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Joint Maintenance Strategy Optimization for Railway Bogie Wheelset

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Abstract: A wheelset is one of the most severely worn components of railway bogies. Its health condition has a significant impact on the safety and comfort of railway trains. Moreover, wheelset maintenance costs account for a sizeable part of the railway operating company. Therefore, it is essential to investigate executable maintenance strategies for wheelsets. In this paper, a maintenance strategy that combines periodic inspection and preventive maintenance is proposed. The wheel flange and wheel diameter deterioration models are established by the compound Poisson process based on the cumulative shock from the failure mechanism, and the Weibull distribution is adopted for modeling wheel tread failure probability. The age reduction factor is introduced to describe the maintenance effect. Then, the joint maintenance optimization model is constructed to determine the periodic inspection interval and the reprofiling strategy, with the objective of achieving a minimum maintenance cost rate and with the wheel flange thickness failure as the failure risk constraint. Lastly, a case study is provided, and the results show that, compared with the two conventional maintenance strategies with fixed inspection periods, the maintenance strategy proposed in this paper can reduce the maintenance cost rate by 27.06% and 12.0%, respectively. Moreover, the life span is prolonged by 11.51% and 11.98%, respectively.

Keywords: wheelset; cumulative shock model; joint maintenance strategy; failure risk

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1. Introduction

The railways of the world have entered a new stage since the beginning of the 21st century. More than 500 cities in 77 countries and regions had operated urban railways by the end of 2020. The boom of railways put forward higher requirements for operational safety. Plenty of research has been conducted on various aspects of railway equipment and infrastructure, such as the robust safety design of rail vehicle systems [1,2], railway infrastructure management adapted RAM analyses [3], the forecast of the failure progression of railway infrastructure turnouts [4] and the interaction between vehicle wheelsets and tracks [5,6]. A wheelset is an important component to ensure the operation and safety of the railway vehicle. It is also necessary to study the maintenance of the wheelset.

As the main component of the bogie, the wheelset is the contact between the vehicle and the rail, which carries the full load of the rolling stock and transmits it to the rail to ensure running and steering. Due to poor service conditions, wheelsets often have defects such as scratches, cracks, spalling, shelling and corrosion. The performance of the wheelset has a crucial impact on running safety and comfort [7,8]. The wheel flange and tread deterioration are vital factors affecting wheel performance [9]. When the wheel flange is excessively worn, the strength of the wheelset decreases, and it even causes derailment in severe cases, threatening running safety. Trains are subjected to severe shock vibration and stress concentration due to tread wear. This reduces the operation stability of the vehicle, accelerates the damage of vehicle components, causes rail damage and brings losses to railway operations. Reprofiling is the primary method of wheelset maintenance, which recovers the shape of the wheelset. For every 1 mm increase in wheel flange thickness, the wheel diameter is reduced by about 4.2 mm [10]. Reasonable inspection intervals and reprofiling strategies of the wheel can avoid excess maintenance [11]. The cost of wheelset maintenance comprises a large percent of the maintenance budget for all railway operating companies [12]. Improving wheelset maintenance strategies can not only improve the safety level of rail transit, but it can also reduce maintenance costs.

Research on wheelset maintenance strategies has been paid more and more attention in recent years. Some research focuses on wear prediction models and theories to forecast and assist maintenance decision making [13,14]. Chang et al. [15] presented an early-stage data-based maintenance strategy to optimize maintenance intervals. Wheelset reprofiling strategies are developed based on wear analysis [16,17]. Data-driven analysis methods exploit information in historical data and are easy to implement. However, data-driven approaches are more focused on making the model fit the data, and the deterioration mechanism is rarely considered in these papers. Wheelset maintenance strategies are investigated by analyzing wheelset reliability [18]. Umamaheswari et al. [19] proposed a maintenance model that considers wheelset aging and degradation for optimal inspection times. Lin et al. [20] used classical and Bayesian semi-parametric degradation approaches to establish a hazard regression model to optimize wheel maintenance intervals. Reliability modeling is the foundation of maintenance optimization. Nevertheless, discussion of reprofiling strategies increases the difficulty of the reliability analysis of the wheelset, and thus, little attention is paid to reprofiling policies for wheelsets in this respect. Additionally, maintenance measures are developed by researching wheelset status evaluations. A data-driven evaluation model and an experience-based evaluation method for wheelsets support the maintenance decision [21,22]. The method can consider the wheelset status comprehensively. However, the evaluation process, such as selecting key indicators, is susceptible to subjective effects. To the best of our knowledge, only a small portion of the literature considers optimizing maintenance intervals and reprofiling strategies at the same time.

In addition, revealing degradation mechanisms and understanding the wear law are important aspects of maintenance optimization research. According to a large number of studies on failure mechanisms, the essence of equipment performance degradation is caused by external shocks. To characterize the changes in performance through external shocks, Kijima et al. [23] proposed a shock model that is more physical and intuitive. Thus far, equipment reliability and maintenance strategy optimization have been studied based on the cumulative shock model [24–29]. Equipment is subject to the competing failure of internal-based deterioration and external-based shocks to optimize preventive maintenance thresholds and periodic inspection intervals [30–33]. The shock model can solve the problem of maintenance optimization very well. However, few studies on maintenance strategy optimization consider the impact of external shocks on wheelsets. E [34] et al. modeled within the framework of the semi-Markov decision-making process, considering wear and external shocks and optimizing the repair strategy. Liang [35] optimized the periodic inspection interval for high-speed train wheelsets, which are subject to internal degradation and external shocks. The wear of wheels during running is mainly caused by the interaction between rails. It is assumed that the degradation of wheelsets is caused by external shocks. The continuous action of external conditions leads to the wear of wheel and needs to be reprofiled.

Combined with practical applications, a joint maintenance strategy that combines periodic inspection and preventive maintenance is proposed to improve the existing maintenance mode. For the proposed strategy, the decision variables of periodic inspection intervals and reprofiling thresholds are optimized to prolong service life and reduce maintenance cost rates. The main contributions of the work are summarized as follows: (1) A novel joint maintenance strategy is proposed by analyzing the wear of wheel flanges and diameters and the effect of maintenance according to the actual maintenance mode of daily inspection and periodic inspection. (2) Three typical failures are comprehensively considered, which are wheel flange wear, wheel diameter wear and tread failure. The age-reduction factor is introduced to describe the effect of imperfect maintenance, and the joint maintenance probability model is established based on the cumulative shock model. (3) The inspection interval and reprofiling control strategy are optimized with the objective of achieving a minimum cost rate and with the wheel flange thickness failure as the failure risk constraint. Detailed steps are provided to illustrate the effectiveness and necessity of the proposed maintenance strategy in the case study.

The paper is organized as follows. Section 2 describes the cause of wheelset wear, analyzes the effect of maintenance and proposes a new maintenance strategy. In Section 3, the joint maintenance probability model and failure risk constraint of the wheelset are constructed. Numerical calculations and an analysis of the results are given in Section 4. The final section summarizes the work and suggests future research directions.

2. Problem Statement

The contact and action between the rolling stock and the rail are carried out through the bogie. The bogie supports the entire train body and enables the train to run fast on the rail. The structure of the bogie is shown in Figure 1. The bogie is mainly composed of the frame, wheelset, axle box, primary suspension, secondary suspension, traction motor and braking unit. The wheelset is an essential part of the bogie and transfers the vehicle weight to the rails directly. Traction and braking force are generated by wheel–rail adhesion, and the vehicle is guided on the rail.

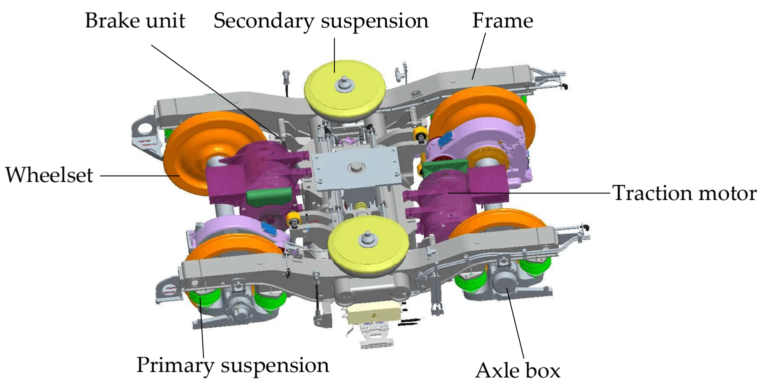

Figure 1. The bogie structure.

The dimensions of important structures of the wheel need to be measured during maintenance. Referring to the standard [36], the wheel profile and main parameters are shown in Figure 2. The wheel diameter is donated as D. The thickness and height of the flange are indicated as Sd and Sh, respectively. Qr stands for the flange gradient. The wheelset is one of the main components of the bogie and also one of the most severely worn parts of the vehicle [37]. For rail transit, the wear of the wheel flange mainly occurs in the curved section. The outer flange of the wheel is in contact with the inner side of the outer rail, and the reactive force on the rail leads to flange wear. Wheel diameter wear is mainly caused by contact with the rail during operation. Moreover, the interaction between the brake shoe and the wheel when the train is braking also causes wear, which is more severe for vehicles that frequently brake and stop [38]. Wheel diameter wear rate and flange thickness, as well as flange thickness wear rate and diameter, are independent of each other [14,35,38]. The wear of the wheel flange and the diameter can be regarded as uncorrelated. Wheel tread scratches are generally caused by wheels sliding on the rail. Strong friction between the tread and the rail generates a high temperature. It is easy to cause small pieces of metal on the tread to fall off or lift up and to cause the tread to spall after the temperature cools down.

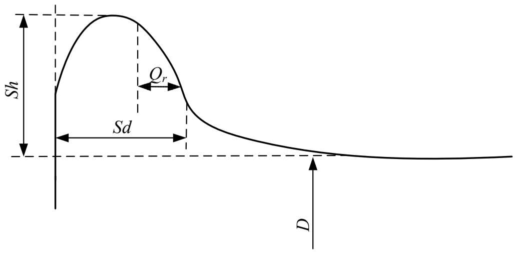

Figure 2. The profile and main parameters of the wheel.

Different vehicle models have strict regulations on wheel parameters. It is required that the vehicle wheelsets of railway operation companies are publicly available, and details are shown in Table 1 [39]. Wheel flange and diameter dimensions decrease with wear during operation. Therefore, wheelset parameters are inspected periodically to ensure the safe operation of the train. Reprofiling is performed when the wheel flange thickness is smaller than the specified requirements. Reprofiling leads to increased wheel flange thickness and reduced wheel diameter [34,37,40], as shown in Figure 3. Preventive maintenance is adopted to the wheel before reaching the threshold to avoid derailment.

Table 1. Standard dimensions for the use of wheelsets.

Items	The Limit
Wheel diameter	790~860 mm
Wheel flange height	28~33 mm
Wheel flange thickness	26~32 mm
Qr value	6.5~12.7 mm
Inside distance of wheelset	1353 (+3, −2) mm

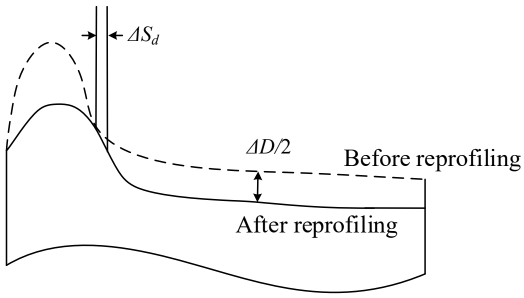

Figure 3. Effects of reprofiling on the wheelset's profile, flange thickness and diameter.

To facilitate the description and research of the problem, the following assumptions are given.

(1) The wear of the wheel flange and diameter is caused by external shocks, and they are uncorrelated with each other.
(2) In this study, it is assumed that the depot is not equipped to automatically measure wheelsets. The wheel flange thickness and wheel diameter are measured manually, and the measurements are recorded by maintenance personnel. The internal failures of the wheel are not considered.
(3) A maintenance cycle starts at the end of the previous periodic inspection and ends at the start of the next periodic inspection, and the interval of a maintenance cycle is Δt.
(4) The effect of reprofiling is imperfect. Reprofiling only recovers the wheel profile but does not eliminate the failure mechanism. After reprofiling, the condition of the wheel is between 'repaired as new' and 'repaired as old'.

Based on the wear mechanisms of the wheel flange and diameter, it is assumed that the external shocks on them are uncorrelated. For many depots, manual inspection is required due to the high cost of automatic measuring equipment and the problem of measurement accuracy. Maintenance can improve a wheelset's condition, but it is difficult to restore the wheelset to the same condition as a new one, so the effect of reprofiling is assumed to be imperfect.

In recent times, according to maintenance regulations [39], the wheel mainly adopts daily inspection and periodic inspection. During daily inspection, also known as patrol inspection, the tread surface damage is checked when the train returns to the depot. Periodic inspection cleans and measures the main parameters of the wheel. The wheel is reprofiled if the flange thickness is less than the specified limit or if the train runs to a specified mileage. This maintenance strategy ensures the normal operation of the wheelset. However, the service life of the wheel can be reduced by frequent reprofiling.

Based on the periodic inspection mode, reprofiling is implemented according to the status of the wheel. The maintenance strategy of periodic inspection and preventive maintenance is as follows.

(1) The wheelset is inspected daily to check its surface. If there is damage exceeding the limit on the tread, the wheel is reprofiled to eliminate surface damage and to recover the thickness of the flange to Sd_H, where Sd_H is the wheel flange thickness threshold after reprofiling.
(2) The wheelset is periodically cleaned, measured and maintained. If the wheel flange thickness is greater than Sd_L, normal maintenance is performed to remove defects on the flange and tread, such as burrs and sharp edges, where Sd_L is the wheel flange thickness threshold to trigger preventive reprofiling. If the thickness of the flange is less than Sd_L, preventive reprofiling is carried out to restore the thickness of the flange to Sd_H.
(3) If it is found that the wheel diameter is less than the threshold during periodic inspection, the wheel is replaced.

These two maintenance strategies are compared in Table 2.

Table 2. Comparison of maintenance strategies.

The Current Maintenance Strategy	The Proposed Joint Maintenance Strategy
Reprofiling the wheel periodically	Reprofiling depending on the wheel condition
Reprofiling when the thickness of the flange is less than the threshold	Reprofiling when the thickness of the flange is less than Sd_L
Reprofiling recovers the wheel to the standard profile	Reprofiling recovers the flange thickness to Sd_H
Both of them need daily inspection, and reprofiling is carried out in time when the tread surface is a failure. The wheel is replaced when the diameter is below the usage limit.	

In the joint maintenance strategy, the states of the wheel are working mode, daily inspection, periodic inspection, corrective maintenance, preventive maintenance and replacement. The events experienced by the wheel from the beginning to the end are shown in Figure 4, with details as follows:

(1) Working event o: The wheel is in normal working condition.
(2) Daily inspection event x: The vehicle returns to the depot, and the wheel surfaces are inspected for damages.
(3) Periodic inspection event s: The wheel is periodically cleaned, maintenance is performed and the main parameters of the wheel are measured.
(4) Corrective maintenance event c: When it is found that the wheel tread damage or wear exceeds the limit during daily inspection, reprofiling is carried out to eliminate the surface damage.
(5) Preventive maintenance event r: If the thickness of the wheel flange is below Sd_L, reprofiling is implemented to restore the thickness of the flange to Sd_H.
(6) Replacement event g: When the wheel diameter reaches the use limit, the wheelset is replaced.

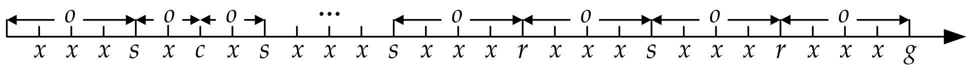

Figure 4. Events in the life of the wheel adopting the joint maintenance strategy.

When the joint maintenance strategy is implemented, the degradation process of the wheel is discontinuous. Wear on the wheel flange and diameter is cumulative during vehicle operation. Reprofiling eliminates the tread surface damage and increases the wheel flange thickness but reduces the wheel diameter. Figures 5 and 6 show the wheel flange thickness and diameter deterioration processes, respectively.

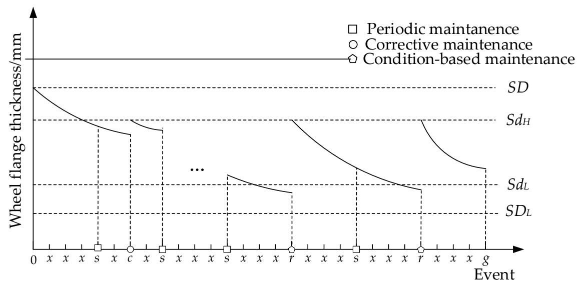

Figure 5. The wheel flange deterioration process.

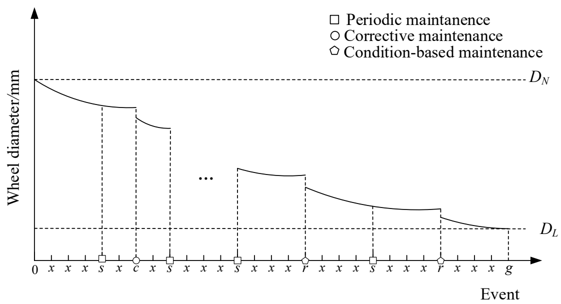

Figure 6. The wheel diameter deterioration process.

As shown in Figure 5, SD is the initial thickness of the wheel, and SD_L represents the floor of the thickness of the wheel flange. Once it is lower than SD_L, the wheelset cannot be put into service and must be reprofiled. Sd_L and Sd_H are the reprofiling strategy threshold. In Figure 6, D_N represents the diameter value of a new wheelset. The wheelset is replaced as soon as the diameter is smaller than D_L. As can be seen from the two figures, daily inspection and periodic inspection cannot eliminate wheel deterioration. If the damages on the tread surface do not exceed the limit during daily inspection, it is kept as it is. Otherwise, corrective maintenance is performed. During periodic inspection, if the flange thickness does not reach the lower limit Sd_L, only cleaning and maintenance are performed. The reduction in the wheel diameter is the wear caused during running. If the thickness of the flange is at or below Sd_L, preventive reprofiling is carried out to restore the flange thickness to Sd_H, and the wheel diameter decreases correspondingly. The effect of preventive maintenance is imperfect. With the reprofiling number increasing, the maintenance effect decreases, and the failure probability increases gradually.

3. Optimization Model of the Joint Maintenance Strategy

In this section, the failure mechanism of the wheel flange and wheelset diameter are explained by the cumulative damage model. It is assumed that the wheel flange and diameter change are caused by external action, and the amount of change accumulates gradually. Failure occurs when the threshold is exceeded. The tread failure can be described by a failure probability function. In order to establish a model of performance degradation of the joint maintenance strategy, it is necessary to describe the rule of external action and the deterioration rule of the wheel flange and wheel diameter.

3.1. Deterioration Model

Considering the wear of the wheel caused by stochastic shock, the non-homogeneous Poisson process describes the general stochastic process well and is often used to express the rule of external action [26]. The number of shocks $N(t)$ occurring within the time interval $(t_{k-1},\ t]$ satisfies

$$P(N(t)=n)=\frac{\left[at^b-at_{k-1}{}^b\right]^n}{n!}\exp\left\{-\left[at^b-at_{k-1}{}^b\right]\right\},\ n=0,1,2,\ldots \tag{1}$$

where $a>0$ represents the number of external shocks per unit of time, and $a=a^f$ and $a=a^d$ represent the wheel flange and diameter. $b>0$ represents the relationship between the number of shocks and the working time. Similarly, $b=b^f$ and $b=b^d$ indicate the wheel flange and diameter, respectively.

The damage of each shock obeys the same and independent normal distribution, i.e., the amount of deterioration caused by the i-th shock is $Y_i\ \sim\ (\mu,\sigma^2)$, $\mu=\mu^f$ and $\mu=\mu^d$ indicate the mean value of the amount of deterioration of each shock on wheel flange and diameter and $\sigma^2=(\sigma^f)^2$ and $\sigma^2=(\sigma^d)^2$ indicate the variance of the performance degradation of each shock of the wheel flange and diameter, respectively. Generally, the number of shocks to the wheelset and the amount of deterioration affected by the shock is related to the operating environment and the wheel manufacturing level, so they can be considered independent of each other.

Therefore, the deterioration $X_{t_{k-1},t}$ in time interval $(t_{k-1},\ t]$ is the accumulation of Y_i caused by $N_{t_{k-1},t}$ shocks, shown as

$$X_{t_{k-1},t}=\sum_{i=1}^{N_{t_{k-1},t}}Y_i \tag{2}$$

The stochastic process $X_{t_{k-1},t}$ is a compound Poisson process. Using the total probability formula, the distribution function of the deterioration $X_{t_{k-1},t}$ is expressed as

$$P(X_{t_{k-1},t}<x)=P(\sum_{i=1}^{N_{t_{k-1},t}}Y_i<x)=\sum_{n=1}^{\infty}P(N_{t_{k-1},t}=n)P(\sum_{i=1}^{n}Y<x) \tag{3}$$

In order to facilitate the calculation, $X_{t_{k-1},t}$ is considered a linear combination of the infinite number of independent normal distributions. Considering the central limit theorem, it can approximately be expressed as a normal distribution, i.e.,

$$X_{t_{k-1},t}\ \sim N(\mu_{t_{k-1},t},\sigma^2_{t_{k-1},t}) \tag{4}$$

where $\mu_{t_{k-1},t}$ and $\sigma^2_{t_{k-1},t}$ are the time-varying parameters. Based on the characteristic of the compound Poisson process, $\mu_{t_{k-1},t}=a\mu\left(t^b-t^b_{k-1}\right)$, ${\sigma^2}_{t_{k-1},t}=a(\mu^2+\sigma^2)\left(t^b-t^b_{k-1}\right)$.

For the wheel flange and wheel diameter, the amount of deterioration within the $i-$th maintenance cycle is $X^f_{(i-1)\Delta t,i\Delta t}$ and $X^d_{(i-1)\Delta t,i\Delta t}$, respectively. If the last reprofiling of the wheel is in the k-th $(1\le k<i)$ maintenance cycle, the daily inspection and the periodic inspection cannot eliminate the deterioration. The wear of the wheel flange at time $i\Delta t$ is the accumulation within $(k\Delta t,\ i\Delta t]$, i.e., $\sum_{j=k}^{i-1}X^f_{j\Delta t,(j+1)\Delta t}$. For the wheel diameter, the wear is accumulated continuously, and the amount of deterioration at time $i\Delta t$ is $\sum_{j=1}^{i}X^d_{(j-1)\Delta t,j\Delta t}$.

The tread receives damage such as scratches due to sliding or other reasons during wheel running, and the wheel surface is inspected after the vehicle returns back to the depot. Assuming that the surface failure of the wheelset follows the Weibull distribution, the probability density function is $f(t)$, as follows:

$$f(t) = \frac{m}{\eta}\left(\frac{t}{\eta}\right)^{m-1}\exp\left[-\left(\frac{t}{\eta}\right)^{m}\right], t \geq 0 \tag{5}$$

where m is the shape parameter, and η is the scale parameter.

3.2. Joint Maintenance Probability Model

In this subsection, a classical age-reduction factor is introduced to describe the effect of imperfect maintenance. The age-reduction factor was first proposed by Kijima et al. [41], which assumes that maintenance can shorten the actual working time. The age-reduction factor of the j-th imperfect maintenance is $\alpha_j \in [0, 1]$. If $\alpha_j = 0$, it means that the maintenance restores the actual working time to zero. If $\alpha_j = 1$, it means that the maintenance cannot shorten the actual working time. The time of the j-th imperfect maintenance is denoted as $i_j\Delta t$ (j = 1, 2, 3, . . .), and the actual working time T_j is expressed as

$$\begin{cases} T_1 = \alpha_1 i_1 \Delta t \\ T_2 = \alpha_2[T_1 + (i_2 - i_1)\Delta t] = \alpha_2 i_2 \Delta t + \alpha_2(\alpha_1 - 1)i_1\Delta t \\ T_3 = \alpha_3[T_2 + (i_3 - i_2)\Delta t] = \alpha_3 i_3 \Delta t + \alpha_3(\alpha_2 - 1)i_2\Delta t + \alpha_3\alpha_2(\alpha_1 - 1)i_1\Delta t \\ \vdots \end{cases} \tag{6}$$

3.2.1. Maintenance Probability Model

In the proposed maintenance strategy, there are two main reasons for reprofiling the wheel. One is that the wheel tread's surface damage exceeds the limit and is found at daily inspection. The other is that the flange thickness is below Sd_L at periodic inspection. In the following, the probabilities of these two cases are investigated in detail.

Let Sd^m and Sd_b donate the flange thickness after the m-th reprofiling and the flange thickness measured before reprofiling. Generally, $Sd^m = Sd_H$, and the value of Sd^m may be different if Sd_b is greater than Sd_H at corrective maintenance.

(1) The probability of preventive maintenance subjected to the last preventive maintenance

Event $A(i^{m-1}\Delta t,\ i^m\Delta t)$ indicates that the amount of flange thickness degradation detected at the periodic inspection time $i^m\Delta t$ is greater than or equal to Lp, and the m-th preventive maintenance must be performed, where Lp is the maximum allowable flange thickness degradation amount for preventive maintenance, i.e., $Lp = Sd^{m-1} - Sd_L$. In particular, $Lp = SD - Sd_L$ at the first maintenance. After the ($m-1$)-th preventive maintenance, the actual age of the wheel is returned to T_{m-1}. The deterioration of the wheel flange thickness in the interval $(i^{m-1}\Delta t,\ (i^m - 1)\Delta t]$ is all less than Lp, and there is no failure on the tread surface.

With respect to the relationship between the two maintenance times $i^{m-1}\Delta t$ and $(i^m - 1)\Delta t$, the discussion is divided into two cases.

① If $i^m = i^{m-1}+1$, it means that the two maintenance cycles are adjacent. In this case, event $A(i^{m-1}\Delta t,\ i^m\Delta t)$ satisfies

$$\begin{aligned} A(i^{m-1}\Delta t, i^m\Delta t) &= \{Lp \leq X(i^m\Delta t), Q(i^{m-1}\Delta t, i^m\Delta t), i^m\Delta t = (i^{m-1}+1)\Delta t \\ &= \{Lp \leq X_{T_{m-1}, T_{m-1}+\Delta t}, Q(T_{m-1}, T_{m-1}+\Delta t) \end{aligned}$$

where $X(i\Delta t)$ represents the wear amount of the flange thickness detected at time $i\Delta t$, and $Q(t_1,\ t_2)$ represents that the tread surface damage does not exceed the limit during the time interval $(t_1,\ t_2]$.

② If $i^m > i^{m-1}+1$, it means that the two preventive maintenance periods are not adjacent to each other, and the deterioration of the wheel flange is less than *Lp* in the time interval $(i^{m-1}\Delta t,\ (i^m - 1)\Delta t]$. Event $A(i^{m-1}\Delta t,\ i^m\Delta t)$ satisfies

$$\begin{aligned} A(i^{m-1}\Delta t, i^m\Delta t) &= \begin{cases} Lp \le X(i^m\Delta t), X((i^{m-1}+1)\Delta t) < Lp, \ldots, X((i^m-1)\Delta t) < Lp, \\ Q(i^{m-1}\Delta t, i^m\Delta t), i^{m-1}+1 < i^m \end{cases} \\ &= \begin{cases} Lp \le X_{T_{m-1},T_{m-1}+(i^m-i^{m-1})\Delta t}, X_{T_{m-1},T_{m-1}+\Delta t} < Lp, \ldots, X_{T_{m-1},T_{m-1}+(i^m-i^{m-1}-1)\Delta t} < Lp, \\ Q(T_{m-1}, T_{m-1}+(i^m\Delta t - i^{m-1}\Delta t)) \end{cases} \end{aligned}$$

To summarize, by adapting the deterioration rule and the age-reduction factor, the probability $P(A(i^{m-1}\Delta t,\ i^m\Delta t))$ of the event $A(i^{m-1}\Delta t,\ i^m\Delta t)$ is obtained as

$$P(A(i^{m-1}\Delta t, i^m\Delta t)) = \begin{cases} \left[1-\Phi\left(\frac{LP-\mu^f_{T_{m-1},\ T_{m-1}+\Delta t}}{\sigma^f_{T_{m-1},T_{m-1}+\Delta t}}\right)\right]\cdot\left[1-\int_{T_{m-1}}^{T_{m-1}+\Delta t} f(t)dt\right], i^{m-1} = i^{m-1}+1 \\ \left[1-\Phi\left(\frac{Lp-\mu^f_{T_{m-1},T_{m-1}+(i^m-i^{m-1})\Delta t}}{\sigma^f_{T_{m-1},T_{m-1}+(i^m-i^{m-1})\Delta t}}\right)\right] \\ \prod\limits_{p=1}^{i^m-i^{m-1}} \Phi\left(\frac{Lp-\mu^f_{T_{m-1},T_{m-1}+(i^{m-1}+p)\Delta t}}{\sigma^f_{T_{m-1},T_{m-1}+(i^{m-1}+p)\Delta t}}\right)\cdot\left[1-\int_{T_{m-1}}^{T_{m-1}+(i^m-i^{m-1})\Delta t} f(t)dt\right], i^m_j > i^{m-1}+1 \end{cases} \tag{7}$$

(2) The probability of corrective maintenance subjected to the last preventive maintenance

Event $B(i^{m-1}\Delta t,\ t^m)$ represents that the last preventive maintenance is performed at time $i^{m-1}\Delta t$, and time $t^m \in ((k-1)\Delta t, k\Delta t)$. The damage on the tread surface is beyond the limit, and reprofiling is adopted.

Similarly, based on the relationship between $i^{m-1}\Delta t$ and $k\Delta t$, two cases are considered in the following.

① If $i^{m-1} = k-1$, it denotes that, in the next maintenance cycle after preventive maintenance, corrective maintenance is required, and event $B(i^{m-1}\Delta t,\ t^m)$ meets

$$\begin{aligned} B(i^{m-1}\Delta t, t^m) &= \{\overline{Q}((k-1)\Delta t, t^m), i^{m-1} = k-1 \\ &= \{\overline{Q}(T_{m-1}, T_{m-1}+t^m - i^{m-1}\Delta t) \end{aligned}$$

where $\overline{Q}(t_1, t_2)$ represents that, from time t_1 to t_2, tread surface damage is found to be over the limit.

② If $i^{m-1} < k-1$, it means that the cycle in which the tread surface failure occurs is not adjacent to the i^{m-1}-th cycle. Event $B(i^{m-1}\Delta t,\ t^m)$ satisfies

$$\begin{aligned} B(i^{m-1}\Delta t, t^m) &= \begin{cases} X((i^{m-1}+1)\Delta t) < Lp, \ldots, X((k-1)\Delta t) < Lp, i^m\Delta t < (k-1)\Delta t < t^m \\ \overline{Q}(i^{m-1}\Delta t, t^m) \end{cases} \\ &= \left\{X_{T_{m-1},T_{m-1}+\Delta t} < Lp, \ldots, X_{T_{m-1},T_{m-1}+(k-1-i^{m-1})\Delta t} < Lp, \overline{Q}(T_{m-1}, T_{m-1}+t^m - i^m\Delta t)\right. \end{aligned}$$

To summarize, the probability $P(B(i^{m-1}\Delta t,\ t^m))$ of the event $B(i^{m-1}\Delta t,\ t^m)$ is obtained as

$$P(B(i^{m-1}\Delta t, t^m)) = \begin{cases} \int_{T_{m-1}}^{T_{m-1}+(t^m-i^{m-1}\Delta t)} f(t)dt, i^{m-1} = k-1 \\ \prod\limits_{p=1}^{k-1-i^{m-1}} \Phi\left(\frac{Lp-\mu^f_{T_{m-1},T_{m-1}+p\Delta t}}{\sigma^f_{T_{m-1},T_{m-1}+p\Delta t}}\right)\int_{T_{m-1}}^{T_{m-1}+(t^m-i^{m-1}\Delta t)} f(t)dt, i^{m-1} < k-1 \end{cases} \tag{8}$$

(3) The probability of preventive maintenance subjected to the last corrective maintenance

Event $A(t^{m-1},\ i^m\Delta t)$ represents the $(m-1)$-th corrective maintenance at time t^{m-1}, where $t^{m-1} \in ((k-1)\Delta t,\ k\Delta t]$, and the actual age is returned to T_{m-1}. In the time interval $[k\Delta t,\ (i^m - 1)\Delta t]$, the flange thickness deterioration is less than *Lp*. Preventive maintenance is required when the amount of flange thickness deterioration is greater than or equal to *Lp* at time $i^m\Delta t$.

Considering the relationship between $k\Delta t$ and $i^m\Delta t$, it can be divided into the following two cases.

① If $k = i^m$, it means that corrective maintenance is performed in the same cycle as preventive maintenance, and the event $A(t^{m-1}, i^m\Delta t)$ is denoted as

$$\begin{aligned} A(t^{m-1}, i^m\Delta t) &= \{Lp \le X(i^m\Delta t), Q(t^{m-1}, i^m\Delta t), t^{m-1} < k\Delta t = i^m\Delta t \\ &= \{Lp \le X_{T_{m-1}, T_{m-1}+(i^m\Delta t - t^{m-1})}, Q(T_{m-1}, T_{m-1} + (i^m\Delta t - t^{m-1})) \end{aligned}$$

② If $k < i^m$, it indicates that corrective maintenance is in a different cycle from preventive maintenance. In this case, event $A(t^{m-1}, i^m\Delta t)$ satisfies

$$\begin{aligned} A(t^{m-1}, i^m\Delta t) = & \{Lp \le X(i^m\Delta t), X(k\Delta t) < Lp, \ldots, X((i^m-1)\Delta t) < Lp, Q(t^{m-1}, i^m\Delta t), t^{m-1} < k\Delta t < i^m\Delta t \\ & = \{Lp \le X_{T_{m-1}, T_{m-1}+(i^m\Delta t - t^{m-1})}, X_{T_{m-1}, T_{m-1}+(k\Delta t - t^{m-1})} < Lp, \ldots, X_{T_{m-1}, T_{m-1}+((i^m-1)\Delta t - t^{m-1})} < Lp, Q(T_{m-1}, T_{m-1} + i^m\Delta t - t^{m-1}) \end{aligned}$$

Therefore, the probability $P(A(t^{m-1}, i^m\Delta t))$ of event $A(t^{m-1}, i^m\Delta t)$ is expressed as

$$P\big(A\big(t^{m-1}, i^m\Delta t\big)\big) = \begin{cases} \left[1 - \Phi\left(\dfrac{Lp - \mu^f_{T_{m-1},\, T_{m-1}+(i^m\Delta - t^{m-1})}}{\sigma^f_{T_{m-1}, T_{m-1}+(i^m\Delta - t^{m-1})}}\right)\right] \cdot \left[1 - \int_{T_{m-1}}^{T_{m-1}+(i^m\Delta - t^{m-1})} f(t)dt\right], k = i^m \\ \left[1 - \Phi\left(\dfrac{Lp - \mu^f_{T_{m-1}, T_{m-1}+(i^m\Delta - t^{m-1})}}{\sigma^f_{T_{m-1}, T_{m-1}+(i^m\Delta - t^{m-1})}}\right)\right] \\ \prod\limits_{p=1}^{i^m - k} \Phi\left(\dfrac{Lp - \mu^f_{T_{m-1}, T_{m-1}+[(k-1+p)\Delta t - t^{m-1}]}}{\sigma^f_{T_{m-1}, T_{m-1}+[(k-1+p)\Delta t - t^{m-1}]}}\right) \cdot \left[1 - \int_{T_{m-1}}^{T_{m-1}+[(k-1+p)\Delta t - t^{m-1}]} f(t)dt\right], t^{m-1} < k\Delta t < i^m\Delta t \end{cases} \tag{9}$$

(4) The probability of corrective maintenance subjected to the last corrective maintenance

Event $B(t^{m-1}\Delta t, t^m)$ performs the $(m-1)$-th maintenance at time t^{m-1}, where $t^{m-1} \in ((k_1 - 1)\cdot t, k_1\Delta t]$, and the actual service age is returned to T_{m-1}. At time $t^m \in ((k_2 - 1)\cdot t, k_2\Delta t]$, the damage on the wheel tread is over the limit, and the m-th reprofiling is performed.

The relationship between k_1 and k_2 has two cases, which are discussed in the following.

① If $k_1 = k_2$, the two corrective maintenances are in the same cycle.

$$\begin{aligned} B(t^{m-1}, t^m) &= \{\overline{Q}(t^{m-1}, t^m), k_1 = k_2 \\ &= \{\overline{Q}(T_{m-1}, T_{m-1} + t^m - t^{m-1}) \end{aligned}$$

② If $k_2 \ge k_1 + 1$, the corrective maintenance cycles are different.

$$\begin{aligned} B(t^{m-1}, t^m) &= \{X(k_1\Delta t) < Lp, \ldots, X((k_2-1)\Delta t) < Lp, \overline{Q}(t^{m-1}, t^m), t^{m-1} < k_1\Delta t \le (k_2-1)\Delta t < t^m \\ &= \begin{cases} X_{T_{m-1}, T_{m-1}+(k_1\Delta t - t^{m-1})} < Lp, \ldots, X_{T_{m-1}, T_{m-1}+[(k_2-1)\Delta t - t^{m-1}]} < Lp, \\ \overline{Q}(T_{m-1}, T_{m-1} + t^m - t^{m-1}) \end{cases} \end{aligned}$$

In the above discussion, the probability of event $B(t^{m-1}\Delta t, t^m)$ is

$$P(B(t^{m-1}, t^m)) = \begin{cases} \int_{T_{m-1}}^{T_{m-1}+(t^m - t^{m-1})} f(t)dt, k_1 = k_2 \\ \prod\limits_{p=1}^{k_2 - k_1} \Phi\left(\dfrac{Lp - \mu^f_{T_{m-1}, T_{m-1}+[(k_1-1+p)\Delta t - t^{m-1}]}}{\sigma^f_{T_{m-1}, T_{m-1}+[(k_1-1+p)\Delta t - t^{m-1}]}}\right) \cdot \int_{T_{m-1}}^{T_{m-1}+(t^m - t^{m-1})} f(t)dt, t^{m-1} < k_1\Delta t \le (k_2-1)\Delta t < t^m \end{cases} \tag{10}$$

When the train operation with the wheel flange thickness is below the usage limit, it brings damage and hidden loss to the vehicle. Therefore, the probability that the wheel flange thickness exceeds the limit during a maintenance cycle needs to be calculated.

(5) The probability of the wheel flange thickness exceeding the limit under the condition of the last preventive maintenance

Event $C(i^{m-1}\Delta t, i^m\Delta t)$ means that the $(m-1)$-th preventive maintenance is performed at time $i^{m-1}\Delta t$, and the actual age is returned to T_{m-1}. In the i^m-th cycle, the wheel flange thickness deterioration exceeds Ls, where Ls is the maximum wear value of the wheel flange thickness. Similarly, $Ls = Sd^{m-1} - Sd_L$. In particular, $Ls = SD - Sd_L$ at the first maintenance.

The following two cases are discussed, which are determined by the two times, $i^{m-1}\Delta t$ and $i^m\Delta t$.

① If $i^m = i^{m-1}+1$, it represents that the wheel flange thickness exceeds the limit in the adjacent maintenance cycle after the last reprofiling, and event $C(i^{m-1}\Delta t,\ i^m\Delta t)$ satisfies

$$\begin{aligned} C(i^{m-1}\Delta t, i^m\Delta t) &= \{Ls \le X(i^m\Delta t), Q(i^{m-1}, i^m), i^m\Delta t = (i^{m-1}+1)\Delta t \\ &= \{Ls \le X_{T_{m-1},T_{m-1}+\Delta t}, Q(T_{m-1}, T_{m-1}+\Delta t) \end{aligned}$$

② If $i^m > i^{m-1}+1$, it means that the maintenance cycle of the previous reprofiling is not adjacent to the i^m-th maintenance cycle.

$$\begin{aligned} C(i^{m-1}\Delta t, i^m\Delta t) &= \begin{cases} Ls \le X(i^m\Delta t), X((i^{m-1}+1)\Delta t) < Lp, \ldots, X((i^m-1)\Delta t) < Lp, \\ Q(i^{m-1}\Delta t, i^m\Delta t), i^{m-1}+1 < i^m \end{cases} \\ &= \begin{cases} Ls \le X_{T_{m-1},T_{m-1}+(i^m-i^{m-1})\Delta t}, X_{T_{m-1},T_{m-1}+\Delta t} < Lp, \ldots, X_{T_{m-1},T_{m-1}+(i^m-i^{m-1}-1)\Delta t} < Lp, \\ Q(T_{m-1}, T_{m-1}+(i^m-i^{m-1})\Delta t) \end{cases} \end{aligned}$$

Combining the above two situations, the probability of $P(C(i^{m-1}\Delta t,\ i^m\Delta t))$ satisfies

$$P\Big(C\Big(i^{m-1}\Delta t, i^m\Delta t\Big)\Big) = \begin{cases} \left[1-\Phi\left(\dfrac{Ls-\mu^f_{T_{m-1},\ T_{m-1}+\Delta t}}{\sigma^f_{T_{m-1},T_{m-1}+\Delta t}}\right)\right]\cdot\left[1-\int_{T_{m-1}}^{T_{m-1}+\Delta t} f(t)dt\right], i^{m-1}+1 \\ \left[1-\Phi\left(\dfrac{Ls-\mu^f_{T_{m-1},T_{m-1}+(i^m-i^{m-1})\Delta t}}{\sigma^f_{T_{m-1},T_{m-1}+(i^m-i^{m-1})\Delta t}}\right)\right] \\ \prod\limits_{p=1}^{i^m-i^{m-1}} \Phi\left(\dfrac{Lp-\mu^f_{T_{m-1},T_{m-1}+(i^{m-1}+p)\Delta t}}{\sigma^f_{T_{m-1},T_{m-1}+(i^{m-1}+p)\Delta t}}\right)\cdot\left[1-\int_{T_{m-1}}^{T_{m-1}+(i^m-i^{m-1})\Delta t} f(t)dt\right], i^m > i^{m-1}+1 \end{cases} \quad (11)$$

(6) The probability of the wheel flange thickness exceeding the limit under the condition of the last corrective maintenance

Event $C(t^{m-1},\ i^m\Delta t)$ represents the $(m-1)$-th maintenance at time t^{m-1}, and $t^{m-1} \in ((k-1)\Delta t,\ k\Delta t]$. In the time interval $[k\Delta t, (i^m-1)\Delta t]$, the wheel flange thickness deterioration is less than Lp, reaching or exceeding Ls in the i^m-th maintenance cycle.

According to the relationship between $k\Delta t$ and $i^m\Delta t$, the following two cases are discussed.

① If $k = i^m$, corrective maintenance is in the same cycle as wheel flange thickness and is over the limit, and event $C(t^{m-1}, i^m\Delta t)$ meets

$$\begin{aligned} C(t^{m-1}, i^m\Delta t) &= \{Ls \le X(i^m\Delta t), Q(t^{m-1}, i^m\Delta t), t^{m-1} < k\Delta t = i^m\Delta t \\ &= \left\{Ls \le X_{T_{m-1},T_{m-1}+(i^m\Delta t-t^{m-1})}, Q(T_{m-1}, T_{m-1}+(i^m\Delta t - t^{m-1}))\right. \end{aligned}$$

② If $k < i^m$, it denotes that the cycle in which the corrective maintenance is performed is not the same as the wheel flange thickness exceeding the limit. Event $C(t^{m-1},\ i^m\Delta t)$ satisfies

$$\begin{aligned} C(t^{m-1}, i^m\Delta t) &= \{Ls \le X(i^m\Delta t), X(k\Delta t) < Lp, \ldots, X((i^m-1)\Delta t) < Lp, Q(t^{m-1}, i^m\Delta t), t^{m-1} < k\Delta t < i^m\Delta t \\ &= \left\{Ls \le X_{T_{m-1},T_{m-1}+(i^m\Delta t-t^{m-1})}, X_{T_{m-1},T_{m-1}+(k\Delta t-t^{m-1})} < Lp, \ldots, X_{T_{m-1},T_{m-1}+((i^m-1)\Delta t-t^{m-1})} < Lp, Q(T_{m-1}, T_{m-1}+i^m\Delta t - t^{m-1})\right. \end{aligned}$$

To summarize, the occurrence probability $P(C(t^{m-1},\ i^m\Delta t))$ is obtained as

$$P\big(C\big(t^{m-1}, i^m\Delta t\big)\big) = \begin{cases} \left[1-\Phi\left(\dfrac{Ls-\mu^f_{T_{m-1},\ T_{m-1}+(i^m\Delta t-t^{m-1})}}{\sigma^f_{T_{m-1},T_{m-1}+(i^m\Delta t-t^{m-1})}}\right)\right]\cdot\left[1-\int_{T_{m-1}}^{T_{m-1}+(i^m\Delta t-t^{m-1})} f(t)dt\right], k\Delta t = i^m\Delta t \\ \left[1-\Phi\left(\dfrac{Ls-\mu^f_{T_{m-1},T_{m-1}+(i^m\Delta t-t^{m-1})}}{\sigma^f_{T_{m-1},T_{m-1}+(i^m\Delta t-t^{m-1})}}\right)\right] \\ \prod\limits_{p=1}^{i^m-k}\Phi\left(\dfrac{Lp-\mu^f_{T_{m-1},T_{m-1}+[(k-1+p)\Delta t-t^{m-1}]}}{\sigma^f_{T_{m-1},T_{m-1}+[(k-1+p)\Delta t-t^{m-1}]}}\right)\cdot\left(1-\int_{T_{m-1}}^{T_{m-1}+(i^m\Delta t-t^{m-1})} f(t)dt\right), t^{m-1} < k\Delta t < i^m\Delta t \end{cases} \quad (12)$$

In addition, the probability that the diameter of the wheel exceeds the limit also needs to be calculated.

(7) The probability of the wheel diameter exceeding the limit

When carrying out correction maintenance, if $Sd_b > Sd_H$, the wheel diameter reduction by reprofiling is about 1.5~2 mm to eliminate the tread surface damage. Otherwise, the reduction in the wheel diameter is $(Sd_H - Sd_b) \times B$, where B is the proportional coefficient, representing the cost of the wheel diameter to restore 1 mm of flange thickness. Therefore, the reduction in the diameter at the m-th reprofiling is

$$Cut(m) = \begin{cases} 1.5 \sim 2, Sd_b \geq Sd_H \\ (Sd_H - Sd_b) \times B, Sd_b < Sd_H \end{cases}$$

Event $L(t^{m-1}, i\Delta t)$ means that the $(m-1)$-th reprofiling occurs at time t^{m-1}, where $t \in ((k-1)\Delta t, k\Delta t]$, and the actual age is returned to T_{m-1}. The amount of wheel diameter deterioration exceeds the limit Lsw^l at time $i\Delta t$. The wheel diameter limit threshold Lsw^l is updated after the wheel is reprofiled at t^{m-1}. The initial limit is Lsw, and $Lsw^l = Lsw - \sum\limits_{j=1}^{m-1}\mathrm{Cut}(j)$.

The relationship between the two maintenance cycles is discussed in the following, which has three cases.

① If $i = k$, the reprofiling and periodic inspection occur in the same cycle, and then

$$\begin{aligned} L(t^{m-1}, i\Delta t) &= \left\{ Xw(t, k\Delta t) \geq Lsw^l, k = i \right. \\ &= \left\{ X^d_{t^{m-1}, i\Delta t - t^{m-1}} \geq Lsw^l \right. \end{aligned}$$

where $Xw(t_1, t_2)$ represents the wear amount of the wheel diameter in the time interval (t_1, t_2).

② If $i = k+1$, the cycle in which the reprofiling is carried out is adjacent to the periodic inspection cycle.

$$\begin{aligned} L(t^{m-1}, i\Delta t) &= \left\{ Xw(t^{m-1}, i\Delta t) \geq Lsw^l, Xw(t^{m-1}, k\Delta t) < Lsw^l, i = k+1 \right. \\ &= \left\{ X^d_{T_{m-1}, T_{m-1}+(i\Delta t - t^{m-1})} \geq Lsw^l, X^d_{T_{m-1}, T_{m-1}+(k\Delta t - t^{m-1})} < Lsw^l \right. \end{aligned}$$

③ If $i > k+1$, the period of reprofiling is not adjacent to the periodic inspection cycle.

$$\begin{aligned} L(t^{m-1}, i\Delta t) &= \left\{ Xw(t^{m-1}, i\Delta t) \geq Lsw^l, Xw(t^{m-1}, k\Delta t) < Lsw^l, \ldots, Xw(t^{m-1}, (i-1)\Delta t) < Lsw^l, i > k+1 \right. \\ &= \left\{ X^d_{T_{m-1}, T_{m-1}+(i\Delta t - t^{m-1})} \geq Lsw^l, X^d_{T_{m-1}, T_{m-1}+(k\Delta t - t^{m-1})} < Lsw^l, \ldots, X^d_{T_{m-1}, T_{m-1}+((i-1)\Delta t - t^{m-1})} < Lsw^l \right. \end{aligned}$$

According to the theory in the last section, the probability $P(L(t^{m-1}, i\Delta t))$ is obtained as

$$P\left(L\left(t^{m-1}, i\Delta t\right)\right) = \begin{cases} 1-\Phi\left(\frac{Lsw^{l}-\mu^{d}_{T_{m-1},\ T_{m-1}+(i\Delta t-t^{m-1})}}{\sigma^{d}_{T_{m-1},T_{m-1}+(i\Delta t-t^{m-1})}}\right),\ k=i \\ \left[1-\Phi\left(\frac{Lsw^{l}-\mu^{d}_{T_{m-1},T_{m-1}+(i\Delta t-t^{m-1})}}{\sigma^{d}_{T_{m-1},T_{m-1}+(i\Delta t-t^{m-1})}}\right)\right]\cdot\Phi\left(\frac{Lsw^{l}-\mu^{d}_{T_{m-1},T_{m-1}+(k\Delta t-t^{m-1})}}{\sigma^{d}_{T_{m-1},T_{m-1}+(k\Delta t-t^{m-1})}}\right), k=i-1 \\ \left[1,-,\Phi,\left(\frac{Lsw^{l}-\mu^{d}_{T_{m-1},T_{m-1}+(i\Delta t-t^{m-1})}}{\sigma^{d}_{T_{m-1},T_{m-1}+(i\Delta t-t^{m-1})}}\right)\right]\cdot\prod_{p=1}^{i-k}\Phi\left(\frac{Lsw^{l}-\mu^{d}_{T_{m-1},T_{m-1}+[(k-1+p)\Delta t-t^{m-1}]}}{\sigma^{f}_{T_{m-1},T_{m-1}+[(k-1+p)\Delta t-t^{m-1}]}}\right), t^{m-1}<k\Delta t<(i-1)\Delta t \end{cases} \tag{13}$$

Event $A(i^{m}\,\Delta t)$ is denoted as the m-th preventive maintenance at time $i^{m}\Delta t$, and event $B(t^{m})$ is denoted as the m-th corrective maintenance at time t^{m}. The recursion relations between the m-th maintenance and the $(m-1)$-th maintenance is

$$P(A(i^{m}\Delta t)) = \sum_{i^{m-1}=m-1}^{i^{m}-1} P(A(i^{m-1}\Delta t))\cdot P(A(i^{m-1}\Delta t, i^{m}\Delta t)) + \sum_{k=1}^{i^{m}}\int_{(k-1)\Delta t}^{k\Delta t} dP(B(t^{m-1}))\cdot P(A(t^{m-1}, i^{m}\Delta t)) \tag{14}$$

$$P(B(t^{m})) = \sum_{i^{m-1}=m-1}^{i^{m}-1} P(A(i^{m-1}\Delta t))\cdot P(B(i^{m-1}\Delta t, t^{m})) + \sum_{k=1}^{t^{m}}\int_{(k-1)\Delta t}^{k\Delta t} dP(B(t^{m-1}))\cdot P(B(t^{m-1}, t^{m})) \tag{15}$$

In particular, $P(A(i^{1}\Delta t)) = P(A(0, i^{1}\Delta t))$, $P(B(t^{1})) = P(B(0, t^{1}))$.

The $P(L(i^{m}\,\Delta t))$ is the probability that the wheel diameter deterioration exceeds the limit in the i-th cycle after the m-th reprofiling.

$$P(L(i^{m}\Delta t)) = \sum_{i^{m}=m}^{i-1} P(A(i^{m}\Delta t))\cdot P(L(i^{m}\Delta t, i\Delta t)) + \sum_{k=1}^{i}\int_{(k-1)\Delta t}^{k\Delta t} dP(B(t^{m}))\cdot P(L(t^{m}, i\Delta t)) \tag{16}$$

3.2.2. The Risk of Wheel Flange Failure

The wheel flange thickness exceeding the limit during vehicle operation may cause serious accidents. The risk of failure is introduced to control the risk. Failure risk refers to the conditional probability of failure in the future under the current normal working conditions. It reflects the possibility of failure in real time, which is helpful for users to adjust the usage plan according to the actual usage needs. The failure risk of the wheel flange at any time s refers to the probability that, under the condition of normal operation at time s, the flange thickness is over the limit after continuous operation time t. It is defined by

$$Risk(s) = \frac{\Pr(\text{The wheel flange thickness over the limit during the time period } (s, s+t])}{\Pr(\text{Normal operation to time } s)}$$

where t is the time that the user wants the wheelset to continue working, which is often related to the task time. RAMS includes reliability, availability, maintainability and safety. For the urban rail transit system, the availability of the system is guaranteed by ensuring the reliability of the system and by improving the maintainability of the system. Combined with the key prevention of failures that may endanger safety, the safety of the system is guaranteed. The failure risk controls the failure probability of the flange thickness exceeding the limit to ensure the safety of wheelset operation.

(1) Event $D(t^{m},\ i\Delta t)$ is denoted as the m-th imperfect maintenance of the wheel at time t^{m}, where $t^{m}\in((k-1)\,\Delta t,\ k\Delta t])$, and it works to time $i\Delta t$, normally. Considering the relationship between maintenance time $k\Delta t$ and time $i\Delta t$, the following cases are discussed.

① If $i=k$, it means that the reprofiling is in the same cycle with periodic inspection, and the wheel flange thickness deterioration does not exceed the limit Ls.

$$\begin{aligned} D(t^{m}, i\Delta t) &= \{X(i\Delta t) < Ls, Q(t^{m}, i\Delta t), k=i \\ &= \{X_{T_{m-1},T_{m-1}+i\Delta t-t^{m}} < Ls, Q(T_{m-1}, T_{m-1}+i\Delta t-t^{m}) \end{aligned}$$

② If $i > k$, the cycle in which the reprofiling is performed is not in the same cycle as $i\Delta t$.

$$\begin{aligned} D(t^m, i\Delta t) &= \{X(k\Delta t) < Ls, X((k+1)\Delta t) < Ls, \ldots, X(i\Delta t) < Ls, Q(t^m, i\Delta t), i > k \\ &= \left\{X_{T_{m-1},T_{m-1}+k\Delta t-t^m} < Ls, X_{T_{m-1},T_{m-1}+(k+1)\Delta t-t^m} < Ls, \ldots, X_{T_{m-1},T_{m-1}+i\Delta t-t^m} < Ls, Q(T_{m-1}, T_{m-1}+i\Delta t-t^m)\right) \end{aligned}$$

The probability $P(D(t^m, i\Delta t))$ of event $D(t^m, i\Delta t)$ is

$$P\left(D\left(t^{m-1} i\Delta t\right)\right) = \begin{cases} \Phi\left(\frac{Ls-\mu^f_{T_{m-1}, T_{m-1}+i\Delta t-t^m}}{\sigma^f_{T_{m-1},T_{m-1}+i\Delta t-t^m}}\right)\cdot\left[1-\int_{T_{m-1}}^{T_{m-1}+i\Delta t-t^m} f(t)dt\right], i = k \\ \prod_{p=0}^{i-k} \Phi\left(\frac{Ls-\mu^f_{T_{m-1},T_{m-1}+(k+p)\Delta t-t^m}}{\sigma^f_{T_{m-1},T_{m-1}+(k+p)\Delta t-t^m}}\right)\cdot\left[1-\int_{T_{m-1}}^{T_{m-1}+i\Delta t-t^m} f(t)dt\right], i > k \end{cases} \quad (17)$$

(2) Event $D(i\Delta t, i^1\Delta t, i^2\Delta t, \ldots, i^m\Delta t)$ indicates that the wheel has been reprofiled in the i^1-th, i^2-th, … , i^m-th maintenance cycle, and the wheel flange thickness exceeds SD_L until time $i\Delta t$. Using equation (17), $P(D(i\Delta t, i^1\Delta t, i^2\Delta t, \ldots, i^m\Delta t))$ can be obtained as

$$P(D(i\Delta t, i^1\Delta t, i^2\Delta t, \ldots, i^m\Delta t) = P(D(i^0\Delta t, i^1\Delta t))P(D(t^1, i^2\Delta t))P(D(t^2, i^3\Delta t))\ldots P(D(t^m, i\Delta t)) \quad (18)$$

(3) Event $E((i+1)\Delta t, i^1\Delta t, i^2\Delta t, \ldots, i^m\Delta t)$ is denoted as the event that occurs after the wheel is reprofiled in the i^1-th, i^2-th, … , i^m-th maintenance cycles, and in the interval $(i\Delta t,(i+1)\Delta t]$, the flange thickness occurs over the limit.

$$P(E((i+1)\Delta t, i^1\Delta t, i^2\Delta t, \ldots, i^m\Delta t)) = P(F(0, t^1))P(F(t^1, t^2))\ldots P(F(t^{m-1}, t^m))P(C(t^m, (i+1)\Delta t)) \quad (19)$$

where $F\left(t^{k-1}, t^k\right)$ indicates that the k-th reprofiling is performed at time t^k, and the $(k-1)$-th reprofiling is implemented at time t^{k-1}, where $t^k \in ((i^k-1)\Delta t, i^k\Delta t]$, $k = 1, 2, \ldots$. The value of $F\left(t^{k-1}, t^k\right)$ is the probability value that the wheel needs to be reprofiled, as discussed in the previous sections (1)–(4), and the specific situation corresponds to its value.

$Risk^m(i\Delta t)$ is used to represent the failure risk at time $i\Delta t$ after the m-th maintenance.

$$Risk^m(i\Delta t) = \frac{P(E((i+1)\Delta t, i^1\Delta t, i^2\Delta t, \ldots, i^m\Delta t))}{P(D(i\Delta t, i^1\Delta t, i^2\Delta t, \ldots, i^m\Delta t)} \quad (20)$$

For any time $i\Delta t$, the maximum value of the failure risk of the wheel at time $i\Delta t$ under different maintenance conditions is $Risk(i\Delta t)$, which satisfies

$$Risk(i\Delta t) = \max\{Risk^m(i\Delta t), m = 0, 1, 2, \ldots, N\} \quad (21)$$

3.3. Optimization Model

When the wheel diameter reaches the limit, the wheelset is replaced. The lifespan of a wheelset is from the start of use to replacement, and reprofiling is carried out N times in total. Hence, the life expectancy $E(t)$ for the wheelset is

$$E(t) = \sum_{i=1}^{\infty} i\Delta t P(L(i^N\Delta t)) \quad (22)$$

The cost of each daily inspection is C_i CNY/10^4 km, and the cost of periodic inspection is C_m CNY/time. Preventive maintenance and corrective maintenance are C_p CNY/time and C_c CNY/time, respectively. The loss cost for the wheel that is operated with the wheel flange thickness that exceeds the limit is C_l CNY/10^4 km, and the cost of replacing the wheelset is C_r CNY. The expected cost is

$$E(C) = C_i E(t) + C_m E(t)/\Delta t + \sum_{k=1}^{\infty}\sum_{j=1}^{N}\left(P(A(k^j\Delta t))\cdot C_p + \int_{(k-1)\Delta t}^{k\Delta t} dP(B(t^j))\cdot C_c + P(C(k^j\Delta t))\cdot C_l\right) + C_r \quad (23)$$

The minimum cost rate within the life cycle of the wheelset is taken as the objective, and the optimization model is established.

$$\begin{cases} \min C(t|\Delta t, Sd_L, Sd_H) \\ C(t|\Delta t) = \frac{E(C)}{E(t)} \\ Risk(i\Delta t) < \alpha \end{cases} \quad (24)$$

where α is the maximum allowable value of risk failure.

4. Numerical Analysis

In this section, a numerical case is constructed to illustrate the effectiveness of the proposed combined maintenance strategy model used for wheelset maintenance. The data were collected from a railway operating company.

4.1. Parameter Setting

Taking a wheel as an example, according to the maintenance regulations, the wheel flange thickness limit is in the range of 26 mm $\leq Sd \leq$ 32 mm, and the wheel diameter should be in the range of 790 mm $\leq D \leq$ 860 mm. In addition, damages such as scratches and spalling of the tread may cause the wheel to be emergency reprofiled and polished. The inspection and maintenance standards for the wheel tread are stated in Table 3 [39].

Table 3. Tread repair standards.

Item	Check Items	The Reprofiling Standard
Tread	Check the wheel tread for scratches and spalling	1. Scratch limit: More than one scratch length $\leq$ 75 mm The length of more than two scratches is between 50~75 mm The length of more than four scratches is between 25~50 mm Damage depth > 0.8 mm 2. Spalling limit: One spalling length $\leq$ 30 mm Two spalling damage lengths $\leq$ 20 mm Spalling depth $\leq$ 1 mm

The actual historical measurement data of the vehicle wheel were studied, and the wheel diameter and wheel flange thickness were measured over a period of time, as shown in Figure 7.

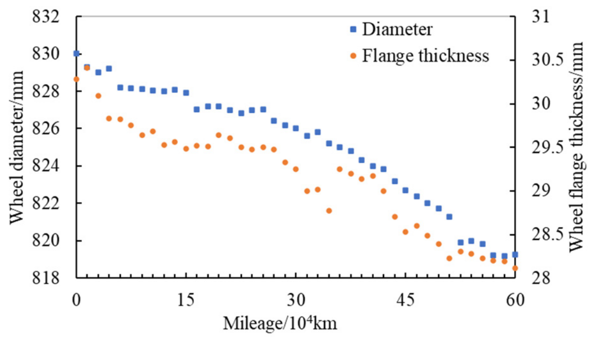

Figure 7. Measurement data of wheel diameter and flange thickness.

The parameters of the shock deterioration model are estimated by maximum likelihood estimation. The degradation value at time t_i (i = 1, 2, 3, ...) is y_i. By combining parameters, the performance degradation rule can be expressed as

$$P(y_i \leq Y) = \Phi(\frac{Y - g(t^b - T_j^b)}{\sqrt{h(t^b - T_j^b)}}) \quad (25)$$

where g and h are the mean and variance changes in a unit of time, respectively, i.e., $g = a\mu$ and $h = a\left(\mu^2 + \sigma^2\right)$. b indicates the relationship between the number of shocks and the working time. Y is the degradation threshold, and T is the time of the last maintenance. Therefore, if parameters g, h and b are determined, the likelihood function can be constructed and simplified it to obtain

$$\begin{array}{l} L(g, h, w \Big| (t_1, y_1), (t_2, y_2), \ldots, (t_j, y_j)) = \frac{1}{2^j \pi^j} [\frac{1}{h^j {t_1}^b ({t_2}^b - {t_1}^b) \ldots ({t_j}^b - {t_{j-1}}^b)}]^{1/2} \\ \exp\left\{ -\frac{1}{2h} [(\frac{{y_1}^2}{{t_1}^b} + \frac{{y_2}^2}{({t_2}^b - {t_1}^b)} + \ldots + \frac{{y_j}^2}{({t_j}^b - {t_{j-1}}^b)}) + g^2 {t_j}^b - 2g \sum_{i=1}^{g} y_i] \right\} \end{array} \quad (26)$$

The logarithm at both ends of the above equation, as well as the derivative of parameters g, h and w, are taken. The derivative formula is set to zero, and the following Equation (27) is obtained. The equations are solved to obtain the estimated value $\hat{g}$, $\hat{h}$ and $\hat{b}$.

$$\left\{ \begin{array}{l} g t_j^b - \sum_{i=1}^{g} y_i = 0 \\ jh - 2g \sum_{i=1}^{g} y_i - g^2 t_j^b + \left(\frac{{y_1}^2}{{t_1}^b} + \frac{{y_2}^2}{{t_2}^b - {t_1}^b} + \ldots + \frac{{y_j}^2}{{t_j}^b - {t_{j-1}}^b} \right) = 0 \\ g^2 t_j^b \ln t_j + h \left(\ln t_1 + \frac{{t_2}^b \ln t_2 - {t_1}^b \ln t_1}{{t_2}^b - {t_1}^b} + \ldots + \frac{{t_j}^b \ln t_j - {t_{j-1}}^b \ln t_{j-1}}{{t_2}^b - {t_1}^b} \right) \\ - \left[\frac{\ln t_1}{{t_1}^b} {y_1}^2 + \frac{{t_2}^b \ln t_2 - {t_1}^b \ln t_1}{\left({t_2}^b - {t_1}^b\right)^2} {y_2}^2 + \ldots + \frac{{t_j}^b \ln t_j - {t_{j-1}}^b \ln t_{j-1}}{\left({t_j}^b - {t_{j-1}}^b\right)^2} {y_1}^2 \right] = 0 \end{array} \right. \quad (27)$$

For the estimation of the relevant parameters, the time-varying parameters of the wheel flange deteriorated by the cumulative shock model are

$$\mu_{t_{k-1,t}}^f = 0.0075 \times \left(t^{1.3525} - t_{k-1}^{1.3525}\right), \left(\sigma_{t_{k-1,t}}^f\right)^2 = 1.6221 \times 10^{-4} \left(t^{1.3525} - t_{k-1}^{1.3525}\right)$$

The parameters of the diameter are

$$\mu_{t_{k-1,t}}^d = 0.0884 \times (t^{1.0935} - t_{k-1}^{1.0935}\Big), \ (\sigma_{t_{k-1,t}}^d\Big)^2 = 0.0025 \times 10^{-4} (t^{1.0935} - t_{k-1}^{1.0935}\Big)$$

The parameters of the Weibull distribution of the tread surface failure function are obtained by the maximum likelihood estimation of the censored data. The probability density function of wheel tread failure is

$$f(t) = 0.0073 \times (\frac{t}{213.4})^{0.55} \times \exp[-(\frac{t}{213.4})^{1.55}\Big].$$

According to the maintenance standard of the wheelset, the limit threshold Ls of the flange thickness is 6, and the limit threshold Lsw of the diameter is 70. The diameter of the wheel and the cost to restore the flange thickness fluctuate due to manual operation or other reasons. It is assumed that the proportional coefficient B follows a uniform distribution on the interval [4,6]. Generally, in the case of a few imperfect maintenance times, it can be approximated that the age-reduction factor is linearly related to the maintenance times. Assuming that the age-reduction factor is satisfied,

$$\alpha_j = \begin{cases} 0.3 + \frac{j-1}{20}, j \leq 15 \\ 1, \quad j \geq 16 \end{cases}$$

where *j* represents the number of imperfect maintenance times, and as the number increases, the effect on the wheelset gradually decreases. It is considered that it is almost impossible for the failure to occur when the probability of failure is less than 10^{-6}, so the maximum allowable value of the failure risk α is set to 10^{-6}. The flange thickness is ensured to be not out of the limit during running. The proposed control strategy for wheel flange thickness (Sd_L, Sd_H) is shown in Figure 8.

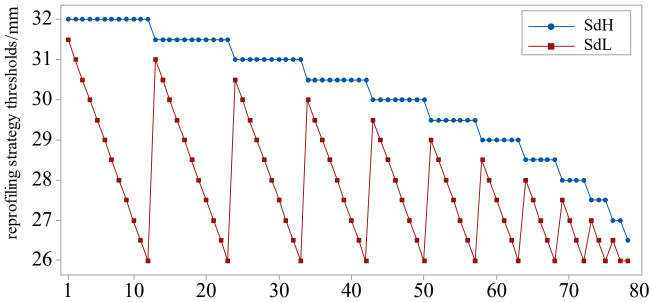

Figure 8. Threshold values of wheel flange reprofiling strategy.

According to some of the literature [42,43] and actual research, the values of different maintenance costs are as shown in Table 4. The cost of the depreciation of reprofiling equipment is not considered here.

Table 4. The value of the different maintenance costs.

Items	C_i	C_m	C_p	C_c	C_l	C_r
The Cost Value	20	50	400	1060	50,000	12,000

4.2. Numerical Results for Wheelset Maintenance

In order to ensure safe operation and prolong the service time of the wheel, with the above performance parameters, the maintenance cycle of the wheel and the reprofiling threshold of the flange thickness are optimized. With the constraint of failure risk, the change rule of the maintenance cost rate with the maintenance cycle and reprofiling strategy is obtained, as shown in Figure 9. Comparing Figure 9 with Figure 7, it can be seen that some reprofiling strategies are filtered out due to the failure risk constraints. The optimal maintenance interval is Δt = 6. The selected reprofiling strategy is (Sd_L, Sd_H) = (28.5, 31), the minimum maintenance cost rate is $C^*(t)$ = 78.12 and the number of reprofiles is 5, i.e., in practical operation, when the mileage of the train running reaches 60,000 km, periodic inspection is performed. When the thickness of the wheel flange is less than 28.5 mm, it is reprofiled and restored to 31 mm. The expected mileage with the proposed maintenance strategy is 284.99×10^4 km.

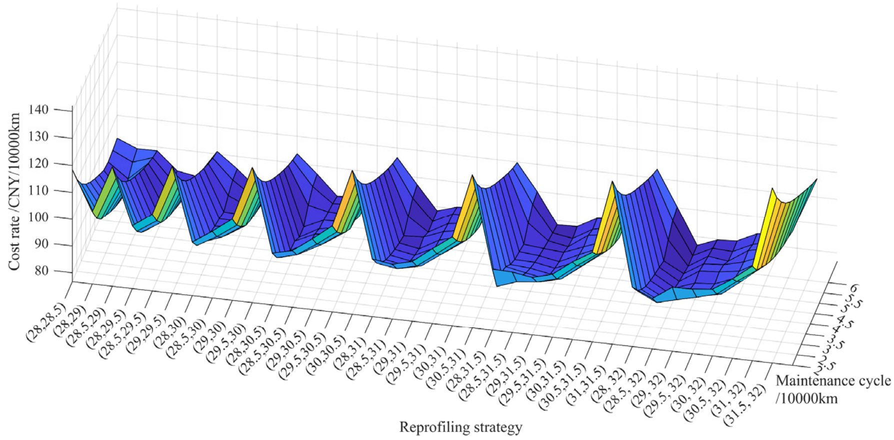

Figure 9. Variation curve of cost rate with maintenance cycles and reprofiling strategies.

By observing the trend of maintenance cost rates in Figure 8, it can be seen that, when Δt and the Sd_H are fixed, the cost rate declines first and then rises with the decrease in Sd_L, or it keeps declining due to some reprofiling control strategies being filtered out. Frequent reprofiling leads to higher maintenance costs when Sd_H and Sd_L are close. When the gap between Sd_H and Sd_L is large, the reduction in wheel diameter is also large for each reprofiling. This results in wheelset life reduction and a cost rate increase.

The variation in the expected life of the wheel is shown in Figure 10. It can be seen that the trend of the expected life of the wheelset is roughly opposite to that of the maintenance cost rate trend. There are small fluctuations in the middle due to reasons such as the random value of the proportional coefficient.

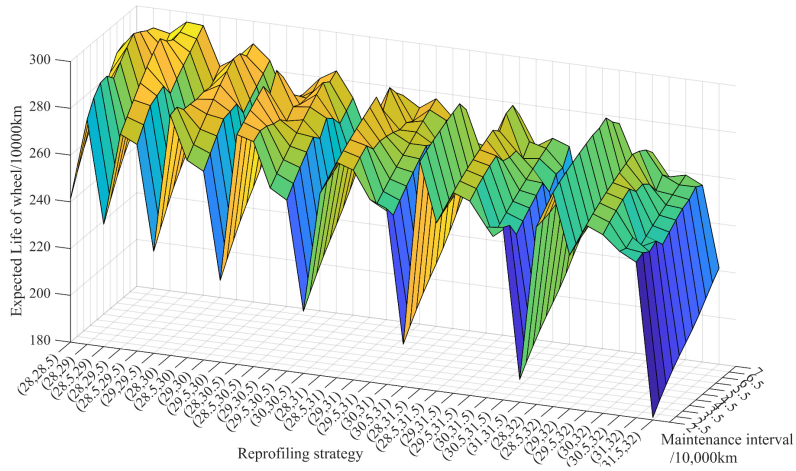

Figure 10. Variation curve of expected life with maintenance cycles and maintenance strategies.

4.3. Analysis of Optimization Results

To further illustrate the applicability of the proposed maintenance strategy optimization model, the results of different maintenance strategies and the numerical changes during the optimization process were further analyzed.

The target value of the proposed maintenance strategy was analyzed quantitatively. It can be seen from Table 5 that the choice of reprofiling strategy has an influence on the number of reprofiling. When the profiling strategy selects (31.5, 32), it needs to be reprofiled 14 times in total, and when choosing (30, 32), only 6 reprofiles are performed on the wheel. Frequent reprofiling causes a heavy work burden for maintenance personnel and affects maintenance planning.

Table 5. Results under different maintenance strategies.

Number	Maintenance Strategy Value			Reprofiling Times/Time	Expected Life/10^4 km	Cost Rate/(CNY/10^4 km)
	Δt	Sd_L	Sd_H			
1	5	31.5	32	14	204.61	116.02
2	5	31	32	10	250.23	94.33
3	5	30.5	32	7	251.40	89.24
4	5	30	32	6	263.64	85.12
5	5	29.5	32	5	271.59	84.03
6	5	29	32	4	255.92	83.45
7	5	28.5	32	4	281.80	78.77
8	5	28	32	4	259.85	83.13
9	5	30.5	31	14	224.90	108.49
10	5	30	31	10	267.77	90.31
11	5	29.5	31	7	267.77	85.81
12	5	29	31	6	271.54	83.71
13	5	28.5	31	5	285.71	79.63
14	5	28	31	4	269.37	80.96
15	6	30.5	31	14	234.65	103.65
16	6	30	31	10	263.44	89.68
17	6	29.5	31	7	272.69	83.18
18	6	29	31	6	270.92	82.20
19	6	28.5	31	5	284.99	78.12

The impact of maintenance intervals on cost rates was further analyzed, as shown in Figure 11, and when the reprofiling strategy (Sd_L, Sd_H) = (28.5, 31) is fixed, with the length of the maintenance interval increasing, the maintenance cost rate also decreases first and then rises. Especially when the maintenance interval gradually increases from Δt = 2 to Δt = 4.5, the influence of the maintenance interval on the cost rate is evident. When the maintenance interval is short, frequent inspections need more expense. As the maintenance interval increases to a certain point, the effect of prolonging the life decreases, and the probability of the wheel flange thickness exceeding the limit increases, resulting in a higher maintenance cost rate.

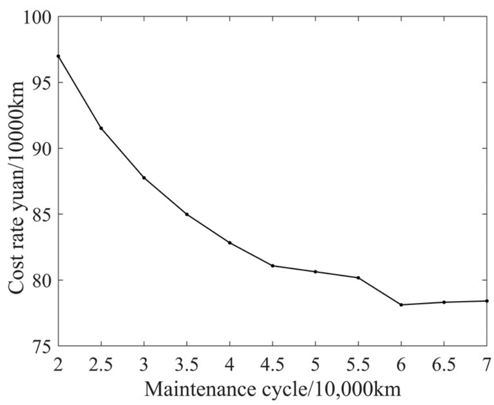

Figure 11. The curve of cost rate changing with the maintenance cycle.

As the mileage increases, preventive and corrective maintenance probability increase, as shown in Figures 12 and 13. With the increase in reprofiling times, the probability of preventive maintenance continues to rise, and the probability of corrective maintenance decreases. Preventive maintenance eliminates tread surface damage and restore the wheel status. Since the effect of preventive maintenance is imperfect, the interval between two preventive maintenances is shortened gradually, reducing the failure probability of the tread surface.

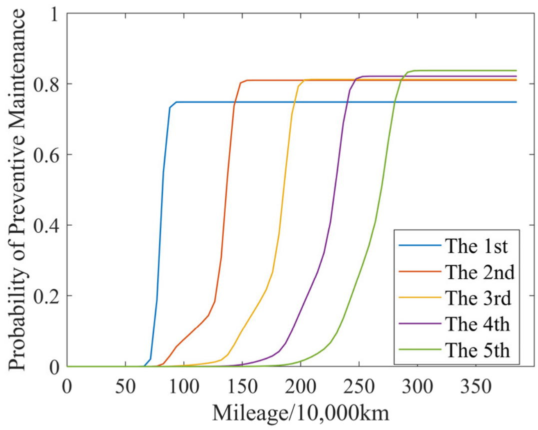

Figure 12. The curve of the preventive maintenance probability.

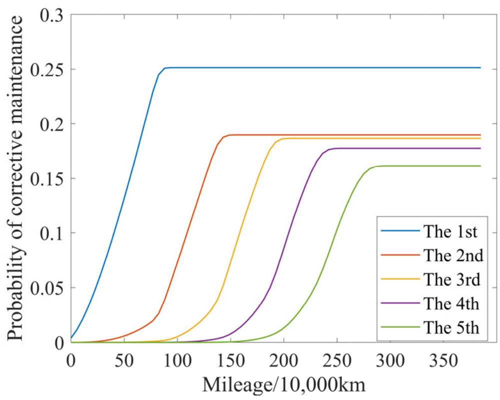

Figure 13. The curve of the corrective maintenance probability.

To analyze the impact of the risk of failure on the maintenance joint strategy, Table 6 gives the optimal results under different failure risk values. With the increase in the risk of failure α, the maintenance interval and life expectancy of the wheelset increase, and the maintenance cost rate is reduced, as a lesser probability of failure risk tends to cost more.

Table 6. Comparison of optimal strategies under different fault risks.

Number	Interval/Δt	Reprofiling Strategy/ (Sd_L, Sd_H)	The Risk of Failure/α	The Number of Reprofiling/Times	Expected Life/ 10,000 km	Cost Rate/ (CNY/10,000 km)
1	6	(28.5, 31)	10^{-6}	5	284.99	78.12
2	6.5	(28, 31)	10^{-4}	5	287.70	77.13
3	7	(28, 31)	10^{-2}	5	287.83	76.58

4.4. Cost Comparison

The proposed maintenance strategy was compared with two periodic inspection strategies. These two strategies are stated below.

Strategy 1: Daily inspection of the wheelset, cleaned and maintained every 3×10^4 km. The wheelset is reprofiled if the wheel flange thickness exceeds the limit. The wheelset is periodically reprofiled every 20×10^4 km. If the wheel diameter exceeds the limit, the wheelset is replaced.

Strategy 2: Daily inspection, periodic cleaning and maintenance every 3×10^4 km. When the flange thickness exceeds the limit and reprofiling is implemented, the wheelset is replaced if the wheel diameter exceeds the limit.

The obtained results were compared with the above two maintenance strategies, as shown in Table 7. Comparing the proposed joint maintenance strategy results with Strategy

1, life expectancy increased by 11.51%, and the cost rate decreased by 27.06%. Compared with Strategy 2, life expectancy increased by 11.98%, and the cost rate decreased by 12.0%. It can be seen that the proposed maintenance strategy prolongs life expectancy and reduces the cost rate effectively.

Table 7. Comparison of the cost rate and the expected life of different maintenance strategies.

Maintenance Strategies	Cost Rate/(CNY/10,000 km)	Expected Life/10,000 km	Number of Reprofiling/Times
Strategy 1	107.1	255.57	12.17
Strategy 2	88.77	254.5	2
The joint maintenance strategy	78.12	284.99	5

4.5. Discussion

The wheelset is a very important and seriously worn part of rail vehicles, so attention should be paid to the maintenance of the wheelset. For wheelset maintenance, the determination of the inspection interval and the selection of the reprofiling threshold have a great influence on the life of the wheelset, the maintenance cost and the number of reprofiles, which can be seen in Table 4. Through calculation, the strategy proposed in this paper can prolong the service life of wheelsets and reduce the cost rate. Some studies on wheelset maintenance strategies only focus on the wheelset reprofiling strategy. For example, in the literature [44], the proposed reprofiling strategy can improve the life of the wheelset by 58.82%. Since it does not consider the factors of periodic maintenance and the influence of tread damage that needs to be reprofiled in time, the proportion of life improvement is relatively high.

Cumulative shock models are often used in reliability analyses, life predictions and maintenance optimizations in recent years. It has been proven to be useful and efficient in maintenance optimization. In this paper, the wear values of the wheel diameter and wheel flange thickness obtained by the cumulative shock model were compared with the measurements, as shown in Figures 14 and 15. It can be seen that the analytical values of the wheel diameter and the wheel flange are relatively close to their actual values. The method based on the shock model can describe the deterioration of the wheel diameter and flange well.

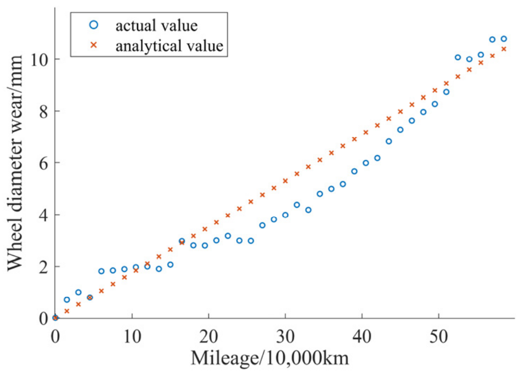

Figure 14. Numerical comparison chart of wheel diameter deterioration.

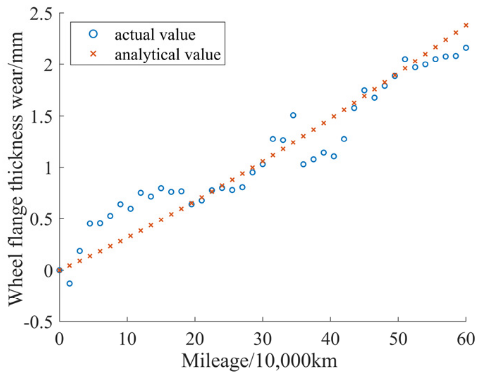

Figure 15. Numerical comparison chart of wheel flange thickness deterioration.

5. Conclusions

In this paper, a maintenance strategy that combines periodic and preventive maintenance is developed by analyzing the characteristics and correlation of wheel flange and wheel diameter wear. Starting from the failure mechanism of the wheel wear and based on the cumulative shock model, a deterioration model of wheel flanges and diameters are established using the compound Poisson process. The Weibull distribution is used to describe the failure probability of the tread surface. The wear of the wheel flange, wheel diameter and tread surface failures are considered comprehensively. After that, the maintenance strategy optimization model is established to minimize the cost rate, taking the wheel flange thickness over the limit as the risk failure constraint. Finally, the validity of the model is verified by a numerical analysis. The results show that the proposed strategy can prolong the life of the wheelset and can reduce the maintenance cost rate, and it can ensure that the wheel flange thickness is kept in a safe state. The wheel does not need to be reprofiled frequently, improving maintenance efficiency. It has both practical application value and also economic significance for the maintenance work of railway vehicles.

The developed maintenance strategy provides an alternative for the practical maintenance of railway bogie wheelsets. However, further expansion of this work can be conducted. In this paper, parameters such as the *Sh* and *Qr* values of the wheelset are not considered. In line with the actual situation of wheelset maintenance, future research can consider flange height and *Qr* value to develop maintenance strategy optimizations. This paper only studies the maintenance strategy of one wheelset, and the constraints on the wheel diameter and the difference between wheels on the same axle, bogie or rolling stock can also be further explored. Furthermore, it does not consider the internal degradation of the wheelset. Wheelset maintenance optimization for competitive failures can be further investigated.

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