

## Article

# Storage Reliability Assessment Method for Aerospace Electromagnetic Relay Based on Belief Reliability Theory

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**Abstract:** The aerospace electromagnetic relay (AEMR) is a key electronic component in aerospace and weaponry systems. It usually lacks sufficient test data to conduct an effective storage reliability assessment at its early development stage. Thus, this paper introduces the theory of belief reliability, a new theory in the field of reliability engineering. Under its theoretical framework, firstly, through the analysis of the storage degradation mechanism of AEMR, the performance degradation characterization parameters are selected to build a storage degradation model. Then, the failure criterion conditions of AEMR are analyzed, and the degradation characterization parameters are used as the 'smaller the better' performance parameters to build a margin equation. Then, the margin equation is combined with the storage degradation model, and the uncertainties of the model parameters are quantified to complete the belief reliability model of AEMR. Finally, a certain AEMR is used as the object for validation. In solving the belief reliability model, the manufacturing information of the product, the degradation simulation data, and the test data are fully utilized to solve the model parameters by utilizing the uncertainty maximum likelihood estimation (UMLE) method. The results show that the method can obtain more accurate assessment results with small test data samples, and the MAE is reduced, compared to only simulation data, by 29.3%. By analyzing the uncertainty of the model parameters, it is found that the main sensitive factor affecting the storage reliability of batch aerospace relays is the initial release time. It was also found that the accuracy of the calculations could be significantly improved by considering the uncertainty of the threshold values when calculating.

**Keywords:** aerospace electromagnetic relay; storage reliability; belief reliability; uncertainty theory



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## 1. Introduction

At the beginning of the last century, the concept of reliability emerged with the development of the modern industry [1]. Before the emergence of this concept, the probability theory was widely applied to solve problems in engineering practice. Therefore, the reliability discipline has been closely integrated with probability theory since its birth and has developed to solve many practical issues in the field of reliability engineering. Probability theory describes the probability of something occurring by its frequency. It is known, from Bernoulli's law of large numbers, that frequency can be equal to probability as long as there are enough trials of independent events. Thus, the traditional probabilistic approach cannot effectively solve the reliability engineering problem when the number of tests is small or the sample size is insufficient [2,3]. Limited by the fact that it only uses objective data to cognize events, in order to use human experience and knowledge effectively, different evaluation methods using subjective cognition, such as Bayesian theory, evidence theory, and fuzzy set theory, have been developed [4]. Although such methods have achieved good results in some fields, with the development of modern industry, today's reliability engineering still exists with some issues: 1. Products tend to be complex; 2. Enterprises are highly competitive, greatly compressing the reliability test cycle of products, resulting in a reduced test sample size and difficulty obtaining adequate data; 3. Uncertainties cannot be ignored but cannot be effectively measured.

Therefore, modern reliability engineering urgently needs a new mathematical theory to solve the new problems arising from the development. In 2007, the uncertainty theory was proposed by Liu [5,6], forming a scientific and self-consistent mathematical axiom system with a set of innovative and convenient algorithms, providing a new method to solve the problem of small samples and uncertainty. Furthermore, in response to the issues faced by modern reliability engineering, Kang [7–9] created the theory of belief reliability by combining traditional probability theory, uncertainty theory, and reliability science. Since it was proposed, many scholars have researched the theory's application, and many successful cases have been produced successively. For example, Zu [10] modeled and estimated the uncertainty of fatigue life with small sample fatigue test data based on uncertainty theory. Li [11] developed a fatigue crack extension reliable life model based on uncertainty theory and belief reliability theory. It concluded, through analysis, that uncertainty theory could get more stable reliability evaluation results when considering epistemic uncertainty, and it found that material dispersion significantly affects fatigue life. Zhang [12] et al. proposed a belief reliability modeling method for the creep-fatigue of radiators, based on uncertainty theory for the tube-and-belt radiator of automobile engine, and gave improvement suggestions by analyzing the uncertainty of model parameters. Li [13] et al. proposed a belief reliability modeling method, based on assembly accuracy of spaceborne synthetic aperture radar (SAR), when facing the practical engineering problem of how to improve the service performance of SAR. By analyzing the multi-source uncertainty, theoretical guidance is provided to carry out the mechanism assembly, which effectively enhances the assembly accuracy and efficiency and enhances performance. Yu [14] et al. proposed a gear reliability modeling method based on belief reliability, which further improved the accuracy of the evaluation results by considering both inherent uncertainty and cognitive uncertainty. Chen [15] et al. proposed a belief reliability modeling and analysis method for harmonic gear transmission efficiency and discovered the main factors affecting transmission efficiency and reliability by analyzing and quantifying multiple sources of uncertainty. Kang [16] et al. proposed a function-oriented design and optimization method of belief reliability based on performance margin. They proved that the uncertainty analysis results of the belief reliability theory could guide product design by applying it in the design phase of torsion spring electrical connectors. The above research results show that the belief reliability theory has good applicability for evaluating different products. It can give valid evaluation results for small sample test data, and considering uncertainty can give guidance suggestions for product improvement.

AEMRs are widely used in vital positions in aerospace and weaponry systems due to their high reliability, long life, and solid on–off performance. They are mainly used to undertake the system power supply, distribution, and timing logic control. Therefore, once one fails, it will undoubtedly produce a devastating blow to the whole system [17]. At the same time, because AEMRs are the least reliable of the many military electronics components, it is reasonable to assume that the reliability of AEMRs determines the reliability of the aerospace and weapon system, based on the Bucket theory. However, similar to missiles, torpedoes and other long-storage weapons and equipment, from production to use, often go through a storage period of up to ten years [18]. During the storage process, even in a relatively stable environment, external stresses inevitably affect the AEMRs, resulting in internal degradation and eventual failure [19]. Thus, the storage reliability of aerospace relays is of great importance for effective weapon systems and even national security. Therefore, it is necessary to study AEMRs' storage reliability assessment technology.

With the development of manufacturing process technology, electronic devices are becoming more and more reliable. AEMRs are no exception, and even if storage accelerated life testing is used for them, it is difficult to achieve effective evaluation in the short term. Especially for products in the early stages of development, the time, labor, and material costs associated with multiple rounds of design modifications and large batch reliability testing of the related products are incalculable. Therefore, for the design phase of AEMR, it is not friendly to use traditional methods for reliability assessment. Ye [20] and Wang [21]

modeled the performance degradation of sensitive parameters, such as suction time and contact resistance, starting from the degradation mechanism, which laid the foundation for the storage reliability assessment work. However, the “one-sample-one-assessment” model is unsuitable for generalization, and the calculation is very tedious when facing the reliability assessment of batch AEMRs. Therefore, a stochastic process-based approach [22,23] to storage reliability assessment was further proposed. Assuming that the model parameters obey a normal-gamma distribution to reflect the individual differences between batches, a storage reliability assessment model for batch AEMRs is established. However, the storage reliability of batch AEMRs is not only affected by individual heterogeneity but also by uncertainty, due to unclear knowledge of the degradation process and the inherent uncertainty of the product itself. Therefore, measuring the multiple sources of uncertainty, in the development stage of AEMRs, will play an important role in the modeling.

As seen above, with the development of modern industry, the problems faced in the field of reliability engineering tend to become more complex, with many influencing factors of uncertainty. The proposed belief reliability theory provides new solutions to problems in the reliability discipline field. Some scholars have combined theory and practice, and through some exploratory attempts, they have shown good application effects on some products with high-reliability requirements. By modeling and evaluating products based on belief reliability theory, on the one hand, effective evaluation results can be obtained with small sample test data. On the other hand, the influence of multiple sources of uncertainty can be analyzed to give a practical impetus to the product optimization work. In the case of AEMR, the subject of this paper, the problem of small samples of test data and the failure of existing models to quantify multiple sources of uncertainty is also present in the development phase.

Therefore, this paper introduces a new theory of belief reliability and gives a new storage reliability assessment method for AEMR. It establishes a belief reliability assessment model by considering multiple sources of uncertainty from the degradation mechanism. In the practical case, analyzing the influence of the uncertainty of model parameters provides the suggestion for AEMR’s design and evaluation.

## 2. Related Theoretical Foundation and Modeling Framework

This section will provide an introduction to the theory underlying the work in this paper, which includes uncertainty theory [6], important definitions and theorems of belief reliability theory, and belief reliability modeling framework.

### 2.1. Uncertainty Theory

**Definition 1.** *Uncertainty measure: Let  $\Gamma$  be a nonempty set,  $L$  be a  $\sigma$ -algebra over  $\Gamma$ , and the element  $\Lambda$  in  $L$  be called an event. Uncertainty measure  $M$  is a set function from  $L$  to  $[0, 1]$  satisfying the following axioms.*

**Axiom 1.** (Normality axiom): for the universal set  $\Gamma$ ,  $M\{\Gamma\} = 1$ .

**Axiom 2.** (Duality axiom):  $M\{\Lambda\} + M\{\Lambda^c\} = 1$ , for any event  $\Lambda$ , where  $\Lambda^c$  is the complementary set of  $\Lambda$ .

**Axiom 3.** (Subadditivity axiom): For a countable sequence of events  $\Lambda_1, \Lambda_2, \dots$

$$M\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} M\{\Lambda_i\} \quad (1)$$

**Axiom 4.** (Product axiom): For a column of uncertain space  $\{\Gamma_k, L_k, M_k\}$ ,  $k = 1, 2, \dots$

$$M\left\{\prod_{i=1}^{\infty} \Lambda_k\right\} = \min_{k=1}^{\infty} M_k\{\Lambda_k\} \quad (2)$$

where  $\Lambda_k$  is an arbitrary event chosen from  $L_k, k = 1, 2, \dots$ .

**Definition 2.** *Uncertainty variable:* let  $\xi$  be a function from an indeterminate space  $\{\Gamma, L, M\}$  to a set  $R$  of real numbers if for any Borel set  $B$ , the set  $\{\xi \in B\} = \{\Gamma \in \Gamma | \xi(\Gamma) \in B\}$  is an event, so  $\xi$  is said uncertain variable.

**Definition 3.** *Uncertainty distribution:* let  $\xi$  be an uncertain variable, then the function  $\Phi(x) = M\{\xi \leq x\}$  is called uncertain distribution.

**Definition 4.** *Inverse Uncertainty distribution:* let  $\xi$  denote an uncertain variable with a regular uncertainty distribution  $\Phi(x)$ , then the inverse function  $\Phi^{-1}(\alpha)$  of  $\Phi(x)$  will ride the inverse uncertainty distribution of  $\xi$ .

The uncertainty algorithm is as follows: let  $\xi_1, \xi_2, \dots, \xi_n$  be a column of independent canonical uncertain variables, and its uncertainty distribution is  $\Phi_1, \Phi_2, \dots, \Phi_n$ . If the function  $f(x_1, x_2, \dots, x_n)$  is monotonically increasing with respect to  $x_1, x_2, \dots, x_n$ , and strictly monotonically decreasing with respect to  $x_{m+1}, x_{m+2}, \dots, x_n$ , then the uncertainty distribution of the uncertain variable  $f(\xi_1, \xi_2, \dots, \xi_n)$  is the following equation:

$$\Phi^{-1}(\alpha) = f^{-1}(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \dots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \dots, \Phi_n^{-1}(1 - \alpha)) \tag{3}$$

### 2.2. Belief Reliability Theory

The theory of belief reliability follows the three most basic principles of reliability science: Margin reliability principle, Degenerate eternity principle, Uncertainty principle [24]. These three principles can be expressed by the following three equations:

1. Margin equation:  $E = G(P, P_{th})$ .
2. Degradation equation:  $P = F(X, Y, t, T)$ .
3. Measurement equation:  $R(t) = \mu(E > 0)$ .

The margin equation, which represents the amount of the object performance allowance and the failure criterion, corresponds to the allowance reliability principle among them. The allowance is essentially the distance from the performance characteristic  $P$  to the performance threshold  $P_{th}$ . If the margin is greater than 0, the product is reliable. The performance of the object  $F$  and the system of intrinsic properties  $X$ , extrinsic properties  $Y$ , and physical time  $t$  occur irreversibly along the degradation time vector  $T$ , according to the degradation equation, which also corresponds to the principle of degradation eternity. The degradation equation describes the degradation law of product determinism. The metric equation describes the quantification of uncertainties in the allowance equation and relates to the uncertainty principle, providing the product reliability in accordance. As a result, it is possible to assume that the reliability is derived by quantifying the uncertainty using the law of certainty.

### 2.3. Belief Reliability Modeling Framework

Firstly, the degradation mechanism of AEMR is analyzed, and the release time is extracted as the critical performance degradation characterization parameter for the stress relaxation of reeds. Second, the storage degradation model of AEMR is established based on the degradation eternity principle. Then, based on the margin reliability principle, the margin model is established by combining the performance degradation threshold and combining the margin model with the AEMR's model of storage degradation to establish the margin degradation model. Finally, the uncertainty analysis is carried out from the perspectives of inherent conditions and the cognitive influence of the margin model. The uncertainty is quantified based on uncertainty theory to establish a belief reliability model describing the storage degradation of AEMR. The whole framework is shown in Figure 1.

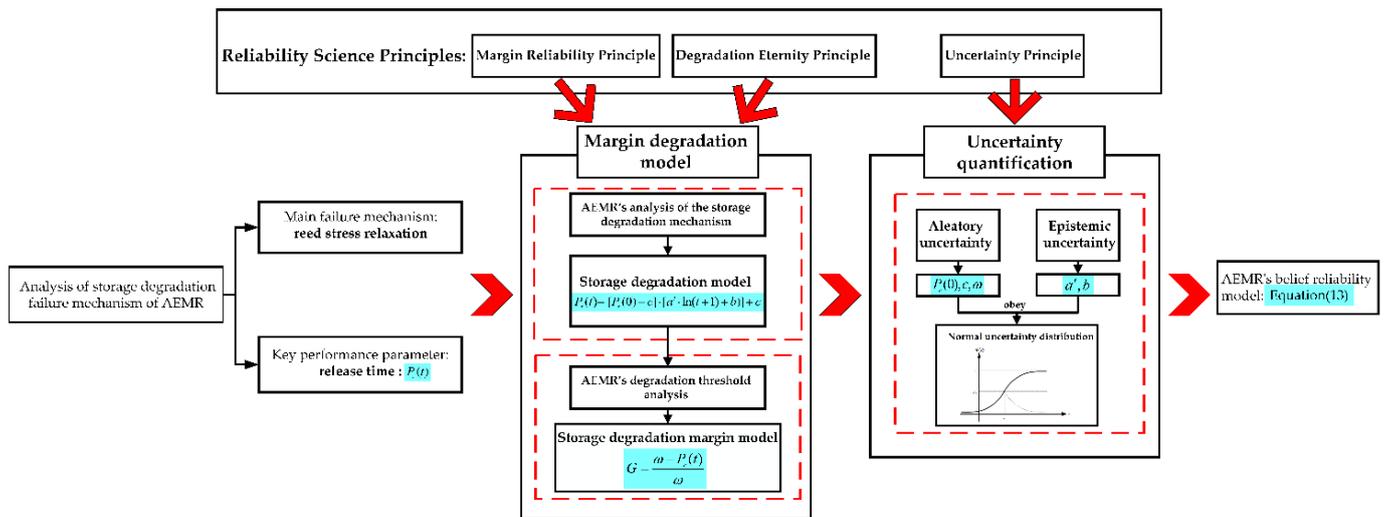


Figure 1. AEMR’s belief reliability modeling framework.

### 3. Storage Degradation Belief Reliability Model for AEMR

In this section, a reliability model to evaluate the storage degradation of AEMR based on the belief reliability and uncertainty theory will be developed, mainly including the analysis of the storage degradation mechanism of AEMR, the modeling process based on the belief reliability theory, the uncertainty characterization of parameters based on the uncertainty theory, and the method to obtain the distribution parameters.

#### 3.1. Analysis of the Storage Degradation Mechanism of Aerospace Relays

During long-term storage, the internal reeds, coils, bobbins, enameled wires, and magnetic materials of AEMR can degrade to a certain degree, affecting their movement and contact characteristics. The relay will fail when the movement or contact characteristics are outside the allowable range. However, for the AEMR in storage, the leading cause of failure is the stress relaxation of the reeds, which reduces the reaction force and affects the reaction force characteristics, increasing the release time during operation and, finally, leading to contact failure.

As the reed is sealed in the internal component of the relay, it is not easy to open and test the stress relaxation state of the reed in the actual storage environment. Therefore, this paper obtains the relationship between the two by conducting finite element simulation to verify the effect of reed reaction force variation on release time. The result is shown in Figure 2.

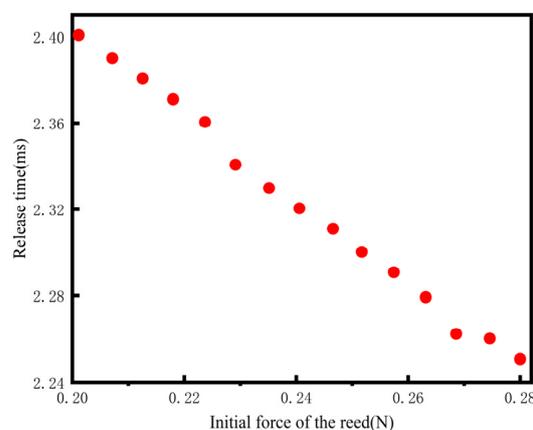


Figure 2. Relationship between initial force and release time.

The simulation results show an approximately linear relationship between the external performance parameter (release time  $P_r$ ) and the underlying performance parameter (reed initial force  $F_{ini}$ ) within a certain range of reed reaction force reduction. Therefore, this paper uses Equation (4) to express the relationship between the two:

$$P_r = k \cdot F_{ini} + c \tag{4}$$

where  $k, c$  are model coefficients related to the design and manufacturing process, which characterize the variability between individual relays of the same batch.

From the above analysis, it is clear that the stress relaxation occurring in the internal reed during the storage period of the AEMR is the fundamental reason for the reduction in the reaction force. Therefore, in this paper, the initial force is used as the characterization parameter of the mechanical reaction force, and the Larson–Miller coefficient is used to describe the storage degradation process of the reed under the joint action of temperature stress (Kelvin) and time (hours), and the corresponding physical model of failure is obtained as shown:

$$\begin{aligned} \frac{F_{ini}(t,T)}{F_{ini}(0)} &= a \cdot \Theta + b \\ &= a \cdot T(\ln t + C) + b \end{aligned} \tag{5}$$

where  $F_{ini}(t, T)$  is the reed initial force value at temperature stress  $T$  stored up to moment  $t$ ;  $F_{ini}(0)$  is the reed initial force value at moment 0;  $a, b$  is the model coefficient, which characterizes the effect of temperature and time on the treatment of the storage degradation process;  $\Theta$  is the Larson–Miller coefficient, expressed as  $T(\ln t + C)$ ;  $C$  is a constant, usually taken as 20.

Consider the following reasons: first, the ambient temperature of the AEMR is more stable during storage; second, the subsequent need to quantify the impact of the initial distribution of the performance parameters at the moment of 0 on the reliability of this paper requires replacing  $\ln t$  in  $\Theta$  with  $\ln(t + 1)$ . Therefore, Equation (5) is further simplified as:

$$\frac{F_{ini}(t)}{F_{ini}(0)} = a' \cdot \ln(t + 1) + b \tag{6}$$

Then, the storage degradation model of the relay release time is obtained by substituting Equation (6) into Equation (4), as follows:

$$\begin{aligned} P_r(t) &= k \cdot F_{ini}(t) + c \\ &= k \cdot F_{ini}(0) \cdot [a' \cdot \ln(t + 1) + b] + c \\ &= [P_r(0) - c] \cdot [a' \cdot \ln(t + 1) + b] + c \end{aligned} \tag{7}$$

where  $P_r(t)$  is the release time value from storage at constant temperature stress  $T$  to moment  $t$ ;  $P_r(0)$  is the initial release time value at moment 0.

Based on the meaning of  $P_r(0)$  and  $\{a', b, c\}$  in Equation (9), it is clear that  $P_r(0)$  is a fixed value for a specific relay sample, but due to individual differences between samples,  $P_r(0)$  is a random effect factor in the model that characterizes the storage degradation process of a batch of relays.  $c$ , a model factor related to individual relay differences, is also a random effect factor. The model coefficients  $a'$  and  $b$  are determined by the reed material, storage conditions, and stress relaxation mechanism.

### 3.2. Reliability Function

Based on the above analysis of the storage degradation mechanism of the relay, the release time is selected as the performance degradation parameter. In the following, the performance parameters will be classified into three categories according to the qualified form of the failure threshold  $P_{th}$  on the performance parameter  $P$ :

Smaller-the-better performance parameter: the product fails when and only when  $P \geq P_{th}$ .

Larger-the-better performance parameter: the product fails when and only when  $P \leq P_{th}$ .

Nominal-the-type performance parameter: the product fails when and only when  $P \geq P_{th,L}$  or  $P \leq P_{th,U}$ .

Therefore, the margin equation, constructed as follows, is shown:

$$G(P, P_{th}) = \begin{cases} \frac{P_{th}-P}{P_{th}}, & \text{smaller the better} \\ \frac{P-P_{th}}{P_{th}}, & \text{larger the better} \\ \min\left(\frac{P_{th,U}-P}{P_{th,U}}, \frac{P-P_{th,L}}{P_{th,L}}\right), & \text{nominal the better} \end{cases} \quad (8)$$

Based on the failure mechanism of a relay during storage, it is generally believed that a contact failure will occur when the stress relaxation-induced release time increases to a certain threshold value. Therefore, the growth of the release time to a certain threshold value can be used as a judgment condition for relay storage failure, and the release time  $P_r(t)$  is used as the expected small performance parameter to construct the margin equation, where  $\omega$  is the failure threshold value.

$$G = \frac{\omega - P_r(t)}{\omega} \quad (9)$$

It is considered reliable when the margin  $G \geq 0$ , so the reliability equation can be established as:

$$\begin{aligned} R(t) &= M\left(\frac{P_r(t)-\omega}{\omega} \leq 0\right) \\ &= M\left(\frac{[P_r(0)-c] \cdot [a' \cdot \ln(t+1)+b]}{\omega} + c - \omega \leq 0\right) \\ &= M([P_r(0) - c] \cdot [a' \cdot \ln(t + 1) + b]) + c - \omega \leq 0 \end{aligned} \quad (10)$$

### 3.3. Uncertainty Representation of Parameters

Usually, belief reliability modeling requires quantifying the uncertainty of its model parameters, so the possible uncertainty of the parameters of the following equation needs to be further discussed after obtaining Equation (11).

$$[P_r(0) - c] \cdot [a' \cdot \ln(t + 1) + b] + c - \omega \leq 0 \quad (11)$$

There are two possible parameter uncertainty cases: aleatory and epistemic uncertainty. The essence of aleatory uncertainty is the inherent uncertainty of the object world, which can also be called random uncertainty. Cognitive uncertainty is caused by the limitation of the objective laws, due to the limited cognitive ability of humans. The parameters in Equation (11) have aleatory uncertainty and epistemic uncertainty due to the dispersion of materials, the volatility of working conditions and environment, and the incompleteness of cognition. Therefore, the parameters in Equation (11) are classified according to their uncertainties.

From the above,  $P_r(0)$  and  $c$  are the randomly influenced parameters in the degradation model, which are, in essence, the random uncertainties caused by dimensional tolerances, assembly errors, etc. Therefore, the uncertainty types of  $P_r(0)$  and  $c$  are aleatory uncertainties. For model parameters,  $a'$  and  $b$ , the reed material and storage conditions are usually relatively fixed in the same batch of stored AEMRs. Therefore, the randomness of  $a'$  and  $b$  is mainly due to insufficient knowledge of the objective change law of the stress relaxation mechanism, so its uncertainty type is epistemic uncertainty.

Both of the above uncertainties can be quantitatively characterized by uncertainty distributions, which are considered to obey the normal uncertainty distribution. An uncertain variable  $\zeta$  is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \left(\frac{\pi(e-x)}{\sqrt{3d}}\right)\right)^{-1}, x \in R \tag{12}$$

denoted by  $N(e, d)$  where  $e$  and  $d$  are real numbers with  $d > 0$  [6]. The normal uncertainty distribution is shown in Figure 3.

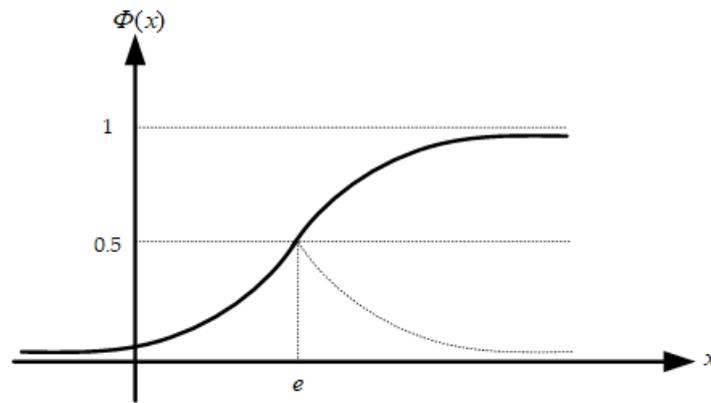


Figure 3. Normal uncertainty distribution.

Further, when considering the uncertainty of all parameters, Equation (10) can be expressed, according to Theorem 1 (Operator’s Rule), as:

$$R(t) = M([P_r(0) - c] \cdot [a' \cdot \ln(t + 1) + b]) + c - \omega \leq 0) \tag{13}$$

$$= \alpha \{ [\Phi_{P_r(0)}^{-1}(\alpha) - \Phi_c^{-1}(\alpha)] [\Phi_{a'}^{-1}(\alpha) \cdot \ln(t + 1) + \Phi_b^{-1}(\alpha)] + \Phi_c^{-1}(\alpha) - \Phi_\omega^{-1}(1 - \alpha) \}$$

where the inverse uncertainty distribution of the uncertain normal distribution is:

$$\Phi^{-1}(\alpha) = e + \frac{\sqrt{3d}}{\pi} \ln \frac{\alpha}{1 - \alpha} \tag{14}$$

### 3.4. Acquired Method of Distribution Parameters

In practical engineering, for storage reliability assessment of batch AEMRs, traditional methods are often based on probabilistic statistics requiring large test data samples. It is not friendly for products in the early design stage because it often requires multiple rounds of modification and optimization. Therefore, If we want to achieve effective evaluation by large sample data, it will undoubtedly increase the time and development cost. It is difficult to keep the timeliness of product launches in the fierce competition with peers. However, in the belief reliability theory, by considering the uncertainty distribution of parameters, it could estimate uncertain parameters by virtue of a small amount of test data to realize the work of storage reliability assessment. When the parameters obey the normal uncertainty distribution, the parameter estimation can be performed using the UMLE. According to axiom 4 of the uncertainty theory, the likelihood function in the UMLE method is the derivative of the uncertain distribution function taken as minor, which is different from the probability density function product in probability theory. Thus, for a sample satisfying the uncertain normal distribution  $N(e, d)$ , the UMLE function is

$$L = \wedge_{i=1}^m \Phi'(x_i|e, d), i = 1, 2, \dots, m \tag{15}$$

where the symbol  $\wedge$  is taken to mean small, and the specific expression for  $\Phi'(x_i|e, d)$  is shown below.

$$\Phi'(x_i|e, d) = \frac{\frac{\pi}{\sqrt{3d}} \exp\left(\frac{\pi(e-x_i)}{\sqrt{3d}}\right)}{\left(1 + \exp\left(\frac{\pi(e-x_i)}{\sqrt{3d}}\right)\right)^2} \tag{16}$$

Belief reliability theory also provides solutions for situations where test and measurement data are unavailable. The unknown parameters can be determined by drawing on previous engineering experience in the development of similar products, as well as by expert designation. However, it is difficult to reflect the heterogeneity of existing specific products by the approach using previous engineering experience, while the approach using expert experience is slightly subjective, and it is difficult to ensure the objectivity of the assessment. Therefore, in this paper, we hope to use the manufacturing information of the product to realize the transformation of the initial distribution of process data and performance parameters, based on the functional relationship between the structural characteristics of the relay and the underlying performance parameters. At the same time, with the help of finite element simulation and the circuit simulation technology of relays, the virtual test of storage degradation of batch products is realized by conducting a multi-physical field simulation to obtain a sufficient amount of degradation simulation data. The data obtained will be more realistic and objective by combining the actual manufacturing information of the product and will apply to the case of multiple revisions at the beginning of the product design.

#### 4. Case Study

The AEMR is small but has both electromagnetic and mechanical mechanisms, so its overall structure is complex, which also determines its failure mechanism. Therefore, the storage degradation test, using multiple stresses, can best reflect the actual degradation state of the AEMR. However, conducting a multi-stress storage degradation test will increase the difficulty of the test. At the same time, when multiple degradation mechanisms are coupled, the degradation trend of the collected data may not be obvious, resulting in the loss of value of the data and even some other unexpected consequences. In fact, the environment of AEMR is relatively stable during storage. Only the temperature, as continuous stress, is affecting the storage state of AEMR and accelerating the process of physical and chemical changes [17].

Therefore, in this section, the temperature was chosen as the accelerating stress for certain types of AEMR, and it carried out the constant stress accelerated storage degradation test. It is used to obtain degradation data to verify the validity of the storage reliability assessment method proposed in this paper. A total of 10 AEMRs, with a coil voltage of 28V, were used for the test. The specific procedure of the test is, first, to place the AEMRs with no load in the incubator, as shown in Figure 4 and set the temperature to 170 degrees Celsius for an accelerated storage degradation test. Then, take them out every 20 h, place them back to room temperature, and use the developed time parameter test system to measure them. When the measurement frequency reaches 200, they will not be measured but placed in the constant incubator until the AEMRs fail. The degradation data obtained from the test are shown in Figure 5.



Figure 4. Temperature chamber.

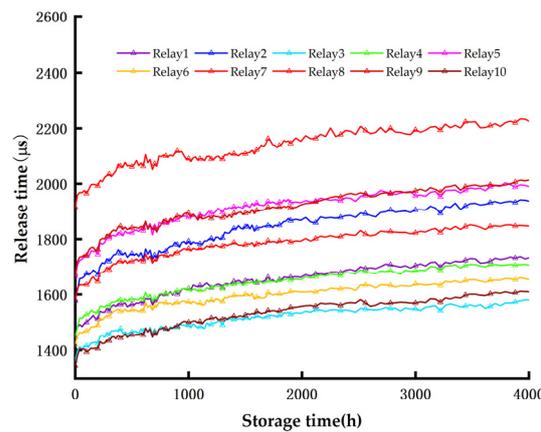


Figure 5. The data of AEMR’s storage degradation.

4.1. Distribution and the Curve of Reliability

Usually, the expert experience can be used to specify or use historical data of similar products to solve the model parameter distribution. However, this makes it difficult to ensure the objectivity of the final evaluation results and is not applicable when facing different products. Therefore, we hope to effectively use the relevant manufacturing information of the product and transform it into the basis for solving the model parameters in this paper.

First, based on the AEMR’s design and manufacturing process data, an approximate calculation model of release time versus initial reed force is constructed.

$$\begin{cases} P_r = f(\Delta\Theta) \\ F_{ini} = f'(\Delta\Theta) \end{cases} \quad (17)$$

where  $\Delta\Theta = \{\Delta\Theta_1, \Delta\Theta_2, \dots, \Delta\Theta_j\}$  denotes the variation of  $j$  design or the manufacturing process data that determine  $P_r$  or  $F_{ini}$ .

Then, according to the range of parameter fluctuations allowed in the relay manufacturing process, random sampling is carried out to obtain  $j$  sets of  $\Delta\Theta$ , and they are substituted into Equation (17) for calculation. It results in  $j$  sets of  $F_{ini}$  that can be used to determine the  $P_r(0)$  distribution parameters. Based on the  $j$  sets of  $\Delta\Theta$ , further batch AEMR’s virtual samples are generated, and its storage degradation simulation is carried out to obtain the data, as shown in Figure 6.

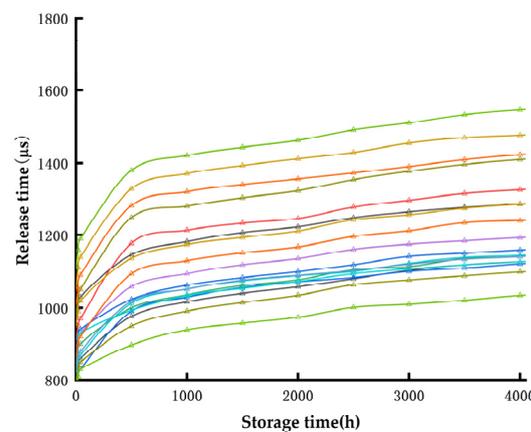


Figure 6. The simulation data of AEMR’s storage degradation.

Finally, the obtained data are used for UMLE to acquire the distribution of the model parameters. Table 1 shows the statistics of the manufacturing process data situation, and Table 2 shows the parameter distribution.

**Table 1.** Statistics on the actual measurement of manufacturing process data.

Serial Number	Measuring Components	Measured Parameters
1	pedestal	
2	bracket	
3	armature, armature shaft	
4	putter & glass Ball	
5	yoke	outline dimension
6	contact spring mount angle piece	
7	moving and static reeds	
8	moving and static contact	
9	magnetic isolation gasket	
10	coil and core	
11	electromagnetic system components	mounting dimension
12	contact spring system components	

**Table 2.** Distribution of model parameters.

Parameter	<i>e</i>	<i>d</i>	Acquired Method
$P_r(0)/\mu s$	882.6	65.1	Manufacture information
<i>c</i>	487.2	77.1	
<i>a'</i>	$1.5 \times 10^{-1}$	$6 \times 10^{-2}$	Simulation data
<i>b</i>	$7.5 \times 10^{-1}$	$3.5 \times 10^{-1}$	

For products at the early stage of development, it is difficult to go with the method of obtaining data by conducting a large number of sample storage degradation tests, for reliability assessment, with the help of mathematical and statistical methods. However, this problem can be circumvented based on belief reliability theory, so in this paper, only five sets of AEMR failure data are used to solve the failure threshold distribution parameters by UML estimation. Table 3 shows the small sample failure data. The distribution result of the contact failure threshold of AEMR was obtained as:  $\omega \sim N(1718.2, 104.8)$ .

**Table 3.** Small sample failure threshold.

Parameter	Sample Number				
	1	2	3	4	5
$\omega/\mu s$	1613.4	1747.8	1680.1	1895.4	2057.8

After all the uncertainty distribution parameters of the degradation model parameters are obtained, the storage reliability curve of the batch of AEMRs can be solved according to the reliability function, and the curve in Figure 7 shows the reliability life  $t_{0.9} = 691.6$  h,  $t_{0.8} = 7896.1$  h. From the trend of the curve, it can be seen that the product's reliability decreases at the fastest rate in the early stage and gradually levels off in the middle and late stages, which also coincides with the law of stress relaxation.

In order to further verify the effectiveness and accuracy of the method, the performance degradation modeling, based on the stochastic process, is carried out in this paper. The reliability evaluation is carried out using experimental (M1) and simulation (M2) data, respectively. The contrast of M1, M2, and M3 is shown in Figure 8. By comparison with M1, it can be seen that the error of curve M3 is smaller than that of M2. However, the trend of curve M3 decreases faster in the early stage, mainly because the degradation modeling only considers the factor of stress relaxation and ignores the influence of coupling with other influencing factors on the degradation process. The main reasons for the deviations in the results, based on the simulation data, may be errors in the approximate modeling process, the fact that the simulation was carried out with only a single consideration of

reed degradation, and that the simulation does not reflect the real storage environment. In the following, to illustrate the accuracy of the proposed method, mean absolute error (MAE) is evaluated as a quantitative index. The MAEs between M3 and M1 and between M2 and M1 were calculated to be 0.029 and 0.041, respectively, which showed that, in the process of storage reliability assessment modeling, by considering the uncertainty of multiple parameters and fusing the product manufacturing process information with the batch product degradation simulation data, the assessment error was reduced by 29.3% compared with the method based on simulation data only. The effectiveness and accuracy of the proposed method are demonstrated. In the next section, the impact of different parameter uncertainties on the batch storage reliability will be quantified.

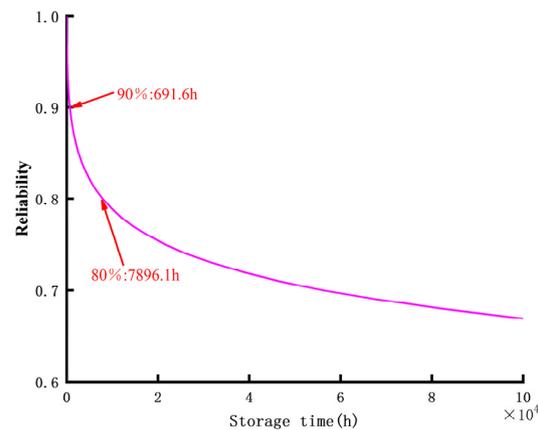


Figure 7. The curve of reliability.

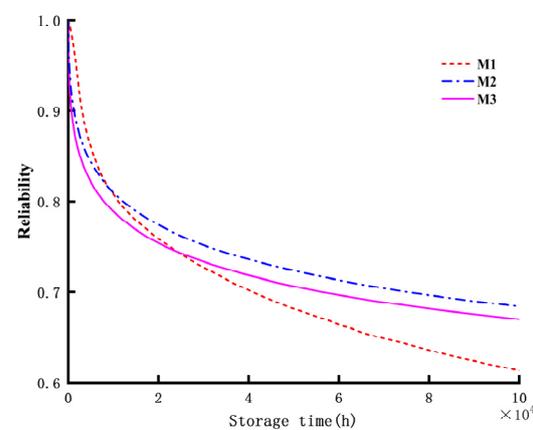


Figure 8. The contrast of M1, M2, and M3.

#### 4.2. Influence of Parameter Uncertainty

Since many uncertainties influence the product degradation process, the relationship between parameter uncertainty and batch storage reliability can be quantified to guide subsequent product optimization. Therefore, this section will analyze the impact on storage reliability by considering the magnitude of uncertainty of different parameters. Relays are usually made of consistent materials and the storage environment is relatively stable, so the uncertainty effects of model parameters  $a'$  and  $b$  are not analyzed, and their distribution is kept fixed. Instead, parameters  $P_r(0)$ ,  $c$ , and  $\omega$  affect the batch product reliability assessment and are considered separately.

Firstly, the influence of the uncertainty of parameter  $P_r(0)$  on the batch product is considered, and the distribution parameters  $d$  of  $P_r(0)$  are empirically made equal to 21.7, 13.1, and 8.1, respectively, while the other parameters keep the initial distribution unchanged. The evaluation results are shown in Figure 9. When  $d$  is equal to 21.7, 13.1, and 8.1, the MAEs between it and the reliability curves based on the test data are 0.081,

0.098, and 0.109, respectively. This can be seen from the figure that the prediction accuracy gradually decreases as the uncertainty decreases. The main reason is that the uncertainty of the parameters decreases, which makes the distribution of  $P_r(0)$  unable to cover the batch products, thus decreasing the accuracy of the evaluation results. However, from the perspective of improving the AEMR's reliability, the uncertainty of  $P_r(0)$  is smaller, which means the consistency of the initial state of the batch product will be better, and the distribution of  $P_r(0)$  is mainly related to the assembly error and the manufacturing process data. If we want to improve the storage reliability of the batch product, we have to modify and optimize the assembly process, the assembly technology and the external characteristics of the product so that the reliability of the batch products can be effective.

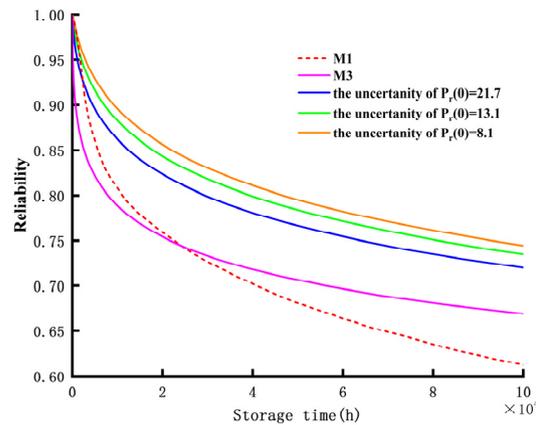


Figure 9. Influence of reliability assessment by considered the uncertainty of  $P_r(0)$ .

The model parameter  $c$  mainly reflects the individual differences of the same batch of relay products. This part was assessed by fixing the distribution of other parameters and changing the distribution parameter  $d$  of  $c$  to 25.7, 15.4, and 9.6, respectively, for reliability assessment. The assessment results are shown in Figure 10a. According to the observation curve in the figure, on the one hand, it can be seen that the uncertainty of parameter  $c$  is reduced to a certain degree. Then it will not further expand with the uncertainty reduction on the assessment results. On the other hand, it can be seen that the assessment results, obtained by reducing the uncertainty of the parameter  $c$ , aggravate the reliability degradation in the early rapid degradation stage, which dramatically affects the calculation accuracy of the reliability assessment work performed in the early stage of product life. The most important thing for a product is the accuracy of the pre-life prediction, thus showing that the influence of the uncertainty of parameter  $c$  cannot be ignored.

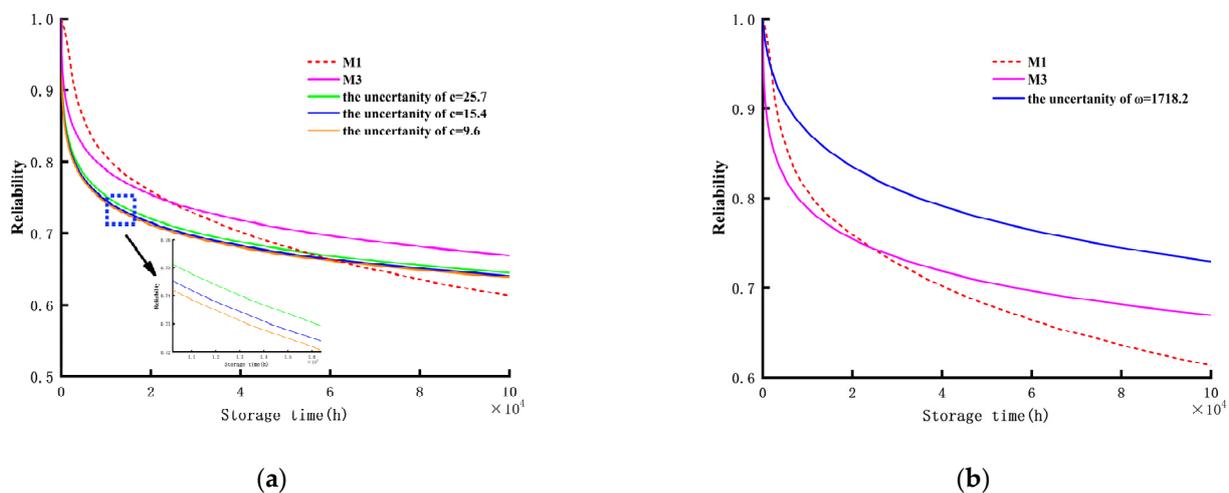


Figure 10. (a) Influence of reliability assessment by considering the uncertainty of  $c$ ; (b) Influence of reliability assessment by considering the uncertainty of  $\omega$ .

For the product failure threshold  $\omega$ , when evaluating the same batch of products of a given model, the engineer usually gives a fixed value for calculation. However, in the actual product production process, due to the different distribution of the initial state of the product, heterogeneity will arise between batches of products, and thus, the failure threshold will vary from product to product. Therefore, considering the uncertainty of the failure threshold in the evaluation process is expected to improve the calculation accuracy. Since the uncertainty of the failure threshold has been considered in the modeling above, in this part,  $\omega$  is taken as a fixed value, and then, the reliability is calculated. The results are shown in Figure 10b, from which it can be seen that the uncertainty of the parameter  $\omega$  has not been considered, resulting in a large deviation of the reliability evaluation result, and its MAE is 0.091, a 207% increase compared to the MAE between M3 and M1. It can be seen that considering the influence of the uncertainty of the failure threshold, in the process of a storage reliability assessment of batch products, can reflect the influence of the heterogeneity of batch products, so more accurate assessment results can be obtained.

## 5. Conclusions

In this paper, we proposed a new method of storage reliability for AEMR based on belief reliability theory. Under the framework of the theory of assured reliability, the method firstly establishes a margin degradation model to describe the relationship between the underlying parameters and the performance degradation characterization parameters by analyzing the AEMR storage degradation failure mechanism. Then, based on uncertainty theory, uncertainties such as product initial state dispersion, individual heterogeneity, and cognitive incompleteness are quantified to construct the belief reliability model. Finally, in the process of the storage reliability solution, multiple sources of information, such as manufacturing process data, degradation simulation data, and test failure data of AEMR, are used to estimate the model parameters to make the assessment results more objective.

A case study was used to verify the validity and applicability of the method, and the following conclusions were reached.

1. With the full use of multiple sources of information, a more accurate result can be obtained by using only five sets of failure data samples for the assessment, significantly reducing the assessment cost. The accuracy of the results is improved, and the MAE is reduced by 29.3% when compared to the evaluation method using only simulation data.
2. By measuring the influence of uncertainties in different model parameters, it was clarified, on the one hand, that the main sensitive factor affecting the storage reliability of batches is the initial release time, which provides a direction for optimization in the product design phase. On the other hand, the importance of the uncertainty of the failure threshold, for the accuracy of the storage reliability assessment, is pointed out. When the threshold's uncertainty is considered, the assessment results are more accurate.

As mentioned above, the method proposed in this paper offers a new solution to the problem that AEMRs in the design phase cannot carry out a valid reliability assessment without sufficient degradation data. It can also provide a clear direction for product optimization based on uncertainty analysis. It is also a potential reference for the reliability assessment of other electrical products of the same type.

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