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Optimal Multi-Antenna Transmission for the Cooperative Non-Orthogonal Multiple-Access System

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Abstract: We investigate the beamforming for the multi antenna cooperative non-orthogonal multiple access (NOMA) system, where an access point (AP) delivers messages for multiple user terminals (UT) with successive interference cancellation (SIC) reception method. Some UTs with multiple antennas cooperate with the AP transmission to improve the diversity and the average power performance. We formally present two optimal beamforming schemes at the AP and at the cooperative UTs. One scheme has no power limitation for the cooperative UTs, while the other one does have such limitation. We guarantee that the rank one beamformer is sufficient to achieve the optimal points so that the proposed schemes have rank one semi-definite programming (SDP) structure. Simulation results show the performance gain of the multi-antenna cooperative NOMA schemes in the sense of diversity and the average power.

Keywords: beamforming; non-orthogonal multiple access; cooperation; diversity gain

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1. Introduction

Reducing the network latency and accommodating the vast amount of devices competing for network resources will be key requirements for the future sixth generation (6G) wireless systems. The importance of non-orthogonal multiple access (NOMA) technologies [1] lies in this aspect and many studies have been reported so far. Unlike the code domain NOMA, the power domain NOMA in Reference [2–4] does not rely on any particular multiple access scheme in applying the NOMA principle, where multiple messages are superposed in a single spectral resource. Different from the counterpart of orthogonal multiple access (OMA), the received signals of the NOMA schemes are not free of the inter-user interference (IUI) so that the successive interference cancellation (SIC) is a popular approach to remove the IUI. In addition to the SIC, efficient resource allocation is also crucial to harvest the true benefit of the NOMA system [5–8]. The additional complexity from the SIC and the resource allocation pays back by allowing more user terminals (UT) to share the network resources and by the increase in spectral efficiency. NOMA can be applied to millimeter wave band communications [9–11].

Another way to enhance the NOMA system is to embed the multiple antennas and the related technologies to exploit the spatial dimensions [3,4,12–19] into the performance improvement. Even the security of the NOMA network can be enhanced [20] through the multi-antenna technologies. The multiple input multiple output (MIMO) case, where both the access point (AP) and the UTs are equipped with multiple antennas, allows for the clustering of UTs so that the signals of different clusters are separated and the UTs of each cluster form an independent NOMA channel [13–15]. In multiple input single output

(MISO) case, where only the AP is equipped with multiple antennas, such clustering is not possible; thus, the AP beamforming serves single channel NOMA UTs only [12,21,22]. Only partial results of the optimal beamforming for the MISO-NOMA system are available in Reference [21]. In the cooperative NOMA system, where some UTs participate in helping the AP transmission [23–27] so that the performance of NOMA system is improved in diversity sense and in average transmit power sense.

This article considers the beamforming in the cooperative NOMA system, where multiple antennas are adopted at the AP and at the helping UTs to further enhance the performance of NOMA system. To the author's best knowledge, the beamforming problem for the cooperative NOMA system has not been discussed much, while the design criteria and actual beamforming schemes are essential to efficiently utilize the spatial dimension into the benefit of transmission. Therefore, we formally present the optimal beamformer design criterion for the AP and the cooperative UTs in the cooperative NOMA system and develop two optimal beamformers (BFs) schemes depending on the power constraint applied at the UTs. These schemes encompass the non-cooperative NOMA when no UT collaborates with the AP. First, we discuss the sufficiency of rank one beamforming scheme in achieving the optimal points so that the ranks of the beamformer matrices at the AP and at the cooperative UTs all need simple rank one structure. This leads to an optimal beamforming criterion with the sum of signal strength terms at all UTs to be equalized. The rank one structure and the derived criterion suggests that the semi-definite programming (SDP) is well suited for the given optimization problem; thus, we propose two beamforming schemes based on optimization with the rank one constraint. In the numerical experiments, these two beamforming schemes perform very closely regardless of the UT power constraint, while they show advantage both in diversity and in average power compared to the non-cooperative NOMA. In other words, they either need less power in achieving a certain outage level or achieve a better outage level with the same power consumption.

Section 1 presents the system model of the proposed cooperative NOMA. Section 2 two beamforming schemes are presented after a discussion on the optimal condition of the cooperative NOMA. Section 3 presents relevant numerical results. Finally, Section 4 concludes this paper.

Notations: The notations $\mathbf{A}^H$ and $\mathbf{A}^T$ are the Hermitian transpose and the transpose of a matrix $\mathbf{A}$, respectively. $\text{Tr}[\mathbf{A}]$ takes the trace of the matrix $\mathbf{A}$ and $\text{rank}\{\mathbf{A}\}$ returns the rank of the matrix $\mathbf{A}$. $\|\mathbf{a}\|$ denotes the ℓ_2 -norm of a vector $\mathbf{a}$. The notation $\mathbf{A}_{i,j}$ represents the element at the i -th row and the j -th column of $\mathbf{A}$. The notation $\text{diag}[\mathbf{v}]$ produces a diagonal matrix with the elements of the vector $\mathbf{v}$ on its diagonal. The vector notations $\mathbf{0}_k$ and $\mathbf{1}_j$ represent all zero vector with k elements and all one vector with j elements, respectively. $\mathcal{CN}(\mathbf{0}, \mathbf{C})$ denotes the complex white Gaussian distribution of random vector with zero mean vector $\mathbf{0}$ and the covariance matrix $\mathbf{C}$. $\mathbb{C}^N$ denotes the N dimensional complex vector space. Finally, $\mathbb{E}_n[x]$ takes the expectation of x with respect to n .

2. System Model

Consider the cooperative NOMA system in Figure 1, where a multiple-antenna AP transmits messages to J NOMA UT. Among these J UTs, L ($L \leq J$) UTs are also equipped with multiple-antennas to cooperate with the AP transmission in half duplex mode. In theory, up to $J - 1$ UTs can join the cooperation to make $J - 1$ cooperation stages. However, more stages require more cooperation time slots ($L + 1$ time slots are required in total); thus, the performance gain, such as diversity from additional stages, comes at the cost of the increased spectral resource. Therefore, one or two ($L = 1$ or 2) stages can be considered to be a practical candidate stage number. The remaining $J - L$ single antenna UTs only receive the cooperative NOMA transmissions during $L + 1$ time slots. The AP uses an $M \times 1$ AP beamformer vector $\mathbf{w}_A$, where M denotes the number of transmit antennas at the AP. The cooperative UTs are equipped with N antennas to receive and to transmit. The decode-and-forward (DF) protocol-based cooperative UTs apply $N \times 1$ unit-norm receive

beamforming vectors $\mathbf{w}_{R,\ell}$, $\ell = 1, \dots, L$ to the received signals to decode the messages from the AP. If the ℓ -th cooperative UT decodes the messages, the signal is re-encoded before being beamformed by the $N \times 1$ transmit vector $\mathbf{w}_{T,\ell}$. The MIMO channel vector from the AP to the ℓ -th cooperative UT is denoted by the $N \times M$ matrix $\mathbf{H}_\ell$ and the MISO channel from the AP to the j -th non-cooperative UT is denoted by the $1 \times M$ vector $\mathbf{h}_j$. In addition, the MIMO channel from the ℓ -th the cooperative UT to the m -th cooperative UT is denoted by the $N \times N$ matrix $\mathbf{G}_{m,\ell}$ and the MISO channel from the ℓ -th cooperative UT to the j -th non-cooperative UT ($j = L + 1, \dots, J$) is denoted by the $1 \times N$ vector $\mathbf{g}_{j,\ell}$. Note that all the elements of $\mathbf{H}_\ell$, $\mathbf{h}_j$, $\mathbf{G}_{m,\ell}$, and $\mathbf{g}_{j,\ell}$ are independent and identically distributed random variables with $\mathcal{CN}(0, 1)$. We assume that full channel state information (CSI) is available at the AP so that all the beamformer vector design is conducted at the AP.

Figure 1. The cooperative non-orthogonal multiple access (NOMA) system with L cooperating user terminals (UTs) among J NOMA UTs. The cooperating UTs use L separate time slots for the cooperation phases.

At the n -th time slot, the AP transmits a composite message toward J NOMA UTs. Recall that there are $J - L$ non-cooperative UTs, where the received signals at the ℓ -th cooperative UT and j -th non-cooperative UT are expressed as

$$\begin{aligned} \mathbf{y}_\ell(n) &= \mathbf{H}_\ell \mathbf{w}_A x(n) + \mathbf{n}_\ell(n) \\ y_j(n) &= \mathbf{h}_j \mathbf{w}_A x(n) + n_j(n), \end{aligned} \quad (1)$$

where $x(n) = \sum_{j=1}^J x_j(n)$ is the composite AP message symbol with $\mathbb{E}[|x(n)|^2] = 1$. The message $x_j(n)$ is destined to the j -th UT with $\mathbb{E}[|x_j(n)|^2] = P_j$ and $\sum_{j=1}^J P_j = 1$. The 1-st cooperative UT applies the receive beamforming to the signal in (1) as $\mathbf{w}_{R,1}^H \mathbf{y}_1(n)$, decodes J messages with the SIC principle from $x_1(n)$ to $x_J(n)$, re-encodes J message signals $x(n) = \sum_{j=1}^J x_j(n)$, and transmits $x(n)$ with beamforming as $\mathbf{w}_{T,1} x(n)$ at the time slot $n + 1$. Note that the j -th cooperating UT of conventional cooperative NOMA systems [25–27] relays only the superposition of $x_{j+1}(n), \dots, x_J(n)$, while all the cooperating UTs in this paper repeat the superposition of $x_1(n), \dots, x_J(n)$. This fact helps each UT to decode easily with the SIC principle, and the beamformer design criteria in the sequel to be easily derived. The vector $\mathbf{n}_\ell(n)$ denotes an $N \times 1$ additive noise vector at the ℓ -th cooperative UT antennas with $\mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ distribution, while $n_j(n)$ denotes the additive noise at the

j -th UT antenna with $\mathcal{CN}(0, 1)$ distribution. Then, the received signal at the j -th UT at the time slot $n + 1$ can be written as

$$\begin{aligned} \mathbf{y}_j(n+1) &= \mathbf{G}_{j,1} \mathbf{w}_{T,1} x(n) + \mathbf{n}_j(n+1), \quad j = 2, \dots, L \\ y_j(n+1) &= \mathbf{g}_{j,1} \mathbf{w}_{T,1} x(n) + n_j(n+1), \quad j = L+1, \dots, J, \end{aligned} \quad (2)$$

where $\mathbf{n}_j(n+1)$ and $n_j(n+1)$ are the additive noise signals at the j -th UT with $\mathcal{CN}(0, \sigma^2 \mathbf{I}_N)$ and $\mathcal{CN}(0, \sigma^2)$ distributions, respectively. If $L \geq 2$, the 2-nd cooperative UT again starts decoding the messages from the observations in (1) and $\mathbf{w}_{R,2}^H \mathbf{y}_2(n+1)$ according to the SIC manner. Then, it re-encodes the J messages and transmits the message $x(n)$ in the time slot $n+2$. Using j previous observations, similar processes are repeated until the L -th cooperative UT, which requires L time slots and the cooperation ends after the $(n+L)$ -th time slot. It may look unnecessary for the ℓ -th cooperative UT to retransmit all the J messages including the message signal destined to itself after it decodes the J messages. However, in NOMA with SIC decoding, each UT needs to decode all the message signals with indices preceding the index of the UT's own signal; thus, it is reasonable for the cooperative UTs to repeat all the message signals to help the decoding at the UTs.

The j -th ($j \geq L+1$) UT has $L+1$ observations from the time slots from n in (1) to $n+L$ so that it can decode the messages $x_1(n), \dots, x_j(n)$ in SIC manner. Assuming the maximum ratio combining (MRC) of those observations, the signal-to-interference-plus-noise ratio (SINR) for the signal $x_k(n)$, $k \leq j$ is as follows:

$$\gamma_k^j = \frac{[|\mathbf{h}_j \mathbf{w}_A|^2 + \sum_{\ell=1}^L |\mathbf{g}_{j,\ell} \mathbf{w}_{T,\ell}|^2] P_k}{[|\mathbf{h}_j \mathbf{w}_A|^2 + \sum_{\ell=1}^L |\mathbf{g}_{j,\ell} \mathbf{w}_{T,\ell}|^2] \sum_{m=k+1}^J P_m + \sigma^2}. \quad (3)$$

On the other hand, at the ℓ -th cooperative UT, there are ℓ observations for the decoding of $x_k(n)$. Assuming MRC with $\mathbf{w}_{R,\ell} = (\mathbf{H}_\ell \mathbf{w}_A)^H / \|\mathbf{H}_\ell \mathbf{w}_A\|$ at slot n and $\mathbf{w}_{R,\ell} = (\mathbf{G}_{\ell,m} \mathbf{w}_{T,m})^H / \|\mathbf{G}_{\ell,m} \mathbf{w}_{T,m}\|$ at slot $n+m$, ($m \leq L$), the SINR for $x_k(n)$ at the cooperative UT is given as follows

$$\gamma_k^\ell = \frac{[\|\mathbf{H}_\ell \mathbf{w}_A\|^2 + \sum_{m=1}^{\ell-1} \|\mathbf{G}_{\ell,m} \mathbf{w}_{T,m}\|^2] P_k}{[\|\mathbf{H}_\ell \mathbf{w}_A\|^2 + \sum_{m=1}^{\ell-1} \|\mathbf{G}_{\ell,m} \mathbf{w}_{T,m}\|^2] \sum_{m=k+1}^J P_m + \sigma^2}. \quad (4)$$

3. Beamformer Design

Here, we design a beamformer set $\mathbf{w}_A$ and $\mathbf{w}_{T,1}, \dots, \mathbf{w}_{T,L}$ with the MRC for $\mathbf{w}_{R,\ell}$ as defined in the previous section. Note that all these beamformers are assumed to be rank one, for which the following Section 3.1 provides a proof using the result of Reference [21] that such rank one assumption of the beamformers for our system model is sufficient to achieve the optimum points. The decoding conditions based on the SINR expressions in (3) and (4) result in a design method as is discussed in Reference [12] for the conventional NOMA system without the cooperative UT. In Reference [12], it is shown that the AP beamformer for a NOMA system maximizes the minimum channel gain ($\min_j |\mathbf{h}_j \mathbf{w}_A|$). This design principle is similar to that for the multicast system [28], which is not so surprising since message signals of a NOMA system should be decoded at most of the UTs under the SIC rule. We formally extend this result to the beamformer set optimization in the cooperative NOMA case.

3.1. The Optimality of Rank One Beamformer

In Reference [21], the optimal condition for the two user MISO NOMA transmission system is presented as

$$\frac{P_1 |\mathbf{h}_1 \mathbf{v}_1|^2}{P_2 |\mathbf{h}_1 \mathbf{v}_2|^2 + \sigma_1^2} = \frac{P_1 |\mathbf{h}_2 \mathbf{v}_1|^2}{P_2 |\mathbf{h}_2 \mathbf{v}_2|^2 + \sigma_2^2}, \quad (5)$$

where $\mathbf{v}_i$, $i = 1, 2$ are the $M \times 1$ pre-coders for $x_i(n)$ and σ_i^2 , $i = 1, 2$ are the additive noise powers at the UTs. The channel vectors $\mathbf{h}_j$, $j = 1, 2$ are $1 \times M$ vectors toward the UTs from the AP. It is easy to see that the condition in (5) is met with $|\mathbf{h}_1 \mathbf{v}_1| = |\mathbf{h}_2 \mathbf{v}_1|$ and $|\mathbf{h}_1 \mathbf{v}_2| = |\mathbf{h}_2 \mathbf{v}_2|$. Note that the condition in (5) comes from the minimum operator taken for the SINRs of two NOMA UTs, which enforces those SINRs to be equalized at the optimum points.

Extending the same reasoning to the cases of J ($J > 2$) users, the optimal points should satisfy the conditions $|\mathbf{h}_1 \mathbf{v}_k| = |\mathbf{h}_2 \mathbf{v}_k| = \dots = |\mathbf{h}_J \mathbf{v}_k|$ for $k = 1, \dots, J$ due to the minimum operator and the following equalization enforcement. From these conditions, it is obvious that rank one pre-coder ($\mathbf{v}_1 = \dots = \mathbf{v}_J$) is sufficient to achieve these conditions regardless of J .

3.2. Design Principle

Following the SIC rule, the decoding condition at the ℓ -th cooperative UT is $\gamma_k^\ell \geq \gamma_k^*$, $k = 1, \dots, J$. Here, $\gamma_k^* = 2^{r_k} - 1$ is the threshold for decoding the k -th message signal and r_k is the data rate of x_k . On the other hand, at the j -th non-cooperative UT, the SIC decoding condition is given as

$$\gamma_k^j \geq \gamma_k^*, \quad k = 1, \dots, j. \quad (6)$$

Collecting the sum of signal strength terms at the k -th UT, we define α_k in (7) with the minimum operator.

$$\alpha_k = \begin{cases} \min[\|\mathbf{H}_1 \mathbf{w}_A\|^2, \dots, \|\mathbf{H}_L \mathbf{w}_A\|^2 + \sum_{m=1}^{L-1} \|\mathbf{G}_{L,m} \mathbf{w}_{T,m}\|^2, \min_{m=k, \dots, J} \{|\mathbf{h}_m \mathbf{w}_A|^2 + \sum_{\ell=1}^L |\mathbf{g}_{m,\ell} \mathbf{w}_{T,\ell}|^2\}], \\ \quad L < k \leq J, \\ \min[\|\mathbf{H}_1 \mathbf{w}_A\|^2, \dots, \|\mathbf{H}_L \mathbf{w}_A\|^2 + \sum_{m=1}^{L-1} \|\mathbf{G}_{L,m} \mathbf{w}_{T,m}\|^2, \min_{m=L+1, \dots, J} \{|\mathbf{h}_m \mathbf{w}_A|^2 + \sum_{\ell=1}^L |\mathbf{g}_{m,\ell} \mathbf{w}_{T,\ell}|^2\}], \\ \quad 1 \leq k \leq L. \end{cases} \quad (7)$$

Proposition 1 reveals the design principle of $\mathbf{w}_A$ and $\mathbf{w}_{T,1}, \dots, \mathbf{w}_{T,L}$ for the cooperative NOMA system.

Proposition 1. To maximize the total rates $\sum_{k=1}^J r_k^*$, the $L+1$ beamformer vectors $\mathbf{w}_A$ and $\mathbf{w}_{T,1}, \dots, \mathbf{w}_{T,L}$ should be designed such that α_k , $k = 1, \dots, J$ should be maximized by equalizing all the terms in the minimum operation so that $\alpha_1 = \alpha_2 = \dots = \alpha_J$.

Proof. The condition in (6) can be rearranged as

$$\min_{m=k, k+1, \dots, J} \gamma_k^m \geq \gamma_k^*, \quad k = 1, \dots, J. \quad (8)$$

Together with the condition at the cooperative UTs, we have for $x_k(n)$ to be decoded at the cooperative UTs and the UTs that need to decode it

$$\begin{aligned} \min_{m=1, 2, \dots, J} \gamma_k^m &\geq \gamma_k^*, \quad k \leq L, \\ \min_{m=1, \dots, L, k, k+1, \dots, J} \gamma_k^m &\geq \gamma_k^*, \quad L < k \leq J. \end{aligned} \quad (9)$$

Note that the SINR expressions in (3) and (4) indicate that γ_k^m , ($m > L$) is a monotonic function of $|\mathbf{h}_m \mathbf{w}_A|^2 + \sum_{\ell=1}^L |\mathbf{g}_{m,\ell} \mathbf{w}_{T,\ell}|^2$ and γ_k^ℓ , ($\ell \leq L$) is a monotonic function of $\|\mathbf{H}_\ell \mathbf{w}_A\|^2 + \sum_{m=1}^{\ell-1} \|\mathbf{G}_{\ell,m} \mathbf{w}_{T,m}\|^2$. The definitions of α_k 's in (7) allow us to re-write (9) concisely as

$$\frac{\alpha_k P_k}{\alpha_k \sum_{m=k+1}^{J-1} P_m + 1} \geq \gamma_k^*, \quad k = 1, \dots, J. \quad (10)$$

Therefore, the rate r_k^* on the right-hand side of (10) is maximized by maximizing α_k ; hence, maximizing the total rates $\sum_{k=1}^J r_k^*$ corresponds to maximizing α_k , $k = 1, \dots, J$. Here, from the $k = 1$ case to the $k = L$ case, produce the minimum of α_k as $\alpha_1 = \dots = \alpha_L$

by including all the gain terms of UTs in the minimum operation. Without any further constraint on the beamformer, the optimum beamforming condition of maximizing the minimum of two variables is equaling the two variables. Therefore, applying this fact from α_J to α_1 one-by-one assures that the last statement of the Proposition holds. $\square$

Defining $\beta_k = \min_{m=k, \dots, J} \{|\mathbf{h}_m \mathbf{w}_A|^2\}$, the design principle of $\mathbf{w}_A$ for the non-cooperative NOMA system appeared in Reference [12] can be easily re-derived as $|\mathbf{h}_1 \mathbf{w}_A|^2 = \dots = |\mathbf{h}_J \mathbf{w}_A|^2$ and $\beta_1 = \dots = \beta_J$ equivalently. From the principle in Proposition 1, we may contrive a scheme to maximize the rates (equivalently r_k^*) with a fixed power or to minimize the power with fixed rates. We follow the latter approach in the next subsection.

3.3. Two Designs

According to the design principles of the cooperative NOMA in Proposition 1, we may propose two feasible designs in this subsection. In addition, a simple modification of the designs results in a design for the non-cooperative NOMA system. Since Proposition 1 states that all the sums of channel gain terms reaching the UTs are required to be equalized for the optimal points, such condition can be met with SDP based approaches through the following steps. Note that the typical application of SDP is composed of linear matrix inequalities (LMI) with semi definite parameter matrices [28] so that we can build a parameter matrix $\mathbf{W}$ from the beamforming vectors and then the optimality condition in Proposition 1 can be absorbed in the LMIs as follows. First, we define the $(M + LN) \times 1$ vector set of $\tilde{\mathbf{h}}_j = [\mathbf{h}_j^T, \mathbf{g}_{j,1}^T, \dots, \mathbf{g}_{j,L}^T]^T$, $j = L + 1, \dots, J$ and the $N \times (M + (L - 1)N)$ concatenated matrices of $\mathcal{H}_1 = [\mathbf{H}_1, \mathbf{0}_N, \dots, \mathbf{0}_N]$, $\mathcal{H}_2 = [\mathbf{H}_2, \mathbf{G}_{2,1}, \mathbf{0}_N, \dots, \mathbf{0}_N]$, $\dots$, $\mathcal{H}_L = [\mathbf{H}_L, \mathbf{G}_{L,1}, \dots, \mathbf{G}_{L,L-1}]$, where $\mathbf{0}_N$ is the $N \times N$ zero matrix. With another definition of the $(M + LN) \times 1$ concatenated beamformer vector $\mathbf{w} = [\mathbf{w}_A^T, \mathbf{w}_{T,1}^T, \dots, \mathbf{w}_{T,L}^T]^T$, we may form the following SDP.

$$\begin{aligned} \min_{\mathbf{W} \in \mathbb{C}^{M+N}} \quad & \text{Tr}[\mathbf{W}] \\ \text{s.t.} \quad & \text{Tr}[\tilde{\mathbf{H}}_j \mathbf{W}] \geq \gamma^*, j = L + 1, \dots, J \\ & \text{Tr}[\mathcal{H}_j^H \mathcal{H}_j \mathbf{W}] \geq \gamma^*, j = 1, 2, \dots, L \\ & \text{rank}\{\mathbf{W}\} = 1. \end{aligned} \quad (11)$$

Note that the last rank constraint requires the rank reduction process as in Reference [28] before the solutions $\mathbf{w}_A$ and $\mathbf{w}_{T,\ell}$ are available. Here, we use the matrix definitions $\tilde{\mathbf{H}}_j = \tilde{\mathbf{h}}_j \tilde{\mathbf{h}}_j^H$ and $\mathbf{W} = \mathbf{w} \mathbf{w}^H$; γ^* is the threshold uniformly applied to all UT channels to control the total throughput $\sum_{k=0}^{J-1} r_k^*$. The SDP in (11) has the same structure as the one in Reference [28] due to the similarity between the NOMA system and the multicast system.

A defect of the problem in (11) is that we have no control over the balance of the AP power ($\|\mathbf{w}_A\|^2$) and the cooperative UT power ($\|\mathbf{w}_{T,\ell}\|^2$) in the solution of $(\mathbf{w})$. Since the cooperative UTs have power limitation in practice, we apply additional constraints on the cooperative UT power of the problem in (11) so that we can have better control on the division of the power. We modify (11) as follows.

$$\begin{aligned} \min_{\mathbf{W} \in \mathbb{C}^{M+N}} \quad & \text{Tr}[\mathbf{W}] \\ \text{s.t.} \quad & \text{Tr}[\mathbf{E}_\ell \mathbf{W} \mathbf{E}_\ell] \leq P_\ell, \ell = 1, \dots, L \\ & \text{Tr}[\tilde{\mathbf{H}}_j \mathbf{W}] \geq \gamma^*, j = L + 1, \dots, J \\ & \text{Tr}[\mathcal{H}_j^H \mathcal{H}_j \mathbf{W}] \geq \gamma^*, j = 1, 2, \dots, L \\ & \text{rank}\{\mathbf{W}\} = 1. \end{aligned} \quad (12)$$

Here, $\mathbf{E}_\ell = \text{diag}[\mathbf{0}_{N\ell}, \mathbf{1}_N, \mathbf{0}_{N(L-\ell)}]$ and P_ℓ is the desired power level ($P_{d,\ell}$) of the ℓ -th cooperative UT. Again, the rank reduction produces a $\mathbf{w}_A$ that complies with $\mathbf{w}_{T,\ell}$, $\ell = 1, \dots, L$ having the desired cooperative UT power. Compared to the SDP in (11), the power control capability of (12) is earned at the expense of some performance loss since the feasible region of (12) is a subset of that in (11).

Note that a slight modification of (11) leads to the SDP for the beamforming in the non-cooperative NOMA system as

$$\begin{aligned} \min_{\mathbf{W}_A \in \mathbb{C}^M} \quad & \text{Tr}[\mathbf{W}_A] \\ \text{s.t.} \quad & \text{Tr}[\tilde{\mathbf{H}}_j \mathbf{W}_A] \geq \gamma^*, j = 1, 2, \dots, J. \end{aligned} \quad (13)$$

Here, we assume that the UTs from $j = 1$ to $j = L$ are also equipped with single antenna just like the other UTs. Again, the rank reduction produces the beamformer $\mathbf{W}_A$. Note that all three algorithms above rely on the SDP approach, in which complexity with the most popular interior point method is expressed as $O(n \log(\frac{n}{\epsilon}))$. Therefore, the numerical results presented in Section 4 reflect the theoretically achievable performance through the cooperation and the beamforming. We leave the study on computationally less demanding schemes as a future research topic discussed in the conclusion section.

4. Numerical Results

In this section, we analyze the performance of the proposed two beamformer schemes and of the non-cooperative scheme. Since the sum rates of the NOMA systems depend on the resource allocation, as well, we compare in Figure 2 the outage probabilities of these three schemes with different sets of antenna configurations to compare the proposed cooperative beamformer schemes with the non-cooperative scheme, where the parameter definitions and simulation conditions used are summarized in Table 1. Here, the outage event is defined as the case when the total transmit power of the AP and the cooperative UTs required for a scheme exceeds a given power level threshold. In Figure 2, some of the curves are close-by since two curves are almost indistinguishable for the right most curves and so are the three curves in the left most curves. Since all the three curves exhibit the same slope of the outage when $M = N = 1$, it is easy to see that the power gain by beamforming is more noticeable than the diversity gain in this case. It is the case of $M = 1$ and $N = 4$, where the diversity gain from the cooperation is maximized compared to the non-cooperative scheme. On the other hand, the case of cooperative $M = 4$ and $N = 1$ exhibits the same diversity performance since the diversity gain is acquired from the sum of channel gains at UTs and all the UTs experience the sum channels of the same diversity order. Again, the power gain is more prominent when $M = 4$ and $N = 1$. Overall, the cooperative beamforming schemes provide better diversity performance than the non-cooperative one and the power gain is guaranteed even when the diversity is not available. In all the cases, the scheme with lightly power constrained cooperative UTs loses negligible amount of outage performance compared to the case absent of such constraint.

In Figures 3 and 4, we compare the average power curves of proposed schemes to achieve the threshold (γ^*) for different antenna settings. In Figure 3, the average power curves against N with fixed M s are plotted, while those against M with fix N s appear in Figure 4. Both M and N are similarly helpful in reducing the average power to achieve the same threshold of Figure 2. Note that the average power curve of the non-cooperative scheme with $N = 3$ in Figure 4, is collapsed with that of optimal cooperative scheme with $N = 1$. Again, the cooperation schemes provide much saving in the power consumption and the one with the UT power constraint does not require noticeably more power than the one without the constraint. Note that the UT power constrained one needs little more power than the one without such constraint when either M or N is low. These results reflect that the beamforming suggested in this paper further boosts the performance gains of the cooperative NOMA system, as well as the gain from pure cooperation. Finally, the gain we observe here is acquired at the expense of additional spectral resource used for the cooperation. Therefore, a trade-off exists between the cooperation gain and the additional spectral resource.

Figure 2. The outage probabilities of the proposed schemes against the signal-to-interference-plus-noise ratio (SINR) with different M and N . Here, $J = 4$ and $\gamma^* = 10$ for all schemes; $L = 1$ and $P_{d,1} = \frac{\gamma^*}{2}$ for the cooperative schemes. “coop.PC” represents the cooperative NOMA with UT power constraint.

Figure 3. The average power of the proposed schemes against N . Here, $J = 4$ and $\gamma^* = 10$ for all schemes; $L = 1$ and $P_{d,1} = \frac{\gamma^*}{2}$ for the cooperative schemes. “coop.PC” represents the cooperative NOMA with UT power constraint.

Figure 4. The average power of the proposed schemes against M . Here, $J = 4$ and $\gamma^* = 10$ for all schemes; $L = 1$ and $P_{d,1} = \frac{\gamma^*}{2}$ for the cooperative schemes. “coop.PC” represents the cooperative NOMA with UT power constraint.

Table 1. Definitions and simulation conditions.

M	AP antenna number.	N	cooperative UT antenna number.
L	number of cooperative UTs.	J	Total number of UTs
$\mathbf{h}_j$	AP to the j -th non cooperative UT channel	$\mathbf{H}_j$	AP to the j -th cooperative UT channel.
$\mathbf{g}_{m,n}$	n -th cooperative UT to the m -th non cooperative UT channel.	$\mathbf{G}_{m,n}$	n -th cooperative UT to the m -th cooperative UT channel.
50,000	channel generation for outage probability.	2000	channel generation for average power.

5. Conclusions

We provide two optimal beamforming schemes for the AP and the cooperative UTs in multi antenna cooperative NOMA systems resulting from the derived optimal BF criterion. One scheme with power limitation of cooperative UTs considers the practical implementation restriction, while the other one has no such limitation. These schemes have rank one structure and rely on the semi-definite programming (SDP) optimization to achieve the optimal points. Simulation results show that the proposed schemes efficiently utilize the spatial dimensions from the multiple antennas and improve the performances of cooperative NOMA systems both in the diversity and in the average transmit power of the system. The power limited cooperative UT scheme exhibits only a slight performance loss compared to the one without such power limitation. Here, we consider decode-and-forward relaying for the cooperative UTs, while the study on the amplify-and-forward UT can be considered in the future. In addition, a study on practical schemes with less computational complexity is an important future research challenge. Finally, the impact of antenna design [29–31] on the performance also attracts the research focus.

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