

## Article

# Chaotic Communication System with Symmetry-Based Modulation

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**Abstract:** Communication systems based on chaotic synchronization are gaining interest in the area of secure and covert data transmission. In this paper, a novel digital communication technique based on a coherent chaotic data transmission approach is proposed. In general, this technique resembles the well-known approach based on the modulation of nonlinearity parameters. The key idea of this study is to modulate a signal by varying not the system parameter but the symmetry coefficient in discrete chaotic models obtained by the special numerical integration method. For this purpose, the self-adjoint semi-implicit integration method of order 2 is used to obtain discrete master and slave models of the considered chaotic oscillator. The experimental results explicitly show that, like during parameter modulation, transmitting and receiving oscillators may completely synchronize only if the symmetry coefficients are equal in both systems. The architecture of the communication system based on the proposed modulation is presented. The practical applicability of the approach is confirmed by transmitting a test binary sequence between the transmitter and receiver models and preliminary benchmarking of the obtained communication system. Since the symmetry coefficient modulation does not significantly impact the chaotic behavior of the transmitting digital system, its better suitability for covert messaging was experimentally confirmed by comparing it with the parameter modulation technique.

**Keywords:** chaotic communication system; symmetric integration; variable symmetry; covert communication



**Citation:** Karimov, T.; Rybin, V.; Kolev, G.; Rodionova, E.; Butusov, D. Chaotic Communication System with Symmetry-Based Modulation. *Appl. Sci.* **2021**, *11*, 3698. <https://doi.org/10.3390/app11083698>

Academic Editor: Christos Bouras

Received: 5 April 2021  
Accepted: 16 April 2021  
Published: 20 April 2021

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## 1. Introduction

Digital modulation is a widely used technique in modern communication systems, for example, in a direct down-conversion software-defined radio (DDC SDR) which is in high demand on today's market [1]. Digital modulation provides opportunities that are not possible for neither analog nor binary modulation. For example, it allows using arbitrary carriers, such as ultra-wideband signals and, which is of particular interest, chaotic signals, as well as applying advanced digital signal processing algorithms.

The idea of using chaotic oscillators to implement communication systems goes back to the discovery of the chaotic synchronization phenomenon. In the early work by Cuomo et al. published in 1993 [2], a secure communication system is proposed using based on two circuits implementing the Lorenz chaotic oscillator. The authors present two possible data transmission schemes: masking of the analog signal by chaotic signal, and bifurcation parameter modulation suitable for binary data transfer. This work stated the fundamental possibility of such type of communication. A similar solution based on the Chua circuit was described by Dedieu et al. [3]. One of the first papers that used chaotic communication for the transmission of complex signals such as speech was published by the Saratov nonlinear dynamics team [4].

Since these pioneering works, a large number of methods for modulating chaotic signals had been developed, based on different possible types of synchronization, including full, generalized, and phase synchronization. The prospective application of chaotic communication include wired [5], wireless [6], optical [7], and underwater acoustic [8] communication. The influence of noise and multipath propagation on the chaotic communication quality and the transfer rate had been extensively studied in the last three decades. Ren et al. [9] have shown analytically that the spectra of Lyapunov exponents preserve in a chaotic signal after being transmitted through a wireless channel. Authors state that this physical property provides excellent performance of the fully chaos-based wireless communication system in the presence of bounded noise and the relative motion between transmitter and receiver. Riaz et al. [7] listed some established advantages of direct chaotic communication, such as security, secrecy, potentially high data transfer rate, and the existence of a larger number of addresses in the channel, which allows increasing the transmission energy without a significant increase in interference. The high transfer rate may be achieved due to the possibility to encode several different symbols in a short waveform. The authors mentioned an experimentally reached rate of 1 Gbps with a bit error rate (BER) of  $10^{-7}$  when transferring data over optical fiber. Besides, many researchers have focused their research exclusively on the possibility of secure and covert data transmission using chaos [5,10].

Among the proposed chaotic communication techniques, two main classes may be distinguished—coherent and non-coherent systems. Coherent systems are based on the synchronization of chaotic oscillators, while non-coherent systems perform data recovery by detecting the features of the received chaotic signal without reproducing the signal on the receiver side. For both of these classes, several different modulation methods have been invented. Advanced non-coherent modulation is robust against linear and non-linear waveform distortions [8,11]. Coherent modulation systems are more sensitive to distortions and disturbances, but provide simpler ways of data transferring, as well as a higher potential level of security and secrecy. Therefore the enhancing of coherent data modulation schemes is an essential task in chaos applications [11].

In this paper, we propose a new coherent chaotic transmission method by modulating the symmetry parameter of the finite-difference model of a continuous chaotic system, obtained by symmetric semi-implicit integration, which is used for chaotic waveform generation. We will call this approach a *symmetry coefficient modulation* (SCM). The proposed type of modulation assumes the benefits of the fully chaos-based communication following from the preservation of the Lyapunov spectrum during signal transmission in the medium. The key feature of the SCM technique is its good suitability for covert communication.

The main contribution of the study is as follows:

1. A novel modulation technique exploiting the geometric properties of the discrete chaotic models obtained by symmetric integration is presented.
2. The proposed approach allows avoiding changes in phase space and spectral characteristics of the carrier because chaotic oscillators do not bifurcate during SCM.
3. The possibility of data transfer by symmetry-based modulation is experimentally shown. The throughput of the communication channel in the absence of disturbances is estimated.

The rest of the paper is organized as follows—in Section 2, the investigated chaotic system and the applied numerical method are described. Then, the architecture of SCM-based communication systems is proposed. Section 3 presents the results of numerical simulation and performance evaluation. Section 4 discusses some practical implementation issues, and Section 5 concludes the paper.

## 2. Materials and Methods

### 2.1. Synchronization of Chaotic Systems

Synchronization of two or more chaotic systems in a master–slave configuration can be performed using the different methods such as Pecora–Carroll common signal

linking [12], Hamiltonian forms [13] or the Open-Plus-Close-Loop approach (OPCL) [14]. In this study, the communication system is implemented using the classical Pecora-Carroll method. This technique involves the addition of the synchronization error multiplied by a parameter, so-called the synchronization coefficient, into one or more lines of the slave equation system.

Let us consider an autonomous  $n$ -dimensional chaotic system:

$$\dot{x} = f(x). \tag{1}$$

Split the system into two parts:

$$\begin{aligned} \dot{u} &= g(u, w), \\ \dot{w} &= h(u, w), \end{aligned} \tag{2}$$

where

$$\begin{aligned} u &= (x_1, \dots, x_m)^\top, \\ w &= (x_{m+1}, \dots, x_n)^\top, \\ g &= (f_1(x), \dots, f_m(x))^\top, \\ h &= (f_{m+1}(x), \dots, f_n(x))^\top, \end{aligned}$$

and  $m, n \in \mathbb{N}, m \leq n - 1$ . Now, one can construct new subsystem  $\hat{h}$  with independent variable subset  $\hat{w}$ , and involving variable subset  $u$  from the first system:

$$\begin{aligned} \dot{u} &= g(u, w), \\ \dot{w} &= h(u, w), \\ \dot{\hat{w}} &= \hat{h}(u, \hat{w}). \end{aligned} \tag{3}$$

Then, the difference  $\Delta w = \hat{w} - w$  tends to zero  $\Delta w \rightarrow 0$  as  $t \rightarrow 0$  due to the chaotic synchronization phenomenon [12], that is the generalization of the Haken’s slaving principle [15].

Applying Pecora-Carroll synchronization [12] to the well-known Rössler system [16] one can obtain the following equation:

$$\begin{aligned} \dot{x} &= -y - z; \\ \dot{y} &= x + ay; \\ \dot{z} &= b + z(x - c). \end{aligned} \tag{4}$$

Here,  $a, b$ , and  $c$  are the system parameters. In the study, let nonlinear parameters be  $a = 0.2, b = 0.2, c = 5.7$ . The ordinary differential equation (ODE) for master Rössler system is as follows:

$$\begin{aligned} \dot{x}_1 &= -y_1 - z_1; \\ \dot{y}_1 &= x_1 + a \cdot y_1; \\ \dot{z}_1 &= b + z_1 \cdot (x_1 - c); \end{aligned} \tag{5}$$

and the slave equation is:

$$\begin{aligned} \dot{x}_2 &= -y_2 - z_2 + k \cdot (x_1 - x_2); \\ \dot{y}_2 &= x_2 + a \cdot y_2; \\ \dot{z}_2 &= b + z_2 \cdot (x_2 - c); \end{aligned} \tag{6}$$

where  $k$  is a synchronization coefficient. For further examples of the Rössler systems synchronized by the Pecora-Carroll method refer to [17].

### 2.2. Semi-Implicit Symmetric Integration Method and Variable Symmetry Approach

The basic concept of diagonally-implicit numerical integration methods, or shortly D-methods, was reported in present works [18–20]. Such methods hold the computational

efficiency of explicit methods possessing higher stability and precision, as well as preservation of some geometrical properties of the solution while simulating conservative systems. The mentioned features were first achieved in Euler-Cromer and Stormer-Verlet semi-implicit algorithms, though they were created for solving Hamiltonian systems. The diagonal implicitness can be resolved by the simple iterations method [20], which makes it possible to expand the applicability of D-methods to a broader class of dynamical systems. The only limitation here is that D methods exist only for systems of dimension  $N \geq 2$ , degenerating for first-order systems to explicit and implicit Euler methods.

The composition D-method, known as the self-adjoint semi-implicit method or composition D-method (CD), is a particular case of diagonally implicit methods. The CD method proved to be well-suited for solving chaotic systems since it prevents chaotic trajectories of the discrete system from slipping to periodic or quasi-chaotic orbits during long-term simulation. Moreover, it does not suppress chaos in both conservative and dissipative systems [20].

Let us consider the CD method  $\Psi$  with integration step size  $h$ :

$$\Psi_h = \Phi_{h/2} \circ \Phi_{h/2}^* \tag{7}$$

It is a composition of a pair of basic adjoint D-methods  $\Phi_{h/2}$  and  $\Phi_{h/2}^*$  taken with halved step size  $h/2$ . In a discrete moment of time  $t_n$ , where the solution  $x_n$  is already known, one can apply the pair of methods  $\Phi_{h/2}$  consequently to obtain  $x_{n+1}$ .

Having a dynamical system of order  $N \geq 2$

$$\dot{\mathbf{x}} = f(\mathbf{x}), \mathbf{x} = (x_1, x_2, \dots, x_N)^T, \tag{8}$$

one should split it into  $N$  parts, for example for case  $N = 2$ :

$$\begin{aligned} \dot{u} &= f_u(u, w); \\ \dot{w} &= f_w(u, w). \end{aligned} \tag{9}$$

Then, the first adjoint method  $\Phi_{h/2}$  is:

$$\begin{aligned} u_{n+\frac{1}{2}} &= u_n + \frac{h}{2} \cdot f_u(u_n, w_n); \\ w_{n+\frac{1}{2}} &= w_n + \frac{h}{2} \cdot f_w(u_{n+\frac{1}{2}}, w), \end{aligned} \tag{10}$$

and the second adjoint method  $\Phi_{h/2}^*$  is:

$$\begin{aligned} w_{n+1} &= w_{n+\frac{1}{2}} + \frac{h}{2} \cdot f_w(u_{n+\frac{1}{2}}, w_{n+1}); \\ u_{n+1} &= u_{n+\frac{1}{2}} + \frac{h}{2} \cdot f_u(u_{n+1}, w_{n+1}). \end{aligned} \tag{11}$$

The first adjoint method is fully explicit, and the second adjoint method contains implicitness in the diagonal elements of the system matrix.

Note that the division of time step  $h$  directly by 2 in Equation (7) is just a special case of more generalized integrator. Let us introduce symmetry coefficient  $s \in (0, 1)$  and apply it to step size  $h$  to split it into two arbitrary parts:

$$\begin{aligned} h_1 &= h \cdot s; \\ h_2 &= h \cdot (1 - s). \end{aligned} \tag{12}$$

Then, a family of adjoint semi-implicit methods with variable symmetry, or VSCD methods, appears:

$$\Psi_{h,s} = \Phi_{h_1} \circ \Phi_{h_2} \tag{13}$$

These methods retain some of the basic properties of the CD method and provide the affine transform of the simulated system phase space without breaking the chaotic regime. Graphical interpretation of a certain VSCD method is presented in Figure 1.

In paper [21], Tutueva et al. discovered that master–slave synchronization with neglectable error cannot be observed if the symmetry coefficient of master and slave VSCD solvers are not equal. As an alternative of data transmission by the bifurcation parameter modulation [2,3], the symmetry coefficient modulation for data transfer was investigated. The possibility of adaptive control of symmetry coefficients was also shown, making integration methods of the VSCD family suitable to be applied to the design of digital chaos-based communication systems.

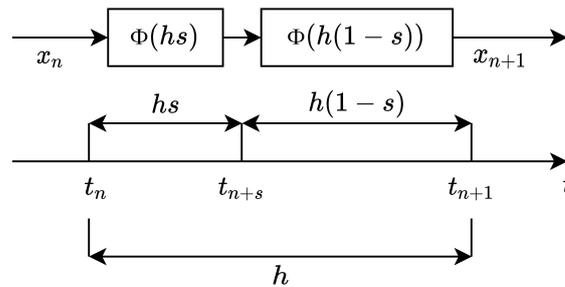


Figure 1. One integration step of the semi-implicit method with variable symmetry.

The discrete model of Rössler master system obtained by applying the VSCD method is as follows:

$$\begin{aligned}
 x_{n+s} &= x_n + h_1 \cdot (-y_n - z_n); \\
 y_{n+s} &= y_n + h_1 \cdot (x_{n+s} + a \cdot y_{n+s}); \\
 z_{n+s} &= z_n + h_1 \cdot (b + z_{n+s} \cdot (x_{n+s} - c)); \\
 z_{n+1} &= z_{n+s} + h_2 \cdot (b + z_{n+s} \cdot (x_{n+s} - c)); \\
 y_{n+1} &= y_{n+s} + h_2 \cdot (x_{n+s} + a \cdot y_{n+s}); \\
 x_{n+1} &= x_{n+s} + h_2 \cdot (-y_{n+1} - z_{n+1}).
 \end{aligned}
 \tag{14}$$

Then the implicitness can be resolved analytically:

$$\begin{aligned}
 x_{n+s} &= x_n + h_1 \cdot (-y_n - z_n); \\
 y_{n+s} &= (y_n + h_1 \cdot x_{n+s}) \cdot (1 - a \cdot h_1)^{-1}; \\
 z_{n+s} &= (z_n + h_1 \cdot b) \cdot (1 - h_1 \cdot (x_{n+s} - c))^{-1}; \\
 z_{n+1} &= z_{n+s} + h_2 \cdot (b + z_{n+s} \cdot (x_{n+s} - c)); \\
 y_{n+1} &= y_{n+s} + h_2 \cdot (x_{n+s} + a \cdot y_{n+s}); \\
 x_{n+1} &= x_{n+s} + h_2 \cdot (-y_{n+1} - z_{n+1}).
 \end{aligned}
 \tag{15}$$

The finite-difference model of the slave system is as follows:

$$\begin{aligned}
 x_{n+s} &= x_n + h_1 \cdot (-y_n - z_n + k \cdot (w_n - x_n)); \\
 y_{n+s} &= y_n + h_1 \cdot (x_{n+s} + a \cdot y_{n+s}); \\
 z_{n+s} &= z_n + h_1 \cdot (b + z_{n+s} \cdot (x_{n+s} - c)); \\
 z_{n+1} &= z_{n+s} + h_2 \cdot (b + z_{n+s} \cdot (x_{n+s} - c)); \\
 y_{n+1} &= y_{n+s} + h_2 \cdot (x_{n+s} + a \cdot y_{n+s}); \\
 x_{n+1} &= x_{n+s} + h_2 \cdot (-y_{n+1} - z_{n+1} + k \cdot (w_n - x_n)).
 \end{aligned}
 \tag{16}$$

Finally, the slave system model with resolved implicitness is:

$$\begin{aligned}
 x_{n+s} &= x_n + h_1 \cdot (-y_n - z_n + k \cdot (w_n - x_n)); \\
 y_{n+s} &= (y_n + h_1 \cdot x_{n+s}) \cdot (1 - a \cdot h_1)^{-1}; \\
 z_{n+s} &= (z_n + h_1 \cdot b) \cdot (1 - h_1 \cdot (x_{n+s} - c))^{-1}; \\
 z_{n+1} &= z_{n+s} + h_2 \cdot (b + z_{n+s} \cdot (x_{n+s} - c)); \\
 y_{n+1} &= y_{n+s} + h_2 \cdot (x_{n+s} + a \cdot y_{n+s}); \\
 x_{n+1} &= x_{n+s} + h_2 \cdot (-y_{n+1} - z_{n+1} + k \cdot (w_n - x_n)).
 \end{aligned}
 \tag{17}$$

### 2.3. Communication System Architecture

The proposed symmetry coefficient modulation approach is similar to the chaotic parameter modulation method [22]. The transmission of data is performed as follows. First, the message is encoded into a set of symbols corresponding to a defined non-binary  $n$ -symbols alphabet. Each symbol of the alphabet corresponds to a certain value of symmetry coefficient. The set of symbols forms an information message  $m(t)$  which affects the symmetry coefficient  $s$  on the transmitter side. The receiver consists of several slave systems, each with a specific symmetry coefficient corresponding to a particular symbol. The signal  $x(t)$  in the channel is applied to all receiver systems, and a synchronization error  $\Delta x$  between  $x(t)$  and each receiver response  $x_i^*(t), i \in [1, N]$  is calculated. When the value of synchronization error  $\Delta x$  on one of the receivers goes below a certain threshold, it is treated like a receiving of the symbol conformed to the symmetry coefficient on this receiver system. Thus, the accepted message  $m^*(t)$  is decoded symbol-by-symbol.

The physical receiver may be represented by a single digital device, where systems with different symmetry coefficients are executed (Figure 2), or a set of devices, where a certain symmetry coefficient corresponds to the certain device (Figure 3). In the second case, to transmit a message, an asynchronous protocol can be used, in which various duration of the synchronized and non-synchronized states will denote different symbols, in the simplest case, binary 0 and 1.

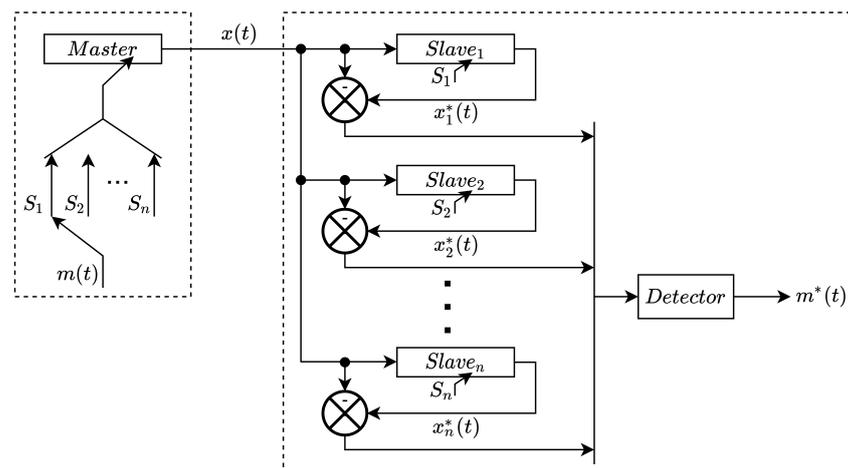


Figure 2. Block-diagram of the peer-to-peer SCM communication system with alphabet of  $n$  symbols.

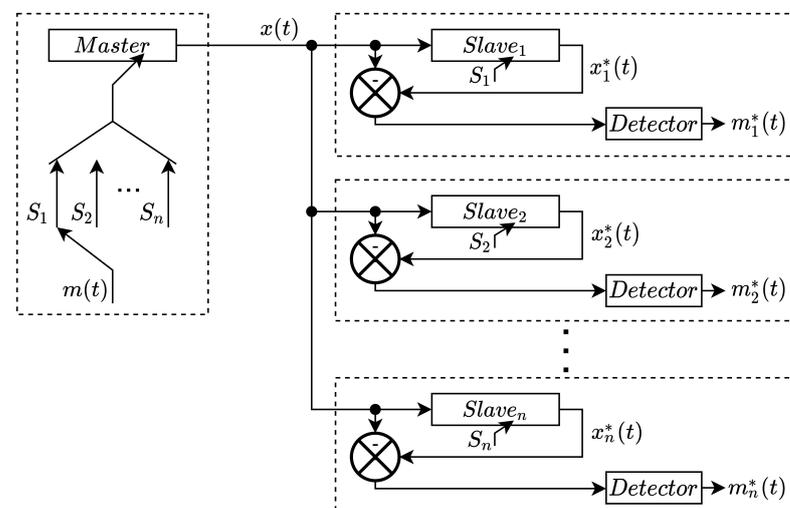
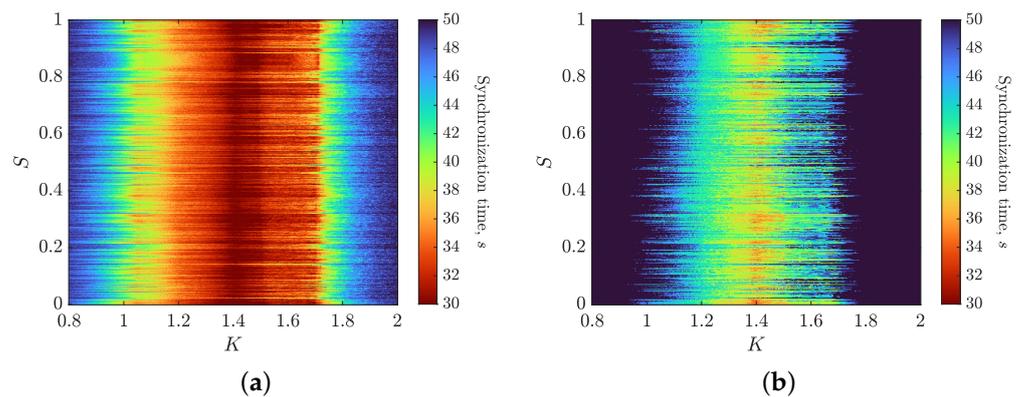


Figure 3. Block-diagram of one-to-many SCM communication system suitable for asynchronous binary coding with  $n$  receivers.

### 3. Experimental Results

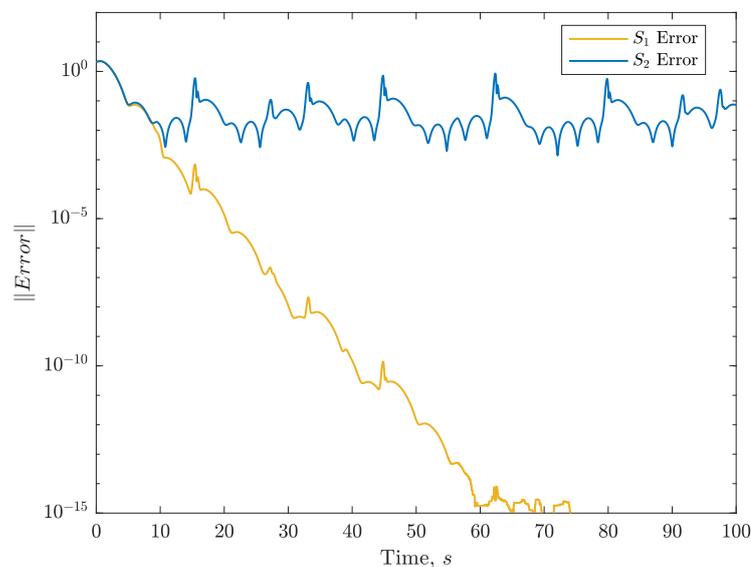
#### 3.1. SCM Transfer Analysis

Let us estimate the synchronization time for various initial conditions and determine the optimal symmetry and synchronization coefficients using average and maximum synchronization time. All computer experiments were performed with the extended floating-point data type using NI LabVIEW 2020 and MATLAB 2021a software. The results for two sets of different initial conditions for master and slave systems with size 200 are shown in Figure 4. One can see, the value of the symmetry coefficient has almost no effect on the synchronization time. For obtained results, one can conclude that the optimal value of  $k$  for Rössler system equals 1.4.



**Figure 4.** The dependence between average (a) and maximum (b) synchronization time and the symmetry and synchronization coefficients.

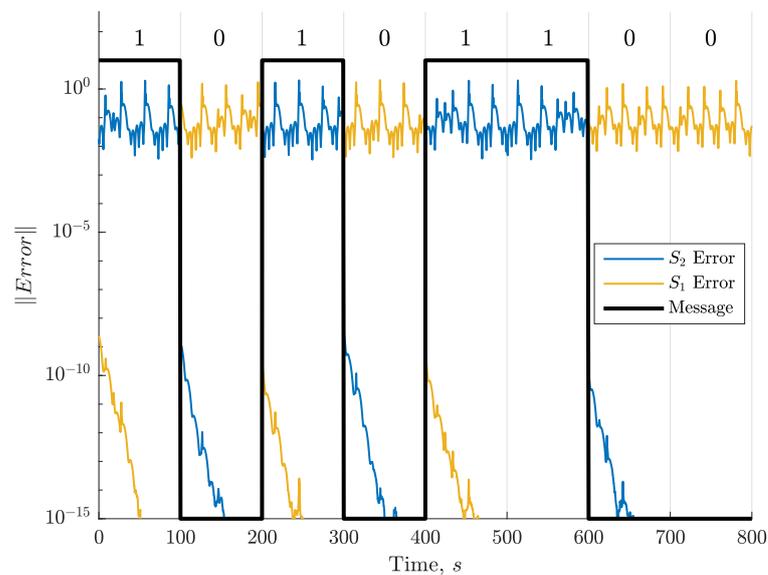
Consider a case of simple peer-to-peer data transfer based on synchronized discrete Rössler systems with adaptive symmetry. If the binary message is encoded by a two-symbol alphabet, then the transmitter is modulated with two different symmetry coefficients  $S_1$  and  $S_2$ . For example, one can take  $S_1 = 0.4$  and  $S_2 = 0.6$  to denote logical 1 and 0. In idle state, master oscillates with  $S_{idle} = 0.5$ . This value is important for the data transfer rate, as will be demonstrated further. To receive the message, a system consisting of two slave oscillators in a peer-to-peer configuration (see Figure 2) is needed. The synchronization error behavior in receiver systems during transmission of the logical 1 is shown in Figure 5.



**Figure 5.** Normalized synchronization errors at receiver systems during a symbol transmission associated with value  $S_1$  at the transmitter.

Note that here the Rössler system is considered in its natural time-scale (4). The system was simulated with the time step  $h = 0.01$  and required 100 s of simulation time per symbol. With this time step, the maximal synchronization accuracy for the double-precision data type was achieved within  $6 \cdot 10^3$  samples. An important observation here is that synchronization error  $\Delta x$  decreases exponentially over time. In a real communication system, chaotic oscillators can be executed with an almost arbitrary frequency depending only on a sample rate. If the transmission is carried out with a sampling frequency of 1 GHz, then the minimal synchronization error will be achieved in 6  $\mu$ s.

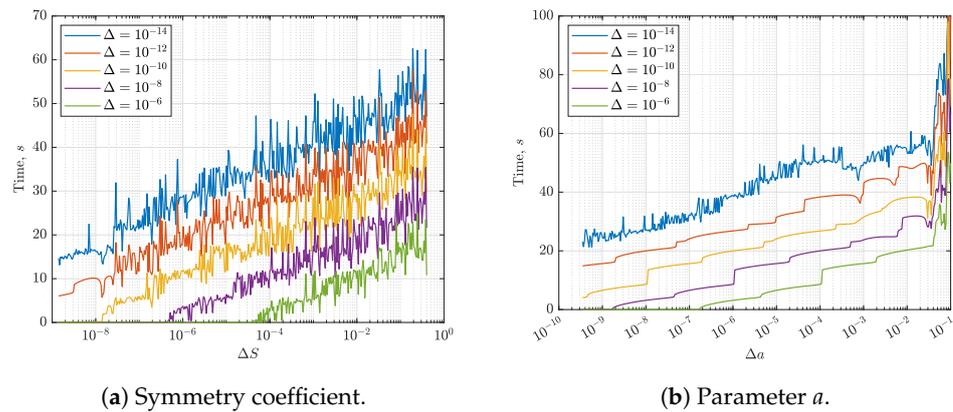
Let us illustrate the transmission of binary messages using the proposed modulation technique. The synchronization error behavior while transmitting the message “10101100” is shown in Figure 6.



**Figure 6.** The behavior of synchronization errors at the receiver during transmission of message “10101100”.

From a practical point of view, achieving the minimum possible synchronization error value during the transmission is unnecessary and even almost impossible due to the noise presented in a real communication channel. Meanwhile, it is sufficient that the synchronization error of one of the receivers responsible for the symbol associated with a specific value of  $S$  should fall below a certain threshold  $\Delta x_0$ , for example,  $\Delta x_0 = 10^{-6}$ . The second practical consideration is that for a case of the dense channel occupation, it is of interest to choose the smallest possible difference  $\Delta$  between the values of the symmetry coefficients associated with the symbols:  $\Delta S = S_i - S_j \rightarrow 0$ . An additional advantage of such an approach is that the trajectories of the master and slave systems with slightly different values of the symmetry coefficients do not diverge much, and when a certain symbol is transmitted, the error decreases below the specified threshold  $\Delta = x_{master} - x_{slave}$  faster.

By choosing different values of the synchronization accuracy threshold  $\Delta x$  and different delta values between the significant and idle synchronization coefficient  $\Delta S$  and between the significant and idle parameter  $\Delta a$ , one can obtain the plot shown in Figure 7. The noise in the data appears due to the influence of the initial conditions at the moment of the synchronization beginning. Thus, the scatter of values on each line shows the best and worst cases of synchronization time.



**Figure 7.** Dependence of synchronization time from the threshold synchronization error  $\Delta x$ .

### 3.2. Comparing the SCM Approach with Parameter-Based Modulation

To compare the proposed method of chaotic modulation with existing solutions, one can use bifurcation diagrams and bifurcation spectrograms. The concept of a bifurcation diagram includes several ways of plotting a phase variable on one axis and a parameter on another. One of the simplest ways to plot the bifurcation diagram using the time series is to use a peak detector [23].

The bifurcation spectrogram is calculated using the windowed Fourier transform. The signal  $s(t)$  is divided into parts, which usually overlap, and then a Fourier transform is performed to calculate the power spectral density  $P$  for each part:

$$\begin{aligned} S(t) &= FFT(s(t)), \\ P &= 20 \log_{20}(|S(t)|^2). \end{aligned} \tag{18}$$

The obtained spectra are plotted in one image. Each vertical line corresponds to a single spectrum. Unlike the widely used time-based spectrogram, the bifurcation spectrogram is plotted not depending on time, but depending on the value of the certain system parameter. The bifurcation parameter or the symmetry coefficient may serve as a parameter plotted along the X-axis of the bifurcation spectrogram.

The above-mentioned analysis techniques allow investigating one of the main disadvantages of communication systems based on parameter modulation for secure and covert data transmission, namely, the conspicuousness of the parameter switching process. Such switching can be detected by an interceptor while analyzing the signal spectrum. In contrast, modulation based on the symmetry coefficient switching does not affect the behavior of the system. The bifurcation and spectral diagrams of the parameters and the symmetry coefficient of the Rössler system are shown in the Figures 8a–d and 9a–d respectively. Note that symmetry coefficient modulation almost does not affect the system dynamics in comparison with system parameters modulation as it was theoretically predicted.

The comparison of steps from  $h = 0.05$  s to  $h = 0.001$  s is given in Figure 10. The smaller is the integration step, the smaller is the influence of the symmetry coefficient on the system dynamics. While decreasing the discretization step, the discrete system becomes closer to the continuous prototype, thus the specific values of the numerical integration method parameters lose their influence on the solution.

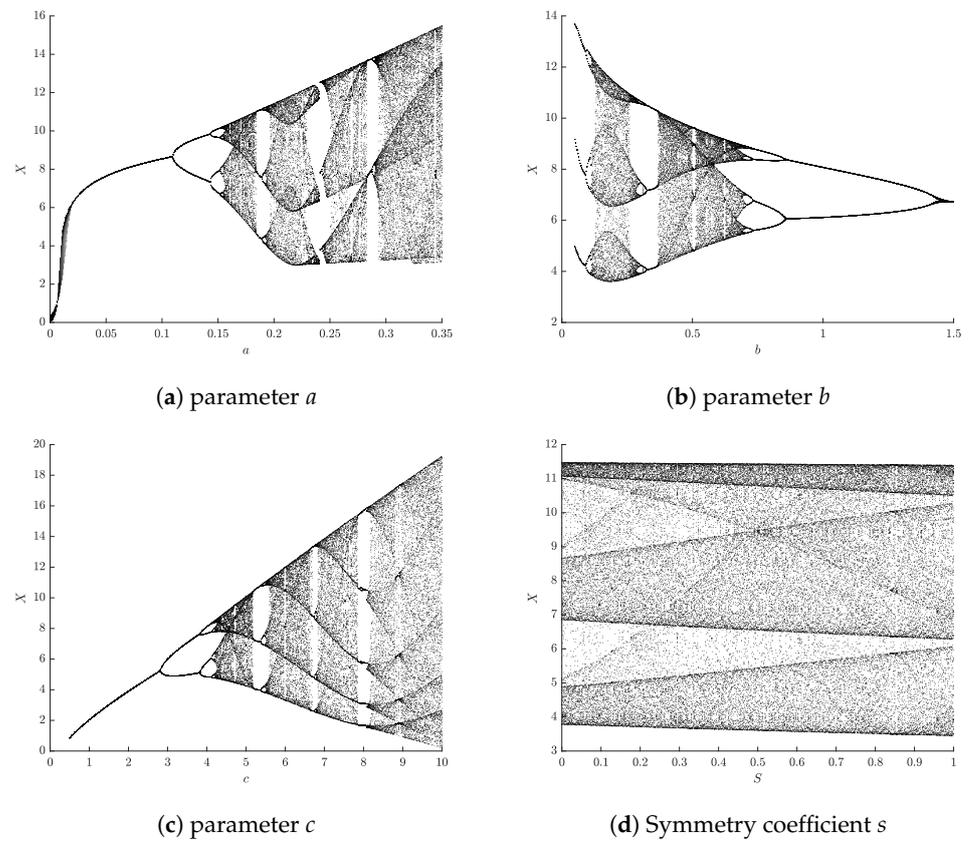


Figure 8. Bifurcation diagrams for Rössler system simulated with  $h = 0.01$  s.

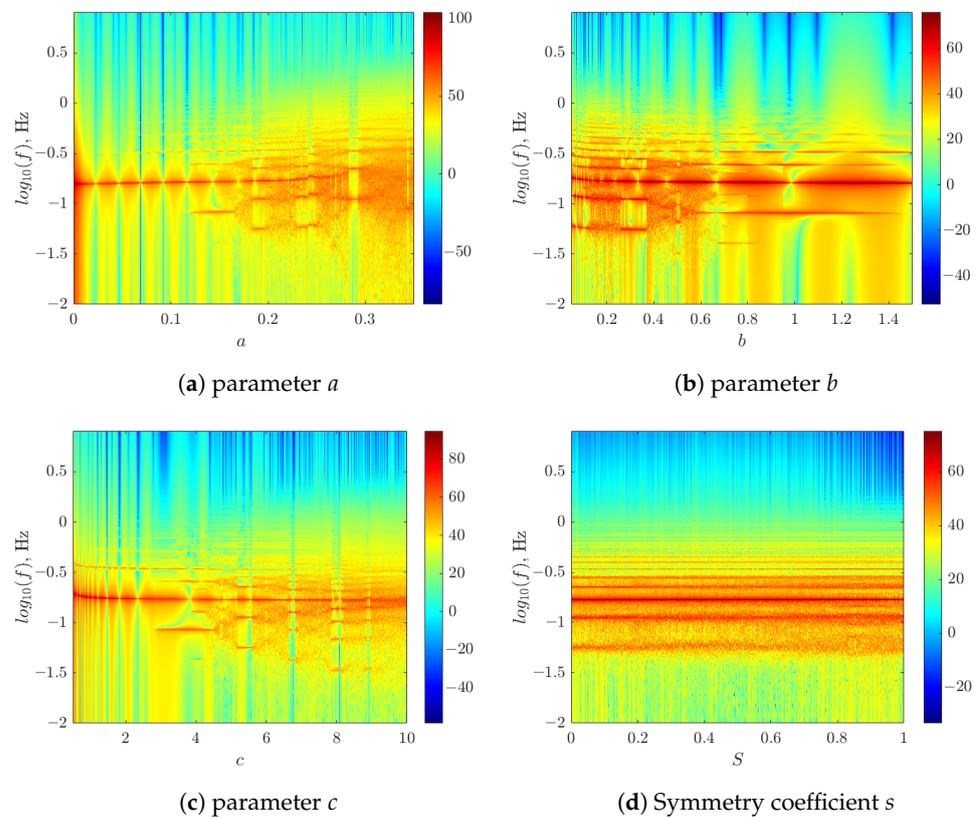
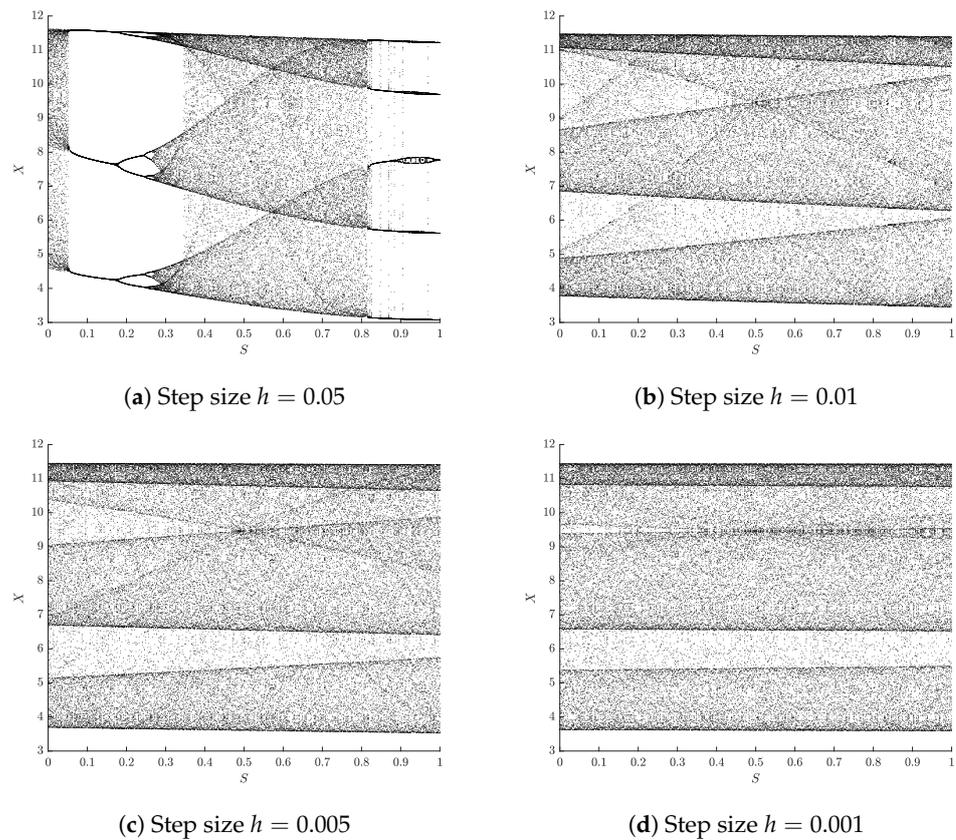
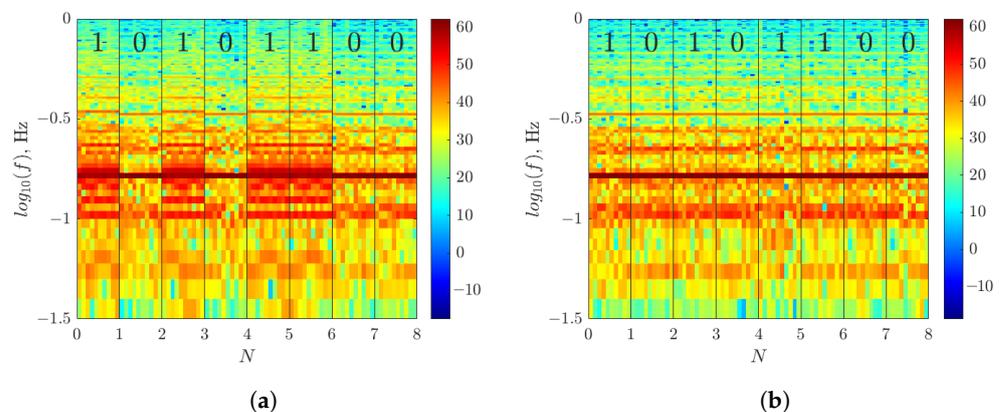


Figure 9. Spectrum diagrams for Rössler system simulated with  $h = 0.01$  s.



**Figure 10.** Bifurcation diagrams of the Rössler system for the symmetry coefficient variation at different numerical integration steps.

The revealed properties make data transmission by varying the symmetry coefficient much more secretive than transmission by modulating the bifurcation parameter. An illustrative example of transmitting the message “1010110” is shown in Figure 11. When a message is transmitted by modulating a parameter, specific patterns can be seen on the spectrogram. Meanwhile, the symmetry-based modulation allows the transmitted symbols to remain undistinguished. One can see, for symbol 1 there are areas where the most marked frequency 0.2–0.6 Hz, which is not the case when transmitting a symbol 0, during the message transmission using parameter modulation. However, such areas are not observed during the message transmission using symmetry coefficient modulation.



**Figure 11.** Spectrograms for message transmission using parameter modulation (a) and symmetry coefficient modulation (b). For parameter modulation, values  $a_1 = 0.2$  and  $a_2 = 0.22$  were switched while values of  $b = 0.2$  and  $c = 5.7$  were constant. For SCM case, symmetry coefficients were  $S_1 = 0.4$  and  $S_2 = 0.6$ , respectively.

#### 4. Discussion

The proposed symmetry coefficient modulation can be used as a basic technology for the practical development of communication systems. Note that in this paper only an alphabet of two symbols was considered, while there can be many more of them by analogy with quadrature modulation, in which there can be 1024 or more symbols. For practical systems, it is of interest to estimate the channel density at different signal-to-noise ratio (SNR) levels and the data transfer rate. In this article, we fundamentally do not consider the issues of noise in the channel, since chaos-based coherent systems are mostly used with noise reduction at the receiver side [11]. As for the transmission rate, it can be calculated from the data presented in Figure 7. The results are summarized in Table 1. To fill this table, one should take the worst of achieved synchronization times for a given  $\Delta S$  or  $\Delta a$ , time step  $h = 0.01$  s to find the corresponding number of samples, and estimate the time needed to transfer these samples having 1 GHz of sampling frequency. The resulting table approximately shows some realistic data transfer rates, which are possible with the SCM approach using an alphabet of 2 symbols.

**Table 1.** The comparison between dependence of the binary data transfer rate on the difference between symmetry coefficients and parameters  $a$  with a defined minimum synchronization error  $\Delta x = 10^{-6}$  ( $f_{\text{samp}} = 1$  GHz).

Symmetry Coefficient		Parameter $a$	
$\Delta S$	Transfer Rate, Mbit/s	$\Delta a$	Transfer Rate, Mbit/s
0.1	0.4	0.1	0.18
0.01	0.5	0.01	0.47
0.001	0.666	0.001	0.6

An example in Table 1 shows that SCM may outperform the parameter modulation in terms of the transmission rate, having another advantage in its spectrum stability resulting in a higher level of secrecy. A more detailed comparison of these systems, as well as with systems without chaos, is not the subject of this study, since transfer rate depends on many practically important factors: the used alphabet, the information encoding method, the number of users in the communication channel, the SNR, the physical transmission medium, and so forth. Some numerical characteristics of implemented chaotic communication systems can be found in the Introduction. Another important practical characteristic of a communication system is a bit error rate (BER). Likewise, the study of the communication system noise resistance is not the subject of this study, since it depends on the encoding method and the denoising techniques. Meanwhile, the reader may be interested in BER estimation for communication systems utilizing chaos, and this issue is addressed in several studies. For example, Kaddoum et al. [24] compare chaos-based DS-CDMA (direct-sequence code division multiple access) system with a conventional DS-CDMA system based on Gold codes and finds that the chaos-based DS-CDMA system outperforms the conventional DS-CDMA system when the spreading factor is low. Rulkov et al. [25] estimate BER of binary phase shift keying (BPSK) and chaotic-pulse-position modulation (CPPM) communication systems and finds that the chaotic communication system is slightly inferior to the conventional one.

#### 5. Conclusions

The key results of the reported study can be summarized as follows:

1. The proposed novel digital communication method based on the synchronization of chaotic systems (symmetry coefficient modulation, SCM) allows the transmission of binary-coded messages. The underlying theoretical concept of using semi-implicit models with variable symmetry coefficient for chaotic signals generation is experimentally verified.
2. It is shown that, unlike the bifurcation parameters, the switching of symmetry coefficient does not affect the behavior of the transmitting chaotic system, making the

symmetry-based modulation more suitable for covert messaging. A presented comparison with parameter-based modulation confirmed these theoretical assumptions.

3. We explicitly show that the transfer of the parameter by the modulation method leads to noticeable changes in the spectrum of systems even with a slight (10%) change in the bifurcation parameter, while a significant (more than 50%) change in the symmetry coefficient does not result in the significant difference in the spectrogram.

Our further research will be dedicated to the study of the channel noise influence on transmission and BER, as well as the development of efficient methods of noise reduction. The simulation of the attack to the proposed secure communication system by detecting spectral patterns and studying more known types of chaotic signal modulation is of great interest as well. In addition, a comparison of the proposed method with other coherent and non-coherent types of chaotic coupling and harmonic signal modulation will be performed.

**Author Contributions:** Conceptualization, D.B.; data curation, V.R. and G.K.; formal analysis, T.K. and D.B.; funding acquisition, T.K.; investigation, T.K. and V.R.; methodology, D.B.; project administration, T.K.; resources, G.K. and E.R.; software, V.R., G.K. and E.R.; supervision, D.B.; validation, V.R., E.R. and D.B.; visualization, T.K. and G.K.; writing—original draft, T.K., V.R. and D.B.; writing—review and editing, E.R. and D.B. All authors have read and agreed to the published version of the manuscript.

**Funding:** This study was supported by the grant of the Russian Science Foundation (RSF), project 20-79-10334.

**Informed Consent Statement:** Not applicable.

**Conflicts of Interest:** The authors declare no conflict of interest.

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