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Predefined-Time Nonsingular Attitude Control for Vertical-Takeoff Horizontal-Landing Reusable Launch Vehicle

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Abstract: This paper presents a novel predefined-time nonsingular tracking control system for a vertical-takeoff horizontal-landing (VTHL) reusable launch vehicle (RLV) in the face of parameter uncertainties, model couplings and external disturbances. Firstly, this paper proposes a novel predefined-time prescribed performance function (PTPPF) with desired steady-state and transient performance. The convergence time of PTPPF from the transient state to the steady state can be flexibly adjusted by changing one parameter. Moreover, the decreasing rate of PTPPF in the transient phase can also be adjusted by changing one parameter on the premise of not changing the convergence time of PPF to reach steady state. A novel predefined-time terminal sliding mode surface (SMS) is designed to avoid the singularity, and the attitude tracking errors on SMS are predefined-time stable. By utilizing PTPPF and error transformation, this paper designs a novel nonsingular sliding mode controller to guarantee the attitudes of RLV with desired tracking performance. Without using piecewise functions, the phenomenon of singularity can be avoided. The Lyapunov method is used to verify the stability of the controller. Lastly, a numerical simulation is presented to validate the efficiency of the proposed controller.

Keywords: reusable launch vehicle; attitude tracking control; predefined-time prescribed performance function; predefined-time terminal sliding mode surface; nonsingular sliding mode controller



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1. Introduction

As a fast-response, reliable, and low-cost vehicle to access space, the vertical-takeoff horizontal-landing (VTHL) reusable launch vehicle (RLV) has become a research hotspot in recent years [1,2]. The shape diagram of the VTHL RLV studied in this paper is shown in Figure 1. However, due to its complex temperature variation and wide reentry flight envelope, RLV is prone to parameter uncertainties and complex aerodynamic characteristics in the reentry phase, which makes controller design a challenging task. Hence, an advanced reentry attitude control scheme with sufficient robustness and fast convergence rate is significant for RLV to achieve a successful mission.

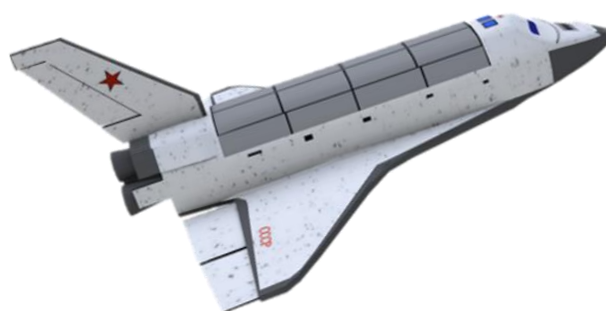


Figure 1. VTHL reusable launch vehicle.

A variety of control methods have been developed to improve the flight quality of reentry vehicles. Song et al. [3] proposed a fractional-order PID control method for hypersonic vehicle attitude control. Ming et al. [4] proposed an active disturbance rejection controller (ADRC) for an air-breathing supersonic vehicle with a new smooth function. Mao et al. [5] proposed an adaptive fuzzy H-infinity controller for RLV under parameter uncertainties and external disturbances. Zhang et al. [6] designed a backstepping technique and combined a nonlinear observer for hypersonic vehicle, which realized stable tracking of attitude. Xu et al. [7] investigated a controller for hypersonic flight vehicle with robust adaptive neural networks. Compared with the above methods, sliding mode control (SMC), which is robust to disturbances and insensitive to parameter uncertainties, has attracted the most interest [8,9].

The peculiarity of SMC is to adjust the attitude tracking errors of RLV toward a predefined sliding surface and then avoids them leaving this surface. As it is known, the traditional SMC merely ensures the asymptotic stability of the control system. Then, a finite-time SMC with solid robustness against internal and external disturbances was proposed by Lu et al. [10] and Hall et al. [11] to obtain finite-time convergence of states. Since the initial condition of the system determines the settling time of finite-time SMC. It may destroy the desired fast convergence performance when the initial values are unknown in practice. Thus, the fixed-time SMC (FxSMC), which has achieved great development in the past several years, can ensure that the system convergence time is independent of the initial conditions [12,13]. Zhang et al. [14] presented a fixed-time nonsingular terminal SMC for RLV. Ju et al. [15] presented an FxSMC to ensure that the vehicle attitudes track the desired command in a fixed time. Liang et al. [16] designed a robust controller for RLV based on a FxSMC. However, the convergence time of the FxSMC is actually obtained after complex calculation rather than known before controller design, which increases the offline workload. Thus, the predefined-time stability has been widely studied in recent years [17]. The upper bound of the settling time is displayed explicitly as an adjustable control parameter [18]. Torres et al. [19] designed a formulation for robust predefined-time stability of second-order system and proved the reliability of the scheme through simulation. Zhang et al. [20] applied a predefined-time SMC to a launch vehicle attitude control. A robust predefined-time SMC was proposed with external disturbances [21]. However, the application of these predefined-time SMC methods is limited by a singular problem, and the studies on nonsingular predefined-time SMC have rarely been reported.

There is insufficient attention paid to the system transient tracking performance, although the above controllers can guarantee good steady-state control performance. Prescribed performance control, which consists of prescribed performance function (PPF) and error transformation, ensures that the attitude tracking errors converge at a specified rate to a preset arbitrarily small set of residuals while guaranteeing that the maximum overshoot value is smaller than the predefined value. More recently, Bu et al. [22] proposed a prescribed performance controller to obtain an expected transient performance for hypersonic vehicle. Zhang et al. [23] designed a controller with prescribed transient performance to solve attitude control problem. Luo et al. [24] proposed a low-complexity prescribed performance method for vehicles under unknown dynamics errors and external disturbances. However, the above PPF cannot freely adjust its decreasing rate in the transient process without changing the time to reach the steady state. This makes it difficult for PPF to meet different task requirements in practical applications.

To overcome the drawbacks mentioned above, a novel nonsingular predefined-time sliding mode control method for VTHL RLV is proposed. The principal innovations are summarized as follows:

- A novel predefined-time PPF (PTPPF) is proposed. The convergence time in the transient phase of PTPPF to the steady state can be flexibly adjusted by changing one parameter. Moreover, the decreasing rate of PTPPF in transient phase can also be adjusted by changing one parameter on the premise of not changing the convergence time of PPF to reach steady state.

- A novel predefined-time sliding mode surface (SMS) is proposed. When the attitude tracking errors moving on the SMS, they can converge to the origin within a predefined time, and the value of convergence time can be adjusted by one parameter.
- Based on PTPPF and the novel SMS, the RLV control system with a novel nonsingular controller is proposed to prevent the attitude tracking errors from exceeding the prescribed performance bounds. Meanwhile, the singularity can be avoided directly without using piecewise continuous functions.

The remainder of this paper is organized as follows. The dynamic model of RLV and preliminaries are introduced in Section 2. The PTPPF and the error transformation are proposed in Section 3. In Section 4, the nonsingular predefined-time SMS and controller are proposed. The simulation results are shown in Section 5, and Section 6 presents the conclusions of this study.

2. Problem Formulation and Preliminary

2.1. Problem Statement

In this paper, the attitude model of RLV in the reentry phase is described as follows [12]:

$$\begin{cases} \dot{\alpha} = \frac{\sin \sigma}{\cos \beta} \left[\dot{\psi} \cos \gamma - \dot{\theta} \sin \psi \sin \gamma + (\dot{\varphi} + \omega_E)(\cos \theta \cos \psi \sin \gamma - \sin \theta \cos \gamma) \right] \\ \quad - \frac{\cos \sigma}{\cos \beta} \left[\dot{\gamma} - \dot{\theta} \cos \psi - (\dot{\varphi} + \omega_E) \cos \theta \sin \psi \right] \\ \quad - p \cos \alpha \tan \beta + q - r \sin \alpha \tan \beta \\ \dot{\beta} = p \sin \alpha - r \cos \alpha + \sin \sigma \left[\dot{\gamma} - \dot{\theta} \cos \psi + (\dot{\varphi} + \omega_E) \cos \theta \sin \psi \right] \\ \quad + \cos \sigma \left[\dot{\psi} \cos \gamma - \dot{\theta} \sin \psi \sin \gamma - (\dot{\varphi} + \omega_E)(\cos \theta \cos \psi \sin \gamma - \sin \theta \cos \gamma) \right] \\ \dot{\sigma} = -p \cos \alpha \cos \beta - q \sin \beta - r \sin \alpha \cos \beta + \dot{\alpha} \sin \beta - \dot{\psi} \sin \gamma \\ \quad - \dot{\theta} \sin \psi \cos \gamma + (\dot{\varphi} + \omega_E)(\cos \theta \cos \psi \cos \gamma + \sin \theta \sin \gamma) \\ \dot{p} = \frac{J_{zz} M_x}{J_{xx} J_{zz} - J_{xz}^2} + \frac{J_{xz} M_z}{J_{xx} J_{zz} - J_{xz}^2} + \frac{(J_{xx} - J_{yy} + J_{zz}) J_{xz}}{J_{xx} J_{zz} - J_{xz}^2} p q + \frac{(J_{yy} - J_{zz}) J_{zz} - J_{xz}^2}{J_{xx} J_{zz} - J_{xz}^2} q r \\ \dot{q} = \frac{M_y}{J_{yy}} + \frac{J_{xz}(r^2 - p^2)}{J_{yy}} + \frac{(J_{zz} - J_{xx})}{J_{yy}} p r \\ \dot{r} = \frac{J_{xz} M_x}{J_{xx} J_{zz} - J_{xz}^2} + \frac{J_{xx} M_z}{J_{xx} J_{zz} - J_{xz}^2} + \frac{(J_{xx} - J_{yy} + J_{zz}) J_{xz}}{J_{xx} J_{zz} - J_{xz}^2} p q + \frac{(J_{yy} - J_{zz} - J_{xx}) J_{xz}}{J_{xx} J_{zz} - J_{xz}^2} q r \end{cases}, \quad (1)$$

where θ and φ represent the latitude and the longitude, respectively. q , r , and p are the pitch angular rate, yaw angular rate, and roll angular rate, respectively. β , α , and σ represent the sideslip, attack, and bank angles, respectively. ψ and γ denote the heading and flight path angles, respectively. J_{ij} ($i = x, y, z; j = x, y, z$) denotes the moment of inertia, M_i ($i = x, y, z$) is the external lumped moment, and ω_E is the Earth's rotation velocity.

Equation (1) can be written as

$$\begin{cases} \dot{\Lambda} = R\omega + f \\ J\dot{\omega} = \Omega J\omega + M_{air} + M_c + \Delta d \end{cases}, \quad (2)$$

where $\Lambda = [\alpha, \beta, \sigma]^T$, and $\omega = [p, q, r]^T$. Δd represents the external disturbances and parameter uncertainties. The inertia matrix is $J \in \mathbb{R}^{3 \times 3}$, while $M_c \in \mathbb{R}^3$ denotes the control moment. The matrices R and Ω and the aerodynamic moment M_{air} are represented as

$$R = \begin{bmatrix} -\cos \alpha \tan \beta & 1 & -\sin \alpha \tan \beta \\ \sin \alpha & 0 & -\cos \alpha \\ -\cos \alpha \cos \beta & -\sin \beta & -\sin \alpha \cos \beta \end{bmatrix}, \quad (3)$$

$$\Omega = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}, \quad (4)$$

$$\mathbf{M}_{air} = \begin{bmatrix} \left(\frac{m_x^p L_r}{V} + m_x^\beta \beta \right) q_0 S_r L_r \\ \left(\frac{m_y^q L_r}{V} + m_y^\alpha \alpha \right) q_0 S_r L_r \\ \left(\frac{m_z^r L_r}{V} + m_z^\beta \beta \right) q_0 S_r L_r \end{bmatrix}, \quad (5)$$

where the air density is ρ , and V is the velocity of RLV. m_x^β , m_y^α , and m_z^β are the static stability moment coefficients of RLV; the damping moment coefficients are m_x^p , m_y^q , and m_z^r . $q_0 = 0.5\rho V^2$. The reference area of RLV is denoted as S_r , and the reference length of RLV is denoted as L_r .

Notation 1. For the given vector $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$, define $\text{sig}^a \mathbf{x} = [|x_1|^a \text{sign}(x_1), |x_2|^a \text{sign}(x_2), \dots, |x_n|^a \text{sign}(x_n)]^T$ and $|\mathbf{x}|^a = [|x_1|^a, |x_2|^a, \dots, |x_n|^a]^T$, where $\text{sign}(\cdot)$ denotes the sign function and a is a real number.

Assumption 1. The external disturbances, parameter uncertainties and their variation rates are assumed to be bounded $|\Delta \mathbf{d}| \leq \mathbf{L}_d$, $|\Delta \dot{\mathbf{d}}| \leq \mathbf{L}_{dd}$, where $\mathbf{L}_d$ and $\mathbf{L}_{dd}$ are unknown positive vectors [25].

Denote $\mathbf{z}_1 = \boldsymbol{\Lambda} - \boldsymbol{\Lambda}_C$ and $\mathbf{z}_2 = \dot{\boldsymbol{\Lambda}} - \dot{\boldsymbol{\Lambda}}_C$. $\boldsymbol{\Lambda}_C = [\alpha_C, \beta_C, \sigma_C]^T$ is the guidance command vector. Then, the attitude tracking error model of RLV can be established as follows:

$$\begin{cases} \dot{\mathbf{z}}_1 = \mathbf{z}_2 \\ \dot{\mathbf{z}}_2 = \mathbf{d} + (\ddot{\mathbf{R}}\mathbf{R}^{-1} + \mathbf{R}\mathbf{J}^{-1}\boldsymbol{\Omega}\mathbf{J}\mathbf{R}^{-1})\mathbf{z}_2 + \mathbf{R}\mathbf{J}^{-1}(\mathbf{M}_c + \mathbf{M}_{air}) + (\ddot{\mathbf{R}}\mathbf{R}^{-1} + \mathbf{R}\mathbf{J}^{-1}\boldsymbol{\Omega}\mathbf{J}\mathbf{R}^{-1})\dot{\boldsymbol{\Lambda}}_C - \ddot{\boldsymbol{\Lambda}}_C \end{cases}, \quad (6)$$

where $\mathbf{d} = -\ddot{\mathbf{R}}\mathbf{R}^{-1}\mathbf{f} + \mathbf{R}\mathbf{J}^{-1}\boldsymbol{\Omega}\mathbf{J}\mathbf{R}^{-1}\mathbf{f} + \mathbf{R}\mathbf{J}^{-1}\Delta \mathbf{d} + \dot{\mathbf{f}}$.

Control Objective: In the presence of the external disturbances and parameter uncertainties, this paper aimed to design a novel nonsingular predefined-time controller for RLV. $\mathbf{z}_1$ can reach a predefined small set of residuals within predefined time. $\mathbf{z}_1$ can always be constrained within the range specified by PTPPF. Meanwhile, the control input $\mathbf{M}_c$ is nonsingular.

2.2. Preliminary

Consider the autonomous dynamical system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}; \boldsymbol{\varphi}), \mathbf{x}(0) = 0, \quad (7)$$

where $\mathbf{x} \in \mathbb{R}^n$ denotes the state, and $\mathbf{f}: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a nonlinear locally Lebesgue-integrable function. $\boldsymbol{\varphi} \in \mathbb{R}^l$ represents the tunable parameters.

Definition 1 ([26]). The origin of Equation (7) is said to be globally stable in predefined time if, for any $T_c \in \mathbb{R}_+$, there exists some $\boldsymbol{\varphi} \in \mathbb{R}^l$ such that $\|\mathbf{x}(t)\| = 0$ holds for $\forall t \geq T_c$ regardless of $\mathbf{x}(t_0)$, where $\|\cdot\|$ stands for the Euclidean norm.

Definition 2 ([27]). A smooth function $\rho(t): \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is called a performance function if both of the following conditions are satisfied:

- (1) $\rho(t)$ is decreasing and positive;
- (2) $\lim_{t \rightarrow \infty} \rho(t) = \rho_\infty > 0$.

Lemma 1 ([28]). For the system in Equation (7), assume that the Lyapunov function $V(\mathbf{x}): \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ satisfies

$$\dot{V}(\mathbf{x}) \leq -\frac{\pi}{aT_p} \left(V^{1-\frac{a}{2}}(\mathbf{x}) + V^{1+\frac{a}{2}}(\mathbf{x}) \right), \quad (8)$$

where $T_p > 0$ and $0 < a < 1$ are positive constants. Then, this system is globally stable in predefined time, and T_p is the convergence time.

Theorem 1. For the system in Equation (7), if the radially unbounded Lyapunov function $V(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ satisfies

$$\dot{V}(\mathbf{x}) \leq -\frac{\sqrt{3}\pi + 2\sqrt{3}\arctan\frac{1}{\sqrt{3}}}{(3+2\alpha)T_c} \left(V^{1/2-\alpha/3}(\mathbf{x}) + V^{2+2\alpha/3}(\mathbf{x}) \right), \quad (9)$$

where $-1.5 < \alpha < 1.5$, $T_c > 0$, then the system is globally stable in predefined time. Meanwhile, the convergence time is T_c .

Proof of Theorem 1. Please refer to Appendix A. $\square$

Lemma 2. For any $y_i > 0$ ($i = 1, 2, \dots, n$), there is

$$\sum_{i=1}^n y_i^{\bar{\gamma}} \geq \left(\sum_{i=1}^n y_i \right)^{\bar{\gamma}}, \text{ if } 0 < \bar{\gamma} \leq 1 \quad \sum_{i=1}^n y_i^{\bar{\gamma}} \geq n^{1-\bar{\gamma}} \left(\sum_{i=1}^n y_i \right)^{\bar{\gamma}}, \text{ if } \bar{\gamma} > 1, \quad (10)$$

where $\bar{\gamma} > 0$ [29].

3. PTPPF and Error Transformation

In this section, a PTPPF is designed, and then the inequality constraint is converted to an unconstrained one, laying a foundation for the nonsingular predefined-time attitude controller design.

3.1. Performance Function Design

To ensure that the transient and steady performance of the tracking errors evolves within the reasonable boundaries, the description of the prescribed performance of the z_1 is expressed as

$$-\rho_i(t) < z_{1i}(t) < \rho_i(t), \quad (11)$$

where $i = 1, 2, 3$, and $\rho_i(t)$ denotes a novel PTPPF, which is described as follows:

$$\rho_i(t) = \begin{cases} a_1 + a_2 \arctan\left(\frac{t}{T_f}\right) + a_3 e^{-k\frac{t}{T_f}} + a_4 e^{-k\left(\frac{t}{T_f}\right)^2} & , 0 \leq t \leq T_f, \\ \rho_{i\infty} & , t > T_f \end{cases} \quad (12)$$

$$\begin{aligned} a_1 &= \rho_{i0} + 3a_4, & a_2 &= -4ke^{-k}a_4 \\ a_3 &= -4a_4, & a_4 &= \frac{\rho_{i\infty} - \rho_{i0}}{3 - \pi ke^{-k} - 3e^{-k}}, \end{aligned} \quad (13)$$

where ρ_{i0} is the maximum overshoot, and $\rho_{i\infty}$ is the prescribed steady-state boundary. ρ_{i0} and $\rho_{i\infty}$ can be flexibly adjusted according to the control requirements. k is a positive constant, $0.5 < k < 3$, which determines the decreasing rate of ρ_i when $0 \leq t \leq T_f$. T_f denotes the prescribed terminal time that ρ_i reaches $\rho_{i\infty}$.

Theorem 2. The PTPPF $\rho_i(t)$ satisfies $\dot{\rho}_i(t) < 0$ when $0 \leq t \leq T_f$ and $\lim_{t \rightarrow T_f^-} \rho_i = \lim_{t \rightarrow T_f^+} \rho_i = \rho_{i\infty}$. Meanwhile, it has a continuous first-order derivative with respect to time.

Proof of Theorem 2. When $0 \leq t \leq T_f$, the derivative of Equation (12) is

$$\begin{aligned} \dot{\rho}_i &= \frac{a_2}{T_f(1+(t/T_f)^2)} - \frac{ka_3}{T_f} e^{-k\frac{t}{T_f}} - \frac{2ka_4t}{T_f^2} e^{-k\left(\frac{t}{T_f}\right)^2} \\ &= -\frac{4ke^{-k}a_4}{T_f(1+(t/T_f)^2)} + \frac{4ka_4}{T_f} e^{-k\frac{t}{T_f}} - \frac{2ka_4t}{T_f^2} e^{-k\left(\frac{t}{T_f}\right)^2}, \end{aligned} \quad (14)$$

$$\ddot{\rho}_i = \frac{-2a_2t}{T_f^3 \left(1 + \left(t/T_f\right)^2\right)^2} - \frac{k^2a_3}{T_f^2} e^{-k\frac{t}{T_f}} - \frac{2ka_4}{T_f^2} e^{-k\left(\frac{t}{T_f}\right)^2} + \frac{4k^2a_4t^2}{T_f^4} e^{-k\left(\frac{t}{T_f}\right)^2}. \quad (15)$$

According to Equation (13), it is obvious that $\lim_{t \rightarrow T_f^-} \rho_i = \lim_{t \rightarrow T_f^+} \rho_i$ and $\lim_{t \rightarrow T_f^-} \dot{\rho}_i = \lim_{t \rightarrow T_f^+} \dot{\rho}_i = 0$. The PTPPF and its first-order derivative are continuous with respect to time.

Considering Equation (13) and the range of k , it can be calculated that $a_4 < 0$. To reduce computational cost, the proof of $\dot{\rho}_i < 0$ can be converted to $y > 0$ on $[0, T_f]$, where y is

$$y = -\frac{4ke^{-k}}{T_f \left(1 + \left(t/T_f\right)^2\right)} + \frac{4k}{T_f} e^{-k\frac{t}{T_f}} - \frac{2kt}{T_f^2} e^{-k\left(\frac{t}{T_f}\right)^2}, \quad 0 \leq t < T_f. \quad (16)$$

Let $x = t/T_f$; Equation (16) can then be simplified as follows:

$$y = -\frac{4ke^{-k}}{T_f(1+x^2)} + \frac{4k}{T_f} e^{-kx} - \frac{2kx}{T_f} e^{-kx^2}, \quad 0 \leq x < 1, \quad (17)$$

where $y(0) > 0$ and $y(1) = 0$. Under the condition $0 \leq x < 1$, the derivative of y is obtained as

$$\dot{y} = \frac{8ke^{-k}x}{T_f(1+x^2)^2} - \frac{4k^2}{T_f} e^{-kx} - \frac{2k}{T_f} e^{-kx^2} + \frac{4k^2x^2}{T_f} e^{-kx^2}, \quad (18)$$

$$\ddot{y} = \frac{8ke^{-k}}{T_f} \left(\frac{1}{(1+x^2)^2} + \frac{4x^2}{(1+x^2)^3} \right) + \frac{4k^2}{T_f} \left[(ke^{-kx} - kx^3e^{-kx^2}) + (3xe^{-kx} - kx^3e^{-kx^2}) \right]. \quad (19)$$

We can get that $\ddot{y} > 0$, $\dot{y}(0) < 0$, and $\dot{y}(1) = 0$ when $0 \leq x < 1$. This means that $\dot{y}$ is in a trend of monotone increasing, and $\dot{y}$ will not be greater than zero because of $\dot{y}(1) = 0$. As a result, y is monotonically decreasing and greater than zero in $0 \leq x < 1$. Specifically, it can be concluded that $\dot{\rho}_i < 0$ for all $0 \leq t \leq T_f$. Thus, the proof of Theorem 2 is complete. $\square$

Some studies have been conducted to affirm the property of the proposed PTPPF. The parameters are given as $\rho_{i0} = 10$ and $\rho_{i\infty} = 0.01$. In Figure 2, we can see that the variate ρ_i converges to the prescribed steady-state error range within a predefined time T_f . Figure 3 shows that $\rho_i(t)$ satisfies $\dot{\rho}_i(t) < 0$ when $0 \leq t \leq T_f$ and $\lim_{t \rightarrow T_f^-} \rho_i = \lim_{t \rightarrow T_f^+} \rho_i = \rho_{i\infty}$. The decreasing rate in the transient stage can be increased by selecting a larger value k . The decreasing rate of PTPPF in the transient phase can be adjusted by changing one parameter k on the premise of not changing the convergence time of PPF to reach steady state.

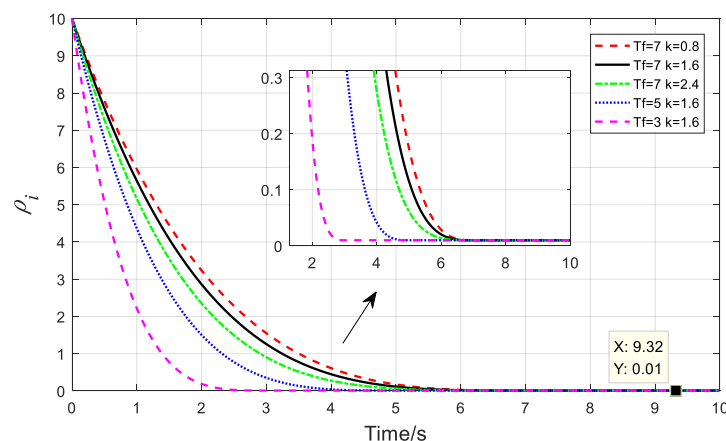


Figure 2. The simulation results with different T_f and k .

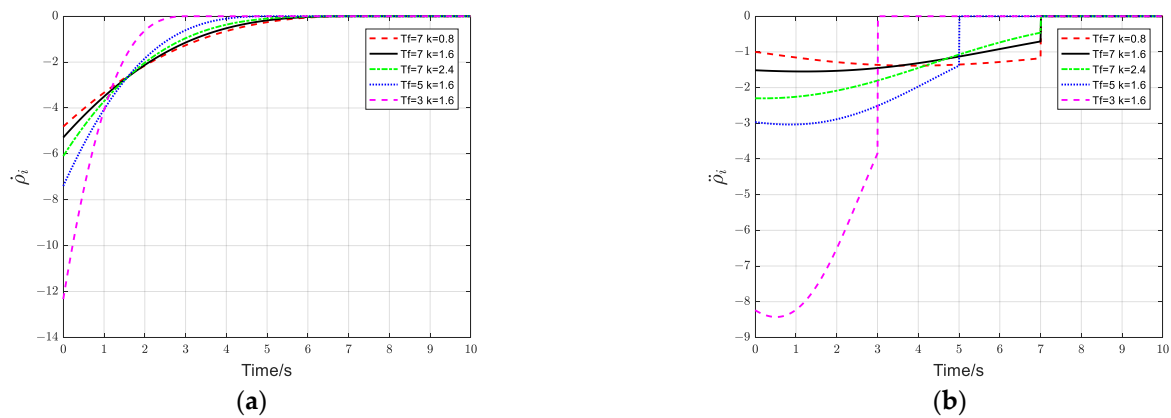


Figure 3. The simulation results of PTPPF: (a) the curves of $\dot{\rho}_i$ for five cases; (b) the curves of $\ddot{\rho}_i$ for five cases.

Remark 1. The function ρ_i has a simple polynomial form. The time to reach $\rho_{i\infty}$ is prescribed explicitly by one parameter T_f , which can be adjusted to meet practical application requirements. Unlike the existing performance functions in [30–32], the decreasing rate of ρ_i can be adjusted by one parameter k when $0 \leq t \leq T_f$.

3.2. Transformation Method

As is known to all, it is very difficult to design a controller because of the attitude tracking error constraint in Equation (11). To avoid this issue, the following transformation method is adopted to transform the constrained problem into an unconstrained one [20]:

$$\sigma_{ei}(t) = \frac{z_{1i}(t)}{\rho_i(t)}, \quad i = 1, 2, 3. \quad (20)$$

In this paper, we define the equivalent unconstrained equation as

$$e_{1i} = \tan\left(\frac{\pi}{2}\sigma_{ei}(t)\right). \quad (21)$$

Taking the first and second derivatives of e_i , there is

$$\begin{aligned} \dot{e}_{1i} &= A_i \left(z_{2i} - \frac{\dot{\rho}_i}{\rho_i} z_{1i} \right) \\ \ddot{e}_{1i} &= P_i \dot{e}_{1i} + A_i (\ddot{z}_{2i} - \Phi_i) \end{aligned} \quad i = 1, 2, 3, \quad (22)$$

where $A_i = \frac{\pi}{2\rho_i}(1 + e_{1i}^2)$, $P_i = \frac{2\pi}{2\rho_i} \left(z_{2i} - \frac{z_{1i}\dot{\rho}_i}{\rho_i} \right) e_{1i} - \frac{\dot{\rho}_i}{\rho_i}$, and $\Phi_i = \frac{z_{2i}\dot{\rho}_i\rho_i + z_{1i}\ddot{\rho}_i\rho_i - z_{1i}\dot{\rho}_i^2}{\rho_i^2}$. Equation (22) can be rewritten as

$$\begin{aligned} \dot{e}_1 &= A \left(z_2 - \text{diag}\left(\frac{\dot{\rho}_i}{\rho_i}\right) z_1 \right) = e_2 \\ \dot{e}_2 &= P \dot{e}_1 + A (\ddot{z}_2 - \Phi) \\ &= P \dot{e}_1 + A \left(\begin{aligned} &(\ddot{R}R^{-1} + RJ^{-1}\Omega JR^{-1})z_2 + RJ^{-1}(M_c + M_{air}) \\ &+ (\ddot{R}R^{-1} + RJ^{-1}\Omega JR^{-1})\dot{\Lambda}_C - \ddot{\Lambda}_C - \Phi \end{aligned} \right) + D, \end{aligned} \quad (23)$$

where $D = Ad$, $e_1 = [e_{11}, e_{12}, e_{13}]^T$.

Remark 2. After error transformation, the inequality-constrained problem is transformed into an equality unconstrained one. The smooth function $\frac{2}{\pi}\arctan(e_{1i})$ has the properties $\lim_{e_{1i} \rightarrow +\infty} \frac{2}{\pi}\arctan(e_{1i}) = 1$ and $\lim_{e_{1i} \rightarrow -\infty} \frac{2}{\pi}\arctan(e_{1i}) = -1$, $i = 1, 2, 3$. The transformed error function in Equation (21) ensures that the steady-state tracking errors can converge to zero when the transformed error e_1 approaches zero.

4. Nonsingular Predefined-Time Attitude Controller Design

In the traditional predefined-time terminal SMC design process, it can be seen that the negative fractional power term always appears in the control law, which may result in a singularity if $e_1 = 0, e_2 \neq 0$ [20]. Thus, in this section, a novel nonsingular terminal sliding controller without using piecewise continuous functions is proposed.

Firstly, a novel predefined-time terminal SMS is expressed as follows:

$$S = \text{sig}^{2+2a/3} e_1 + \text{sig}^{\frac{12+4a}{3-2a}} \left(\frac{(3+2a)T_c}{\sqrt{3}\pi + 2\sqrt{3}\arctan(1/\sqrt{3})} e_2 + \text{sig}^{2+2a/3} e_1 \right), \quad (24)$$

where $T_c > 0$ and $-1.5 < a < -0.75$.

Theorem 3. *The phenomenon of the singularity is avoided. Once the tracking error e_1 and its derivative e_2 make $S = [S_1, S_2, S_3]^T = 0$, e_1 and e_2 will converge to zero within the predefined time T_c .*

Proof of Theorem 3. When $e_1 = [e_{11}, e_{12}, e_{13}]^T$ and e_2 evolve on the expected SMS, we have

$$\begin{aligned} \text{sig}^{1/2-a/3} e_{1i} + \frac{(3+2a)T_c}{\sqrt{3}\pi + 2\sqrt{3}\arctan \frac{1}{\sqrt{3}}} e_{2i} + \text{sig}^{2+2a/3} e_{1i} &= 0 \\ \Downarrow \\ -\frac{(3+2a)T_c}{\sqrt{3}\pi + 2\sqrt{3}\arctan \frac{1}{\sqrt{3}}} e_{2i} &= \text{sig}^{2+2a/3} e_{1i} + \text{sig}^{1/2-a/3} e_{1i} \quad , \\ \Downarrow \\ e_{2i} \text{sign}(e_{1i}) &= -\frac{\sqrt{3}\pi + 2\sqrt{3}\arctan \frac{1}{\sqrt{3}}}{(3+2a)T_c} \left(|e_{1i}|^{2+2a/3} + |e_{1i}|^{1/2-a/3} \right) \end{aligned} \quad (25)$$

where $i = 1, 2, 3$. Choosing the Lyapunov function, $V_i = |e_{1i}|$ and the derivative of V_i can be expressed as

$$\begin{aligned} \dot{V}_i &= e_{2i} \text{sign}(e_{1i}) \\ &= -\frac{\sqrt{3}\pi + 2\sqrt{3}\arctan \frac{1}{\sqrt{3}}}{(3+2a)T_c} \left(|e_{1i}|^{2+2a/3} + |e_{1i}|^{1/2-a/3} \right) . \\ &\leq -\frac{\sqrt{3}\pi + 2\sqrt{3}\arctan \frac{1}{\sqrt{3}}}{(3+2a)T_c} \left(V_i^{1/2-a/3} + V_i^{2+2a/3} \right) \end{aligned} \quad (26)$$

By applying Theorem 1, the proposed nonsingular SMS is predefined-time stable within T_c . Meanwhile, e_1 and e_2 will reach the SMS $S = 0$, whose proof is given in Theorem 4. The proof of Theorem 3 is completed. $\square$

The simulation below affirms the effectiveness of the proposed predefined-time SMS. The different values of a and T_c are taken when the initial values satisfy $e_{1i} = 1, e_{2i} = 0$, and $S_i = 0, i = 1, 2, 3$. In Figure 4, we can see that e_{1i} converges to zero within T_c .

Then, a predefined-time extended state observer (PTESO) is introduced, which can effectively estimate the disturbances D . The form of PTESO is expressed as follows:

$$\begin{aligned} \varphi_1 &= e_1 - \hat{e}_1 \\ \dot{\hat{e}}_1 &= e_2 + \Psi \zeta \bar{\Xi} l_1 \varphi_1 + (1 - \Psi) k_1 Y^{1/3} \text{sig}^{2/3}(\varphi_1) \\ \dot{\hat{e}}_2 &= \hat{\Delta} + \Psi \zeta^2 \bar{\Xi}^2 l_2 \varphi_1 + (1 - \Psi) k_2 Y^{2/3} \text{sig}^{1/3}(\varphi_1) + H + U' \\ \dot{\hat{\Delta}} &= \Psi \zeta^3 \bar{\Xi}^3 l_3 \varphi_1 + (1 - \Psi) k_3 Y \text{sig}^0(\varphi_1) \end{aligned} \quad (27)$$

where $H = P\dot{e}_1 + A \begin{pmatrix} (\dot{R}R^{-1} + RJ^{-1}\Omega JR^{-1})z_2 - \ddot{\Delta}_C - \Phi \\ + (\dot{R}R^{-1} + RJ^{-1}\Omega JR^{-1})\dot{\Delta}_C \end{pmatrix}$, $U = ARJ^{-1}(M_C + M_{air})$,

$\bar{\Xi} = \sec^2\left(\frac{\pi t}{2\bar{t}_f}\right)$, $t \in [0, \bar{t}_f)$, and $\bar{t}_f$ is the convergence time. The time switching function is

$$\Psi = \begin{cases} 1, & t \in [0, \bar{t}_f) \\ 0, & t \in [\bar{t}_f, +\infty) \end{cases}. \quad (28)$$

Y is a constant to be designed. $k_{1,2,3}$ are the parameters satisfying $k_3 > Y$, $k_1 = 3.34k_3^{1/3}$, and $k_2 = 5.3k_3^{2/3}$. The stability of the PTESO can be referenced in Cui's paper [33]. According to the previous work, the proposed PTESO has high disturbance estimation accuracy, i.e., $\lim_{t > \bar{t}_f} \|\hat{\Delta} - D\| = 0$.

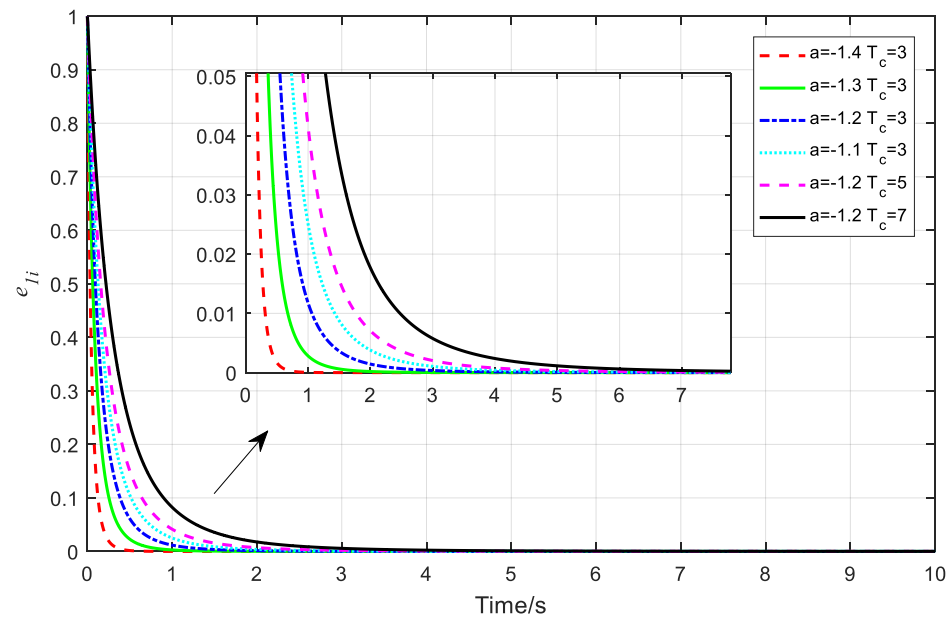


Figure 4. The simulation results of the tracking errors.

Considering the predefined-time terminal SMS in Equation(24), the novel nonsingular controller can be designed as

$$M_c = -\left(ARJ^{-1}\right)^{-1} \begin{pmatrix} A\left(\left(\dot{R}R^{-1} + RJ^{-1}\Omega JR^{-1}\right)z_2 + \left(\dot{R}R^{-1} + RJ^{-1}\Omega JR^{-1}\right)\dot{\Lambda}_C - \ddot{\Lambda}_C - \Phi\right) \\ + \Theta\left(2 + \frac{2a}{3}\right)\text{diag}\left(|e_{1i}|^{1+2a/3}\right)(\Gamma + e_2) \\ + \frac{\pi}{2\eta T_p}\text{diag}\left(|S_i|^{1-\eta/2} + 3^{\eta/2}|S_i|^{1+\eta/2}\right)\text{sign}(S) + Pe_1 + \hat{\Delta} \end{pmatrix} - M_{air}, \quad (29)$$

$$\Gamma = \frac{3-2a}{12+4a}\Theta\text{sig}^{\frac{-8a-6}{3-2a}}\left(e_2/\Theta + \text{sig}^{2+\frac{2}{3}a}e_1\right) + \frac{3-2a}{12+4a}\Theta\text{sig}\left(e_2/\Theta + \text{sig}^{2+\frac{2}{3}a}e_1\right), \quad (30)$$

where $\Theta = \frac{\sqrt{3}\pi + 2\sqrt{3}\arctan\frac{1}{\sqrt{3}}}{(3+2a)T_c}$, $i = 1, 2, 3$. $0 < \eta < 1$, $T_p > \bar{t}_f > 0$.

The controller in Equation (29) and the auxiliary function Γ are nonsingular because $-1.5 < a < -0.75$. By combining Equation (24) and Equation (29), the following equation is established:

$$\frac{12+4a}{3-2a}\left|e_2/\Theta + \text{sig}^{2+\frac{2}{3}a}e_1\right|^{\frac{9+6a}{3-2a}}\Gamma = e_2 + \Theta S. \quad (31)$$

Theorem 4. Under Assumption 1, if the controller is designed as Equation (29), then e_1 and e_2 will reach $S = 0$.

Proof of Theorem 4. Firstly, differentiating S leads to

$$\begin{aligned}
\dot{S} &= (2 + \frac{2}{3}a) \text{diag}(|e_{1i}|^{1+2a/3}) e_2 + \Phi \left(\frac{\dot{e}_2}{\Theta} + (2 + \frac{2a}{3}) \text{diag}(|e_{1i}|^{1+2a/3}) e_2 \right) \\
&= (2 + \frac{2}{3}a) \text{diag}(|e_{1i}|^{1+2a/3}) e_2 + \Phi \left(\frac{1}{\Theta} \left(P\dot{e}_1 + A \begin{pmatrix} (\dot{R}R^{-1} + RJ^{-1}\Omega JR^{-1})z_2 + RJ^{-1}(M_c + M_{air}) \\ + (\dot{R}R^{-1} + RJ^{-1}\Omega JR^{-1})\dot{\Lambda}_C - \dot{\Lambda}_C - \Phi \end{pmatrix} + D \right) \right. \\
&\quad \left. + (2 + \frac{2a}{3}) \text{diag}(|e_{1i}|^{1+2a/3}) e_2 \right) \\
&= (2 + \frac{2}{3}a) \text{diag}(|e_{1i}|^{1+2a/3}) e_2 + \Phi \left(\frac{1}{\Theta} \left(\Theta(2 + \frac{2a}{3}) \text{diag}(|e_{1i}|^{1+2a/3}) (\Gamma + e_2) \right. \right. \\
&\quad \left. \left. + \frac{\pi}{2\eta T_p} \text{diag}(|S_i|^{1-\eta/2} + 3^{\eta/2}|S_i|^{1+\eta/2}) \text{sign}(S) - \hat{\Delta} + D \right) \right. \\
&\quad \left. + (2 + \frac{2a}{3}) \text{diag}(|e_{1i}|^{1+2a/3}) e_2 \right) \\
&= -(2 + \frac{2}{3}a) \text{diag}(|e_{1i}|^{1+2a/3}) \left(\Theta \text{sig}^{2+2a/3}(e_1) + \Theta \text{sig}^{\frac{12+4a}{3-2a}} \left(\frac{1}{\Theta} e_2 + \text{sig}^{2+2a/3}(e_1) \right) \right) \\
&\quad + \frac{1}{\Theta} \Phi \left(-\frac{\pi}{2\eta T_p} \text{diag}(|S_i|^{1-\eta/2} + 3^{\eta/2}|S_i|^{1+\eta/2}) \text{sign}(S) - \hat{\Delta} + D \right) \\
&= \frac{1}{\Theta} \Phi \left(-\frac{\pi}{2\eta T_p} \text{diag}(|S_i|^{1-\eta/2} + 3^{\eta/2}|S_i|^{1+\eta/2}) \text{sign}(S) - \hat{\Delta} + D \right) - (2 + \frac{2}{3}a) \text{diag}(|e_{1i}|^{1+2a/3}) S,
\end{aligned} \tag{32}$$

where $\Phi = \frac{12+4a}{3-2a} \text{diag} \left(\left| \frac{e_{2i}}{\Theta} + \text{sig}^{2+2a/3}(e_{1i}) \right|^{\frac{9+2a}{3-2a}} \right)$, $i = 1, 2, 3$.

Choosing the Lyapunov function yields

$$V_S = |S_1| + |S_2| + |S_3|, \tag{33}$$

where $S = [S_1, S_2, S_3]^T$. By differentiating Equation (33), on the basis of Lemma 2, we have

$$\begin{aligned}
\dot{V}_S &= \dot{|S_1|} + \dot{|S_2|} + \dot{|S_3|} \\
&= \text{sign}(S)^T \dot{S} \\
&= \text{sign}(S)^T \left(\frac{1}{\Theta} \Phi \left(-\frac{\pi}{2\eta T_p} \text{diag}(|S_i|^{1-\eta/2} + 3^{\eta/2}|S_i|^{1+\eta/2}) \text{sign}(S) - \hat{\Delta} + D \right) \right. \\
&\quad \left. - (2 + \frac{2}{3}a) \text{diag}(|e_{1i}|^{1+2a/3}) S \right) \\
&\leq -\min \left(\frac{1}{\Theta} \Phi_{ii} \right) \frac{\pi}{2\eta T_p} \left(V_S^{1-\eta/2} + V_S^{1+\eta/2} \right) + \frac{1}{\Theta} \text{sign}(S)^T \Phi (-\hat{\Delta} + D)
\end{aligned} \tag{34}$$

According to Equation (27), the disturbances D can be precisely estimated by PTESO when $\bar{t}_f < t < T_p$. Specifically, the following inequality holds.

$$\begin{aligned}
\dot{V}_S &\leq -\min \left(\frac{1}{\Theta} \Phi_{ii} \right) \frac{\pi}{2\eta T_p} \left(V_S^{1-\eta/2} + V_S^{1+\eta/2} \right) \\
&\leq -\frac{1}{\Theta} \Phi_{jj} \frac{\pi}{2\eta T_p} \left(V_S^{1-\eta/2} + V_S^{1+\eta/2} \right) \\
&\leq G \left(-\frac{\pi}{2\eta T_p} \left(V_S^{1-\eta/2} + V_S^{1+\eta/2} \right) \right),
\end{aligned} \tag{35}$$

where $i = 1, 2, 3$. $j = i$ when $\Phi_{jj} = \min(\Phi_{ii})$. Then, we define $\psi = \frac{e_{2j}}{\Theta} + \text{sig}^{2+2a/3}(e_{1j})$, $G = \frac{1}{\Theta} \Phi_{jj} = \frac{12+4a}{3-2a} \frac{(3+2a)T_c}{\sqrt{3\pi+2\sqrt{3}\arctan\frac{1}{\sqrt{3}}}} |\psi|^{\frac{9+2a}{3-2a}}$. Thus, the state spaces are divided into two areas, i.e., $\Omega_1 = \{(e_{1j}, e_{2j}) | G \geq 1\}$ and $\Omega_2 = \{(e_{1j}, e_{2j}) | G < 1\}$. If the system states are in Ω_1 , we have

$$\begin{aligned}
\dot{V}_S &\leq G \left(-\frac{\pi}{2\eta T_p} \left(V_S^{1-\eta/2} + V_S^{1+\eta/2} \right) \right) \\
&\leq -\frac{\pi}{2\eta T_p} \left(V_S^{1-\eta/2} + V_S^{1+\eta/2} \right).
\end{aligned} \tag{36}$$

According to Lemma 1, the attitude tracking errors will reach $S = 0$ or Ω_2 within a predefined time. If the system states are in Ω_2 when $\psi \neq 0$, the SMS $S = 0$ is still an attractor. The remainder of the study demonstrates that the curve $\psi = 0$ is unattractive except for $S = 0$. There is $\Gamma = 0$ when $\psi = 0$, and the control command M_c is described as

$$M_c = -\left(ARJ^{-1}\right)^{-1} \begin{pmatrix} A\left(\left(\dot{R}R^{-1} + RJ^{-1}\Omega JR^{-1}\right)z_2 + \left(\dot{R}R^{-1} + RJ^{-1}\Omega JR^{-1}\right)\dot{\Lambda}_C - \ddot{\Lambda}_C - \Phi\right) \\ + \Theta\left(2 + \frac{2a}{3}\right)\text{diag}\left(|e_{1i}|^{1+2a/3}\right)e_2 \\ + \frac{\pi}{2\eta T_p}\text{diag}\left(|S_i|^{1-\eta/2} + 3^{\eta/2}|S_i|^{1+\eta/2}\right)\text{sign}(S) + P\dot{e}_1 + \hat{\Delta} \end{pmatrix} - M_{air}. \quad (37)$$

Differentiating ψ yields

$$\dot{\psi} = -\frac{\pi}{2\eta T_p}\left(|S_j|^{1-\eta/2} + 3^{\eta/2}|S_j|^{1+\eta/2}\right)\text{sign}(S_j). \quad (38)$$

It is obvious that $\dot{\psi} < 0$ for $S_j > 0$ and $\dot{\psi} > 0$ for $S_j < 0$. Therefore, the curve $\psi = 0$ is not an attractor except for $S = 0$; the attitude tracking errors will transgress the area Ω_2 into Ω_1 monotonically. $\square$

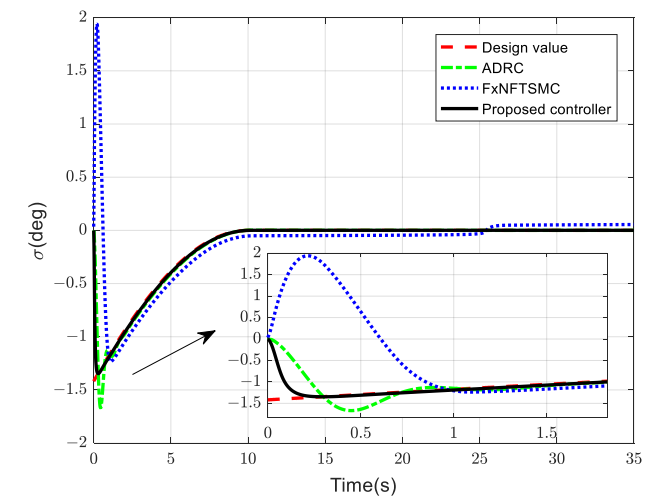
Remark 3. The transient and steady-state performance of the attitude tracking errors can be sufficiently guaranteed under the proposed PTPPF. Compared with the traditional nonsingular predefined-time controller [34,35], the singularity can be avoided without using piecewise continuous functions.

Remark 4. It is theoretically possible to design a very short convergence time for RLV attitude control, but this will lead to a very large control moment. Therefore, in practical application, the convergence time needs to be reasonably designed according to the mission requirement and the ability of RLV control system.

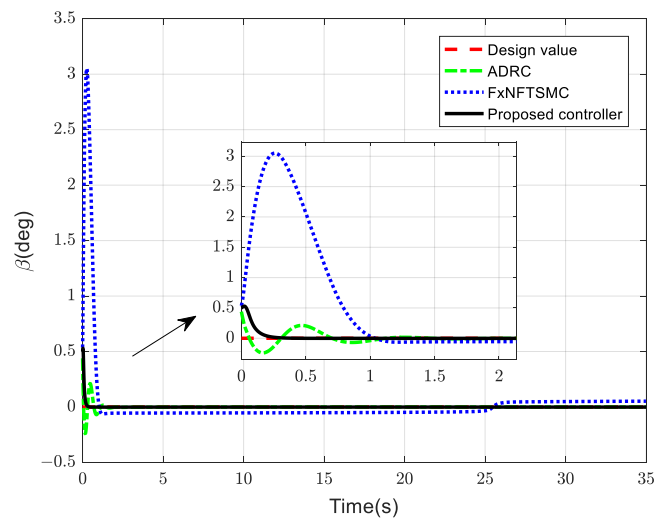
5. Simulation and Analysis

The simulation of a 6-DOF RLV dynamic is conducted in this section. The parameters of RLV are $J_{xx} = 130700 \text{ kg} \cdot \text{m}^2$, $J_{yy} = 1507000 \text{ kg} \cdot \text{m}^2$, $J_{zz} = 1493000 \text{ kg} \cdot \text{m}^2$, and $J_{xz} = 116000 \text{ kg} \cdot \text{m}^2$. The cross-section area and reference length are $S_{\text{ref}} = 73 \text{ m}^2$ and $L_r = 40 \text{ m}$, respectively. The initial state variables in the simulation are given as follows: $r = 6457 \text{ km}$, $\theta = -17.32^\circ$, $\varphi = 105.01^\circ$, $\sigma = 0.01^\circ$, $\beta = 0.49^\circ$, $\alpha = 45.01^\circ$, $p = 0.0007 \text{ rad/s}$, $q = 0.0 \text{ rad/s}$, $r = 0.0698 \text{ rad/s}$, $v = 2916 \text{ m/s}$, and $m = 23000 \text{ kg}$. Moreover, the parameters of PTPPF are set as $\rho_{i0} = 0.2\text{rad}$, $\rho_{i\infty} = 0.005\text{rad}$, $k = 1.6$, and $T_f = 1$. The parameters of the controller are $a = -1.2$, $T_c = 0.5$, $n = 0.5$, and $T_p = 0.5$. The parameter uncertainties of aerodynamic coefficients in the simulation are set to 20% bias, while 10% bias is set for the moment of inertia. Then, two different control simulations are conducted for comparison; one is for the fixed-time nonsingular fast terminal sliding mode control (FxNFTSMC) [14], and the other is for active disturbance rejection controller (ADRC) [36]. For FxNFTSMC, let $v_1 = 1.1$, $v_2 = 3.2$, $\lambda_1 = 1.6$, $\lambda_2 = 1.3$, $\gamma_1 = 1.3$, $\gamma_2 = 0.38$, $\chi = 1.5$, and $\lambda = 2.0$. In the ADRC, there is $\beta_1 = (150, 160, 160)$, $\beta_2 = (9.09 \times 10^3, 9.09 \times 10^3, 9.09 \times 10^3)$, $\beta_3 = (1 \times 10^5, 2 \times 10^3, 2 \times 10^3)$, $c = (0.4, 0.4, 0.4)$, $r = (810, 830, 305)$, and $h_0 = (0.1, 0.1, 0.1)$.

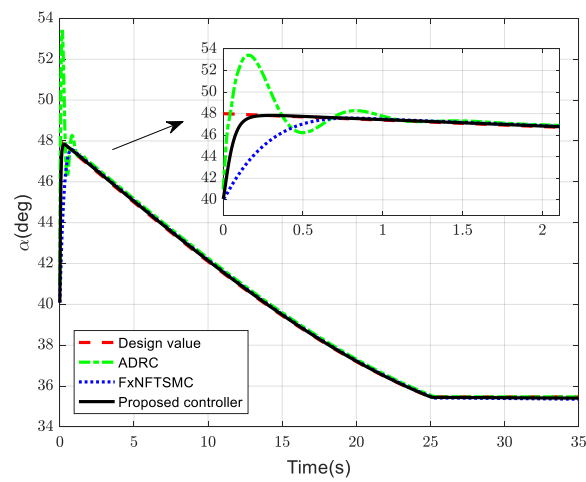
In Figures 5–8, we can see that the attitudes of RLV can track the time-varying guidance command Λ_C in the desired time 1.0 s, and the deviation from guidance commands is less than the predefined value $\rho_{i\infty} = 0.005 \text{ rad}$. Figure 5 shows that the proposed controller has an excellent control effect taking into account the parameter uncertainties, state constraints, and external disturbances. In contrast, the attitude tracking errors of the FxNFTSMC are eliminated to be near zero within 1.45 s, and the convergence time of ADRC is about 1.5 s. Meanwhile, the FxNFTSMC needs a complex calculation of design parameters to obtain the desired convergence time. The proposed controller can directly give the actual convergence time, which avoids the process of obtaining the convergence time of the system through constant iteration and repeated testing in the FxSMC.



(a)



(b)



(c)

Figure 5. Attitude tracking curves of σ , β , α : (a) the roll channel; (b) the yaw channel; (c) the pitch channel.

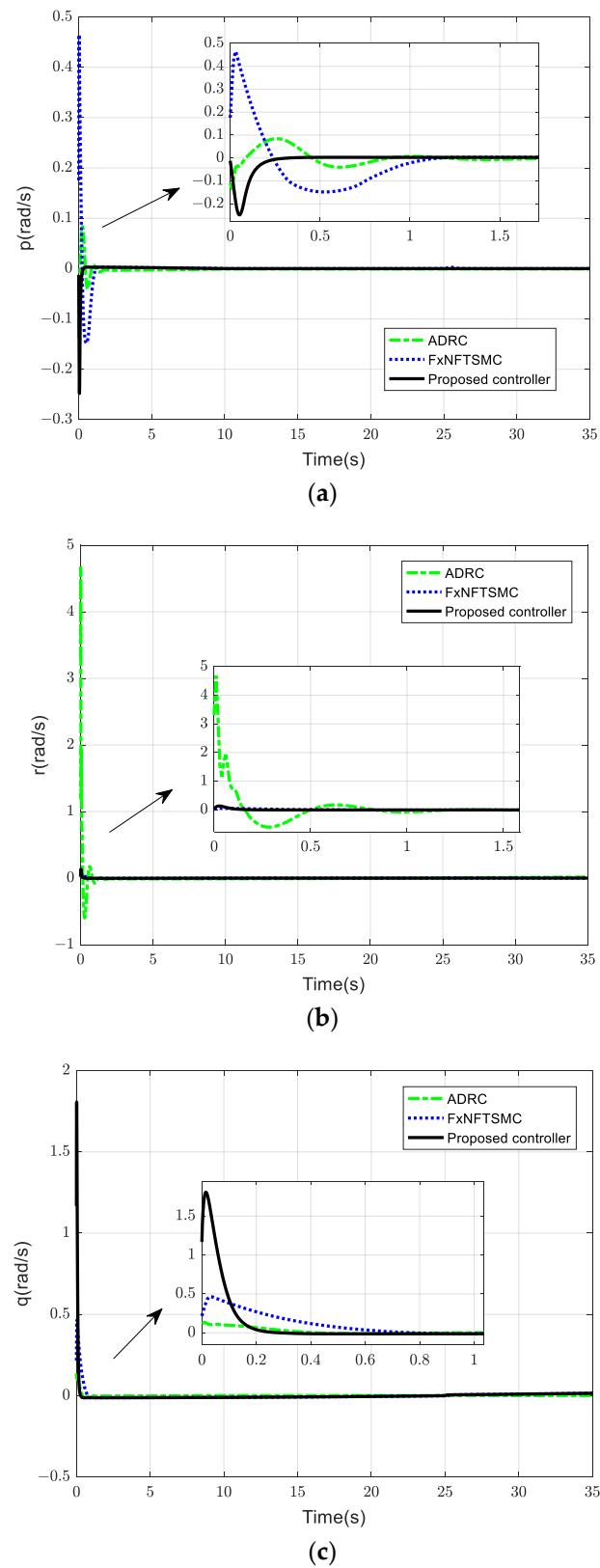
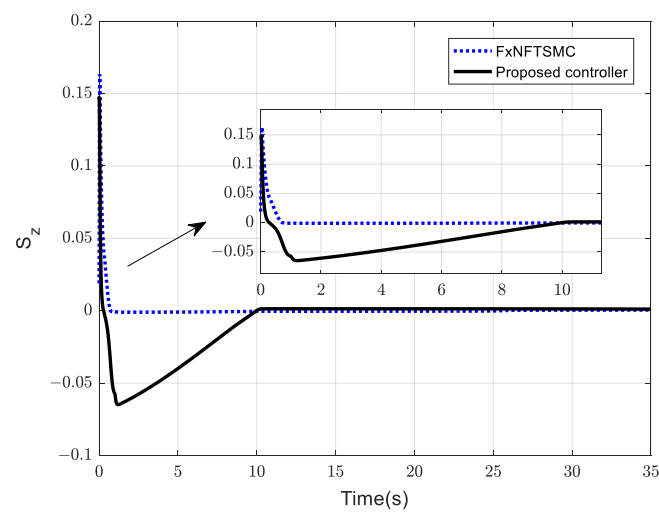
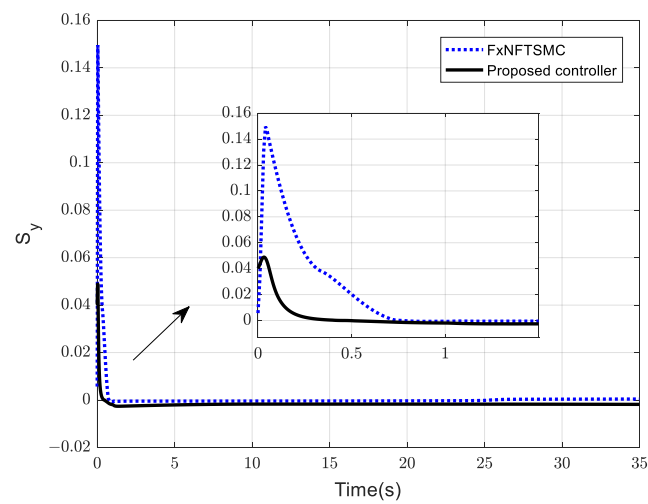


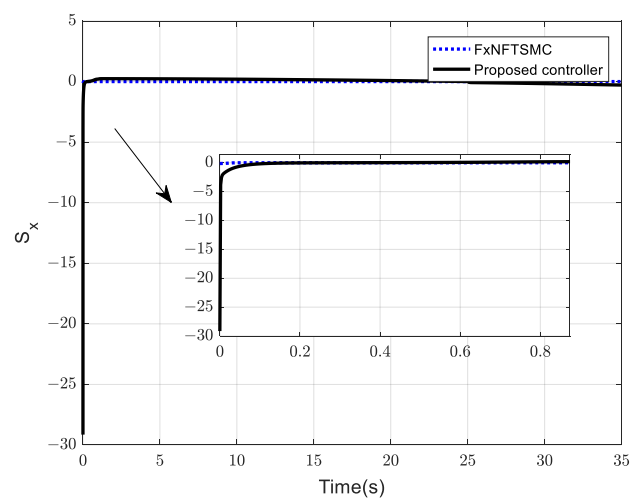
Figure 6. Time responses of angular velocity: (a) the roll channel; (b) the yaw channel; (c) the pitch channel.



(a)

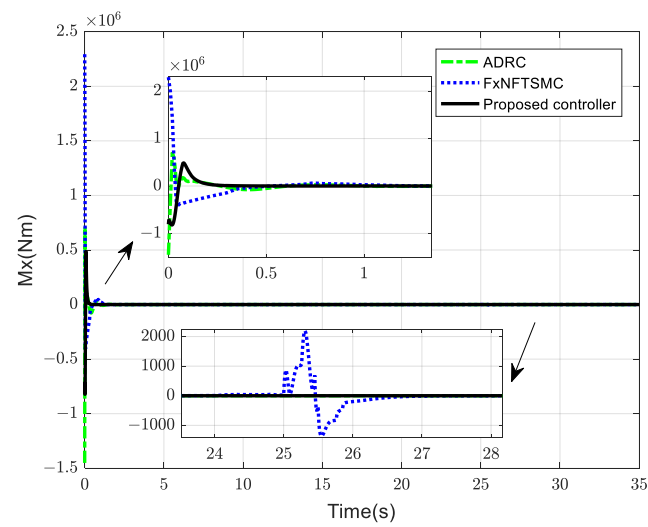


(b)

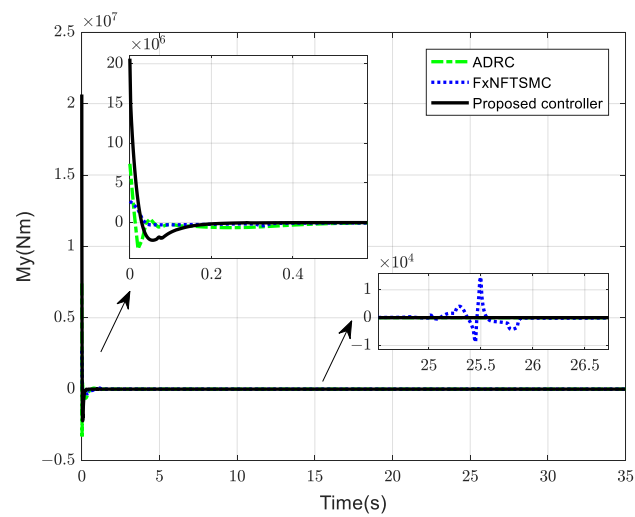


(c)

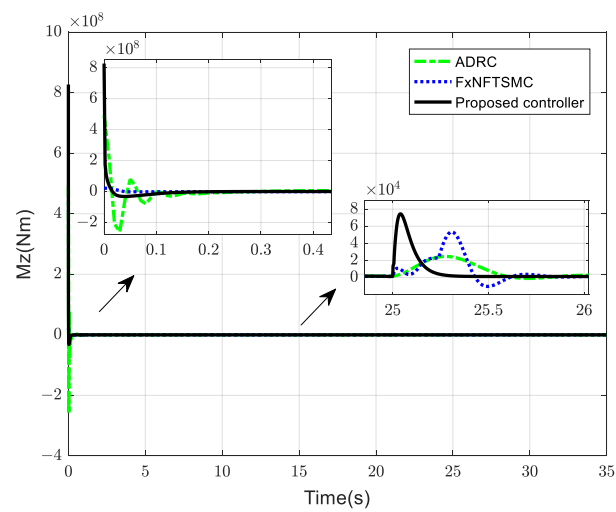
Figure 7. Sliding mode surface of three channels: (a) the roll channel; (b) the yaw channel; (c) the pitch channel.



(a)



(b)



(c)

Figure 8. Control moment of three channels: (a) the roll channel; (b) the yaw channel; (c) the pitch channel.

Figure 7 shows the curve of the SMS, and the control moment is shown in Figure 8. It can be seen that the proposed method can effectively alleviate the chattering phenomenon. The control moment becomes larger at 25 s of simulation time, which is due to a sudden change in the rate of the angle of attack.

In Figures 9 and 10, the value of ρ continuously changes to ρ_∞ in the prescribed time T_f . This indicates the effectiveness of the controller, which can prevent z_1 from exceeding the prescribed bounds.

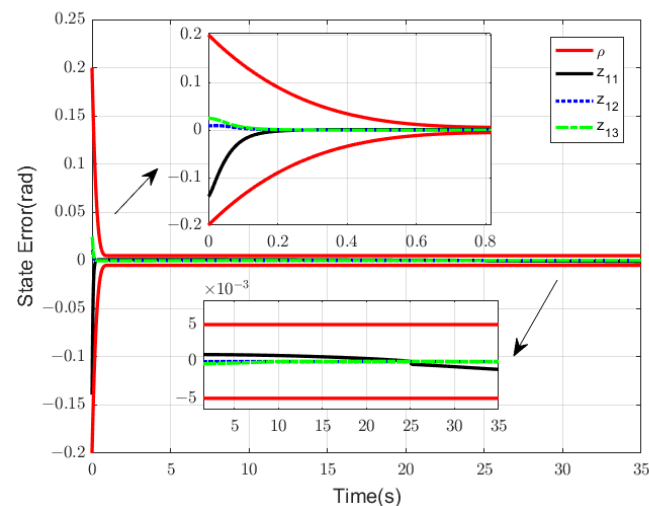


Figure 9. Attitude tracking errors of σ, β, α .

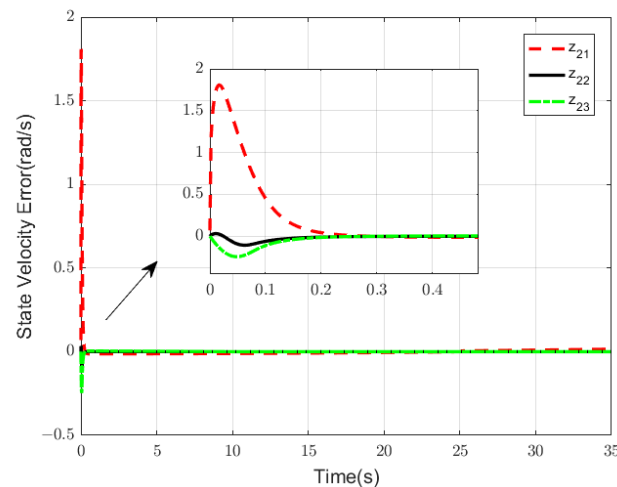


Figure 10. Attitude velocity errors.

6. Conclusions

A novel predefined-time prescribed performance nonsingular sliding mode controller was designed for attitude control of VTHL RLV. A PTPPF is put forward to guarantee that the attitude tracking errors are constrained within the desired transient and steady-state boundaries. On the basis of the error transformation method, the novel predefined-time nonsingular controller can guarantee that the attitude tracking errors converge to the predefined steady-state boundary in a predefined time. Meanwhile, the problem of singularity is avoided without using piecewise continuous functions. The stability of the system was verified using the Lyapunov method. The performance of the proposed method was confirmed using numerical simulations.

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Appendix A

Consider the following equality:

$$\dot{V} = -\frac{\sqrt{3}\pi + 2\sqrt{3}\arctan\frac{1}{\sqrt{3}}}{(3+2\alpha)T_c} \left(V^{1/2-\alpha/3} + V^{2+2\alpha/3} \right). \quad (A1)$$

Integrating Equation (A1) yields

$$\begin{aligned} T(x_0) &= -\int_{V(x_0)}^0 \frac{dV}{\frac{\sqrt{3}\pi + 2\sqrt{3}\arctan\frac{1}{\sqrt{3}}}{(3+2\alpha)T_c} \left(V^{1/2-\alpha/3} + V^{2+2\alpha/3} \right)} \\ &= -\frac{(3+2\alpha)T_c}{\sqrt{3}\pi + 2\sqrt{3}\arctan\frac{1}{\sqrt{3}}} \int_{V(x_0)}^0 \frac{dV}{\left(V^{1/2-\alpha/3} + V^{2+2\alpha/3} \right)} \\ &= -\frac{6T_c}{\sqrt{3}\pi + 2\sqrt{3}\arctan\frac{1}{\sqrt{3}}} \int_{V(x_0)}^0 \frac{dV^{1/2+\alpha/3}}{1+V^{3/2+\alpha}} \\ &= -\frac{6T_c}{\sqrt{3}\pi + 2\sqrt{3}\arctan\frac{1}{\sqrt{3}}} \left[\frac{1}{3} \ln \left| \frac{V^{1/2+\alpha/3}+1}{\sqrt{(V^{1/2+\alpha/3})^2 - V^{1/2+\alpha/3}+1}} \right| + \frac{1}{\sqrt{3}} \arctan \left(\frac{2V^{1/2+\alpha/3}-1}{\sqrt{3}} \right) \right] \Bigg|_{V(x_0)}^0. \end{aligned} \quad (A2)$$

It is apparent that the setting time function $T(x_0)$ is less than T_c because of $\lim_{V(x_0) \rightarrow \infty} T(x_0) = T_c$. Finally, we can conclude that the system in Equation (7) is globally predefined-time stable within predefined-time T_c .

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