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Determining the Quadratic Electro-Optic Coefficients for Polycrystalline $\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3\text{-PbTiO}_3$ (PMN-PT) Using a Polarization-Independent Electro-Optical Laser Beam Steerer

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Abstract: A polarization-independent electro-optical (EO) laser beam steerer based on a bulk relaxor ferroelectric polycrystalline $\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3\text{-PbTiO}_3$ (PMN-PT) was developed in this study to steer light ranging from visible to mid-IR wavelengths. A large number of the resolvable spots was achieved with this EO steerer. A Fourier-transform infrared (FTIR) spectroscopy was employed to determine the refractive index of the polycrystalline PMN-PT over a wide range of optical wavelengths. Besides measuring the transmission of this material, the capacitance bridge analysis was used to characterize the effect of temperature on the dielectric constant of PMN-PT. The performance of the steerer over a variety of wavelengths was simulated using COMSOL Multiphysics. The deflection angle for the wavelengths of 532, 632.8, 1550, and 4500 nm was measured in the lab in terms of mrad.mm/kV at two different temperatures and compared to the simulation results. The quadratic Kerr electro-optic coefficient and the halfwave electric field were determined for those four wavelengths at two different temperatures. The results showed polycrystalline PMN-PT has a large quadratic EO coefficient for visible light, almost as large in the near IR, but drops significantly in the mid-IR. No significant temperature dependency for the EO coefficients was observed for any of those four wavelengths.

Keywords: relaxor ferroelectrics; Kerr effect; beam steerer; polycrystalline PMN-PT; halfwave electric field; Fourier-transform infrared (FTIR) spectroscopy; mid-IR

1. Introduction

The electro-optic (EO) effect refers to producing birefringence in an optical medium in response to a slowly varying external electric field compared to optical frequencies [1,2]. The change in the medium's refractive index can be proportional to the electric field or the square of the electric field. The former is called the linear EO effect, or Pockels effect, observed in crystals with a lack of centrosymmetry such as transparent ferroelectric crystals. The latter is called the nonlinear EO effect, or Kerr effect, observed in almost all transparent materials [3]. The EO materials in which the Pockels effect or linear EO effect is dominant, and hence, they exhibit linear EO response, are called linear EO materials. Nonlinear EO materials are referred to as the materials in which the Kerr effect or nonlinear EO effect is dominant.

Besides the Pockels effect, the ferroelectric transparent crystals also exhibit piezoelectric and pyroelectric effects, and their performance is temperature-dependent [4,5]. After a specific temperature, called the Curie temperature, where the dielectric constant peaks, the material becomes paraelectric, and the Kerr effect becomes dominant [1,5]. Although all transparent materials show a Kerr effect, this effect is very significant in some liquids like nitrobenzene and nitrotoluene and some homogenous solid crystals such as the polycrystalline PMN-PT [5] and paraelectric KTN crystal [6,7], which is the ferroelectric KTN heated above its Curie temperature.

A large Kerr effect is also observed in metal, organic polymeric nanoparticles, and quantum dots [8–10]. Quantum dots are metallic nanoparticles smaller than 10 nm. Thakur and Van Cleave measured the Kerr coefficient of gold nanoparticles in a glass and showed the magnitude of the Kerr coefficients increases rapidly by decreasing the diameter of nanoparticles [8]. They showed the subnanometer size particles exhibit the largest Kerr coefficients of any materials. However, to reach an appreciable optical path delay (OPD), a long nano sample is required. It is also challenging to make the sample thick enough to provide a uniform external electric field and address a large aperture. Furthermore, nanoparticles' Kerr effect is very dispersive since the wavelength should be very close to the onset of the optical absorption due to surface plasmon resonance. Those facts limit the industrial application of nanoparticles, especially when a larger aperture and broader bandwidth of optical frequency are necessary.

The EO effect is used to deflect light in EO beam steerers [1]. The EO steerers refract the light by imposing a phase delay due to changing the medium's refractive index [7]. The steering angle is varied by adjusting the voltage applied on the crystal to electro-optically control the phase delay across the beam cross-section [2,11]. Therefore, a bulk EO crystal beam steerer consists of a transparent EO bulk crystal with electrodes deposited on its surface or the ferroelectric domains engineered to multiple prisms under a variable external electric field [11].

EO beam steerers offer fast and random access beam steering compared to mechanical optical beam steering methods such as polygons, microelectromechanical, or galvanic mirrors, which are limited by mechanical inertia effects. EO beam steering systems are also more efficient and have higher bandwidth than acousto-optic (AO) beam deflectors. The EO scanning method offers high optical efficiency and physical robustness [3,5,7,12–14].

EO beam steerers can be arranged to meet a variety of requirements per application. Appropriate choice of the drive electronics, optical path, and especially the EO crystal must be made to address specific functional requirements. Any change in the crystal, the pattern of the electrodes, and the applied voltage can significantly affect the performance of the EO deflectors. Therefore, the design problem is to find an EO crystal, an optimized electrode arrangement, and an optimized amount of voltage to get the magnitude of desired refractive index change with sufficient speed [3].

Comparing to liquid crystals and other liquids exhibiting an EO effect, the transparent solid electro-optical oxides are the only type of materials featuring both significant and fast EO effects. Solid oxide EO beam steerers can be considered capacitors that are quickly charged with a voltage leading to a bandwidth in the range of a few MHz [3,5,15]. Some new and some widely used solid electro-optical oxides are listed in Table 1, where n is the refractive index, ϵ is dielectric constant or relative permittivity, and r is the linear electro-optical coefficient [1,2,16].

An ideal crystal for EO devices and applications should have a high EO coefficient and high index to provide enough phase delay at a lower applied voltage. It should also have a low dielectric constant to offer a low capacitance and a fast response [3]. Therefore, a crystal with a higher n^3r/ϵ in Table 1 will be more attractive for EO devices. In addition to those criteria, an ideal EO crystal should be inexpensive and available in larger sizes to promote use for a larger aperture in order to offer a larger number of resolvable spots. Furthermore, it should have high transmission and low dependency on temperature and polarization.

The refractive index and EO coefficient of EO crystals vary by wavelength and temperature [17,18]. The dielectric constant of a crystal also varies by temperature [1,4,6]. For example, the EO coefficient of KD*P is almost independent of wavelength, but it varies by temperature. For LiNbO₃, not only the EO coefficient varies with wavelength and temperature but also changes by modulation frequency [4,18]. As seen in Table 1, the EO coefficient of BBO is much smaller than other crystals. However, because its relative permittivity is low, it can handle ultra-high peak power or average power light fluences. In general, if an EO crystal is being used at a temperature far enough from its Curie temperature, its permittivity and EO coefficient are relatively temperature-independent.

Table 1. The properties of some materials with linear electro-optic (EO) response.

Material	n	ϵ	r (pmV ⁻¹)	n^3r/ϵ (pmV ⁻¹)
Ferroelectrics				
LiNbO ₃	2.26	29	31	12.339
LiTaO ₃	2.23	43	31	7.995
KNbO ₃	2.23	55	64	12.904
BaTiO ₃	2.4	1800	1670	12.826
SBN75 (Sr _{0.75} Ba _{0.25} Nb ₂ O ₆)	2.3	1550	1340	10.519
SBN60 (Sr _{0.6} Ba _{0.4} Nb ₂ O ₆)	2.3	750	216	3.504
Ferroelectric KTN (KTa _{0.35} Nb _{0.65} O ₃)	2.3	10,000	8000	9.734
KBN (K ₂ BiNb ₅ O ₁₅)	2.4	510	346	9.379
KBN (K ₂ BiNb ₅ O ₁₅) Ce ³⁺ :Ce ⁴⁺	2.4	509	370	10.049
BSTN	2.4	345	200	8.014
BSKNN-2	2.35	800	380	6.164
BSKNN-3	2.301	800	420	6.396
SCNN	2.321	1740	1200	8.623
BBO (BaB ₂ O ₄)	1.67	6.7	2.7	1.877
Semiconductors				
GaAs	3.5	13.2	1.2	3.898
InP	3.29	12.6	1.45	4.098
CdTe	2.82	9.4	6.8	16.223
Bi ₁₂ SiO ₂₀	2.54	56	5	1.463
Bi ₁₂ GeO ₂₀	2.55	47	3.4	1.200

Casson et al. calculated the EO coefficient of LiTaO₃ in four different wavelengths by measuring the deflection angle of a horn-shaped EO scanner [12]. They concluded that the EO coefficient of LiTaO₃ decreases from 30.2 pm/V for HeNe laser (632 nm) to 27.1 pm/V for diode laser (1558 nm). Although using the EO crystal as an EO scanner and measuring each wavelength's deflection angle is a precise method to determine the EO coefficient of that crystal, they did not consider refractive index variation with wavelength. The deflection angle depends on the EO coefficient and the cube of the refractive index, which is varied by wavelength [1]. In addition, the temperature dependencies of the EO coefficient, refractive index, and permittivity were not measured in that study.

Most EO beam steering systems use transparent single ferroelectric crystals exhibiting a linear EO effect like lithium niobate and lithium tantalate to steer the light beam [12,17]. However, the disadvantages are that those single crystals are highly temperature-dependent and have a low EO effect [5,19]. Therefore, the applied voltage should be very high. Single ferroelectric crystals also need to be poled, and their crystalline orientation should be domain-engineered like multi-prisms [12,17,20]. Furthermore, single ferroelectric crystal steerers are polarization-dependent, and the number of resolvable spots is low because of the small beam size. Those issues have made the EO beam steerers impractical for many industrial applications.

Although the beam after a deflection in an EO steerer can be expanded in a telescope, that will decrease the deflection angle proportional to that beam magnification as follows [11]:

$$\theta_2 = \frac{\theta_1}{M} \quad (1)$$

where θ_2 is the deflection angle after expansion by the factor of M , and θ_1 is the deflection angle before beam expansion. Therefore, deflecting an 80 μ m beam to 125 mrad is equivalent to deflecting a 6 mm beam to 1.67 mrad. The factor that remains constant is sometimes called an angle/aperture product [3].

Instead of the deflection angle, which is not a complete metric for beam steerers, we describe the number of the resolvable spots instead as the metric for light beam steerers as follows [1]:

$$N = \frac{\theta}{\theta_{beam}} \quad (2)$$

where θ is the deflection angle depending on the geometry, optical, crystallography properties of the crystal, the type of EO effect, and the strength of the external electric field, and θ_{beam} is the beam divergence angle in far-field, which is calculated as [1]

$$\theta_{beam} = \frac{\lambda}{\pi w_0} \quad (3)$$

where λ is the wavelength of the light, and w_0 is the radius of the beam, which can usually be up to half of the crystal height (H) or width (W) as

$$Max\ w_0 = 0.5 \times Min\{W, H\} \quad (4)$$

The first EO steerer was introduced by Fowler et al. in 1964 using KH_2PO_4 as the EO crystal [21]. This crystal is paraelectric with tetragonal symmetry at room temperature. Although its Curie temperature is below $-150\ ^\circ C$, which makes its optical properties almost temperature-independent, this crystal is soluble in the water. In addition, a very large voltage should be applied to provide enough phase delay, making it impractical.

Chen et al. employed potassium tantalate niobate as a single EO crystal in their EO scanners [22]. The most used single crystal in EO steerers is $LiTaO_3$. Hisatake et al., Scrymgeour et al., Casson et al., and many more used $LiTaO_3$ in their EO scanners [12,13,17,20,23,24]. Because both of those two crystals are single ferroelectric crystals, their dipole orientations must be adjusted by applying more than 21 kV/mm voltage. In addition, multiple prism-shaped domains must be created to steer a small beam to a few degrees by applying more than 14 kV/mm on the crystal. Furthermore, those crystals are temperature-dependent [5,15], and their EO scanners were polarization-dependent. Polarization dependency makes the scanners deflect the light if it is only polarized in a certain direction. Therefore, polarization dependency limits these scanners' application even in one dimension and makes it impossible to steer the light in two dimensions in one crystal.

Nakamura and Nakamura et al. developed an EO scanner based on the Kerr effect and space-charge-controlled electrical conduction using paraelectric KTN [7,25]. They heated a 0.5 mm thick small KTN single-crystal slab with uniform titanium electrodes up to a few degrees above its Curie temperature to inject electrons into the crystal, taking advantage of the crystal's high permittivity at around its Curie temperature [6]. They reported deflecting an 80 μm beam to 125 mrad by applying 500 V/mm, resulting in less than 20 resolvable spots [7]. Although this scanner uses another physical effect besides the EO effect and has the lowest applied voltage reported so far, it has several limitations. First, KTN does not have an excellent transmission in near IR and mid-IR. Moreover, adjusting the temperature in this scanner is crucial to ensure the KTN is in its paraelectric region and the permittivity is high enough to inject the electrons. In addition, using a crystal at its maximum dielectric constant will result in high loss and slow response. Furthermore, because it is impossible to inject electrons in thick dielectrics, the crystal must always be a fraction of a millimeter thick or less, which significantly limits its beam size and number of resolvable spots. The structure of the scanner and its polarization dependency not only limit its ability to steer the beam to one dimension in many applications but also makes it impossible to steer the beam to both dimensions in one crystal. The only way to steer the light in two dimensions using this scanner is to steer the light to one dimension in a crystal first, then flip the polarization of the light by a halfwave plate and focus it on another crystal to steer it into the other dimension. This inefficient and lengthy procedure does not offer a compact two-dimensional beam steering, limits the deflection angle in both dimensions, and may cause beam clipping and distortion from hitting the crystal wall boundary.

In addition to the index change, the deflection angle of an EO steerer depends on the effective length-to-width ratio of the EO crystal [1,2,13,14,24,26]. Since long single EO crystals are usually not available, the crystals' effective width is reduced to increase the

deflection angle in some studies. McManamon and Ataei suggested cutting the EO crystal like a fishtail [11]. Fang et al. suggested a trapezoid-shaped and horn-shaped crystal to maximize the effective length-to-width ratio and maximize the deflection angle [24]. However, cutting and machining the EO crystal increases the cost of the EO beam steerer. Therefore, if a long EO medium were available, the optimum shape would be a long rectangular shape.

Polycrystalline $\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3\text{-PbTiO}_3$ (PMN-PT) exhibits a much higher EO effect compared to LiNbO_3 and LiTaO_3 and is transparent from visible to mid-IR. This ceramic is relatively inexpensive and available in a larger size [5,19]. In this study, the refractive index of polycrystalline PMN-PT was determined using the Fourier-transform infrared (FTIR) spectroscopy as well as its transmission over a wide range of optical frequencies. A capacitance bridge analysis was performed to characterize the effect of temperature on the dielectric constant of this ceramic as well. A polarization-independent beam steerer was designed using polycrystalline PMN-PT. Its performance was simulated using COMSOL Multiphysics and measured in the lab for the wavelengths of 532, 632.8, 1550, and 4500 nm. The quadratic Kerr EO coefficient and the halfwave electric field for each wavelength were determined by measuring the deflection angle and refractive index associated with each wavelength. The effect of temperature on the EO coefficient in a finite temperature range was also investigated.

2. Materials and Methods

Polycrystalline PMN-PT is a transparent relaxor ferroelectric ceramic [5,15,19]. Unlike single crystals, ceramics are easier, cheaper, and faster to produce by several well-matured techniques like hot-pressing [5]. Ceramics are available in larger sizes compared to single crystals so that they can address a larger aperture. Polycrystalline PMN-PT, like other ceramics, is made of randomly arranged grains or crystals [5,19]. Although each grain may have different orientations, the ceramic's properties are defined by the sum of many grain orientations, making the ceramic isotropic. Therefore, the optical properties of polycrystalline PMN-PT are the same in all directions, even under its Curie temperature. Applying an external electric field makes its isotropic and stable structure distorted and converted to a uniaxial medium in which the external electric field defines the optical axis [1,2,7]. Therefore, polycrystalline PMN-PT can be used as an EO beam steerer, and its EO properties can be defined by measuring and simulating the deflection angles. Since an EO steerer's deflection angles also depend on the medium's refractive index, the refractive index of polycrystalline PMN-PT, its dielectric constant, and its transmission should also be determined.

2.1. Determining the Refractive Index, Transmission, and Dielectric Constant of PMN-PT Ceramic

To design and model an EO beam steerer based on polycrystalline PMN-PT and determine the ceramic's EO coefficient at several wavelengths and temperatures by measuring and/or simulating the deflection angle, the refractive index, transmission, and dielectric constant of this ceramic should be known. The refractive index was determined using FTIR spectroscopy [27–29]. The transmission was determined from Fresnel equations and compared with the data measured in the lab. The dielectric constant was found as a function of temperature through a capacitance bridge analysis (CBA).

2.2. Interference Fringes in FTIR Transmission Spectra

A common, and normally undesirable, artifact of transmission spectroscopy, when applied to thin samples, is the inclusion of interference fringes within the transmission data. Whenever a sample consists of two planar partially-reflective parallel interfaces and the face separation is not insignificant, interference fringes will occur [27]. The phenomenon which gives rise to these interference fringes is the internal reflection, as depicted in Figure 1.

Figure 1. When an optical sample is structured with two parallel partially-reflective optical flats, the incident light (I) contributes to both the reflected (R) and transmitted (T) intensities. The light which is not immediately reflected (i.e., passes the first interface) can be internally reflected and ultimately add interference to either the reflected light or transmitted light. Note that absorbance is not explicitly depicted in this illustration but is assumed to affect both reflectance and transmittance.

The thin sample forms a simple Fabry–Pérot etalon, creating an optical resonator that allows the source light to constructively and destructively interfere with itself. An important parameter when analyzing a Fabry–Pérot etalon is the so-called “finesse”, which determines the fringe contrast of the system and is given as [30]

$$F = \frac{4|r|^2}{(1 - |r|^2)^2} \quad (5)$$

where $|r|^2$ is the surface reflectance (for normally incident light) which is defined as

$$|r|^2 = \left(\frac{n(\nu) - 1}{n(\nu) + 1} \right)^2 \quad (6)$$

High-finesse systems will have correspondingly high fringe contrast, which increases the likelihood that FTIR spectrometers with large spectral bandwidths will resolve them. Great effort, which includes extremely small step sizes and long integration times, is typically needed for a dispersive spectrometer to do the same.

The frequency of the interference fringes is proportional to the thickness of the sample; a thick sample will produce very high frequency fringes (small separation), while a very thin sample will produce very low frequency fringes (large separation). The latter case helps to explain why characterizing thin films using FTIR spectroscopy can be very challenging. When the amplitudes of the interference fringes are strong, they can severely obscure the transmission values. Typically, this is why significant effort is made to reduce the impact of any interference fringes.

If, however, accurate transmission values are not the end-goal of characterization, then the fringes themselves offer valuable information about the sample material. For example, the interference fringe maxima are governed by

$$\nu_m n(\nu_m) = \frac{m}{2d} \quad (7)$$

where ν_m is the wavenumber associated with the fringe of order number m , and d is the sample thickness [31]. If two of the three aforementioned values are known, then the third can be solved by rearranging Equation (7).

2.3. FTIR Analysis of Polycrystalline PMN-PT Sample

The main objective of characterizing the PMN-PT sample using an FTIR spectrometer is to identify the positions of the interference fringe peaks superimposed on the transmission data. Once the peaks have been identified, it will still be necessary to assign the correct order number to each peak to calculate the refractive index at each frequency position

accurately. Since it is not possible to directly observe the zeroth-order fringe (fringe peak at 0 cm^{-1}), it will be necessary to use the measured fringe spacing to extrapolate down to the zeroth-order and assign absolute order numbers to the known fringes. With this methodology in mind, it is desirable to measure as many fringe positions as possible directly. To that end, transmission data were collected using two separate FTIR spectrometers with different but overlapping analysis spectra.

In general, analysis using FTIR spectrometers is as simple as collecting a background scan followed by a scan of the sample itself. For this analysis, it is features of the transmission data, not the transmission values themselves, that are the focus of inspection. Special consideration must therefore be made in an effort to make these features more prominent. The first of these considerations was done by preparing a thin PMN-PT sample. A thin medium is much more likely to exhibit transmission interference fringes than one whose thickness is on the order of many hundreds of wavelengths.

Another important consideration is the incident angle of the source on the sample. Typically, the normally incident transmission of a well-characterized medium can be calculated as [29]

$$T(\nu) = \frac{1}{1 + F \sin^2(2\pi\nu n(\nu)d)} \quad (8)$$

where F is the so-called “finesse” value [30]. If the source light is incident on the sample at any angle other than normal, as occurs at the focal plane of FTIR spectrometers, then the $n(\nu)d$ term from Equation (8) must be substituted with Equation (9), which takes into account the angle of incidence.

$$n(\nu)d \rightarrow \sqrt{n^2(\nu) - n_{Air}^2(\nu) \sin^2(\theta)}d \quad (9)$$

It is possible to reduce this distortion by restricting the source light via an aperture placed some distance in front of the sample. While this will not collimate the incident illumination, it will limit the angular variation such that Equation (8) becomes a good approximation of the transmittance values. An illustration of this modification is included in Figure 2. While it is unavoidable that restricting the source illumination in this manner will reduce the system throughput, it is possible to balance the reduction of output power with the increased signal-to-noise ratio [29]. It will also be necessary to collect all background scans with the aperture installed such that it matches the restriction used during the actual material scans.

Additionally, restricting the source beam spread serves to increase the interference fringe amplitudes. A significant consequence of the unrestricted beam spread in typical FTIR spectral analysis is that fringes at higher frequencies are suppressed; this modification reduces that effect [28]. Figure 3 shows a direct comparison of interference fringes collected using three different aperture sizes to restrict the source beam and illustrates its effect on fringe amplitudes. Apertures were mounted within each FTIR spectrometer to limit the angle of incidence to less than $\pm 3.0^\circ$ from normal. Specific aperture configurations for each FTIR spectrometer are summarized in Table 2.

In addition to the prepared PMN-PT sample, a commercially available LiNbO_3 single crystal thin sample was characterized as a control, and subsequent refractive index calculations were compared to literature values to help support the validity of this methodology (Figure 4). The LiNbO_3 sample used for this purpose is professionally polished and has a measured thickness of $508 \mu\text{m}$.

Figure 2. A comparison of the incident ray bundles during unrestricted FTIR analysis (a) and restricted FTIR analysis (b).

Figure 3. Reducing the aperture diameter, thereby reducing the maximum angle of incidence, serves to amplify the magnitude of the interference fringes.

Figure 4. The LiNbO₃ single crystal control sample (a) and prepared PMN-PT sample (b) arranged with millimeter scales for reference.

Table 2. Aperture size and spacing used for each FTIR spectrometer to amplify interference fringe amplitude.

Spectrometer	Aperture Diameter (mm)	Aperture Distance (mm)	Maximum Angle of Incidence (°)
Thermo Scientific	6.0	60.0	2.862
Perkin Elmer	5.0	54.75	2.614

Using Equation (7), it was estimated that the interference fringes of the prepared PMN-PT sample have a period of approximately 5 cm^{-1} . With this in mind, the FTIR spectrometers were configured with a scanning resolution of 0.25 cm^{-1} to produce well-defined fringes in the final transmission spectrum. Additionally, the transmission values were averaged over thirty-two scans to increase waveform smoothness.

While a scan resolution of 0.25 cm^{-1} is sufficient to capture the fringe waveforms, for the purpose of calculating the refractive index, the fringe peak positions must be known more precisely. To that end, the transmission spectra were mathematically characterized with a polynomial fit to more precisely interpolate the location of the fringe peaks. Peak locations measured using this method were expected to have an accuracy of at least two decimal places ($0.01 \text{ cm}^{-1} \rightarrow 0.0001 \text{ m}^{-1}$), supporting an index of refraction calculation out to four decimal places. An example of the fringe peak fitting is depicted in Figure 5.

Figure 5. The accuracy of interference fringe peak location measurements is increased by interpolation via a polynomial fit.

Once the fringe positions have been measured using the polynomial fit method, it is still necessary to assign absolute fringe order numbers. The absolute fringe order for each peak is extrapolated via a linear fit to the zeroth-order. Fringe peak spacing in the frequency domain is nearly linear such that plotting the relative fringe order numbers against the fringe frequency position allows for an accurate determination of the offset from zero (i.e., 0 cm^{-1}). This extrapolation process is depicted in Figure 6.

Of all the critical values necessary for accurate refractive index calculations, proper assignment of the fringe order numbers has the largest impact on the dispersion curve [32]. Successful determination of the fringe order numbers is aided by a large data set; it is at this point that the near-IR and mid-IR collections are integrated, extending the reach of the extrapolation closer to zero. If the assignment is trusted to be accurate, the index of refraction at each fringe peak location is calculated by rearranging Equation (7) as

$$n(\nu_m) = \frac{m}{2\nu_m d} \quad (10)$$

where ν_m , m , and d are the known values.

Figure 6. The measured fringe position was used to extrapolate the zeroth-order fringes (at 0 cm^{−1}) to assign absolute fringe order numbers to each fringe position.

2.4. The Optical Transmission Spectrum of Polycrystalline PMN-PT

For the transparent dielectric slab with a thickness of d , a refractive index of n with considering a normally incident s-polarized plane wave with a vacuum wavelength of λ_0 , using Fresnel's equation, the complex amplitude of the reflected and transmitted beam at the slab's frontside (ρ and τ) and the ones at the slab's backside (ρ' and τ') are calculated as [2]

$$\rho = (1 - n)/(1 + n) \quad (11)$$

$$\tau = 2/(1 + n) \quad (12)$$

$$\rho' = (n - 1)/(n + 1) \quad (13)$$

$$\tau' = 2n/(n + 1) \quad (14)$$

If ϕ is the phase shift associated with a single path of the beam through the slab ($\phi = knd = 2\pi nd/\lambda_0$), the total reflection r and total transmission t of the slab considering all partial reflections and transmissions at the two sides will be

$$r = \rho + \tau\tau'\rho'e^{2i\phi} \sum_{m=0}^{\infty} (\rho'e^{i\phi})^{2m} = \rho + \frac{\tau\tau'\rho'e^{2i\phi}}{1 - \rho'^2e^{2i\phi}} \quad (15)$$

$$t = \tau\tau'e^{i\phi} \sum_{m=0}^{\infty} (\rho'e^{i\phi})^{2m} = \frac{\tau\tau'e^{i\phi}}{1 - \rho'^2e^{2i\phi}} \quad (16)$$

If the slab is thin enough to ignore absorption, the total transmission of the slab can be calculated as

$$T = 1 - \left(\rho + \frac{\tau\tau'\rho'e^{2i\phi}}{1 - \rho'^2e^{2i\phi}} \right)^2 \quad (17)$$

By determining the refractive index spectrum using FTIR, the optical transmission spectrum of PMN-PT ceramic can be calculated by Equation (17). In order to verify the calculation results, the experimental set-up shown in Figure 7 was developed in the lab. A 1.5 mm × 5 mm × 5 mm polycrystalline PMN-PT slab was polished and used as the sample in that set-up. The transmission of that PMN-PT slab is measured by dividing the intensity of 532 nm, 632.8 nm, and 1550 nm lights, measured by the power meter, with the slab in its place by the intensity without the slab in the pathway.

Figure 7. Schematic of the experimental set-up used for measuring the transmission of a 1.5 mm thick PMN-PT ceramic slab at three different wavelengths (532 nm, 632.8 nm, and 1550 nm). Those three measurements were used to verify the three points of the transmission spectrum calculated by Equation (17) using the refractive indices defined by the FTIR method.

2.5. The Experimental Set-up for Dielectric Constant Measurements

In this study, impedance measurements were performed to determine the dielectric constant of the device under test (DUT) as well as various other electrical properties. Special AC bridge circuits are used to accurately measure the complex impedance values for the DUT. More specifically, capacitance bridges such as the Schering Bridge (Figure 8) were used to assess the capacitance values of the DUT and subsequently calculate the dielectric constant as

$$\kappa = \epsilon_r = \frac{\epsilon_m}{\epsilon_0} = \frac{C_m}{C_0} \quad (18)$$

where C_m and C_0 are the capacitances of the DUT and the air-gapped specimen cell, respectively.

Figure 8. The capacitance bridge circuit is a useful AC bridge circuit that allows for the precise measurement of impedance values through balancing (unifying) the voltages at nodes B and D.

Implementation of the capacitance bridge is greatly simplified through the use of advanced LCR measurement devices. These meters are purpose-built to measure the inductance (L), capacitance (C), and resistance (R) of connected devices using a variable AC source. Many LCR meters utilize four BNC connections to independently measure current and voltage. These four connections can be configured as an auto-balancing bridge circuit which quickly and automatically determines the capacitance value for a DUT (Figure 9).

For the calculations of the dielectric constant, a thin unpolished sample of PMN-PT (approximately 12 mm × 10 mm × 0.5 mm) was characterized using a capacitance bridge apparatus. Capacitance values of the PMN-PT sample at multiple temperatures were compared to “open-cell” capacitance to determine the dielectric constant as a function of temperature.

Figure 9. Connection configuration when using an LCR meter as a capacitance bridge to test the capacitance of a device under test (DUT).

The PMN-PT samples for dielectric measurement require uniformity but can be unpolished. Beginning with one of the thin slices taken from the bulk sample segment, the pointed ends were removed. Additionally, it was necessary to remove the outer edge of the sample due to poor homogeneity in this area which is a result of the hot-press manufacturing process. Following the sample shaping, conductive silver paint was applied to the two parallel faces to serve as electrodes for impedance measurements (Figure 10).

Figure 10. The dielectric sample was prepared from a bulk sample thin slice (a), and silver paint was applied to each face to serve as electrodes (b).

The conductive coating applied to either face should help to ensure that the electric field is distributed evenly across the area of the sample. The capacitance bridge can be implemented using a high-accuracy inductance capacitance resistance (LCR) meter operating at 1 kHz.

Again, LiNbO₃ single crystal samples served as a control group to support the validity of this methodology. Due to the birefringent nature of LiNbO₃ single crystals, the dielectric constant of the material is orientation-dependent. For thoroughness, the capacitance bridge method was used to calculate the dielectric constant for both x-cut (11) and z-cut (33) LiNbO₃ samples at room temperature. Literature values for the dielectric constants of x-cut and z-cut LiNbO₃ crystals are given as 85.2 and 28.7, respectively [33]. If the dielectric constant of a material is known, an estimate of the capacitance can be calculated as

$$C_m = \epsilon_0 \epsilon_r \frac{A}{d} \quad (19)$$

where A and d are the area and thickness of the material, respectively, the resulting calculations for the expected capacitance values of the two LiNbO₃ samples are summarized in Table 3, which also includes the physical parameters needed for Equation (19).

Table 3. Expectation values for the capacitance measurements of x-cut and z-cut lithium niobate (LiNbO_3) single crystal samples.

Parameter	LiNbO_3 (x-cut)	LiNbO_3 (z-cut)
Area (cm^2)	1.6445	1.0
Thickness (mm)	1.0	0.5
Calculated Capacitance (pF)	124.0	50.8

With expected capacitance values for the LiNbO_3 samples in hand, measurements can be made using the LCR meter. Electrical connections are established with the samples by inserting them between conductive plates that are wired directly to the LCR meter terminals. Measuring the capacitance with the LiNbO_3 sample between the plates establishes the material capacitance, C_m . To measure the open-cell capacitance, C_0 , the LiNbO_3 sample is removed, and the plates are held apart by the thickness of the sample with the same area of overlap. These two measurements cannot be immediately used for calculating the dielectric constant. They must first be corrected for the open circuit capacitance of the LCR meter. Separating the two conductive plates by a sufficient distance results in a small capacitance measurement that acts as the relative zero of the system. To correct the material and open-cell measurements, this open circuit value simply needs to be subtracted from both measurements. Following these steps, the necessary capacitance values for both LiNbO_3 samples were recorded and the dielectric constants calculated as shown in Table 4.

Table 4. Measured capacitance values and resulting dielectric constants calculations for multiple orientations of lithium niobate (LiNbO_3) single crystals.

Parameter	LiNbO_3 (x-cut)	LiNbO_3 (z-cut)
Material Capacitance (pF)	113.20	48.55
Open-Cell Capacitance (pF)	1.35	1.63
Calculated Dielectric Constant	83.73	29.75
Accepted Dielectric Constant	85.2	28.7
Percent Error (%)	1.73	3.66

As shown above, in Table 4, the process of calculating the dielectric constant from the measured capacitances is accurate but not precise. The LCR meter used is expected to provide measurements with $\pm 0.3\%$ accuracy, and thus it is likely that the discrepancy is a result of human error. Precision in calculating the dielectric constant is dependent on precisely measuring the open-cell capacitance, which is strongly proportional to the separation of the conductive plates. A small difference between open-cell separation and the DUT thickness can significantly affect the final dielectric constant calculation. With this limitation in mind, it stands to reason that the calculated dielectric constant values still serve to describe the relative magnitude and not precise measurements.

For dielectric characterization of the PMN-PT sample, the previous experimental setup is modified such that the measurement cell is contained within a Peltier heating stage. Recording the capacitance values at various temperatures allows for an observation of the effect of temperature on the dielectric constant of the material. The Peltier heat stage used for this experimentation is capable of maintaining temperatures between 0°C and 120°C without needing chilled water so that capacitance values will be measured over that range in 5°C increments.

2.6. Designing an EO Beam Steerer Based on Polycrystalline PMN-PT

The light steerer designed in this study was based on the EO effect in a $7.5\text{ mm} \times 7.5\text{ mm} \times 40\text{ mm}$ PMN-PT ceramic, which is considered isotropic, as discussed before.

The propagation of an optical wave in a crystal can be described in terms of the relative impermeability tensor as $B_{ij} = 1/n_{ij}^2 = \epsilon_0/\epsilon_{ij}$, where ϵ_0 is the permittivity of free space and ϵ_{ij} is the permittivity tensor. The B_{ij} is a symmetric tensor because the ϵ_{ij} is a symmetric

tensor for a lossless and inactive medium. The two directions of dielectric polarization and the refractive indices can be calculated by indicatrix or index ellipsoid as [1]

$$B_{ij}x_i x_j = 1 \quad (20)$$

where i and j are 1, 2, and 3 and represent the principal axes of a crystal which are the directions in the crystal where the electric field (E) and displacement vectors (D) are parallel. If we consider x , y , and z respectively for the subscripts of 1, 2, and 3, the ellipsoid index can be expanded as follows:

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad (21)$$

where n_x , n_y , and n_z are the refractive indices for the incident light polarized in the directions of x , y , and z , respectively.

The quantum theory of solids says the indicatrix depends on the charge distribution in the crystal. The external electric field (E) and stress (σ) can change that distribution and result in a change in the shape, size, and orientation of the index ellipsoid specified by electro-optic and photoelastic effects on ΔB_{ij} as follows [34]:

$$B_{ij}(E) - B_{ij}(0) = \Delta B_{ij} = r_{ijk}E_k + s_{ijkl}E_k E_l + \pi_{ijkl}\sigma_{kl} \quad (22)$$

where r_{ijk} and s_{ijkl} are the Pockels electro-optic and Kerr electro-optic tensors, and π_{ijkl} is the piezo-optical tensor. Note that the stress field is related to the strain tensor (T_{rs}) as per Hook's law [34];

$$\sigma_{kl} = c_{klrs}T_{rs} \quad (23)$$

and

$$\pi_{ijkl} = p_{ijrs}\Omega_{rslk} \quad (24)$$

where p_{ijrs} and Ω_{rslk} are elasto-optical tensor and compliances, respectively. Although the direct photoelastic effect vanishes when no direct stress is applied to the crystal (free crystal), mechanical constraints still can exist because of the converse piezoelectric effect. That is because the electric field will cause a strain by the converse piezoelectric effect, and therefore, the indicatrix will get modified by the photoelastic effect, which is called the secondary effect.

The strain produced by an electric field is calculated as $d_{kij} E_k$, where d_{kij} is the piezoelectric tensor. Therefore, the primary and secondary electro-optic effects on a free crystal ($\sigma = 0$) can be calculated as

$$B_{ij}(E) - B_{ij}(0) = \Delta B_{ij} = r_{ijk}E_k + s_{ijkl}E_k E_l + (pd_{kij})E_k \quad (25)$$

Because the relative impermeability tensor is symmetric for inactive and lossless medium, the i and j subscripts can be permuted. This will reduce the number of independent elements of r_{ijk} from 27 to 18 and from 81 to 36 for s_{ijkl} . The same thing is applied for piezoelectric tensor except for some factor of 2. Therefore, the change in indicatrix can be written for convenience as follows [34]:

$$B_{ij}(E) - B_{ij}(0) = \Delta B_{ij} = r_{ij}E_j + s_{ijkl}E_j E_k + (pd_{ki})E_k \quad (26)$$

where

$$p_{mn} = \pi_{mr}c_{rn}(m, n = 1, 2, \dots, 6) \quad (27)$$

$$\pi_{mr} = \pi_{ijkl} (n = 1, 2, 3) \quad (28)$$

$$\pi_{mr} = 2\pi_{ijkl} (n = 4, 5, 6) \quad (29)$$

As discussed, although each individual grain of PMN-PT ceramics is not centrosymmetric, the polycrystalline nature makes the PMN-PT ceramic isotropic and, thus, polarization-

independent. Therefore, the r_{ij} and d_{ki} in Equation (26) are zero, and the Kerr effect is the main factor in altering the refractive index. By contracting the indices as 11:1, 22:2, 33:3, 23:4, 13:5, and 12:6, the impermeability tensor becomes

$$B = \begin{pmatrix} B_1 & B_6 & B_5 \\ B_6 & B_2 & B_4 \\ B_5 & B_4 & B_3 \end{pmatrix} \quad (30)$$

The Kerr effect tensor for an isotropic medium is given as [1]

$$s = \begin{pmatrix} s_{11} & s_{12} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{12} & s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5(s_{11} - s_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5(s_{11} - s_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5(s_{11} - s_{12}) \end{pmatrix} \quad (31)$$

When a polycrystalline PMN-PT gets under a slowly varying external electric field, its molecules get aligned with the field, and the isotropic ceramic becomes birefringent, behaving like a uniaxial crystal in which the electrical field defines the optical axis. The change in impermeability tensor by applying an electric field on the PMN-PT ceramic will be

$$\begin{pmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \\ \Delta B_4 \\ \Delta B_5 \\ \Delta B_6 \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{12} & s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5(s_{11} - s_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5(s_{11} - s_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5(s_{11} - s_{12}) \end{pmatrix} \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_yE_z \\ 2E_xE_z \\ 2E_xE_y \end{pmatrix} \quad (32)$$

Assuming the external electric field is in the direction of the z-axis ($E = E_z \hat{z}$), the indicatrix of PMN-PT ceramic will be defined as

$$x^2 \left(\frac{1}{n^2} + s_{12}E^2 \right) + y^2 \left(\frac{1}{n^2} + s_{12}E^2 \right) + z^2 \left(\frac{1}{n^2} + s_{11}E^2 \right) = 1 \quad (33)$$

$$\frac{x^2 + y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 \quad (34)$$

where n_o is the refractive index associated with ordinary light, and n_e is the refractive index seen by an extraordinary light propagating through the PMN-PT. Hence, if the light is polarized perpendicular to the external electric field direction, the index change it sees will be

$$\Delta n = n_o - n = -0.5n^3 s_{12}E^2 \quad (35)$$

The EO scanner designed in this study had the light propagate perpendicular to the external electric field direction and was polarized parallel to the optical axis defined by that field. Therefore, the index change seen by the light can be described as

$$\Delta n = n_e - n = -0.5n^3 s_{11}E^2 \quad (36)$$

The birefringence (Δn) imposes relative phase retardation (Γ) to the light propagating through that medium by the length of L as

$$\Gamma = \frac{2\pi\Delta nL}{\lambda} = -\frac{\pi n^3 s_{11}E^2L}{\lambda} \quad (37)$$

therefore, the electric field that causes π radians of phase retardation will be

$$E_{\pi} = \sqrt{\frac{\lambda}{n^3 s_{11} L}} \quad (38)$$

The external electric field (E) is approximated as the applied voltage divided by the thickness of the PMN-PT ceramic ($E = V/d$), which was 7.5 mm ($d = 7.5$ mm) thick. The s_{11} is the Kerr coefficient of polycrystalline PMN-PT, which varies by wavelength and temperature. This coefficient can be defined by measuring the EO steerer's deflection angle associated with each wavelength at different temperatures.

The polycrystalline PMN-PT is isotropic and polarization-independent. Unlike single ferroelectric crystals, no crystalline orientation adjustment and no domain-engineered multi-prisms are required to provide the OPD for linearly polarized light propagating through the crystal.

The most simple structure of an EO steerer we can build with the PMN-PT ceramic is a triangular prism with uniform electrodes deposited on two opposite sides, as seen in Figure 11.

Figure 11. A triangular prism-shaped EO scanner. (W is the width, L is the length, and H is the height of the triangular prism).

Before applying the voltage on the electrodes, according to Snell's law, we have

$$\theta + A = \sin^{-1}(n \sin A) \quad (39)$$

After applying the voltage, the isotropic medium becomes uniaxial. The light is polarized along the external electric field. Therefore the deflection angle change will be

$$\Delta\theta = \frac{\Delta n \sin A}{\sqrt{1 - n^2 \sin^2 A}} \quad (40)$$

where Δn is given by Equation (36). To prevent a total internal reflection, $|n \sin A|$ must be smaller than 1.

Instead of having a triangular prism with uniform electrodes, a rectangular prism with triangular electrodes can be considered. The non-uniform electrodes should be deposited on the two opposite sides of the rectangular PMN-PT ceramic (7.5 mm $\times$ 7.5 mm $\times$ 40 mm) to impose a non-uniform electric field distribution and index change to deflect the light. Figure 12 depicts the arbitrary electrodes on the rectangular PMN-PT ceramic. Figure 13 shows the refraction of the light at the dielectric interfaces.

Because the electric field should be nonuniformly distributed to steer the light, the number of electrode interfaces (N) should be equal to or larger than 1 ($N \geq 1$).

Suppose the number of interfaces (N) was small and the PMN-PT ceramic length was long. In that case, the incident angle of an interface might get wide enough to enable a total internal reflection, at least at some voltages applied on the electrode. Therefore, the number of interfaces and the electrode geometry should be designed to avoid the total internal reflections and partial back-reflections.

Figure 12. The arbitrary electrodes on the rectangular PMN-PT ceramic (top view).

Figure 13. The refraction of the light at interfaces.

When the light hits the electrode interface, the region with a lower refractive index, no total internal reflection should occur. Therefore, we should have

$$\cos^{-1}\left(1 + \frac{\Delta n}{n}\right) \leq \alpha_i = \tan^{-1}\left(\frac{W}{l_i}\right) \quad (41)$$

Once a ray is reflected back from an electrode interface, it is considered a loss because even if it is partially reflected back again, it will not get steered to the desired angle. Therefore, the total partial back-reflections through the medium (TPBR) should be minimized.

$$TPBR = \sum_{i=1}^N R_i = \sum_{i=1}^N \left(\frac{n \sin \alpha_i - (n + \Delta n) \sin(\alpha_i - \theta'_i)}{n \sin \alpha_i + (n + \Delta n) \sin(\alpha_i - \theta'_i)} \right)^2 \quad (42)$$

Finally, the Δn and the ceramic's length should be large enough to give the desired steering angle (θ). As seen in Figure 13, Snell's law at each interface states

$$(n + \Delta n) \cos(\alpha_j) = n \cos(\alpha_j + \theta'_j) \quad (43)$$

Assuming the deflection angle inside the medium (θ'_i) is small and $\Delta n \ll n$, we will have

$$n \sin \theta'_i \sin \alpha_j \Delta n \theta'_i \sin \alpha_j = -\Delta n \cos \alpha_j \quad (44)$$

Hence, the total deflection angle after N interfaces inside the ceramic will be

$$\theta' = -\frac{\Delta n}{n} \sum_{j=1}^N \cot \alpha_j = -\frac{\Delta n}{n} \sum_{j=1}^N \frac{l_j}{W} = -\frac{L \Delta n}{W n} \quad (45)$$

The displacement of the beam (δ) at the output of that rectangular prism and the pivot point (pp), which is the point where the beam deflects to the angle of (θ'), will be

$$\delta = \frac{\theta' L}{2} = -\frac{L^2 \Delta n}{2Wn} \quad (46)$$

$$pp = \frac{\delta}{\theta'} \quad (47)$$

The external deflection angle (θ) is related to the internal reflection angle by Snell's law. If θ is small enough, it can be calculated as follows:

$$\theta = n\theta' = -\frac{L\Delta n}{W} = -\frac{OPD}{W} = \frac{L}{2W} n^3 s_{11} E^2 \quad (48)$$

As seen, for a rectangular prism PMN-PT ceramic deflector with a dimension of $WXHXL$ (where the width is W , the height is H , and the length is L), it is possible to grade the external electric field linearly and make an OPD in the linearly polarized light with the spot size of w_0 (see Equation (4)) propagating through the crystal. That OPD makes the beam deflect inside the crystal to the angle of θ' and the angle of θ outside the crystal. By measuring the angle of θ for various wavelengths, the dispersing of s_{11} in that ceramic can be determined.

The EO steerers' deflection angle for each wavelength at a specific voltage and temperature can be measured by keeping the light hitting the camera's center by displacing the camera up or down on the y -axis as seen in Figure 14. By making that measurement at two different image planes with a distance of d apart, since the $\Delta y \ll d$, the deflection angle for that specific wavelength, voltage, and temperature can be defined as

$$\theta = \frac{\Delta y}{d} \quad (49)$$

Figure 14. The experimental set-up for characterization of the EO beam steerer based on polycrystalline PMN-PT.

In order to measure the deflection angle for the various wavelengths in terms of $\text{mrad}\cdot\text{mm}/\text{kV}$ in the lab and calculate the EO coefficient for each wavelength under different temperatures, the external electric field applied to the ceramic must be steady or DC. We also applied an AC external electric field to illustrate the beam's deflection angle variation by varying the applied voltage. We uploaded that illustrative video as the supplementary material of this paper.

3. Results and Discussion

The dispersion curve of PMN-PT ceramics defined in this study using FTIR spectrometry could not be compared to literature because it has not been previously reported. Instead, we could validate our method's precision by taking a $508\ \mu\text{m}$ thick LiNbO_3 slab as a control sample, defining its dispersion curve, and comparing it to the literature. Figure 15 compares the calculated refractive index values to the values reported in the literature [35].

Figure 15a indicates that the experimental and accepted refractive index data are exceptionally well-matched, and that is why the two curves in Figure 15a are indistinguishable. Expanding one-tenth of the spectrum, the slight variation in values can begin to be seen in Figure 15b. The amount of precision achieved in the calculation of the LiNbO_3 dispersion curve indicates that the methodology for calculating the refractive index of the PMN-PT sample is quite sound. The calculated dispersion curve for the PMN-PT sample after combining the data from the NIR and MIR transmission spectra is shown in Figure 16.

Figure 15. A comparison of the calculated refractive index values to literature values for the lithium niobate control sample. (a) full-view; (b) zoomed-view to show differences.

Figure 16. The calculated dispersion curve for the PMN-PT sample after combining the data from the NIR and MIR transmission spectra.

The PMN-PT ceramics transmission for an s-polarized wave was determined using the calculated dispersion curve (Figure 17), as discussed in the previous section. Again, because the transmission of polycrystalline PMN-PT in such a wide wavelength range was not previously reported, we could not compare the results to the values reported in the literature. Instead, the transmission of a 1.5 mm × 5 mm × 5 mm polycrystalline PMN-PT slab was measured in the lab (see Figure 7) for three wavelengths, and the results were compared to the calculated values as seen in Table 5. Table 5 shows the measured transmission values agree with the calculated ones within 8.8% errors or less.

Figure 17. The calculated transmission spectrum of polycrystalline PMN-PT.

Table 5. The calculated and measured transmission and absorbance of a 1.5 mm thick polycrystalline PMN-PT slab for three different wavelengths.

Wavelength (nm)	Calculated Transmission (%)	Measured Transmission (%)	Error (%)	Absorbance
532	61.8	60.1	2.75	0.22
632.8	68.7	65.8	4.22	0.18
1550	82.1	74.9	8.77	0.13

The absorbance or optical density, OD , can be calculated from the measured transmission, T , as

$$OD = \log\left(\frac{1}{T}\right) \quad (50)$$

The magnitudes of absorbance of the PMN-PT sample for those three wavelengths have also been approximated and given in Table 5.

The dielectric constant was calculated for each temperature using Equation (18) after collecting the capacitance values over the range, once for material capacitance and again for open-cell capacitance. The resulting dielectric values for polycrystalline PMN-PT are graphed against temperature in Figure 18. As seen in Figure 18, the maximum dielectric constant achieved was approximately 12,000, and the peak dielectric constant is reached at or near 25 °C. Therefore, the Curie temperature of this ceramic is at room temperature. It should be noted that a higher dielectric constant increases the loss and reduces the speed of response.

The optimum number of interfaces (N) for the 7.5 mm × 7.5 mm × 40 mm polycrystalline PMN-PT considering no internal reflection and minimum partial back-reflection was defined as two. Figure 19 shows the polycrystalline PMN-PT under the voltage in the lab. A 1 kV/mm DC electric field was applied to the PMN-PT ceramic electrodes to measure the deflection angle using Equation (49). A 1 kV/mm AC electric field was also applied to illustrate this scanner's beam steering, as seen in this paper's supplementary materials. This EO beam steerer was used to steer the light from visible to mid-IR without any orientation adjustment or domain-engineered multi-prisms required.

Figure 18. The dielectric constant for the PMN-PT ceramic as a function of temperature. The peak dielectric constant of 11,688.9 is reached at room temperature, approximately 25 °C.

Figure 19. Light deflection test with the 7.5 mm × 7.5 mm × 40 mm polycrystalline PMN-PT with voltage electrodes.

The deflection angle for the wavelengths of 532, 632.8, 1550, and 4500 nm was measured in the lab in terms of mrad.mm/kV at 25 °C and 40 °C temperatures. The results are given in Table 6. The measured deflection angles and the calculated refractive indices can define the EO coefficients of the polycrystalline PMN-PT using Equation (48). This EO beam steerer was also modeled in COMSOL Multiphysics to simulate its performance and compare it to the measured values. The calculated EO coefficients, the refractive indices, and the measured permittivity were used as the COMSOL model's inputs. The voltage and electric field distribution, the birefringence, and deflection angles were the COMSOL model outputs, as seen in Figures 20–22.

Figure 20. (a) Voltage (b) electric field (E_z) simulation of the EO steerer based on PMN-PT ceramic under 1 kV/mm at 25 °C using a COMSOL model.

Figure 21. The birefringence simulation of the EO steerer based on PMN-PT ceramic under 1 kV/mm at 25 °C using a COMSOL model for (a) 532 nm, (b) 633 nm, (c) 1550 nm, and (d) 4500 nm.

Figure 22. The deflection angle simulation of the EO steerer based on PMN-PT ceramic under 1 kV/mm at 25 °C using a COMSOL model for (a) 532 nm, (b) 633 nm, (c) 1550 nm, and (d) 4500 nm.

Table 6. The measured and calculated wavelength and temperature-dependent parameters of the EO beam steerer based on polycrystalline PMN-PT.

Parameter	Unit	532 nm	633 nm	1550 nm	4500 nm
The refractive index	-	2.4744	2.4635	2.4573	2.4619
The measured deflection angle at 25 °C	mrad.mm/kV	63.35	61.6	54.88	8.68
The measured deflection angle at 40 °C	mrad.mm/kV	62.66	60.96	54.37	8.59
The measured halfwave electric field at 25 °C *	V/μm	0.124	0.136	0.228	NA
The measured halfwave electric field at 40 °C *	V/μm	0.128	0.141	0.232	NA
Number of resolvable spots at 25 °C	-	1403	1146	417	23
Number of resolvable spots at 40 °C	-	1388	1135	413	22
The EO coefficient (s_{11}) at 25 °C	m^2/V^2	1.57×10^{-15}	1.55×10^{-15}	1.39×10^{-15}	2.18×10^{-16}
The EO coefficient (s_{11}) at 40 °C	m^2/V^2	1.55×10^{-15}	1.53×10^{-15}	1.37×10^{-15}	2.16×10^{-16}
The calculated halfwave electric field at 25 °C *	V/μm	0.122	0.135	0.224	0.960
The calculated halfwave electric field at 40 °C *	V/μm	0.123	0.136	0.225	0.965
The calculated deflection angle at 25 °C	mrad.mm/kV	65.85	63.51	56.85	8.91
The calculated deflection angle at 40 °C	mrad.mm/kV	64.91	62.96	55.96	8.78

* The optical path was 1.5 mm.

As seen in Table 6, the measured deflection angles for four wavelengths at 25 °C and 40 °C temperatures could also define the number of resolvable spots using Equation (2). The EO coefficients and the refractive indices can define the halfwave electric field using Equation (38). The halfwave electric field for the three different wavelengths at 25 °C and 40 °C was also measured in the lab. As seen in Table 6, the results are comparable to the calculated values.

The wavelength-dependent birefringence (Δn) defines the phase retardance for those four wavelengths using Equation (37), as seen in Figure 23.

The results show that although the EO coefficient of PMN-PT ceramic is not strongly varying by the temperature, it drops significantly from 15.68×10^{-16} for green laser (532 nm) to 2.18×10^{-16} for mid-IR (4500 nm). That resulted in reducing the number of resolvable spots of the EO beam steerer designed in this study from 1403 for 532 nm to 23 for 4500 nm at 25 °C.

Figure 23. The phase retardance versus applied electric field on PMN-PT ceramic for four wavelengths (532, 633, 1550, and 4500 nm).

Table 7 compares the visible light (633 nm) and near-IR light (1.55 μm) deflections associated with polycrystalline PMN-PT measured in this study to the ones measured for LiTaO₃ by Casson et al. [12,17].

Table 7. Comparing the performance of polycrystalline PMN-PT to LiTaO₃ in deflection light at visible and near-IR.

Wavelength (μm)	Deflection (mrad.mm/kV)	
	LiTaO ₃	Polycrystalline PMN-PT
0.633	9.41	61.6
1.55	7.78	54.88

As seen, polycrystalline PMN-PT performs about seven times better than LiTaO₃ at those two wavelengths, and yet, as shown, PMN-PT ceramic is polarization-independent and almost temperature-independent.

4. Conclusions

This paper offered a robust approach for accurately measuring the dispersion curve of polycrystalline PMN-PT based on interference patterns in FTIR spectroscopy in a wide range of optical wavelengths for the first time. The dispersion curve was used to define the transmission of this ceramic. The dielectric constant of the PMN-PT ceramic was defined as a function of temperature through a capacitance bridge configuration and a high-accuracy LCR meter. The Curie temperature of PMN-PT ceramic was found close to the room temperature, where the dielectric constant peaks at about 12,000.

A polarization-independent EO laser beam steerer based on a 7.5 mm $\times$ 7.5 mm $\times$ 40 mm polycrystalline PMN-PT was designed in this study to steer light from visible to mid-IR. The optimum number of interfaces for this EO beam steerer was found as two, considering no internal reflection and minimum partial back-reflections.

The only EO beam steerer that does not need crystalline orientation adjustment so far was the one offered by Nakamura from NTT Photonics Laboratories in Japan [7]. However, since that steerer was based on charge injection and it is impossible to inject charges to a thick medium uniformly, the medium must be a fraction of a millimeter thick. Therefore, that EO steerer could steer only an 80 μm laser beam and resolved only 20 spots. For the first time, this paper offered an EO beam steerer, which was similarly polarization-independent and did not need any crystalline orientation adjustment, but it could resolve

up to 1400 spots. The voltage per millimeter applied on the EO medium in this paper was only two times more than the one presented by Nakamura, but the number of the resolvable spots was 70 times more.

The deflection angle for the wavelengths of 532, 632.8, 1550, and 4500 nm was measured in the lab in terms of mrad·mm/kV at 25 °C and 40 °C temperatures to define the EO coefficients associated with those four wavelengths at two different temperatures for the first time.

The calculated EO coefficients, refractive indices, transmission, and dielectric constant were used as inputs of a simulation model developed with COMSOL Multiphysics. The simulation model was used to calculate the voltage and electric field distribution as well as the birefringence. The EO beam steerer's performance, including the deflection angles, was simulated and compared to the measured values. The EO coefficients and the refractive indices were used to define the halfwave electric field, and the results were compared to the measured values. The phase retardance associated with those four wavelengths was determined.

The results show that the EO coefficient of PMN-PT ceramic is not strongly varying by the temperature, but it drops from 15.68×10^{-16} for green laser (532 nm) to 2.18×10^{-16} for mid-IR (4500 nm). That significant drop in the EO coefficient reduced the number of resolvable spots of the EO beam steerer, developed in this study, from 1403 to 23.

Although the result shows the EO coefficient of bulk PMN-PT ceramics is an order of magnitude smaller than gold nanoparticles measured by Thakur and Van Cleave [8] at 633 nm, the bulk PMN-PT offers other benefits like higher transmission at mid-IR, very low thermal dependency, less EO dispersion and ability to address larger apertures.

Comparing the polycrystalline PMN-PT with LiTaO₃, which is the most used single crystal for EO beam steering, has shown that PMN-PT ceramic performs about seven times better than LiTaO₃. Nevertheless, PMN-PT ceramic is polarization-independent. Therefore, unlike LiTaO₃ single crystal, it does not need orientation adjustment and domain-engineered multi prisms to be used as an electro-optical beam steerer. The PMN-PT ceramic is also easier, cheaper, and faster to produce and is available in larger sizes than most EO single crystals.

Supplementary Materials: The following are available online at <https://www.mdpi.com/article/10.3390/app11083313/s1>, Video S1: Electro-optical 633 nm laser beam steering with the 7.5 mm × 7.5 mm × 40 mm polycrystalline PMN-PT under 1 kV/mm external electric field at 1 Hz.

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