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Solving Aggregate Production Planning Problems: An Extended TOPSIS Approach

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Abstract: Aggregate production planning (APP) was developed for solving the problem of determining production, inventory, and workforce levels to meet fluctuating demand requirements over a planning horizon. In this work, multiple objectives were considered to determine the most effective means of satisfying forecasted demand by adjusting production rates, hiring and layoffs, inventory levels, overtime work, back orders, and other controllable variables. An extended technique for order preference via the similarity ideal solution (TOPSIS) approach was developed. It was formulated to solve this complicated, multi-objective APP decision problem. Compromise (ideal solution) control minimized the measure of distance, providing which of the closest solutions has the shortest distance from a positive ideal solution (PIS) and the longest distance from a negative ideal solution (NIS). The proposed method can transform multiple objectives into two objectives. The bi-objective problem can then be solved by balancing satisfaction using a max–min operator for resolving the conflict between the new criteria based on PIS and NIS. Finally, an application example demonstrated the proposed model's applicability to practical APP decision problems.

Keywords: aggregate production planning; compromise programming; multicriteria decision-making; TOPSIS



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1. Introduction

Optimal production, safety inventory, and manpower are concurrently determined given a set of manufacturing resources and limitations, Baykasoglu [1] mentioned; the APP spans the production planning time to a range of 2 to 18 months, during which the planners may take into consideration each product category the overall production levels to meet predictable fluctuating demands, which is justified from an aggregate point of view [2]. At the same time, such APP can provide production information to effectively utilize an organization's resources to satisfy various demands. Aggregate production planning determines not only which output levels have to be met and what appropriate input mix of resources should be used, but also how to meet prescribed requirements at minimum cost without violating capacity limits [3].

It is essential to aggregate the information for it is almost impossible to consider every detail related to the production planning without sacrificing its effectiveness in the long run. This planning approach can calculate an aggregate number of production problems, such as average item, weight, volume, production time, or dollar value. Production quantities for

each item or item group can be specifically determined upon the creation of the aggregate production plan that includes anticipated constraints. One of the benefits of an APP is that it provides multiple selections of mixes that contains either one or a combination of strategies to respond to demand fluctuation, such as overtime/idle time production rate adjustment, workforce size regulation, level production plus inventory policy, or sales shortfalls [4–6]. In addition, managers may have the option of using subcontracting as a suitable alternative for part of the production schedule to reduce internal capacity shortages.

This work proposes a multi-objective planning (MOP) method to determine the most effective means of satisfying forecast demand by adjusting production rates, hiring and layoffs, inventory levels, overtime work, back orders, and other controllable variables. The objective functions of this APP decision problem are to minimize total production costs, carrying and backordering costs, also changing workforce levels under production constraints.

While pervasive approaches—such as global criteria methods, goal programming, fuzzy programming and interactive approaches—consider only the single criterion of shortest distance from goal(s) or the positive ideal solution (PIS), the technique for order preference via the similarity ideal solution (TOPSIS) provides a broader principle of compromise for solving multiple criteria, decision-making problems. The compromise control (ideal solution) minimizes the distance measure, provided that the nearest solution has the shortest distance to the positive ideal solution (PIS) and the longest distance to the negative ideal solution (NIS). The proposed method transforms multiple objectives into two objectives. The bi-objective problem can then be solved by balancing satisfaction through a max–min operator to resolve the conflict between the new criteria based on PIS and NIS. The development of the proposed approach is motivated by the following facts: (1) it combines MOP and TOPSIS to provide an easy way to solve a complex APP problem; (2) it can be efficiently coded when the problem is large in scale; and (3) this combined decomposition-based method gives better results than traditional methods for solving MODM problems [7]. The efficiency of the decomposition-based method increases sharply with the size of the problem.

This work is organized as follows. First, Section 2 presents some approaches to solving APP problems. Section 3 presents the formulation of the model. Section 4 provides a guide for developing the optimal overall production plan using ideal-solution principles. A case study in Section 5 demonstrates the applicability of the proposed model to practical APP decision problems for developing the optimal overall production plan. Finally, conclusions are drawn in Section 6.

2. Literature Review

Being well-known as a decision-making tool, this APP still has imperfections similar to many real-life problems or real-life itself. A way to dissolve such an issue is by using particular quadratic programming (e.g. Holt et al. [8]), and numerous models have spawned henceforth [9–16], among which subjective criteria are rare in the majority and rather transformed into quantitative ones [17]. Aydin and Tirkolaee [18] conducted a systematic review of the literature on APP over the last 50 years. The main objective is to relate current APP research to sustainable development using digital technologies. They discussed the limitations of existing research and gave suggestions to overcome them in the era of Industry 4.0. More insight into APP research that focuses on sustainability and circularity as well as future research directions are also provided. Alazemi et al. [19] proposed a 3-phase fuzzy-based framework to minimize product completion time under fuzzy conditions, which can be applied to small and medium-sized supply chains in developing countries. Some effective techniques have also been used to solve this complicated problem occasionally. These successful methods are discussed in the following subsections.

2.1. Robust Optimization

An APP is based on parameters with uncertain values in many production environments. Since product demand can be uncertain, the robustness of such planning

relies on incorporating uncertain parameters into production scheduling model to address such uncertainty and to meet production demands. Leung et al. [20] developed a solid optimization model to resolve a multisite APP problem in an uncertain environment. Kazemi-Zanjani et al. [21] on the other hand proposed another optimization approach as a possible method in a manufacturing environment but with random yields instead. Al-e-Hashem et al.'s [4] vigorous optimization model is designated to handle multiple products and locations in APP problems, where two conflicting objectives are simultaneously considered as well as the volatile nature of the supply chain. The supply chain cost parameters and demand fluctuations can be unreliable; the problem can then be transformed into a linear multi-objective problem.

2.2. Mixed Integer/Integer Linear Programming

Mixed integer programming is widely used in recent researches. Dhaenens-flipo and Finke [22] created a mixed-integer linear programming-based planning model in a multi-firm, multi-product, and multi-period environment. Park [23] proposed an integrated transportation and production planning model using mixed-integer linear programming in an environment that contains multiple locations, numerous readers, various products, and aggregate phases. The author also presented a sub-model that generates inputs to another sub-model from its outputs that has a transportation planning purpose and a general objective of maximizing total profit using the same technique as Dhaenens-flipo and Finke [22]. Da Silva et al. [24] presented an APP model applied to a Portuguese firm that produces construction materials and developed a multi-criteria, mixed-integer linear programming model with the following performance criteria: profit maximization, order timeliness, and workforce stability. It includes certain operational features, such as partial inflexibility of the workforce, legal constraints on workload, size of the workforce, workers in training, and production and inventory capacity. The purpose is to determine the number and types of workers, overtime hours, inventory level for each product category, and subcontracting necessities. Rizk et al. [25] proposed a mixed-integer, linear programming model for production and distribution planning in a manufacturing environment with a single production facility and multiple distribution centers.

2.3. Nonlinear Programming

To satisfy multiple conflicting objectives, such as multiple products, phasic production, more than 1 periods, and/or numerous unmeasurable goals, Chen and Lee [26] formulated a mixed-integer, nonlinear programming problem that can be used on a multi-echelon supply chain network with unpredictable market demand and variable product prices. It can provide solutions to issues such as fair profit distribution, secure inventory levels, maximum customer service levels, and the robustness of decisions with respect to uncertain product demand. This model simultaneously accounts for compromised product price preference levels of from the sellers' and buyers' perspectives. Lababidi et al. [27] also created a deterministic, mixed nonlinear integer programming model to optimize supply chain system resources by minimizing total production costs and raw material procurement, as well as loss costs, transportation costs, reorder costs, and labor change costs while maximizing overall customer satisfaction in terms of inventory levels, demand, labor levels, machine capacity, and warehouse space.

2.4. Fuzzy Mathematical Programming

Classical mathematical programming may not be perfect for solving APP problems. Wang and Fang [28] discussed these limitations and proposed a fuzzy linear programming model that considers pricing, subcontracting costs, workforce sizes, manufacturing scales, and market feedback using fuzzy set theory. Fung et al. [29] modeled multiproduct APP problems with rough demand, uncertain capacity, and financial constraints. With these unclear demand, and vague production capacities, a fuzzy production-inventory equation for a single period, and a dynamic equation for equilibrium are developed as fuzzy /soft

equations, representing the possibilities of meeting market demand. Using this formulation and interpretation, a fuzzy multi-product aggregate production-planning model was built, which introduces its solution-implementing parametric programming, best balance, and interactive techniques to meet different scenarios based on different decision preferences. Chen and Huang [30] proposed an approach to finding the membership function of fuzzy minimum total cost of APP problems with fuzzy parameters. This approach applies α -cuts and Zadeh's extension principle to transform the fuzzy APP model into a family of crisp APP models that can be described by a pair of mathematical programs. The lower and upper bounds of the α -cuts of the fuzzy minimum total cost for different probability levels α were calculated to derive the approximated membership function; the corresponding optimal aggregate production schedules were also provided.

2.5. Algorithms for Solving Large-Scale Problems

Pradenas et al. [31] developed a mathematical model and a heuristic procedure based on a tabu search for the aggregate production scheduling problem, set in a sawmill, to determine the quantities of different tree trunk (product) types and use different cutting (manufacturing) schedules. Aliev et al. [32] designed a fuzzy-integrated multi-period and multi-product production and distribution model for the supply chain. The model was built in the form of fuzzy programming, and a genetic algorithm provided the solution. These along with interactive aggregate can provide a reasonable tradeoff between maximizing of profit and fill rate. Jain and Palekar [33] presented and compared several heuristics for generating input data for solving APP problems through configuration-based formulations. Computational experiments show that large, real-world problems can be solved in a reasonable time frame using heuristics and commercial optimization software, such as CPLEX. Ramezani et al. [34] have developed a mixed-integer, linear programming (MILP) model to solve NP-hard problems based on genetic and tabu search algorithms in production systems, which generates multiple different products with demands that require a stock-up of inventory. Zare, H. et al. [35] concluded this best by suggesting that decision-makers are enabled to put together "environmental, organizational, and managerial factors" and derive multiple objectives and priorities via goal planning. Based on the literature review, a multi-objective model (MOP) was constructed. The proposed approach extends the TOPSIS method to solve the complex, multi-objective constrained production planning process problem faced by many manufacturing firms.

3. Problem Formulation

The APP problem can be described as follows. It is assumed that a firm produces a single product to satisfy market demand over the planning horizon. The problem is to determine the most effective manufacturing mix that satisfies the projected demand by adjusting production rates, workforce sizes and loads, inventory levels, order status, supplier collaboration, and other controllable variables. The objective functions of this APP decision problem are to maximize sales revenue, to minimize total production costs, and to minimize repair costs as well. Thus, the APP problem can be derived into a mathematical model under the following assumptions:

Notations

The notations used in the model are given as follows.

Parameters

- sr_i = sales revenue for product i (\$/unit)
- pc_i = production cost of regular time for product i (\$/unit)
- oc_i = production cost of overtime for product i (\$/unit)
- cc_i = inventory carrying costs for product i in each period (\$/unit period)
- bc_i = stock-out cost for product i in period t (\$/unit period)
- lc_t = regular payroll cost per worker in period t (\$/man hour)

hc_t = cost of hiring one worker in period t (\$/man-day)
 fc_t = cost of firing one worker in period t (\$/man-day)
 rc_i = repair cost for product i (\$/unit)
 ρ_i = defect rate for product i
 k_i = labor time for product i (man-hour/unit)
 δ = regular working hours per worker per day
 β = fraction of regular workforce available for overtime use in period t
 W_0 = initial workforce level (man-day)
 W_{tmax} = maximum workforce level available in period t
 t = planning horizon or number of periods
 I_0 = initial inventory level (units)
 B_0 = initial backorder level (units)
 D_{itmax} = maximum demand for product i in period t (units)
 D_{itmin} = minimum demand for product i in period t (units)
 M_t = regular time machine capacity in period t (machine-hour)
 θ_{Mt} = fraction of regular machine capacity available for overtime use in period t
 b_i = machine time for product i (machine-hour/unit)
 $\bar{I}_t$ = the storage space limitation in period t
 α_i = fraction of subcontracting output available for product i
 C_{tmax} = maximum capacity limitation of subcontracting in period t
 Q_{tMOQ} = minimum order quantity for subcontracting in period t

Decision Variables

I_{it} = inventory level for product i in period t (units)
 B_{it} = backorder level for product i in period t (units)
 RP_{it} = the unit of regular time production for product i in period t
 OP_{it} = the unit of overtime production for product i in period t
 OS_{it} = the unit of subcontracting production for product i in period t
 W_t = the number of workers in period t
 H_t = the number of workers hired in period t
 L_t = the number of workers laid off in period t

Objective Functions

This proposed APP problem considers three objectives: maximizing sales revenue, minimizing total production cost, and minimizing repair cost. The sales revenue indicates the market share. When market share is a main concern for a company, maximizing sales revenue becomes a primary objective in the APP problem. Repair costs can represent inner failure costs in the production process. In many cases, inner failure costs may be a chief goal because they impact the utilization of production resources. When inner failure costs are high, a company may be forced to use overtime production or more resources. In these cases, repair costs should be considered a separate objective, instead of a component of production costs. In this case, minimizing total production costs, maximizing sales revenue, and minimizing repair costs are all important considerations for the case company. Therefore, it is more appropriate to model them as three separate objectives so that the APP model can find a Pareto optimum that balances these three goals. Thus, we formulate a three-objective, multiple-period APP model for the case study as follows:

Maximize Sales Revenue

$$\max Z_1 = \sum_{i=1}^N \sum_{t=1}^T sr_i \times (I_{it-1} - B_{it-1} + RP_{it} + OP_{it} + OS_{it} - I_{it} + B_{it}) \quad (1)$$

Minimize Production Costs

$$\min Z_2 = \sum_{i=1}^N \sum_{t=1}^T (pc_i \times RP_{it} + oc_i \times OP_{it} + sc_i \times OS_{it} + cc_i \times I_{it} + bc_i \times B_{it}) + \sum_{t=1}^T (hc_t \times H_t + fc_t \times L_t) + \sum_{t=1}^T lc_t \times W_t \quad (2)$$

Minimize Repair Costs

$$\min Z_3 = \sum_{i=1}^N \sum_{t=1}^T \rho_i \times rc_i \times (RP_{it} + OP_{it}) \quad (3)$$

The first objective function in (1) is to achieve the highest possible return from the quantities generated by regular production, overtime production, and contract production, including inventories and back orders. The second objective function in (2) is to minimize production costs, which contain three components. The first component involves production costs, subcontracting costs, inventory, and backorder level costs. The second component entails extra production loading or how many workers would have to be laid off to reduce overhead. The last component is the labor costs associated with regular-time workers. The third objective function in (3) considers the quality issue. Defect rates differ slightly across products. Management has set an acceptable (if not desirable) amount that the company is willing to pay in each period for repair costs. The first objective function is to maximize sales revenue and increase production quantities to achieve the highest possible goal. However, attaining the highest possible revenue may result in increased production costs in the second objective function. Similarly, increased repair costs may raise total expenses, which impacts total sales revenue. Most conflicts are generated by improving each objective function. Attempting to enhance each objective function may result in further conflicts. Hence, this paper proposes a compromised solution to solve this problem.

Constraints

After the three objective functions formulated in the previous section, nine constraints related to the APP model were set up as follows.

$$W_t \leq W_{tmax} \quad (4)$$

$$W_t = W_{t-1} + H_t - L_t, t \in T \quad (5)$$

$$H_t \times L_t = 0, t \in T \quad (6)$$

$$\sum_{i=1}^N k_i \times RP_{it} \leq \delta \times W_t, t \in T \quad (7)$$

$$\sum_{i=1}^N k_i \times OP_{it} \leq \delta \times \beta \times W_t, t \in T \quad (8)$$

$$I_{it-1} - B_{it-1} + RP_{it} + OP_{it} + OS_{it} - I_{it} + B_{it} \leq D_{itmax} \quad (9)$$

$i \in I, t \in T$

$$RP_{it} + OP_{it} + OS_{it} + I_{it-1} - B_{it-1} \geq D_{itmin} \quad (10)$$

$i \in I, t \in T$

$$I_{it} \times B_{it} = 0, i \in I, t \in T \quad (11)$$

$$\sum_{i=1}^N b_i \times RP_{it} \leq M_t, t \in T \quad (12)$$

$$\sum_{i=1}^N b_i \times OP_{it} \leq \theta_{Mt} M_t, t \in T \quad (13)$$

$$\sum_{i=1}^N I_{it} \leq \bar{I}_t, t \in T \quad (14)$$

$$Q_{tMOQ} \leq \sum_{i=1}^N \alpha_i OS_{it} \leq C_{tmax} \quad (15)$$

$$RP_{it}, OP_{it}, OS_{it}, I_{it}, B_{it}, W_t, H_t, \bar{I}_t \geq 0, \quad i \in I, t \in T \quad (16)$$

Constraint (4) ensures that the limit of the maximum available labor is respected in each period. Constraint (5) ensures that the available workforce in each period is equal to the workforce in the previous period, plus or minus any change in the workforce in the current period. The change in headcount may be due to the hiring of additional workers or the laying off of redundant workers. Constraint (6) states that $H_t \times L_t = 0$ because either net hiring or net laying off of workers occurs in a period but not both simultaneously. Constraints (7) and (8) ensure that the hours worked to produce products during regular and overtime hours are limited to the available regular and overtime hours. Constraints (9) and (10) specify that the quantity of products sold is between the minimum known demand and the maximum forecast demand. Constraint (11) ensures that either inventory or backorders are included in the solution, but not both. Constraints (12) and (13) ensure that the machine times for manufacturing the products are limited to the regular and overtime machine capacity in the manufacturing facility. Constraint (14) ensures that the inventory does not exceed the maximum storage space limit. Constraint (15) ensures that procurement quantities can meet the minimum order quantity, but do not exceed the maximum supplier capacity. Constraint (16) ensures that all decision variables are non-negative.

4. Model Solution

4.1. d_p Distance Concept

To solve the MOP problems, initial reference points must be established to obtain compromise programming, such as the global criteria method, goal programming, fuzzy programming, and the interactive approach. With given reference points, the MOP problems can then be solved in the way that is closest to the ideal point. Thus, the problem is how to measure the distance to the ideal point. Target programming measures this distance using the weighted sum of the absolute distance to the given targets. The d_p metric defines the distance between two points, z and z^+ (ideal point), in k -dimensional space as:

$$d_p = \left\{ \sum_{j=1}^k [z_j^+ - z_j]^p \right\}^{1/p}, \text{ where } p \geq 1, \quad (17)$$

Based on the distance concept, the compromise programming (CP) model [36,37] follows goal-seeking behavior, looking for the closest solution to the ideal point in terms of a distance function, given by:

$$d_p(x) = \left\{ \sum_{j=1}^k w_j^p [z_j^+ - z_j(x)]^p \right\}^{1/p}, \text{ where } p \geq 1, \quad (18)$$

where p is the order of the norm, w_j are the weights given to the normalized deviations, X is the set of feasible alternatives, and $z^+ = (z_1^+, z_2^+, \dots, z_k^+)$ is the ideal point ($z_j^+ = \text{opt}\{z_j(x), x \in X\}$).

The parameter p is a factor that balances group utility with maximum individual regret. As the value of p increases, the distance d_p decreases, i.e., $d_1 \geq d_2 \geq \dots \geq d_p$, and greater emphasis is given more importance in forming the total. Specifically, $p = 1$ means that all these deviations are equally important, while $p = 2$ means that these deviations are weighted proportionally, with the largest deviation having the greatest weight. Finally, for $p = \infty$, the largest deviation completely dominates the distance determination [38,39].

$$d_\infty = \max_j \{w_j | z_j^+ - z_j(x)\}, j \in k \quad (19)$$

However, because of the incommensurability among these objectives, it is not possible to use the above distance family directly. To remove the effects of the incommensurability, we have need to normalize the distance (18) by taking the ideal point [40,41] as:

$$d_p = \left\{ \sum_{j=1}^k w_j^p \left[\frac{z_j^+ - z_j(x)}{z_j^+} \right]^p \right\}^{1/p}, \text{ where } p \geq 1 \quad (20)$$

The amount of d_p decreases when parameter p increases.

To obtain a compromise solution, the global criteria method for MOP problems calculates the distance value, where the ideal solution is the reference point. The problem now is how to solve the following auxiliary problem [39]:

$$\min_{x \in S} d_p = \left\{ \sum_{j=1}^k w_j^p \left[\frac{z_j(x^+) - z_j(x)}{z_j(x^+)} \right]^p \right\}^{1/p}, \quad (21)$$

where x^+ , $j \in k$ is the positive ideal solution, and $p = 1, \dots, \infty$.

In the next section, the concept of TOPSIS is extended for MODM problems to obtain a compromise solution for MOP problems.

4.2. TOPSIS for Solving MOP Problems

Hwang and Yoon [34] proposed TOPSIS (technique for order preference by similarity to ideal solution). Usually, the solutions based on the positive ideal solution (PIS) are different from the solutions based on the negative ideal solution (NIS). Therefore, both $PIS(x^*)$ and $NIS(x^*)$ can be used to normalize the distance family as follows [7,37,38]:

$$d_p = \left\{ \sum_{j=1}^k w_j^p \left[\frac{z_j^+ - z_j(x)}{z_j^+ - z_j^-} \right]^p \right\}^{1/p}, \text{ where } p \geq 1 \quad (22)$$

where z_j^+ and z_j^- are the maximum and minimum values of the function $z_j(x)$, respectively.

They include the positive ideal solution (PIS) and the negative ideal solution (NIS) for solving multi-attribute decision making (MADM) problems. Using a similar concept, the principle of TOPSIS for MODM is to determine the optimal solution such that it minimizes the distance to PIS and maximizes the distance to NIS. Suppose there is the following MOP problem:

$$\max/\min \quad [z_1(x), z_2(x), \dots, z_k(x)] \quad (23)$$

$$\text{s.t. } X = \{x \in X | g_i(x) \leq, =, \geq b_i, \\ \forall i = 1, \dots, m, x \geq 0, x \in R^k\} \quad (24)$$

where

$z_j(x)$ = benefit objective for maximization $j \in J$

$z_i(x)$ = cost objective for minimization $i \in I$

The ideal and anti-ideal values of a maximization objective are calculated using (25) under the problem constraints. The same values of a minimization objective are computed from (26) under the constraints of the problem.

$$z^+ = \left\{ \max_{x \in X} z_j(x), j = 1, 2, \dots, k \right\}, \\ z^- = \left\{ \min_{x \in X} z_j(x), j = 1, 2, \dots, k \right\} \quad \text{or} \quad (25)$$

$$z^+ = \left\{ \min_{x \in X} z_i(x), i = 1, 2, \dots, k \right\}, \\ z^- = \left\{ \max_{x \in X} z_i(x), i = 1, 2, \dots, k \right\} \quad (26)$$

where $j \in J, i \in I, z^+ = \{z_1^+, \dots, z_k^+\}$, and $z^- = \{z_1^-, \dots, z_k^-\}$ is a set of individual positive (negative) ideal solutions and is a point in the k -dimensional objective functional space. Using the PIS and the NIS, the following distance functions are obtained:

$$d_p^{PIS} = \left\{ \sum_{j \in k_1} w_j^p \left[\frac{z_j^+ - z_j(x)}{z_j^+ - z_j^-} \right]^p + \sum_{i \in k_2} w_i^p \left[\frac{z_i(x) - z_i^+}{z_i^- - z_i^+} \right]^p \right\}^{1/p} \quad (27)$$

and

$$d_p^{NIS} = \left\{ \sum_{j \in k_1} w_j^p \left[\frac{z_j(x) - z_j^-}{z_j^+ - z_j^-} \right]^p + \sum_{i \in k_2} w_i^p \left[\frac{z_i^- - z_i(x)}{z_i^- - z_i^+} \right]^p \right\}^{1/p} \quad (28)$$

where $w_j, w_i, j = 1, 2, \dots, k_1$, and $i = 1, 2, \dots, k_2$ are the relative weights of the objectives, k_1 denotes the number of maximizing (or minimizing) objective functions, k_2 represents the number of minimizing (or maximizing) objective functions and $p = 1, \dots, \infty$.

To obtain a compromise solution, the MOP problem is transformed into the following bi-objective problem with two justifiable goals:

$$\min d_p^{PIS}(x) \quad (29)$$

$$\max d_p^{NIS}(x) \quad (30)$$

$$\begin{aligned} \text{s.t. } & X = \{x \in X | g_i(x) \leq, =, \geq b_i, \\ & \forall i = 1, \dots, m, x \geq 0, x \in R^k\}, \end{aligned} \quad (31)$$

where $p = 1, 2, \dots, \infty, d_p^{PIS}$, and d_p^{NIS} represent the distances of the d_p -metric from the PIS and from the NIS, respectively. Since there are usually conflicts with the individual targets, it is difficult to achieve their individual optima simultaneously. Therefore, the membership functions $\mu_1(x)$ and $\mu_2(x)$ are used to represent the satisfactory level of the bi-objective functions and use the max-min operation [42,43] to control the equivalent model, given the same values of γ :

$$\max \gamma \quad (32)$$

$$\begin{aligned} \text{s.t. } & \mu_1(x) \geq \gamma \\ & \mu_2(x) \geq \gamma \\ & x \in X \end{aligned} \quad (33)$$

where $\gamma = \min(\mu_1, \mu_2)$ is the minimal satisfaction level for the two criteria of shortest distance from PIS and farthest distance from NIS. The membership functions $\mu_1(x)$ and $\mu_2(x)$ are shown in Equations (34) and (35), respectively.

5. Model Implementation

5.1. Data Description

In this section, we describe the application of the proposed approach to a real-world aggregate production planning problem. Founded in 1989, the company is the leading provider of complete broadband access solutions for internet service providers, enterprises, and home users. The company has more than 3200 employees and sales offices worldwide. The company's 2008 revenue was more than US\$479 million. Production facilities are located in Taiwan and mainland China. The company produces two products (Wireless LAN and Ethernet Switch) from its Taiwan manufacturing plants to fulfill demand. Based on company reports, a six-period planning horizon is determined. The regular workday is 8 (δ) man-hours. The regular payroll is NT\$150 (l_{ct}) in each period. The costs associated with hiring and firing are NT\$50 (hc_t) and NT\$40 (fc_t) per worker per day, respectively. Production costs for overtime are limited to a maximum of 30% (β) of regular-time production. It is also assumed that there is no beginning inventory (I_{i0}) or backorder (B_{i0}) for each product. The storage space limitation of the inventory does not exceed 2000 m^3 ($\bar{I}$). Table 1 shows the sales revenue of each product and the production cost. Table 2 presents the related

operating data. The minimum order quantity of a subcontractor is 1000 units (Q_{tMOQ}), but the capacity does not exceed 5000 units (C_{tmax}) in each period. Table 3 depicts the maximum workforce and fraction of machine capacity. The regular time machine capacity in each period (M_t) is 12,000 h. Finally, Table 4 displays the maximum forecast demand and minimum known demand.

Table 1. Sales Revenue and Production Costs.

Product (i)	Sales Revenue (sr_i)	Production Costs, Regular Time (pc_i)	Production Costs, Overtime (oc_i)	Subcontracting Costs (sc_i)	Inventory Costs (cc_i)	Backorder Costs (bc_i)
1	100	50	70	90	50	45
2	200	60	80	110	55	50

Table 2. Related Operating Data.

Product (i)	Labor Time for Product (k_i)	Machine Time (b_i)	Subcontracting Output Fraction (α_i)	Repair Cost (rc_i)	Defect Rate (ρ_i)
1	2	0.3	0.5	30	5%
2	3	0.4	0.6	45	3%

Table 3. Maximum Work Force and Fraction of Machine Capacity.

	Period (t)					
	1	2	3	4	5	6
W_{tmax}	450	500	350	550	400	500
θ_{Mt}	0.5	0.4	0.5	0.4	0.6	0.6

Table 4. Maximum forecast demand and minimum known demand.

	Period (t)					
	1	2	3	4	5	6
Product: 1 (Wireless LAN)						
Maximum forecast demand 1	6000	7000	5500	5000	4500	6000
Minimum known demand 1	2000	2500	1500	1300	1000	2000
Product: 2 (Ethernet Switch)						
Maximum forecast demand 2	5500	6000	6500	4000	5000	3500
Minimum known demand 2	1500	2000	2300	2000	1000	800

5.2. Problem Solving

Using the data presented in the previous subsection, LINGO—a commercial optimization software—is used to solve this model. A traditional linear programming approach with a single objective in (23)–(26) was used to solve a set of single positive ideal solutions and negative ideal solutions. Then, the payoff table PIS and the payoff table NIS constructed in Tables 5 and 6, respectively, were obtained.

Table 5. PIS Payoff Table.

		Z_1	Z_2	Z_3
Max	Z_1	9,500,000 ⁺	6,488,440	22,790
Min	Z_2	1,697,000	1,642,785 ⁺	3318
Min	Z_3	5,000,000	3,858,000	0 ⁺

PIS: $z^+ = (9,500,000, 1,642,785, 0)$.

Table 6. NIS Payoff Table.

		Z_1	Z_2	Z_3
Min	Z_1	1,677,200 –	1,654,480	3318
Max	Z_2	9,500,000	8,494,955 –	18,389
Max	Z_3	7,776,000	5,183,500	33,930 –

NIS: $z^- = (1,677,200, 8,494,955, 33,930)$.

Next, the information from the PIS and NIS payoff tables was used to transform (27) and (28). Thus, d_p^{PIS} and d_p^{NIS} in (36) and (37) were acquired.

$$\mu_1(x_k) = \begin{cases} 1 & \text{if } d_p^{PIS}(x_k) > (d_p^{PIS})^+ \\ 1 - \frac{d_p^{PIS}(x_k) - (d_p^{PIS})^+}{(d_p^{PIS})^- - (d_p^{PIS})^+} & \text{if } (d_p^{PIS})^- \leq d_p^{PIS}(x_k) \leq (d_p^{PIS})^+ \\ 0 & \text{if } d_p^{PIS}(x_k) < (d_p^{PIS})^- \end{cases} \quad (34)$$

$$\mu_2(x_k) = \begin{cases} 1 & \text{if } d_p^{NIS}(x_k) > (d_p^{NIS})^+ \\ 1 - \frac{(d_p^{NIS})^+ - d_p^{NIS}(x_k)}{(d_p^{NIS})^+ - (d_p^{NIS})^-} & \text{if } (d_p^{NIS})^- \leq d_p^{NIS}(x_k) \leq (d_p^{NIS})^+ \\ 0 & \text{if } d_p^{NIS}(x_k) < (d_p^{NIS})^- \end{cases} \quad (35)$$

$$d_p^{PIS} = \left\{ \begin{array}{l} (\sum_{i=1}^2 \sum_{t=1}^6 w_1^p \times [(9,500,000 - Z_1) / (9,500,000 - 1,677,200)]^p + \\ \sum_{i=1}^2 \sum_{t=1}^6 w_2^p \times [(Z_2 - 1,642,785) / (8,494,955 - 1,642,785)]^p + \\ \sum_{i=1}^2 \sum_{t=1}^6 w_3^p \times [(Z_3 - 0) / (33,930 - 0)]^p \end{array} \right\}^{1/p} \quad (36)$$

$$d_p^{NIS} = \left\{ \begin{array}{l} (\sum_{i=1}^2 \sum_{t=1}^6 w_1^p \times [(Z_1 - 1,677,200) / (9,500,000 - 1,677,200)]^p + \\ \sum_{i=1}^2 \sum_{t=1}^6 w_2^p \times [(8,494,955 - Z_2) / (8,494,955 - 1,642,785)]^p + \\ \sum_{i=1}^2 \sum_{t=1}^6 w_3^p \times [(33,930 - Z_3) / (33,930 - 0)]^p \end{array} \right\}^{1/p} \quad (37)$$

To obtain numerical solutions in (29)–(31), the weights of d_p^{PIS} and d_p^{NIS} are set as $w_1 = 0.4$, $w_2 = 0.3$, and $w_3 = 0.3$. According to Lai et al.'s study [39], increasing in p increases the influence of objectives with large weights on the measurement distances. Thus, the value of p is related to the preferences of decision-makers. In this study, $p = 1$ was set to obtain the problem as either concave or convex. If the preference for the p value that is significantly different from 1 can be efficiently solved with multiple local optima, then the other p values can be set in the future. Table 7 showed the PIS Payoff Table for this case.

Table 7. PIS Payoff Table.

	d_1^{PIS}	d_1^{NIS}
$Min d_1^{PIS}$	1.35858	1.49444
$Max d_1^{NIS}$	3.78877	1.43973

Finally, the membership functions $\mu_1(x)$ and $\mu_2(x)$ of (29)–(31) can be used to represent the satisfactory level of bi-objective functions and use the max–min operation to control the corresponding model. This results in the following formulation can be obtained:

$$\max \quad \gamma \quad (38)$$

$$\begin{aligned} \text{s.t.} \quad & (d_1^{PIS}(x) - 1.35858) / (2.4302) \geq \gamma \\ & (1.43973 - d_1^{NIS}(x)) / (-0.05474) \geq \gamma \\ & x \in X \end{aligned} \quad (39)$$

When the results of d_1^{PIS} and d_1^{NIS} are calculated, the problems of (38) and (39) can be solved. In Table 7, the maximum satisfactory level (γ_{max}) is 0.7120 to achieve the

compromise solution for the production plan in Table 8. From the system of Equations (36), (37) and (3), three values Z_1 , Z_2 , Z_3 are found. The sales revenue (Z_1), production costs (Z_2), and repair costs (Z_3) are NT\$7,662,900, NT\$4,779,525, and NT\$10,458, respectively.

Table 8. Compromise Solution for the Production Plan.

	Period (t)					
	1	2	3	4	5	6
Product: 1 (Wireless LAN)						
Regular time production	734	753	128	633	485	305
Overtime production	10	102	194	207	26	127
Subcontracting level	2143	1662	1870	492	491	2978
Inventory level	0	0	0	0	0	0
Backorder level	0	0	0	0	0	0
Product: 2 (Ethernet Switch)						
Regular time production	61	207	848	0	743	871
Overtime production	317	240	3	88	97	156
Subcontracting level	5122	5553	5661	3900	2721	2290
Inventory level	0	0	12	0	0	0
Backorder level	0	0	0	0	1439	1622
Workforce level	450	500	350	550	400	500
Hiring	0	50	0	200	0	100
Laying off	50	0	150	0	150	0

5.3. Performance Analysis

To evaluate the performance of the proposed approach, the d_p distance concept was used to develop an evaluation method; (19)–(21) were considered to determine the degree of closeness of the result of the extended TOPSIS approach to the ideal solution. The distance functions can be defined as follows.

$$d_{\infty} = \max \left\{ w_j \left| z_j^+ - z_j(x) \right|, j \in k \right\}$$

and

$$d_p = \left\{ \sum_{j=1}^k w_j^p \left[\frac{z_j^+ - z_j(x)}{z_j^+} \right]^p \right\}^{1/p}, p \geq 1, j \in k$$

where d_p represents the degree of approximation of the preferred compromise solution vector X to the optimal solution vector with respect to the k th objective function. Here, $w = (w_1, w_2, \dots, w_k)$ is the vector of objective demand levels. The power p represents a distance parameter $1 \leq p \leq \infty$.

Finally, the solution of the illustrative example is considered using different methods. The target programming approach (a) yields the following results: $Z_1 = 4,750,000$, $Z_2 = 2,616,120$, and $Z_3 = 3785$. The fuzzy goal programming approach (b) [16] gives the following results: $Z_1 = 3,350,000$, $Z_2 = 2,679,250$, and $Z_3 = 3785$. The compromise programming (CP) approach (c) [41] yields $Z_1 = 7,530,000$, $Z_2 = 4,706,250$, and $Z_3 = 1019$. On the other hand, the proposed approach (d) obtains the solution $Z_1 = 7,662,900$, $Z_2 = 4,779,525$, and $Z_3 = 10,458$. Table 9 compares the results obtained by three existing methods and the proposed approach. In the given example, it is assumed that $w_1 = 0.4$, $w_2 = 0.3$, and $w_3 = 0.3$.

From Table 9 and Figure 1, it is clear that approaches (c) and (d) offer preferred solutions that are better than the solutions obtained by approaches (a) and (b) for all distance functions d_1 , d_2 , and d_{∞} . The profits and sales revenues resulting from approaches (c) and (d) are also higher than those obtained by the other two approaches. In addition, further comparison between approaches (c) and (d) reveals that the profit of the solution obtained by the proposed approach (d) is higher than that resulting from approach (c). The sales revenue (market share) in the solution is 132,900 units higher than that obtained

by approach (c). Moreover, the distances d_1 and d_2 are shorter in the solution, while d_∞ is the same for the two approaches. The shorter the distances d_1 and d_2 are, the better the performance. Thus, these outcomes show that the proposed approach has good performance. It can be concluded that the extended TOPSIS compromise solution is more suitable than the other approaches. The comparison between different approaches is illustrated in Figure 1.

Table 9. Comparison of Solutions Obtained by Four Different Approaches.

	(a) GP	(b) Fuzzy GP	(c) CP	(d) Proposed Approach	(e) Optimal Solution
Z_1	4,750,000	3,350,000	7,530,000	7,662,900	9,500,000
Z_2	2,616,120	2,679,250	4,706,250	4,779,525	1,642,785
Z_3	3785	3785	1019	10,458	0
Profit	2,130,095	666,965	2,822,731	2,872,917	-
d_1	0.8116	1.1504	0.5999	0.5928	-
d_2	0.5123	0.8017	0.3729	0.3714	-
d_∞	0.4000	0.7343	0.3000	0.3000	-

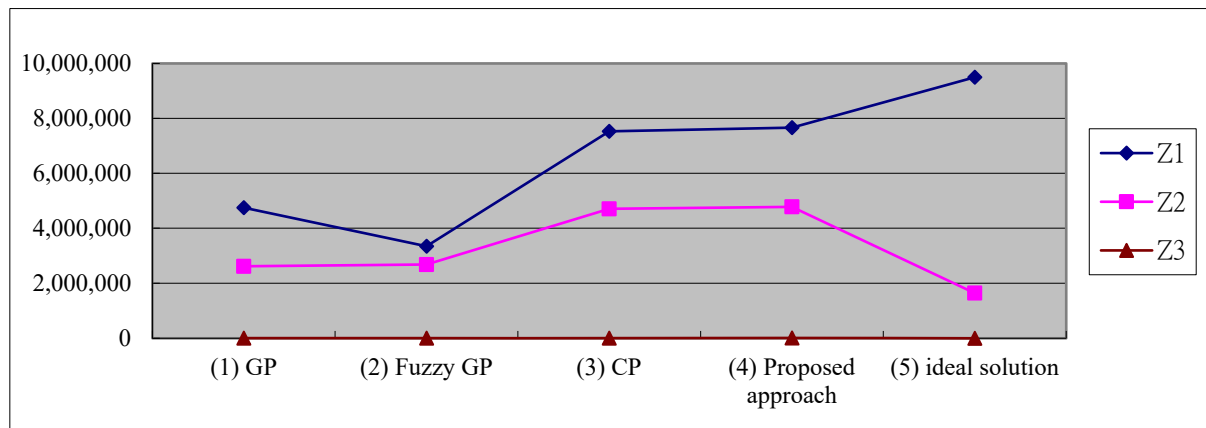


Figure 1. Graphical representation of the solutions by different approaches.

6. Conclusions

In this paper, the TOPSIS compromise solution method was extended to solve programming problems with multiple objectives. The APP model, combined with a TOPSIS approach, provides an effective way to find a satisfactory solution to such problems. In generally, TOPSIS provides a broader principle of compromise for solving multiple-criteria decision making problems. It converts k-objectives, which are conflicting and incompatible with each other, into two-objectives (the shortest distance from PIS and the longest distance from NIS), which are compatible and conflicting with each other in most cases. The bi-objective problem can then be solved using membership functions to represent the satisfaction level for both criteria and obtain a TOPSIS compromise solution via a second-order compromise. The max–min operator is then considered a suitable approach to solve the conflict between the new criteria (the shortest distance from the PIS and the longest distance from the NIS) [39].

The proposed APP model provides useful information to the industry from the determined optimal overall production plan. Using this model, decision-makers can work out the optimal number of temporary workers to hire at the beginning of each period, the optimal number of temporary workers to be laid off at the end of each period, and the number of temporary workers that should be available in each period. This information is useful for managing the human resources of a manufacturing company. The output of this model also provides information on the optimal number of overtime man-hours of permanent and temporary workers during workdays and holidays, inventory levels, and

the number of regular working hours for some periods. The related cost elements and total costs are presented for financial consideration by top management.

In this paper, four advantages of TOPSIS have been addressed: (i) sound logic that is the rationale in solving multi-objective decision problems; (ii) a scalar value that considers both the best and worst ideal solutions simultaneously; (iii) a simple calculation process that can be easily programmed into a spreadsheet; and (iv) the performance measures compared to other approaches can be visualized on a polyhedron.

The proposed general APP model is applicable to a wide range of industries in which the output per period can be adjusted by changing the number of jobs and the number of workers, and by applying overtime. However, the proposed APP model is not suitable for multi-plant and multi-product problems because the model's design is suitable for single-product, single-plant problems. A new APP model can be constructed with different sets of constraints to handle problems with multiple plants and multi-product. We recommend that further studies be conducted to develop APP models that meet the specific requirements of multi-plant and multi-product problems. Appropriate methods for disaggregating the aggregate plan into the total production plan should also be devised based on the situations and requirements of the industries.

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