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Multiple-Criteria Fuzzy Optimization of the Heat Treatment Processes for Two Steel Rolled Products

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Abstract: This paper presents a developed method for fuzzy multiple-criteria optimization of the rolled-steel heat treatment processes in the modern metallurgical plant. At the first stage of the study, by means of passive industrial experiments or a mathematical simulation of heat transfer processes, and using statistical methods, the regression dependencies of the output parameters of process quality on the input variables that are technological parameters are established. Then, based on the quality parameters, membership functions are formed that represent local criteria of the process quality, and their ranks are calculated using the matrix of pairwise comparisons. The practically useful methodology of the fuzzy multiple-criteria optimization of technological processes is proposed. To illustrate this methodology's practical efficiency, the solutions of two optimization problems are found by maximizing the global criterion that aggregates local criteria using their ranks. It is shown that the efficiency of the obtained optimal heat treatment modes significantly exceeds the efficiency of the technology used earlier in the plant.

Keywords: fuzzy optimization; multiple criteria; heat-treatment processes; steel-rolled products

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1. Introduction

In this paper, we will consider fuzzy optimization as the multiple-criteria optimization problem with fuzzy local criteria and constraints of different levels of importance (weights). As we have some experience in this field, we can say that, regardless of the analyzed branch of technology—economy, medicine, etc.—in the solution of fuzzy optimization tasks, we will encounter all or some of the following problems:

1. Aggregated presentation of local criteria under different types of uncertainty. Local criteria can be built based on measured parameters, such as technological ones, or with the use of expert subjective estimations, e.g., a verbal assessment of the investment project's scientific or innovative levels. The known property of people's thinking is that experts usually try to provide estimates at the verbal level of consideration, avoiding quantitative ones, as they their opinions are unreliable. Therefore, the problem of the synthesized treatment of local criteria characterized by different types of uncertainty becomes actual [1–4]. However, people's experience and intuition are very important in the formulation of optimization tasks, and should not be lost in analysis. Of course, uncertainty is an inevitable component of such types of information, which should not be ignored; on the contrary, it should be studied carefully, evaluated and included in the optimization process. This uncertainty is of subjective nature. Therefore, it cannot adequately be presented by means of a traditional probabilistic approach. Therefore, a justified methodology, providing an opportunity for synthesized assessments of uncertainty factors, that makes it possible to design comparable local criteria based on measurable quantitative parameters as well as verbal assessments, is needed. To solve this problem, the methods of the fuzzy set theory, developed for dealing

with uncertainties of a subjective origin, may be used with great efficiency. Therefore, in real-world optimization problems, we deal with two groups of local criteria: objective criteria based on measured parameters, and subjective criteria based on the verbal expert assessments. Therefore, we have a problem in the development of an appropriate common quantitative scale for representing objective and subjective criteria;

2. Generalization of experts opinions.

In the formulation of especially important fuzzy optimization tasks, groups of experts from corresponding fields are often involved in defining the membership functions of the local criteria used, and their relative importance (weights). To aggregate experts' opinions, the methods developed in the framework of the Group Multiple-Criteria Decision-Making group (MCDM) [5–7] may successfully be applied. These methods provide a compromise between expert assessments, including the cases when they are presented in a verbal form. The modern methods of the hesitant fuzzy set theory [8] may be applied when different numbers of experts can be invited to define the local criteria or their weights [9];

3. Local criteria aggregation.

The real-world optimization problems can involve broad sets of local criteria that need to be analyzed simultaneously in each step of an optimization process. It is clear that the corresponding optimizing algorithm should somewhat reflect human possibilities to make this analysis. Unfortunately, a person's ability to carry out such an analysis is severely limited by the well-known, strict, experimentally justified law of applied psychology, according to which a person can discern no more than 7 or minus 2 classes or grades on any scale of characteristics. If the number of classes is greater than this, neighboring classes begin to merge and cannot confidently be distinguished [10,11]. To solve this problem, an appropriate method for the aggregation of local criteria, taking their weights into account, can be applied to build a generalized criterion. Therefore, the problem of choosing a relevant aggregation method is of continuing interest, since it is directly related to the practical optimization problems. The most-used aggregation method is the weighted sum. This is used in many known optimization models as it is the simplest (linear) approach, but, regrettably, is often used often without any previous critical analysis.

On the other hand, in some areas, e.g., environmental modeling, the weighted sum aggregation is forbidden from use [12]. The reason for this is that, in practice, there are cases when, if any of the local criteria are completely unsatisfied, the alternative in question may be excluded from consideration altogether, although the values of other criteria may be great enough to recompensate the zero value of the single criterion. Therefore, when solving a complex problem with a large number of local criteria, it seems justified to use all possible aggregating modes that are relevant to the problem at hand. If the results obtained using different aggregation methods are similar, then this fact can be considered as a good confirmation of their optimality. Otherwise, an additional analysis of local criteria and their ranking should be carried out;

4. Local criteria weights' establishment.

First, based on our experience, we can say that if an expert considers local criteria to be of equal importance, this usually only means that he/she is not competent enough. In cases when we are dealing with a small number (2 or 3) of local criteria, their weights can be directly established by an expert based on his/her experience and intuition, but if we have more criteria, then, due to the abovementioned psychological law, an expert begins to experience serious difficulties in determining the weights of the criteria. Therefore, the methods based on linguistic pairwise comparison matrices are currently used to obtain the real-valued [13,14] or fuzzy weights of local criteria [15,16];

5. Generalization of aggregation modes.

A natural consequence of problem 3 is the growing interest in methods for generalization (convolution) aggregating operators [17]. For this purpose, the use of the theory of possibilities [18], the weighted sum aggregation of aggregating modes [19] and

the methods of soft sets' theory are proposed [20]. In [21], a critical analysis of the known approaches to the aggregation of aggregating modes is provided and a new method, based on the synthesis of type 2 and level 2 fuzzy sets, which is free of the revealed drawbacks of existing approaches, is developed. This method was successfully used for the solution of the fuzzy multiple-objective optimization problem in logistic [22];

6. Comparing the fuzzy values.

It is worth noting that the problem of comparing interval and fuzzy values plays a key role in the fuzzy optimization. For example, when we are dealing with fuzzy weights of local criteria, we have a fuzzy general criterion to be minimized or maximized. Therefore, the fuzzy values of this criterion should be compared in subsequent optimization steps. There are many definitions of the fuzzy values' order relations proposed in the literature [23,24]. Most often, to compare fuzzy values, different real-valued indices are used [25]. The values of these indices represent an extent to which one fuzzy value is greater/lesser than the other one. In some cases, several indices are used at the same time. A separate group of methods is based on the so-called probabilistic approach to comparing fuzzy values. The attractiveness of this approach is due to the possibility of the presentation of fuzzy values' comparison based on a minimum set of restricting assumptions [26].

Obviously, the list of problems presented above can be extended. For example, we encounter additional problems when the variables defining the fuzzy local criteria used are fuzzy values as well, or the values of membership functions are also represented by fuzzy values.

We will not try to present all possible problems of the fuzzy optimization here: it is simply impossible to do. Therefore, we present only those problems that we have encountered in our own practical activities in the field of fuzzy multiple-criteria optimization of technological processes, as well as in the related field of multiple-criteria decision-making.

The aim of this paper is to present and illustrate the methodology that we have developed in the three last decades, based on our steadily growing scientific and practical experience in developing and embedding optimal technological processes. Therefore, this methodology may be considered as our scientific and practical contribution to the solution to fuzzy multiple-criteria optimization problems.

Therefore, this paper is largely addressed to those practitioners who encounter such problems in work which can be classified as fuzzy multiple-criteria optimization. We hope that this article will help them to adequately navigate the vast sea of proposed methods and publications on this topic with minimal time and material resources.

The rest of the paper is set out as follows.

In Section 2, an informal, but informative introduction to the problem at hand is provided. To expose the basic problems in fuzzy multiple -criteria optimization, a simple, real-world heating-based technology is analyzed. This might help in reading and understanding the paper, especially for those who are not familiar with modern approaches to the solution of optimization tasks. Section 3 presents the solutions to two different fuzzy multiple-criteria optimization problems concerned with the heat-treatment processes for two steel-rolled products. These processes are components of the overall technological chain in the "Belorussian Metallurgical Mini Plant". Section 4 concludes the paper.

2. Informal Presentation of the Problem

To expose the basics of the fuzzy multiple-criteria optimization of technological processes, consider the sample problem of optimizing grilled chicken cooking technology.

The main factor, which almost entirely determines the success of this enterprise, is the time τ a chicken takes to cook.

Three main local criteria directly depend on this parameter of quality: the taste of the chicken $\mu_t(\tau)$, the cost of electricity for cooking $\mu_e(\tau)$, and the waiting time for a gourmet in the queue $\mu_q(\tau)$, see Figure 1.

Figure 1. The problem of grilled-chicken-cooking process-optimizing.

The problem is choosing the optimal cooking time based on a set of local criteria. For the mathematical formalization of local criteria, the desirability functions [27] can be used. These functions change from 0 in areas that are not allowed (undesirable) as values of the parameter of quality, up to a maximum value 1 in the areas of its most preferred values. There are no other restrictions on desirability functions other than continuity and convexity.

Meanwhile, along with the desirability functions introduced by Harrington in 1965 [27], membership functions, as a cornerstone of the fuzzy sets theory, were proposed by Zadeh [28]. It is worth noting that, in the context of optimization problems, the definitions of desirability and membership functions practically coincide. Nevertheless, to formalize local criteria, we will use the membership functions as the rich semantic of the fuzzy sets theory to provide opportunities to reveal and use more complicated types of uncertainty.

Let us turn to our example. Taking into account above assumptions, the economic criterion for energy costs can be represented by the membership function $\mu_e(\tau)$ shown in Figure 1. According to this function, at $\tau < \tau_1$, energy costs are not significant for the company. With the further growth of τ , energy costs increase, which is reflected in the reduction in the membership function values to 0 at $\tau = \tau_8$. At $\tau > \tau_8$, the production becomes unprofitable due to too-high energy costs.

The behavior of the membership function, which formalizes the criterion of waiting for a client in the queue $\mu_q(\tau)$, corresponds to the fact that at $\tau < \tau_2$, almost all clients are ready to wait for cooking. With the further growth in τ , more and more customers leave the queue, because there are competing firms, and with $\tau > \tau_5$, customers consider the waiting time unacceptable.

The form of the membership function of the chicken-taste criterion $\mu_t(\tau)$ is determined by the following considerations. At $\tau < \tau_3$, the chicken will be soggy. With the growth in τ , its taste qualities grow, and in the range of $\tau_4 < \tau < \tau_6$, the chicken is most delicious from the perspective of most customers. At $\tau > \tau_6$, the chicken is already somewhat dry, although there is a sufficient number of fans of fried chicken. At $\tau > \tau_7$, the chicken becomes practically burnt and does not find buyers.

The membership functions obtained on the basis of the above speculative considerations have significantly specific features. In principle, the criterion $\mu_e(\tau)$ can be built on the basis of objective financial analysis data. The criterion $\mu_q(\tau)$ can be constructed based on some statistical data. In contrast, the criterion $\mu_t(\tau)$ is mainly based on subjective assessments of the taste and appearance of a fried chicken.

Moreover, in this situation, it is the subjective criterion of “product taste” that plays a decisive role in the success of a commercial enterprise. The exclusion of this criterion from consideration because of its “subjectivity”, in this case, would make the task of optimizing the operation of the considered food enterprise almost meaningless.

Another source of subjectivity in evaluating the performance of the enterprise is the presence of several local criteria, which are antagonistic. We can see that the improvement in terms of the criterion $\mu_t(\tau)$ is accompanied by a simultaneous deterioration relative to the criteria $\mu_e(\tau)$ and $\mu_q(\tau)$ (Figure 1). This situation, as a rule, has the character of a general pattern. It is well known that the presence of at least two local criteria in the formulation of the optimization problem is inevitably accompanied by the introduction of uncertainties of an inconsistent, subjective nature.

The following sources of subjective uncertainty in this example are the need to quantify the parameters of relative importance of local criteria (weights), and the choice of how to aggregate them into a generalized criterion. To conclude our example, we note that the weight of a chicken is, in general, a random variable with a corresponding frequency distribution.

It is clear that the chicken cooking time should depend on its weight, which makes it necessary to simultaneously take into account both subjective and objective (in this case, statistical) uncertainties in the task formulation.

As a result, the household problem, which at first was considered simple, turns out to be very difficult to solve using formal mathematical approaches. This example illustrates the main features of setting optimization problems when solving real-world problems: multiple criteria; antagonism and non-equivalence of local criteria; the importance of taking criteria based on subjective assessments into account; the need to simultaneously take uncertainties of a different nature into account. In many cases, an additional complexity is the hierarchy of the system of local criteria.

3. Fuzzy Multiple-Criteria Optimization of Heat-Technological Processes in the Rolled Steel Production

In this section, we present the solutions of fuzzy multiple-criteria problems concerned with two different technological processes based on the heat transfer and heat treatment of rolled steel. These processes are the inevitable components of the technological chain in the Belorussian Mini Metallurgical plant, which is a modern enterprise, designed and built by the known Austrian firm “Voest-Alpine”.

The specificity of this plant is that its basic raw material is steel metal scrap. As it usually has a variable chemical composition, timely changes and corrections of the used technologies are needed. Of course, it is very desirable that such adjusted technologies are optimal.

The methodology used for the optimization of real-world technological processes can be presented as follows.

1. Creating a mathematical model of a process, with parameters characterizing the process quality as outputs, and inputs which are technological parameters defining the process quality. If the values of all input and output parameters can be directly measured, such a model can be obtained using the data of a passive production experiment, i.e., recording data continuously coming from the automatic control system and the factory laboratory. Then, these data are used to obtain, by statistical methods, a regression model defining the dependencies of output parameters on input ones.
In many cases, the parameters that determine the quality cannot be directly measured, such as, for example, the temperature in the center of the heated ingot or the thermal stresses in it. To estimate the values of such parameters, complex systems of non-stationary differential equations in finite differences solved by numerical methods are used. Usually, the values of some important parameters of such models, such as, for example, the coefficients of heat exchange with the external environment, are not known in advance. However, their values can be found by parametric identification, i.e., by determining the values that minimize the discrepancies between the calculated data and the data of the industrial experiment. Note that industrial experiments are usually very expensive, but they are easily paid off after the setting of optimal technological modes.

Nevertheless, even the “precise” mathematical model obtained after identification, based on a complex system of differential equations in finite differences, is not very convenient for the frequent solution of optimization problems due to the too-high machine-time expenses needed for calculations by the model. To reduce expenses for optimization, the initial “precise” model is considered to be a real process, and a certain number of numerical experiments (calculations made for predefined different values of input parameters) are provided according to the defined plan of the numerical experiment. Based on the obtained results and using statistical methods, the simple polynomial regression model representing the dependence of the output parameters on the input ones is finally inferred. This model is simple enough to be used for optimization;

2. Establishment of the membership functions of local criteria based on actual technological instructions, as well as on the experience and intuition of technologists;
3. Calculation of the local criteria’s relative importance (weights) based on the linguistic pairwise comparisons matrix using the method developed in [29];
4. Choice of an appropriate method for the local criteria aggregation or a set of relevant methods to build a compromise general criterion;
5. Solution of the optimization task based on the maximization of a general criterion. This methodology can be applied to the optimization of a wide range of technologies, not only to those based on heat transfer processes.

3.1. The Simulation, Parametric Identification and Fuzzy Multiple-Criteria Optimization of the Heating Steel Ingots for the Rolling Process

As the first example of the practical implementation of the developed general methodology for the modeling, identification and optimization, we will consider the multiple-criteria fuzzy problem of optimizing the quality of the ingot-heating process before rolling at rolling mill 850 of the Belorussian Mini Metallurgical Plant.

Ingots are heated in methodical gas furnaces. The methodical furnace with walking beams of mill 850 is designed for continuously heating cast blooms with a cross-section of 250×300 and 300×400 mm with a length of from 2.5 to 5.5 m with a layout step of 150 and 200 mm, respectively. The total length of the furnace is 23 m, according to the location of the burners, and it is divided into seven sections, which can be divided into the methodical, and the so-called welding and homogenizing zones.

Due to the widespread use of automated systems for controlling the processes of heating workpieces, it is necessary to obtain a large amount of data on changes in the thermal and thermal deformation states of workpieces in the furnace. A pure experimental approach to developing economical modes of metal-heating in operating furnaces requires a large number of experiments, which are associated with significant labor, time and monetary costs. As a result, the methods for mathematical modeling and the optimization of metal heating processes in pass-through furnaces are of crucial importance.

The mathematical model for calculating the quality parameters of heating ingots before rolling includes:

- Two-dimensional nonlinear equation of the non-stationary heat transfer in partial derivatives with boundary conditions of the third and fourth kind;
- equation for calculating the thickness of the formed scale;
- The system of equations for calculating thermal stresses.

The model was implemented numerically using a combination of finite-difference and finite-element methods.

The thermal part of the model contains indeterminate parameters: the coefficient of convective heat exchange of the ingot with the heating medium, which, according to the literature data, was assumed to be $30 \text{ W}/(\text{m}^2 \text{ } ^\circ\text{C})$, and the reduced radiation coefficient of the ingot surface- α , identified by the production experiment. As a result of the identification, the values of α were obtained, which vary from zone to zone in the rates

of 1.7–1.9 w/(m² °C⁴) and provide high accuracy in calculating the temperatures in the heated ingot. The equation for calculating the thickness of the formed scale δ was taken as

$$\frac{d\delta}{d\tau} = \frac{(\exp(-10,125/T(\tau) + 7.25))^2}{2\delta}, \quad (1)$$

where $T(\tau)$ is the surface temperature of the ingot.

To assess the risk of cracks in the ingot, along with the stress field, the stress state coefficients $K_\sigma = (\sigma_i - \sigma_T(T))/\sigma_T(T)$ were calculated, where σ_i is the stress intensity, and σ_T is the destructive stresses. The K_σ isolines calculated for the forced mode showed that the tensile stresses that occur in the ingot during heating do not reach the critical values that lead to the appearance of cracks. From the perspective of setting and solving the optimization problem, the obtained result is very important, because it allows us to exclude the stress-state coefficient of the ingot from consideration as a criterion for the quality of the process.

As a result, the quality of the process was evaluated according to three main local criteria: minimizing the scale thickness δ , the accuracy of heating T , and minimizing the maximum temperature difference in the ingot ΔT at the time of unloading. The membership functions of the process local quality criteria are presented in Figure 2.

Figure 2. Membership functions (a–c) of the local criteria used.

Consider the membership function of the heating accuracy criterion (Figure 2b). It can be seen that the optimal temperatures of the wide face of the ingot are in the region of $1190\text{ °C} \leq T \leq 1200\text{ °C}$. The temperatures $T \leq 1150\text{ °C}$ or $T \geq 1250\text{ °C}$ are unacceptable for technological reasons. Intermediate sections are characterized by a decrease in the membership function when T approaches the region of invalid values. The linear law of decrease is adopted here only for reasons of simplicity. If there is any additional information about the behavior of the membership function in the intermediate regions, the regularity of decrease can be changed.

The membership functions of the criteria for the thickness of scale and the temperature difference over the thickness of the ingot during unloading are constructed on the basis of information obtained in numerical experiments, with variables within the range of acceptable values.

The solution to the fuzzy multiple-criteria optimization problem was reduced to finding the optimal thermal operating modes of the furnace, which ensure the maximization of the global quality criterion D for various furnace capacities P and the temperatures T_0 of the metal loading. The capacity P was used in the model to calculate the residence time of the ingot in each of the zones. At the same time, in all cases, the furnace temperature was assumed to be $T = 1200\text{ °C}$ in the homogenization zone, and at the beginning of the unheated zone 700 °C .

As a result, for each fixed pair of P and T_0 , the task was reduced to finding the optimal temperatures of the first T_1 and second T_2 welding zones. The temperature change within each of the zones was assumed to be linear.

The direct use of the mathematical model which is the system of second-order partial differential equations to solve the optimization problem requires too much machine time. Therefore, a two-step approach was used. In the first stage, the original model was reduced using experiment planning methods. In this case, two series of numerical experiments with the original model were used, with the variation in four input variables P , T_0 , T_1 , and T_2 in the technologically acceptable ranges.

In the first series of experiments, the T_0 values were varied in the range of 20–400 °C (cold ingots' landing); in the second series, the T_0 values were varied in the range of 550–700 °C (hot ingots' landing).

As a result of processing the data of numerical experiments for the cold ingots' landing, the following regression dependencies were obtained

$$\delta = 1.7 - 1.73 \cdot 10^{-2}x_4 + 4.1 \cdot 10^{-3}x_2 + 3 \cdot 10^{-3}x_1 + 2 \cdot 10^{-4}x_4^2 - 3.66 \cdot 10^{-5}x_2x_4 - 2.687 \cdot 10^{-5}x_1x_4 + 7.736 \cdot 10^{-6}x_1^2 + 7.387 \cdot 10^{-6}x_2^2 - 1.2 \cdot 10^{-6}x_1x_2 - 2.1 \cdot 10^{-7}x_2x_3 + 3.12 \cdot 10^{-6}x_3 + 8.71 \cdot 10^{-8}x_1x_3 + 1.43 \cdot 10^{-7}x_3x_4. \quad (2)$$

$$T = 1205 + 0.265x_2 - 0.37x_4 - 1.177 \cdot 10^{-2}x_4^2 + 5.75 \cdot 10^{-2}x_1 + 2.5 \cdot 10^{-3}x_1x_4 + 2.2 \cdot 10^{-3}x_2x_4 + 5x_3 + 3.1 \cdot 10^{-4}x_3x_4 - 6.2 \cdot 10^{-5}x_1^2 - 4.8 \cdot 10^{-6}x_3^2 - 9.1 \cdot 10^{-5}x_1x_2 - 1.1 \cdot 10^{-5}x_1x_3 + 5.4 \cdot 10^{-5}x_2^2. \quad (3)$$

$$\Delta T = 9.87 - 1.17 \cdot 10^{-2}x_2x_4 + 2.9 \cdot 10^{-3}x_4^3 - 3 \cdot 10^{-3}x_1x_4 - 3.24 \cdot 10^{-6}x_1^2x_3 + 1.58 \cdot 10^{-3}x_1x_2 - 1.7 \cdot 10^{-4}x_1x_4^2 + 6.3 \cdot 10^{-5}x_1x_2x_4 - 4.3 \cdot 10^{-4}x_3x_4 + 8.66 \cdot 10^{-5}x_2^3 - 3.9 \cdot 10^{-4}x_2x_4^2 - 1.06x_4 + 6.27 \cdot 10^{-5}x_2^2x_4 - 1.2 \cdot 10^{-5}x_3x_4^2 + 2 \cdot 10^{-5}x_1x_2^2 + 7.88 \cdot 10^{-6}x_2x_3x_4 + 5.5 \cdot 10^{-5}x_1x_3 + 3.75 \cdot 10^{-6}x_1x_2x_3 + 1.16 \cdot 10^{-4}x_2x_3 + 2.94 \cdot 10^{-4}x_1^2, \quad (4)$$

where $x_1 = T_1 - 1195$, $x_2 = T_2 - 1235$, $x_3 = T_0 - 217.3$, $x_4 = P - 59.6$.

For the hot ingots' landing case, we have obtained

$$\delta = 1.8 - 1.22 \cdot 10^{-2}x_4 + 3.78 \cdot 10^{-3}x_2 + 4.78 \cdot 10^{-4}x_1 + 1.3 \cdot 10^{-4}x_4^2 - 3.326 \cdot 10^{-5}x_2x_4 - 4.7 \cdot 10^{-6}x_1x_4 + 1.38 \cdot 10^{-6}x_1^2 + 6.9 \cdot 10^{-6}x_2^2 + 4.3 \cdot 10^{-7}x_2x_3 + 9.47 \cdot 10^{-7}x_1x_2. \quad (5)$$

$$T = 1180 + 0.295x_2 - 1.085x_4 - 1.143 \cdot 10^{-2}x_4^2 + 7.4 \cdot 10^{-2}x_1 + 2.68 \cdot 10^{-3}x_1x_4 + 2.774 \cdot 10^{-3}x_2x_4 + 1.88 \cdot 10^{-2}x_3 + 9.63 \cdot 10^{-4}x_3x_4 - 5.1 \cdot 10^{-5}x_1^2 - 9.9 \cdot 10^{-5}x_1x_2. \quad (6)$$

$$\Delta T = 37.47 - 3.7 \cdot 10^{-3}x_2x_4 + 1.8 \cdot 10^{-2}x_4^3 - 0.126x_1 - 0.39x_2 - 4.7 \cdot 10^{-3}x_1x_4 - 5.36x_4 - 3.9 \cdot 10^{-6}x_3^3 - 1.7 \cdot 10^{-3}x_3x_4 + 8.4 \cdot 10^{-5}x_1^2 + 8.84 \cdot 10^{-5}x_2x_4^2 - 1.34 \cdot 10^{-6}x_1x_2x_3 + 1.65 \cdot 10^{-4}x_1x_2 - 3.2 \cdot 10^{-5}x_3x_4^2 + 6.53 \cdot 10^{-5}x_1x_3 + 1.34 \cdot 10^{-6}x_1^2x_4 - 1.44 \cdot 10^{-5}x_1x_4^2 + 2.18 \cdot 10^{-6}x_1x_2x_3 + 3.87 \cdot 10^{-6}x_1x_2x_4 - 4.5 \cdot 10^{-5}x_3^2 + 8.3 \cdot 10^{-5}x_2x_3, \quad (7)$$

where $x_1 = T_1 - 837$, $x_2 = T_2 - 1211$, $x_3 = T_0 - 625$, $x_4 = P - 60$.

The obtained regression dependencies approximate the results obtained using the original model exactly enough for the case of heating steel 45, with errors in the output temperature T estimates not exceeding 1%, and in the estimation of δ and ΔT errors not exceeding 5%. This allows us to use the above regression expressions when solving the optimization problem, instead of the original model. This reduces the cost of machine time by many thousands.

The next step is the calculation of local criteria's relative importance (weights). This was done based on the linguistic pairwise comparison matrix and corresponding expert opinion expressed in verbal form using the method developed in [29] (see its description and analysis in [30]).

As the result, we have obtained the following weights: $\alpha_1 = 0.227$, $\alpha_2 = 1.433$ and $\alpha_3 = 1.34$ for the local criteria μ_δ , μ_T and $\mu_{\Delta T}$, respectively.

Since we have formulated the membership functions of local criteria and calculated their weights, we can discuss the different methods in terms of their aggregation and choose the most appropriate one for the framework of our problem. As is noted in [30,31], the most used aggregation method is the weighted sum, the min and multiplicative methods are not as popular, although, in some fields, the weighted sum is not used at all [12]. Nevertheless, the weighted sum possesses a set of disadvantages, analyzed in [32]. The most important one is the compensatory property (strictly speaking, in some cases, this is a very useful property). This means that if one our criterion is completely unsatisfied, i.e., the value of its members' function is equal to zero, then such a lack of satisfaction may be recompensated in the weighted sum by large values of rest criteria. Therefore, the weighted sum is not an admissible aggregation method in our case, as the complete dissatisfaction (failure) of any local criteria leads automatically to the alarm situation. It is shown in [32] that the min aggregation performs somewhat better than the multiplicative one.

Taking the above reasoning into account, here we will use the min type of aggregation, which, for our task, where all quality indicators δ , T and ΔT depend on the input variables P , T_0 , T_1 and T_2 , can be presented as follows

$$D(P, T_0, T_1, T_2) = \min(\mu_\delta^{\alpha_1}(\delta(P, T_0, T_1, T_2)), \mu_T^{\alpha_2}(T(P, T_0, T_1, T_2)), \mu_{\Delta T}^{\alpha_3}(\Delta T(P, T_0, T_1, T_2))). \quad (8)$$

Then, for each fixed pair P^* , T_0^* , the solution to the optimization problem will be the values of T_1 and T_2 that maximize D , i.e.,

$$(T_1, T_2)_{opt} = \arg \max_{T_1, T_2} (D(P^*, T_0^*, T_1, T_2)) \quad (9)$$

The solution to the above problem was obtained using the known direct random-search method [33], which was adopted to take the features of our task into account. Obviously, there are many other contemporary optimization methods, e.g., genetic algorithms proposed in the literature. However, using different convincing examples, it is shown in [34] that, if we deal with the nonlinear, non-differentiable or non-smooth optimization problem, the direct search methods provide the best results.

The results of solving the optimization problem for the cold ingots' landing are summarized in Table 1.

Table 1. Results of solving the optimization problem for the cold ingot planting.

P , t/h	T_0 , °C	T_1 , °C	T_2 , °C	δ , MM	T , °C	ΔT , °C	μ_δ	μ_T	$\mu_{\Delta T}$	D
60	100	1194	1242	1.33	1206	9.41	0.91	0.85	0.56	0.46
	200	1134	1257	1.46	1207	9.29	0.82	0.82	0.56	0.46
	300	976	1249	1.63	1193	1.64	0.69	1	0.97	0.46
	400	1011	1221	1.71	1189	1.22	0.63	0.99	0.99	0.48
80	100	1245	1260	1.34	1204	4.44	0.90	0.89	0.82	0.98
	200	1299	1236	1.39	1203	2.63	0.85	0.92	0.91	0.92
	300	1076	1275	1.76	1193	1.45	0.59	1	0.48	0.47
	400	1072	1262	1.66	1189	0.62	0.67	0.98	1	0.76

The Table 1 shows that the dependence of the optimal values of T_1 and T_2 on the initial temperature of the metal T_0 is non-monotonic, and no clear correlations could be detected. The effectiveness of the obtained solutions is illustrated in Figure 3, where the distribution of the global criterion in the vicinity of the optimum (solid line) is shown, and, for comparison, the same distribution in the vicinity of the center of the plan (dotted line) is presented as well. The calculations were carried out for the mode $P = 60$ t/h, $T_0 = 400$ °C, $T_1 = 1011$ °C at the optimal mode and for $T_1 = 1100$ °C for the center of the plan.

Figure 3 shows that the optimal solution is significantly better than that taken at an arbitrary point. This confirms the effectiveness of the developed fuzzy multiple-criteria optimization technique. The results of solving the considered optimization problems were used in technological practice to improve the heating modes for rolling in the Belorussian Mini Metallurgical Plant.

Figure 3. Distribution of the global criterion in the vicinity of the found optimum and the center of the plan.

3.2. Fuzzy Multiple-Criteria Optimization of the Wire Rod Heat Treatment Process

Heat treatment is the last operation in the technological chain of wire-rod production at the wire block of the Belorussian Mini Metallurgical plant. The correct choice of heat-treatment mode, taking into account the chemical composition of the metal and its temperature after hot rolling, should provide the required mechanical properties of the wire rod and grain size, with technological restrictions on the values of the operating parameters and the percentage of the main components of the steel composition. The technological scheme of the heat-treatment of rolled products is has two stages. It provides step-by-step water-cooling in four sections, located between the finishing group of stands and the screw-laying machine. To equalize the temperature field along the thickness of the rolled product, gaps (compensation sections) are provided between the sections. The cooling mode is set by turning on the required number of nozzles in the water cooling sections. After cooling from the temperatures of 1010–1070 to 750–900 °C in the first stage, the rolled product reaches the screw-laying machine, where its temperature T is measured in a non-contact way, and flat coils are formed. Then, they are less intensive than in the first stage, of air-cooling on a special conveyor.

The conveyor has 16 cooling zones; the discharge fans are installed under zones 1–8 and 15, 16. The coils of rolled products are moved by a roller conveyor in a special tunnel equipped with insulating covers, which can be closed and opened to adjust the cooling rate.

Thus, the factors that determine the technological mode of the heat-treatment of wire rod are

- Relative water consumption in water-cooling sections $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ measured as a percentage of the maximum flow rate when the injectors are fully engaged;
- Speed of movement of turns on the conveyor v ;
- The relative amount of air ϵ pumped by the fans and measured in fractions of the maximum possible amount;
- The degree of cooling Δ , equal to the proportion of closed thermal insulation covers.

The main indicators of the quality of the process are the following characteristics of the metal of the finished wire rod:

- Tensile strength σ ;
- The relative narrowing ψ ;
- Relative elongation δ ;
- Grain size d ;
- The relative thickness of the decarburized layer θ .

In the first stage of solving the optimization problem, a regression mathematical model was constructed, based on the data of the industrial experiment, linking the output quality

indicators with the factors that determine the technological mode. Since, in the case under consideration, the quality indicators largely depend on the initial chemical composition of the steel, the number of variable factors included the percentage of the main alloying components C and Mn . The two-stage heat treatment process made it possible to simplify its mathematical description. The mathematical model of the water-cooling section was initially presented in the form $T = f(\gamma_1, \gamma_2, \gamma_3, \gamma_4)$, where f is the regression polynomial. However, during the processing of the experimental data, it was found that the model which is simpler in structure— $T = 950.32 - 26.79 \gamma$, where $\gamma = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4$ —provides more accurate results than the original model (the maximum error of this simplified model is 8%; the error of the original model is 30%). The obtained result greatly simplified data-processing, allowing the entire set of factors that determine the water-cooling mode with the single variable T to be replaced. As a result of processing data obtained from industrial experiments on one of the steel grades, multiple-linear-regression equations were obtained, which link the input and output variables

$$\begin{aligned}\sigma &= 1251.74 + 0.14 \cdot T + 1336.35 \cdot C + 195.385 \cdot Mn + 1.988 \cdot \epsilon - 1426.94 \cdot \nu - 1203.94 \cdot \Delta; \\ \delta &= 0.169 - 0.00317 \cdot T - 23.55 \cdot C + 8.77 \cdot Mn + 0.0877 \cdot \epsilon + 29.83 \cdot \nu + 23.97 \cdot \Delta; \\ \psi &= 46.566 + 3.75 \cdot 10^{-3} \cdot T - 4.613 \cdot C + 26.34 \cdot Mn + 0.0814 \cdot \epsilon - 16.327 \cdot \nu - 20.51 \cdot \Delta; \\ d &= 0.349 - 3.749 \cdot 10^{-4} \cdot T - 0.1156 \cdot C + 7.429 \cdot 10^{-2} \cdot Mn - 2.8 \cdot 10^{-4} \cdot \epsilon + 0.189 \cdot \nu + 0.166 \cdot \Delta; \\ \theta &= 1.9 - 3.383 \cdot 10^{-3} \cdot T - 1.423 \cdot C - 0.357 \cdot Mn - 4.47 \cdot 10^{-2} \cdot \epsilon + 4.496 \cdot \nu + 3.487 \cdot \Delta.\end{aligned}\quad (10)$$

The above equations are adequate according to the Fisher criterion; the error in the forecast of the main quality indicators σ , ψ and δ did not exceed 10...15%, which indicates that the obtained model is sufficiently accurate. It should be noted that the attempts made to increase the adequacy of the model by constructing regression dependencies of the second- and higher-orders did not lead to a significant reduction in the error.

The membership functions were used to formalize local criteria and constraints. In Figure 4, the membership functions of the most important quality indicators are presented.

Figure 4. Membership functions of the most important quality indicators.

Similarly, the membership functions of the other indicators of the quality $\mu_\psi(\psi)$, $\mu_d(d)$, $\mu_\delta(\delta)$ were built, and the restrictions on the content of C and Mn : $\mu_C(C)$, $\mu_{Mn}(Mn)$. The usual constraints of the inequality type were imposed on the variables T , ν , ϵ , Δ .

Since the formulated local criteria and constraints are satisfied in practice in various areas of varying process parameters, the optimal solution was found as a compromise between conflicting requirements. To do this, the local criteria and constraints were aggregated into the generalized process-quality criterion, taking the relative importance (weights) of local criteria into account. The generalized process-quality criterion, as seen in the previous subsection, was built using the min operator as follows:

$$D(C, Mn, T, \nu, \epsilon, \Delta) = \min(\mu_\sigma^{\alpha_1}(\sigma(T, Mn, C, \nu, \epsilon, \Delta)), \mu_\delta^{\alpha_2}(\delta(T, Mn, C, \nu, \epsilon, \Delta)), \mu_\psi^{\alpha_3}(\psi(T, Mn, C, \nu, \epsilon, \Delta)), \mu_\theta^{\alpha_4}(\theta(T, Mn, C, \nu, \epsilon, \Delta)), \mu_d^{\alpha_5}(d(T, Mn, C, \nu, \epsilon, \Delta)), \mu_C^{\alpha_6}(C), \mu_{Mn}^{\alpha_7}(Mn)) \quad (11)$$

where $\alpha_1 \dots \alpha_7$ are the weights of the criteria and constraints. They were determined using the linguistic pairwise comparison matrix, as in the previous subsection.

The desired point of the optimum was obtained as follows

$$(C, Mn, T, \nu, \epsilon, \Delta)_{opt} = \arg \max_{C, Mn, T, \nu, \epsilon, \Delta} (D(C, Mn, T, \nu, \epsilon, \Delta)) \quad (12)$$

To find the maximum, we used the known direct random-search method.

The preliminary results of solving the optimization problem showed the presence of a number of competing local optima, and each of them was achieved at values ϵ at the upper limit of the range of acceptable values. This allowed us to reduce the dimension of the factor space by putting in (11) and (12) $\epsilon = \epsilon_{max}$, after which the solution of the problem was obtained again. As the optimal solution, we selected the point that provides the highest value of the criterion D , equal to 0.642, among the identified local maxima. The results of solving the optimization problem presented in Table 2 indicate that the degree of satisfaction of the most important local criteria corresponds well with their initial ranking. A comparison of the distributions of the generalized criterion D in the vicinity of the optimum and the center of the experiment plan (Figures 5 and 6) allows us to confirm the high effectiveness of the obtained solution.

Table 2. The values of the local criteria μ and their relative importance (weights) α in the optimum point.

Criterion	σ	δ	ψ	θ	d	C	Mn
α	2.64	1.65	1.00	0.69	0.45	0.32	0.25
μ	1.00	0.76	0.76	0.73	0.37	0.76	0.78

Note the weak dependence of D in the vicinity of the optimum on the contents of C and Mn in almost all the range of valid values, and the presence of a wide range of the T , in which the criterion D also changes little and is close to the maximum. Figures 5 and 6 show that variations in the factors ν and Δ have the greatest influence on the value of criterion D . Their values can be maintained with high accuracy with the help of the automatic control systems available at the heat-treatment site. The obtained results were used to improve the heat-treatment technology of rolled products after mill 320 of the Belorussian Metallurgical Mini Plant.

Figure 5. Sensitivity of the generalized criterion to the variation in factors: 1— $D(T)$, 2— $D(C)$, 3— $D(Mn)$. (Solid lines—in the vicinity of the optimum; dashed—in the vicinity of the center of the experiment plan).

Figure 6. Sensitivity of the generalized criterion to the variation in factors: 1— $D(v)$, 2— $D(\Delta)$ (Solid lines—in the vicinity of the optimum; dashed—in the vicinity of the center of the experiment plan).

4. Conclusions

This aim of this paper is to present and illustrate the methodology that we have developed in the three last decades based on our steadily growing scientific and practical experience in the development and embedding of optimal technological processes. Therefore, this methodology may be considered as our scientific and practical contribution to the solution of fuzzy multiple-criteria optimization problems.

Therefore, this paper is, to a great extent, addressed to those practitioners who encounter problems which can be classified as fuzzy multiple criteria optimization in their work. We hope that this article will help them to adequately navigate the vast sea of proposed methods and publications on this topic, with minimal time and material resources.

The used methodology for the optimization of real-world technological processes can be presented as follows.

1. Creating a mathematical model of a process, with parameters characterizing the process quality as outputs, and inputs which are technological parameters defining the process quality. If the values of all input and output parameters can be directly measured, such a model can be obtained using the data of the passive production experiment, i.e., recording data continuously coming from the automatic control system and the factory laboratory. Then, these data are used to obtain, by means of statistical methods, a regression model defining the dependencies of output parameters on input ones.
In many cases, the parameters that determine quality cannot be directly measured, such as, for example, the temperature in the center of the heated ingot or the thermal stresses in it. To estimate the values of such parameters, complex systems of non-stationary differential equations in finite differences solved by numerical methods are used. Usually, the values of some important parameters of such models, e.g., the coefficients of heat exchange with the external environment, are not known in advance. However, these values can be found by parametric identification, i.e., by determining the values that minimize the discrepancies between the calculated data and the data of the industrial experiment;
2. Establishment of the membership functions of local criteria based on actual technological instructions as well as on the experience and intuition of technologists;
3. Calculation of the local criterias relative importance (weights) based on the linguistic pairwise comparisons matrix;
4. Choice of an appropriate method for the local criteria aggregation or a set of relevant methods to build a compromise general criterion;

5. Solution to the optimization task based on the maximization of a general criterion. The effectiveness of this methodology is illustrated by the solutions of two different fuzzy multiple-criteria optimization problems concerned with the heat-treatment processes for two steel-rolled products. These processes are components of the overall technological chain in the “Belorussian Metallurgical Mini Plant”.

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