

Article

Micromechanism of the Breakage of Two Spherical Gypsum Particles under Normal–Tangential Contact Conditions

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Featured Application: This paper provides a theoretical basis for the calculation of particle breakage at weak points of coarse-grained soil materials.

Abstract: Contact breakage of particles makes a large difference in the strength of coarse-grained soils, and exploring the characteristics within the process of the breakage is of great significance. Ignoring the influence of particle shape, the micromechanism of two spherical particles breaking under normal–tangential contact conditions was investigated theoretically and experimentally. Through theoretical analysis, the breakage form, the shape and size of the conical core, and the relationship between the normal and tangential forces at crushing were predicted. Particle contact tests of two gypsum spheres were carried out, in which the breakage forms, features of the conical cores and the normal and tangential forces at crushing were recorded for comparison with the predicted values. The test results and the theoretical predictions showed good agreement. Both the analysis and test demonstrate that the presence of tangential forces causes the conical core to assume the shape of an oblique cone, and the breakage form to change. Moreover, with increasing normal contact force, the tangential force needed for crushing increases gradually first and then decreases suddenly.

Keywords: particle contact test; coarse-grained soil; normal–tangential contact; conical core



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1. Introduction

Coarse-grained soils, such as gravel, are used in many fields, including earth dams, harbor facilities, nuclear power waste storage areas, and roadbed materials [1]. As a result of the large particle size, coarse-grained soils, in which the particle sizes are in the range of some cm to some dozen cm, usually have different mechanical properties compared with other soils. Since the 1960s, scholars worldwide have used theoretical analysis [2–7], laboratory tests [8–11] and numerical simulations [12–17] to study the strength and deformation characteristics of coarse-grained soils. In recent years, high-fill engineering projects have become increasingly common due to the construction of dams or embankments for infrastructures in mountainous regions. The problem of particle fragmentation in such projects, which directly affects the reliability of the overall structure, has attracted the attention of a growing number of academics [9,14,18–20].

Common methods used to study the fragmentation of coarse-grained soil particles include the shear test [11,21–25], the triaxial test [9,26–28], the uniaxial compression test [24,29] and the particle contact test [30–32], in which the first three methods are all about the effect of particle breakage on soil properties. The particle contact test was first proposed by Zhou [33], and has been increasingly adopted by scholars to study the mechanical properties of the breakage of a single particle, as research on the macromechanical properties of soils due to particle fragmentation has hit a bottleneck. There are

many factors that influence the crushing characteristics of particles, such as the particle shape [34], material [35], contact form [30–32] and force state [34,36], in which the effect of particle shape has a strong uncertainty due to its randomness. The research on the features of a single particle in the process of contact breakage is still in the initial stages, and some scholars start from the simplest spherical particles to explore the micromechanism of particle breakage. For instance, using a statistical approach, Zhou [35] investigated the spherical particle crushing patterns of three materials, sandstone, limestone and slate, and derived empirical equations for the conical core size and the critical normal force at overall crushing. Through a combination of experiments and numerical simulations, Yu [37] studied the micromechanism of ball–plane normal contact breakage of spherical particles. Current studies on particle contact breakage have mainly focused on experimental studies and numerical simulations, with relatively little theoretical research on this problem.

Theoretical analysis of the internal stress state of spherical rock particles subjected to external forces has been carried out since the 1950s [3,38–41]. Based on the Hertz contact model, many scholars have proposed various models to solve different contact problems, such as Mindlin and Deresiewicz [42]. One of the most popular tests is the compression test of spheres between a pair of diametrical concentrated loads, and the mechanical model proposed by Hiramatsu and Oka [43] has been applied by various authors [40,44–47]. To date, most of the experimental studies on the breakage mechanism of brittle spherical rock materials subjected to external forces have been based on this model. It assumed the brittle rock sphere as an isotropic elastic material and equated loads acting on both ends of the sphere as uniform loads acting on a small area pointing to the center of the sphere, which does not change with the loading. However, during the contact crushing of two spherical particles, local compaction occurs at the contact point due to the stress concentration, resulting in the emergence of a circular plane, which causes the boundary conditions to change as the contact process progresses. The model of a sphere squeezed by a symmetrical pair of concentrated loads cannot be applied to the conditions of contact crushing between two spherical particles since the mechanism of the whole crushing process cannot be revealed. In order to reveal the formation mechanism of the conical core, the Hertz contact model is not suitable for this experiment since it is mainly used to solve static stress distribution in an elastic body, while the formation process of the conical core is a plastic deformation process.

According to the numerical simulation by Yu [37], Niu [48] divided the process of particle contact breakage into three stages—local compaction, elastic deformation and integral crushing—and gave solutions to the force–displacement curve, the conical core size and the critical force at fracture under the condition of ball–ball normal contact, in which only the normal force existed at the contact point. Nevertheless, in actual conditions, there are not only normal forces but also tangential forces at the contact point of two particles. Hence, it is necessary to explore the mechanism of contact fracture between two spherical particles under normal–tangential contact conditions.

In this study, the particle breakage of two spheres under normal–tangential contact conditions is discussed theoretically and experimentally. In the theoretical analysis, the breakage form, the shape and size of the conical core, and the relationship between the normal and tangential forces at failure are studied on the basis of the normal contact test [48]. Then, particle contact tests of two gypsum spheres under normal–tangential contact conditions are carried out to make a comparison with the theory. The theoretical research and experimental results in this paper are in good agreement. This study summarizes the variation rules of the breakage form and the conical core characteristics in different force states, and gives a solution to the normal and tangential forces required for particle crushing.

2. Theoretical Analysis

In this section, we derive a new solution applicable to the situation of normal–tangential contact based on the theoretical analysis of the normal contact test [48]. In

this paper, we assume that the spherical particles are of isotropic brittle material. Before the analysis, the results of the normal contact test are reported here briefly.

2.1. Normal Contact

Niu [48] divided the process of the normal contact breakage of two spherical particles into three stages—local compaction, elastic deformation and integral crushing (see Figure 1)—and within this process, the relationship between the normal force F_n and the displacement of contact point s is as follows:

$$F_n = \begin{cases} \frac{\pi}{3}\delta R s^3 - \frac{\pi}{12}\delta s^4, & (0 < s \leq s_0) \\ \frac{\pi E \sqrt{R^2 - (R - s_0)^2}}{2(1 - \nu^2)}(s - s_0), & (s > s_0 \text{ and } F_n \leq F_{cr}) \end{cases} \quad (1)$$

where δ , E and ν are hardness or crushing modulus, Young's modulus and Poisson's ratio of the material, respectively; R is the radius of the sphere; F_{cr} is the normal force when breakage occurs under normal contact conditions; s_0 is the critical displacement separating the local compaction stage and the elastic deformation stage, and in the following:

$$s_0 = \sqrt[3]{\frac{\alpha}{R} + \sqrt{\frac{\alpha^2}{R^2} + \frac{\alpha^3}{27R^6}}} + \sqrt[3]{\frac{\alpha}{R} - \sqrt{\frac{\alpha^2}{R^2} + \frac{\alpha^3}{27R^6}}}, \quad (2)$$

$$\alpha = \left[\frac{E}{2\delta(1 - \nu^2)} \right]^2 \quad (3)$$

α is a dimensionless parameter determined by Young's modulus, Poisson's ratio and hardness modulus, which has a powerful effect on the size of the conical core.

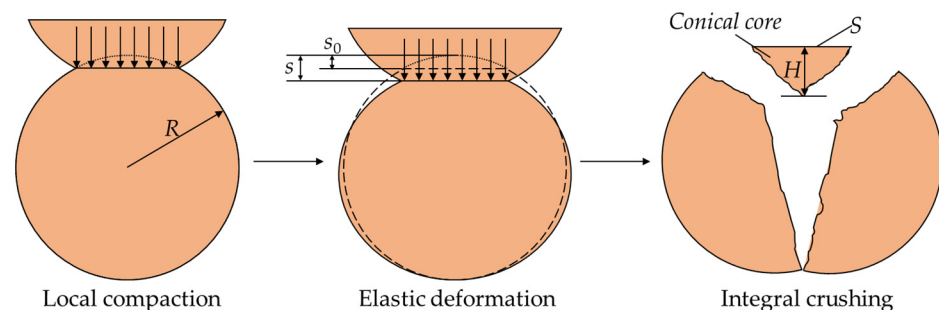


Figure 1. A sketch of the contact crushing process of spherical particles under normal contact conditions. S and H are the bottom area and the height of the conical core, respectively.

In the stage of local compaction, due to stress concentration, the contact point turns into a circular compacted flat region that gradually gets larger with the generation and gradual increase in the contact pressure, while the elastic deformation of the whole sphere is so small to be neglected. As the elastic deformation stage starts, the local compacted region no longer enlarges and all the displacements of the contact point come from the elastic deformation of the whole sphere until the breakage occurs. The solutions of the conical core size are [48], as follows:

$$S = \pi \left[R^2 - (R - s_0)^2 \right], \quad (4)$$

$$H = \sqrt{R^2 - (R - s_0)^2}. \quad (5)$$

2.2. Normal–Tangential Contact

Unlike the normal contact breakage, the breakage under normal–tangential contact conditions needs to be discussed in two cases—the high–normal force level and the low–normal force level. For the case of high–normal force level, the static frictional force between the two spheres is able to reach the tangential force required for breaking, and there is no relative sliding between them before the breakage occurs. On the other hand, for the low–normal force level, relative slippage between the two spheres occurs as the tangential force required for the particle crushing exceeds the maximum static frictional force the contact surface can provide. These two cases are separately discussed in detail in the following subsections.

2.2.1. High–Normal Force Level

In the case of the high–normal force level, the maximum static frictional force on the contact surface between the two spheres is larger than the tangential force required for any one sphere crushing, that is $\mu F_{ncr} > F_{tcr}$, and the breakage occurs without relative sliding between the two particles. F_{ncr} and F_{tcr} are the normal force and the tangential force at crushing, respectively, and μ is the surface sliding friction coefficient of the material. In the stages of local compaction and elastic deformation, only the normal force does work, while the tangential force does not. As a result, the mechanism of these two stages is similar to that under normal contact conditions, and only the integral crushing of the particle is to be discussed next.

The solutions of stresses within the half-space along the line normal to and through the center of the contact area and the displacement at the center of the contact area are of interest as the area of the contact zone is much smaller than the particle size and they lend themselves to simple mathematical manipulation. The normal stress components at a point in the half-space (see Figure 2), under the action of a concentrated force P , are as follows [49]:

$$\sigma_\rho = \frac{P}{2\pi r^2} \left[-\frac{3\rho^2 z}{r^3} + \frac{(1-2\nu)r}{r+z} \right], \quad (6)$$

$$\sigma_\varphi = \frac{(1-2\nu)P}{2\pi r^2} \left(\frac{z}{r} - \frac{r}{r+z} \right), \quad (7)$$

$$\sigma_z = -\frac{3Pz^3}{2\pi r^5}. \quad (8)$$

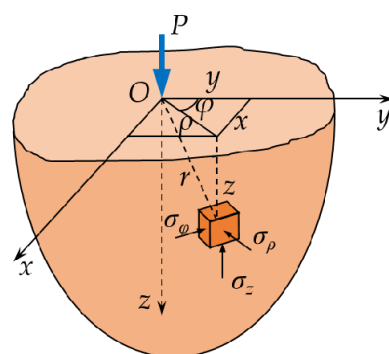


Figure 2. Cylindrical coordinates ρ – φ – z and orthogonal normal stresses σ_ρ , σ_φ and σ_z . ρ and z are the horizontal and vertical distances from the action point of the force P to the location of interest, respectively. φ is the horizontal rotation angle of the location of interest relative to the y -axis.

The cylindrical coordinate system shown in Figure 3a is established with the central point of the compacted area as the origin, and the z -axis is in the same direction as the resultant force F between the two spheres. The stress components at the point M on the z -axis inside the sphere can be obtained by superimposing Equations (6)–(8) on the contact

surface. Figure 3b shows an inclined plane subjected to two vertical concentrated loads P symmetrical to the origin O , in which θ is very small, and the normal stress component σ_z of the point M on the z -axis is as follows:

$$\sigma_z = -\frac{3P(z + \rho \tan \theta)^3}{2\pi[\rho^2 + (z + \rho \tan \theta)^2]^{5/2}} - \frac{3P(z - \rho \tan \theta)^3}{2\pi[\rho^2 + (z - \rho \tan \theta)^2]^{5/2}} \approx -\frac{6Pz^3}{2\pi(\rho^2 + z^2)^{5/2}}, \quad (9)$$

which are close to the calculation results in Figure 3c. The same conclusions are obtained for σ_ρ and σ_φ . Consequently, the stress components at the point M in Figure 3a are obtained through equivalent the boundary condition as a uniform load q acting on a horizontal circular area with radius a , as follows:

$$\sigma_z = q \left[1 - \frac{z^3}{(a^2 + z^2)^{3/2}} \right], \quad (10)$$

$$\sigma_\rho = \sigma_\varphi = \frac{q}{2} \left[(1 - 2\nu) + \frac{z^3}{(a^2 + z^2)^{3/2}} - \frac{2(1 + \nu)z}{(a^2 + z^2)^{1/2}} \right]. \quad (11)$$

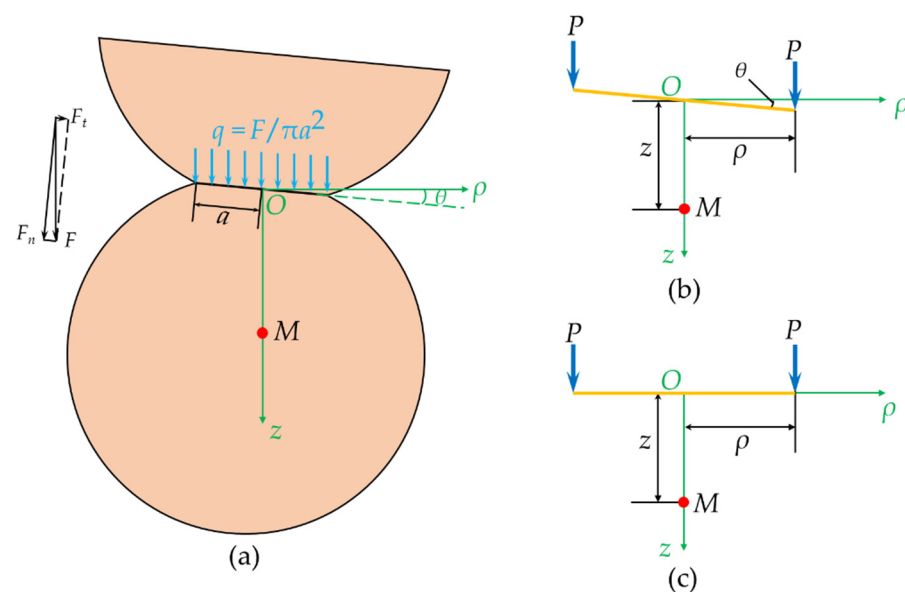


Figure 3. (a) The coordinate system with the center of the compacted area as the coordinate origin. F_n and F_t are the normal and tangential forces between the two spheres, respectively. F is the resultant force of F_n and F_t . $\theta = \tan^{-1} F_t / F_n$; (b) a pair of concentrated loads P symmetrical to point O act on an inclined plane with an inclination angle θ ; (c) a pair of concentrated loads P symmetrical to point O act on a horizontal plane. M is a point on the z -axis.

In this study, the simple two-parameter multiaxial failure criterion for brittle materials proposed by Christensen [50,51] is applied. The criterion states that a material is at failure when the following applies:

$$\frac{\psi K}{\sqrt{3}} \sigma_{ii} + (1 + \psi)^2 s_{ij} s_{ij} > \frac{K^2}{1 + \psi}, \quad (12)$$

where the parameter ψ and K are as follows:

$$\psi = \frac{|\sigma_c|}{\sigma_t} - 1, \quad (13)$$

$$K = \frac{1 + \psi}{\sqrt{3}} |\sigma_c|, \quad (14)$$

respectively. Russell and Muir [46] improved this criterion for ease of application, and stated that a material is at failure when the following applies:

$$K_{mob} > K, \quad (15)$$

$$K_{mob} = -\frac{\sqrt{3}(1 + \psi)^2}{\psi} \frac{s_{ij}s_{ij}}{\sigma_{ii}}. \quad (16)$$

where $s_{ij}s_{ij}$ and σ_{ii} are the second invariant of the deviatoric stress tensor and the first invariant of the stress tensor, respectively. According to Equations (10) and (11), they take the values from the following:

$$s_{ij}s_{ij} = \frac{q^2}{3} \left\{ \nu - \frac{2\nu + 1}{2[1 + (z/a)^2]^{1/2}} + \frac{1}{[1 + (z/a)^2]^{3/2}} - \frac{1}{2} \right\}^2, \quad (17)$$

$$\sigma_{ii} = 2q \left\{ \nu - \frac{2\nu + 1}{2[1 + (z/a)^2]^{1/2}} - \frac{1}{2[1 + (z/a)^2]^{3/2}} + 1 \right\}. \quad (18)$$

By combining Equations (13), (14) and (16)–(18), the plot of K_{mob} within the sphere along the z -axis is shown in Figure 4a. The point at which the K_{mob} value first reaches K is the place where the crack starts from, and is also the vertex of the conical core. The crack expands in the direction of the resultant force between two particles and eventually penetrates the entire sphere, evolving into a rupture surface as shown in Figure 4c. The angle between the rupture surface and the diameter passing through the center of the bottom surface of the conical core is θ , which is called the inclination of the rupture surface. Different from the right conical core in the normal contact test, the conical core in the normal–tangential contact test is an oblique cone with an inclination of θ since its vertex must be on the z -axis, as shown in Figure 4b. The bottom area S of the conical core can still be calculated by Equation (4), while its height is calculating using the following:

$$H = \cos \theta \sqrt{R^2 - (R - s_0)^2}. \quad (19)$$

Through Equations (13)–(18), it is easy to know that the resultant force when integral fragmentation occurs is F_{cr} —the normal force at failure under normal contact conditions—of which the value is determined by the property of the material. Furthermore, the relationship between the normal and tangential forces when integral crushing occurs is as follows:

$$F_{tcr} = \sqrt{F_{cr}^2 - F_{ncr}^2}. \quad (20)$$

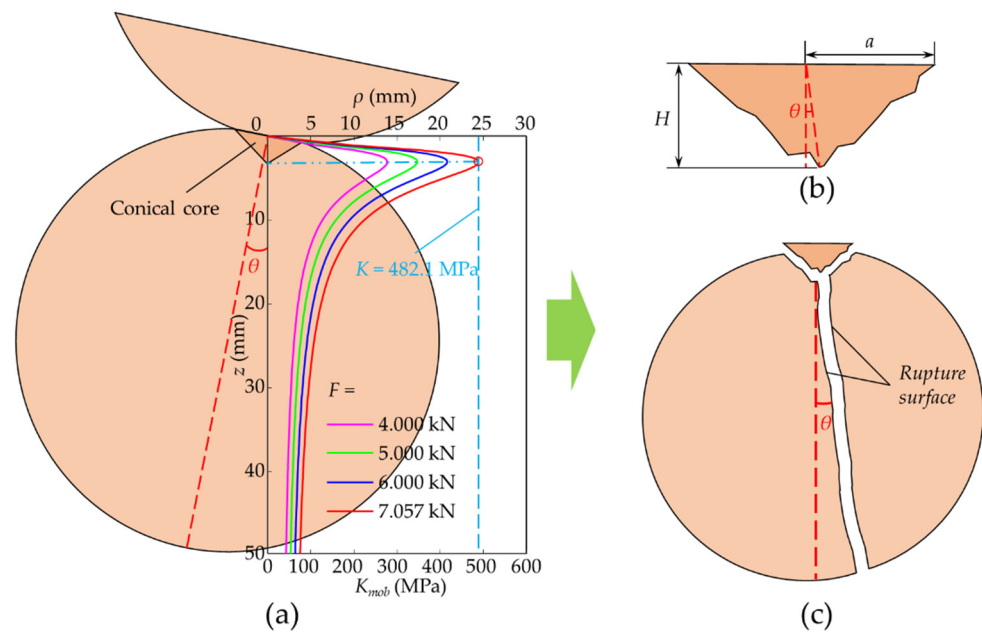


Figure 4. (a) A plot of K_{mob} along the line passing through the center of the compacted area and in the direction of the resultant force for the following material parameters: $\sigma_c = 50$ MPa, $\sigma_t = 3$ MPa, $\nu = 0.3$, $a = 2.7$ mm and $R = 25$ mm; (b) the size and shape of the conical core; (c) a sketch showing the breakage form of a sphere.

2.2.2. Low Normal Force Level

If the maximum static frictional force on the surface of the contact point is less than the tangential force required for either sphere crushing, relative sliding between the two particles will occur before breakage (see Figure 5a), which results in a reduction in the contact area (see Figure 5b). The decrease in contact area leads to the increase in force per unit area on the surface of the contact point. It is assumed that when the relative slip between the two spheres is small, the increase in force per unit area on the surface of the contact point does not cause a secondary occurrence of local compaction. With the increasing uniform load q at the contact surface, the values of K_{mob} at each point inside the sphere along the z -axis increase. Once the K_{mob} value at a point of the inner sphere along the z -axis passing through the center of the bottom surface of the conical core reaches K , the whole sphere crushes (see Figure 5c). Meanwhile, the relationship between the normal and tangential forces at the moment of particle crushing is $F_{tcr} = \mu F_{ncr}$. When the normal force is small enough, the relative slip is large, resulting in a small contact area between the two particles and thus local compaction under the contact surface caused by stress concentration occurs again rather than integral crushing.

To sum up, the relationship between the normal and tangential forces in the normal–tangential contact test when the overall crushing occurs is obtained using the following:

$$F_{tcr} = \begin{cases} \mu F_{ncr}, & (F_{nmin} < F_{ncr} < \frac{\sqrt{F_{cr}^2 - F_{ncr}^2}}{\mu}) \\ \sqrt{F_{cr}^2 - F_{ncr}^2}, & (\frac{\sqrt{F_{cr}^2 - F_{ncr}^2}}{\mu} \leq F_{ncr} \leq F_{cr}) \end{cases} \quad (21)$$

where F_{nmin} is the minimum normal force required for spherical particle crushing under normal–tangential contact conditions. If $F_n < F_{nmin}$, there will be a secondary occurrence of local compaction at the contact point rather than particle breakage. Different from the

normal contact test, both the rupture surface and the conical core have the same inclination that takes the value of the following:

$$\theta = \begin{cases} \tan^{-1} \frac{F_{tcr}}{F_{ncr}}, & (F_{nmin} < F_{ncr} < \frac{\sqrt{F_{cr}^2 - F_{ncr}^2}}{\mu}) \\ \tan^{-1} \mu, & (\frac{\sqrt{F_{cr}^2 - F_{ncr}^2}}{\mu} \leq F_{ncr} \leq F_{cr}) \end{cases} \quad (22)$$

The bottom area and the height of the conical core are solved using Equations (4) and (19), respectively.

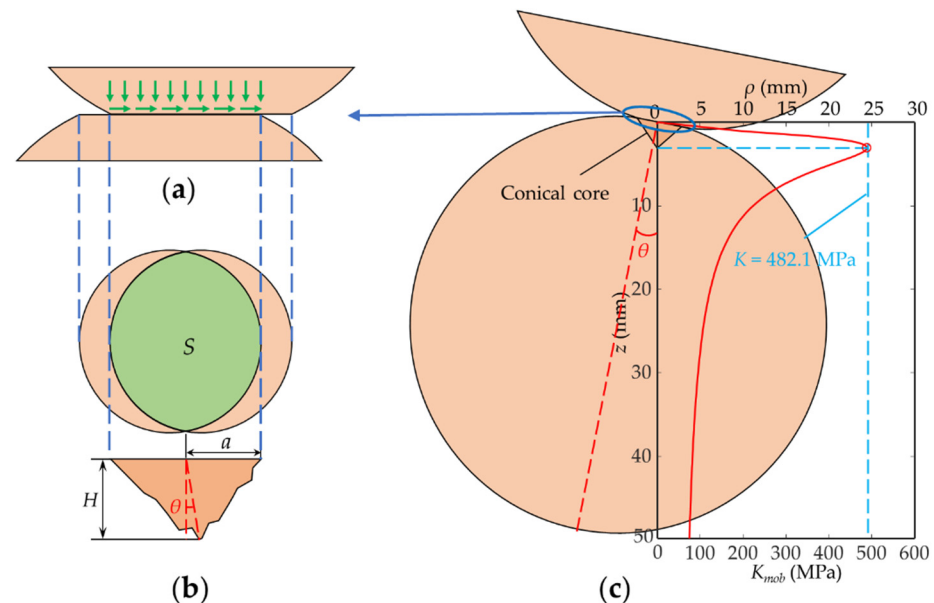


Figure 5. (a) Relative slip occurring on the contact surface; (b) a detailed sketch of the shape and size of the conical core; (c) distribution of K_{mob} values inside the sphere along the z -axis at the moment of breakage. $\sigma_c = 50$ MPa, $\sigma_t = 3$ MPa, $\nu = 0.3$ and $R = 25$ mm.

3. Particle Contact Test

To verify the above theory, the authors performed normal–tangential contact tests between two gypsum spherical particles, which is discussed in detail next.

3.1. Materials and Methods

The particular gypsum used in this study is the Sanxin dental plaster produced by Yuyao Xinshi Gypsum Products Co., Ltd., Yuyao, Zhejiang, China. For 25% water content (defined as the mass of water divided by the mass of dental stone powder), the 1-h and long-term compressive strengths of the plaster made by the dental stone powder are about 35 and 75 MPa, respectively. The reason why gypsum was chosen as the test material was that the sample preparation was simple, the influence of natural cracks in the rock could be eliminated, and the isotropic brittle material could be well simulated. In this experiment, gypsum was made by using water contents of 35% during mixing, and the mixing time and curing time were 5 min and 14 days, respectively. Spheres with a diameter of approximately 48 mm were prepared. This particle size was chosen because if the sphere is too small, it would be hard to observe and measure the shape and size of the conical core, and if it is too large, the exothermic hardening process of the gypsum may induce temperature cracking inside the specimen. The basic mechanical parameters acquired by rock tests of this high-strength gypsum material used in the test are shown in Table 1.

Table 1. Physical parameters of the gypsum material.

UCS ¹	Young's Modulus	Poisson Ratio	UTS ²	Crushing Modulus	Surface Sliding Friction Coefficient
53.1 GPa	5.7 GPa	0.23	3.0 MPa	11.2 GPa	0.32

¹ Unconfined Compressive Strength. ² Unconfined tensile strength.

Among these mechanical parameters, Young's modulus, Poisson ratio and UCS were obtained by compressing solid circular cylinders of the material with a length of 100 mm and a diameter of 50 mm, while UTS was obtained by the Brazilian splitting test of solid circular cylinders of the material with a length of 50 mm and a diameter of 50 mm. Particle contact tests under ball–plane contact conditions were performed to obtain crushing modulus of the spheres [48]. The surface sliding friction coefficient of gypsum material in this test was obtained by the device shown in Figure 6 and the following equation:

$$\mu = \tan \beta - \frac{0.5}{t^2 \cos \beta} \quad (23)$$

All of the results are the means of 5 repeated tests.

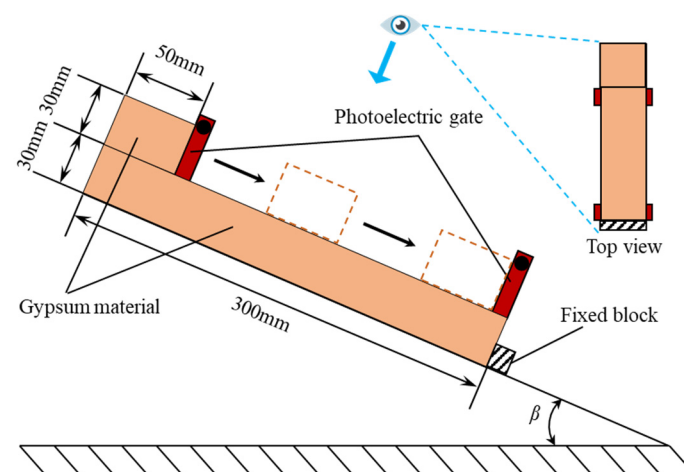


Figure 6. A device for measuring the surface sliding friction coefficient of the material. Both gypsum cubes were fabricated by the same process as the spheres used in the formal test, and the lower cube was fixed on the slope. Two photoelectric gates were used to record the time interval t as the upper gypsum cubes passed through them.

The equipment used in the test was the rock rheological testing system (see Figure 7) in the Key Laboratory of Geotechnical and Underground Engineering, Tongji University, China, which can provide a maximum vertical pressure of 500 kN and a horizontal pressure of 300 kN. The minimum controllable deformation value of this rock rheological testing system is 0.001 mm and the minimum controllable mechanical value is 5 N. The system is equipped with a high-speed static force–displacement acquisition system, and the highest acquisition frequency is 100 times per second. In the present study, the acquisition frequency was set to 5 times per second. The upper and lower spheres are fixed with circular arc-shaped supports.

In order to accurately control and measure the vertical and transverse forces, respectively, the following loading methods are adopted. First, the vertical loading component slowly applies a pressure to the upper sphere to a certain normal force grade F_n ($F_n < F_{cr}$). Then, keep the vertical force unchanged and slowly apply a transverse force to the upper sphere by using the transverse loading component until one of the spheres crushes. The vertical loading component is controlled by force at a rate of 5 N/s, while the transverse loading component is controlled by displacement at a rate of 0.001 mm/s. The air temperature and relative humidity in the laboratory were 28 °C and 49%, respectively.

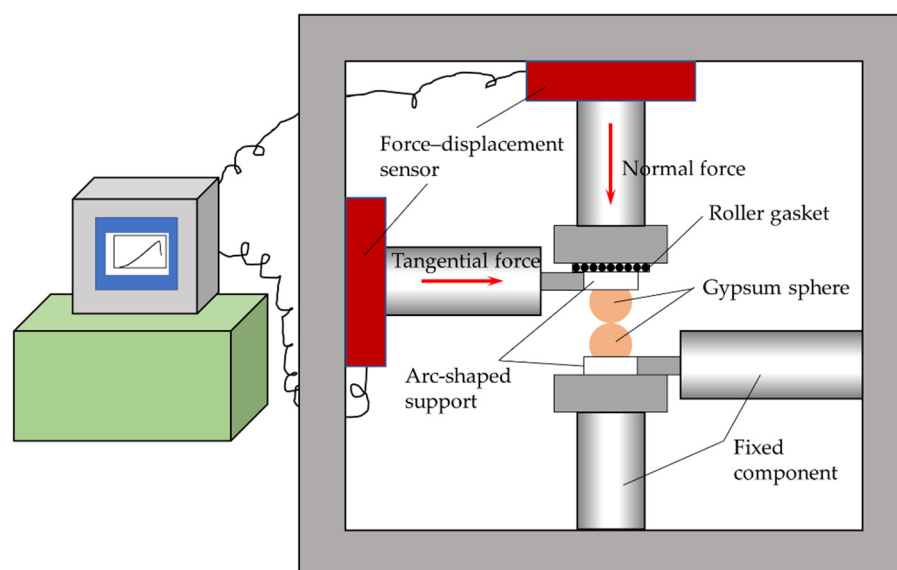


Figure 7. An illustration of the rock rheological testing system. The roller gasket is used to eliminate the influence of friction between the arc-shaped support and the loading component.

Before the formal tests, the authors derived the force required for the sphere crushing under normal contact conditions by combining Equations (13)–(18), and $F_{cr} = 8.021$ kN. Therefore, a total of 9 groups of normal–tangential contact tests were performed, and their normal force grades were 6.016 kN, 6.417 kN, 6.818 kN, 7.219 kN, 7.620 kN, 7.700 kN, 7.780 kN, 7.861 kN and 7.941 kN, separately. For each normal force grade, 3 sets of replicate tests were carried out. In the test, the tangential forces when particle breakage occurred were recorded. Moreover, the inclination of the conical core and the rupture surface, as well as the size of the conical core, were measured after particle breakage occurred.

3.2. Results

Table 2 shows a comparison between the test and the theoretical analysis, and they are in good agreement. The heights of the conical cores are all about 2.9 mm and are not listed here since their changes are very small for different normal force grades. It is clear that in the normal–tangential contact test, the normal force grade has a great influence on the features of particle breakage. In the low–normal force level, the size of the conical core increases with the increasing normal force grade, while the inclination of the conical core and the rupture surface is fixed. In the high–normal force level, the inclination of the conical core and the rupture surface decreases with the increasing normal force grade, and the conical core has a fixed size.

Table 2. Comparison between the predicted and measured values including the inclinations of the rupture surface and the conical core, and the bottom areas of conical core. The heights of the conical cores are not listed due to the small differences.

F_{ncr} (kN)	S (mm ²)	Predicted S (mm ²)	θ (°)	Predicted θ (°)
6.015	19.97	21.18	18.4	17.8
6.416	20.56	22.60	19.3	17.8
6.817	24.91	24.01	15.8	17.8
7.218	23.62	25.43	17.1	17.8
7.619	25.79	26.84	15.6	17.8
7.700	27.33	26.91	14.9	16.3
7.780	25.30	26.91	13.7	14.1
7.860	26.68	26.91	13.1	11.5
7.940	27.34	26.91	10.3	8.1

Figure 8 shows the relationship between F_{ncr} and F_{tcr} , including the theoretical predictions and the data measured in the particle contact test. Obviously, when $F_{ncr} < 7.639$ kN ($\frac{\sqrt{F_{cr}^2 - F_{ncr}^2}}{\mu}$), the relationship between F_{tcr} and F_{ncr} presents a tilted straight line with a slope of $0.32 (\mu)$; when $F_{ncr} \geq 7.639$ kN ($\frac{\sqrt{F_{cr}^2 - F_{ncr}^2}}{\mu}$), the F_{tcr} – F_{ncr} curve is a circular arc centered at the origin of the coordinate. The test under normal contact conditions [48] can be regarded as a specific situation where the tangential force on the contact surface is zero (see the purple circle in Figure 8). From the plot, the data measured in the test are in good agreement with the predicted values. If the normal force is too small, the relative slippage of the two spheres is too large, bringing about a secondary occurrence of local compaction rather than the overall breakage. This situation is too complex and therefore is not discussed in this paper.

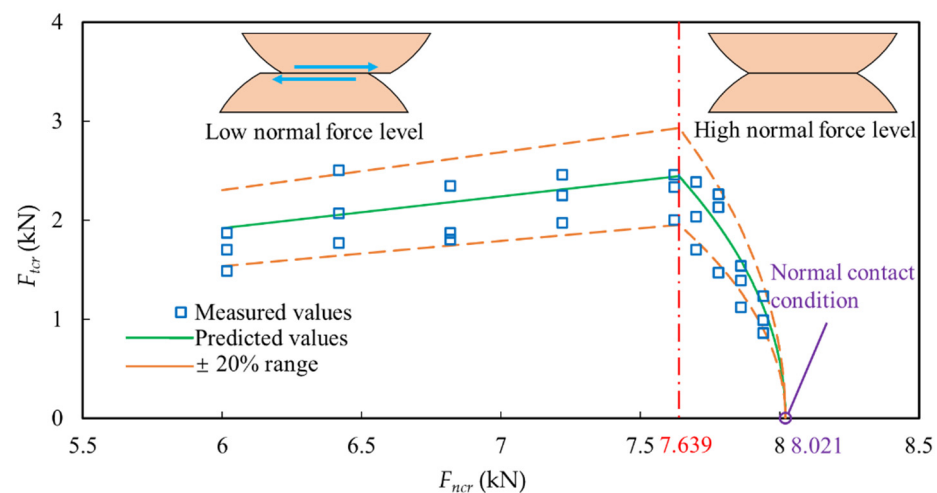


Figure 8. The predicted relationship between the normal and tangential forces at failure as well as the experimental data. The red dot dash line divides the curve into the following two parts: the low-normal force level and the high-normal force level.

4. Discussion

In the present study, particle breakage of two spheres under normal–tangential contact conditions is investigated by theoretical analysis and experimental research. Considering different normal forces, there are the following two types of particle breakage: the high-normal force level in which there is no relative slippage between the spheres, and the low-normal force level in which relative slippage occurs between the spheres.

In the high-normal force level, the size of the conical core is almost the same as that in the normal contact test. The difference is that under normal contact conditions, the conical core is a positive cone, while under normal–tangential contact conditions, it is an inclined cone. Different from the normal contact test, there is an angle between the rupture surface and the diameter passing through the center of the contact point. Furthermore, the conical core and the rupture surface have the same inclination, which becomes smaller with increasing normal force. The tangential force required for particle breaking decreases with the increasing normal force, and the relationship between the normal and tangential forces presents an arc with the origin of the coordinate as the center of the circle.

In the low-normal force level, both the bottom area and the height of the conical core reduce as the normal force decreases, and the bottom is not a circle anymore. The inclination of the conical core and the rupture surface has a fixed value— $\tan^{-1} \mu$. The tangential force when particle breakage occurs is the sliding frictional force between the two spheres, and it increases with the increasing normal force. When the normal force is less than a certain value, and the contact area between the two spheres decreases to a

certain extent due to relative slippage, local compaction occurs again rather than particle breakage, which is not discussed here due to its complexity.

5. Conclusions

In this paper, the micromechanism of the breakage of two spherical gypsum particles under normal–tangential contact conditions is investigated both theoretically and experimentally. The theoretical analysis in this paper is based on the normal contact test. The purpose of particle contact tests is to verify the validity of the theory. The test data are in good agreement with the theoretical prediction. The following conclusions are made:

1. The normal–tangential contact breakage of two spherical particles is divided into two cases depending on whether there is relative sliding—the high–normal force level and the low–normal force level;
2. In the high–normal force level, the size of the conical core does not change with the variation in the contact force, while its inclination decreases with the increasing normal force. In the low–normal force level, the inclination of the conical core does not change with the variation in the contact force, while the size becomes smaller with the decreasing normal force;
3. The rupture surface when the breakage occurs has a fixed inclination in the low–normal force level. However, it decreases with the increasing normal force in the high–normal force level. The rupture surface and conical core always have the same inclination;
4. As the normal force increases, the tangential force required for the overall breakage increases first and then decreases.

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