

Article

Mathematical Modeling of the Manufacturing Sector's Dominant Part as a Base for Automation

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Abstract: The current great expansion of automation and robotics affects a multiplicity of various fields. A prominent example is industry, where the different manufacturing processes and technologies embrace a certain level of automation and robotics. Thus, the use of robotics and automation implementation is part of a rapidly rising trend in industry. The presented paper deals with the manufacturing segment in the context of automation. The main subject is data analysis, with our own subsequent model building and final realization of the prediction corresponding to the machinery and electrical machinery sector as a highly relevant automation driver through the use of mathematical modeling. The design of the model is accompanied by optimization of the particular weights. Determination of the most suitable model is preceded by creating and testing a number of models to decide upon the final one. The construction of the mathematical model pursues the aim of making predictions relating to the machinery and electrical machinery sector for the specific national economy as the concluding investigation step. We apply a polynomial approximation as the research method. The software selected for our purposes is Matlab.

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1. Introduction

At present, industrial manufacturing (see, e.g., in [1–4]) as well as various manufacturing processes and technologies (see, e.g., in [5–7]) involve automation and robotics. Such a reality is not unusual in the context of the current manufacturing sector. Looking at automation in the machine manufacturing field and robotics, the great challenge is signified by working software, especially designing software systems. The next vivid challenge is the question of the platforms' performance. Thus, projection and improvement of the automation and robotics software systems become highly important and emerging issues [8]. Naturally, various actuators and sensors represent an integral part of the automated specialized machines and robots in the contemporary manufacturing systems [9]. Automation and robotics are on the rise in broad-spectrum areas, see, e.g., in [10–12]. Processes and technologies that are strongly influenced by this trend pertain to, inter alia, so-called building production [13]. This field can be perceived as one of the many representatives where robotics and automation have dominant standing. Therefore, construction automation and robotics growth, with related building technologies, are popular subjects for research conducted in numerous scientific papers, see, e.g., in [14–17]. Construction automation technologies or service robot systems appear as essential elements of the future. Considering automation as well as robotics from an overall point of view indicates that mathematical modeling has a strong position and constitutes a solid basis for research, testing, and practical implementations.

This paper proposes our own mathematical model that is designed and suggested for research use in the manufacturing sector. Further, an execution of the prediction corresponding to the machinery and electrical machinery sector is realized on the previous mathematical modeling bases. What is more, the model optimization is accomplished through weight assignment.

Even more contributions can be found in addition to those stated above. In our work, we implement various kinds of dataset, modeling, and forecasting that represent joining an industrial/manufacturing field with an economic background. In compliance with our knowledge, supported by the scientific literature review, the selected modeling technique, accompanied by all other selected (or identified) elements, is uncommon in the area of research presented here. Thus, pointing to the utilized method, as well as widening its great perspectives through our investigation, can be included among the paper's numerous benefits. Future contributions will gain much more precise results, opening possibilities for additional exploration, extensions of the model and specifications, or a certain degree of universality pertaining to our proposed model, with its multiple potential applications.

The paper is structured as follows. Section 2 includes the scrutiny of the many literature sources—manifold research papers, corresponding methodology, and employed data and methods. Section 3 brings the core of our research—findings supported by graphical representations, step-by-step. Finally, Section 4 deals with our achievements, interpretations, contributions, implications, and future research directions.

2. Materials and Methods

Mathematical modeling represents a core for many empirical studies. The versatility of the mathematical methods and tools is indisputable. A huge application area is posed by the manufacturing field.

An irreplaceable role of the mathematical modeling within manufacturing processes is highlighted [18]. Experimentations, investigations, and numeric simulations are stated as reasons for which the mathematical model's elaboration is required. In fact, it signifies the thoughtful comprehension of the main mechanism belonging to such processes. What is more, mathematical models are a basis for the improvement of computational models.

Mathematical modeling in the manufacturing sphere can be found in [19]. The electrochemical micro additive manufacturing technology grounded on the fluidic force microscope was constituted there. The particle conversion process manifested the mathematical modeling. The suggestion of the mathematical model concerned the species flux under the action of pulsed pressure in a newly localized liquid feeding procedure. Another empirical study from this domain is the work in [20], where additive manufacturing is also the principal topic. The authors proposed a universal method for setting up the mathematical spatial uncertainty model. Its fundamentals were based on the gauged geometry of additive manufacturing microstructures. The benefit of this lies in the universal utilization—possible use for parts fabricated by different manufacturing methods or different additive manufacturing processes. There was also uncertainty in [21]. The automobile segment was under scrutiny. The paper dealt with mathematical analysis to the modeling of automobile engine remanufacturing with regard to optimization. The uncertainty mentioned referred to market demand and procurement. Furthermore, solving the practical problems on the subject of automobile engine remanufacturing was integrated into the paper. The next example, where the mathematical modeling is implemented in manufacturing, is the work compiled in [22]. This research was targeted at the needs of the manufacturing companies' leaders. A multi-product fabrication-distribution issue signified the key matter. The survey offered a two-stage single-machine manufacture scheme. Naturally, an application of the mathematical modeling was important in proceeding with the chosen experimental idea. Manufacturing in the context of environmental concerns and sustainability is an increasingly intense topic nowadays. It is an area with potential for the implementation of mathematical modeling. The capital objective established in [23]

reflected discovering the finest product mix of a manufacturing device to maximize its sustainability. A mixed integer nonlinear programming model was designed. Thus, a mathematical programming technique found its use. A mathematical model appeared in [24], where attention was paid to the oscillations of a boring mandrel with a vibration damper connected to the mandrel with a viscoelastic coupling. This model was constructed through differential equations. The fact was underlined that the drafted mathematical model and algorithms for the numeric answer to the particular differential equations gave the possibility for the selection of the most suitable parameters of the boring mandrel damping element.

A strategic part of the manufacturing sector is deputized by the machinery and electrical machinery segment. Many scientific works are dedicated to this segment at different levels and take it as a subject for analysis.

The overview involving electrical machinery with the emphasis on optimization was pursued in [25]. Prospects of the technology that have recently appeared, as well as various questions, tasks, or calls from practice, were discussed. The other study dedicated to the disputed sector is that in [26], where fractional-slot surface permanent-magnet machines were analyzed. Their multiple usage was highlighted, for instance, industrial automation or electrical traction. A proposal for the use of the reliability method which typified manufacturing errors was presented. Considering automation, the authors of [27] reviewed approaches for tracking the machine state and industrial automation serving to plant-wide state inspection of rotating electrical machines. It was pointed out that the ratio of state monitoring expense to equipment expense is recognized as one of the crucial inhibiting agents. This was denoted as being decisive for the adoption of utilizing tracking to manage upkeep for an extensive fleet of electrical machinery. Likewise, automation was put into a center of interest in [28] through diagnostics of electrical drives in robots accompanied by sensor support. The authors established an artificial intelligence model with technical terms of fuzzy inference rule definitions for the recognition of a robot drive's technical terms and an origin for the definition of linguistic variables. Machinery was the manufacturing sector's dominant part with an accent on automation [29]. Computer-aided process planning was presented in the position of very influential element within contemporary manufacturing and was characterized as a great support for automation. That was the reason for its closer inspection. The result of the research was the new integration practice that created a handy computer-aided process planning technique. It was targeted directly on at end users to gain particular benefits. Rotating electrical machines were scrutinized in [30]; more precisely, a diagnosis of their disorders was provided. Such a topic is placed among leading issues looking at modern industrial automation. Upstart methodology was proposed related to multi-label classification for synchronously diagnosing mass disorders, along with assessing the disorder gravity per loud terms. Modeling was run with Electrical Signature Analysis and orthodox vibration data.

A review of the scholarly literature showed that research pertaining to the manufacturing sector often contains the polynomials for modeling purposes. Regarding polynomials, varied types and degrees exist. The work in [31] focused on optimization when a confidence interval-based process optimization technique was offered. For this aim, second-order polynomial regression analysis was applied. Testing was realized on a process dataset collected from a ball mill. The performed simulations led to the conclusion that users acquire an opportunity for comfort selection of the most suitable resolution from the specific set of resolutions. The polynomial regression can be found in [32]. Data-driven digital twins for the operation of technical building services represented the research subject. Implementation was included in the case study that was situated in Germany. The next paper with polynomials is the work in [33]. In the experimental section, two approaches were put into effect. One of them was the polynomial chaos expansion method. It may be summarized that uncertainty quantification and radial turbines were the main points for exploration. A match occurs with the examination of the work in [34]. Uncer-

tainty quantification, as well as a polynomial chaos approach, were situated in both empirical documents. The second one considered a film cooling performance of an industrial gas turbine vane. Findings manifested the interval of confidence on behalf of investigation and probabilistic performance as well. Another area using polynomial chaos is pharmaceutical manufacturing [35]. Within pharmaceutical manufacturing processes, spray-drying was disputed. The method of selection for investigation was a polynomial chaos-based sensitivity analysis. The high efficiency of the stated technique was accentuated. Use of the model of the polynomial chaos expansion occurred in [36], where uncertainty quantification and the consequences of small manufacturing deviations on film cooling were reflected. Again, there is an observable match with studies mentioned previously.

A survey of the empirical works produced a further interesting paper dedicated to the manufacturing sector, together with the possibility to take advantage of modeling through the assistance of polynomials, specifically [37]. The modeling of asymmetric hysteresis behavior and compensation of the piezoelectric actuators were carried out. For this goal, a polynomial-modified Prandtl-Ishlinskii model was chosen. After the model testing phase, the conclusions presupposed the value of running the suggested polynomial-modified pattern. An uncertainty quantification cause is highly actual, mainly in an industrial area. This is confirmed in [38]. Researchers investigated the impact of model discrepancies at the calibration of physical model arguments. What is more, a Bayesian inference structure, plus an effort to fix for model discrepancy, was implemented. A polynomial expansion was introduced. An electric motor model was declared as an employment domain. Numeric screening of the uncertain forced convection of Al₂O₃-water nanofluid laminar flow in a grooved microchannel was realized in [39]. Geometrical variables and material characteristics reported uncertainties which were deputized by intervals. Chebyshev polynomial approximation was picked as a research method. Fifth-order polynomial equations were engaged in [40] alongside constitutive analysis. The object of interest was a high-temperature flow manner of steel for the design of flow stress.

2.1. Methodology

The main subject of the paper is data analysis, with subsequent building of our own model and final realization of the prediction corresponding to the machinery and electrical machinery sector as a highly relevant automation driver by using mathematical modeling. The suggested type of modeling as well as the prediction is unusual for the kind of data and character of topic examined in this study. The research concerns the combination of an industrial and manufacturing part (represented by the machinery and electrical machinery sector with regard to automation) and an economic part (the data are in fact of an economic nature). According to our knowledge, the chosen modeling type is not utilized in such an area. This claim applies to the selected method and likewise to the specific tool. A review of the available scientific literature basically showed two main research directions: On the one hand, are studies solving technical/industrial matters, accompanied by the application of the investigation method suitable for this study category. On the other hand, are studies with economic issues and economic methods. Thus, it is appropriate to argue that our approach is quite rare.

Considering the nature of our paper, the chosen topic, as well as the suggested analytical approach, a few matters should be highlighted. To gain a correct sequence for individual procedures in the research, we propose dividing the study into wider and narrower viewing angles with particular elements and steps. In a broader sense, the proper and detailed selection of the following elements is needed for the purposes of our analysis:

- Country
- Sector
- Indicator
- Period

From a narrower point of view, we propose and follow these concrete steps:

1. Mathematical modeling
2. Selection of individual elements
3. Weight assignment
4. Model type choice
5. Polynomial degree selection
6. Calculation of parameters
7. Prediction execution

The aforementioned steps represent an explicit logical sequence for our entire decision-making, modeling, and for the research itself.

For a better explanation of the methodological procedures, we provide a more detailed specification of the individual steps here. First, the mathematical modeling is stated. This step is included in all parts of our research; it is the fundamental string. The next one is selection of individual elements. Making the decision about country, sector, and indicator alike as considered time period stands behind this step. The following step-weight assignment yields improvement of our model fitting and consequently overall optimization. The model type choice indicates the commencement of the design our model. The polynomial degree selection offers operating with a number of polynomials, whereas after their modeling and testing, a pick of the two best variants is possible. Every previous step, together with the calculation of parameters, led to the execution of the prediction itself. This constitutes our final taken step.

We would like to highlight that a conceptual framework of the methodology is our own suggestion. However, it is based and combined with the approaches used in as standard research domains.

2.2. Data and Methods

The research is grounded in the collection, selection, analysis, and consecutive usage of eligible data for mathematical modeling purposes. The data come from a public and a freely available source: The World Integrated Trade Solution (WITS) [41]. The stated solution offers access to manifold international data with the possibility of sorting them by certain criteria. Various options can be discovered through this software in the form of numerous classifications, statistics, special analyses, development trends, or outputs involving a mixture of different filters. However, we find the pure data the best fundament for our research. Our work is thus based only on straight numbers. Anyway, we consider such options very helpful and do not exclude their usage in future investigations. Another great advantage, and powerful reason to utilize the described solution, is data guarantee. Data are compiled and summed under the patronage of several global organizations (e.g., the World Bank [42] and the World Trade Organization [43]) which represent the elite in the given issue. This creates an extensive, useful, and reliable database.

Reflecting on the proposed methodology and stated elements that are required for mathematical modeling, the pick is made as follows:

- Country: Lithuania
- Sector: Machinery, electrical machinery and parts (Mach and Elec)
- Indicator: Export
- Period: From 2004 to 2018

The regarded figures appertain to Lithuanian exports to the world. In other words, all countries with an export trade partnership with Lithuania are included; international merchandise trade cooperation is included. Exports are reported in thousands of US dollars. The time interval taken into account in the analysis incorporates 2004 and 2018, inclusively. In summary, it is a 15-year period. The WITS database provides data before 2004; however, we reject them for a certain reason. Lithuania became a member of the European Union in 2004 [44]. Therefore, from this year all data are comparable. The same methodology and methods were used in data collection, sorting, and processing, which

was not exactly the same before. Each country had its own partially adjusted procedures. Data for 2018 are the latest up-to-date, complete, and available ones.

The decision about the implementation of mathematical modeling as the most suitable variant is our first one. This is followed by the selection of the model type. After studying the scientific literature and contemplating our research conditions, data, and specifics, we apply polynomials [45]. We assume that the polynomial approximation is an excellent method for building our model. Our research case corresponds to the polynomials in numerical analysis. The concretization of the investigation method (in our case, a closer specification of the polynomial degree that is de facto related to the choice of separate function) occurs later within the testing realization. We consider this fact to be a plus because it allows a wider space for research itself and its more natural development. The whole direction of developing the model is not strictly defined in advance and can therefore be developed more appropriately.

Several software packages exist serving the mathematical design of the model. In our paper, the preferred one is Matlab [46]. In our opinion, the range of possibilities, accompanied by shipshape tools and graphical figurations, is unrivaled. From a wide scale of utilities, we adopt the Matlab Curve Fitting Tool [47]. The toolbox presents an app function for fitting curves and surfaces to data. Thus, we chose the polynomial model type with a Curve Fitting Tool when we looked for the most suitable degree of polynomial.

3. Results

For the sake of better clarity and orientation to the achieved results, we adhere to our proposed steps from the methodology section here. Choices for the first step, mathematical modeling, are all described above (data and methods part). What is more, mathematical modeling is incorporated in each and every suggested step.

The second one is selection of individual elements. In the context of automation, the most important and decisive factor for our research is sector determination. The machinery and electrical machinery sector is specified as a very substantial driver in the automation sphere. Its temporal modification is shown in Figure 1. In addition to the choice of sector, our selection of country, indicator, as well as period is preserved there.

We would like to highlight that all stated figures (Figures 1–7) represent our own processing, the outputs having been achieved by using Matlab [46], and they are based on WITS data [41].

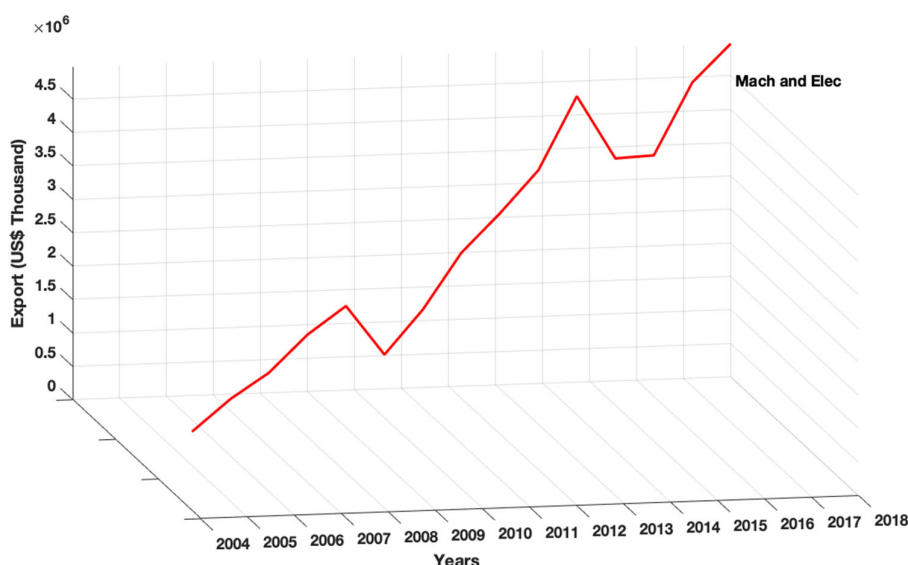


Figure 1. Development of the machinery and electrical machinery.

In general, a growing trend is visible in Figure 1. Moreover, there is a noticeable difference when comparing the values at the beginning of the observed period and at its end.

Concerning Lithuanian overall exports for all product groups, see Figure 2. To highlight the most recent developments, the last five years of our period are marked there.

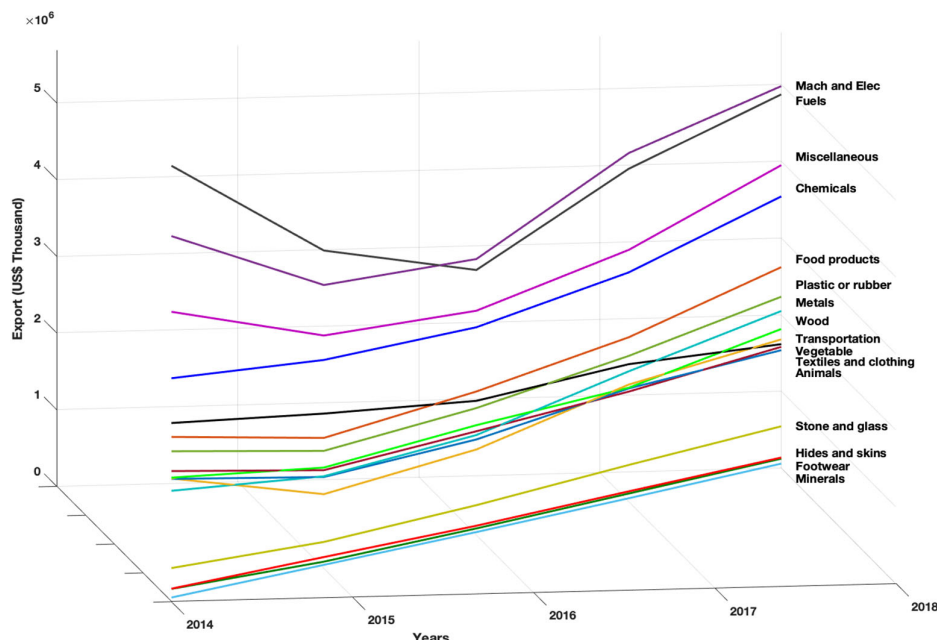


Figure 2. Development of all product groups.

Figure 2 offers distinguished mapping of the strong position regarding our reference segment as a proportion of the country's exports. Moreover, the point at which the leading place changed can be observed. To be precise, we are talking about 2016, when an exchange related to the first two standings occurs. Machinery and electrical machinery overtook fuels, which dropped to the second position. Such status is retained for the additional declared years. Note that from a comprehensive perspective, our selected sector has a relatively advantage over most other exported products. By scrutinizing Figure 2, on the one hand we find that quite a gap is formed between the first two categories (machinery and electrical machinery and fuels) and next two (miscellaneous group, chemicals) on the other hand. The same thing may be claimed for the second sectors mentioned (miscellaneous group and chemicals) and the ensuing set of eight product representatives (food products, plastic or rubber, metals, wood, transportation, vegetable, textiles and clothing, and animals). At the bottom of the export categories is a class of the last four product groups (stone and glass, hides and skins, footwear, and minerals).

It is possible to summarize that all findings resulting from Figures 1 and 2 only confirm the significance and thus the appropriateness of our choice for the sector element.

The third step constitutes the weight assignment. The main point for weighting utilization is to improve our fit. Our research practice uses diverse methods with the aim of dispatching the weights. We decided to follow the handbook of statistical methods—the Engineering Statistics Handbook [48]. The authors declare an approach where less weight is attributed to the less accurate appraisal, and vice versa, in the course of assessing the unknown parameters in the model. We apply weights as a computed weight per each value using the Nonlinear Least Square method operating with the Matlab Curve Fitting Tool with coefficients set from $<0.1, 1>$. Older values, from 2004, have less relevance, and the last available year (2018) has the most relevance for our later prediction. We can compare our older values to the less accurate appraisals and therefore assign less weight.

The subsequent step represents the model type choice. In fact, we are starting to build our model itself. This means creating and testing a number of models to get the right one—that with the most relevant, precise, and highest possible expressive capability. We proceed pursuant to the user's guide of the Curve Fitting Tool for use with Matlab [49]. Working in Matlab with a goal to identify the model type implies de facto defining the function type.

Three strategies exist to build a model: The first one is reviewing scientific literature to find an existing equation for our system or process. The second is based on the first while yielding a combination of multiple competent equations with the resulting new one. The remaining way is model development from the data sets. We create our model directly from the data sets. This objectivity leads us to look for a function with the highest R-square value. Data sets tell us that we have two admissible model alternatives for the purposes of our prediction. Applicable are linear fitting (the degree 1 polynomial) or high-level polynomials (from the degree 4 to 9). Degree 2 and 3 polynomials are not relevant because of their parabolic and hyperbolic running. First, we try a linear fitting with the results affirmed below.

Note that x entails the particular year (x -axis). Our input data are used for the creation of the model, thus for the coefficients' calculations.

Linear model:

$$f(x) = a * (\sin(x - \pi)) + b * ((x - 10)^2) + c \quad (1)$$

Where the coefficients (with 95% confidence bounds) are

$$a = -3.213e+05 \ (-6.7e+05, 2.733e+04) \quad (2)$$

$$b = 65.28 \ (51.32, 79.24) \quad (3)$$

$$c = -2.584e+08 \ (-3.143e+08, -2.026e+08) \quad (4)$$

and goodness of fit is represented by

$$SSE = 2.196e+12 \quad (5)$$

$$R\text{-square} = 0.8978 \quad (6)$$

$$\text{Adjusted } R\text{-square} = 0.8808 \quad (7)$$

$$RMSE = 4.277e+05 \quad (8)$$

We see from the results that linear fitting is not enough for our prediction because the R-square is under 90% of fit efficacy. Therefore, we continue with high-degree polynomial modeling and choosing the right degree of polynomial.

Based on the previous statements, very logically and naturally we pass to the step of polynomial degree selection. We work with several polynomials, from the degree 4 polynomial to the degree 9 polynomial. The findings are marked down closely.

Degree 4 polynomial (Linear model Poly4):

$$f(x) = p1 * x^4 + p2 * x^3 + p3 * x^2 + p4 * x + p5 \quad (9)$$

where x is normalized by mean 2011 and std 4.472, the coefficients (with 95% confidence bounds) are

$$p1 = -2.882e+04 \ (-5.75e+05, 5.174e+05) \quad (10)$$

$$p2 = -7.945e+04 \ (-5.244e+05, 3.655e+05) \quad (11)$$

$$p3 = 1.225e+05 \ (-1.221e+06, 1.466e+06) \quad (12)$$

$$p4 = 1.284e+06 (4.755e+05, 2.092e+06) \quad (13)$$

$$p5 = 2.897e+06 (2.315e+06, 3.48e+06) \quad (14)$$

and efficacy of fit is represented by

$$SSE = 2.855e+12 \quad (15)$$

$$R\text{-square} = 0.8671 \quad (16)$$

$$\text{Adjusted } R\text{-square} = 0.8139 \quad (17)$$

$$RMSE = 5.343e+05 \quad (18)$$

Degree 5 polynomial (Linear model Poly5):

$$f(x) = p1*x^5 + p2*x^4 + p3*x^3 + p4*x^2 + p5*x + p6 \quad (19)$$

where x is normalized by mean 2011 and std 4.472, coefficients (with 95% confidence bounds) are

$$p1 = 323.9 (-17.58, 665.4) \quad (20)$$

$$p2 = -3.257e+06 (-6.691e+06, 1.767e+05) \quad (21)$$

$$p3 = 1.31e+10 (-7.102e+08, 2.691e+10) \quad (22)$$

$$p4 = -2.635e+13 (-5.412e+13, 1.428e+12) \quad (23)$$

$$p5 = 2.649e+16 (-1.435e+15, 5.442e+16) \quad (24)$$

$$p6 = -1.066e+19 (-2.189e+19, 5.769e+17) \quad (25)$$

and efficacy of fit is represented by

$$SSE = 1.902e+12 \quad (26)$$

$$R\text{-square} = 0.9115 \quad (27)$$

$$\text{Adjusted } R\text{-square} = 0.8623 \quad (28)$$

$$RMSE = 4.597e+05 \quad (29)$$

Degree 6 polynomial (Linear model Poly6):

$$f(x) = p1*x^6 + p2*x^5 + p3*x^4 + p4*x^3 + p5*x^2 + p6*x + p7 \quad (30)$$

Where x is normalized by mean 2011 and std 4.472, the coefficients (with 95% confidence bounds) are

$$p1 = 3.838e+05 (-3.971e+05, 1.165e+06) \quad (31)$$

$$p2 = 5.644e+05 (-3.783e+04, 1.167e+06) \quad (32)$$

$$p3 = -1.433e+06 (-4.331e+06, 1.464e+06) \quad (33)$$

$$p4 = -1.788e+06 (-3.653e+06, 7.63e+04) \quad (34)$$

$$p5 = 1.369e+06 (-1.427e+06, 4.166e+06) \quad (35)$$

$$p6 = 2.275e+06 (1.002e+06, 3.548e+06) \quad (36)$$

$$p7 = 2.74e+06 (2.138e+06, 3.343e+06) \quad (37)$$

and efficacy of fit is represented by

$$SSE = 1.637e+12 \quad (38)$$

$$R\text{-square} = 0.9238 \quad (39)$$

$$\text{Adjusted } R\text{-square} = 0.8667 \quad (40)$$

$$RMSE = 4.523e+05 \quad (41)$$

Degree 7 polynomial (Linear model Poly7):

$$f(x) = p1*x^7 + p2*x^6 + p3*x^5 + p4*x^4 + p5*x^3 + p6*x^2 + p7*x + p8 \quad (42)$$

where x is normalized by mean 2011 and std 4.472, coefficients (with 95% confidence bounds) are

$$p1 = -7.435e+05 (-1.684e+06, 1.969e+05) \quad (43)$$

$$p2 = 3.838e+05 (-3.153e+05, 1.083e+06) \quad (44)$$

$$p3 = 3.727e+06 (-3.093e+05, 7.763e+06) \quad (45)$$

$$p4 = -1.433e+06 (-4.027e+06, 1.161e+06) \quad (46)$$

$$p5 = -5.542e+06 (-1.058e+07, -5.093e+05) \quad (47)$$

$$p6 = 1.369e+06 (-1.134e+06, 3.873e+06) \quad (48)$$

$$p7 = 3.356e+06 (1.576e+06, 5.135e+06) \quad (49)$$

$$p8 = 2.74e+06 (2.201e+06, 3.279e+06) \quad (50)$$

and efficacy of fit is represented by

$$SSE = 1.092e+12 \quad (51)$$

$$R\text{-square} = 0.9492 \quad (52)$$

$$\text{Adjusted } R\text{-square} = 0.8984 \quad (53)$$

$$RMSE = 3.949e+05 \quad (54)$$

Degree 8 polynomial (Linear model Poly8):

$$f(x) = p1*x^8 + p2*x^7 + p3*x^6 + p4*x^5 + p5*x^4 + p6*x^3 + p7*x^2 + p8*x + p9 \quad (55)$$

where x is normalized by mean 2011 and std 4.472, the coefficients (with 95% confidence bounds) are

$$p1 = -8.279e+05 (-2.054e+06, 3.981e+05) \quad (56)$$

$$p2 = -7.435e+05 (-1.615e+06, 1.279e+05) \quad (57)$$

$$p3 = 4.363e+06 (-1.565e+06, 1.029e+07) \quad (58)$$

$$p4 = 3.727e+06 (-1.304e+04, 7.467e+06) \quad (59)$$

$$p5 = -7.288e+06 (-1.629e+07, 1.709e+06) \quad (60)$$

$$p6 = -5.542e+06 (-1.021e+07, -8.787e+05) \quad (61)$$

$$p7 = 4.06e+06 (-5.507e+05, 8.67e+06) \quad (62)$$

$$p8 = 3.356e+06 (1.707e+06, 5.004e+06) \quad (63)$$

$$p9 = 2.553e+06 (1.982e+06, 3.124e+06) \quad (64)$$

and efficacy of fit is represented by

$$SSE = 7.503e+11 \quad (65)$$

$$R\text{-square} = 0.9651 \quad (66)$$

$$\text{Adjusted } R\text{-square} = 0.9185 \quad (67)$$

$$RMSE = 3.536e+05 \quad (68)$$

Degree 9 polynomial (Linear model Poly9):

$$f(x) = p1*x^9 + p2*x^8 + p3*x^7 + p4*x^6 + p5*x^5 + p6*x^4 + p7*x^3 + p8*x^2 + p9*x + p10 \quad (69)$$

where x is normalized by mean 2011 and std 4.472, the coefficients (with 95% confidence bounds) are

$$p1 = -2e+05 (-2.287e+06, 1.887e+06) \quad (70)$$

$$p2 = -8.279e+05 (-2.23e+06, 5.746e+05) \quad (71)$$

$$p3 = 3.188e+05 (-1.081e+07, 1.145e+07) \quad (72)$$

$$p4 = 4.363e+06 (-2.418e+06, 1.114e+07) \quad (73)$$

$$p5 = 1.883e+06 (-1.783e+07, 2.16e+07) \quad (74)$$

$$p6 = -7.288e+06 (-1.758e+07, 3.004e+06) \quad (75)$$

$$p7 = -4.383e+06 (-1.761e+07, 8.847e+06) \quad (76)$$

$$p8 = 4.06e+06 (-1.214e+06, 9.333e+06) \quad (77)$$

$$p9 = 3.163e+06 (4.111e+05, 5.916e+06) \quad (78)$$

$$p10 = 2.553e+06 (1.9e+06, 3.207e+06) \quad (79)$$

and efficacy of fit is represented by

$$SSE = 7.413e+11 \quad (80)$$

$$R\text{-square} = 0.9655 \quad (81)$$

$$\text{Adjusted } R\text{-square} = 0.9034 \quad (82)$$

$$\text{RMSE} = 3.85\text{e}+05$$

(83)

We find it interesting to present a graphical representation for the degree 4 polynomial (Figure 3) and similarly the degree 9 polynomial (Figure 4). This allows a stimulating and instantaneous illustrative comparison as well as tracking the differences.

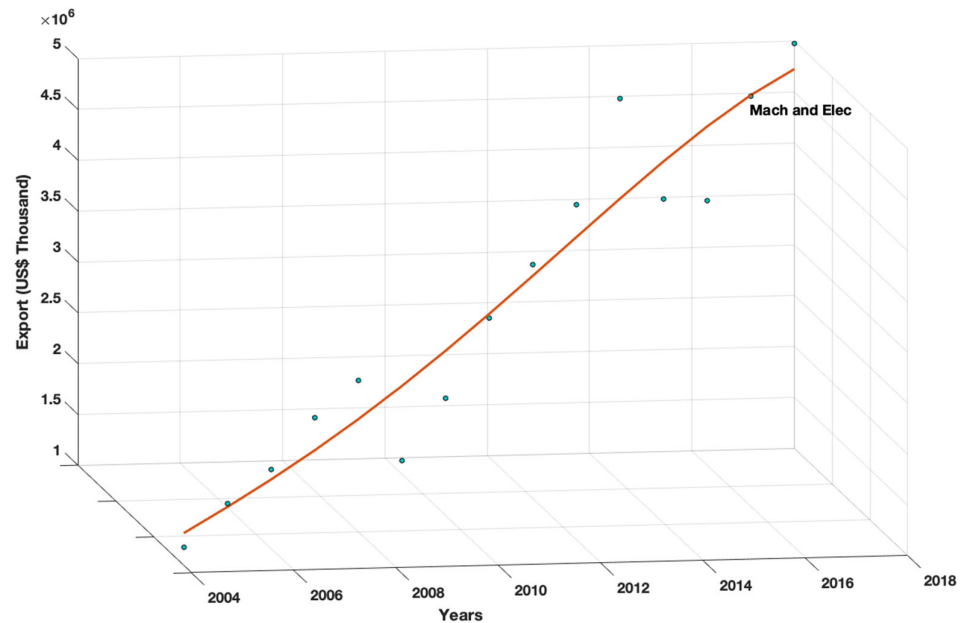


Figure 3. The degree 4 polynomial.

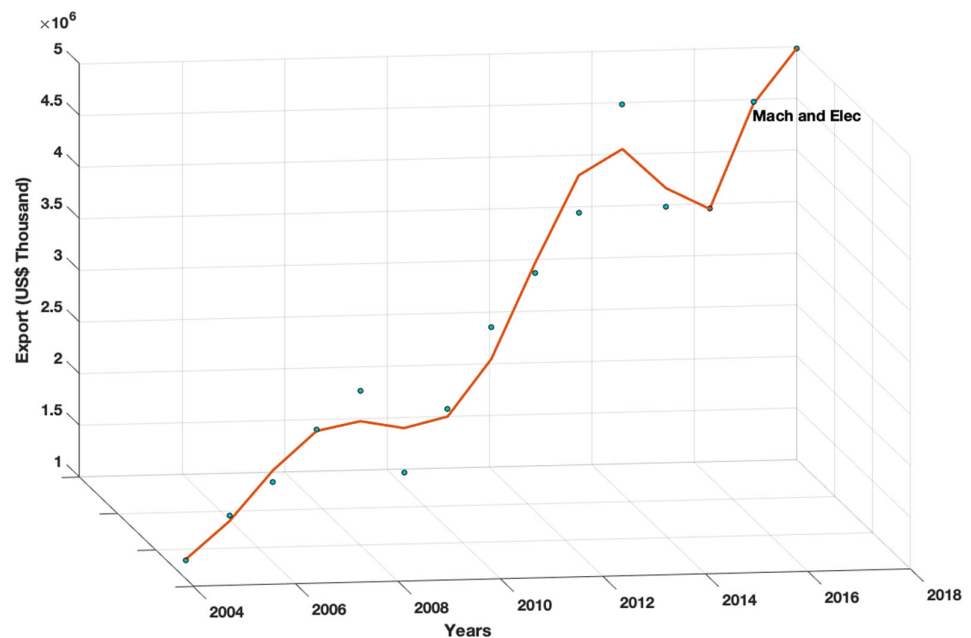


Figure 4. The degree 9 polynomial.

For higher coherence, we demonstrate in Table 1 where the review of tested models and their results is initiated. Altogether, we tested seven mathematical models.

Table 1. The overview of tested models with corresponding results.

Model Type.	SSE	R-Square	Adjusted R-Square	RMSE
Linear fitting	2.196e+12	0.8978	0.8808	4.277e+05
Degree 4 polynomial	2.855e+12	0.8671	0.8139	5.343e+05
Degree 5 polynomial	1.902e+12	0.9115	0.8623	4.597e+05
Degree 6 polynomial	1.637e+12	0.9238	0.8667	4.523e+05
Degree 7 polynomial	1.092e+12	0.9492	0.8984	3.949e+05
Degree 8 polynomial	7.503e+11	0.9651	0.9185	3.536e+05
Degree 9 polynomial	7.413e+11	0.9655	0.9034	3.85e+05

The degree 6 and 7 polynomial models have the best results. These two variants are further considered. In Table 1, they are marked in dark. Even though the degree 8 and 9 polynomials get the better R-square results, this is because of the so-called higher degree polynomials problem. In general, distortion of the results is obtained by calculation over the R-square method, because when you increase the degree of a polynomial, the R-square increases, but the fit does not necessarily correspond to this R-square increase.

The calculation of parameters is a step which is actually an integrated part of our modeling. The exact values are already given for the individual model types. As part of this step, the documentation [42] on behalf of our used tool can be reminded. The coefficients are calculated with 95% confidence of bounds. This implies that our model attempts to fit as close as possible to 95% of the determined data set. The remaining 5% may be redundant and the computation tool should ignore them if they are out of the function range. This set level is enough for our modeling to keep all relevant data, to get the sufficient scope to our model, and to receive the relevant fit together with the prediction.

Describing our results, we move on to the next step—prediction. Execution of the prediction is the final step according to our designed methodology and constitutes the completion of the previous ones. As we stated above, the degree 6 polynomial model and the degree 7 polynomial model report the best outcomes. That is the reason why we test both of them within the prediction. Table 2 gives the prediction of bounds for the degree 6 polynomial as well as the degree 7 polynomial models.

Table 2. Prediction for the degree 6 and the degree 7 polynomial models.

Predicted Bound/Value.	Degree 6 Polynomial (10 ⁶) (optimized under Weights)	Degree 7 Polynomial (10 ⁶)
2019 lowest predicted bound	2.18	−4.16
2019 highest predicted bound	11.06	1.214
2019 predicted value with the most relevance	6.622	3.990
2020 lowest predicted bound	−5.06	−3.787
2020 highest predicted bound	24.43	2.885
2020 predicted value with the most relevance	9.682	−4.511
2021 lowest predicted bound	−21.94	−12.913
2021 highest predicted bound	51.98	6.475
2021 predicted value with the most relevance	15.020	−32.187
2022 lowest predicted bound	−55.26	−333.54
2022 highest predicted bound	102.52	133.40
2022 predicted value with the most relevance	23.628	−100.07
2023 lowest predicted bound	−114.76	−740.03

2023 highest predicted bound	188.19	254.68
2023 predicted value with the most relevance	36.715	−242.67

Regarding the difference between the degree 6 and 7 polynomials, the degree 7 polynomial has a better R-square value but a worse RMSE value. This is because the degree 7 polynomial fits our actual data better; nevertheless, the degree 6 polynomial is more suitable for predicting these data. The degree 7 polynomial sways what is further reflected in the degree 8 polynomial and the degree 9 polynomial, too. Therefore, these models are not suitable for prediction realization (see Table 1). Due to the mentioned swaying, the degree 7 polynomial is already basically a sinusoid. Such a state is improper for predicting our kind of data—it is impossible to go into negative numbers. At this spot, we track figures pertaining to the rows discussing predicted value with the most relevance (see Table 2). They represent the substantive values for our results. In Table 2, they are highlighted in dark. Thus, the prediction unsuitability of the degree 7 polynomial, despite a better R-square, can be clearly seen from Table 2.

All analysis, modeling, and testing point to the degree 6 polynomial as our final model type. We accept it for prediction of the machinery and electrical machinery sector. Figure 5 shows the degree 6 polynomial optimized under weights.

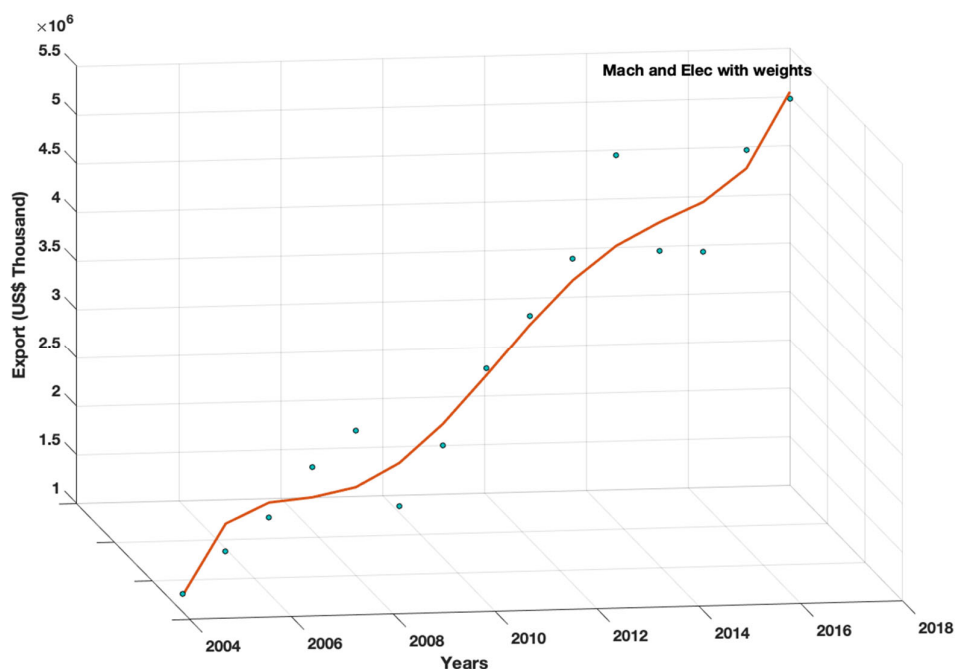


Figure 5. The degree 6 polynomial with weights.

The predicted development using our mathematical model is exhibited in Figure 6; it gives the final predicted values including weights.

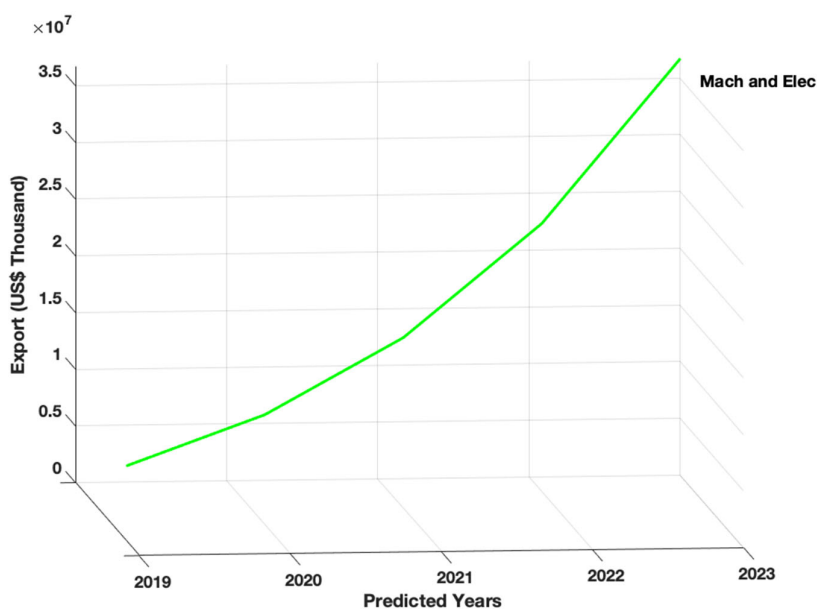


Figure 6. Prediction for machinery and electrical machinery based on our model.

The summarized visualization of the machinery and electrical machinery export for the selected period is demonstrated in Figure 7.

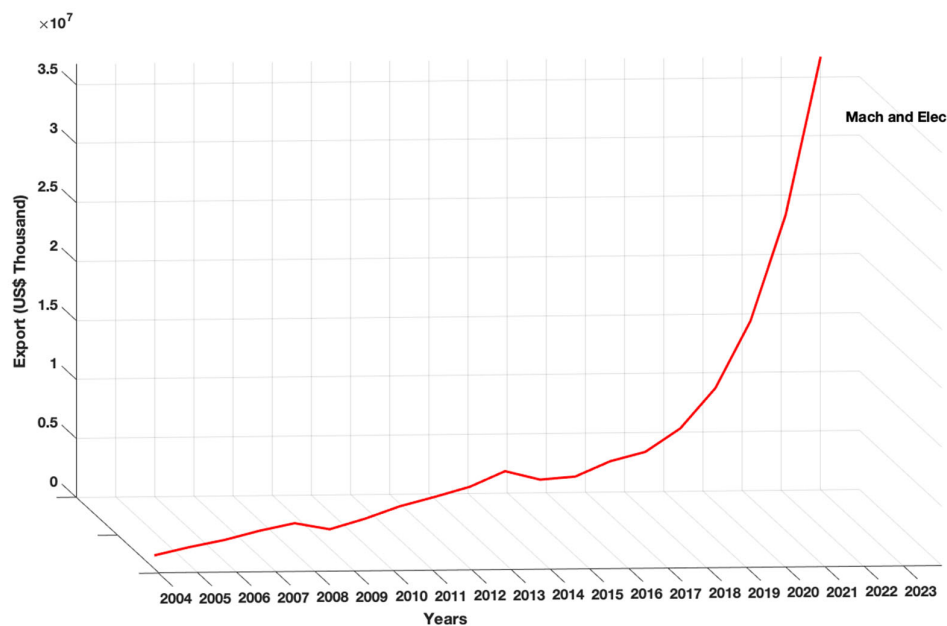


Figure 7. Development of the machinery and electrical machinery sector covering 2004–2023.

4. Discussion

Automation and robotics are becoming an increasingly more essential part of the many different fields. Our research is aimed at the manufacturing sector and its strategic part—machinery and electrical machinery segment. It is perceived as a very substantial driver in the automation sphere.

The paper brings several interesting benefits. One of the contributions of this work is showing and expanding the potential of the investigation method ordinarily applied for

technical and industrial fields and through its implementation for data discussing the manufacturing sector's selected part, however with an economical nature (export data). The decision about the modeling type, which is not ordinarily used with respect to the character as well as data sets of our examination, is not a random choice. It has its justification. The purpose, and also another benefit of the article, is receiving significantly more accurate results utilizing this type of modeling compared with implementing some of the standardly employed methods from an economic or statistical area. What is more, the opportunity for additional questions of the investigation arises here. The next different sectors, indicators, countries, or specific units are enabled to then be tackled. Furthermore, various economic tasks can be solved following our mathematical modeling. All of the mentioned possibilities may be realized, even in combination with the mutual comparison of their individual outputs.

Further contribution and the core of our research represents the model building accompanied by optimization over weights. Determination of the most suitable model was preceded by creating and testing a number of models to obtain the proper one. We tested a total of seven specimens. The mathematical model was constructed with the aim to accomplish the prediction of the selected sector of the specific national economy. We fulfilled this stated goal, as well.

Looking at our final results, we declare a clear and strongly growing trend with regard to the Lithuanian export of the machinery and electrical machinery. It can be said from the perspective of practice that the more this sector succeeds, the more companies will introduce and invest in automation and robotics. Nowadays, an indisputable fact is the existence of a turbulent business environment. This applies to the particular companies, sectors, countries, or various associations of countries. Thus, it is valid at micro level as well as macro level, and worldwide too. In such a connection, the usage of our paper's content and findings is applicable in many different domains and their subdomains, for instance, the question of competitiveness (staying competitive, gaining the competitive advantage), investments, decision-making, strategic management (planning), introduction of new technologies, partnerships, manufacturing processes, foreign business, industrial software (automation and robotics), and much more.

Our scrutiny offers some interesting matters. Therefore, several extensions come into play in the future. We can mention the addition of other indicators or mathematical procedures, thereby ensuring the refinement of the resulting model. The use of further tools and algorithms offered by Matlab is also worth considering. Last but not least, the implementation of additional software, and even potentially a comparison with the results achieved with the help of Matlab and its tools, come into reflection.

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References

- Gresova, E.; Svetlik, J. Modeling within national economy using industry-oriented indicators: Evidence from Czech Republic. *MM Sci. J.* **2020**, *6*, 3892–3895, doi:10.17973/MMSJ.2020_06_2019028.
- Gresova, E.; Svetlik, J. Economic Impact of Industrial Segment in the Selected Country. *Manag. Syst. Prod. Eng.* **2020**, *28*, 312–317, doi:10.2478/mspe-2020-0044.
- Cohen, Y.; Shoval, S.; Faccio, M.; Minto, R. Deploying cobots in collaborative systems: major considerations and productivity analysis. *International Journal of Production Research* **2021**, DOI: 10.1080/00207543.2020.1870758.
- Cohen, Y.; Faccio, M.; Pilati, F.; Yao, X.F. Design and management of digital manufacturing and assembly systems in the Industry 4.0 era. *Int. J. of Adv. Man. Tech.* **2019**, *105*, 3565–3577, DOI 10.1007/s00170-019-04595-0.
- Krenicky, T. Non-contact Study of Surfaces Created Using the AWJ Technology. *Manuf. Technol.* **2015**, *15*, 61–64, doi:10.21062/ujep/x.2015/a/1213-2489/MT/15/1/61.
- Boschetti, G.; Faccio, M.; Minto, R. Control Model for Collaborative Manufacturing: An Integrated Opened Framework for Human-Robot Collaboration. *Mechanisms and Machine Science* **2021**, *91*, 403–413.
- Boschetti, G.; Bottin, M.; Faccio, M.; Minto, R. Multi-robot multi-operator collaborative assembly systems: a performance evaluation model. *J. Intell. Manuf.* **2021**, DOI: 10.1007/s10845-020-01714-7.
- Salic, Z.; Atmojo, U.D.; Park, H.; Chen, A.T.Y.; Kevin, I.; Wang, K. Designing dynamic and collaborative automation and robotics software systems. *IEEE Trans. Ind. Inform.* **2017**, *15*, 540–549, doi:10.1109/TII.2017.2786280.
- Wan, J.; Tang, S.; Shu, Z.; Li, D.; Wang, S.; Imran, M.; Vasilakos, A.V. Software-defined industrial internet of things in the context of industry 4.0. *IEEE Sens. J.* **2016**, *16*, 7373–7380, doi:10.1109/JSEN.2016.2565621.
- Bortolini, M.; Faccio, M.; Galizia, F.G.; Gamberi, M.; Pilati, F. Design, engineering and testing of an innovative adaptive automation assembly system. *Assembly Automation* **2020**, *40*, 531–540, DOI: 10.1108/AA-06-2019-0103.
- Bortolini, M.; Faccio, M.; Galizia, F.G.; Gamberi, M.; Pilati, F. Adaptive Automation Assembly Systems in the Industry 4.0 Era: A Reference Framework and Full-Scale Prototype. *Appl. Sci.* **2021**, *11*, DOI: 10.3390/app11031256.
- Faccio, M.; Bottin, M.; Rosati, G. Collaborative and traditional robotic assembly: a comparison model. *Abbreviated Journal Name* **2019**, *102*, 1355–1372, DOI: 10.1007/s00170-018-03247-z.
- Pan, M.; Linner, T.; Pan, W.; Cheng, H.; Bock, T. A framework of indicators for assessing construction automation and robotics in the sustainability context. *J. Clean. Prod.* **2018**, *182*, 82–95, doi:10.1016/j.jclepro.2018.02.053.
- Bock, T. The future of construction automation: Technological disruption and the upcoming ubiquity of robotics. *Autom. Constr.* **2015**, *59*, 113–121, doi:10.1016/j.autcon.2015.07.022.
- Cai, S.; Ma, Z.; Skibniewski, M.J.; Bao, S. Construction automation and robotics for high-rise buildings over the past decades: A comprehensive review. *Adv. Eng. Inform.* **2019**, *42*, 100989, doi:10.1016/j.aei.2019.100989.
- Rakay, S.; Labant, S.; Bartos, K. Verification of floor planarity by trigonometrical measurement of heights on a 5-storey monolithic building. *Geod. Cartogr.* **2018**, *44*, 14–21, doi:10.3846/gac.2018.269.
- Oke, A.; Aigbavboa, C.; Mabena, S. Effects of automation on construction industry performance. In *Second International Conference on Mechanics, Materials and Structural Engineering*; Atlantis Press: Beijing, China, 2017; pp. 370–374; doi:10.2991/icmmse-17.2017.61.
- Jaluria, Y. Mathematical Modeling of Manufacturing Processes. In *Advanced Materials Processing and Manufacturing, Mechanical Engineering Series*; Springer: Cham, Switzerland, 2018; pp. 19–39; doi:10.1007/978-3-319-76983-7_2.
- Ren, W.; Xu, J.; Lian, Z.; Yu, P.; Yu, H. Modeling and Experimental Study of the Localized Electrochemical Micro Additive Manufacturing Technology Based on the FluidFM. *Materials* **2020**, *13*, 2783, doi:10.3390/ma13122783.
- Kim, N.; Yang, C.; Lee, H.; Aluru, N.R. Spatial Uncertainty Modeling for Surface Roughness of Additively Manufactured Microstructures via Image Segmentation. *Appl. Sci.* **2019**, *9*, 1093, doi:10.3390/app9061093.
- Liao, H.; Deng, Q.; Wang, Y. Optimal Acquisition and Production Policy for End-of-Life Engineering Machinery Recovering in a Joint Manufacturing/Remanufacturing System under Uncertainties in Procurement and Demand. *Sustainability* **2017**, *9*, 338, doi:10.3390/su9030338.
- Chiu, S.W.; Kuo, J.-S.; Chiu, V.; Chiu, Y.-S.P. Cost Minimization for a Multi-Product Fabrication-Distribution Problem with Commonality, Postponement, and Quality Assurance. *Math. Comput. Appl.* **2016**, *21*, 38, doi:10.3390/mca21030038.
- Galal, N.M.; Moneim, A.F.A. A Mathematical Programming Approach to the Optimal Sustainable Product Mix for the Process Industry. *Sustainability* **2015**, *7*, 13085–13103, doi:10.3390/su71013085.
- Sentyakov, K.; Peterka, J.; Smirnov, V.; Bozek, P.; Sviatskii, V. Modeling of Boring Mandrel Working Process with Vibration Damper. *Materials* **2020**, *13*, 1931, doi:10.3390/ma13081931.
- Orosz, T.; Rassolkin, A.; Kallaste, A.; Arsenio, P.; Panek, D.; Kaska, J.; Karban, P. Robust Design Optimization and Emerging Technologies for Electrical Machines: Challenges and Open Problems. *Appl. Sci.* **2020**, *10*, 6653, doi:10.3390/app10196653.
- Torreggiani, A.; Bianchini, C.; Davoli, M.; Bellini, A. Design for Reliability: The Case of Fractional-Slot Surface Permanent-Magnet Machines. *Energies* **2019**, *12*, 1691, doi:10.3390/en12091691.
- Kande, M.; Isaksson, A.J.; Thottappillil, R.; Taylor, N. Rotating Electrical Machine Condition Monitoring Automation—A Review. *Machines* **2017**, *5*, 24, doi:10.3390/machines5040024.
- Nikitin, Y.; Božek, P.; Peterka, J. Logical–Linguistic Model of Diagnostics of Electric Drives with Sensors Support. *Sensors* **2020**, *20*, 4429, doi:10.3390/s20164429.

29. Nguyen, T.K.; Phung, L.X.; Bui, N.-T. Novel Integration of CAPP in a G-Code Generation Module Using Macro Programming for CNC Application. *Machines* **2020**, *8*, 61, doi:10.3390/machines8040061.
30. Dineva, A.; Mosavi, A.; Gyimesi, M.; Vajda, I.; Nabipour, N.; Rabczuk, T. Fault Diagnosis of Rotating Electrical Machines Using Multi-Label Classification. *Appl. Sci.* **2019**, *9*, 5086, doi:10.3390/app9235086.
31. Yu, J.; Yang, S.; Kim, J.; Lee, Y.; Lim, K.-T.; Kim, S.; Ryu, S.-S.; Jeong, H. A Confidence Interval-Based Process Optimization Method Using Second-Order Polynomial Regression Analysis. *Processes* **2020**, *8*, 1206, doi:10.3390/pr8101206.
32. Blume, C.; Blume, S.; Thiede, S.; Herrmann, C. Data-Driven Digital Twins for Technical Building Services Operation in Factories: A Cooling Tower Case Study. *J. Manuf. Mater. Process.* **2020**, *4*, 97, doi:10.3390/jmmp4040097.
33. Cravero, C.; Ottonello, A. Uncertainty Quantification Methodologies Applied to the Rotor Tip Clearance Effect in a Twin Scroll Radial Turbine. *Fluids* **2020**, *5*, 114, doi:10.3390/fluids5030114.
34. Gamannossi, A.; Amerini, A.; Mazzei, L.; Bacci, T.; Poggiali, M.; Andreini, A. Uncertainty Quantification of Film Cooling Performance of an Industrial Gas Turbine Vane. *Entropy* **2020**, *22*, 16, doi:10.3390/e22010016.
35. Bhonsale, S.; Munoz Lopez, C.A.; Van Impe, J. Global Sensitivity Analysis of a Spray Drying Process. *Processes* **2019**, *7*, 562, doi:10.3390/pr7090562.
36. Shi, W.; Chen, P.; Li, X.; Ren, J.; Jiang, H. Uncertainty Quantification of the Effects of Small Manufacturing Deviations on Film Cooling: A Fan-Shaped Hole. *Aerospace* **2019**, *6*, 46, doi:10.3390/aerospace6040046.
37. Wang, W.; Wang, J.; Chen, Z.; Wang, R.; Lu, K.; Sang, Z.; Ju, B. Research on Asymmetric Hysteresis Modeling and Compensation of Piezoelectric Actuators with PMPI Model. *Micromachines* **2020**, *11*, 357, doi:10.3390/mi11040357.
38. John, D.N.; Schick, M.; Heuveline, V. Learning Model Discrepancy of an Electric Motor with Bayesian Inference. *Proceedings* **2019**, *33*, 11, doi:10.3390/proceedings2019033011.
39. Zheng, Z.; Jing, Q.; Xie, Y.; Zhang, D. An Investigation on the Forced Convection of Al₂O₃-water Nanofluid Laminar Flow in a Microchannel under Interval Uncertainties. *Appl. Sci.* **2019**, *9*, 432, doi:10.3390/app9030432.
40. Lei, B.; Chen, G.; Liu, K.; Wang, X.; Jiang, X.; Pan, J.; Shi, Q. Constitutive Analysis on High-Temperature Flow Behavior of 3Cr-1Si-1Ni Ultra-High Strength Steel for Modeling of Flow Stress. *Metals* **2019**, *9*, 42, doi:10.3390/met9010042.
41. World Integrated Trade Solution. Available online: <https://wits.worldbank.org/> (accessed on 25 September 2020).
42. World Bank Group. Available online: <https://www.worldbank.org/en/what-we-do> (accessed on 25 September 2020).
43. World Trade Organization. Available online: https://www.wto.org/english/thewto_e/thewto_e.htm (accessed on 25 September 2020).
44. Ministry of Foreign Affairs of the Republic of Lithuania. Available online: <https://urm.lt/default/en/foreign-policy/lithuania-in-the-region-and-the-world/lithuania-member-of-the-european-union/membership> (accessed on 25 September 2020).
45. Polynomials. Available online: https://www.mathworks.com/help/matlab/polynomials.html?s_tid=srchtitle (accessed on 10 October 2020).
46. Matlab. Available online: <https://www.mathworks.com/> (accessed on 10 October 2020).
47. Curve Fitting Toolbox. Available online: <https://www.mathworks.com/help/curvefit/> (accessed on 10 October 2020).
48. Engineering Statistics Handbook. Available online: <https://www.itl.nist.gov/div898/handbook/index.htm> (accessed on 10 October 2020).
49. User's Guide—Curve Fitting Toolbox for Use with Matlab. Available online: http://cda.psych.uiuc.edu/matlab_pdf/curvefit.pdf (accessed on 10 October 2020).