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Article

The Regression Model and the Problem of Inventory Centralization: Is the "Square Root Law" Applicable?

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Abstract: The research problem undertaken by the authors of this article concerns the optimization of the size of the distribution network (the number of warehouses). The authors developed regression models, which are an alternative to the classical "Square Root law" optimization formula. The models were built for the two distributions of demand most commonly used in the literature: Gaussian and Gamma distribution. They allow the calculation of the level of inventory with a given number of warehouses and the level of stock availability as a measure of logistic customer service. The aim was to create a useful tool for decision-makers in companies. The models were elaborated on the base of the simulations carried out for various products (loading parameters, value), sales volumes, number of warehouses, and different standard deviations. Both regression models were statistically significant; the coefficients of determination are relevant. A slightly better value was obtained in the case of Gaussian distribution. The results obtained with the use of the classic "Square Root law" were in some cases quite similar. However, the type of distribution and the variability of demand, measured by standard deviation, have a significant influence here. Thus, the authors believe that the models developed may give more accurate results and that the "Square Root law" formula should be modified taking into account the characteristics of the demand. After completing the regression models with cost components, the total costs were calculated for selected cases of product groups (food, electronics, garments), different levels of the availability of stocks, and different number of warehouses. As it turned out, centralization may not necessarily be the optimal strategy for the most expensive goods. Loading parameters are also important, as they have a significant impact on the costs of storage and, above all, transport.

Keywords: costs of transport; economic efficiency of logistics processes; regression models; simulation

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1. Introduction

Determining the amount of stock in the supply chain is one of the optimization problems in the area of logistics (trade-off situation).

If the number of warehouses is reduced, the inventory level may be, with the same service level (measured by the availability of inventory for customers), lower than in a larger distribution network. Although in decentralized distribution, warehouses are located close to customers, the failure to adjust the level of inventories to the actual demand may result in a shortage of goods stored in them. This means that it is possible to offer the same or even a higher level of logistics customer service and reduce the costs of maintaining inventories in one central warehouse.

The costs of maintaining inventories are related both to the level of inventories in terms of quantity and their value: costs of money which is frozen in stocks, loans, losses due to their maintenance, theft, etc. As for storage costs, they are mainly related to the amount of stored goods and their storage susceptibility. When using warehousing services, warehousing costs can be related to the level of inventory in a comparable way to the cost of maintaining the inventory. However, a company offering warehousing services may offer discounts to large customers. It can also offer a fixed rate for renting warehouse space. In addition, for more expensive goods and goods difficult to store, there may be a need to provide appropriate storage conditions. In the case of these costs, the parameters of the loads (weight, volume) as well as the method of packaging and the loading units used (e.g., pallets) are also important.

In the case of an owned warehouse, the cost structure will be different than in the case of warehouse services; fixed costs (independent of the stock level) will have the largest share. However, the degree of the utilization of storage space is important, especially if, due to centralization, its use will be more efficient than in a decentralized network. The efficiency of the warehouse operation depends to a great extent on the organization of the storage and the technologies used both for storing and moving loads in the warehouse (e.g., combustion or electric trucks).

The solutions used in warehouses, on the other hand, have an impact on the costs as well as on the level of logistics service, because the efficiency of the warehouse processes also affects the time to fulfil customer orders. To some extent, it may also affect the efficiency of transport processes by reducing the waiting time of transport means for loading. Moreover, there will be fewer manipulations in the case of a central warehouse (reloading, moving inside a warehouse) than in the case of a network of warehouses.

The efficiency of the logistics processes depends on the distance at which deliveries are made. However, this impact is not clear-cut, taking into consideration that in the case of centralization strategy, a given company may decide to use a more efficient transport solution (transport and handling technology, faster transport, e.g., air). It depends also on the form of ownership of transport: owned or a hired and whether a company cooperates with transport companies, forwarding companies, or specialized logistics operators who are able to deliver small loads frequently, quickly and on time, using their own distribution network e.g. with the use of cross docking.

Due to the reduction of the number of warehouses, not only the internal costs of these deliveries may be higher, but also the external costs of transport. However, this impact is not unequivocal for similar reasons to those mentioned above. The increase of the efficiency of transport processes in the case of the centralization strategy (e.g., thanks to the cooperation with logistics operators) may also contribute to lowering the external costs of transport. External costs are also generated in warehouses and they also may be lower in a central warehouse, due to the greater efficiency of warehouse operations, less manipulation, and the use of more efficient technologies.

The issue of the influence of the centralization strategy on the efficiency of logistics processes and external costs has already been the subject of research by one of the authors of this article [1]. The considerations presented in this article are based partly on the results of that research, but are largely an extension of them. The authors have developed a simulation model that is relatively (compared to many optimization models) an accurate reflection of the real logistics processes. The model takes into account various parameters of these processes and various factors of their efficiency. For this reason, this model is very complex and a possible user would have to have the appropriate knowledge and be trained in its use. Moreover, such an extensive simulation program requires efficient equipment.

For this reason, based on the simulation results obtained from this simulation model, the authors developed the regression models that can be useful for making decisions about the number of warehouses in the distribution network. Such models may allow for the assessment of the effects of centralization or decentralization of warehousing from the point of view of both a given company (total costs of logistics processes, profitability) and the natural environment (external costs of these processes).

2. Literature Review

According to Kotler [2], inventory management refers to all the activities involved in developing and managing the inventory levels of raw materials, semi-finished materials (working progress), and finished good so that adequate supplies are available and the costs of over or under stocks are low. The first and the most famous model for calculating the optimal size of a delivery of goods to a warehouse was the formula of the Economic Order Quantity (EOQ) elaborated by Harris and Wilsons's in 1913. The EOQ model is a very popular and successful model for managing the supply chain. Many studies have been completed about this model [3]. There are many reasons for the popularity of the model such as simplicity and ease of use, despite its limitations. Its modified versions are to be used not only to optimize processes in the field of material procurement, but also in production [4]. Horowitz [5] took into account the inflation rate of the classic EOQ model, without which the optimal batch size may be drastically underestimated.

The paper by N. Thinakaran et al. [6] contains a survey on the inventory models of Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) with a full and partial backorder condition. Primarily, most of the paper was focused on a full backorder model or lost sales for the inventory model, with important parameters such as cycle time varying, stock demand and lot size, and substitutable items.

However, models containing the EOQ formula include a few impractical assumptions, which simplify the model too much [7]. Lin and Chung [8] take into consideration an instance with warehouses and additionally query the assumptions of the EOQ model: "Although the traditional EOQ models are still widely used in industry, practitioners frequently question validities of assumptions of these models such that their use encounters challenges and difficulties."

2.1. Centralization vs. Decentralization Strategies

The effect of the number of warehouses on the stock levels in a distribution network has been the subject of research for many years. However, the literature on this problem is sparse. It is generally known that decreasing the amount of stock and, therefore, the implementation of a centralization strategy favors the reduction of inventory levels throughout the distribution network and improves the availability of stocks in the warehouses (better logistics customer service). At the same level of service, when the number of warehouses is reduced, the level of inventory in the entire network decreases. However, it should be borne in mind that this formula is only used to calculate the level of the safety margin, which is only a part of the total inventory [9,10].

On the other hand reducing the number of warehouses results in higher transport costs-longer transport distances and possibly smaller batches of shipments to end customers, which is considered the most expensive part of transportation [11,12].

There are a number of works that have dealt with a similar problem, e.g., Miranda and Garrido [13], who suggest a Lagrangian relaxation-based heuristics with the use of the subgradient method. Sourirajan et al. [14] took into consideration a supply chain with a production facility and multiple retailers. Another article that takes into consideration the costs of transportation is that of D.-J. Lee and I.-J. Jeon [15]. They consider a semi-centralized inventory system with costs of transportation.

The results of the studies concerning the centralization of stocks vary widely. The literature presents a wide range of recommendations for various external conditions [16]. Arya et al. [17] suggest that decentralization may prove effective when a firm relies on the external suppliers and strategically manages inventory, while Daugherty et al. [18] demonstrate that centralization strategy is conducive to the implementation of innovations in logistics services. On the other hand, Duan and Liao [19] show that centralized control is beneficial for supply chains with the unstable demand patterns, and also that centralized decision-making presents the best solution [20]. Moreover, there are some works that claim that production quantity, research and development level, and profit are highest if a centralized and decentralized supply chain exist in parallel [21].

The industry sector and type of business activity is a very important factor. The centralized stock method may be effective in businesses, in which inventory maintenance costs and their storage are extremely excessive, such as in the food business [22]. The literature rarely refers to the size of companies in relation to the benefits of centralization of warehousing. According to Pedersen et al. [23] small and medium companies have a smaller number of resources, but also fewer skills, which translates into smaller benefits when using warehouse centralization. Schmitt et al. [24] examined both a decentralized and a centralized storage system in the context of future costs and their changes. They showed that if there are supply disruptions and the demand is constant, decentralization of storage decreases the risk and should, therefore, also decrease the costs. The results of these studies contradict the classical views according to which, when demand is random and supply is constant, then using storage centralization is the best choice due to the risk-pooling effect. At the strategic level, the decentralization of storage can also provide a greater decision-making resilience.

Also other problems were the subject of the conducted research. Yu et al. [25] considered how the use of different numbers of suppliers is affected by supply disruptions. Articles by Qi [26] and Sawik [27] address the topic of supplier selection under uncertainty. The aim of the thesis of Petersson and Sturesson [28] was to optimize, with the use of simulation modelling (in a public transport company), inventory management in the centralized spare parts warehouse. The findings that were obtained in the article demonstrated that the storage centralization strategy is more effective than the decentralization strategy. For the centralized approach, procurement costs as well as storage costs were found to be higher. Corts et al. [22] compared the arguments for both centralization and decentralization of warehouses. Centralization reduces fixed storage costs [11], involves less capital, and reduces the number of warehouse workers. Abrahamsson [29] proved that inventory centralization does not necessarily lead to longer delivery times, even if the average distance to customers increases. In addition, inventory centralization enables more effective quality control and greater inventory visibility [30]. On the other hand, arguments in favor of decentralization mainly relate to advantages in terms of delivery costs and times [31] and greater closeness to customers.. Furthermore, Axsäer [32] wrote that "in practice, decentralization of control is generally more attractive because a centralized system requires strategic planning and alignment throughout the chain." Nevertheless, as Fleischmann (2016) points out, in applying the Square Root law, we are limited by certain assumptions [33]. For example, that replenishment of inventory, both cyclical and safety stock, is done in economic order quantities (EOQ). Despite this, for many years no one has contested the application of the EOQ method.

2.2. Square Root Formula Limitations

The relationship between the level of safety stocks and the number of warehouses in the distribution network reflects the "The Square Root Law", which is presented in the form of the following mathematical formula [9,10]:

$$X_2 = X_1 \cdot \sqrt{\frac{n_2}{n_1}}$$

n_1 = number of existing facilities;
n_2 = number of future facilities;
X_1 = existing inventory;
X_2 = future inventory.

Since the level of the customer service is included (measured by the availability of inventory), this means that it can also be used to evaluate the impact of warehouse centralization policies on the level of logistics customer service.

However, in the scientific literature, one can find a critical approach to the applicability of the mentioned formula. Considering the different economic sectors, the obtained outcomes are highly varied. In the work of Oeser and Romano [34], it was shown that the formula does not perform well in companies in the manufacturing sector and does better in companies in the trading sector. Additionally, another paper by this author [35] shows that it overestimates the savings that result from centralizing warehouses.

First of all, the formula allows the calculation of the level of safety stocks before and after centralization. On the other hand, in some papers, there is a suggestion that cyclical stock and safety stock can be considered together [31,33,36,37] and that a significant aspect for the effective application of this approach is transportation: the quality and the cost of transportation service [38].

In addition, warehouses may accumulate excess inventory that results from erroneous forecasts, and the ability to adjust to actual demand may be higher in the case of centralization strategies.

Some authors note that the minimization of the external costs could be aided by a minimal number of warehouses, in the sense of using less of certain resources such as energy [39]. This impact is not clear. The operation of big and extremely high-tech warehouses can result in a growing consumption of energy (heating or cooling, lighting, air conditioning) and growing CO_2 emission [40]. However, this impact also depends on certain properties of the building, e.g., the number of doors and windows, insulation, and efficacy of the heating and cooling system, as well as external factors such as properties of stored goods and outside temperature [41], the technology used [42], the parameters of the used equipment (motor power, energy consumption), and also the operating mode, e.g., the speed of movement in the warehouse and the organization of work in the warehouse [43].

Current analyses show that about 10% of global carbon emissions come from logistics supply chains. The significant lighting, cooling,+ or heating energy required in storage accounts for almost 21% of the total logistics costs. According to a study, the total electricity consumption of warehouse facilities has decreased from 1976 to 2017, influenced by technological developments that resulted in the production of green equipment (e.g., air-circulation, lighting or ventilation systems, etc.) [44].

The reduction of energy consumption is considered to be the most important factor for reducing external storage costs, which can be achieved, for example, by using the appropriate equipment in the picking area [45–47].

Milewski [1] developed a simulation model of the total costs of centralization strategies in both micro (enterprise) and macro (external costs) terms. The results of simulations indicate that while the effects of a particular strategy may vary, in many cases the influence on both the efficiency of the logistics processes and the external costs of transportation can be significant.

This paper presents the results of simulations that are an extension of this model. The authors conducted tests for more variants and for two types of demand deviations: the Gaussian and Gamma distributions.

3. Materials and Methods

Using a distribution network model, the authors conducted computer simulations for various alternatives, the effects of which formed the basis for developing a regression model. The assumption was that deliveries to warehouses (distributions centers) are performed on a daily basis, immediately after production.

First of all, in order to develop the regression model, simulations had to be carried out (using the model mentioned above) to find the relationship between the following variables:

The dependent variable is:

y—Average Stocks in the Distribution Centers.

The independent variables are:

x_1—Yearly sales;
x_2—Standard deviation of average sales in one warehouse;
x_3—No. of distribution centers;
x_4—Customer service level in distribution centers

The significant scientific novelty of this article is the presentation of a new methodology that, through calculations from a computer simulation model, allows for the development of a simplified multiple regression model for calculating average inventory levels.

Multiple regression is a statistical method of which the main aim is to quantify the relationship between multiple independent variables and the dependent variable [48]. Even if there is no direct relationship between the variables, one can seek to link them by means of a mathematical equation that, under certain assumptions, allows one to predict quantities determined on the basis of knowledge of other variables. Regression is one of the most commonly used methods when applied to problem solving in technical sciences, economics, and management [49]. The concept of the developed methodology is presented in Figure 1.

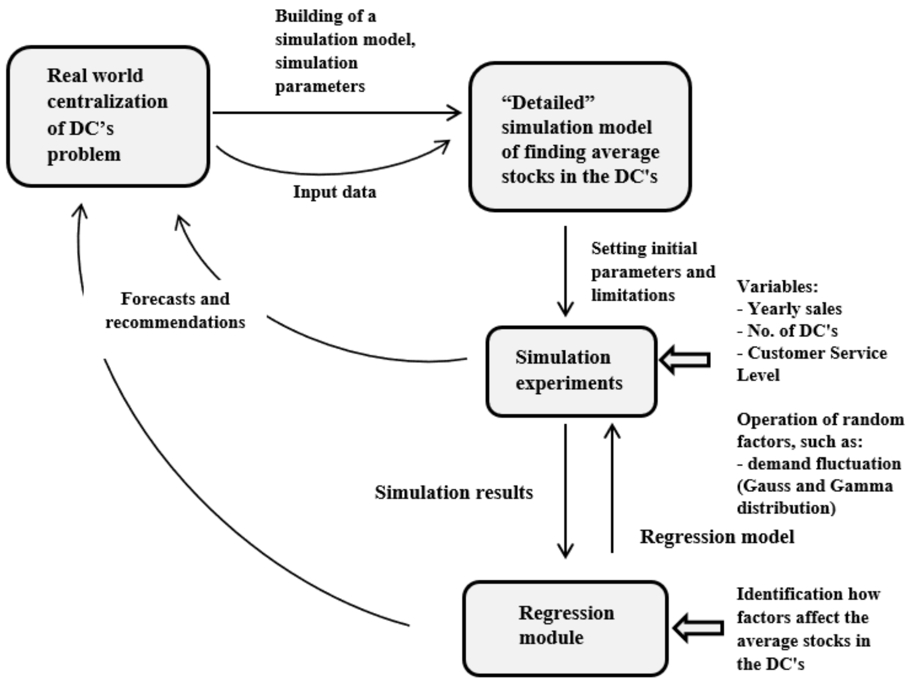

Figure 1. Conceptual framework scheme of simulation modeling with a regression module for finding average stocks in the distribution centers.

The concept is based on the assumption that from the results obtained by a complex simulation model, which takes into consideration a large number of variables as well as random factors, it is possible to obtain a simpler formula to be used in practice by managers. Such a model (formula) allows one to obtain the answer to what should be the average level of stocks under given conditions, which would allow managers to easily implement it in practice. When analyzing real phenomena and processes, even in relatively simple situations, we are not able to explain them fully. Therefore, when describing interdependencies between them, we usually use some simplified models of real interdependencies. By "model" we can, therefore, mean a useful form of presentation of empirical data. When approaching the process of building a model, we have to accept a certain compromise between oversimplifying reality and wanting to include too much detailed data. One of the ways to describe such interdependencies can be the well-known method of multiple regression, which is used in this paper.

In this paper, the regression approach was used to explain the result of changes in the independent variables (x_1 to x_4) on the dependent variable (y). Two analyses were conducted separately for two types of demand modeling in the simulation model: the model using the Gaussian distribution and the model using the Gamma distribution; these are the two most commonly used models in the literature for demand analysis [50]. For the model using the Gamma distribution, all four independent variables were found to be statistically significant. Table 1 shows significance tests with effect sizes, *p*-value, and *t*-test for variables x_1, x_2, x_3, and x_4.

Table 1. Significance tests for independent variables in a regression model for the Gaussian distribution.

	t-Test	*p*-Value
x_1—Yearly sales	11.48637	<0.00001
x_2—Standard deviation of average sales in one warehouse	23.95595	<0.00001
x_3—No. of distribution centers	11.86944	<0.00001
x_4—Customer service level in distribution centers	1.98158	0.048205

Source: Own calculations.

From Table 1, one can observe that all variables in the regression analysis are statistically significant; the *p*-values are distinctly smaller than 0.05. Additionally, an assessment of the significance of the structural parameters was carried out. To check the significance of structural parameters b_0, b_1,..., b_n, the *t*-test was used, where the *t* statistic has a *t*-test distribution with *n*-*k*-1 degrees of freedom.

Null and alternative hypotheses are:

H_0: *$b_i = 0$ (no linear relationship);*

H_1: *$b_i \neq 0$ (linear relationship does exist).*

The critical region is two-sided with the critical value that we read from the tables, the *t*-test distribution for a fixed level of significance, and *n*-*k*-1 degrees of freedom. If the value of *t* is in the critical region (the calculated value of *t* > value of *t* from a table), then we have to reject H_0 in favor of H_1. Otherwise, there is no basis to reject H_0. In the case of structural parameters from Table 1, all values of *t* are in the critical region; thus, a relationship does exist.

The regression model for *y* (average stocks in the distribution centers) is as follows:

$$y = -10221.2327311 + 0.0027863344633x_1 + 16.6820813869x_2 + 131.355655244x_3 + 9865.91831311x_4.$$

A similar situation can be seen in Table 2, which presents significance tests along with effect sizes for the second case of the analyzed type of demand modeling, the Gamma distribution. In this case, regression analysis indicated that the set of all variables x_1, x_2, x_3, and x_4 is statistically significant.

Table 2. Significance tests for independent variables in a regression model for the Gamma distribution.

	t-Test	*p*-Value
x_1—Yearly sales	10.35361	<0.00001
x_2—Standard deviation of average sales in one warehouse	9.210843	<0.00001
x_3—No. of distribution centers	10.6077	<0.00001
x_4—Customer service level in distribution centers	−8.60058	<0.00001

Source: Own calculations.

From Table 2, one can observe that all variables are statistically significant; the *p*-values are distinctly smaller than 0.05. To check the significance of structural parameters in the table, there are also *t*-test values; in each case, they are in the critical region; thus a relationship does exist.

The regression model for y (average stocks in the distribution centers) in the case of the Gamma distribution is as follows:

$$y = -55738.8618951 + 0.00629264047071x_1 + 7.08365631632x_2 + 795.996410504x_3 + 54119.5613891x_4.$$

Furthermore, model verification was carried out to check whether the econometric models were acceptable. The verification process included:

- The coefficient of determination, R^2, is interpreted as the part of the variance of the dependent variable that can be explained by the variance of the independent variable;
- The properties of residuals (random components) of the regression model: conformity to normal distribution, occurrence of autocorrelation of model residuals (Durbin–Watson test);
- The analysis of variance (F-statistic at low p-value). The analysis allows the determination of the uncertainty associated with the predicted values of the variable, which may deviate from the average the higher the variance is. The variance as the mean square deviation of the value of a random variable from its mean value is a measure of dispersion of possible values of the variable and is an indispensable tool for testing the significance of the whole equation [49].

The statistical analyses were performed using the STATISTICA package (StatSoft), assuming a significance level of 0.05. The outcomes are presented in Table 3.

Table 3. Verification of the regression model.

	R^2	F	p-Value	d Durbin–Watson
The Gaussian distribution	0.905694	978.1901	<0.00001	1.9013
The Gamma distribution	0.840391	663.1129	<0.00001	1.8976

Source: own calculations.

It can be seen from Table 3 that the R^2 parameter shows that the model using the Gaussian distribution explains more than 90% of the variation in the average stocks in the distribution centers, while in the case of the Gamma distribution, it allows explaining about 84% of the total variance. In both cases, the results are statistically significant, as allowed by the observation of the F statistic along with a low p-value. The analysis of the residuals of the analyzed models confirmed their validity. The value of the Durbin–Watson test statistic allows one to conclude that there is no autocorrelation of residuals in the obtained model.

Summarizing these results and the value of the t-test in Tables 1 and 2, it can be concluded that both regression models are statistically significant; a slightly better value is obtained by the model created using demand modeling with the Gaussian distribution, especially the parameter R^2, which explains as much as 90% of the variance of the model.

4. Impact of the Centralization Strategy on the Effectiveness of the Distribution

The regression model can then be used to simulate the cost-effectiveness of using a storage strategy.

Calculations using this model were carried out for three hypothetical products from the following industries: food, electronics, and clothing; various standard deviations of demand, and two distributions of demand: Gaussian and Gamma.

First, calculations were carried out, the purpose of which was to check to what extent the results coincided with the results obtained using the “Square Root” formula (Tables 4 and 5 and Figures 2 and 3). The starting point is a decentralized system with nine warehouses.

Table 4. Changes of levels of safety stocks for the Gaussian distribution with the use of Regression Model (RM) Square Root Formula (SRF) [pallets].

Yearly Sales [Pallets/Year]		60,000	Customer Service Level in Distribution Centers			99.15%
Method	**Stand Dev/Average Sales**	**Number of Warehouses**				
		9	**5**	**4**	**3**	**2**	**1**
RM	15%	2670	2144	2013	1882	1750	1619
SRF	15%	2670	1990	1780	1541	1259	890
RM	30%	4338	3812	3681	3550	3418	3287
SRF	30%	4338	3233	2892	2504	2045	1446

Source: own calculations.

Table 5. Changes of levels of safety stocks for the Gamma distribution with the use of Regression Model (RM) Square Root Formula (SRF) [pallets].

Yearly Sales [Pallets/Year]			60,000	Customer Service Level in Distribution Centers		99.15%
Method	**Stand Dev/Average Sales**	**Number of Warehouses**					
		9	**5**	**4**	**3**	**2**	**1**
RM	115%	6908	3724	2928	2132	1336	540
SRF	115%	6908	5149	4605	3988	3256	2303
RM	30%	7616	4432	3636	2840	2044	1248
SRF	30%	7616	5677	5077	4397	3590	2539

Source: own calculations.

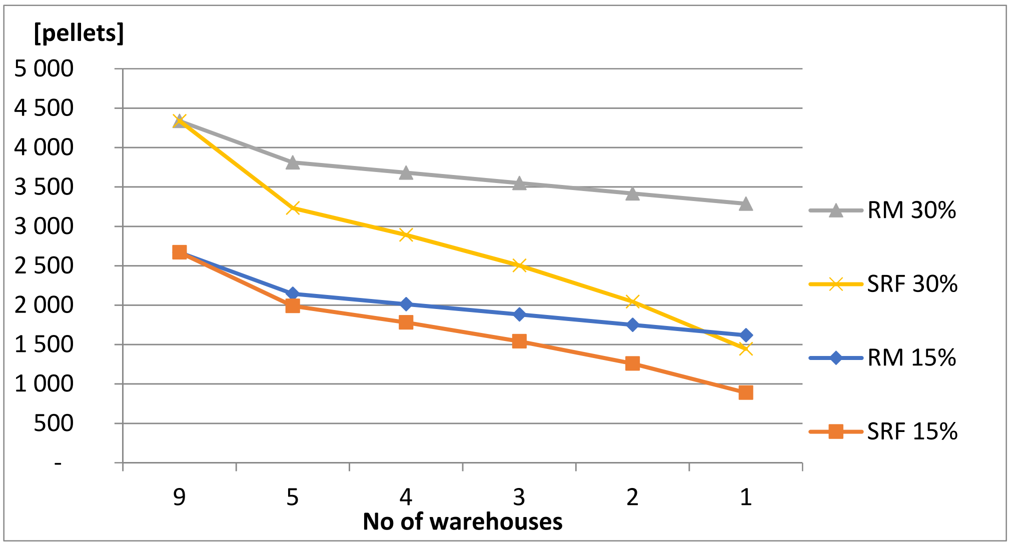

Figure 2. Levels of stocks—simulation vs. "Square Root Formula" (Gaussian distribution).

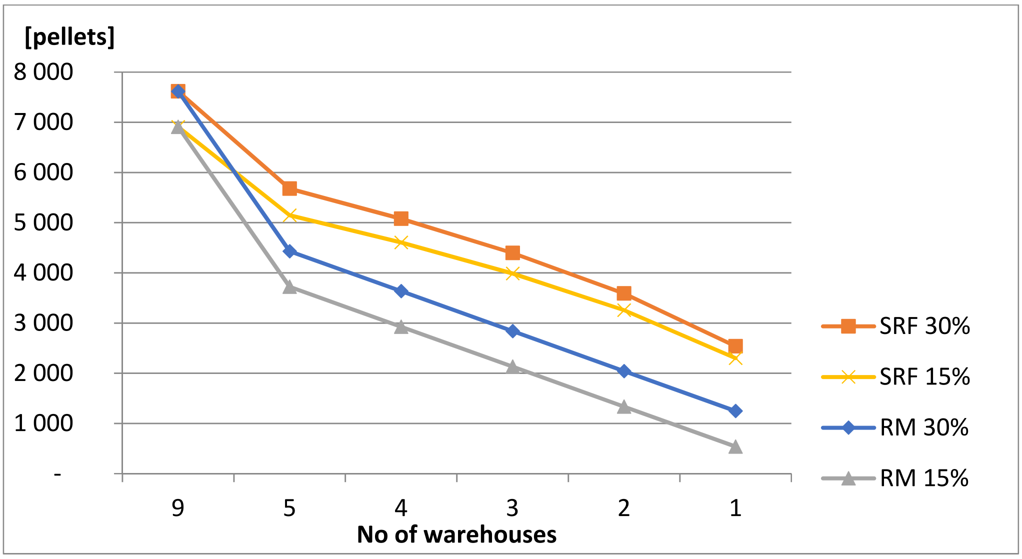

Figure 3. Levels of stocks—simulation vs. "Square Root Formula" (Gamma distribution).

In both distributions, even with small standard deviations (15% of the average demand), the levels of safety stocks calculated using the regression model decreased significantly with the reduction of the number of warehouses. However, the greater benefits of centralization are in the Gamma distribution. In the Gaussian distribution, the safety stock in one central warehouse is 50.1% of the stock in a system consisting of nine warehouses. In the case of the Gamma distribution, this stock drops to 12.8%. Even greater differences are visible when the sales fluctuations are greater (the standard deviation is 30% of the average demand). In the case of Gauss, the stock is only 80.71% (thus, the benefits of centralization are smaller here). In the Gamma distribution, with greater fluctuations in demand, the benefits are also smaller, but are still greater than in the case of Gauss, because the safety stock in the central warehouse is 32.8% of the inventory in the decentralized system.

The results of the calculations of the safety stock level obtained by the regression model (RM) were then compared with the results calculated using the "Square Root Formula" (SRF). In the case of the normal distribution with small fluctuations in sales (15% of average demand), the inventory levels calculated using both methods are very similar, although the differences between the two become larger with fewer warehouses. Inventory levels already vary significantly with greater volatility in demand (30% of average demand). Therefore, it could be concluded that perhaps the Square Root Formula could be used to calculate the level of the safety margin after some modification of this formula, which would take into consideration the demand volatility. However, the results were completely different when the Gamma distribution was in demand. The results obtained with both methods are most similar to each other, not with small changes in scale (and therefore unlike in Gauss), but with large changes in sales (Table 5 and Figure 3). Moreover, with large deviations, the results for Gamma are more similar to each other than for Gauss, which may give rise to a supposition that the "Square Root law" was perhaps developed for this type of demand. This issue should be the subject of further research.

The calculated inventory levels can then be used to calculate the total cost of distribution using the following formula:

$$Cd = Ctw + Ci + Cw + Cls + Ctc$$

where:

Cd—The total costs of distribution;
Ctw—Transport costs to warehouses;
Ci—Inventory costs;

Cw—Warehousing costs;
Cls—The costs of lost sales;
Ctc—Transport costs from warehouses to customers.

The costs shown should be calculated as following:

Ci = Average inventory level × Cost of maintaining inventories × Unit cost of production;
Cw = Average inventory level × Unit cost of warehousing;
Cls = Sales price × Sales volume × (100%-Level of Service);
Ctc = Transport performance [tkm/year] × Freight rate [EUR/tkm] × Distance.

Average inventory level = Cycle Stock + Safety Stock + Excess Stock.

The parameters used in the calculations are presented in Tables 6 and 7. The results of the calculations for Gaussian are presented in Tables 8 and 9 and Figures 4–9.

Table 6. Parameters for calculations.

Products	Food	Electronics	Garment
Yearly sales [pallets/year]	200,000	200,000	200,000
Items per pallet	500	60	600
Weight of a commodity [kg/item]	1.65	0.90	0.30
Value of a commodity [EUR/item]	0.67	111.86	44.74
Transport [EUR/ton/km]			
Deliveries to warehouses	0.03	0.49	0.15
Final deliveries to customers	0.07	1.04	0.31
Direct deliveries to customers	0.05	0.84	0.25
Capital costs in inventories		30%	
Warehousing [EUR/pallet]		0.22	

Source: own assumptions.

Table 7. Distance to Warehouses.

Location	Distance [km]
Rotterdam	1
Amsterdam	82
Eindhoven	108
Dortmund	251
Bremen	388
Paris	448
Warsaw	1229
Naples	1846
Lisbon	21,776

Source: own assumptions.

In order to ensure comparability, the same annual demand for 400,000 pallets was assumed for all product groups. The products, however, differ in terms of volume, weight and value. Transport rates vary depending on the size of the shipment and the distance.

In the case of cheap food, centralization is not a good strategy. The optimal solution is four warehouses, regardless of the distribution of demand. The differences between the costs in the different strategies are not large and the impact of changes in sales measured by standard deviation is also small.

Table 8. Total costs of distribution (Gauss) [M EUR/year].

Yearly Sales [Pallets/Year]		**60,000**	**Customer Service Level in Distribution Centers**			**99.15%**
No. of Warehouses	**9**	**5**	**4**	**3**	**2**	**1**
St. dev/Av. sales			Food			
115%	16.95	***16.71***	16.75	17.12	17.75	26.74
30%	17.24	***17.00***	17.04	17.41	18.04	27.02
St. dev/Av. sales			Electronics			
115%	23.33	21.94	***21.67***	21.70	22.00	30.82
30%	26.95	25.49	25.20	***25.16***	25.39	34.27
St. dev/Av. sales			Garment			
115%	43.29	34.69	33.70	32.91	***32.41***	40.81
30%	57.33	46.21	45.11	44.10	***43.39***	52.00

Source: own calculations.

Table 9. Total costs of distribution (Gamma) [M EUR/year].

Yearly Sales [Pallets/Year]		**60,000**	**Customer Service Level in Distribution Centers**			**99.15%**
No. of Warehouses	**9**	**5**	**4**	**3**	**2**	**1**
St. dev/Av. Sales			Food			
15%	17.68	16.78	***16.70***	16.96	17.47	26.34
30%	17.81	16.90	***16.83***	17.08	17.59	26.47
St. dev/Av. Sales			Electronics			
15%	32.53	25.10	23.40	22.01	***20.95***	28.37
30%	34.07	26.61	24.90	23.48	***22.39***	29.84
St. dev/Av. sales			Garment			
15%	78.96	51.19	44.42	37.92	***31.92***	34.79
30%	84.93	57.04	50.22	43.61	***37.50***	40.48

Source: own calculations.

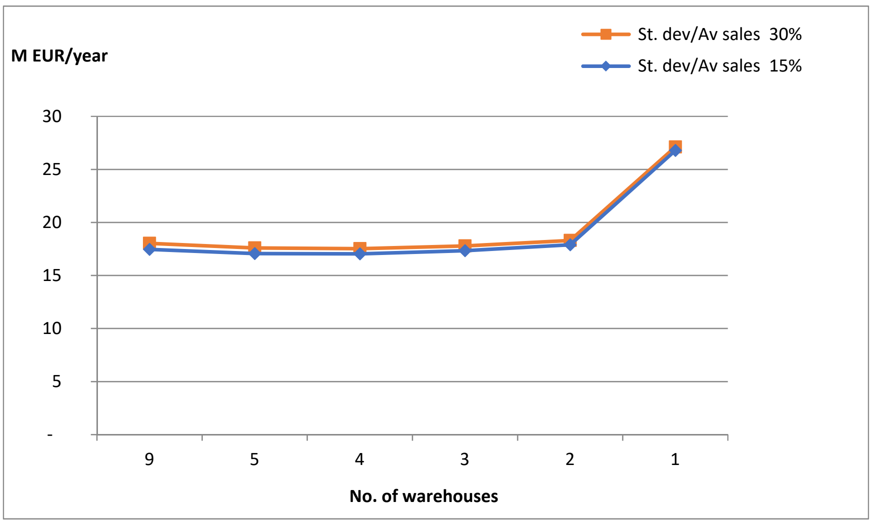

Figure 4. Impact of the number of warehouses and distances of deliveries on the total costs of distribution (Gaussian distribution, agriculture products).

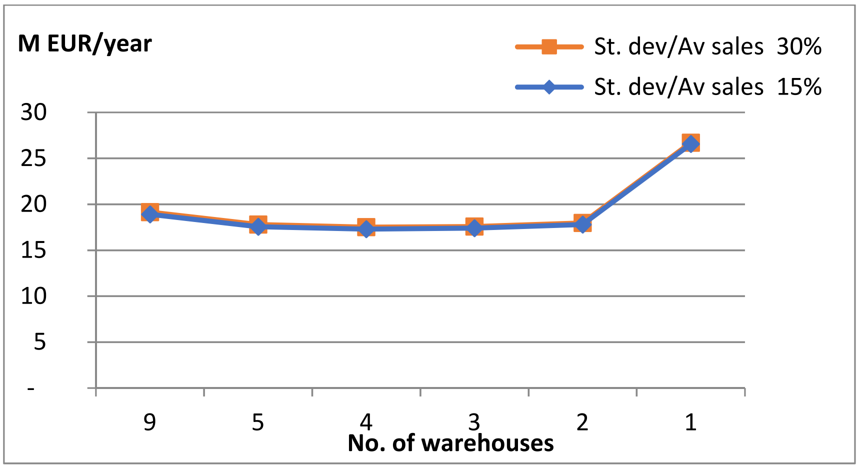

Figure 5. Impact of the number of warehouses and distances of deliveries on the total costs of distribution (Gamma distribution, agriculture products).

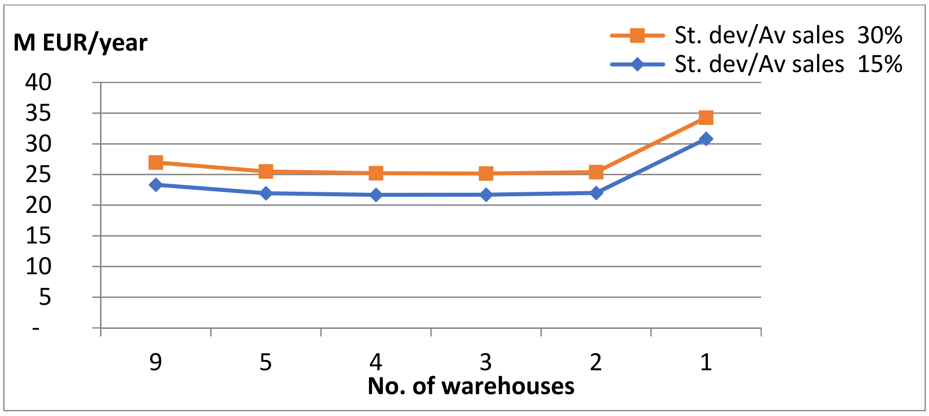

Figure 6. Impact of the number of warehouses and distances of deliveries on the total costs of distribution (Gaussian distribution, electronics products).

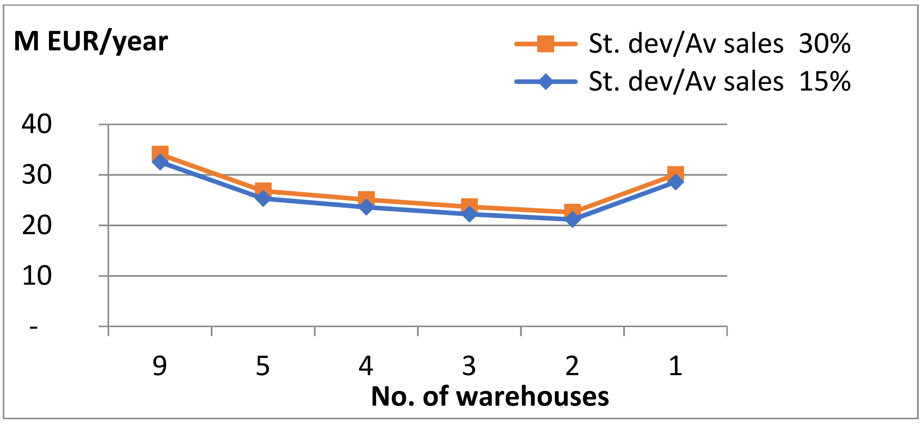

Figure 7. Impact of the number of warehouses and distances of deliveries on the total costs of distribution (Gamma distribution, electronics products).

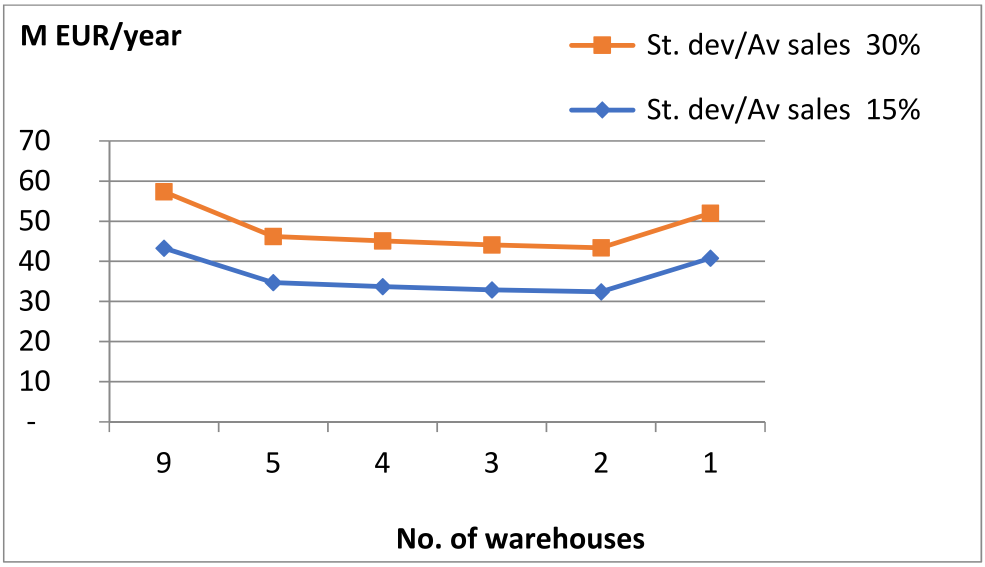

Figure 8. Impact of the number of warehouses and distances of deliveries on the total costs of distribution (Gaussian distribution, garments).

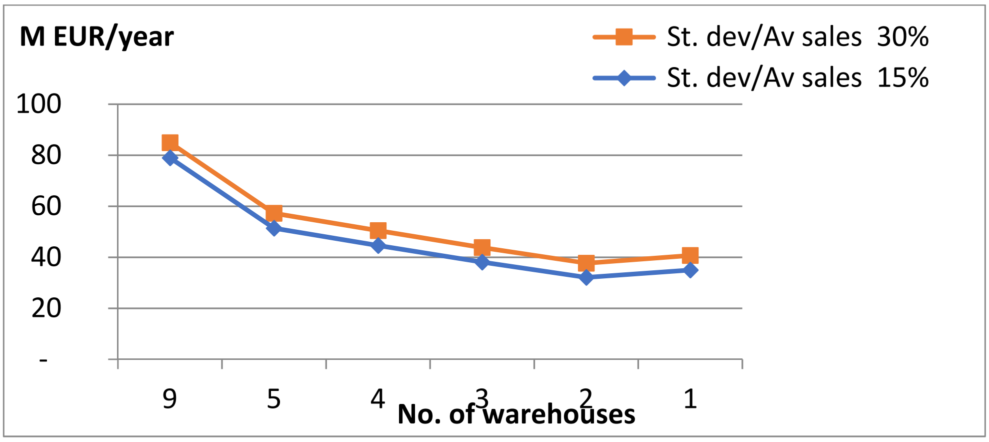

Figure 9. Impact of the number of warehouses and distances of deliveries on the total costs of distribution (Gamma distribution, garments).

A slightly greater impact on sales volatility is visible in the case of electronic products. The optimal number of warehouses is also four, regardless of the size of demand fluctuations, but only in the case of the Gaussian distribution. In the case of the Gamma distribution, the optimal size of the warehouses is smaller and, moreover, the influence of the demand fluctuations is visible; the optimal number of warehouses is three for 15% of the standard deviation and two for larger deviations (30%). The cost impact of choosing a storage strategy is also greater, which may be influenced by the greater value of electronic products.

The largest cost differences are visible in the case of clothing; moreover, in the case of these products, the most profitable strategy is centralization. In the case of Gauss, the optimal number of warehouses is three when the sales fluctuations are small and two when the sales fluctuations are larger. In the case of Gamma, these are two and one, respectively.

The results may seem surprising to some extent. It could be expected that the most favorable centralization strategy would be the most advantageous for the most expensive (electronic) products. Meanwhile, this strategy has turned out to be beneficial for clothing. However, these results can be interpreted quite easily. The total costs of distribution are influenced not only by the value of goods, but also by cargo parameters that affect the costs of storage and transport.

The simulations also took into consideration the impact of the presence of excess inventories resulting from incorrect sales forecasts. If we assume that the excess inventory for one central warehouse is 15% of inventories and for nine warehouses, 100%, then in all variants of the type of probability distribution and sales fluctuations, the results obtained by both methods are very similar. Therefore, it can be hypothesized that the Square Root Formula was developed for a situation where, in the case of decentralization of storage, excess stocks increased to a large extent.

The model, after modification, can also be used to calculate external costs, both transport and storage (Table 10). If the external costs of storage for nine warehouses account for 50% of the external costs of transport, the total costs of the centralization strategy would be the lowest in the case of the Gamma distribution, regardless of the fluctuations in demand (Tables 11 and 12).

According to the latest studies, external transport costs are even lower—[51], so the cost-effectiveness of the centralized warehousing would be even higher when taking into consideration the external costs.

Table 10. External unit costs (interbank EURO-4).

Cost Component	**A Vehicle**			
	<7.5 *t*	**7.5–16 *t***	**16–32 *t***	**>32 *t***
[€ct per vkm]	16.28	17.58	17.58	19.59
[€ct per tkm]	4.40	2.51	2.03	0.75

Source: own calculations based on [52].

Table 11. Total costs of distribution (Gauss) with external costs [MM EUR/year].

Share of External Costs of Warehousing			**50%**			
Yearly Sales [Pallets/Year]		**60,000**	**Customer Service Level in Distribution Centers**			**99.5%**
No. of Warehouses	**9**	**5**	**4**	**3**	**2**	**1**
St. dev/Av sales			Food			
15%	23.5	***22.6***	22.5	22.8	23.6	37.3
30%	24.1	23.3	***23.1***	23.4	24.2	37.8
St. dev/Av sales			Electronics			
15%	23.7	22.3	***22.0***	22.1	22.4	31.5
30%	27.3	25.9	25.6	***25.5***	25.8	35.0
St. dev/Av sales			Garments			
15%	44.6	35.9	34.9	34.2	***33.7***	43.2
30.00%	58.6	47.5	46.4	45.4	***44.8***	54.5

Source: own calculations.

Table 12. Total costs of distribution (Gamma) with external costs [MM EUR/year].

Share of External Costs of Warehousing			**50%**			
Yearly Sales [Pallets/Year]		**60,000**	**Customer Service Level in Distribution Centers**			**99.5%**
No. of Warehouses	**9**	**5**	**4**	**3**	**2**	**1**
St. dev/Av sales			Food			
15%	***21.4***	37.8	36.2	35.1	34.2	38.2
30%	***21.6***	38.3	36.8	35.6	34.8	38.7
St. dev/Av sales			0			
15%	***35.0***	45.7	43.0	40.4	38.0	40.6
30%	***36.6***	47.6	44.9	42.3	40.0	42.7
St. dev/Av sales			Garments			
15%	81.4	71.8	64.0	56.3	49.0	***47.0***
30.00%	87.4	78.0	70.2	62.4	55.1	***53.3***

Source: own calculations.

5. Results

The research problem undertaken by the authors of this article concerns the optimization of the size of a distribution network (the number of warehouses). In the authors' opinion, the optimization formulas may be too simplified and do not take into account the complex relations of logistic processes. The alternative is the simulation method, which, however, requires the use of very complex simulation models, the construction and application of which may be too complicated for a potential user who does not have theoretical background. The intention of the authors was therefore to develop a model that could be a simple and useful tool for making decisions.

That is why the authors developed a regression model that allows the calculation of the level of inventory with a given number of warehouses and the level of stock availability as a measure of logistic customer service. This model is an alternative to the classical "Square Root law" formula. Such a model (after taking into account the costs) can be used to calculate the total costs and, consequently, to determine the optimal number of warehouses in the distribution network. The optimization criterion is the minimal total costs, including the costs of logistics processes (costs of maintaining inventories and warehousing, transport) and the costs of lost sales, which result from the level of customer logistics service. It also allows the calculation of the impact of the warehousing centralization strategy on the external costs of the logistics processes.

First, the simulations were carried out for various products (different loading parameters, values), sales volumes, number of warehouses, for the two distributions of demand most commonly used in the literature—Gaussian and Gamma distributions—and with different standard deviations. Both regression models were statistically significant; the coefficients of determination were relevant. A slightly better value was obtained in the case of the Gaussian distribution.

In both distributions, even with small standard deviations (15% of the average demand), the levels of safety stocks calculated using the regression model decrease significantly with a reduction of the number of warehouses. However, there are greater benefits of centralization in the Gamma distribution (stock are reduced by over 92%) than in the Gaussian distribution (below 40%). Slightly smaller savings for Gamma are obtained when the demand fluctuations are greater (standard deviations—30%), but they are still significant (−83.61%) and still better than in Gaussian distribution (−24.22%).

This can be interpreted as follows - that the warehousing centralization strategy is effective when demand is relatively less stable. On the other hand, apparently when changes in demand are large, the cost of maintaining inventories in both strategies increases. However, even in this case centralization is beneficial.

The results of the calculations of the safety stock level obtained by the regression model ("RM") were then compared with the results calculated using the "Square Root Formula" (SRF). According to the "Law of the Square Root", the reduction in the level of inventories during centralization should amount to −66.67%. This is more than in the regression model for the Gaussian distribution but less than in the Gamma distribution.

6. Conclusions

The simulation results show that the type of distribution and the variability of demand, measured by standard deviation, have a significant influence. Thus, the authors' conclusion is that the models developed may give more accurate results and that the "Square Root Formula" should be modified taking into account the characteristics of the demand. However, when the excess inventories resulting from inaccurate demand forecasts were included (100% for nine warehouses and 15 for one central warehouse), the results obtained with both methods (simulation and optimization) were similar for all variants. It can therefore be concluded that the classical formula of the "Square Root" was probably developed for such a (specific) situation. Actually, a negative consequence of decentralization of storage may be an excessive level of inventories. It is easier to forecast demand at the level of a central warehouse serving a larger market than for local markets, as evidenced by case studies of companies that have implemented such a strategy.

Finally, after completing the regression models with cost components, the total costs were calculated on selected cases of product groups (food, electronics, clothes) and the number of warehouses with the lowest total costs was found.

The models developed by the authors (simulation and regression) and simulations and calculations carried out with their use allow the identification of several interesting relationships. Optimizing the size of a distribution network can actually have a significant impact on a company's profitability. Cost savings ranged from −24.3% to −37.5% for Gaussian distributions and were even greater for Gamma, from −33.7% to −59.3%. However, as it turned out, centralization may not necessarily be the optimal strategy for the most expensive goods (high inventory maintenance costs and lost sales). Loading parameters are also important, as they have a significant impact on the costs of storage and, above all, transport.

The results of the simulations proved that it is profitable to conduct research on methods of optimization of the distribution network. However, instead of creating a complex mathematical model that would reflect in great detail the complex interconnecting relationships, parameters of logistics processes, and the efficiency of these processes, regression models can be used.

7. Recommendations

It is difficult to state at the present stage of the research what is the actual usefulness of the "Square Root law" and whether it can be modified. Perhaps several formulas should be developed for, e.g., different demand probability distributions. The simulations were performed for various standard deviations of demand. It can be seen that the type of the demand distribution is an important factor. Therefore, the authors suggest that extensive research should be conducted on the variability and predictability of demand for various products in different markets. This problem in itself, namely, what the actual distribution of the demand is for different types of consumer goods, is very interesting, not only in connection with the problem of optimizing the distribution network. Knowledge on this subject would be useful in the development of optimization or simulation models to solve various problems in the field of logistics.

Another aspect that should be considered is the quantity of an order. In the simulations carried out by the authors, it was assumed that the goods are sent to distribution warehouses immediately after production, so the delivery is sized to the daily demand. If they were first collected prior to shipment in order to be consolidated, then the sizes of shipments should also be included in the regression model.

Secondly, the model should be able to take into account excess stocks resulting from wrong forecasts in the case of a decentralized distribution network. The share of this type of inventory, however, can only be assessed by a given user, because it is an individual matter in a given company, resulting, e.g., from the specificity of a given product or the market in which a given company operates, as well as from ineffective methods of inventory control.

The authors would like to emphasize strongly that the idea of their study was not to criticize the "Square Root" formula. The simulation or regression methods have the advantage that it allows taking into account various factors and calculating the total inventory (i.e. the size of the delivery sizes and excess inventory), and not only the safety stock, as it is in the "Square Root" formula. The law of the "square root", however, was very important in the history of science, because it drew attention to a very interesting relationship between the number of warehouses and the level of stocks. It was also an inspiration for the research undertaken by the authors.

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