Article

Instability Study of Magnetic Journal Bearing under S-CO₂ Condition

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Abstract: A supercritical CO₂ (S-CO₂)-cooled Brayton cycle is under development for distributed power applications for remote regions. In order to successfully develop it, issues of controlling shaft levitation with bearings have to be solved. From several studies, magnetic bearings have been suggested for reliable levitation performance with reduced cost and complexity. However, several studies on magnetic bearing show that instability issues under high-pressure fluid and high-speed operating conditions may exist. The purpose of this research is to provide background for understanding the instability of magnetic bearings under S-CO₂ conditions and propose functional requirements of the magnetic bearing. Thus, the rotating shaft with magnetic bearings operating under high pressure fluid was first analyzed. To test the theory, a magnetic bearing test rig was constructed. By comparing experimental data to the analysis results, the analysis results were verified. Therefore, the analysis results can be used for predicting instability in the future and can contribute to the development of better magnetic bearing controllers.

Keywords: supercritical carbon dioxide Brayton cycle (S-CO₂ Brayton cycle); magnetic bearing; instability

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1. Introduction

There are many reasons why the supercritical CO₂ (S-CO₂) power cycle technology has been attracting attention from many researchers over the years. First, the S-CO₂ Brayton cycle can attain competitive efficiency (~50 %) with moderate temperature heat sources (450~750 °C) [1]. Its components are compact, and arrangement is simple. This leads to small footprints of the power system. These benefits mainly come from the specific properties of S-CO₂. These characteristics of an S-CO₂ power system make it possible to apply distributed generation [2].

However, in order to stay competitive with the S-CO₂ power cycle in the distributed power generation market, developing a bearing technology appropriate for the power cycle has been proven to be a major technical challenge [3]. To levitate the rotating shaft in the turbomachinery, a bearing system is necessary. Thus, design of a high-speed S-CO₂ turbomachine with a stable bearing system has always been accentuated for successful development of the S-CO₂ power cycle. A bearing system should be properly selected because of the power of an S-CO₂ power cycle, as shown in Figure 1 [4]. The gas foil bearing technology is very challenging to levitate a shaft for power systems above 3 MWe. For wide applications of S-CO₂ power systems, enhancing bearing performance is necessary. Hydrodynamic oil bearing and hydrostatic bearing can handle higher load than other bearing technologies. However, because most oils dissolve in S-CO₂, maintaining the purity of S-CO₂ without complex systems can be a failure for long-term reliable cycle operation [5,6]. Thus, magnetic bearing technology can be a favorable option for an S-CO₂ cycle with distributed generation with a long maintenance span.

TM Feature	Power (MWe)						
	0.3	1.0	3.0	10	30	100	300
TM Speed/Size	75,000 / 5 cm		30,000 / 14 cm		10,000 / 40cm		3600 / 1.2 m
Turbine type	Single stage		Radial		Multi stage		
					Axial		Multi stage
	Single stage		Radial		Multi stage		
					single stage		Axial Multi stage
Bearings	Gas Foil			Hydrodynamic oil			
				Magnetic		Hydrostatic	

Figure 1. Bearing options for supercritical carbon dioxide (S-CO₂) Brayton cycles for various power scales [4].

An active-control magnetic bearing (AMB) levitates the rotating shaft without direct contact by generating magnetic force using electromagnets. Recent AMBs are deal with many rotordynamic issues such as vibration suppression, stability control, and condition monitoring for flexible rotor-bearing systems [7]. Using a hybrid magnetic bearing is one solution for dealing with rotordynamic and power loss issues [8,9]. There has also been research on magnetic bearing fault detection and diagnosis for reducing the rotor instability [10].

Until now, the magnetic bearing research has mostly been dealing with the air gap condition. With the air gap condition, the fluid force is assumed to be negligible compared to the magnetic force and magnetic loss. However, under the S-CO₂ condition, this hypothesis has not been tested and validated before, to the best of the authors' knowledge. If magnetic bearings were to be used under S-CO₂ conditions, this hypothesis has to be also tested and verified. If this hypothesis is tested, the fluid force has to be considered, although a new control technology could overcome this issue; magnetic bearings can offer better S-CO₂ power system architecture. This is because higher bearing capacity can be achieved without an oil recovery system, which greatly simplifies the current S-CO₂ power system.

There have been several experiments of the radial AMB under S-CO₂ condition [11]. However, from these experiments, shaft control under an S-CO₂ environment was not reported as successful. This study repeatedly addresses the instability problem of high-speed operation tests under high pressure S-CO₂ conditions. Test data obtained from the Korea Atomic Energy Research Institute (KAERI) and Korea Advanced Institute of Science and Technology (KAIST) are shown in Figure 2 [12].

Figure 2. Magnetic bearing operation data from the Korea Atomic Energy Research Institute (KAERI) and Korea Advanced Institute of Science and Technology (KAIST): (a) under S-CO₂ conditions at 14,000 rpm (43.42 °C, 78 bar); (b) under room temperature and pressure at 30,000 rpm [12].

The instability causes the clearance between shaft and stator to disappear and leads to contact; therefore, the rotation speed cannot be increased in S-CO₂ conditions. On the contrary, in normal air conditions, higher rotation speed was achieved, and the magnetic bearing stably maintained clearance during the test. The gap between the shaft and the magnetic bearing was also filled with the working fluid; therefore, it can be concluded that the instability problem is related to the fluid conditions in the gap, such as pressure and density of the fluid. There have been several studies to suppress the oil whirl/whip to improve the stability [7] or considering the air gap as a disturbance for magnetic flux variation [13]. However, unlike the lubricant oil or air gap, the influence of S-CO₂ on the rotor-bearing system has not yet been investigated thoroughly. Therefore, the instability problem will be viewed with thin film lubrication first to analyze the interaction among S-CO₂ lubricant, bearing, and shaft.

The main challenges of using magnetic bearings under the S-CO₂ conditions and suppressing instability are: (i) scarcity of the experimental data; and (ii) insufficient understanding of how S-CO₂ can affect the dynamic loading of the rotor that has to be compensated by the magnetic bearing. In order to overcome these challenges, this paper first tries to develop a simplified lubrication model to understand the S-CO₂ effects on the rotor, and follows with a presentation of preliminary data. A correlation between the experimental results and the lubrication model predictions is attempted and presented in the paper. This correlation seems to verify the lubrication model and assists in finding the cause of instability. Eventually, the model could contribute to resolving the contemporary challenges.

In this paper, the lubrication model for S-CO₂ is first developed from the Reynolds equation by following the well-established method in the previous studies [14–16]. From the results of the lubrication model, the instability source is predicted. In order to test the hypothesis, an S-CO₂ experimental facility for magnetic bearing was designed and constructed. Finally, the correlations between obtained experimental results and lubrication model's predictions are discussed.

2. Magnetic Bearing Analysis for Instability Study

2.1. Magnetic Bearing Lubrication Model Development

The AMB in this paper was a hetero-polar 8-pole type. Eight electromagnets were located symmetrically with 400 μm clearance with respect to the air gap. The electromagnets were placed to exert the magnetic field perpendicularly to the shaft surface, as shown in Figure 3. The fluid flows through a thin film and gaps between electromagnets.

Figure 3. Cross section of a radial active-control magnetic bearing (AMB): (a) electromagnets in the magnetic bearing [17]; (b) possible secondary flow between the electromagnets.

To understand the phenomenon which causes unstable shaft levitation, the research on how the working fluid affects the rotating shaft becomes important because this phenomenon seems to depend on the operating conditions and the working fluid. Fluid influences the rotating shaft primarily with pressure distribution around the shaft. It is noted that a real magnetic bearing has a gap between electromagnets, as shown in Figure 3. Due to this gap, there can be a secondary flow occurring within the gap that can affect the pressure distribution of the shaft. However, to simplify the model development process, this gap is not modeled in this paper. In the future, the secondary flow effect can be studied by changing the magnetic bearing configuration.

To calculate the lubrication force exerted to the shaft, the pressure distribution around the shaft should be obtained. The method to describe the pressure distribution of the thin film fluid is suggested from many references by using the Reynolds equation [18]. This equation is obtained by substituting the velocity distribution from the Navier–Stokes equation into the continuity equation. By assuming uniform condition in axial direction due to fast rotation, the Reynolds equation with short bearing theory can be used as shown in Equations (1) and (2). It is noted that the adopted model is just the first step to model the magnetic bearing operating under the S-CO₂ conditions, and in the future more sophisticated models will be used to better understand and interpret the data.

The turbulent flow in bearings is studied based on the Taylor vortices, which become unstable to non-axisymmetric disturbance and distorted when the Taylor number, Ta , increases above its critical value, Ta_C . With the rotor and bearing design, if Ta reaches its critical value before the Re reaches 2000, the transition to vortex flow occurs. However, if the Re exceeds 2000 while Ta is still less than Ta_C , then the transition directly to turbulent flow occurs [19]. The Reynolds number in the bearing was higher than 50,000; therefore, the bearings were considered to be operating under fully turbulent flow. The turbulence effect was considered with the Ng-Pan model as described in Equation (2) [20]. The variables in the equations are fluid density ρ , pressure p , shaft surface's rotating direction velocity u , dynamic viscosity μ , time t , and clearance h .

$$\frac{\partial}{\partial X} \left(\frac{\rho h^3}{k_x \mu} \frac{\partial p}{\partial X} \right) = \frac{\partial(\rho h)}{\partial t} + \frac{1}{2} \frac{\partial(\rho h u)}{\partial X}, \quad (1)$$

$$k_x = 12 + 0.0388(Re)^{0.8} \quad (2)$$

To utilize Reynolds equation, the coordinate system should be determined as shown in Figure 4a. Furthermore, the shaft position and motion with this coordinate system should be prescribed. The shaft motion is assumed to be revolving with the same frequency of rotation. With this assumption, remaining information needed for the shaft position is only the biased length, e . The clearance h is described with R_2 , e and θ , as shown in Equation (3).

$$h = R_2 - e * \cos\theta - \sqrt{R_1^2 - e^2 \sin^2\theta}, \quad (3)$$

The governing equation (Equation (1)) was numerically solved with the finite difference method (FDM). The thin film geometry for the FDM is shown Figure 4b. Spatially uniform thermal properties were substituted into Equation (1). Then, the pressure distribution could be obtained by iterative calculation by updating the thermal properties based on pressure distribution in the previous step.

For the defined nodes shown in Figure 4b, the governing equation is simplified in matrix form. At first, the variables are bound as $k = \frac{\rho h^3}{k_x \mu}$ (left hand side), and $c = \rho h$, $d = \frac{\rho h u}{2}$ (right hand side). From this, governing Equation (1) becomes Equation (4).

$$\frac{\partial}{\partial X} \left(k \frac{\partial p}{\partial X} \right) = \frac{\partial c}{\partial t} + \frac{\partial d}{\partial X}, \quad (4)$$

Figure 4. Geometry of the unbalanced shaft and the stator: (a) geometry description; (b) coordinate system for the finite difference method (FDM).

To form a matrix, the variables are denoted as vectors, such as in Equations (5)–(8).

$$P(i) = P_i, \quad (5)$$

$$K(i) = \frac{\rho_i h_i^3}{k_X \mu}, \quad (6)$$

$$C(i) = \rho_i h_i, \quad C_-(i) = \rho_{i-1} h_{i-1}, \quad (7)$$

$$D(i) = \frac{\rho_i h_i u_i}{2}, \quad (8)$$

By setting the time step dt to be equal to dx/u , the C_- variable can describe the C vector with a time step. The spatial derivatives are denoted as diagonal matrices $\frac{\partial^2}{\partial X^2} = A$ and $\frac{\partial}{\partial X} = B$, as shown in Equations (9) and (10).

$$A(i,j) = \begin{cases} \frac{1}{(\Delta X)^2} & \text{if } j = i \pm 1 \\ -\frac{2}{(\Delta X)^2} & \text{if } j = i \\ 0 & \text{for else} \end{cases}, \quad (9)$$

$$B(i,j) = \begin{cases} \frac{1}{2\Delta X} & \text{if } j = i + 1 \\ -\frac{1}{2\Delta X} & \text{if } j = i - 1 \\ 0 & \text{for else} \end{cases}, \quad (10)$$

As a result, Equation (4) now becomes Equation (11).

$$P = \text{inv}(\text{diag}(BK)B + \text{diag}(K)A) * \left(\frac{1}{dt}(C - C_-) + BD \right) \quad (11)$$

Forces acting on the shaft are now calculated as below from the obtained pressure distribution. F_r is the force exerted on the shaft with the biased direction ($\theta = 0^\circ$), and F_t is the force with a direction of $\theta = 90^\circ$

$$\begin{aligned} F_r &= \sum \cos(\theta_i) * p_i * \Delta X * l \\ F_t &= \sum \sin(\theta_i) * p_i * \Delta X * l \end{aligned} \quad (12)$$

To consider fluid's compressibility, linear approximations of the thermal properties [21] are not considered because the thermal properties of S-CO₂ are nonlinear, near the pseudo-critical line. Instead, this model iteratively calculates by updating the thermal properties

from pressure distribution changes in previous steps until it converges. These processes are described in the flow chart shown in Figure 5. The code was implemented in MATLAB.

Figure 5. Flow chart of the lubrication model.

2.2. Magnetic Bearing Lubrication Model Analysis

To find the instability source, results from stable (air) and unstable (S-CO₂) operating conditions were compared. The inlet pressure range was selected to cover from 1 bar to 100 bar for the comparison. The temperature range was from room temperature (25 °C) to 70 °C. The rotational speed range was from 10,000 rpm to 30,000 rpm. The eccentricity ratio, ε , is the ratio between the eccentricity, e , and the averaged gap distance, $c = R_2 - R_1$. The range for ε was between 0.01 and 0.1.

From the lubrication model, the pressure distributions for several conditions were obtained. Figure 6 shows part of the calculation results. Figure 6 shows that larger differences in peak and valley values occur for pressure distribution when the eccentricity ratio is higher. This is because the skewed shaft forms larger density and physical wedges, contributing to larger pressure differences with respect to the angle [18]. From the obtained pressure distribution for various pressure and temperature conditions, forces were calculated. F_r and F_t of CO₂ and air for 30,000 rpm are shown in Figures 7 and 8, respectively.

Figure 6. Pressure distribution around a rotating shaft: (a) 7.5 MPa, 30 °C CO₂ inlet and 30,000 rpm conditions with various eccentricity ratio. (b) description of fluid force exerted on the shaft caused by pressure distribution

Figure 7. (a) F_r and (b) F_t of CO₂ for various conditions (30,000 rpm, $\varepsilon = 0.05$).

Figure 8. (a) F_r and (b) F_t of air for various conditions (30,000 rpm, $\varepsilon = 0.05$).

The calculated F_r and F_t values under various conditions for CO₂ show that operating conditions can potentially affect the stability due to the magnitude and directional change of forces under different fluid conditions. From Figure 7, it is shown that F_r acts in the opposite direction of the shaft position, and the F_t consistently acts in the $\vec{\omega} \times \vec{e}$ direction; thus, the attitude angle which is the shaft position when the net force is in the opposite direction of gravity will have a value between 1.5π and 2π . From the results, it is concluded that F_r in the S-CO₂ condition has a peak value at the pseudo-critical line, so F_r will dramatically change around it. In the case of F_t , it seems to be primarily proportional to the density.

In contrast, rapid change of F_r and F_t near the pseudo-critical line is not shown for an air-lubricated condition. From the comparison, the reason seems to be a nearly constant and low density of air, which is different from CO₂. Further calculations with high-density air (case 1) and S-CO₂ conditions (case 2) were performed using the developed lubrication model to analyze the influence of the dramatic change in F_r near the pseudocritical line.

The obtained results imply that forces acting on the shaft are influenced either by the fluid's high density or density gradient. To identify the major factor for the result, high-density air conditions under high pressure as well as low-density air under atmospheric conditions were evaluated with the developed model to contrast with CO₂ cases. To

explicitly show the influence of density and density gradient terms on the forces, the right-hand side (RHS) of Equation (3) is further separated, as shown in Equation (13).

$$\frac{\partial}{\partial X} \left(\frac{\rho h^3}{k_x \mu} \frac{\partial p}{\partial X} \right) = \frac{\partial(\rho h)}{\partial t} + \left(\frac{hu}{2} \right) \frac{\partial \rho}{\partial X} + \left(\frac{u}{2} \frac{\partial h}{\partial X} \right) \rho, \quad (13)$$

Each term on the RHS was spatially analyzed to be compared, as shown in Figure 9. The change in $\left(\frac{hu}{2} \right) \frac{\partial \rho}{\partial X}$ value with angular position is described in Figure 9a, and the change in $\left(\frac{u}{2} \frac{\partial h}{\partial X} \right) \rho$ value in Figure 9b. In Figure 9a,b, it is presented that the difference between case 1 and case 2 is larger in $\left(\frac{hu}{2} \right) \frac{\partial \rho}{\partial X}$ than $\left(\frac{u}{2} \frac{\partial h}{\partial X} \right) \rho$. This is because $\left(\frac{u}{2} \frac{\partial h}{\partial X} \right) \rho$ is less affected by $\frac{\partial \rho}{\partial X}$ when ρ itself is large. In conclusion, the pressure distribution is dominated by the density's spatial change under high-density conditions.

Figure 9. The distributions of (a) $\left(\frac{hu}{2} \right) \frac{\partial \rho}{\partial X}$, (b) $\left(\frac{u}{2} \frac{\partial h}{\partial X} \right) \rho$, (c) $\frac{\partial(\rho h)}{\partial t}$, and (d) the left-hand side (LHS) value of Equation (13) around the shaft with $\varepsilon = 0.25$ and 30,000 rpm.

The value of $\frac{\partial(\rho h)}{\partial t}$ is shown in Figure 9c as well, and it did not show any difference between air and CO₂. The left-hand side (LHS) around the shaft is shown in Figure 9d. The forces are summarized in Table 1. From Figure 9b,c, the influences of density and its temporal change in S-CO₂ conditions and air are very close to each other, showing that these terms are not the source of difference between air and CO₂. However, the spatial change of density in Figure 9a shows substantial differences, which causes the phase and magnitude differences in the forces. The combined results are shown in Figure 9d. From Table 1, it can be concluded that F_r grows due to the density change. This can explain the tendency in Figure 7a. Furthermore, because these forces are highly sensitive to the fluid properties, additional analysis regarding the fluid properties is required to increase the accuracy of the lubrication model.

Table 1. Forces acting on the shaft with various conditions ($\varepsilon = 0.25$ and 30,000 rpm)

Thermal Condition	F_r (N)	F_t (N)
Air at 8 MPa, -153.7°C	35.3	−1433.6
CO ₂ at 8 MPa, 30°C	265.2	−1344.7

3. Experiments with Magnetic Bearings Under S-CO₂ Conditions

3.1. Magnetic Bearing Experiment

The purposes of the magnetic bearing experiment were obtaining the shaft trajectory data and finding the correlation with the lubrication model prediction. After analyzing the correlation, the required AMB performance to levitate the shaft for prescribed operating conditions can be suggested.

From the analysis, it was shown that the fluid force is sensitive to the temperature and pressure. Therefore, the experiment was performed while considering those properties under control. The pre-existing S-CO₂ pressurizing experiment (S-CO₂PE) facility in KAIST was utilized to achieve appropriate experimental conditions for studying the magnetic bearing. S-CO₂PE consists of an S-CO₂ pump, a printed circuit heat exchanger (PCHE)-type pre-cooler and a water-cooling loop to control the temperature and the pressure. This facility is shown in Figure 10.

Figure 10. S-CO₂ power cycle experimental facility (S-CO₂PE) in KAIST.

The AMB test rig was constructed as shown in Figure 11. The test rig was a compressor mockup consisting of an AMB and its controller. The AMB has a displacement sensor for active control; therefore, this test rig had the advantage of obtaining shaft position data in real time. By removing the impeller from the compressor mockup, only the effect of the thin film surrounding the rotating shaft was analyzed. The main test data were the shaft position data from the displacement sensor in the AMB and the electric current into the electromagnets.

Figure 11. Design drawing of the test compressor with the AMB.

The relationship between the operation speed and the shaft balance with AMB is shown in Figure 12, which is the compressor's Campbell diagram. From this figure, the test speed 30,000 rpm is far from the critical speed, which is around 90,000 rpm.

Figure 12. Campbell diagram of test compressor.

To reach appropriate test conditions, the AMB test rig was connected to the S-CO₂PE, as shown in Figures 13 and 14. The booster pump was used to increase the inlet pressure of the S-CO₂PE. The pump generated flow to the AMB test rig and compensated the pressure drop. The generated heat was dissipated by the water-cooling system.

Figure 13. AMB and the compressor system for S-CO₂.

Figure 14. Layout of the magnetic bearing instability experiment.

The displacement sensor used in the magnetic bearing was an inductance type which used an AC coil and its inductance corresponded with the distance between the coil probe and the target. The output signal of the sensor had a sensitivity of $\pm 400 \mu\text{m}/2 \text{ V}$ with 16 bit. From the sensor test, the noise level was observed to be less than $1 \mu\text{m}$ of the RMS (Root Mean Square) and $3 \mu\text{m}$ of the peak. While the eddy current sensor's performance was sensitive to the sheet resistance change with different temperature, the inductance sensor was stable.

3.2. Data Analysis Method

The relationships between the unstable shaft rotation and the forces on the shaft were analyzed. The experimental results were analyzed from the shaft trajectory data and the electromagnet current data. The trajectory data were dynamically utilized to calculate the net force, F_{net} , on the shaft. Three major forces formed F_{net} : force from the lubricating fluid, F_{LUB} ; magnetic force from the bearing, F_{MB} ; and centrifugal force due to rotation of the unbalanced mass, F_{UB} . The free body diagram for those forces is displayed in Figure 15.

Figure 15. Free body diagram of the rotating shaft under AMB control.

To measure F_{net} , the mass center of the shaft was obtained. The shaft had an unbalanced mass center; therefore, the shaft's mass center has phase, β and distance, d , as shown in Figure 16.

Figure 16. Description of the unbalanced mass center.

β can be obtained by using Equation (14) [22].

$$\beta = \tan^{-1} \frac{2\xi f}{1 - f^2} \quad (14)$$

Frequency, f , was a known value. The damping ratio, ξ , was iteratively defined with F_{net} as the termination criterion. The d from the cross-section center could be obtained from the shaft design information. From the shaft design information, force due to unbalance (F_{UB}) was the same as the weight of the shaft when it operated at 37,000 rpm, as described in Equation (15). d can be obtained from this relationship.

$$F_{UB} = m * d * \omega_{37,000 \text{ RPM}}^2 = m * g, \quad (15)$$

To analyze the source of instability, other forces were calculated. The F_{MB} was calculated as a sum of forces from each electromagnet. The magnitude of each force, f , was calculated with Equation (16). The permeability of CO₂, μ , barely changed from μ_0 , even if CO₂ was flowing with high speed or had a large density variation. Thus, the magnetic field's affection to the hydrodynamic field was reasonably smaller than the shaft friction. Therefore, the interaction between the magnetic field and hydrodynamic field was ignored in this paper.

$$f = \frac{B^2 A_g}{2\mu_0} = \frac{\mu^2 N^2 I^2 A_g}{2\mu_0 I_g^2}, \quad (16)$$

F_{MB} had several losses; therefore, the F_{MB} in Equation (16) was calibrated under the vacuum test. The vacuum test created an environment to nullify F_{LUB} , leaving only the gravitational force. Finally, F_{LUB} was measured from experiments and it was compared to the calculated values from the lubrication model results to verify the model. F_{LUB} was obtained by subtracting the calibrated F_{MB} , F_{UB} , and gravity from the measured F_{net} obtained from the shaft trajectory data.

3.3. Results

S-CO₂ tests were performed under four different conditions: test 1: 8 MPa and 36 °C, (350 kg/m³); test 2: 9 MPa and 44 °C (350 kg/m³); test 3: 8 MPa and 40 °C (280 kg/m³); and test 4: 9 MPa and 49 °C, (290 kg/m³). These four conditions were selected to test the bearing under two different densities and two different pressure conditions. This is presented in Figure 17.

Figure 17. CO₂ density map for test condition.

The shaft trajectory data under S-CO₂ conditions are shown in Figure 18a and show that the shaft does not keep a single revolving center under S-CO₂ conditions when the rpm increases. Therefore, the changing revolution center and shaft's relative position is traced in real time for obtaining the β .

Figure 18. Shaft trajectory data from the (a) air test and (b) S-CO₂ test 1 at 30,000 rpm.

From the data, F_{LUB} and F_{MB} were calculated. The F_{LUB} values with respect to the shaft's position are shown in Figure 19, and F_{MB} values are in Figure 20. From Figure 19, the F_{LUB} acts in a direction almost parallel to the shaft velocity. The difference between test 1 and test 4 is that the radial components of F_{LUB} , $F_{LUB,r}$ in tests 1 and 2 are changed more frequently than in tests 3 and 4. This can be explained with Figure 7a, which describes F_r with density change. From Figure 20a, F_{MB} has constant force to work against the shaft weight, $F_{gravity}$. F_{MB} plus $F_{gravity}$ is plotted in Figure 20b to eliminate the constant force of the AMB. From Figure 20b, the F_{MB} acts as a second-order system which has stiffness and damping. However, F_{LUB} (Figure 19) does not act as a restoring force as F_{MB} does. In rotor dynamics, the force acting like F_{LUB} is follower-force from the cross-coupled stiffness, which can increase the energy of the shaft motion with time [23].

The force magnitudes with various ε values during the experiments are summarized in Figure 21. From the fitted line, it is shown that F_{LUB} is proportional to the ε , but the relationship is weak and the F_{LUB} has a higher magnitude in test cases 1 and 2 than cases 3 and 4. It is believed to be due to dramatic density change near the pseudo-critical line because the test cases 1 and 2 have more scattering of the data. The R^2 values of test 1, test 2, test 3, and test 4 are 0.1624, 0.1708, 0.1950, and 0.2370, respectively. It is inferred that the reason for this difference is due to the frequent change in $F_{LUB,r}$ due to density sensitivity which is shown in Figures 7 and 19.

The F_{MB} versus ε had a similar distribution with F_{LUB} , as shown in Figure 22. The reason for this wide distribution is inferred to be a response to unstable F_{LUB} . However, F_{MB} was not significantly difference among these four tests, unlike F_{LUB} . It seems that the performance of the AMB used in the experiment reached the limit to react to S-CO₂'s F_{LUB} .

Figure 19. F_{LUB} distribution vs. shaft position from the 30,000 rpm S-CO₂ test 1 (a); test 2 (b); test 3 (c); and test 4 (d).

Figure 20. (a) F_{MB} and (b) $F_{MB} - F_{gravity}$ distribution vs. shaft position from the 30,000 rpm S-CO₂ test 1.

(a)

(b)

(c)

(d)

Figure 21. F_{LUB} distribution vs. ϵ from 30,000 rpm S-CO₂ test 1 (a), test 2 (b), test 3 (c), and test 4 (d).

(a)

(b)

Figure 22. *Cont.*

Figure 22. F_{MB} distribution vs. ϵ from the 30,000 rpm S-CO₂ test 1 (a), test 2 (b), test 3 (c), and test 4 (d).

4. Summary and Discussion

In this paper, a lubrication model under S-CO₂ conditions was developed and compared with air conditions to investigate the instability of bearings under S-CO₂. From the S-CO₂ to air comparison, it was shown that the bearing rotating under S-CO₂ conditions can have more issues, which is caused by the property change of S-CO₂ near the critical point and pseudo-critical line. From the model results, the AMB operating under S-CO₂ conditions was tested while thermal properties were being controlled. In these experiments, the lubrication model revealed that thermal properties of S-CO₂ can affect the AMB performance. The model predicted that high density and large density gradient changes can affect the shaft motion by affecting the magnitude and direction of the F_{LUB} value. The obtained F_{LUB} from tests 1 and 2 showed substantial scattering with respect to the eccentricity due to unstable rotation. The observed phenomenon seemed to be due to the high sensitivity of the density of S-CO₂ with respect to the pressure and temperature changes.

The presented experimental data show the decomposition of forces acting on the rotor under the S-CO₂ conditions with magnetic bearings supporting the rotor. To the best of the authors' knowledge, this is the first study to actually present how various forces act on a rotor under S-CO₂ conditions. However, because this study is still at the early stage, further improvements of the experiment are necessary to better understand the phenomenon as well as to accumulate more data under different operating conditions to expand the database. Moreover, the lubrication analysis model has ideal assumptions, and each assumption has to be carefully re-evaluated to improve the model prediction capability. For instance, assumptions such as a single revolving center, uniform temperature distribution, and smooth geometry have to be scrutinized and improved where necessary.

In the future, to overcome those shortcomings, the thermal properties inside the bearing should be measured to reveal clearer relationships between fluid properties and the destabilizing forces applied to the shaft. In addition, the non-ideal geometry of the electromagnets, which is considered to be another disturbance source, has to be modified in further study to match the model more closely to the experiment. By implementing the suggested improvements, it is expected that the unstable region can be clearly defined and the required performance or design of the AMB can be more precisely determined.

The dynamics of the shaft must also be investigated. Moreover, the relationship between F_{LUB} and shaft motion can be better described as a second-order system. The trajectory of the shaft can be predicted from the data and compared to the experimental results. After validation, this model can be utilized in various S-CO₂ cycle systems. Then, required performance and control logic of the AMB can be suggested for each S-CO₂ cycle system.

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