

Article

The Impact of Fracture Persistence and Intact Rock Bridge Failure on the In Situ Block Area Distribution

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Abstract: The in situ block size distribution is an essential characteristic of fractured rock masses and impacts the assessment of rockfall hazards and other fields of rock mechanics. The block size distribution can be estimated rather easily for fully persistent fractures, but it is a challenge to determine this parameter when non-persistent fractures in a rock mass should be considered. In many approaches, the block size distribution is estimated by assuming that the fractures are fully persistent, resulting in an underestimation of the block sizes for many fracture geometries. In addition, the block size distribution is influenced by intact rock bridge failure, especially in rock masses with non-persistent fractures, either in a short-term perspective during a slope failure event when the rock mass increasingly disintegrates or in a long-term view when the rock mass progressively weakens. The quantification of intact rock bridge failure in a rock mass is highly complex, comprising fracture coalescence and crack growth driven by time-dependent changes of the in situ stresses due to thermal, freezing-thawing, and pore water pressure fluctuations. This contribution presents stochastic analyses of the two-dimensional in situ block area distribution and the mean block area of non-persistent fracture networks. The applied 2D discrete fracture network approach takes into account the potential failure of intact rock bridges based on a pre-defined threshold length and relies on input parameters that can be easily measured in the field by classical discontinuity mapping methods (e.g., scanline mapping). In addition, on the basis of these discrete fracture network analyses, an empirical relationship was determined between (i) the mean block area for persistent fractures, (ii) the mean block area for non-persistent fractures, and (iii) the mean interconnectivity factor. The further adaptation of this 2D approach to 3D block geometries is discussed on the basis of general considerations. The calculations carried out in this contribution highlight the large impact of non-persistent fractures and intact rock bridge failure for rock mass characterization, e.g., rockfall assessment.

Keywords: rockfall; rock mass characterization; in situ block area distribution; discrete fracture network



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1. Introduction

Rockfalls and rock avalanches are natural hazards in mountainous regions. They can cause severe damage to settlements and infrastructure as well as serious injuries and fatalities due to their extremely high velocities and runout distances [1]. Rockfalls are classified as types of rock slope failure characterized by the detachment of single or clusters of individual rock blocks from a steep slope followed by a rapid down-slope motion by falling, bouncing, rolling and sliding [2,3]. Fragmental rockfall is related to the movement of individual fragments which interact with the substrate and thus can be simulated by physical models based on rigid body ballistics. The dynamic interaction of individual blocks of a rockfall event is minor and has no relevant impact on the runout behavior. Rock avalanches are moving in a flow-like manner as masses of fragments that have a strong dynamic interaction of blocks [1]. Many of these events, particularly

the large ones, show unexpected long runout distances. From a modelling perspective, this type of movement requires a completely different mechanical approach, which is considering the rapid coherent flowing mass by granular flow models (e.g., [4]). By nature, there is no distinct boundary between rock avalanches and rockfalls, but rather a gradual transition influenced by the rock type, structural setting, rock fragmentation, substrate properties, and slope topography of the runout path. Nevertheless, also for rock avalanches, single or clusters of blocks can separate during motion from the flowing mass resulting in movements of individual fragments like rockfall events.

Although there is a completely different movement mechanism between these two types of rock slope failure, both are influenced by geometrical rock mass characteristics of the discontinuity network, such as orientation and number of sets, frequency and spacing, size and persistence, and intact rock bridges of the failure area. These parameters influence one important rock mass parameter termed the in situ block size distribution (IBSD [5]). Discontinuities intersecting a rock mass are causing separated rock blocks with sizes ranging from mm^3 in crushed rocks to several m^3 in massive rocks. The IBSD influences the size of the runout area and impact energy of the rockfall and rock avalanche events in case of a failure. Thus, the IBSD in the source area is a crucial rock mass property for rockfall/rock avalanche hazard assessment comprising both runout modelling and designing of protective measures [6–10].

Generally, the non-persistence of the fractures increases the percentage of bigger blocks compared to a discrete fracture network (DFN) with similar fracture set spacings and orientations but persistent fractures (Figure 1). However, for a rock mass with non-persistent fractures, intact rock bridge failure between pre-existing fractures can have a major impact on the IBSD. Fracture coalescence and crack growth driven by time-dependent changes of the in situ stresses due to thermal, freezing-thawing, and pore water pressure fluctuations, etc., can lead to the failure of intact rock bridges between the fractures, provided that the intact rock bridge is small enough [11]. This commonly observed process usually leads to a decrease of the resulting block sizes of a DFN, the stability of rock slopes and an increase in hydraulic connectivity, and consequently, conductivity. The impact of rock bridge failure of non-persistent fractures on the resulting block sizes as well as the shape of the cumulative block size distribution curve has not been investigated in detail so far because of the difficulty in assessing various relevant parameters in the field.

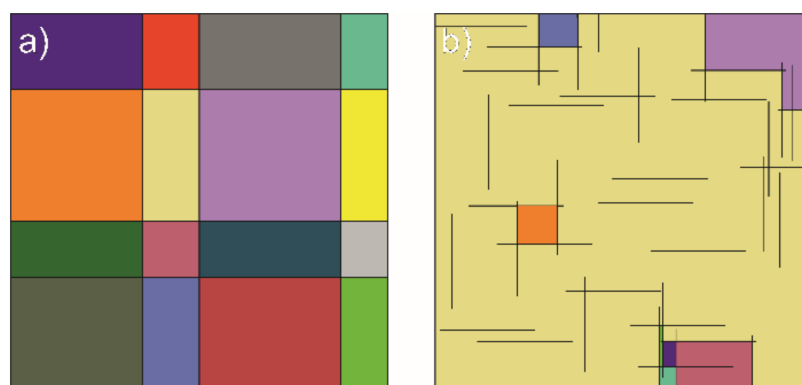


Figure 1. Schematic trace maps illustrating the difference between (a) persistent fractures and (b) non-persistent fractures (black lines), and their impact on the resulting block areas/sizes (colored areas). Both trace maps are simulated by assuming the same mean 1-D fracture frequency.

This contribution presents stochastic analyses of the two-dimensional in situ block area distribution and the mean block area of a non-persistent fracture network based on simulated DFNs. Due to the large amount of time required for the calculations, this article focuses on two-dimensional fracture networks. Furthermore, the applied 2D DFN approach takes into account the potential failure of intact rock bridges based on pre-defined threshold lengths in two dimensions. On the basis of these stochastic DFN analyses, an

empirical relationship between (a) the mean block area for persistent fractures, (b) the mean block area for non-persistent fractures, and (c) the interconnectivity factor according to [12] was determined. These relationships are based on fracture parameters which can be determined in the field by scanline surveys. The further adaptation of this 2D approach to 3D block geometries is discussed on the basis of an approach proposed by [13,14].

2. Theoretical Background

The IBSD of a natural or man-made fractured rock slope is influenced by (a) the orientation and number of fracture sets, (b) the frequency and spacing, and (c) the persistence (size) of the fractures. Natural meso-scale fractures (i.e., joints) are often non-persistent and therefore characterized by finite fracture trace lengths or sizes. According to [5], persistence is defined as the areal extent or size of a fracture within a plane. In 2D, persistence can be estimated by observing the discontinuity trace lengths on the surface of rock exposures. Non-persistent fractures often show terminations within the intact rock which geometrically lead to intact rock bridges of variable extend. These rock bridges affect the overall rock mass strength, given that the intact rock is usually characterized by higher strengths and thus has a major impact on the stability of rock masses [15], but it is nearly impossible to map them at in situ conditions. It is obvious that rock mass failure occurs much easier if fractures are fully persistent and rock blocks are already formed as if pre-existing intact rock bridges have to fail either suddenly or in the long-term by sub-critical crack growth [11,15,16].

In many cases, the block area A_0 [m²] or block volume V_0 [m³] is estimated either on the basis of numerical DFN modelling or analytically by determining the mean normal set spacing of three major fracture sets (s_1 , s_2 , and s_3 in [m]) and the angle between these fracture sets (γ_{11} , γ_{13} , and γ_{23} in [°]). According to [14], the mean block area and block volume assuming persistent fractures is given by:

$$A_0 = \frac{s_1 \cdot s_2}{\sin(\gamma_{12})} \quad (1)$$

$$V_0 = \frac{s_1 \cdot s_2 \cdot s_3}{\sin(\gamma_{12}) \cdot \sin(\gamma_{13}) \cdot \sin(\gamma_{23})} \quad (2)$$

This common analytical approach based on the assumption of persistent fractures cause an underestimation of the block areas/volumes of non-persistent fracture sets (Figure 1). This could have adverse impacts for rockfall hazard assessments, such as underestimated run out length and block energies. In [13], the authors validated the supposition of [17], suggesting that the mean block area and block volume of rock masses with non-persistent fracture sets is related to the fracture persistence according to,

$$A_b = \frac{s_1 \cdot s_2}{\sin(\gamma_{12}) \cdot \sqrt{p_1 \cdot p_2}} \quad (3)$$

$$V_b = \frac{s_1 \cdot s_2 \cdot s_3}{\sin(\gamma_{12}) \cdot \sin(\gamma_{13}) \cdot \sin(\gamma_{23}) \cdot \sqrt[3]{p_1 \cdot p_2 \cdot p_3}} \quad (4)$$

where A_b and V_b are the block area and block volume assuming non-persistent fractures, respectively, and p_1 , p_2 , and p_3 are persistence factors in the range between 0 and 1. Subscripts 1 to 3 represent the assigned number to fracture sets. Assuming a Veneziano fracture network model [18,19], i.e., Poisson line and Poisson plane processes, the persistence factor is given as the ratio between the accumulated fracture trace length in a sampling plane to the total characteristic length of the rock mass under consideration (i.e., the sampling plane). Alternative approaches to estimating the persistence factor have been presented by, e.g., [14,19], but they are all based on ratios between fracture and intact rock bridge geometries, which is a ratio that is usually very difficult to measure during practical campaigns in the field.

Validation of these equations by [13] based on stochastic analyses using numerical fracture network simulation tools implemented in the discrete element codes UDEC and 3DEC (Itasca), respectively, suggest that simulated block sizes only differ by <1% and around 4% from the Equations (3) and (4), respectively. However, the persistence factors p_1 , p_2 and p_3 require a ratio between the sum of the trace lengths of the joint sections in a plane and the length of the intact rock bridges in between. While the trace length distribution of individual fracture sets usually can be acquired in the field by window and scanline mapping, a measurement of the length of the intact rock bridges is still a big challenge and nearly impossible. Furthermore, persistent fracture traces/planes are generally not aligned one by one along a plane but rather are randomly distributed within the rock mass [20]. An additional limitation of some DFN modelling software products is the requirement of at least one fully persistent fracture set or a spacing parameter that is valid for the Veneziano model only. Besides that, for many cases, the applied persistence factor [17] does not consider the relation to other fracture sets and is a simplification concerning its impact for IBSD considerations. In practice, the determined IBSD is only a rough estimate because all mentioned restrictions do not consider natural conditions and limitations. Consequently, a method of analyzing the IBSD of a rock mass composed of non-persistent fractures on the basis of geometrical parameters that can be easily acquired in the field is needed. Typically, only the orientation, spacing and trace lengths of fractures are acquired in the field concerning fracture geometries (e.g., by applying the widely accepted scanline mapping method [21]). Similar parameters of large datasets can be acquired by non-contact survey methods, such as photogrammetry and terrestrial laser scanning, e.g., [22–27]. The application of the interconnectivity index, I_{ij} , proposed by [12], as an alternative parameter to the persistence factor, is a possibility for such a parameter, which can be acquired based on the above-mentioned field parameters. The interconnectivity index is independent of the knowledge of the lengths of intact rock bridges and can be determined between two fracture sets within a rock mass.

$$I_{ij} = \frac{l_i}{s_j} \cdot \sin(\gamma_{ij}) \quad (5)$$

where l_i is the mean trace length of the fracture set i [m]; s_j is the mean normal set spacing of fracture set j [m]; γ_{ij} ($i \neq j$) is the average angle in [°] between the two fracture sets i and j . For a fractured rock mass containing several fracture sets, the total interconnectivity index (I_i) for each fracture set can be determined by summation of the interconnectivity indexes,

$$I_i = \sum_{j=1}^n I_{ij} (i \neq j) \quad (6)$$

where n is the number of fracture sets. Although the index does not rely on a theoretical background, it provides a useful parameter to characterize interconnectivity based on fracture orientation, trace length and spacing. From a practical perspective, the index is easier to determine for a rock mass than the persistence factor proposed by [17]. Nevertheless, the resulting values for the interconnectivity index may show a much larger range than the persistence factor (i.e., $0 < p \leq 1$ while $0 < I_i < \infty$). Consequently, p and I_i are not comparable, and thus I_i cannot be directly implemented in Equations (3) and (4). However, so far, no equations were established to implement the interconnectivity index for the determination of an IBSD.

Generally, DFN modelling is the most appropriate approach to analyze IBSDs of rock masses. Currently, there are only a few software products, e.g., FracMan (Golder 2021), UDEC, 3DEC (Itasca 2021), ELFEN (Rockfield 2021) [28–30], mostly commercial and proprietary codes, available that are able to calculate the block sizes by DFN modelling. Most of them have limitations in considering rock bridge failure. The adaptation of these proprietary codes to implement rock bridge failure is challenging. While there have been recent improvements in using DFN for open source code analyses, especially in the field of

hydrogeology [31,32], the simulation of the IBSD for various scenarios (e.g., including rock bridge failure) still needs further investigation.

3. Methodology

3.1. Discrete Fracture Network Generation

A total of about 1500 models with random fracture networks (DFN) were generated to analyze their 2D block areas. These models comprise non-persistent fractures and are generated by using MATLAB (R2015b, Mathworks) [33]. Fractures are represented by their traces, i.e., lines (see Figures 1b and 2). Stochastic analyses were performed on the basis of all simulations by varying values for each input parameter of the simulations, comprising of fracture orientation, the number of sets, linear frequency along the normal to the i^{th} fracture set, and trace length. For all simulations, a discrete fracture network with two fracture sets was generated for a $50 \text{ m} \times 50 \text{ m}$ study region. In this study, the four outer boundaries of the region were defined as fully persistent fractures in contrast to other approaches (e.g., [34]).

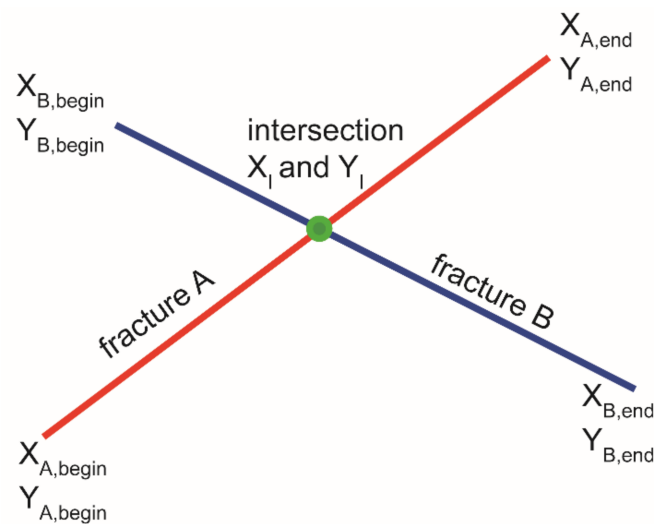


Figure 2. Determination of the intersection of two intersecting fractures.

The number of fractures, n_f [–], which are generated in each of the $50 \text{ m} \times 50 \text{ m}$ study regions, is based on the linear fracture frequency, λ_i [m^{-1}], according to the following equation:

$$n_f = \lambda_i \cdot L_y \cdot \frac{L_x}{t_l} \quad (7)$$

where L_x and L_y are the lengths of the analyzed region in the x - and y -direction [m], respectively, and t_l is the mean fracture trace length [m]. The linear fracture frequency is usually measured by the application of scanlines and is similar to P10 proposed by [35], which is perpendicular to the mean fracture set orientation. According to [36], the linear fracture frequency is the reciprocal of the normal set spacing of a fracture set. The central locations of all fractures within the analyzed study region were determined randomly by a 2D Poisson process, which causes a negative exponential distribution of normal set spacing values. This type of probability distribution was chosen because it is able to represent many naturally occurring fracture spacing patterns (e.g., [14,37–39]). Other common distributions would be, e.g., the logarithmic normal (e.g., [40–42]) or fractal (e.g., [43–45]) distributions, which can be generated by a Kolmogorov process or a scale-invariant process, respectively [46].

The following geometric restrictions were implemented in the random fracture generation for the two fracture sets of the approx. 1500 models: (a) mean values for the fracture trace length (0.5–12 m), (b) linear fracture frequency (0.15–3 m^{-1}), and (c) fracture orien-

tation (0–90°). As an additional limitation, a minimum difference of the angle between the two sets was defined as 15°. Based on the mean orientation of the fracture set and a standard deviation of $\text{Std} = 5$, the fracture orientation of each fracture of a set was simulated randomly on the basis of a normal probability distribution (e.g., [47]). In addition, the individual fracture trace lengths were generated with the randomly defined mean trace length and a negative exponential probability density distribution (e.g., [12,36,48]). Fractures simulated and located partly outside of the 50×50 m study region (central locations of fractures were set to be located always within the boundaries of the study region though) were deleted and not considered for further analyses, i.e., the trace map was cropped along the outer boundary of the analyzed region.

3.2. Block Area Calculation

The in situ block area distributions of the non-persistent fractures in the DFNs were calculated as followed:

First, the coordinates of each intersection between two fracture traces (lines) are determined by the MATLAB function *lineSegmentIntersect* (Ref. [49], Figure 2).

The X and Y coordinates of the intersection, X_I and Y_I , are given by:

$$X_I = X_{A,begin} + u_A \cdot (X_{A,end} - X_{A,begin}) \quad (8)$$

$$Y_I = Y_{A,begin} + u_A \cdot (Y_{A,end} - Y_{A,begin}) \quad (9)$$

where $X_{A,begin}$, $X_{A,end}$, $Y_{A,begin}$, and $Y_{A,end}$ are the X and Y coordinates of the beginning and termination of fracture A , and the slope, u_A , is given by:

$$u_A = \frac{(X_{B,end} - X_{B,begin}) \cdot (Y_{A,begin} - Y_{B,begin}) - (Y_{B,end} - Y_{B,begin}) \cdot (X_{A,begin} - X_{B,begin})}{(Y_{B,end} - Y_{B,begin}) \cdot (X_{A,end} - X_{A,begin}) - (X_{B,end} - X_{B,begin}) \cdot (Y_{A,end} - Y_{A,begin})} \quad (10)$$

In the next step, the lines are checked if they share an adjacency matrix (e.g., see [50]) by using the built-in MATLAB function *adjacency*. Points or line segments that are not part of any adjacency matrix are removed for further calculation steps.

For each of the adjacency matrices, closed polygons are determined, which result from crossing the fracture traces (i.e., lines). Closed polygons, which are formed from the combination of more than one other closed polygons, are excluded, i.e., only the smallest possible polygons are counted.

Next, the surface area, A , of these polygons are then calculated by the built-in MATLAB function *polyarea* according to:

$$A = [(X_1 + X_2) \cdot (Y_1 - Y_2) + (X_2 + X_3) \cdot (Y_2 - Y_3) + \dots + (X_n + X_1) \cdot (Y_n - Y_1)] / 2 \quad (11)$$

where X and Y are the coordinates of the vertices and n is the number of vertices.

When the surface areas of all polygons of an adjacency matrix are calculated, this procedure is repeated for the next adjacency matrix until no more exist.

This algorithm produces a list of block areas of all polygons, whereby the total sum of all block areas is equal to the area of the study region, herein assumed as $50 \text{ m} \times 50 \text{ m}$ (i.e., 2500 m^2). Only the smallest possible polygons are counted in this way.

3.3. Implementation of Rock Bridge Failure

Depending on the investigated rock mass scenario, a discrete fracture network consists of the pre-existing fractures but also of fractures that are newly formed due to rock bridge failure during the failure process.

In this study, approximately 600 different fracture network scenarios where rock bridges may fail and coalesce with two pre-existing fracture sets were simulated. In nature, fracture coalescence between two pre-existing fractures is characterized by tensile (i.e., wing cracks, en echelon) and/or shear (secondary cracks) failure modes, according

to the stress state in the rock mass (e.g., [51]). Thus, the newly formed fractures do not simply grow as single fractures aligned in the direction of the shortest distance. In order to avoid too much complexity and the short fracture coalescence length assumed herein, a simplified approach based on fracture propagation in the direction of the shortest distance between two pre-existing fractures is chosen. As an assumption and simplification, a new fracture will be formed between two pre-existing fractures if the shortest rock bridge thickness is below a pre-defined threshold. For this simulation the threshold value of the rock bridge was set to (i) 0.05 m, (ii) 0.1 m, (iii) 0.2 m, (iv) 0.3 m, and (v) 0.4 m, respectively. The presented approach does not consider the complexity of fracture coalescence due to stress heterogeneities or anisotropies and does not consider the impact of non-planarity of fractures on the block area distribution.

Considering the new, due to the intact rock bridge failure formed fractures, the shortest distance between the two fractures is calculated according to the MATLAB function *DistBetween2Segment* [52]. This function uses a geometric approach to find the unique vector, which is characterized as the smallest possible length between the two pre-existing fractures [53]. In the case that the distance between two fractures is smaller than the threshold value of the rock bridges (i.e., here specified as 0.05 m, 0.1 m, 0.2 m, 0.3 m, or 0.4 m), a new fracture representing rock bridge failure is generated along the line representing the shortest measured length (red dotted lines in Figure 3). Restrictions, where rock bridge failure is not possible, are summarized in Figure 3 and comprise: (1) a failure distance that is larger than the threshold length, (2) a new coalescing fracture crossing a different fracture in between, and (3) two existing fractures, which are already intersecting. Concerning the example of Figure 3, three closed polygons are formed (green, magenta, and turquoise background colors). It should be noted that, for example, a merged polygon consisting of the magenta, turquoise, and green polygons, is not considered to be a valid polygon in this study. The block areas are then calculated by using a method similar to the method described above.

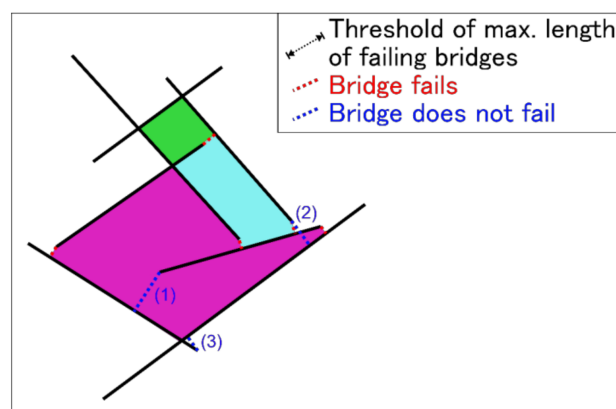


Figure 3. Schematic illustration showing the rock bridge failure procedure between pre-existing fractures (black lines) and the development of closed polygons.

3.4. Monte Carlo Simulations for Persistent Fractures

The block area distribution of discrete fracture networks based on non-persistent fractures were compared with the common approach, which assumes persistent fractures (Equation (1)). Monte Carlo simulations of Equation (1) were performed to estimate the entire distribution of block areas instead of unique values only. Therefore, values obtained randomly from normal and negative exponential probability density distributions for fracture orientation and spacing were applied, respectively.

4. Results

4.1. Block Areas Assuming Persistent Versus Non-Persistent Fractures

For each of the more than 600 simulations, block areas of the DFN were calculated and statistically evaluated. Generally, the block area analyses show cumulative distributions, which are typically gained when the fracture frequency follows a negative exponential probability distribution. Exemplarily, Figure 4 shows block area distribution simulations based on three different input scenarios. Simulation scenario A is based on a mean angle between the two fracture sets of 85.6° , a frequency and trace length of set #1 of 3.4 m^{-1} and 10.0 m, and a frequency and trace length of set #2 of 2.8 m^{-1} and 13.5 m, respectively. For simulation scenario B, a mean angle between the two fracture sets of 61.6° , a frequency and trace length of set #1 of 0.9 m^{-1} and 5.0 m, and a frequency and trace length of set #2 of 2.9 m^{-1} and 7.7 m was implemented. For simulation scenario C, the mean angle between the two fracture sets were further reduced to 27.5° . The frequency and trace length of set #1 was set to 1.1 m^{-1} and 2.1 m and of set #2 to 1.0 m^{-1} and 7.2 m, respectively. The shape of the cumulative block area distribution curves is similar for non-persistent and persistent fractures, although the slopes of the distribution curves for non-persistent fractures are marginally less steep for all simulations (Figure 4). Depending on the implemented fracture network geometry, the simulated mean block areas based on non-persistent fractures are between 1.1 to 6.2 times larger than the corresponding block areas, which were estimated by Equation (1) and the Monte Carlo approach.

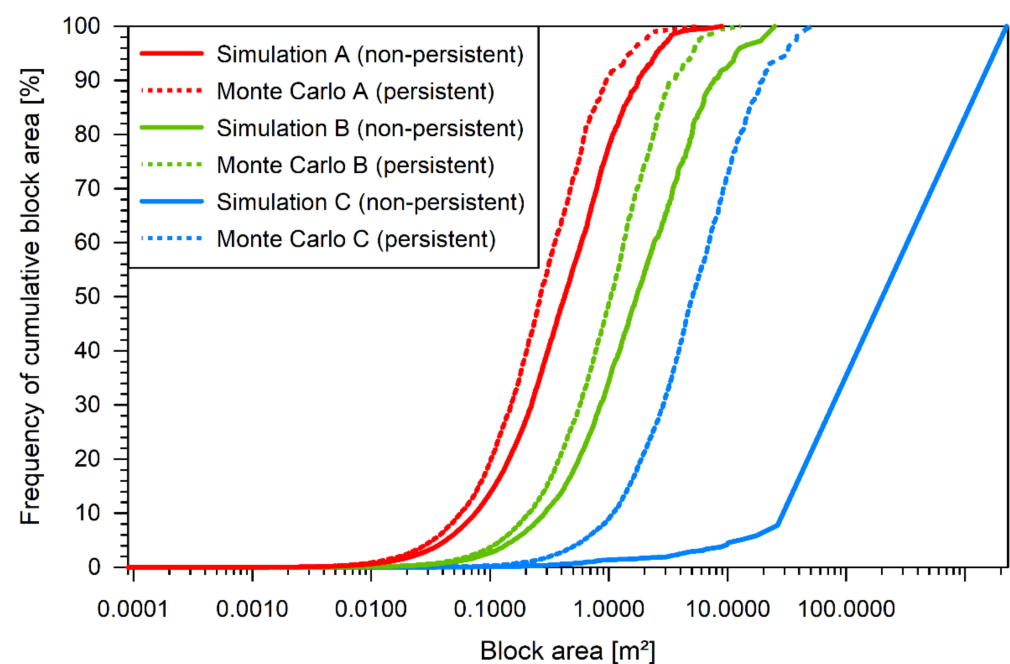


Figure 4. Selected examples of block area simulations, assuming non-persistent fractures (solid lines) and block area distributions, assuming persistent fractures (Monte Carlo simulation based on Equation (1), dashed lines).

In contrast to the suggestions by [12] for the calculation of the total interconnectivity index (I_i), a new factor termed as the geometric mean interconnectivity factor (I_g) was defined, which was averaged from the two interconnectivity indexes I_{12} and I_{21} .

$$I_g = \frac{I_{12} + I_{21}}{2} \quad (12)$$

This modification was done because it allows making correlations with only one interconnectivity factor instead of two. The difference between the simulated mean block areas with non-persistent fractures (A_b) and the mean block areas (A_0), assum-

ing persistent fractures (see Equation (1)) for a given fracture network, was determined. Figure 5 illustrates a clear correlation between the mean A_0/A_b ratio and the geometric mean interconnectivity factor (I_g). Remarkably, the mean A_0/A_b ratio does not converge towards 1, even for high values of the interconnectivity factor. Consequently, the simulated block areas (A_b) are larger than those that were obtained by the simplified calculation according to equation (1), even for very long fracture dimensions. All geometric mean interconnectivity factors greater than approximately 20 result in an A_0/A_b ratio of approximately 0.65–0.85, indicating that A_b is 15–35% smaller than A_0 for the analyzed DFN simulations.

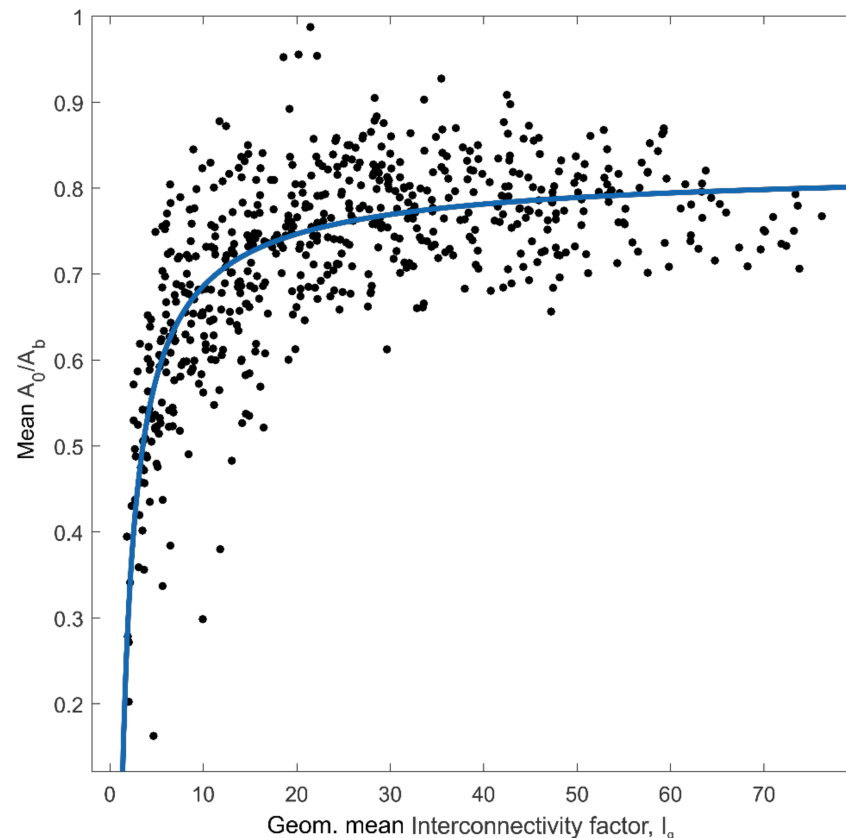


Figure 5. Correlation between the geometric mean interconnectivity factor (I_g) and the mean A_0/A_b ratio based on about 600 simulations with different input values for fracture frequency, orientation, and trace length (blue line: power law curve fit).

An empirical relationship based on non-linear curve fitting (Figure 5) was obtained between the mean A_0/A_b ratio and the geometric mean interconnectivity factor (I_g). Curve fitting according to a power law showed the best fit to the data set, resulting to the empirical Equation (13):

$$\frac{A_0}{A_b} = a \cdot I_g^b + c \quad (13)$$

For this simulation, empirical fitting parameters of $a = -0.9001$, $b = -0.8012$, and $c = 0.8283$, as well as a coefficient of determination of $R^2 = 0.9902$ are determined. Consequently, the mean block area A_b for a non-persistent fracture network can be estimated by:

$$A_b = \frac{\frac{s_1 s_2}{\sin(\gamma_{12})}}{a \cdot I_g^b + c} \quad (14)$$

Figure 5 further illustrates that the mean A_0/A_b ratio of the DFNs with small I_g values (i.e., low interconnectivity factor) are characterized by a large range of variation. Higher I_g values lead to minor variations.

4.2. Impact of Rock Bridge Failure on Block Areas

For each geometrical fracture network scenario (i.e., variation of fracture orientation, trace length and frequency), a total of six DFNs were generated by setting the maximum threshold length of potential rock bridge failures to (a) 0 m (i.e., no rock bridge failure), (b) 0.05 m, (c) 0.1 m, (d) 0.2 m, (e) 0.3 m, and (f) 0.4 m, respectively. For each rock bridge failure length, surface areas were calculated and analyzed.

Depending on the geometrical parameters of the DFN, i.e., trace length, fracture frequency, and fracture orientation, consideration of rock bridge failure can have a minor or major impact on the resulting block areas (Figure 6). All simulations of the selected example shown in Figure 6 are characterized by the same fracture network, but varied regarding the threshold lengths for rock bridge failure from (a) 0 m, (b) 0.05 m, (c) 0.1 m, (d) 0.2 m, (e) 0.3 m, and (f) 0.4 m. The illustrated fracture networks were generated by implementing two fracture sets. Fracture set #1 has a mean dip angle of 1.16° , with a standard deviation of $\text{std} = 5$, a mean trace length of 5.65 m, and a mean 1D frequency of 0.74 m^{-1} . Fracture set #2 is based on a mean dip angle of 94.72° ($\text{std} = 5$), a mean trace length of 2.33 m, and a mean 1D frequency of 1.20 m^{-1} .

In comparison to scenarios without rock bridge failure, the scenarios with rock bridge failure lead to cumulative distribution curves shifting towards smaller block areas. However, the shape of the cumulative distribution curves (Figure 7) between these scenarios remains almost similar. Thus, rock bridge failure affects block areas of all sizes, and there is no linear decrease in block areas with increasing maximum length of rock bridge failure. However, the shift along the x -axis of the cumulative block area distribution curves in the graph of Figure 7 for the individual threshold values is approximately in the same order of magnitude. Implementation of rock bridge failure decreases the number of blocks with unrealistic complex shapes, which is dominating for larger block areas. As a result of implementing rock bridge failure in the simulations, blocks are representing more natural shapes and less bounding surfaces. In addition, it was observed that rock bridge failure has a stronger impact on the resulting block area for DFNs with smaller interconnectivity factors than those with larger interconnectivity factors. Comparing similar DFNs, the resulting mean block areas of simulations without rock bridge failure are approximately five times larger than those obtained for simulations considering rock bridge failure and implementing a maximum rock bridge failure length of 0.4 m.

For DFNs with rock bridge failure thresholds larger than 0 m, no correlation between the geometric mean interconnectivity factor and the ratio between the mean block area (A_0) assuming persistent fractures and the mean block area (A_b) considering non-persistent fractures (in contrast to simulations in Figure 5) is observed. Instead, the impact of the threshold length on the resulting mean block area, A_{br} , is a function of both the geometric mean interconnectivity factor and the mean block area with a threshold level of 0 m (A_{nb} , Figure 8). For large I_g values (approximately > 10), the ratio between the mean block area without rock bridge failure, A_{nb} , and the mean block area with rock bridge failure, A_{br} , is nearly constant and varies only between 1.0 and 1.5, depending on the maximum length of rock bridge failure. This suggests that the block areas do not decrease with the implementation of rock bridge failure if I_g is larger than approximately 10. The power law of Equation (15) fits the data set and shows a clear correlation trend (Figure 8).

$$\frac{A_{nb}}{A_{br}} = a \cdot I_g^b + c \quad (15)$$

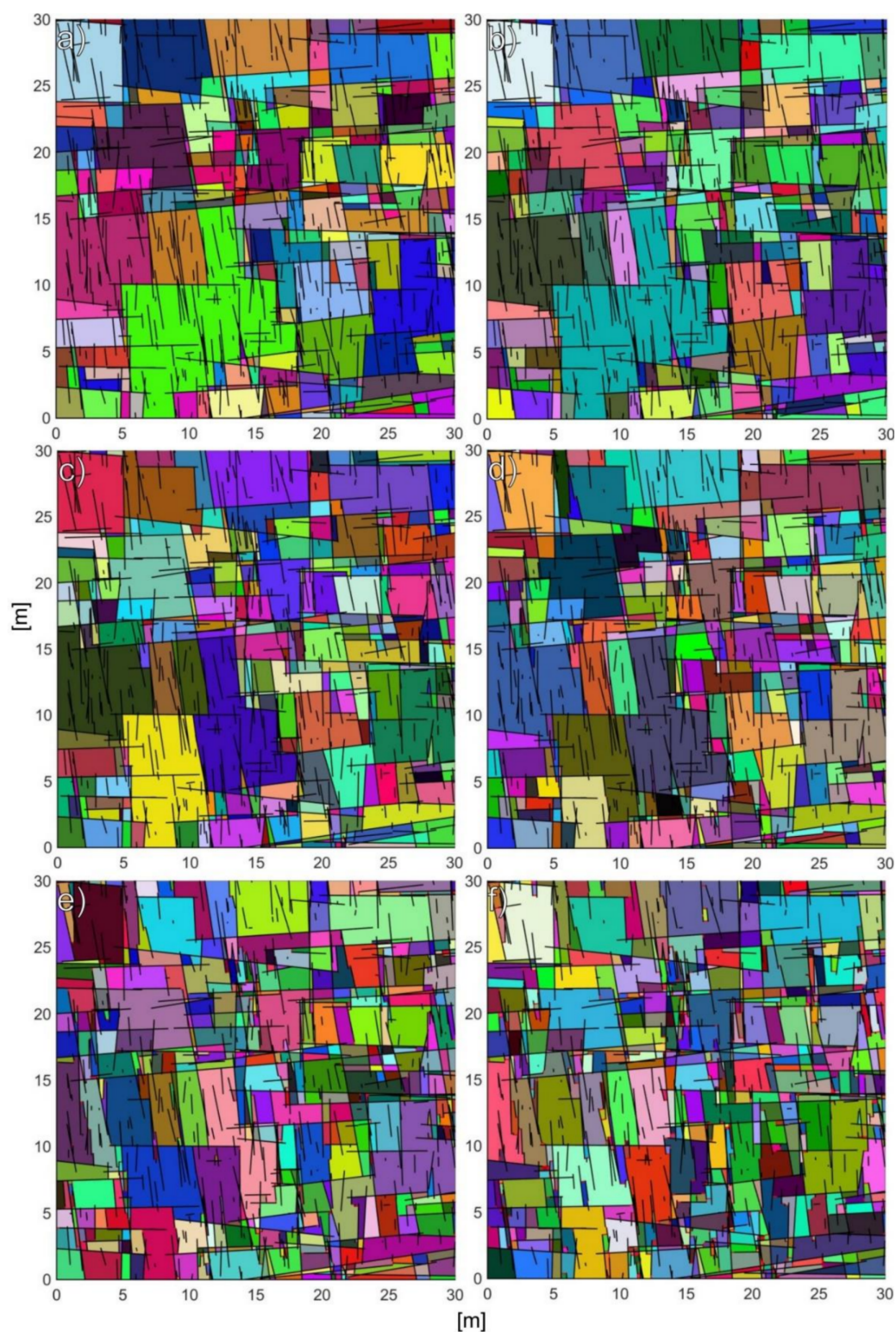


Figure 6. 2D discrete fracture networks showing 30×30 m detail views, extracted out of the 50×50 m total models. Areas generated by intersecting fracture traces (black lines) are shown and randomly colored. All models (a–f) are characterized by the same fracture network but varied regarding the threshold lengths for rock bridge failure.

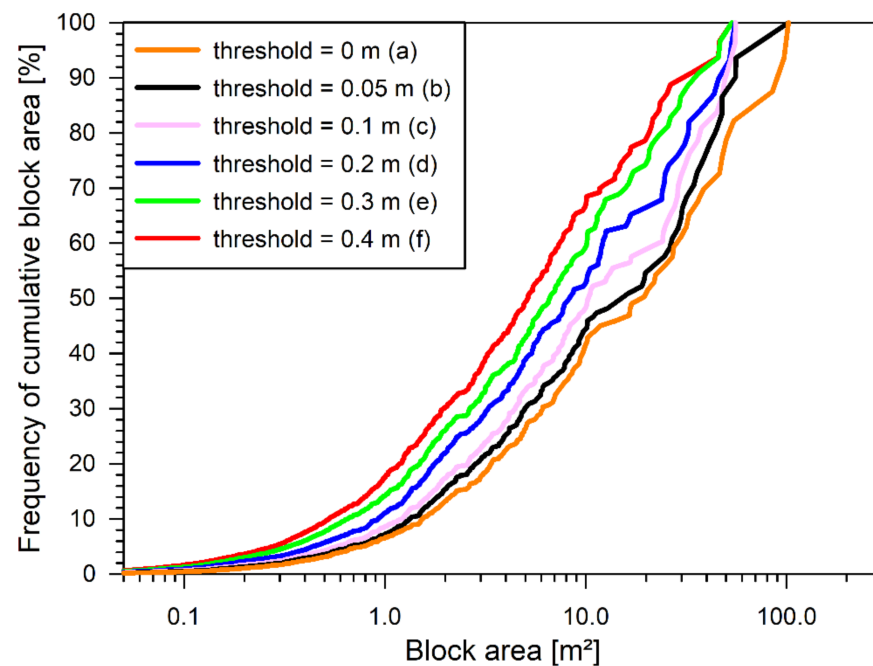


Figure 7. Cumulative block area distribution of the six simulation examples presented in Figure 6.

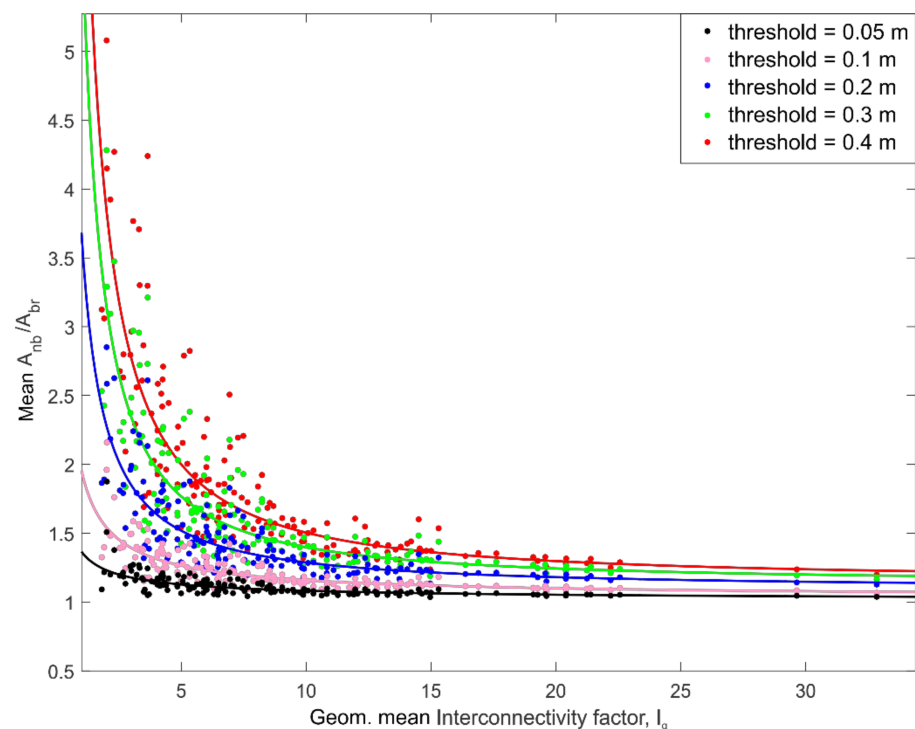


Figure 8. Correlation between the geometric mean interconnectivity factor, I_g , and the ratio between the mean surface areas, A_{nb}/A_{br} (color-coded according to different thresholds). Power-law fit for the various data sets of similar threshold lengths are illustrated as lines and color-coded according to the lengths.

However, scattering is high for very low values (around $I_g < 2$). This could be an indication that the defined study area with a size of $50 \text{ m} \times 50 \text{ m}$ was set too small (i.e., the representative elementary area is larger than $50 \text{ m} \times 50 \text{ m}$) in order to enable comprehensive statistical analyses of these DFNs.

Parameters (a, b, and c) of the empirical power-law describing the correlation between the ratio A_{nb}/A_{br} according to a threshold of 0.05 m, 0.1 m, 0.2 m, 0.3 m, and 0.4 m for rock bridge failure with the geometric mean interconnectivity factor are presented in Table 1. The coefficient of determination, R^2 , increases with longer threshold lengths.

Table 1. Fitting parameters for Equation (15) of the power law in relationship to the threshold length.

Threshold Length	a	b	c	R^2
0.05 m	0.360	−0.668	1.005	0.972
0.10 m	0.924	−0.842	1.026	0.980
0.20 m	2.575	−1.115	1.089	0.982
0.30 m	4.823	−1.263	1.134	0.984
0.40 m	6.443	−1.259	1.149	0.985

Consequently, it is possible to estimate the mean block areas of non-persistent fractures based on the combination of Equations (14) and (15) if the fracture orientations, fracture frequencies, trace lengths as well as the maximum length of rock bridge failure is known (Note: A_{nb} equals A_b).

5. Discussion

5.1. Implications for Using the Results

This study focuses on the difference in rock block areas between models, assuming persistent fractures and non-persistent fractures. It is widely accepted that fractures terminating within rock blocks are influencing the overall rock mass properties. Rock bridge failure and block formation become important for the determination of rockfall hazards, quarry production, and others. Thus, the implementation of potential rock bridge failure in this study provides a further step to account for this problem. However, the approach herein is simple and is based solely on a defined threshold length for rock bridges that are prone to fail. Other relevant factors, such as the impact of in situ stresses, dynamic loading (e.g., rockfall impact forces or blasting energy), or rock anisotropy and heterogeneity, were not considered in this study. Furthermore, the setting of representative threshold length values for rock bridge failures is difficult and requires preceding studies on selected well-exposed rock faces, ideally recently formed failure surfaces of rockfall events.

It should be noted that the methodical approach used in this study does not exclude smaller blocks enclosed totally by larger blocks (e.g., the orange block within the large beige block in Figure 1). This may or may not produce representative results, depending on the application of the in situ block area distribution. In addition, the simulation can produce complex block area shapes, which may not gravitationally release from a rock face due to partial blocking by the remaining rock mass. In Figure 6a, for example, the large, irregular light green block area highlights the problem, which occurs mainly for DFNs with small interconnectivity factors. The consequences of this phenomenon were not considered in this study but may be relevant, especially for rockfall studies.

5.2. Adaptation from Block Area to Block Volume Analyses

One major limitation of this study is that the analyses of the block distributions were done only two-dimensional. Within this framework, this limitation was accepted because, even in 2D, a high number of simulations were required to perform a reliable statistical analysis. 3D simulations would have been very time-consuming, and starting initially with a 2D approach allows for a first and crucial validation of the methodical approach.

Conceptually, if a third fracture set is added, an estimation of the mean 3D block volume (V_b [m^3]) on the basis of the 2D analyses can be done according to following general mathematical relation:

$$V_b = A_b \cdot \frac{s_3}{\cos \gamma_{3D}} \quad (16)$$

where A_b is the simulated mean block area considering non-persistent fractures [m^2]; s_3 is the mean normal set spacing [m]; γ_{3D} is the angle between the mean normal to fracture set #3 and the normal vector to the A_b -plane for fracture sets #1 and #2. If the dip direction, α_d , and dip angle, β_d , of a fracture set is known, the trend and plunge of its normal can be found from the following expressions:

$$\alpha_n = \alpha_d \pm 180 \text{ with } 0 \leq \alpha_d \leq 360 \quad (17)$$

$$\beta_n = 90 - \beta_d \text{ with } 0 \leq \beta_d \leq 90 \quad (18)$$

Accordingly, the normal vector to a plane (i.e., fracture set) is determined from the trend, α_n , and the plunge, β_n .

$$n = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} \cos \alpha_n \cos \beta_n \\ \sin \alpha_n \cos \beta_n \\ \sin \beta_n \end{bmatrix} \quad (19)$$

The orientation of the A_b -plane is clearly defined by the normal vector of fracture set #1 (n_{set1}) and fracture set #2 (n_{set2}). Furthermore, the normal vector, n_{Ab} , to the A_b -plane is calculated by the cross product of the normal vectors of fracture sets #1 and #2.

$$n_{Ab} = n_{set1} \times n_{set2} \quad (20)$$

The angle, γ_{3D} , between the normal vector of fracture set #3 and the normal vector to the A_b -plane is determined by the dot product:

$$\cos \gamma_{3D} = \frac{n_{Ab} \cdot n_{set3}}{|n_{Ab}| \cdot |n_{set3}|} \quad (21)$$

The limitations of this approach are manifold and comprise that fracture set #3 is fully persistent and that the 3D shape of the fractures, i.e., polygonal or circular, are not taken into account. Furthermore, scanline mapping or window sampling based on laser scanning or photogrammetric methods adds additional biases in the data set. These biases comprise that the observed fracture dimensions do not necessarily represent the actual fracture dimensions, given that they are locally obscured. The intact rock bridge failure that can be considered for fracture sets #1 and #2 cannot be considered for fracture set #3. The occurrence of more than three fracture sets cannot be considered as well.

Besides that, the difference between the trace length in 2D and the areal extent of the fractures in 3D must be considered. For example, three orthogonal fracture sets with circular-shaped fractures, trace lengths and frequencies of 1 m and 1 m^{-1} , respectively, can form a closed block area of 1 m^2 but cannot form a block volume of 1 m^3 . Consequently, the shape of the fractures must be known to be able to determine the IBSD in 3D. Large, bias-corrected datasets from fracture mapping by non-contact mapping methods may be the best option to gain all parameters required for 3D simulations.

However, these considerations have not been validated yet. Nonetheless, they highlight that the general impact of non-persistent fractures on the IBSD is even larger in 3D than in 2D. Further, intact rock bridge failure must play an important factor when comparing the block sizes and shapes of rockfall deposits with the in situ fracture set geometry.

Apart from this simplified comparison between 2D and 3D fracture networks, additional research is needed to prove and further develop the herein proposed approach for the applicability to 3D rock masses. For example, an extension of Wang's equation method [54] for non-persistent fractures to estimate the entire in situ block size distribution (IBSD) might be possible in the future by implementing the interconnectivity factor and adaptation of the correction factors. However, a comprehensive review of the 2D approach in terms of quality and practicality should be carried out before further development is pursued.

6. Conclusions

This contribution presents the approach and statistical results of 2D block areas of DFNs, which are formed by the intersection of non-persistent, non-parallel fracture traces. Besides that, the impact of rock bridge failure on the block area distribution is investigated statistically.

On the basis of stochastic analyses performing 600 simulations by using the MATLAB code, a correlation was found between the interconnectivity factor also see [12] and the difference between the mean block area assuming non-persistent fractures and the mean block area assuming simplified persistent fractures. This correlation allowed the development of an empirical relationship to estimate the mean block area based on the mean fracture set orientation, fracture set trace length and fracture set spacing.

Varying the threshold length of rock bridge failure in simulations with similar DFNs showed an approximately uniform shift of the cumulative distribution curves towards smaller block areas by increasing the rock bridge failure threshold length. Furthermore, there is a correlation between the ratio of mean block area with rock bridge failure to the mean block area without rock bridge failure and the geometric mean interconnectivity factor.

These empirical equations require input parameters only, which can be acquired in the field easily with great accuracy by applying classical fracture mapping methods (e.g., scanline surveys). These parameters comprise (i) the mean fracture set orientation (and its standard deviation), (ii) the normal set spacing, and (iii) the mean fracture set trace length. This direct relation to in situ measurements of fracture data represents a benefit of this approach compared to other DFN modelling approaches.

As a basis for DFN simulations to investigate block sizes in 3D, general mathematical equations are discussed to adapt the 2D analyses for 3D block volume problems.

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