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A SkSP-R Plan under the Assumption of Gompertz Distribution

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Abstract: In this study, we designed a time-truncated SkSP-R sampling plan when the lifetime of units follows a Gompertz distribution (GmzD). The GmzD is briefly discussed. All of the plan parameters were obtained using a two-point approach, which is based on limiting quality level (LQL) and acceptable quality level (AQL). Moreover, operating characteristic (OC) values were calculated for the determined value of the plan parameters by using the OC function of the SkSP-R. Applications of two real life situations in engineering were presented to illustrate the applicability of the offered sampling inspection plan. It was found that the new SkSP-R sampling inspection plan can be used efficiently in the field.

Keywords: sampling inspection plan; Gompertz distribution; operating characteristic; average sample number; acceptable quality level

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1. Introduction

The industry directly depends on customers and their experiences about the product quality. Thus, the quality of the product is the most important factor in making the product more demanding to the customers by manufacturing units. To provide consumers a positive impression of product quality, a manufacturer or wholesaler selects the highest-quality product from a lot and distributes it to customers.. For the selection of best quality product, one may use 100 percent inspection, but 100 percent inspection is not possible due to time, money, labor, etc., constraints. In addition to 100 percent inspection, we have a path between 100 percent inspection and no inspection that is known as acceptance sampling inspection plan (ASIP). Various types of ASIPs are presented in the literature; namely; attribute ASIP and variable ASIP, Single ASIP (SASIP), double ASIP (DASIP), multiple ASIP (MASIP), sequential ASIP (SeASIP), group ASIP (GASIP), and skip-lot ASIP (SkASIP) are included in attribute ASIP, while sampling plans are based on variables using perfect measurements of quality characteristics.

Some of the authors who have developed several ASIPs for different probability distributions include the following: Ref. [1] for gamma distribution, Ref. [2] for for normal and lognormal distributions, Ref. [3] for Birnbaum Saunders model, Ref. [4] for inverse Rayleigh distribution, Ref. [5] for generalized Rayleigh distribution, Ref. [6] for generalized Birnbaum Saunders model, Ref. [7] for generalized exponential distribution, Ref. [8] for generalized inverted exponential distribution, Ref. [9] for for SASIP based on generalized half-normal distribution and [10] for SASIP and DASIP to the transmuted Rayleigh distribution, Ref. [11] for chain sampling plan for variables inspection, Ref. [12] for selection of chain sampling plans ChSP-1 and ChSP-(0,1). Moreover, see [13–30] for other works in ASP.

We explored the literature of SQC and found that this is the first study developing SkSP-R under the GmzD. In addition, we placed a strong emphasis on the suggested plan's real implementation, and we employed two data sets to accomplish this goal. The structure of this paper is classified as follows: In Section 2, GmzD is presented with its main properties. The design of the offered SkSP-R plan for GmzD is placed in Section 3 along with illustration. The description of tables is provided in Section 4. Real data set examples for application purposes are presented in Section 5. Section 6 summarizes the main finding and concludes the paper.

2. Gompertz Distribution

The Gompertz distribution is introduced by Benjamin Gompertz in 1825 and showed the importance of GmzD by considering human mortality and actuarial tables. The GmzD is a an extended version of the exponential distribution and it has a relationship with double exponential, exponential, generalized logistic, Weibull, and extreme value (Gumbel) distributions. The probability density function (pdf) and cumulative distribution function cdf of the GmzD are provided below, respectively, as:

$$f(x) = \frac{\theta}{\alpha} e^{x/\alpha} e^{-\theta(e^{x/\alpha}-1)}; x > 0, \theta > 0, \alpha > 0, \tag{1}$$

and

$$F(x) = 1 - e^{-\theta(e^{x/\alpha}-1)}. \tag{2}$$

The mean of the GmzD is as follows.

$$\mu = \alpha e^{\theta} \Gamma(0, \theta). \tag{3}$$

The hazard rate and reliability functions of the GmzD are, respectively, given by the following.

$$H(x) = \frac{\theta e^{x/\alpha}}{\alpha}, \text{and } R(x) = e^{\theta - \theta e^{x/\alpha}}.$$

The plots of the probability density and cumulative distribution functions of the GmzD are provided in Figure 1 for some selected parameters. Moreover, the plots of the hazard rate and reliability functions of the GmzD are provided in Figure 2 for various parameters choices. Figure 2 revealed the flexibility of the GmzD in accommodating several shapes of the hazard rate and reliability functions.

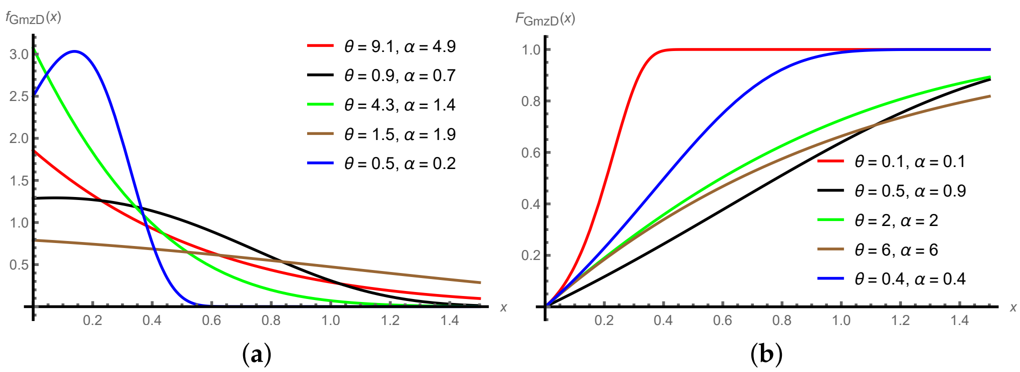

Figure 1. Plots of pdf (**a**) and cdf (**b**) of the GmzD for various values of parameters.

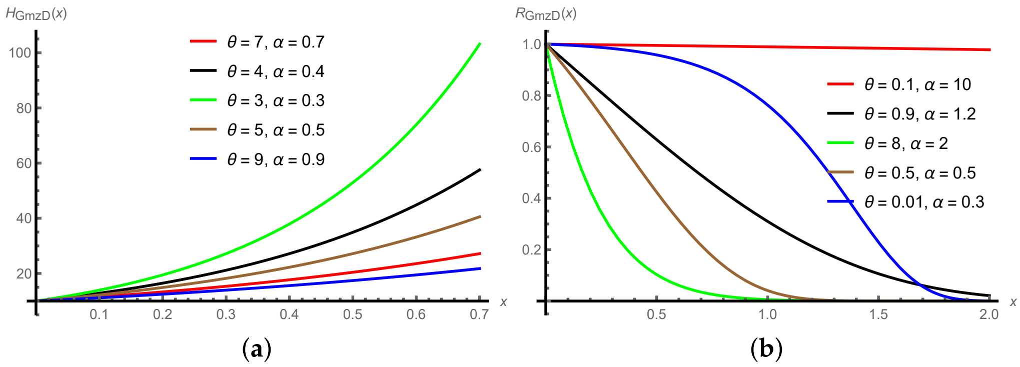

Figure 2. Plots of hazard rate (**a**) and reliability (**b**) functions of the GmzD for various values of parameters.

3. Design of the SkSP-R Plan for GmzD

This section defines and discusses the SkSP-R for the GmzD distribution considered in this study . SkSP-R is introduced by [31] and showed its advantages over some other popular ASIPs. The SkSP-R plan parameters are $(n, c, i, f, k$, and $m)$. The procedure of time truncated SkSP-R can be described as follows:

1. Begin with the normal inspection using the reference plan, and then place n items on test for prefixed time t_0. Notice and count the number of sample items that failed before the experiment duration, say, d. If $d \leq c$, then accept the lot and reject it if $d > c$.
2. Stop the normal inspection and utilize the skipping inspection (SI) if i successive units are accepted under normal inspection based on time truncated life tests..
3. Within SI, inspect only a fraction f of lots that is randomly selected. SI is continued until a sampled lot is rejected.
4. If a lot is not accepted after k consecutively sampled lots have been accepted, then the resampling procedure is employed for the immediate next lot as given below (Step 5).
5. Within the resampling technique, conduct the inspection based on the reference plan and continue SI if the lot is accepted. If the lot is not accepted, resampling is performed m times and the lot is rejected if it has not accepted on $(m-1)$st resubmission.
6. If a lot is not accepted based on resampling scheme, then directly revert to the normal inspection (Step 1).
7. Remove or correct all the nonconforming units found with conforming units in the rejected lots.

The average sample number (ASN) of the SkSP-R is as follows:

$$ASN = \frac{nfQP^{i+k} - nfP^k(1-P^i)(1-Q^m) + nf}{P^i(1+fQP^k) + f(1-P^i)[1-P^k(1-Q^m)]}, \tag{4}$$

and the OC function probability od acceptance of the proposed SkSP-R is as follows:

$$P_a = \frac{(1-f)P^i + fP^k(P^i - P)(1-Q^m) + fP}{P^i(1+fQP^k) + f(1-P^i)[1-P^k(1-Q^m)]}, \tag{5}$$

where $P = \sum_{j=0}^{c} \binom{n}{j} p^j (1-p)^{n-j}$, and $p = F(t_0)$ is the CDF of GmzD, which can be modified in terms of termination ratio and quality ratio as follows.

$$p = 1 - e^{-\theta[e^{(t_0/\mu_0)\times(e^{\theta}\Gamma(0,\theta)/\mu/\mu_0)}-1]}. \tag{6}$$

Now, we employed a two-point strategy to determine plan parameters; in this approach, the OC curve passes through both AQL and LQL. As a result, the optimization problem for determining plan parameters using the two-point technique AQL and LQL, as well as the optimization problem, is the following:

$$ASN = \frac{nfQP^{i+k} - nfP^k(1-P^i)(1-Q^m) + nf}{P^i(1+fQP^k) + f(1-P^i)[1-P^k(1-Q^m)]}, \quad (7)$$

$$\frac{fP_0 + (1-f)P_0^i + fP_0^k(P_0^i - P_0)(1-Q_0^m)}{f(1-P_0^i)[1-P_0^k(1-Q_0^m)] + P_0^i(1+fQ_0P_0^k)} \geq 1-\alpha_p, \quad (8)$$

$$\frac{fP_1 + (1-f)P_1^i + fP_1^k(P_1^i - P_1)(1-Q_1^m)}{f(1-P_1^i)[1-P_1^k(1-Q_1^m)] + P_1^i(1+fQ_1P_1^k)} \leq \beta, \quad (9)$$

where P_0 and P_1 are probabilities at AQL and LQL, with Q_0 and Q_1 being $1-P_0$ and $1-P_1$, respectively. Our aim is to minimize the ASN of proposed SkSP-R where ASN depends on the sample size n. Therefore, we minimize the sample size by using the above mentioned optimization problem (Equations (6)–(8)).

4. Description of Tables

To demonstrate how the proposed SkSP-R would be implemented, some tables are presented and discussed for various plan parameters . The necessary tables for values of $\theta = 2,3,4,5$, $m = 2$, $\beta = 0.25, 0.05, 0.10, 0.01$, $\alpha_p = 0.10$, termination ratio $t_0/\mu_0 = 0.5, 0.75$, $m = 2$, and quality ratio (μ/μ_0 is $2,3,4,5,6,7$, and 8) are computed. Plan parameters and probability of lot acceptance for $\theta = 2,3,4,5$ are placed in Tables 1–4, respectively, under the assumption of the proposed plan. In most circumstances, as the termination time grows, the sample size decreases, and this pattern holds true for any value of θ, β and μ/μ_0. For each value of θ and β, if there are decreases from 0.25 to 0.01 and increases in quality ratio μ/μ_0, then the sample sizes increases for each considered set up.

Similar trends are observed regarding sample size for other choices of $\theta = 3,4,5$ and $m = 2$. Moreover, we have other important aspect associated with the ASN, where it is found that addedthe ASN follows the same pattern as sample size for all considered setups. In case of any selection of θ, the probability of acceptance of the submitted lot under the assumptions of new plan is greater than 0.90 for all values of quality ratio μ/μ_0.

Table 1. Plan parameters of SkSP-R for GmzD with $\theta = 2$ and $m = 2$.

		$a = t_0/\mu_0 = 0.5$							$a = t_0/\mu_0 = 0.75$						
β	μ/μ_0	n	c	i	f	k	$P_{acg}(p)$	ASN	n	c	i	f	k	$P_{acg}(p)$	ASN
0.25	2	34	10	2	0.5	1	0.9535057	27.97392	24	10	2	0.5	1	0.9553009	20.06413
	3	31	9	2	0.5	1	0.9959384	25.66619	22	9	2	0.5	1	0.9960654	18.66366
	4	28	8	2	0.5	1	0.9990554	21.41822	19	8	2	0.5	1	0.9994029	15.21178
	5	24	7	2	0.5	1	0.9996647	19.1525	17	7	2	0.5	1	0.9996928	13.84022
	6	21	6	2	0.5	1	0.9997141	16.87444	15	6	2	0.5	1	0.9997168	12.43753
	7	18	5	2	0.5	1	0.9996190	14.58213	13	5	2	0.5	1	0.9995891	10.99879
	8	15	4	2	0.5	1	0.9992690	12.27287	11	4	2	0.5	1	0.9991400	9.517559
0.10	2	68	18	2	0.5	1	0.9625093	66.06291	47	18	2	0.5	1	0.9689919	45.63715
	3	64	17	2	0.5	1	0.9994385	61.85672	45	17	2	0.5	1	0.9994789	43.85736
	4	61	16	2	0.5	1	0.9999713	59.12137	42	16	2	0.5	1	0.9999815	40.61462
	5	58	15	2	0.5	1	0.9999963	56.36989	40	15	2	0.5	1	0.9999976	38.85132
	6	54	14	2	0.5	1	0.9999993	52.20480	38	14	2	0.5	1	0.9999993	37.06218
	7	51	13	2	0.5	1	0.9999997	49.46471	35	13	2	0.5	1	0.9999998	33.86162
	8	48	12	2	0.5	1	0.9999998	46.70592	33	12	2	0.5	1	0.9999998	32.0866
0.05	2	114	30	2	0.5	1	0.9895492	113.1479	79	30	2	0.5	1	0.9917155	78.44711
	3	110	29	2	0.5	1	0.9999873	109.0512	76	29	2	0.5	1	0.9999924	75.32948
	4	107	28	2	0.5	1	0.9999999	106.1515	74	28	2	0.5	1	1.0000000	73.42533
	5	104	27	2	0.5	1	1.0000000	103.2452	72	27	2	0.5	1	1.0000000	71.51112
	6	100	26	2	0.5	1	1.0000000	99.16030	69	26	2	0.5	1	1.0000000	68.40539
	7	97	25	2	0.5	1	1.0000000	96.25778	67	25	2	0.5	1	1.0000000	66.49731
	8	93	24	2	0.5	1	1.0000000	92.17513	65	24	2	0.5	1	1.0000000	64.57880
0.01	2	154	38	2	0.5	1	0.9857021	153.9443	106	38	2	0.5	1	0.9893034	105.9631
	3	151	37	2	0.5	1	0.9999931	150.9517	104	37	2	0.5	1	0.999996	103.9696
	4	147	36	2	0.5	1	1.0000000	146.9463	101	36	2	0.5	1	1.0000000	101.9751
	5	144	35	2	0.5	1	1.0000000	143.9537	99	35	2	0.5	1	1.0000000	98.96939
	6	140	34	2	0.5	1	1.0000000	139.9486	97	34	2	0.5	1	1.0000000	96.97509
	7	137	33	2	0.5	1	1.0000000	136.9561	94	33	2	0.5	1	1.0000000	93.96941
	8	133	32	2	0.5	1	1.0000000	132.9513	92	32	2	0.5	1	1.0000000	91.97531

Table 2. Plan parameters of SkSP-R for GmzD with $\theta = 3$ and $m = 2$.

		$a = t_0/\mu_0 = 0.5$							$a = t_0/\mu_0 = 0.75$						
β	μ/μ_0	n	c	i	f	k	$P_{acg}(p)$	ASN	n	c	i	f	k	$P_{acg}(p)$	ASN
0.25	2	32	10	2	0.5	1	0.9538064	25.85492	23	10	2	0.5	1	0.952789	18.9601
	3	29	9	2	0.5	1	0.9960467	23.39951	21	9	2	0.5	1	0.9957579	17.49022
	4	26	8	2	0.5	1	0.9991216	20.95096	19	8	2	0.5	1	0.9989842	16.0018
	5	23	7	2	0.5	1	0.9995944	18.50952	17	7	2	0.5	1	0.9994881	14.49266
	6	20	6	2	0.5	1	0.9996732	16.07547	15	6	2	0.5	1	0.9995471	12.96005
	7	17	5	2	0.5	1	0.9995915	13.64909	12	5	2	0.5	1	0.9996393	9.617745
	8	14	4	2	0.5	1	0.9992689	11.23064	10	4	2	0.5	1	0.9992878	8.138429
0.10	2	64	18	2	0.5	1	0.9623296	61.82625	45	18	2	0.5	1	0.966264	43.48126
	3	61	17	2	0.5	1	0.9992944	59.01565	43	17	2	0.5	1	0.9993888	41.68839
	4	58	16	2	0.5	1	0.9999636	56.20086	41	16	2	0.5	1	0.9999678	39.87927
	5	55	15	2	0.5	1	0.9999954	53.38140	39	15	2	0.5	1	0.9999958	38.05390
	6	52	14	2	0.5	1	0.9999988	50.55674	36	14	2	0.5	1	0.9999993	34.79924
	7	48	13	2	0.5	1	0.9999996	46.32888	34	13	2	0.5	1	0.9999997	32.99749
	8	45	12	2	0.5	1	0.9999997	43.52742	32	12	2	0.5	1	0.9999997	31.17690

Table 2. *Cont.*

		$a = t_0/\mu_0 = 0.5$							$a = t_0/\mu_0 = 0.75$						
β	μ/μ_0	n	c	i	f	k	$P_{acg}(p)$	ASN	n	c	i	f	k	$P_{acg}(p)$	ASN
0.05	2	108	30	2	0.5	1	0.9883012	107.1027	76	30	2	0.5	1	0.9897322	75.41146
	3	105	29	2	0.5	1	0.9999810	104.1692	74	29	2	0.5	1	0.9999846	73.48365
	4	102	28	2	0.5	1	0.9999999	101.2336	72	28	2	0.5	1	0.9999999	71.54970
	5	99	27	2	0.5	1	1.0000000	98.29585	69	27	2	0.5	1	1.0000000	68.42994
	6	95	26	2	0.5	1	1.0000000	94.17864	67	26	2	0.5	1	1.0000000	66.50477
	7	92	25	2	0.5	1	1.0000000	91.24830	65	25	2	0.5	1	1.0000000	64.57284
	8	89	24	2	0.5	1	1.0000000	88.31538	62	24	2	0.5	1	1.0000000	61.45723
0.01	2	147	38	2	0.5	1	0.9813676	146.9547	103	38	2	0.5	1	0.9833812	102.9737
	3	143	37	2	0.5	1	0.9999908	142.9469	100	37	2	0.5	1	0.9999934	99.96625
	4	140	36	2	0.5	1	1.0000000	139.9523	98	36	2	0.5	1	1.0000000	97.97152
	5	137	35	2	0.5	1	1.0000000	136.9574	95	35	2	0.5	1	1.0000000	94.96343
	6	133	34	2	0.5	1	1.0000000	132.9501	93	34	2	0.5	1	1.0000000	92.96930
	7	130	33	2	0.5	1	1.0000000	129.9556	91	33	2	0.5	1	1.0000000	90.97441
	8	127	32	2	0.5	1	1.0000000	126.9607	88	32	2	0.5	1	1.0000000	87.96706

Table 3. Plan parameters of SkSP-R for GmzD with $\theta = 4$ and $m = 2$.

		$a = t_0/\mu_0 = 0.5$							$a = t_0/\mu_0 = 0.75$						
β	μ/μ_0	n	c	i	f	k	$P_{acg}(p)$	ASN	n	c	i	f	k	$P_{acg}(p)$	ASN
0.25	2	31	10	2	0.5	1	0.9529443	24.90356	23	10	2	0.5	1	0.9405765	19.56785
	3	28	9	2	0.5	1	0.9960064	22.35642	21	9	2	0.5	1	0.9943247	18.00976
	4	26	8	2	0.5	1	0.9987942	21.7716	18	8	2	0.5	1	0.9991692	14.39786
	5	23	7	2	0.5	1	0.9994509	19.19575	16	7	2	0.5	1	0.9995955	12.89812
	6	20	6	2	0.5	1	0.9995685	16.63486	14	6	2	0.5	1	0.9996545	11.38677
	7	17	5	2	0.5	1	0.9994774	14.08994	12	5	2	0.5	1	0.9995407	9.861995
	8	14	4	2	0.5	1	0.9990974	11.56217	10	4	2	0.5	1	0.9991243	8.321259
0.10	2	63	18	2	0.5	1	0.9539819	61.27592	45	18	2	0.5	1	0.9529552	43.95910
	3	60	17	2	0.5	1	0.9990669	58.40179	42	17	2	0.5	1	0.999305	40.65008
	4	56	16	2	0.5	1	0.9999619	54.08897	40	16	2	0.5	1	0.9999629	38.82681
	5	53	15	2	0.5	1	0.9999953	51.23819	38	15	2	0.5	1	0.9999951	36.99133
	6	50	14	2	0.5	1	0.9999988	48.38748	36	14	2	0.5	1	0.9999987	35.1434
	7	47	13	2	0.5	1	0.9999995	45.53644	33	13	2	0.5	1	0.9999996	31.88039
	8	44	12	2	0.5	1	0.9999997	42.68460	31	12	2	0.5	1	0.9999997	30.05944
0.05	2	105	30	2	0.5	1	0.9869442	104.1437	75	30	2	0.5	1	0.9864446	74.53027
	3	102	29	2	0.5	1	0.9999773	101.1917	72	29	2	0.5	1	0.9999828	71.39263
	4	99	28	2	0.5	1	0.9999999	98.23924	70	28	2	0.5	1	0.9999999	69.46043
	5	96	27	2	0.5	1	1.0000000	95.28632	68	27	2	0.5	1	1.0000000	67.52334
	6	93	26	2	0.5	1	1.0000000	92.33283	66	26	2	0.5	1	1.0000000	65.58145
	7	90	25	2	0.5	1	1.0000000	89.37870	63	25	2	0.5	1	1.0000000	62.45506
	8	86	24	2	0.5	1	1.0000000	85.25299	61	24	2	0.5	1	1.0000000	60.52318
0.01	2	142	38	2	0.5	1	0.9810519	141.9459	100	38	2	0.5	1	0.9832736	99.96297
	3	139	37	2	0.5	1	0.9999883	138.9502	98	37	2	0.5	1	0.9999906	97.96809
	4	136	36	2	0.5	1	1.0000000	135.9543	96	36	2	0.5	1	1.0000000	95.97265
	5	133	35	2	0.5	1	1.0000000	132.9582	94	35	2	0.5	1	1.0000000	93.97670
	6	130	34	2	0.5	1	1.0000000	129.9619	92	34	2	0.5	1	1.0000000	91.98028
	7	126	33	2	0.5	1	1.0000000	125.9539	89	33	2	0.5	1	1.0000000	88.97385
	8	123	32	2	0.5	1	1.0000000	122.9582	87	32	2	0.5	1	1.0000000	86.97798

Table 4. Plan parameters of SkSP-R for GmzD with $\theta = 5$ and $m = 2$.

		$a = t_0/\mu_0 = 0.5$							$a = t_0/\mu_0 = 0.75$						
β	μ/μ_0	n	c	i	f	k	$P_{acg}(p)$	ASN	n	c	i	f	k	$P_{acg}(p)$	ASN
0.25	2	31	10	2	0.5	1	0.944385	25.62728	22	10	2	0.5	1	0.9519282	17.77898
	3	28	9	2	0.5	1	0.9951027	22.98371	20	9	2	0.5	1	0.99565	16.24248
	4	25	8	2	0.5	1	0.9989351	20.36137	18	8	2	0.5	1	0.9989846	14.69947
	5	22	7	2	0.5	1	0.9995298	17.76231	16	7	2	0.5	1	0.9995094	13.1491
	6	19	6	2	0.5	1	0.9996424	15.18902	14	6	2	0.5	1	0.9995872	11.59023
	7	17	5	2	0.5	1	0.9993861	14.37158	12	5	2	0.5	1	0.9994622	10.02132
	8	14	4	2	0.5	1	0.9989641	11.77523	10	4	2	0.5	1	0.9989975	8.440149
0.10	2	61	18	2	0.5	1	0.9581897	58.92042	44	18	2	0.5	1	0.9546326	42.79859
	3	58	17	2	0.5	1	0.9991801	56.03471	42	17	2	0.5	1	0.9990502	40.94366
	4	55	16	2	0.5	1	0.9999578	53.15181	40	16	2	0.5	1	0.9999473	39.0799
	5	52	15	2	0.5	1	0.9999948	50.27162	37	15	2	0.5	1	0.9999954	35.77937
	6	49	14	2	0.5	1	0.9999987	47.39399	35	14	2	0.5	1	0.9999988	33.94354
	7	46	13	2	0.5	1	0.9999995	44.51875	33	13	2	0.5	1	0.9999995	32.09703
	8	43	12	2	0.5	1	0.9999997	41.64564	31	12	2	0.5	1	0.9999996	30.23946
0.05	2	103	30	2	0.5	1	0.9861437	102.1556	74	30	2	0.5	1	0.9850251	73.54036
	3	100	29	2	0.5	1	0.9999749	99.19212	71	29	2	0.5	1	0.9999796	70.39578
	4	97	28	2	0.5	1	0.9999999	96.22899	69	28	2	0.5	1	0.9999999	68.45795
	5	94	27	2	0.5	1	1.0000000	93.26621	67	27	2	0.5	1	1.0000000	66.51622
	6	91	26	2	0.5	1	1.0000000	90.30373	65	26	2	0.5	1	1.0000000	64.57062
	7	88	25	2	0.5	1	1.0000000	87.34148	63	25	2	0.5	1	1.0000000	62.62121
	8	85	24	2	0.5	1	1.0000000	84.37941	60	24	2	0.5	1	1.0000000	59.49680
0.01	2	140	38	2	0.5	1	0.977879	139.9565	99	38	2	0.5	1	0.9800148	98.96906
	3	137	37	2	0.5	1	0.9999849	136.9594	97	37	2	0.5	1	0.9999871	96.97312
	4	133	36	2	0.5	1	1.0000000	132.9498	93	36	2	0.5	1	1.0000000	93.96386
	5	130	35	2	0.5	1	1.0000000	129.9533	92	35	2	0.5	1	1.0000000	91.9687
	6	127	34	2	0.5	1	1.0000000	126.9567	90	34	2	0.5	1	1.0000000	89.97305
	7	124	33	2	0.5	1	1.0000000	123.9600	88	33	2	0.5	1	1.0000000	87.97693
	8	121	32	2	0.5	1	1.0000000	120.9632	85	32	2	0.5	1	1.0000000	84.96881

5. Real Life Examples

In this part, two real data sets were chosen for the illustration purpose of the proposed SkSP-R plan. To begin, we examine whether data sets have been fitted to the GmzD or not. To accomplish this purpose, several criteria such as Akaike information criterion (AIC), Bayesian information criterion (BIC), and Kolmogorov–Smirnov statistic (KS) goodness-of-fit test value were used. Moreover, the p-value associated with the KS test were considered to support the presented results based on the real data sets. Descriptive statistics summary and model fitting results of both data sets are presented in Tables 5 and 6, respectively.

Table 5. Descriptive summary of data sets.

Data	Minimum	Q_1	Median	Mean	Q_3	Maximum	CS	CK
I	0.020	0.688	1.965	1.770	2.983	3.000	−0.2840467	1.453664
II	0.550	1.375	1.590	1.507	1.685	2.240	−0.8999263	3.923761

Table 6. Measures of goodness-of-fit statistics for both data sets.

Data	Estimates	L-L	AIC	BIC	KS Value	p-Value
I	$\alpha = 1.3509349, \theta = 0.2496109$	−41.34595	86.6919	89.4943	0.18892	0.2346
II	$\alpha = 0.2741801, \theta = 0.0024180$	−14.80810	33.61621	37.90247	0.12676	0.2635

Data I: The data include 30 observations of the times of failures and running times for samples of devices from an eld-tracking study of a larger system. Previously, these data were studied by [32]. The data are as follows: 0.02, 0.10, 0.13, 0.23, 0.23, 0.28, 0.30, 0.65, 0.80, 0.88, 1.06, 1.43, 1.47, 1.73, 1.81, 2.12, 2.45, 2.47, 2.61, 2.66, 2.75, 2.93, 3.00, 3.00, 3.00, 3.00, 3.00, 3.00, 3.00, and 3.00. Figure 3 shows the histogram density, empirical CDF and P-P plot of the GmzD for the first data set.

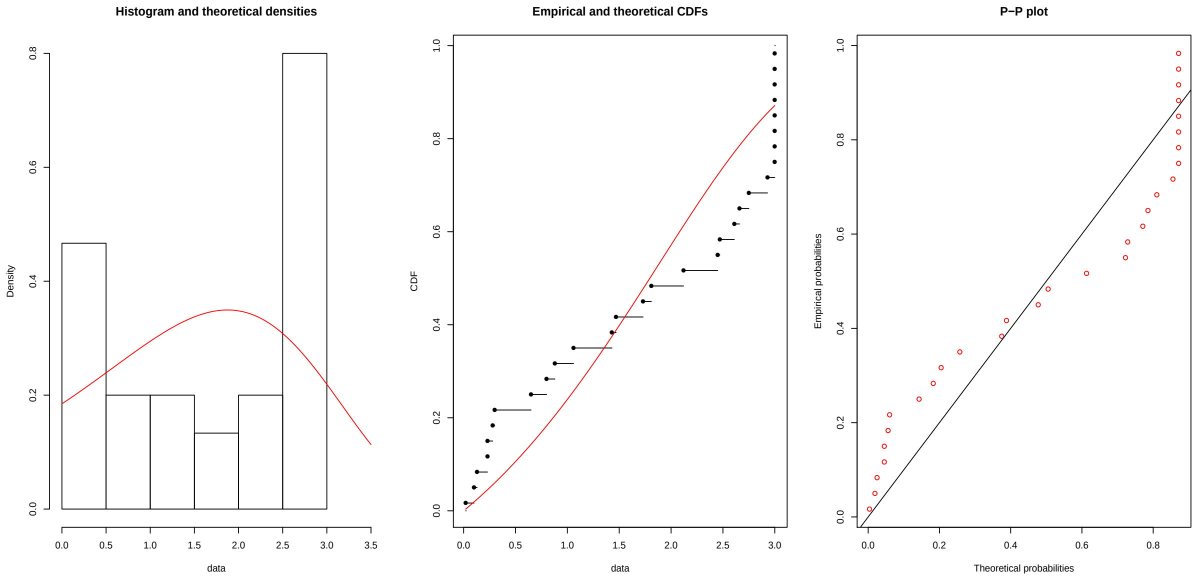

Figure 3. Histogram density, empirical CDF and P-P plot of the GmzD for the first data set.

The estimated parameters α and θ values are 1.3509349 and 0.2496109, respectively, based on Data I. Suppose that a researcher likes to set the mean life μ_0 as 0.4 unit and termination ratio t_0/μ_0 = 0.5; then, based on these values, termination time t_0 is 0.2. For the considered setup, $\alpha = 1.3509349, t_0/\mu_0 = 0.5$, $\beta = 0.25$, and $\alpha_p = 0.05$; the plan parameters of suggested SkSP-R plan are $(n = 28, c = 5, i = 2, f = 0.5, k = 1, m = 2)$; and the process is described follows:

1. Start normal inspection and put $n = 28$ items on test for prefixed time $t_0 = 0.2$. Detect and count the number of sample items that failed before the experiment duration, say, $d = 3$, and $d \leq 5$. Hence, accept the lot.
2. When $i = 2$, consecutive lots are not rejected under normal inspection based on time truncated life test; end the normal inspection and follow SI.
3. Throughout SI, test only a fraction $f = 0.5$ of lots chosen at random. SI is continued up to a point where a sampled lot is rejected.
4. After $k = 1$, where a lot is rejected, consecutively sampled lots are accepted; hence, utilize the resampling method for the immediate next lot as in Step 5.
5. In the resampling technique, perform the inspection based on a reference plan. If the lot is not rejected, then keep SI. If the lot is not accepted, resampling is formed for $m = 2$ times and the lot is rejected if it is not accepted on $(m-1) = 2-1$st resubmission.
6. If a lot is not accepted on resampling scheme, then immediately proceed to the normal inspection provided in Step 1.
7. Remove or correct all the nonconforming items found with asserting units in the rejected lots.

The ASN value is 22.65489. When the quality ratio is $\mu/\mu_0 = 3$, the probability of acceptance of the lot is 0.9882626.

Data II: The data set was obtained by [33]. It consists of 63 observations the strengths of 1.5 cm glass fibers, measured by the National Physical Laboratory, England. The data include the following: 0.77, 0.81, 0.84, 0.93, 1.04, 0.55, 0.74, 1.11, 1.13, 1.24, 1.25, 1.27, 1.28, 1.29, 1.30, 1.36, 1.39, 1.42, 1.48, 1.52, 1.53, 1.54, 1.55, 1.55, 1.48, 1.49, 1.49, 1.50, 1.76, 1.76, 1.77, 1.78, 1.81, 1.82, 1.84, 1.50, 1.51, 1.73, 1.84, 1.89, 2.00, 1.58, 1.59, 1.60, 1.61, 1.61, 1.61, 1.68, 1.68, 1.69, 1.70, 1.70, 1.61, 1.62, 1.62, 1.63, 1.64, 1.66, 1.66, 1.66, 1.67, 2.01, and 2.24. Figure 4 illustrates the histogram density, empirical CDF and P-P plot of the GmzD for the second data set.

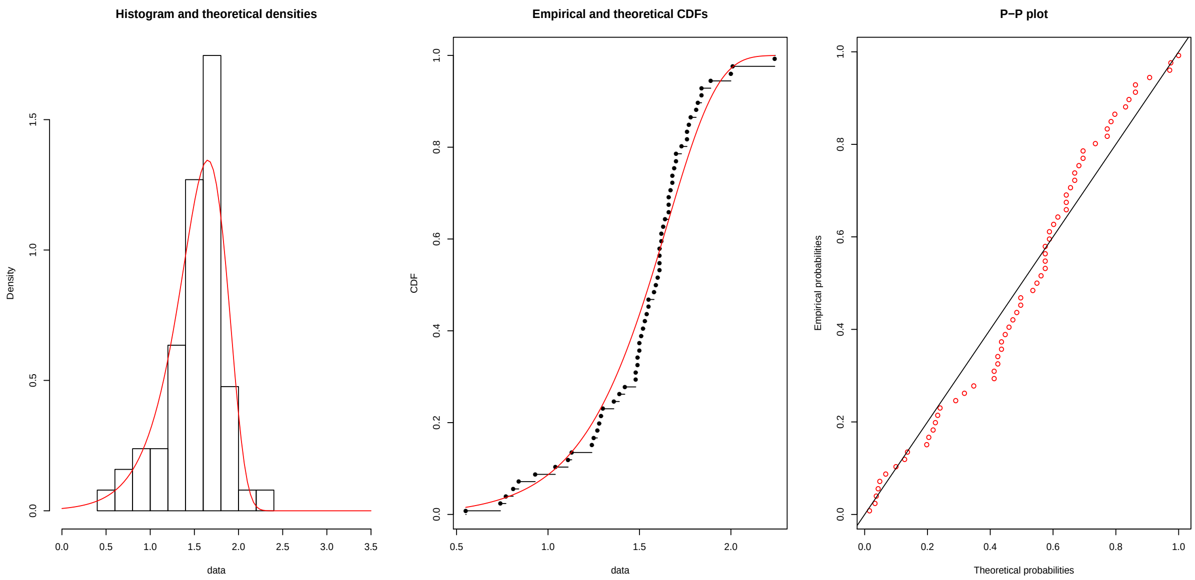

Figure 4. Histogram density, empirical CDF and P-P plot of the GmzD for the second data set.

The estimated values of parameters α and θ are 0.2741801 and 0.0024180, respectively, for the Data II. Suppose that the researcher wants to set the mean life μ_0 to be 1.2 unit and termination ratio t_0/μ_0 as 0.5; then, by using this information, termination time t_0 is 0.6. For the considered setup, $\alpha = 0.2741801, t_0/\mu_0 = 0.5$, $\beta = 0.25, \alpha_p = 0.05$, and the plan parameters of proposed SkSP-R plan are $(n = 54, c = 1, i = 2, f = 0.5, k = 1, m = 2)$; moreover, the process is as follows:

1. Start normal inspection and put $n = 54$ items on the test for prefixed time $t_0 = 0.6$. Detect and count the number of sample items which failed before the experiment duration, say, $d = 1$, and $d \leq 1$. Thus, we accept the lot.
2. When $i = 2$, consecutive lots are accepted under normal inspection based on time truncated life test, and the normal inspection is discontinued. A switch to the skipping inspection is made.
3. During the skipping inspection, inspect only a fraction $f = 0.5$ of lots selected at random. The skipping inspection is continued until a sampled lot is rejected.
4. If the lot is rejected after $k = 1$, consecutively sampled lots are accepted; then, proceed to the resampling procedure for the immediate next lot as in Step 5 provided below.
5. During resampling procedure, perform the inspection using the reference plan. If the lot is accepted, then continue the skipping inspection. Upon the non-acceptance of the lot, resampling is performed for $m = 2$ times and the lot is rejected if it has not been accepted on $(m - 1) = 2 - 1$st resubmission.
6. If a lot is rejected on resampling scheme, then immediately revert to the normal inspection in Step 1.
7. Remove or correct all nonconforming units found with conforming units in the rejected lots.

The ASN value is 43.26171. When the quality ratio is $\mu/\mu_0 = 3$, then probability of acceptance of lot is 0.9676758.

The above explained real life examples show the superiority of the proposed SkSP-R plan and how one can use it in real life situations.

6. Conclusions

The purpose of this paper is to develop a new SkSP-R for GmzD. We have discussed the GmzD characteristics with mean properties. The SkSP-R design for GmzD is presented in this study along with an optimization problem that aids in determining the suggested SkSP-R plan parameters. The necessary tables of the proposed plan are provided and discussed for various values of the distribution parameter θ. Two real life examples were used to support the suggested SkSP-R plan's applicability in real life scenarios. It turned out that industrialists or engineers can use the proffered tables to control the quality of the product. The results in this paper can be modefied based on ranked set sampling method as a future work [34–36].

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