

Article

Seismic Active Earth Pressure of Limited Backfill with Curved Slip Surface Considering Intermediate Principal Stress

Hui Liu , Dezhi Kong *, Wensong Gan and Bingjie Wang

School of Civil Engineering and Architecture, Henan University, Kaifeng 475004, China; liuhui_1102@126.com (H.L.); ganwen_song@126.com (W.G.); wangbingjie0511@163.com (B.W.)

* Correspondence: 10160005@vip.henu.edu.cn

Abstract: The traditional method for seismic earth pressure calculation has certain limitations for retaining structures under complex conditions. For example, when the soil width is small, the results obtained by the traditional method will be much larger. Therefore, this paper assumes that the soil slip surface is a logarithmic spiral. Based on the plane strain unified strength theory formula, while also considering the soil arching effects and tension cracks, the analytical solutions of the lateral earth pressure coefficient and the active earth pressure under the earthquake action were deduced. The mechanism and distribution of seismic active earth pressure with limited width were discussed in terms of some relevant parameters. The results indicated that the seismic active earth pressure presented a “convex” nonlinear distribution along the retaining structure. As the contribution of the intermediate principal stress increased, the strength limit of the material was effectively utilized, and the earth pressure was reduced by 22.96%. The resultant force increased as the horizontal seismic coefficient increased. However, this effect was no longer evident when the wall–soil friction angle was close to the internal friction angle. The resultant force action point increased with the wall–soil friction angle, and it should be noted that $h_a > H/3$ was true when $\delta/\varphi_0 > 0.55$. Finally, by drawing a comparison with previous studies, we verified that the method proposed in this paper is reasonable and can provide a new idea for subsequent 3D seismic earth pressure research.



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Keywords: active earth pressure; intermediate principal stress; seismic; curved slip surface; limited backfill

1. Introduction

With the development of construction technology and the increase of underground building density, the soil behind deep foundation pit retaining structures will be restricted by many other types of underground structures. For example, in an existing deep basement, underground pipe gallery, or subway station, the soil is contained within a limited width. Earthquake is also a factor that must be considered. Retaining structures in high-intensity seismic areas are constantly facing the severe test of earthquake damage. Therefore, for the safety and economy of the retaining structure, the accurate estimation of the seismic earth pressure of the limited backfill is very important. Traditional Rankine [1], Coulomb [2], and Mononobe–Okabe [3,4] theories are based on semi-infinite space. The slip surface is assumed as a plane, and the effects of the intermediate principal stress and soil arching effects are not considered. Some scholars have also realized this problem, and many theoretical and experimental studies have been carried out.

At present, the earth pressure distributions obtained by some theories commonly used in engineering are linear. However, related experimental and theoretical studies [5,6] have shown that the distribution is actually nonlinear. For soil with limited width, the magnitude and distribution of earth pressure given by some experiments and numerical studies [7–9] are also quite different from the traditional earth pressure theory. Tang and Chen [10] established a general calculation model and obtained an analytical solution for limited backfill. Xie et al. [11] studied the active earth pressure of the retaining wall near

the rock face and proposed an analytical solution which can more accurately predict the earth pressure of the retaining wall near the rock face. Moreover, the influence of soil arching cannot be ignored. Since the earliest discovery of the arching effect by Terzaghi [12], many scholars have introduced soil arching effects into the calculation of earth pressure. Handy [13] proposed the lateral earth pressure coefficient and a formula for the nonlinear active earth pressure. Take and Valsangkar [14] conducted a series of centrifugal tests on the earth pressure of a retaining wall with a limited backfill width. Paik and Salgado [15] derived a method for the calculation of the lateral pressure coefficient based on soil arching effects. Zhao and Zhu [16] introduced the soil arching effects into limited backfill, and the results obtained were closer to the numerical analysis, which has a certain engineering reference value.

As mentioned earlier, traditional earth pressure theories are based on the assumption of a planar fracture surface. However, some studies [17,18] have shown that the slip surface of limited backfill is actually closer to a curve. Yang et al. [19,20] and He et al. [21] conducted experimental and theoretical studies of the limited backfill slip surface. The results showed that the slip surface is a logarithmic spiral, and the theoretical results were in good agreement with the experimental results. Seismic action must be paid sufficient attention. Based on Rankine's theory, Iskander et al. [22] analyzed the seismic active earth pressure of cohesive soil in semi-infinite space using a pseudo-static method. The rationality of the method was verified by comparison with previous studies. Yang and Zhang [23] used the model of the combination of logarithmic spiral and tension cracks. The energy method was used to analyze the seismic active earth pressure under different crack types. In some statistics about the damage of supporting structures, the influence of earthquake action is very significant [24,25]. However, there are few studies on the seismic earth pressure of limited backfill.

Although the above works carried out studies on different aspects of earth pressure, they are all based on the traditional Mohr–Coulomb theory—that is, the effect of intermediate principal stress is ignored. The soil behind the wall is actually in a three-dimensional (3D) stress state. Li et al. [26,27] studied the active earth pressure coefficient in the 3D state and pointed out that consideration of the 3D effect of the soil will produce a more economical solution. Since the unified strength theory [28–30] was proposed, the important influence of intermediate principal stress on material failure has been considered. Zhang et al. [31] proposed an improved Rankine earth pressure formula based on the unified strength theory. The results were lower than those obtained using the classic Rankine theory, indicating that the unified strength theory can further exert the strength potential of materials. Jiao et al. [32] introduced the intermediate principal stress into the calculation of the semi-infinite earth pressure. The soil arching effects were considered. The results showed that the proposed method could effectively reduce material consumption and save engineering costs. Shao et al. [33] considered the relationship between the residual strength effect and the strength parameters of unsaturated loess, established the seismic passive earth pressure formula of the loess filling based on the modified plane strain unified strength theory.

In summary, the current research on seismic earth pressure mainly has the following deficiencies: (1) The influence of intermediate principal stress is ignored; (2) research on limited backfill needs to be supplemented; (3) most of the existing research is still based on the assumption of a planar slip surface; and (4) there is little research on cohesive soil. Therefore, on the basis of previous studies, this paper assumes that the slip surface is a curve, considering the influence of intermediate principal stress, soil arching effects, and tension cracks. The active lateral pressure coefficient and the earth pressure formula of limited backfill under earthquake action are deduced, and the sensitivity of the main parameters is analyzed through calculation examples.

2. Unified Strength Theory Formula under Plane Strain State

The intermediate principal stress has an important influence on the failure of different materials. The twin-shear unified strength theory was proposed, containing all the stress components acting on the orthogonal octahedron element and different effects on material failure [30]. In addition, it can adapt to geotechnical materials with different tensile and compressive strengths and related to hydrostatic stress, and makes up for the deficiencies of the Mohr–Coulomb theory. This theory collects a series of failure criteria, covering all areas between the upper and lower limits, and has good applicability to different materials. The principal stress form of the twin-shear unified strength theory is expressed as:

$$\begin{cases} f = \sigma_1 - \frac{\alpha(b\sigma_2 + \sigma_3)}{1+b} = \sigma_t \left(\sigma_2 \leq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \right) \\ f' = \frac{(b\sigma_2 + \sigma_1)}{1+b} - \alpha\sigma_3 = \sigma_t \left(\sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \right) \end{cases}, \quad (1)$$

$$\alpha = \frac{\sigma_t}{\sigma_c} = \frac{1 - \sin \varphi_0}{1 + \sin \varphi_0}, \sigma_t = \frac{2c_0 \cos \varphi_0}{1 + \sin \varphi_0}, \quad (2)$$

where b is the intermediate principal stress weight coefficient, which can express the influence of the intermediate principal stress on the yield of the material, and $0 \leq b \leq 1$ [29]; $\sigma_1, \sigma_2, \sigma_3$ are three principal stresses; σ_t and σ_c are the material tensile and compressive stress, respectively; α is the material tension–compression ratio parameter; and c_0 and φ_0 are the soil cohesion and internal friction angle obtained from the triaxial test, respectively.

The π plane limit lines of the unified strength theory are shown in Figure 1. The unified strength theory can degenerate to more calculation criteria under different b values. The limit lines are convex when $0 \leq b \leq 1$. When $b < 0$ or $b > 1$, the limit lines are non-convex. Therefore, the unified strength theory is not limited by the traditional convexity theory. In addition to forming a series of convex limit surfaces, it can also form a series of non-convex limit surfaces, which can be flexibly adapted to various materials. The innermost is the single-shear strength theory—that is, the Mohr–Coulomb strength theory ($b = 0$). The outermost is the twin-shear strength theory ($b = 1$).

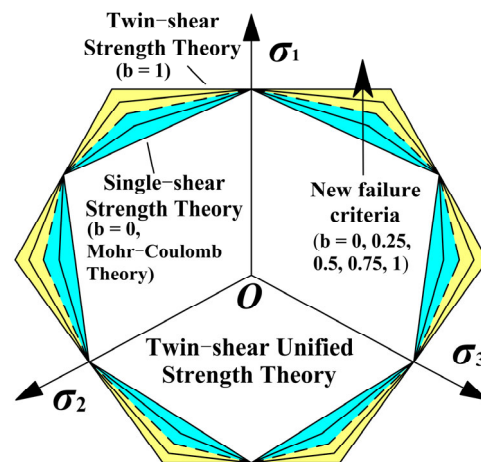


Figure 1. Yield loci of the unified strength theory.

Because the intermediate principal stress σ_2 is between the maximum principal stress σ_1 and the minimum principal stress σ_3 , the following relationship is given:

$$\sigma_2 = \frac{m(\sigma_1 + \sigma_3)}{2}, \quad (3)$$

where m is the intermediate principal stress coefficient. The elastic solution of the plane strain problem can be known: $\varepsilon_y = 0$. Therefore, m can be obtained by the generalized Hooke's law: $0 < m \leq 1$. In plane strain, volume deformation in the plastic state is ignored, that is, $m = 1$. Moreover, when $m = 1, b = 0, 0.5, 1$, the twin-shear unified strength theory

is a special case or linear approximation of Tresca, Mises, and twin-shear stress yield criteria. Different b values can reflect different intermediate principal stress effects. Therefore, $b = 0, 0.5, 1$ is taken as the representative value in subsequent analysis, combined with the influence of the intermediate principal stress. By combining Equations (1) and (2), the modified soil friction angle φ_t and cohesion c_t are determined as:

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2} \sin \varphi_t + c_t \cos \varphi_t, \quad (4)$$

$$\begin{cases} \varphi_t = \arcsin \left[\frac{2(1+b) \sin \varphi_0}{2+b(1+\sin \varphi_0)} \right] \\ c_t = \frac{2(1+b)c_0 \cos \varphi_0}{2+b(1+\sin \varphi_0)} \cdot \frac{1}{\cos \varphi_t} \end{cases}. \quad (5)$$

3. Slip Surface Equation of Limited Backfill

This paper is based on the theoretical and experimental research of Wang et al. [34] and Yang and Tang [19]. The cohesive backfill has a limited width. The slip surface of the backfill is assumed to be a logarithmic spiral, as shown in Figure 2.

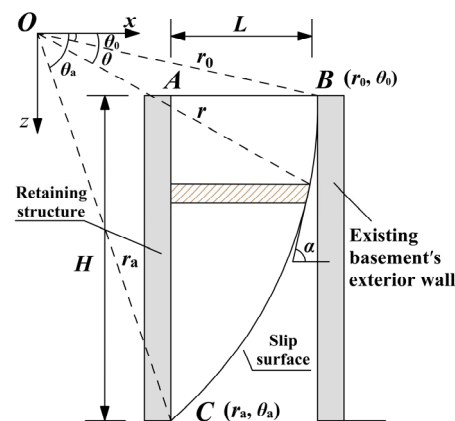


Figure 2. Sketch of logarithmic spiral slip surface.

In Figure 2, L is the retained soil width; H is the retaining structure height; α is the angle between the tangent line at any point on the slip surface and the horizontal plane; and θ and r are the logarithmic spiral angle and polar radius at any point on the slip surface, respectively. (r_0, θ_0) and (r_a, θ_a) are the polar coordinates of the pending point.

The slip surface equation is given by:

$$r(\theta) = r_0 \exp[(\theta - \theta_0) \tan \varphi_t]. \quad (6)$$

The corresponding rectangular coordinate equation can be expressed as:

$$\begin{cases} x = r_0 \exp[(\theta - \theta_0) \tan \varphi_t] \cos \theta \\ z = r_0 \exp[(\theta - \theta_0) \tan \varphi_t] \sin \theta \end{cases}. \quad (7)$$

The slope of any point on the logarithmic spiral is:

$$k = dz/dx = 1/\tan(\varphi_t - \theta). \quad (8)$$

Then, the angle between the tangent line at any point on the slip surface and the horizontal plane is:

$$\alpha = \frac{\pi}{2} + \varphi_t - \theta. \quad (9)$$

θ_a at point C can be determined as:

$$\theta_a = \frac{\pi}{2} + \varphi_t - \theta_{cr}, \quad (10)$$

where θ_{cr} is the Coulomb failure angle when the soil reaches the active limit state, and $\theta_{cr} = \pi/4 + \varphi_t/2$. At this time, the active earth pressure required is biased towards safety [20].

4. Active Earth Pressure Analysis under Seismic Action

Earth pressure analysis under seismic action is a dynamic problem. In this paper, the pseudo-static method is used to deal with the calculation of dynamic earth pressure—that is, the Mononobe–Okabe theory. The seismic inertial force is applied to the sliding soil and then combined with the sliding soil to form an equivalent weight. It is made into a static problem. In order to simplify the calculation, the following assumptions are made:

1. The soil surface is level.
2. The supporting structure is rigid, and its deformation is not considered.
3. The influence of the foundation pressure in the basement of the adjacent building is not considered [35].
4. The horizontal seismic inertial force points to the retaining structure, and the vertical seismic inertial force points upwards.
5. The effect of the earthquake on the basic mechanical properties of soil is not considered.

4.1. Seismic Active Lateral Pressure Coefficient for Cohesive Soil

Soil arching effects exist objectively [36,37], especially in limited soil and deep foundation pits. In the system under consideration in the present study, due to the lateral movement of the retaining structure and the influence of the wall–soil friction angle, the stress redistribution phenomenon will be more obvious (Figure 3a). In addition, this research is aimed at cohesive soil. Based on the research of Paik and Salgado [15], and combined with the Mohr circle for initial liquefaction given by Richards et al. [38], the stress Mohr circle when the soil behind the retaining structure reaches the active limit state under the earthquake is shown in Figure 3b.

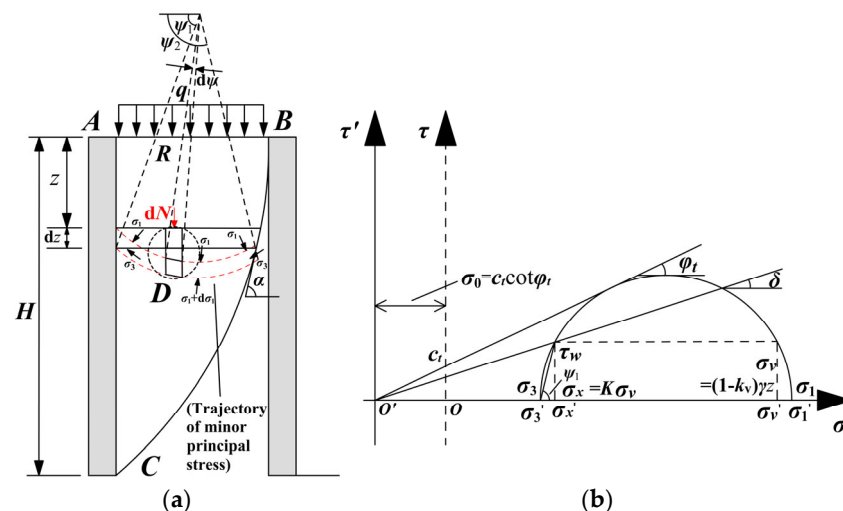


Figure 3. Arching effects in limited backfill: (a) trajectory of minor principal stress; (b) Mohr circle for stresses at the retaining structure.

In Figure 3, ψ_1 and ψ_2 are the angles between the minor principal stress surface and the horizontal plane at the retaining structure and the slip surface, respectively; R is the radius of the minor principal stress arch; z is the distance from any point inside the soil to the surface; dz is the thickness of the thin-layer element; q is the uniform surcharge on the soil surface; dN is the vertical force at any point of the minor principal stress trajectory at depth z ; τ_w is the shear stress on the retaining structure; δ is the wall–soil friction angle; k_v

is the vertical seismic coefficient; K is the lateral earth pressure coefficient; and the other parameters are as mentioned above.

From Figure 3b, the following relationship is given:

$$\sigma' = \sigma + \sigma_0 = \sigma + c_t \cot \varphi_t, \quad (11)$$

where σ and σ' are the stress values in the new and old coordinate systems, respectively.

When the soil reaches the active limit equilibrium state, combined with the stress Mohr circle, the principal stress deflection angle in the soil at the retaining structure can be obtained [15]:

$$\psi_1 = \tan^{-1} \left[\frac{1 - K_a + \sqrt{(1 - K_a)^2 - 4K_a \tan^2 \delta}}{2K_a \tan \delta} \right], \quad (12)$$

where K_a is the Rankine active earth pressure coefficient, and $K_a = \tan^2(\pi/4 - \varphi_t/2)$.

The angle between the action surface of the major principal stress and the failure surface is $\beta_f = \pi/4 + \varphi_t/2$. Therefore, the angle between the minor principal stress surface and the horizontal plane at the slip surface can be given by:

$$\psi_2 = \frac{\pi}{2} - \beta_f + \alpha = \frac{3\pi}{4} + \frac{\varphi_t}{2} - \theta. \quad (13)$$

As shown in Figure 3a, the average vertical stress acting on the thin-layer element can be obtained:

$$\bar{\sigma}'_v = \frac{\int_{\psi_1}^{\psi_2} dN}{R(\cos \psi_1 - \cos \psi_2)} = \sigma'_1 \left[1 - \frac{1 - K_a}{3} (\cos \psi_1 + \cos \psi_2 + \cos \psi_1 \cos \psi_2) \right]. \quad (14)$$

Additionally, from Figure 3b, and considering the uniform surcharge on the soil surface, the following is obtained:

$$\begin{cases} \sigma'_x = \sigma'_1 \cos^2 \psi_1 + \sigma'_3 \sin^2 \psi_1 \\ \sigma_1 = (1 - k_v)(\gamma z + q) \end{cases}, \quad (15)$$

where γ is the unit weight of the retained soil. Combining Equations (14) and (15), the seismic active lateral pressure coefficient for cohesive soil can be obtained:

$$K = \frac{\sigma_x}{\bar{\sigma}_v} = \frac{\sigma'_x - \sigma_0}{\bar{\sigma}'_v - \sigma_0} = \frac{\cos^2 \psi_1 + K_a \sin^2 \psi_1 - \frac{c_t}{[(1 - k_v)(\gamma z + q)] \tan \varphi_t + c_t}}{\frac{[(1 - k_v)(\gamma z + q)] \tan \varphi_t}{[(1 - k_v)(\gamma z + q)] \tan \varphi_t + c_t} - \frac{(1 - K_a)}{3} (\cos^2 \psi_1 + \cos^2 \psi_2 + \cos \psi_1 \cos \psi_2)}. \quad (16)$$

In order to facilitate subsequent calculations, the following relationship is given:

$$K = \frac{\sigma_x}{\bar{\sigma}_v} = \frac{\sigma'_x - \sigma_0}{\bar{\sigma}'_v - \sigma_0} = \frac{K'(\bar{\sigma}_v + \sigma_0) - \sigma_0}{\bar{\sigma}_v} = K' + \frac{(K' - 1)\sigma_0}{\bar{\sigma}_v}, \quad (17)$$

where K' is the non-cohesive soil lateral earth pressure coefficient, which can be expressed as:

$$K' = \frac{3(\cos^2 \psi_1 + K_a \sin^2 \psi_1)}{3 - (1 - K_a)(\cos^2 \psi_1 + \cos^2 \psi_2 + \cos \psi_1 \cos \psi_2)}. \quad (18)$$

The influence of tension cracks in cohesive soil is a problem that must be considered. The height of tension cracks under earthquake ($K = 0$):

$$z_c = \frac{1}{\gamma} \left[\frac{c_t}{(1 - k_v) \tan \varphi_t} \left(\frac{1}{\cos^2 \psi_1 + K_a \sin^2 \psi_1} - 1 \right) - q \right], \quad (19)$$

according to Equation (19), when $z_c < 0$, $z_c = 0$.

4.2. Establishment of Seismic Active Earth Pressure Formula

The thin-layer element method is used in this part. The selected element is shown in Figure 4. When the element thickness dz is sufficiently small, $\overline{bc}$ can be approximated as a straight line.

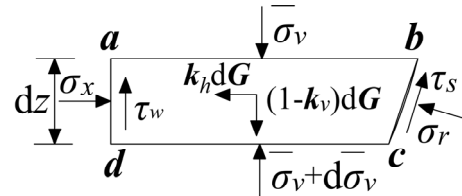


Figure 4. Horizontal thin-layer element model.

In Figure 4, $\bar{\sigma}_v$ and $\bar{\sigma}_v + d\bar{\sigma}_v$ are the average vertical stress acting on the top and bottom of the thin-layer element, respectively; k_h and k_v are the horizontal and vertical seismic coefficients, respectively; dG is the self-weight of the thin-layer element; σ_x is the horizontal reaction force of the retaining structure; σ_r is the reaction force of the lower soil body to the upper soil; and τ_s is the shear stress at the slip surface.

According to Figures 2 and 4, the geometric relationship of the element is:

$$\begin{cases} \overline{ab} = r_0 \{ \exp[(\theta - \theta_0) \tan \varphi_t] \cos \theta - \exp[(\theta_a - \theta_0) \tan \varphi_t] \cos \theta_a \} \\ \overline{cd} = \overline{ab} - \cot \alpha dz; \overline{ad} = dz; \overline{bc} = \csc \alpha dz \end{cases} \quad (20)$$

The self-weight of the element is (higher-order infinitesimals omitted):

$$dG = \gamma S_{abcd} = \gamma \overline{abd} dz. \quad (21)$$

The shear stresses on both sides of the element are taken as:

$$\begin{cases} \tau_w = c_a + \sigma_x \tan \delta = (c_t \cot \varphi_t + \sigma_x) \tan \delta \\ \tau_s = c_t + \sigma_r \tan \varphi_t \end{cases}, \quad (22)$$

where c_a is the adhesion between the retaining structure and soil.

The stress equilibrium equation in the horizontal and vertical directions of the thin-layer element are:

$$\begin{cases} \sigma_x \overline{ad} + \tau_s \cos \alpha \overline{bc} = k_h dG + \sigma_r \overline{bc} \sin \alpha \\ \bar{\sigma}_v \overline{ab} + (1 - k_v) dG = (\tau_s \sin \alpha + \sigma_r \cos \alpha) \overline{bc} + (\bar{\sigma}_v + d\bar{\sigma}_v) \overline{cd} + \tau_w \overline{ad} \end{cases} \quad (23)$$

By combining Equations (20)–(23) and with higher-order infinitesimals omitted, the following differential equation is obtained:

$$\frac{d\bar{\sigma}_v}{dz} + A_1 \bar{\sigma}_v = A_2, \quad (24)$$

where

$$\begin{cases} A_1 = \frac{K'(\tan \delta + \tan \theta) - \tan(\theta - \varphi_t)}{\overline{ab}} \\ A_2 = (1 + k_h \tan \theta - k_v) \gamma - \frac{c_t \{ 1 + K' \cot \varphi_t (\tan \delta + \tan \theta) + \tan \theta [\tan(\theta - \varphi_t) - \cot \varphi_t] \}}{\overline{ab}} \end{cases} \quad (25)$$

Equation (24) is solved with the following boundary condition, $\bar{\sigma}_v|_{z=z_c+r_0 \sin \theta_0} = (1 - k_v)(\gamma z_c + q)$, yielding:

$$\bar{\sigma}_v = B \exp[A_1(z_1 - z)] + A_2/A_1, \quad (26)$$

where

$$\begin{cases} B = (1 - k_v)(\gamma z_c + q) - A_2/A_1 \\ z_1 = z_c + r_0 \sin \theta_0 \end{cases} \quad (27)$$

In summary, the distribution formula of the seismic active earth pressure can be expressed as:

$$\sigma_x = K\{B \exp[A_1(z_1 - z)] + A_2/A_1\}. \quad (28)$$

4.3. Resultant Force and Its Action Point

By combining Equations (16) and (26), the horizontal active earth pressure resultant force can be obtained as:

$$\begin{aligned} E_x &= \int_{z_1}^{z_2} K \bar{\sigma}_v dz = \int_{z_1}^{z_2} K' \bar{\sigma}_v + (K' - 1) \sigma_0 dz \\ &= \frac{K'}{A_1} \{B[1 - \exp[A_1(z_1 - z_2)]] + A_2(z_2 - z_1)\} + (K' - 1)(z_2 - z_1) \sigma_0 \end{aligned} \quad (29)$$

where $z_2 = r_a \sin \theta_a$; θ can be determined as $\theta \in (\theta_0, \theta_a)$. Using the optimization algorithm to find the number that makes E_x achieve the maximum value is the required θ , and the horizontal active earth pressure resultant force can also be obtained accordingly.

The total overturning moment of the retaining structure can be expressed as:

$$\begin{aligned} M &= \int_{z_1}^{z_2} K \bar{\sigma}_v (z_2 - z_1) dz = \int_{z_1}^{z_2} [K' \bar{\sigma}_v + (K' - 1) \sigma_0] (z_2 - z_1) dz \\ &= \frac{K'}{A_1} \left\{ \left[B + \frac{A_2(z_2 - z_1)}{2} \right] (z_2 - z_1) - \frac{B[1 - \exp[A_1(z_1 - z_2)]]}{A_1} \right\} + \frac{(K' - 1)(z_2 - z_1)^2 \sigma_0}{2} \end{aligned} \quad (30)$$

The resultant force and its action point are respectively determined as:

$$E_a = E_x / \cos \delta, \quad (31)$$

$$h_a = M / E_x. \quad (32)$$

Using the formula in this paper to calculate the active earth pressure of limited backfill, the main calculation process diagram is shown in Figure 5.

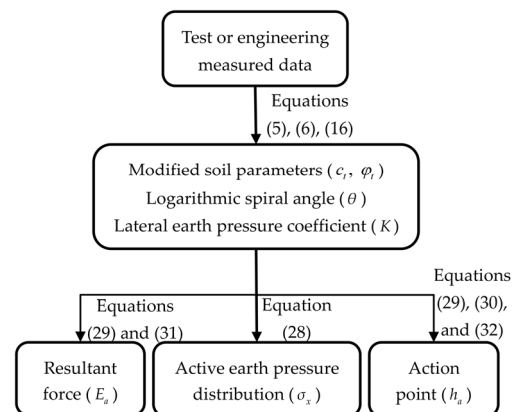


Figure 5. Flow chart of active earth pressure calculation.

5. Parametric Sensitivity Analysis

Relevant important parameters are discussed according to the listed calculation route. In the following analysis, the basic parameters of the soil were uniformly selected as unit weight $\gamma = 18 \text{ kN/m}^3$, cohesion $c_0 = 10 \text{ kPa}$, internal friction angle $\varphi_0 = 20^\circ$, retaining structure height $H = 10 \text{ m}$, soil width $L = 3 \text{ m}$, uniform surcharge $q = 15 \text{ kPa}$. We mainly discuss the influence of intermediate principal stress weight coefficient b , horizontal and vertical seismic coefficients k_h and k_v , wall–soil friction angle δ , and soil width L on the earth pressure distribution, lateral earth pressure coefficient, active earth pressure resultant force, and its action point.

5.1. Active Earth Pressure Distribution

We assumed that the wall–soil $\delta = \varphi_0/2$, and that the intermediate principal stress weight coefficient $b = 0, 0.5, 1$. According to the Specification of Seismic Design for Highway Engineering (JTG B02-2013) [39], the following parameter values were set: horizontal seismic coefficient $k_h = 0, 0.2$ (level VIII), 0.4 (level IX); vertical seismic coefficient $k_v = 0, 0.5k_h, k_h$. The horizontal seismic active earth pressure distributed along the height of the retaining structure is shown in Figure 6. Horizontal active earth pressure and depth are dimensionless.

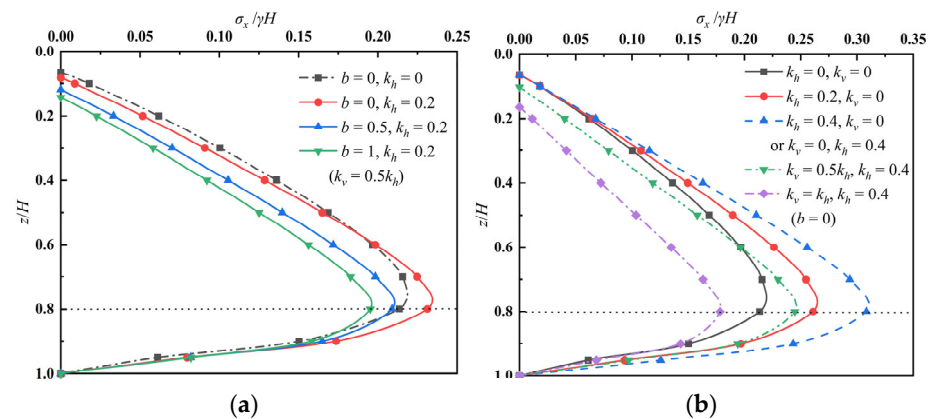


Figure 6. Distribution of horizontal seismic active earth pressure: (a) $b - \sigma_x$; (b) $k_h - \sigma_x$ and $k_v - \sigma_x$.

Figure 6a shows the horizontal active earth pressure distribution under different intermediate principal stress weight coefficients. Where $k_v = 0.5k_h$, it can be seen that as the b values increased, the horizontal active earth pressure gradually decreased. It is worth noting that in the working condition without earthquake action (i.e., $k_h = 0, k_v = 0$), the intermediate principal stress was not considered. The obvious difference indicates that the seismic action significantly influenced the earth pressure compared with other working conditions. In addition, it can be seen from Figure 6b that when $b = 0$, the horizontal active earth pressure gradually increased with the horizontal seismic coefficient, and the trend of non-linearity increased sharply. The main gaps were concentrated in the middle and lower parts of the retaining structure. On the contrary, as the vertical seismic coefficient increased, the horizontal active earth pressure demonstrated a clear and gradual decreasing trend. Under several working conditions, the distribution of active earth pressure presented the same trend, and the decreasing points were all near $0.8H$.

5.2. Lateral Seismic Active Earth Pressure Coefficient

The lateral seismic active earth pressure coefficient is the key to calculating the earth pressure, which is affected by many factors. That is:

$$K = f(c_0, \varphi_0, \delta, k_v, z, \gamma, q). \quad (33)$$

In this study, $\delta = \varphi_0/2$; $b = 0, 1$. The change curves of K under different depths z and vertical seismic coefficient k_v are shown in Figure 7. It can be seen that under several working conditions, along the depth direction, K showed a sharp increasing trend in the middle and upper parts of the retaining structure, and then the increase rate tended to become flat. Affected by the tension cracks, K had a negative value within a certain range of the soil surface, and the overall coefficient was smaller than the Rankine earth pressure coefficient. In addition, K gradually decreased with the increase of b and k_v . When $k_v = 0.4$, the reduction was the most obvious.

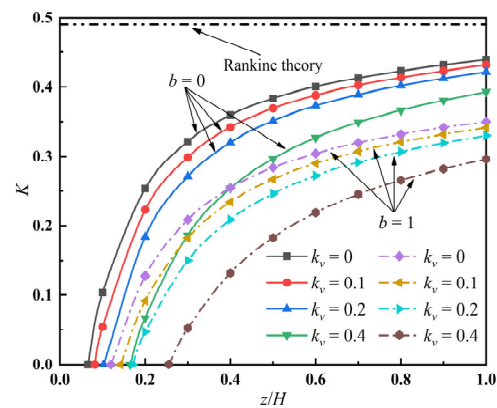


Figure 7. Influence of z and k_v on K .

5.3. Active Earth Pressure Resultant Force

In this study, $b = 0, 1$; $\delta \leq \varphi_0$. The change curves of resultant force E_a under different wall–soil friction angles δ and horizontal and vertical seismic coefficients k_h, k_v are shown in Figure 8. It can be seen from Figure 8a that as b and δ increased, E_a gradually decreased. Under the same earthquake conditions, the resultant earth pressure of $b = 1$ was lower than that of $b = 0$ by 22.96%. On the contrary, E_a increased as k_h increased. However, with the increase of δ , this amplitude gradually decreased. When δ was close to the internal friction angle φ_0 , the influence of k_h on E_a was no longer very obvious. In Figure 8b, E_a had the same changing trend as b, δ , and k_v increased—that is, it gradually decreased in all cases.

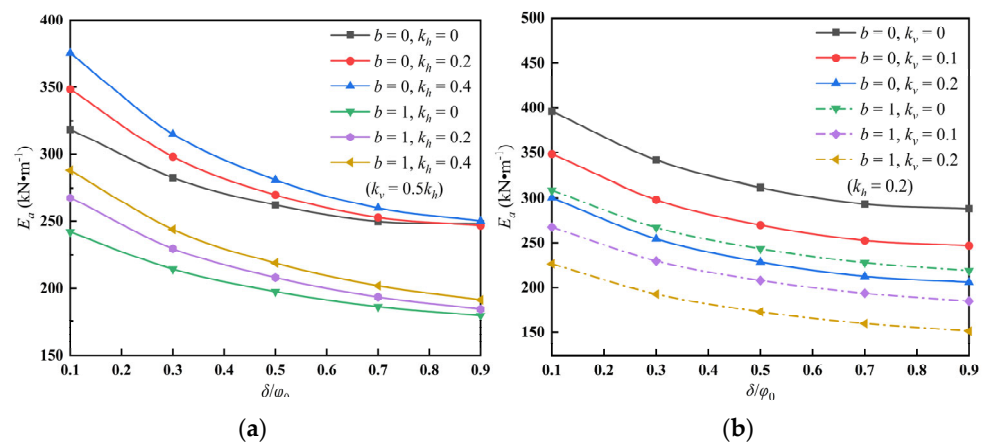


Figure 8. Influence of δ and k_h, k_v on E_a : (a) $\delta - E_a$ and $k_h - E_a$; (b) $\delta - E_a$ and $k_v - E_a$.

5.4. Resultant Force Action Point

We assumed that $b = 0, 0.5, 1$. The change curves of the resultant force action point h_a/H under different horizontal and vertical seismic coefficients k_h, k_v ($\delta = \varphi_0/2$) and wall–soil friction angles δ ($\delta \leq \varphi_0$) are shown in Figure 9. It is clear from Figure 9a,b that h_a/H decreased with increasing b, k_h , and k_v . The decreasing trend was approximately linear with increasing k_h , while a nonlinear trend was observed for increasing k_v . When $b = 0$, the initial reduction points of the two working conditions were located near the traditional earth pressure resultant point $H/3$. In Figure 9c, unlike the seismic coefficient, h_a/H gradually increased as δ increased. When δ was small, the resultant force action points under several working conditions were less than $H/3$. However, when $b = 0$ and $\delta/\varphi_0 > 0.55$, situations where the resultant force action points were greater than $H/3$ began to appear.

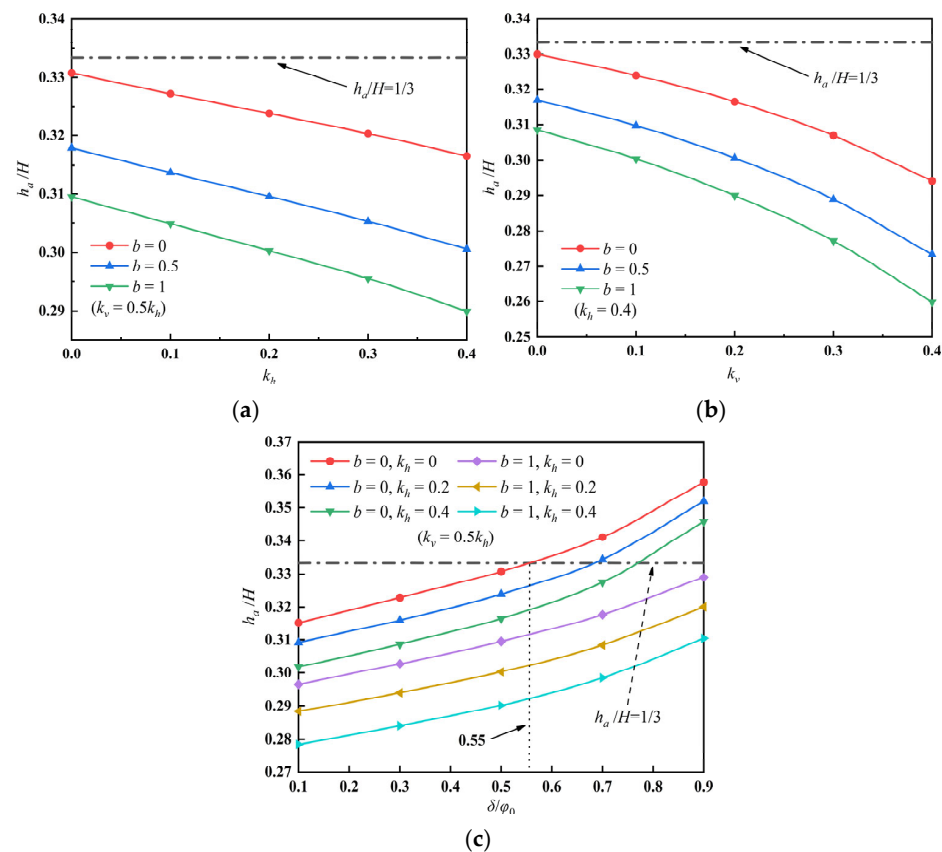


Figure 9. Influence of k_h , k_v , and δ on h_a : (a) $k_h - h_a$; (b) $k_v - h_a$; (c) $\delta - h_a$.

6. Comparison and Verification

In order to further illustrate the applicability of the method in this paper, the research results of Lin et al. [40] were selected for comparison. The relevant parameters were assumed to be: $b = 0, 0.5, 1$; $\gamma = 18 \text{ kN/m}^3$; $c_0 = 10 \text{ kPa}$; $\varphi_0 = 20^\circ$; $H = 10 \text{ m}$; $L = 3 \text{ m}$; $q = 15 \text{ kPa}$; $k_h = 0.2$, and $k_v = 0.1$. The horizontal earth pressure distributions of several methods are shown in Figure 10. The traditional Mononobe–Okabe method leads to a linear distribution, while the results of Lin et al. [40] and the method in this paper both demonstrate nonlinear distributions. At the same time, based on the results of shaking-table tests of seismic earth pressure on retaining walls under different foundation conditions by Qu and Zhang [24], it is found that a “convex” nonlinear distribution in the lower part of the retaining structure is realistic. It can also be seen that the results obtained by the traditional Mononobe–Okabe method were much larger than the results of this paper and those of Lin et al. [40]. Compared with the results obtained by Lin et al. [40], the main gap was concentrated in the lower part of the retaining structure. This may be explained by the fact that the intermediate principal stress and soil arching effect were not considered, and because the linear slip surface was used simultaneously. However, both displayed the same trend, and the earth pressure reduction point was near $0.8H$, which is consistent with the previous analysis. This also shows that the method proposed in this paper is reasonable.

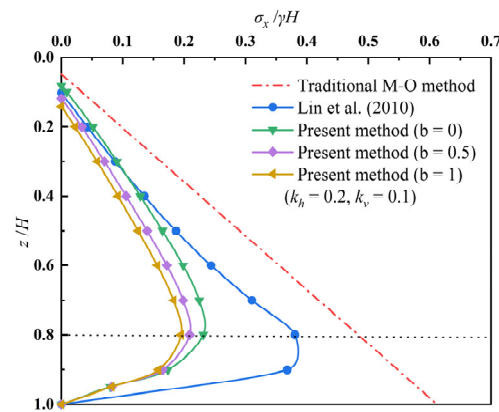


Figure 10. Comparison of active earth pressure in previous studies.

7. Discussion

This study is the first attempt to apply the twin-shear unified strength theory to calculate the seismic active earth pressure under limited backfill. The influence of a curved slip surface is also considered, which has very important theoretical significance for the seismic earth pressure of limited backfill and the subsequent 3D earth pressure research. In the parameter sensitivity and comparative analysis, the earth pressure distribution form, the intermediate principal stress, the seismic coefficient, and the wall–soil friction angle were discussed.

The horizontal earth pressure presented a “convex” nonlinear distribution, which is consistent with the measured results obtained from the shaking-table test of a retaining structure by Qu and Zhang [24]. Lin et al. [40] also carried out theoretical research on seismic active earth pressure and obtained the same distribution form based on a semi-infinite space assumption. However, these previous studies did not comprehensively consider the effects of limited backfill width, intermediate principal stress, and soil arching effects. Therefore, there are still some obvious deviations between the results obtained in this research and previous studies, mainly in the middle and lower parts of the retaining structure. Nevertheless, the overall trend is consistent, and they are all different from the linear distribution of the classical Mononobe–Okabe theory [3,4] or a simple extension of it [22], indicating that the method proposed in this paper is reasonable. The seismic active earth pressure gradually decreased with the intermediate principal stress weight coefficient, by up to 22.96%. This indicates that the intermediate principal stress has a positive effect on further exerting the strength limit of the material, which presents good agreement with previous earth pressure studies considering the intermediate principal stress [32,33]. The lateral earth pressure coefficient K deduced in this paper was affected by the vertical seismic coefficient k_v . K gradually decreased with k_v , which is different from the research by Yang and Zhang [23] on the influence of the horizontal seismic coefficient on the earth pressure coefficient. The resultant force action point h_a increased nonlinearly with the wall–soil friction angle δ , which also indirectly indicates that h_a is significantly affected by the soil arching effects. It is worth noting that h_a began to be greater than the classic resultant force action point $H/3$ when $\delta/\varphi_0 > 0.55$. The anti-overturning stability should be strengthened in retaining structure design.

It should be pointed out that the earth pressure is an approximate calculation. Regardless of which theory is adopted, there will still be a substantial difference between the theoretical value and the actual situation. In addition, the method proposed in this paper is a generalized model. From the perspective of safety, it assumes that the logarithmic spiral angle at the bottom of the retaining structure is $\theta_a = \pi/2 + \varphi_t - \theta_{cr}$, which makes the formula in this paper have certain limitations—that is, the critical width of the limited backfill under the curved slip surface cannot be given. At present, measurement data on the seismic earth pressure of soil with a limited width is very scarce. The formulas in this study need to be further verified and improved by many experiments in the future.

8. Conclusions

In soil with limited width, it is valuable to explore the action mechanism and distribution characteristics of active earth pressure under the action of intermediate principal stress and earthquake. Based on previous studies, the default slip surface is a logarithmic spiral. This study was carried out under complex conditions based on the plane strain twin-shear unified strength theory formula and considering the effects of soil arching and tension cracks. Three conclusions are summarized as below:

Due to the contribution of the intermediate principal stress, the lateral earth pressure coefficient, the horizontal active earth pressure, the resultant force, and the action point gradually decreased with the increase of the intermediate principal stress weight coefficient b . When $b = 1$, the strength potential of the material could be increased by 22.96% at most under the same seismic conditions. Therefore, reasonable consideration of the influence of the intermediate principal stress can reduce the consumption of materials to a certain extent, which has considerable economic benefits in engineering.

The effect of earthquake on earth pressure is very significant. As the horizontal seismic coefficient k_h increased, the active earth pressure σ_x and the resultant force E_a gradually increased, while the action point h_a gradually decreased. As the vertical seismic coefficient k_v increased, the lateral earth pressure coefficient K gradually decreased—when $k_v = 0.4$, the reduction of K was the most obvious. In addition, as the wall–soil friction angle δ increased, the soil arching effects became increasingly obvious, E_a gradually decreased, and h_a gradually increased. It is worth noting that in the calculations in this paper, $h_a > H/3$ when $b = 0$ and $\delta/\varphi_0 > 0.55$. At this time, it is necessary to strengthen the anti-overturning stability when designing retaining structures.

The pseudo-static method was used in this study to deal with the dynamic earth pressure under earthquake action when soil has a limited width, which is equivalent to a further improvement of the Mononobe–Okabe method. Through comparison with previous studies, we found that both methods presented a “convex” nonlinear distribution along the retaining structure, and the decreasing points were all near $0.8H$. This indicates that the method proposed in this paper is reasonable and can provide a new approach for the subsequent study of 3D seismic earth pressure considering the intermediate principal stress.

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