-precision micro-coordinate measuring machine: Multiprobe measurement system for measuring yaw and straightness motion error of XY linear stage. *Precis. Eng.* **2011**, *35*, 424–430. [CrossRef]
19. Wit, C.C.D.; Lischinsky, P. Adaptive Friction Compensation with Partially Known Dynamic Friction Model. *Int. J. Adapt. Control Signal Processing* **1997**, *11*, 65–80. [CrossRef]
20. Cheng, D.J.; Yang, W.S.; Park, J.H.; Park, T.J.; Kim, S.J.; Kim, G.H.; Park, C.H. Friction Experiment of Linear Motion Roller Guide THK SRG25. *Int. J. Precis. Eng. Manuf.* **2014**, *15*, 545–551. [CrossRef]
21. Low, K.S.; Keck, M.T. Advanced Precision Linear Stage for Industrial Automation Application. *IEEE Trans. Instrum. Meas.* **2003**, *52*, 785–789.

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22. GEKKO, Python Package. Available online: <https://gekko.readthedocs.io/en/latest/index.html#> (accessed on 19 November 2021).
 23. Li, H.; Le, M.D.; Gong, Z.M.; Lin, W. Motion Profile Design to Reduce Residual Vibration of High-Speed Positioning Stages. *IEEE/ASME Trans. Mechatron.* **2009**, *14*, 264–269.